12 Compound units

This chapter is going to show you

- How to solve problems involving speed
- How to calculate and use density
- How to solve problems involving compound units
- How to calculate unit prices and use them to find value for money

What you should already know

- What is meant by direct proportion
- Standard units of length, mass, capacity and volume
- How to calculate the volumes of simple shapes

About this chapter

We measure speed in units such as metres per second or kilometres per hour. We measure the rate of flow of fuel from a pump in litres per second or litres per minute. These are examples of compound units which are created when other units are combined. They often occur when two variables are in direct proportion but in this chapter you will look at other examples too. For example, units that are useful for comparing costs of different size packets of a particular item.

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12.1 Speed

Learning objective

- To understand and use measures of speed

Keywords	
speed	compound unit

A formula for speed is $speed = \frac{distance}{time}$

Jasmine is running at a constant **speed**. She runs 100 metres in 20 seconds.

Jasmine's speed is $\frac{100}{20} = 5$ m/s.

The units are m/s (metres per second) because the distance is in metres and the time is in seconds. This is an example of a **compound unit** which involves other units – in this case metres and seconds.

The units of speed depend on the units used to measure the distance and the time.

Example 1

Work out the speed in the following cases. Include units in your answers.

a A car travels 45 km in 30 minutes.

b A runner runs 10 kilometres in 1 hour and 45 minutes.

Solution

a $speed = \frac{distance}{time}$

$= \frac{45}{30} = 1.5$ km/minute The units are km/minute because the time is in minutes.

An alternative answer is $\frac{45}{0.5} = 90$ km/h since 30 minutes = 0.5 hours.

b $speed = \frac{distance}{time}$

$= \frac{10}{1.75} = 5.71$ km/h One hour 45 minutes is $1\frac{3}{4}$ hours or 1.75 hours.

The answer has been rounded to 2 decimal places.

In km/minute, the answer is $\frac{10}{105} = 0.095$ km/minute because 1 hour 45 minutes is 105 minutes.

Example 2

A car is travelling at a constant speed of 30 m/s.

a How far does the car travel in one minute?

b How long does the car take to travel one kilometre?

Solution

a $speed = \frac{distance}{time}$ In this case the speed is 30 m/s and the time is 60 s.

$30 = \frac{d}{60}$ The time must be in seconds. Use *d* for distance.

(*Continued*)

$30 \times 60 = d$ Multiply by 60 to solve the equation.

$d = 1800$

The distance is 1800 m or 1.8 km. You must include the units.

b $speed = \frac{distance}{time}$ In this case the distance is 1000 m.

$30 = \frac{1000}{t}$ The distance must be in metres. Use t for time.

$30t = 1000$ First multiply by t.

$t = \frac{1000}{30} = 33.33 \ldots$ Now divide by 30.

It takes 33 seconds. It is sensible to round the answer to the nearest second.

Exercise 12.1

1 A marathon runner runs 40 km in $2\frac{1}{2}$ hours. Work out his speed in km/h.

2 A train is travelling at a constant speed. It takes 30 minutes to travel 45 km.

a Work out the speed in km/minute.

b Work out the speed in km/h.

3 Calculate the speed in each case. Put units in each answer.

a Peter runs 320 metres in 50 seconds.

b A car travels 15 km in 10 minutes.

c A plane flies 400 km in half an hour.

d A cyclist travels 1500 m in 4 minutes.

4 Matthew is cycling at 18 km/h.
Calculate how far he travels in

a 1 hour **b** 4 hours **c** 1.5 hours **d** 2 hours and 30 minutes.

5 Natalie can swim at a constant speed of 3 km/h.

Calculate how far she can swim in

a 1.5 hours **b** $\frac{1}{2}$ hour **c** $\frac{1}{4}$ hour **d** 1 minute.

6 Calculate the distance travelled in each case.

a Sharon walks at 3 m/s for two minutes.

b Nathan drives at 80 km/h for 15 minutes.

c A plane flies at 700 km/h for 4.5 hours.

d A snail moves at 0.2 m/minute for 150 seconds.

7 The top speed of a sprinter is 8 m/s.

Calculate the time it takes at that speed to sprint

a 40 m **b** 80 m **c** 50 m **d** 200 m.

8 Calculate the times taken for these journeys.

a 40 km at 120 km/h

b 22 km at 4 km/h

c 200 m at 5 m/s

d 5 km at 4 m/s

9 Anita is walking in the countryside at 6 km/h.

a Copy and complete this table.

Time (t hours)	0.5	1	1.5	2	2.5
Distance (d km)					

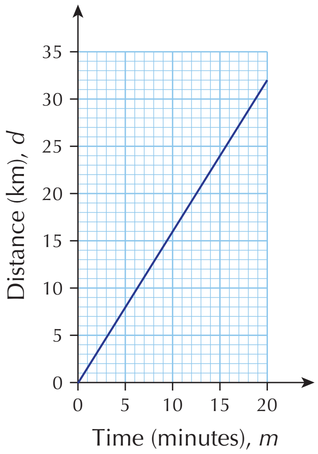
b Work out how long it takes Anita to travel 8 km.

10 A plane is flying at 850 km/h.

a Calculate how far the plane flies in $3\frac{1}{2}$ hours.

b Calculate the time the plane takes to fly 5000 km.

11 This graph shows the journey of a car.



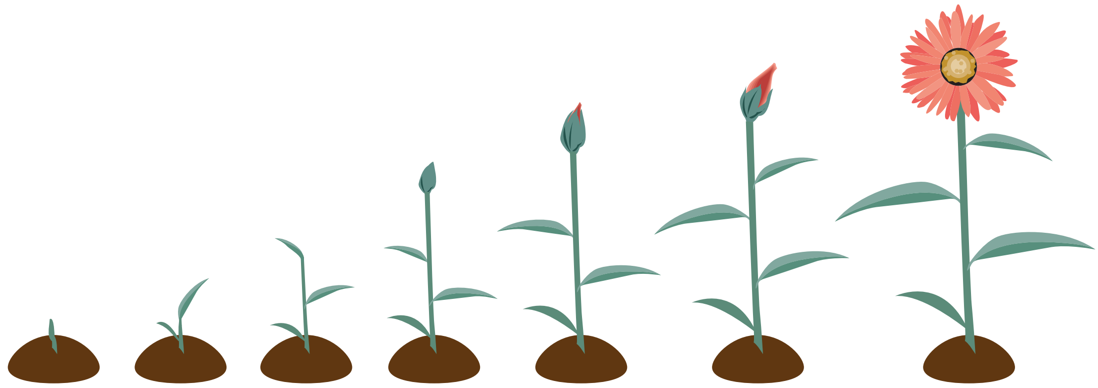
a How far does the car travel in 20 minutes?

b Explain how you know the car is travelling at a constant speed.

c Work out the speed of the car in km/minute.

d How long does it take the car to travel 50 km?

12 A plant grows 3.6 cm in 2 days. What is the rate of growth in mm/h?



PS **13** The speed of sound is 340 m/s. There is an explosion 2 km away from Sam. Calculate how many seconds will pass before Sam hears the explosion.

Challenge

a The national speed limit in the UK is 70 miles/hour. How fast is that in metres per second?

Here are some facts that might help you.

A distance of five miles is approximately eight kilometres.

There are 1000 metres in a kilometre.

A mile is 1760 yards.

There are 3600 seconds in an hour.

b In built-up areas the speed limit is 30 miles per hour. How fast is that in metres per second?

12.2 More about proportion

Learning objective

- To understand and use density and other compound units

Keywords	
rate	density

The last section looked at one important example of compound units. Another example concerns the **rate** of flow, a measure of how quickly a liquid is flowing.

Example 3

Water is flowing out of a tap. In 5 minutes 24 litres flow out of the tap.

a Work out the rate of flow in litres/minute.

b How long does it take to fill a 7.5 litre bucket?

Solution

a The rate of flow = $\frac{\textit{litres}}{\textit{minutes}}$ number of litres ÷ number of minutes gives litres/minute

$= \frac{24}{5} = 4.8$ litres/minute

b $4.8 = \dfrac{7.5}{m}$ — Call the number of minutes m.

$4.8m = 7.5$ — Multiply by m.

$m = \dfrac{7.5}{4.8} = 1.5625$ — Divide by 4.8.

It takes 1.56 minutes. — Round off the answer. It is just over $1\frac{1}{2}$ minutes.

Another example of a compound unit is **density**.

For a quantity of a substance, $\textit{density} = \dfrac{\textit{mass}}{\textit{volume}}$.

Examples of possible units are g/cm^3 or g/litre.

If you compare equal volumes of different substances, the denser one will be heavier.

Example 4

A piece of iron has a volume of $20\ cm^3$ and a mass of 158 g.

a Calculate the density of iron.

b Calculate the mass of $36\ cm^3$ of iron.

Solution

a $\textit{density} = \dfrac{\textit{mass}}{\textit{volume}} = \dfrac{158}{20}$

$= 7.9\ g/cm^3$ — The units must involve grams and cm^3.

b $\textit{density} = \dfrac{\textit{mass}}{\textit{volume}}$

$7.9 = \dfrac{m}{36}$ — Use the answer from part a. Use m for the mass.

$m = 7.9 \times 36 = 284.4$ — Multiply by 36.

The mass is 284 g.

Exercise 12.2

1. A driver takes 20 seconds to put 56 litres of petrol in a fuel tank. What is the rate of flow of the petrol? Give units in your answer.
2. A tap is dripping. In 20 minutes 0.3 litres drip from the tap. What is the rate in litres/h?

3 The water flows down a stream at a rate of 6 litres/s.

a Calculate how much water flows in one minute.

b Calculate how much water flows in half an hour.

c Work out how long it takes for 1000 litres to flow past a particular point.

4 A shower has a rate of flow of 12 litres/minute.

a Work out how much water is used if Sam has a shower for four minutes.

b Work out how long it takes to use 1 litre of water.

5 Work out the density of the following.

a A piece of metal with a mass of 24 g and a volume of 108 cm^3.

b A piece of wood which has a mass of 400 g and a volume of 320 cm^3.

c A piece of plastic with a mass of 75 g and a volume of 165 cm^3.

6 Balsa wood is used to make model aeroplanes because it has a low density of 0.16 g/cm^3.

a A piece of balsa wood has a volume of 25 cm^3. Calculate its mass.

b Calculate the volume of 1 g of balsa wood.

7 Oxygen has a density of 1.43 g/litre. Work out the mass of 200 litres of oxygen.

8 Oak has a density of about 0.75 g/cm^3.

a Work out the mass of 1000 cm^3 of oak.

b Work out the volume of a piece of oak with a mass of 150 g.

c An oak plank measures 3 cm by 15 cm by 120 cm. Work out its mass.

9 Graphite is used to make pencils. It has a density of 2.3 g/cm^3.

a Work out the mass of 50 g of graphite.

b A pencil contains 1.2 cm^3 of graphite. Work out the mass of graphite in 10 pencils.

c Work out the mass of 100 cm^3 of graphite.

10 A piece of copper has a mass of 45 g and a volume of 5 cm^3.

a Work out the density of copper.

b Work out the volume of 120 g of copper.

c Work out the mass of a cube of copper with a side of 5 cm.

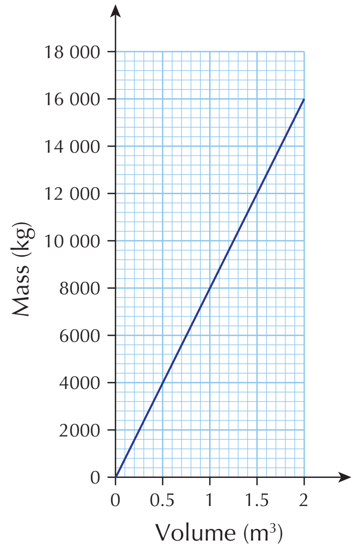
PS **11** The density of gold is 19.3 g/cm^3.

a Work out the mass of 3.5 cm^3 of gold.

b Work out the volume of 3.5 g of gold.

c The largest gold bar in the world has a mass of 250 kg. Work out its volume.

PS **12** This graph shows the connection between mass and volume for a type of steel.



a Use the graph to find the mass of 0.75 m^3. of steel.

b Use the graph to find the volume of 1400 kg of steel.

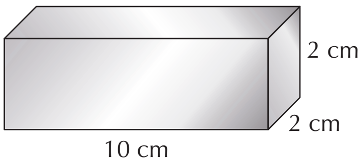
c The mass of a particular type of steel beam is 360 kg.

i Calculate the volume of the beam in m^3.

ii Calculate the volume of the beam in cm^3 (Hint 1 m^3 = 1 000 000 cm^3)

Challenge

A metal cuboid measures 2 cm by 2 cm by 10 cm. It has a mass of 340 g.



a Calculate the volume of the cuboid.

b Calculate the density of the metal.

c The same metal is used to make a cylinder with a diameter of 3 cm and a length of 6 cm. Calculate the mass of the cylinder.

12.3 Unit costs

Learning objective

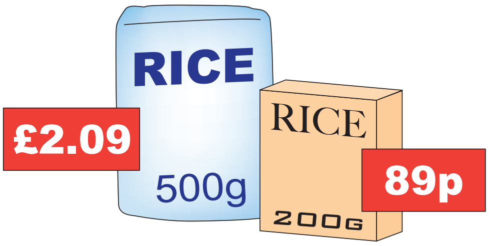
- To understand and use unit pricing

Keyword

unit price

Packets and containers of food and other items are sold in different sizes. If you want to compare the prices of the same item in different sized containers, it is helpful to be able to work out a **unit price**. This means the price of one gram or one litre, or any other suitable unit.

Here are two packets of rice. Which is better value?



- You could find the cost/gram of rice from each packet.

The answers then are £0.89 ÷ 200 = £0.00445 and £2.09 ÷ 500 = £0.00418.

The larger packet is better value as it has the lower cost per gram.

- You could find how much you can buy for 1p.

For the smaller packet the number of grams/p is 200 ÷ 89 = 2.247… g/p.

For the larger packet it is 500 ÷ 209 = 2.392…. g/p.

The larger packet is better value because you can buy more for 1p.

This exercise will give you the chance to practice different methods.

Note: Items in shops often indicate the weight of an item in grams. It is more correct to use the word mass instead of weight. The word mass will be used in this exercise.

Exercise 12.3

1 A bag of pasta has a mass of 250 g and costs 87p.

a Calculate the cost per 100 g.

b Calculate the cost per gram.

c Calculate the number of grams for 1p.

d Calculate the number of grams for £1.

2 A bag of sugar has a mass of 1 kg and costs £2.35.

a Calculate the cost per 100 g.

b Calculate the cost per gram.

c Calculate the number of grams for 1p.

d Calculate the number of grams for £1.

3 Gina buys a 600 g bag of satsumas for £2.40. Work out the cost per kilogram.

4 Raspberries cost £12.50 per kilogram.

Calculate the mass you can buy for £1.00.

FS 5 A 160 g can of tuna costs £1.85. A pack of four cans costs £6.20.

a Work out the cost per 100 g for one can.

b Work out the cost per 100 g if you buy the pack of four cans.

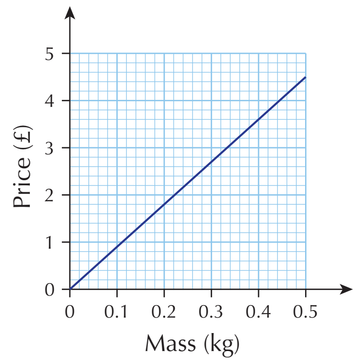
c Which is better value? Give a reason for your answer.

FS 6 You can buy tomato puree in tubes or jars. A shop sells 140 g tubes for 69p and 200 g jars for £1.09.

a Work out the cost per 100 g in a tube of tomato puree.

b Work out which is better value for money. Justify your answer.

7 This graph shows the price of asparagus.



a Use the graph to find the cost of 400 g of asparagus.

b Use the graph to find the mass you can buy for £2.00.

c Explain how the graph shows that cost is proportional to mass.

d Work out the cost of asparagus in £/kg.

FS 8 Aziz buys 700 g of apples for £1.68. Calculate the cost per kilogram.

FS 9 A supermarket sells two cheese selection packs. One has a mass of 490 g and costs £6.00. The other has a mass of 265 g and costs £3.49.
Calculate the cost per kg of each pack.

MR 10 A shop sells cans of pineapple in two sizes. The prices are in this table.

Mass	225 g	435 g
Price	83p	£1.45

Which is better value? Justify your answer.

MR 11 Milk is sold in litres or pints. One pint is 568 ml.

A supermarket sells 1 litre of milk for 95p or 1 pint for 49p.
Which is better value? Justify your answer.

MR 12 A 600 g box of muesli costs £2.65. An 850 g box costs £3.95. Which is better value? Justify your answer.

MR 13 Toothpaste comes in different size tubes. A 125 ml tube costs £2.99 and a 75 ml tube of the same brand costs £1.89.
Which is better value for money? Justify your answer.

PS 14 A 400 g packet of beef costs £4.60. What price would you expect to pay for a 750 g pack? Justify your answer.

MR 15 A pack of four 120 g pots of yogurt costs £2.00. A 450 g pot of the same yogurt costs £1.79.
Show that the large pot is better value.

MR 16 A 350 g block of Cheddar cheese costs £2.90. A 250 g packet of grated Cheddar cheese costs £1.95.
Show that the block of cheese is better value.

MR **17** A 150 g packet of raisins costs £1.09. A 500 g pack of raisins costs £2.55. Which is better value? Justify your answer.

Challenge

A shop sells two sizes of cereal packet.

One is 350 g and costs £1.79 and the other is 575 g and costs £2.85.

They are going to start selling a 750 g pack. They want it to be slightly better value than the other packs. They have asked you to recommend a price. What is your suggestion? Justify your answer.

Ready to progress?

I can given a distance and a time, calculate a speed

I can solve problems involving speed
I can calculate density and solve problems involving density
I can solve problems involving other compound units such as rates of flow
I can calculate unit prices and use them to find value for money

Review questions

1 It takes James 8 minutes to walk to the shops. The distance is 400 m.

a Work out the speed in m/minute.

b How long would it take James to walk 1 km at the same speed?

2 A plane flies at 800 km/h.

a How far does it fly in $1\frac{1}{2}$ hours?

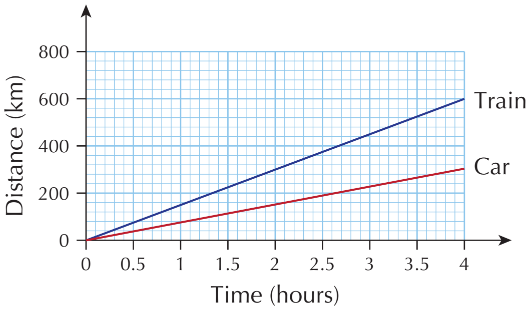
b Calculate the time taken to fly 2000 km.

3 A cheetah can run 100 metres in 4 seconds.

Work out the speed of the cheetah

a in m/s **b** in m/min **c** in km/h

4 This graph shows the distance travelled by a train and a car over a four-hour interval.



a Work out the speed of the train.

b Work out the speed of the car.

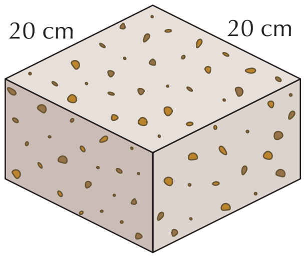
c If the train travels d km in t hours, write a formula for d in terms of t.

5 20 litres of water flows out of a tap every minute. Work out the rate of flow in

a litres/second **b** litres/hour

6 A paddling pool has a capacity of 200 litres. It is filled at a rate of 25 litres/minute. Work out how long it takes to fill the pool.

7 Here is a block of concrete in the shape of a cuboid.



a Calculate the volume of the block.

b The density of concrete is 2.4 g/cm^3. Calculate the mass of the block of concrete.

8 A piece of metal has a volume of 150 cm^3 and a mass of 1.23 kg. Calculate the density of the metal in g/cm^3.

9 Densities can be given in g/cm^3 or in kg/m^3.
1 g/cm^3 = 1000 kg/m^3

a The density of the wood in a tree trunk is 0.85 g/cm^3. Write this in kg/m^3.

b The volume of the tree trunk is 3.8 m^3. Work out the mass of the tree trunk.

10 A 250 g piece of cheese costs £2.10.

a Work out the cost per 100 g of the cheese.

b Work out the cost per kg of the cheese.

c What mass of cheese costs £5.00?

PS **11** The cost of petrol is £1.35 per litre. Petrol flows into the tank of a car at a rate of 1.2 litre/second.

Jess puts petrol into her car for half a minute.

a Work out the quantity of petrol Jess puts in her car.

b Work out the cost of the petrol.

12 A 350 g packet of biscuits costs £1.49. A 450 g packet of the same biscuits costs £1.99. Which is better value? Justify your answer.

13 A box of cereal has a mass of 300 g and costs £2.85.

a Work out the cost per 100 g.

b The manufacture decides to give 25% extra for the same price. Calculate the new cost per 100 g.

Reasoning, Problem Solving

Population density

1. Look at these figures.

	Area (thousand km²)	Population (million)
Sweden	410	10
Germany	360	81

The area of Sweden is 410 000 km². The population of Sweden is 10 000 000.

a Write down the area and the population of Germany.

b The area of Sweden is about 14% larger than the area of Germany. What can you say about the populations of the two countries?

For any country you can work out the $\textit{population density} = \dfrac{\textit{population}}{\textit{area}}$.

This tells you how crowded a country is. The units are people per km².

The population density of Sweden = 10 000 000 ÷ 410 000 = 24 people/km².

c Work out the population density of Germany, to the nearest whole number.

2 Here is a table of 11 more European countries.

Country	Area (thousand km²)	Population (million)
Belgium	31	11
Finland	300	5
France	540	64
Greece	130	11
Ireland	70	5
Italy	300	60
Netherlands	34	17
Poland	310	39
Portugal	92	11
Spain	500	47
United Kingdom	240	64

a Work out the population density for each country. Give your answers to the nearest whole number.

b If you include Sweden and Germany, you now have data for 13 countries. Which country has

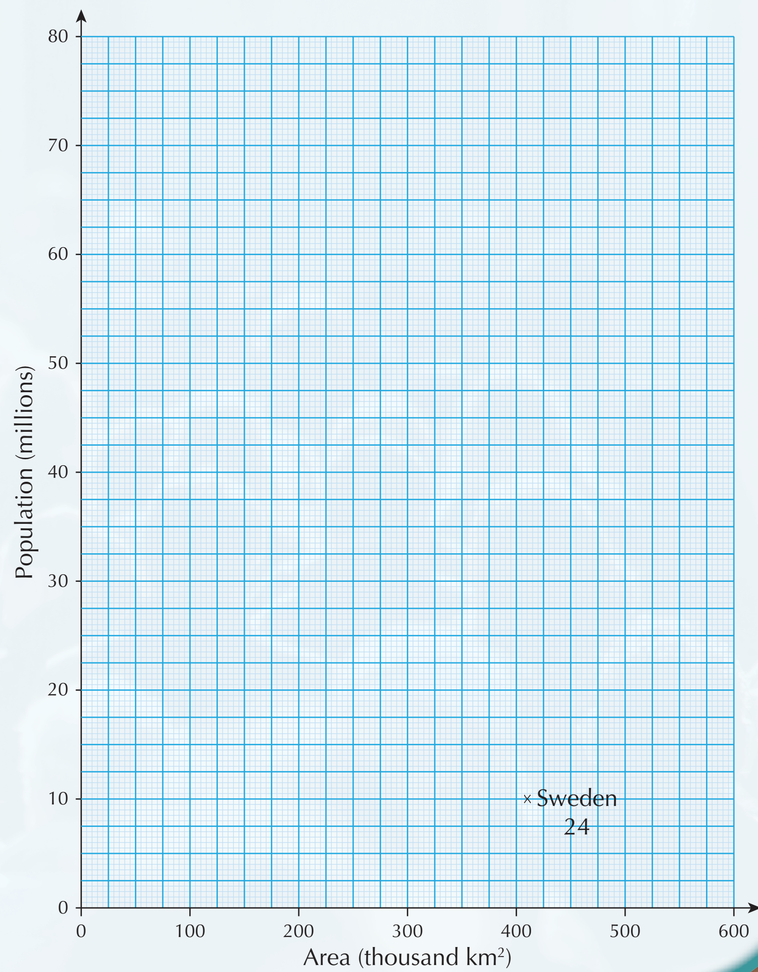
i the largest population? **ii** the largest population density?

iii the smallest population? **iv** the smallest population density?

3. Draw axes like this on graph paper.

Use a scale of 2 cm to 100 000 km² and 2 cm to 10 million people.

Put a cross for each country and label it with the name and population density. Sweden has been done for you.



4. Two countries with very different population densities are Singapore and Australia.

Singapore has an area of only 700 km² and a population of 5 400 000.

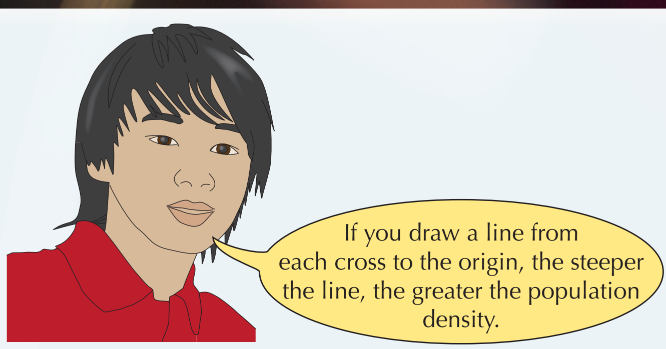
Australia has an area of 7 700 000 km² and a population of 23 000 000.

a Work out the population density of each.

b If you plotted Singapore on your graph which axis would it be closer to?

c If you extended your graph so that you could plot Australia, which axis would it be closer to?

5.



Check that this is the case.