

13

Proportion

This chapter is going to show you:

- how to solve problems involving direct proportion
- graphical and algebraic representations of direct proportion
- how to solve problems involving inverse proportion
- graphical and algebraic representations of inverse proportion.

You should already know:

- how to write a simple formula algebraically
- what a ratio is
- how to draw graphs.

About this chapter

The proportions of the parts of the human body change with age.

For an average baby, the length of the head is about a quarter of the length of the whole body.

For an average adult, the length of the head is about an eighth of the length of the whole body.

Ratios of different parts of the body, such as the length of legs compared to the whole body, vary from person to person. Scientists have carried out research to try to find the body ratios that people find most attractive. They have found that the 'ideal' body shape varies from one country to another.

Leonardo da Vinci thought that the ideal proportions should fit in a circle. He made a drawing called *Vitruvian Man* to show this.



13.1 Direct proportion

Learning objectives

- To understand the meaning of direct proportion
- To find missing values in problems involving proportion

Key words

direct proportion

proportional

When you buy petrol you pay a fixed price for each litre. If you buy twice or three times as much, you pay twice or three times as much as you paid before. This is an example of **direct proportion**.

Two variables (such as the number of litres and the price) are in direct proportion if, when you multiply one by a number (such as 2 or 3 or 0.5) you multiply the other by the same number.

These are some other pairs of variables that are in direct proportion.

- The distance travelled by a car moving at 100 km/hour and the time taken
- The volume of water flowing out of a tap and the time, in seconds, for which the water flows
- The volume of a fizzy drink and the amount of sweetener in it
- The mass of some loose carrots bought in a shop and the cost, in pounds
- The time light has been on and the cost of the electricity used
- The length of a journey, in miles, and the length of the same journey, in kilometres

Example 1

At a petrol station 15 litres of petrol cost £20.40.

a Work out the cost of: **i** 60 litres **ii** 5 litres **iii** 48.5 litres.

b Work out the cost of 1 litre.

c How many litres can you buy for £1?

a It helps to put the numbers in a table.

Petrol (litres)	15	60	5	48.5
Cost (£)	20.40			

i $60 = 15 \times 4$ The number of litres (15) is multiplied by 4, so the cost is also multiplied by 4.

The cost of 60 litres = $\text{£}20.40 \times 4 = \text{£}81.60$.

ii $5 = 15 \div 3$ The number of litres is divided by 3. Do the same to the cost.

The cost of 5 litres is $\text{£}20.40 \div 3 = \text{£}6.80$.

iii You need to work out the multiplier.

If $15 \times ? = 48.5$ then $? = 48.5 \div 15 = 3.233$ to 3 dp.

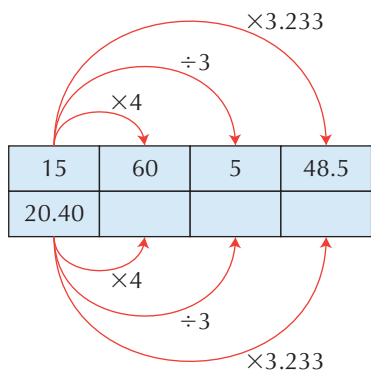
The cost is $20.4 \times 3.233 = \text{£}65.95$.

b 15 litres cost £20.40 so 1 litre costs $\text{£}20.40 \div 15 = \text{£}1.36$.

c £20.40 buys 15 litres so £1 buys $15 \div 20.4 = 0.74$ litres to 2 dp.

Hint

If you multiply or divide the number of litres by any number, you must multiply or divide the cost by the same number.



You say that the number of litres and the cost in pounds are directly **proportional**.
 You can leave out the word 'directly' and just say they are proportional.
 You will learn about another sort of proportion later in this chapter.

Exercise 13A

- 1 A train is travelling at a constant speed.
 The distance travelled is proportional to the time taken.
 In 20 minutes the train travels 48 kilometres.
 Copy and complete this table.

Time taken (minutes)	5	10	20	30	45
Distance (km)			48		

- 2 Jacob buys 350 g of carrots and they cost 84 pence in total.
 Work out the cost of:
a 700 g of carrots **b** 50 g of carrots **c** 250 g of carrots **d** 500 g of carrots.
- 3 250 ml of cola contains 28 g of sugar.
 Work out the amount of sugar in:
a 125 ml of cola **b** 1.5 litres of cola **c** 2 litres of cola **d** 100 ml of cola.
- 4 Rachel knows that 1.5 kg of flour is enough to make four small loaves.
a How much flour is needed to make 10 small loaves?
b How many small loaves can she make from 60 kg of flour?
- 5 A distance of 5 miles is approximately the same as 8 km.
 Copy and complete this table to show equivalent distances.

Miles	5	25	80			
Kilometres	8			20	100	50

- 6** The perimeter of a circle is called the circumference.
The circumference of a circle is proportional to the diameter of the circle.
A circle with a diameter of 3.6 m has a circumference of 11.3 m.
Work out the circumference of a circle with a diameter of:

a 1.8 m **b** 10.8 m **c** 1 m **d** 10 m.

- 7** Water is dripping from a tap at a steady rate.

In 12 minutes there are 70 drips.

- a** Work out the number of drips in one hour.
b Work out the time taken for 1000 drips from the tap.

- FS** **8** In a shop, 100 g of sweets cost 72 pence.

- a** Work out the cost of:
i 250 g of sweets **ii** 1 kg of sweets **iii** 30 g of sweets.
b What mass of sweets can you buy for:
i £1.44 **ii** £2 **iii** £3?

- FS** **9** The exchange rate between pounds (£) and US dollars (US\$) is £43 = US\$65.

- a** Copy and complete this table.

Pounds (£)	43	164		107.50
US dollars (US\$)	65		195	

- b** How many dollars is £1 worth?
c How many pounds is \$1 worth?

- PS** **10** The pressure of a car tyre can be measured in two different units, bar or psi (pounds per square inch).

0.7 bar is the same as 10 psi.

- a** Copy and complete this table to show conversions between the two units.

Bar	0.7			8.4	10.5
psi	10	25	4		

- b** How many bar is 1 psi?
c How many psi is 1 bar?

- PS** **11** The mass of a steel cable is proportional to its length.

A five metre length of a particular cable has a mass of 9.2 kg.

- a** Work out the mass of 100 metres of the cable.
b How long is a cable with a mass of 100 kg?

- FS** **12** This table shows the exchange rate between pounds (£) and New Zealand dollars (NZ\$).

Pounds (£)	50	125
New Zealand dollars (NZ\$)	96	

- a** Work out the missing value.
b Work the ratio of the two amounts of pounds.
c Work out the ratio of the two amounts of dollars.



- 13** Energy content on food labels is given in two different units, kilocalories (kcal) and kilojoules (kJ).

Here is part of a conversion table.

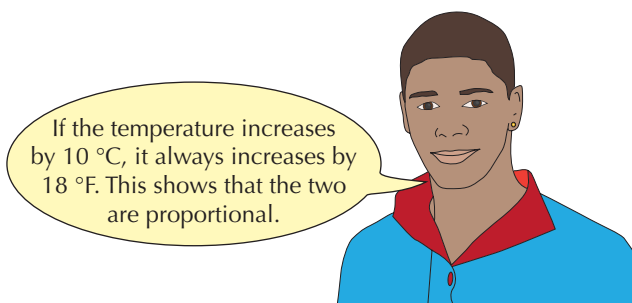
Kilocalories (kcal)	38	
Kilojoules (kJ)	160	240

- a** Work out the missing value.
b Show that the ratio of the two amounts of kilojoules is the same as the ratio of the two amounts of kilocalories.



- 14** Temperature can be measured in degrees Celsius ($^{\circ}\text{C}$) or degrees Fahrenheit ($^{\circ}\text{F}$). Here is a table of values.

Degrees Celsius ($^{\circ}\text{C}$)	30	40	50	60
Degrees Fahrenheit ($^{\circ}\text{F}$)	86	104	122	140



Look at what Ben says. Is he correct? Give a reason for your answer.

Investigation: Age, height and mass

This table is from a US website.

It shows the average height and the average mass of a boy at different ages.

Because it is a US website, the heights are in inches and the masses are in pounds.

Age (years)	2	4	6	8
Average height (inches)	31	37	42	45
Average mass (pounds)	28.4	36.0	46.2	57.2

- A** Is the average height proportional to age? Use numbers from the table to justify your answer.
B Is the average mass proportional to age? Use numbers from the table to justify your answer.
C Is the average mass proportional to the average height? Use numbers from the table to justify your answer.

13.2 Graphs and direct proportion

Learning objective

- To represent direct proportion graphically and algebraically

Key words

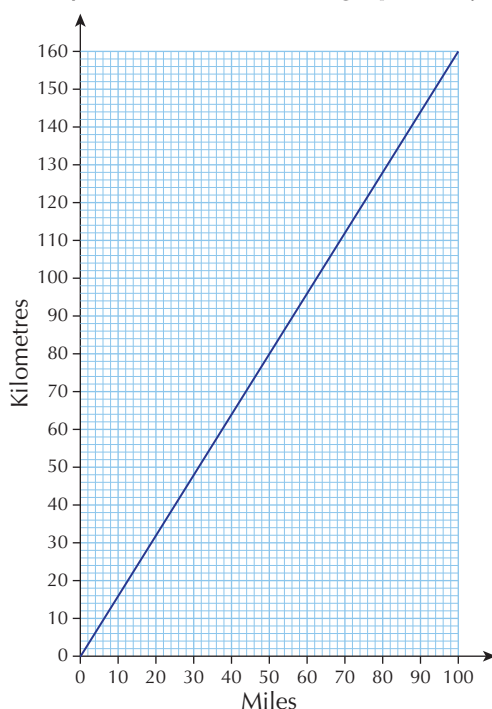
formula

graph

This table shows the relationship between distances measured in miles and in kilometres. They are in direct proportion.

Distance (miles)	20	30	50	60	70	100
Distance (kilometres)	32	48	80	96	112	160

You can plot these values on a **graph** and join them with a line.



There are two things that you should notice.

- The points are in a straight line.
- The line passes through the origin.

A graph of values of two variables in direct proportion always has those properties.

Look back at the pairs of values in the table at the start of this section.

Check that $20 \times 1.6 = 32$ $30 \times 1.6 = 48$ $50 \times 1.6 = 80$

and so on.

If you know that x miles is the same distance as y kilometres, you can write this as a formula:

$$y = 1.6x$$

If two variables, x and y , are in direct proportion you can always write a formula:

$$y = mx$$

where m is a number.

Example 2

Ribbon is sold by the metre. Lucy buys 6 metres and the cost is £5.04.

- a Find a formula for the cost, y pence, of x metres of ribbon.
- b Draw a graph to show the cost of different lengths of ribbon.
- c Use the formula to work out how many metres you can buy for £6. Use the graph to check your answer.

- a The cost is proportional to the length of ribbon, so the formula is $y = mx$.

You need to work out the value of m .

You know that when x is 6 then y is 504 Notice that y is the cost, in pence.

$$504 = m \times 6$$

$$m = 504 \div 6 = 84$$

The formula is $y = 84x$.

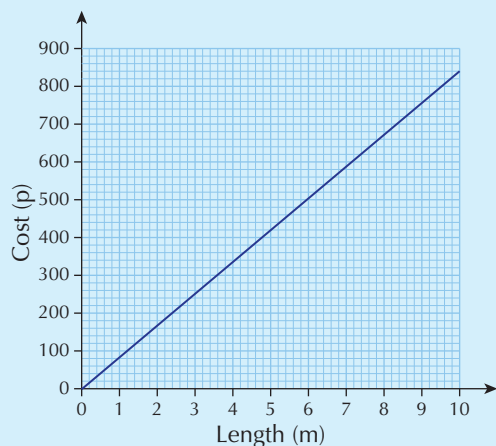
- b Use the formula to find the costs of different lengths.

Multiply the length by 84 to find the cost.

Choose some values for the length.

Length (x metres)	1	2	3	5	7	10
Cost (y pence)	84	168	252	420	588	840

Plot the points on graph paper and join them up.



The points should be in a straight line. The line should go to the origin.

The equation of the line is $y = 84x$.

- c When the cost is £6, then $y = 600$.

Substitute this value into the formula and solve for x .

$$600 = 84x$$

$$x = 600 \div 84 = 7.14$$

£6 will pay for 7.14 metres of ribbon.

Check that the point (7.14, 600) is on the line on the graph.

Exercise 13B

- 1 The perimeter (y cm) of a square of side x cm is given by the formula $y = 4x$.

a Copy and complete this table to show values of x and y .

Side (x cm)	2	4	6	8	10
Perimeter (y cm)					

x cm



b Draw a graph to show the values in the table. Label the axes.

FS

- 2 The cost, $\pounds y$, of x litres of petrol is given by the formula $y = 1.3x$.

a Work out the cost of: i 10 litres of fuel ii 25 litres of fuel.

b Draw a graph to show the cost of fuel.

- 3 A 200 ml glass of a fizzy drink contains 24 g of sugar.

a Copy and complete this table.

Drink (x ml)	200	100	500	1000
Sugar (y g)	24			

b If x ml of fizzy drink contain y g of sugar, show that $y = 0.12x$.

c Draw a graph to show the figures in your table.

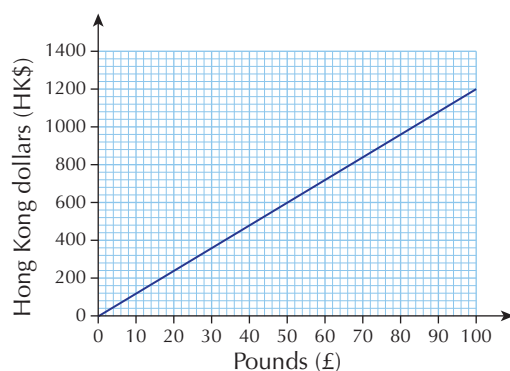
- 4 Max buys 1.5 kg of potatoes and they cost $\pounds 2.40$.

a Work out the cost of 3 kg of potatoes.

b Work out a formula for the cost, $\pounds y$, of x kg of potatoes.

c Draw a graph to show how the cost of potatoes varies with mass. The x -axis should go up to 10 kg.

- 5 This graph shows the exchange rate between pounds (£) and Hong Kong dollars (HK\$).



a Use the graph to complete this table.

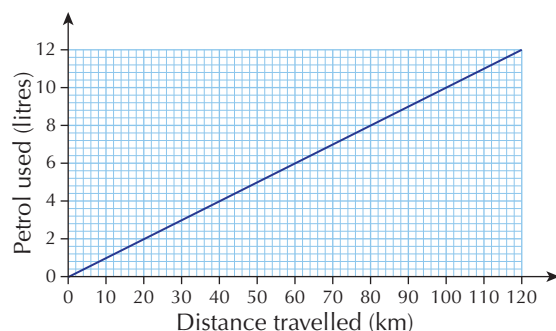
Pounds (£ x)	20	40	80	100
Hong Kong dollars (HK\$ y)				

b Work out a formula for y in terms of x .

c Use the formula to change $\pounds 1320$ into Hong Kong dollars.

d Use the formula to change HK\$7900 into pounds.

- 6 A car is travelling at 80 kilometres per hour (km/h). The graph shows the amounts of petrol used for different distances travelled.



- a Use the graph to complete this table.

Distance (x km)	50	120			
Petrol (y litres)			10	7.5	4.3

- b Write a formula for y in terms of x .

- c The cost of petrol is £1.36 per litre. Work out the cost of travelling 163 km.

PS

- 7 The speed of a car can be measured in metres per second (m/s) or in kilometres per hour (km/h).

A speed of 5 m/s is the same as a speed of 18 km/h.

- a A world class sprinter can run 100 metres in 10 seconds. Work out the average speed of the sprinter in km/h.

- b A speed of y km/h is the same as a speed of x m/s.

Work out a formula for y in terms of x .

- c Use the formula to convert 100 m/s to km/h.

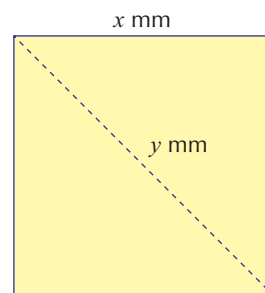
- d The top speed of a Bugati Veyron is 415 km/h.

Work out that speed in m/s.

- 8 The lengths of the side and the diagonal of a square are proportional.

- a This table shows possible values of x and y . Fill in the missing values.

Side (x mm)	5	10	15	20	25
Diagonal (y mm)	7				



- b Work out a formula for y in terms of x .

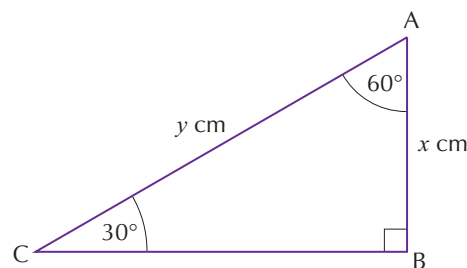
- c Use the formula to find the diagonal of a square when the length of the side is 32 mm.

- d A square piece of card has a diagonal of 63 mm. How long is each side of the square?

- 9 The angles of this triangle are 30° , 60° and 90° .
The lengths of AB and AC are proportional.
The table shows some possible values of x and y .

AB (x cm)	4.1	5.7	
AC (y cm)	8.2		24.3

- Write down a formula for y in terms of x .
- Write down a formula for x in terms of y .
- Work out the missing values in the table.
- What is the ratio of $x : y$ for each pair of values in the table?
- Draw your own triangle with angles of 30° , 60° and 90° , like the one in the diagram. Measure x and y and check that the ratio of $x : y$ is correct.

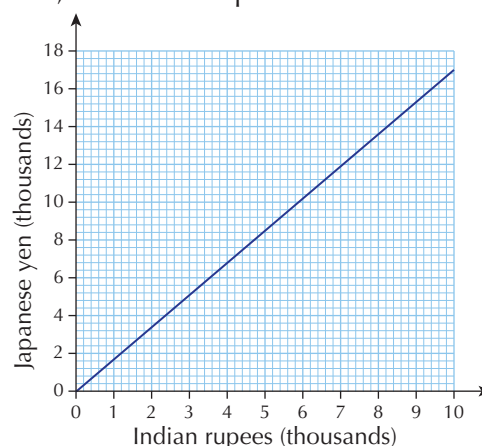


Financial skills: Exchange rates

This graph shows the exchange rate between two currencies, the Indian rupee and the Japanese yen.

The amount x rupees is the same as the amount y yen.

- Work out a formula for y in terms of x .
- How many rupees is 1 yen worth?
- How many yen is 1 rupee worth?
- Work out a formula for x in terms of y .



13.3 Inverse proportion

Learning objectives

- To understand what inverse proportion is
- To use graphical and algebraic representations of inverse proportion

Key word

inverse proportion

Suppose you go on a car journey of 120 kilometres.

The time it takes depends on the speed of the car.

- If the car travels at 60 km/h the journey will take $120 \div 60 = 2$ hours.
- If the car travels at 40 km/h the journey will take $120 \div 40 = 3$ hours.

The table shows the journey times for a car travelling at various speeds.

Speed (x km/h)	20	30	40	50	60
Time (y hours)	6	4	3	2.4	2

It you multiply the speed by any number, you divide the time by the same number.

For example:

- $20 \times 2 = 40$ and $6 \div 2 = 3$
- $40 \times 1.5 = 60$ and $3 \div 1.5 = 2$

The speed (x) and time (y) are in **inverse proportion**. When you multiply one by a number you divide the other by the same number.

In the example above, if you multiply the speed by the time then the answer is always 120, because:

$$\text{speed} \times \text{time} = \text{distance}$$

For example:

- $20 \times 6 = 120$
- $50 \times 2.4 = 120$

If the speed is x km/h and the time is y hours, you can write this formula:

$$xy = 120$$

When x and y are in inverse proportion you can always write a formula:

$$xy = k$$

where k is a number.

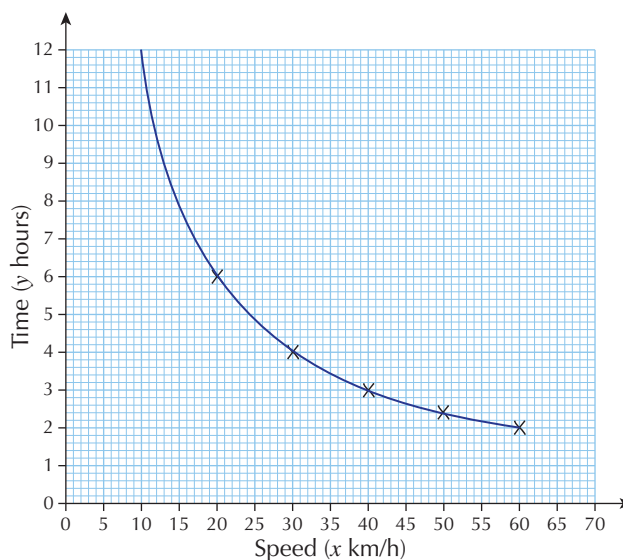
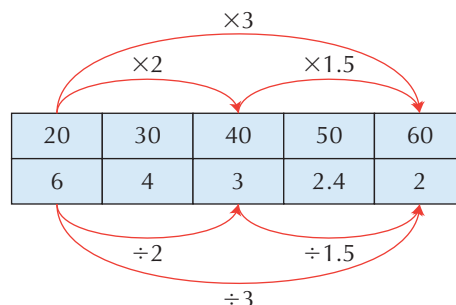
The graph shows the numbers in the table at the start of this section.

The points are not in a straight line. A smooth curve has been drawn through them.

You can read information from the graph in the same way that you can from a straight-line graph. For example, the curve passes through (10, 12). This tells you that, at a speed of 10 km/h, it would take 12 hours to make the journey.

Graphs of inverse proportion are always this shape.

The variables do not have to be x and y . You can use any letters.



Example 3

The area of a rectangular field is 1200 m^2 .

The sides of the field are a metres and b metres long.

a Work out a formula connecting a and b .

b Find the value of a when $b = 40$.

c Find the value of b when $a = 63.5$.

a The area of the field is the lengths of the two sides multiplied together, so the formula is $ab = 1200$.

b When $b = 40$, $a \times 40 = 1200$.

Hence $a = 1200 \div 40 = 30 \text{ m}$.

c When $a = 63.5$, $b \times 63.5 = 1200$.

Hence $b = 1200 \div 63.5 = 18.9 \text{ m to 1 dp}$.

In this case the variables, a and b , are inversely proportional.

Exercise 13C

1 A train is travelling a distance of 600 km .

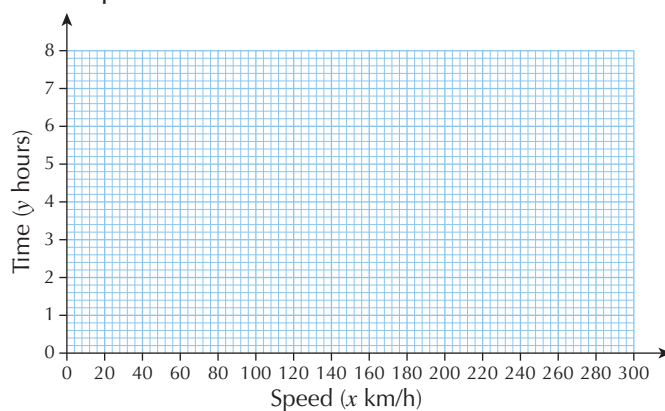
a How long does the train take if it travels at 100 km/h ?

b Copy and complete this table.

Speed ($x \text{ km/h}$)	100	150			300
Time ($y \text{ hours}$)			5	3	

c Write down a formula connecting x and y .

d Draw a pair of axes like this.



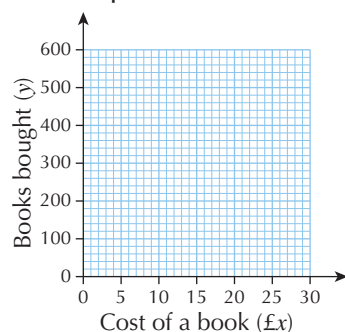
e Plot the points from the table in part **b** and join them with a smooth curve.

- 2 A teacher has £1000 to spend on books.

- Some books cost £10 each. How many can the teacher buy?
- Other books cost £5 each. How many can the teacher buy?
- Copy and complete this table.

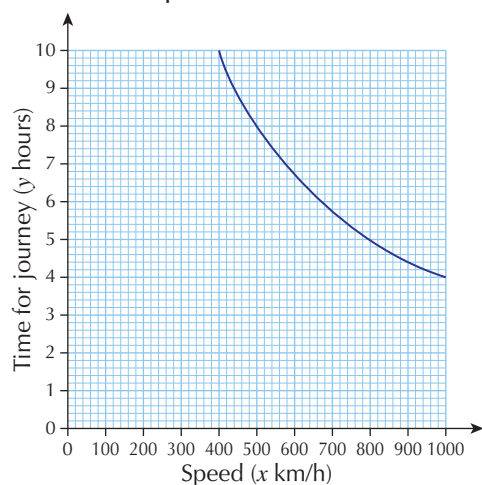
Cost of a book (£ x)	2	4	5	10	20	25
Number bought (y)						

- Show that x and y are inversely proportional.
- Write down a formula connecting x and y .
- Draw a pair of axes like this.



- Draw a graph to show the information in the table.

- 3 The graph shows the time taken by an aeroplane to travel between two airports, at different speeds.



- Use the graph to find the speed when the journey takes 5 hours.
- The time (y hours) is inversely proportional to the speed (x km/h).
Work out a formula connecting x and y .
- Use your formula to find the time taken when the plane travels at 500 km/h.
Check that the graph gives the same answer.

- 4 Some children are measuring the lengths of their paces and how many paces they take to walk 100 metres.

- a If the length of a pace is 0.5 metres, how many paces will they take to walk 100 metres?
- b Copy this table and fill in the missing values.

Length of pace (p metres)	0.5	0.6	0.7	0.8	0.9	1.0
Number of paces (n)						

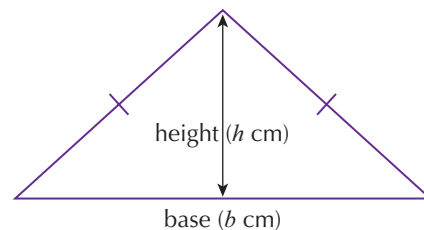
- c Show that p and n are inversely proportional.
- d Write down a formula connecting p and n .
- e Louis takes 130 paces to walk 100 m. How long is Louis' pace?



- 5 An isosceles triangle has an area of 100 cm^2 .

The length of the base (b cm) is inversely proportional to the height (h cm).

When the base is 20 cm then the height is 10 cm.



- a Write down a formula connecting b and h .
- b Copy and complete this table to show possible values of b and h .

Base (b cm)	10	15	20	25	30	35
Height (h cm)			10			

- c Draw a graph to show how the height varies with the base.
- d Work out the base when the height is 4 cm.
- e Work out the base when the base and the height are the same length.



- 6 Some families want to buy play equipment for their local park.

The total cost is £45 000.

The families agree to share the cost between them, equally.

The cost for each family will depend on how many families take part.

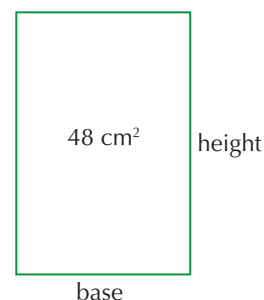
- a Copy and complete this table of values.

Number of families (n)	10	20	30	40	50
Cost for each family (£ c)					

- b Work out a formula connecting n and c . Justify your answer.
- c Draw a graph to show how the cost varies with the number of families.
- d They decide that each family should not pay more than £800. Use your formula to work out the smallest number of families that are needed. Use your graph to confirm your answer.

Activity: Different rectangles, same area

- A Draw two different rectangles with an area of 48 cm^2 .
- B Draw up a table of different possible values for the base and height of a rectangle with an area of 48 cm^2 .
- C Draw a graph to show your values.
- D Write down a formula to show the connection between the base and the height.
Choose letters to use in your formula and explain what they stand for.
- E Use your graph to find the side of a square with an area of 48 cm^2 .



13.4 Comparing direct proportion and inverse proportion

Learning objective

- To recognise direct and inverse proportion and work out missing values

Here is a summary of what you have learnt about direct proportion and inverse proportion.

Direct proportion

x	3	4.5	6	15	20
y	24	36	48	120	160

The variables x and y are directly proportional.

If you multiply (or divide) a value of x by a number, you multiply (or divide) y by the same number.

You can always write $y = mx$ or $\frac{y}{x} = m$ where m is a number.

In this example $y = 8x$ or $\frac{y}{x} = 8$.

Inverse proportion

x	3	4.5	6	15	20
y	24	16	12	4.8	3.6

The variables x and y are inversely proportional.

If you multiply a value of x by a number, you divide y by the same number.

You can always write $xy = k$ where k is a number.

In this example $xy = 72$.

Check that multiplying each pair of numbers does make 72.

The formula could also be written as $x = \frac{72}{y}$ or $y = \frac{72}{x}$.

Example 4

Here are the values of two variables, p and q .

p	20	50
q	90	

- a** Find the missing value when q is directly proportional to p .
b Find the missing value when q is inversely proportional to p .

a $q \div p = 90 \div 20 = 4.5$

$q = 4.5p$ Because they are in direct proportion.

When $p = 50$ then $q = 4.5 \times 50 = 225$.

$q = 225$

- b** Look at the values in the first column.

$pq = 20 \times 90 = 1800$

$pq = 1800$ Because they are in inverse proportion.

When $pq = 1800$ and $p = 50$ then $50 \times q = 1800$.

$q = 1800 \div 50 = 36$

Exercise 13D

- 1** George is walking at a constant speed.

In 10 minutes he walks 750 metres.

- a** He walks d metres in t minutes. Write down a formula for d in terms of t .

- b** Use the formula to work out how far George walks in 28.5 minutes.

- 2** Anne is doing a sponsored walk of 40 km.

- a** Explain why the time taken (t hours) is inversely proportional to her walking speed (w km/h).

- b** Write down a formula connecting t and w .



- 3** These are some tables of values. Say whether they show direct proportion, inverse proportion or neither of these. Give a reason in each case.

If the variables are directly or inversely proportional, work out the formula.

a

x	24	13	42	33
y	84	45.5	147	115.5

b

c	12	30	24	2
d	15	6	7.5	90

c

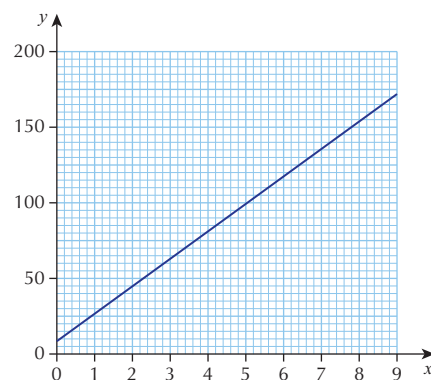
f	3.8	58.9	331.7	204.6
r	0.4	6.2	21.4	13.2

d

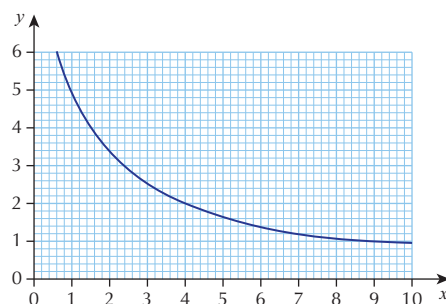
u	0.6	12	1.44	0.45
w	0.6	0.03	0.25	0.8



- 4 Does this graph show direct proportion? Justify your answer.



- 5 Does this graph show inverse proportion? Justify your answer.



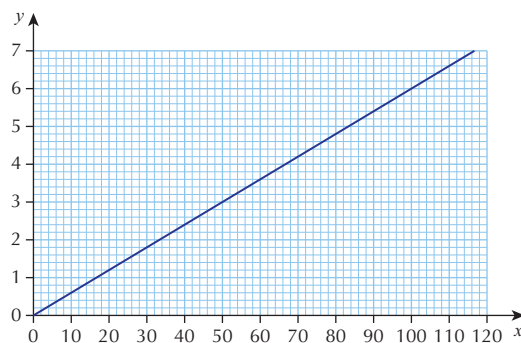
- 6 a Two variables, x and y , are in direct proportion.
When $x = 60$, $y = 0.2$.

Work out a formula for y in terms of x .

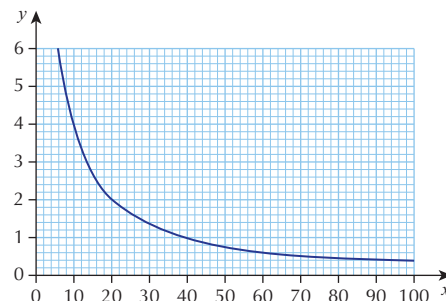
- b Two more variables, x and y , are in inverse proportion.
When $x = 60$, $y = 0.2$.

Work out a formula connecting x and y .

- 7 This graph shows two variables in direct proportion.
Work out a formula for y in terms of x .



- 8 This graph shows two variables in inverse proportion.
Work out a formula for y in terms of x .



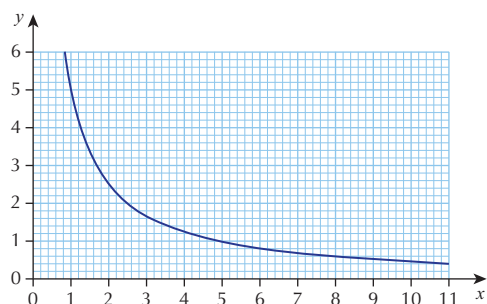
- 9** A fence is put up along one side of a school playground. It is 200 m long. The fence is made from separate panels.

- a** Show that the number of panels needed is inversely proportional to the length of each panel.
b The table shows the costs of panels that are different lengths.

Length (<i>m</i>)	1.5	1.8	2.4
Cost	£8.60	£9.95	£11.15

Work out the total cost for each possible panel and state which panel is the cheapest.

- 10** This graph shows the values of two variables in inverse proportion.



- a** Work out a formula connecting x and y .
b Use your formula to find the value of y when $x = 17.5$.
- 11** The table shows values of r and s .

r	1.6	1.2
s	0.06	?

- a** If s is directly proportional to r , work out:
i the missing value of s **ii** a formula connecting r and s .
b If s is inversely proportional to r , work out:
i the missing value of s **ii** a formula connecting r and s .

Reasoning: Looking for proportion

The perimeter of a rectangle is 20 cm.

- A** If one side of the rectangle is 3 cm, show that the other side is 7 cm.
B Work out three other pairs of possible values for the two sides of the rectangle.
C Draw a graph to show the pairs of values and draw a line through them.
D Are the lengths of the two sides in direct proportion? Justify your answer.
E Are the lengths of the two sides in inverse proportion? Justify your answer.

Ready to progress?



I can recognise and draw a graph for two variables that are in direct proportion.



I can work out a formula connecting two variables that are in direct proportion.
I can work out a formula connecting two variables that are in inverse proportion.
I can recognise and draw a graph for two variables that are in inverse proportion.
I can decide whether two variables are in direct proportion or inverse proportion.

Review questions

- The number of pages of a particular book and the time to read them are proportional. Work out the missing numbers in this table.

Number of pages	20	30	65
Time to read them (minutes)	17		

- A rectangle in which the length is 1.6 times the width is called a 'golden rectangle'. Copy and complete this table to show some possible sizes for a golden rectangle.

Width	10 cm	15 cm	50 cm		
Length			80 cm	1 metre	4 m

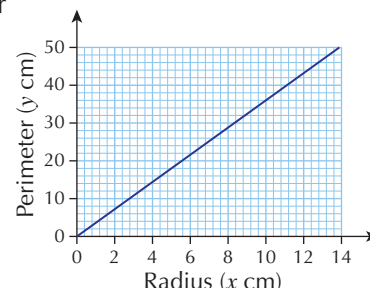
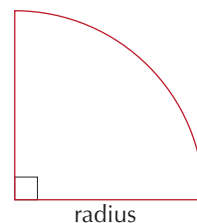
- The mass of five maths text books is 3.48 kg. Work out the mass of 40 books.
- The cost of oranges is proportional to the number bought. Eight oranges cost £2.24. Work out the cost of 13 oranges.
- The average cost of heating the water for a shower is £0.23.
 - Work out the cost, in pounds of:
 - 7 showers (one each day for a week)
 - 30 showers (one each day for a month)
 - 365 showers (one each day for a year).
 - Write down a formula for the cost (£ c) of n showers.
- The cost (£ y) of a taxi journey of x km is given by the formula $y = 1.65x$.
 - Is the cost proportional to the distance travelled? Justify your answer.
 - Work out the cost of a journey of 6.4 km.
 - Ian pays £23.43 for a taxi ride. How far did he travel?
 - Draw a sketch to show the shape of a graph of y against x .

- 7 This shape is called a quadrant. It is a quarter of a circle.

The graph shows the perimeter (y cm) of a quadrant of radius x cm.

- Explain why the graph shows that the perimeter of the quadrant is proportional to the radius.
- Use the graph to work out a formula for the perimeter (y cm) of the quadrant in terms of the radius (x cm).
- Use your formula to find the perimeter of a quadrant of radius 19 cm.
- Use your formula to find the radius of a quadrant with a perimeter of 85 cm.
- The formula for the area (a cm²) of a quadrant with a radius of x cm is $a = 0.785x^2$.

Show that the area is not directly proportional to the radius.



- 8 A survey of sixty-three 50-year olds shows that nineteen of them wear glasses. If the sample in the survey is a fair representation of all 50-year olds, how many people out of 1000 do you expect to wear glasses?
- 9 The time taken to complete a rail journey is inversely proportional to the average speed of the train.

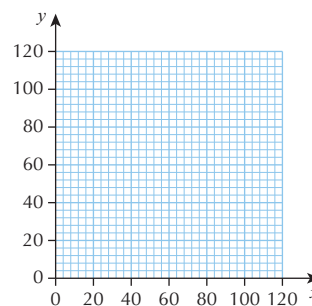
At a speed of 100 km/h the journey takes 4.5 hours.

- Work out the missing values in this table.

Speed (y km/h)	100	80	120
Time (x hours)	4.5		

- Work out a formula connecting x and y in the form $xy = k$ where k is a number.
- 10 The time taken to walk to the end of a playing field is inversely proportional to the walking speed. Amelia walks at 1.6 m/s and takes 174 seconds.
- Work out a formula connecting walking speed (s m/s) and time taken (t seconds).
 - Lily takes 142 seconds. Work out her walking speed.
- 11 Some rectangles have an area of 1200 cm².

- The length of one side of one rectangle is 80 cm. Work out the length of the other side.
- The length of one side of another rectangle is 10 cm. Work out the length of the other side.
- If the lengths of the sides are x cm and y cm, write down a formula connecting x and y .
- Draw a graph to show values of x and y . Use axes like these.



Challenge

Planning a trip

A youth leader is planning a coach trip for some youth club members.

- 1 The total cost of the coach is £1000.
The coach can take up to 53 passengers.
This must be shared equally by the people on the coach.
 - a Work out the cost per person if the coach is full.
 - b Work out the cost per person if there are 25 passengers.
 - c Draw a graph to show how the cost per person varies with the number of people.
- 2 On the motorway the coach travels at a maximum speed of 114 km/h.
 - a How far does the coach travel in 30 minutes at this speed?
 - b How far does the coach travel in 5 minutes at this speed?
 - c Work out a formula for the distance (y km) travelled in x minutes.
 - d Draw a graph to illustrate your formula.

- 3 The time to complete the whole journey is inversely proportional to the average speed. The journey is 240 km.
- Find the time taken, in hours, if the average speed is 80 km/h.
 - Work out a formula connecting the average speed (x km/h) and the time taken (y hours).
 - Draw a graph to show how the time varies with the average speed. You should include average speeds up to 100 km/h.
- 4 The amount of fuel the coach uses varies with the distance. Look at the data in this table. It shows the fuel used when the coach is travelling on the motorway at 90 km/h.

Fuel used (f litres)	93	131	179
Distance travelled (d km)	242	341	465

- Is the relationship between fuel used and distance travelled in direct proportion, in inverse proportion or neither of these?
- Use your answer to part **a** to write a formula for f in terms of d .

The fuel consumption of the coach is measured in litres per kilometre (litres/km).

The fuel consumption varies with speed.

Look at the data in this table.

Speed (km/h)	50	60	70
Fuel consumption (litres/km)	3.8	3.4	3.1

- Is the relationship between speed and fuel consumption in direct proportion, in inverse proportion or neither of these? Give a reason for your answer.