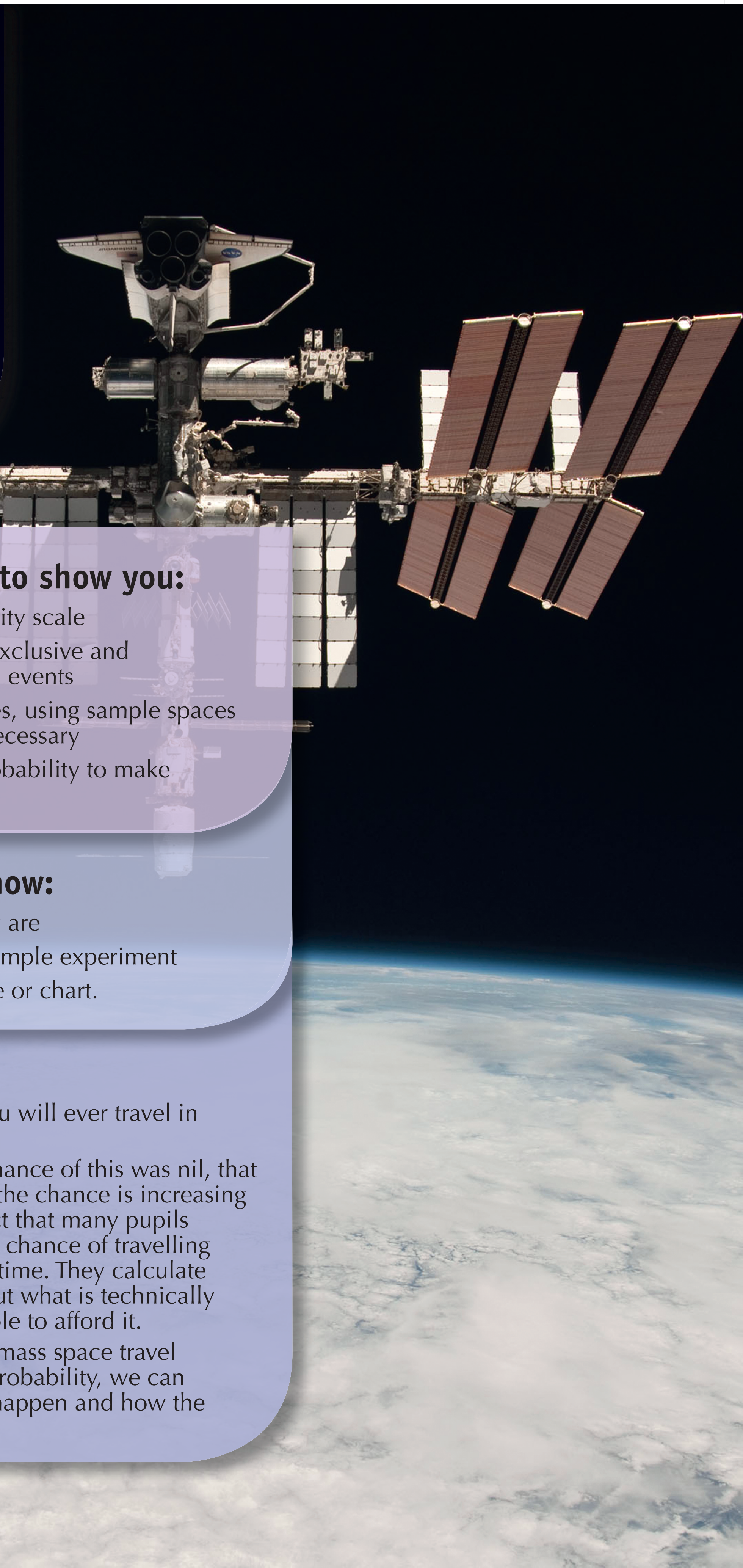
3 Probability



This chapter is going to show you:

- how to work with a probability scale
- how to recognise mutually exclusive and non-exclusive outcomes and events
- how to work out probabilities, using sample spaces and Venn diagrams where necessary
- how to use experimental probability to make predictions.

You should already know:

- what chance and probability are
- how to collect data from a simple experiment
- how to record data in a table or chart.

About this chapter

What is the probability that you will ever travel in space?

One hundred years ago, the chance of this was nil, that is, it was impossible, but now the chance is increasing every decade. Scientists predict that many pupils in schools now will have a fair chance of travelling into space one day in their lifetime. They calculate the probabilities by working out what is technically possible, and who might be able to afford it.

We do not know for certain if mass space travel will happen but, by studying probability, we can understand how likely it is to happen and how the scientists work it out.

3.1 Probability scales

Learning objective

- To use a probability scale to represent a chance

Key words	
equally likely	event
outcome	probability
probability scale	

When you do something such as rolling a dice, this is called an **event**.

The possible results of the event are called its **outcomes**. For example, rolling a dice has six possible outcomes: scoring 1, 2, 3, 4, 5 or 6.

You can use **probability** to decide how likely it is that different outcomes will happen.

Equally likely outcomes

Equally likely outcomes are those that all have the same chance of happening. For example, when you roll a dice, there are six different possible outcomes. This is because it could land so that any one of its six numbers shows on top.

The probability of an equally likely outcome is:

$$\text{P(outcome)} = \frac{\text{the number of ways the outcome could occur}}{\text{the total number of possible outcomes}}$$

There is only one way a normal dice can show a 6 when it lands, so only one of its possible outcomes will be the one you want.

There are six numbers on the faces, so there are six possible outcomes. Therefore:

$$\text{P}(6) = \frac{1}{6}$$

If you throw two dice and add their scores there are several ways that you could score 6, because there is a much larger number of possible outcomes.

Example 1

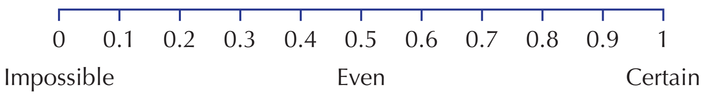
What is the probability of scoring a number less than 5 when you roll a dice?

There are four possible outcomes that give you a number less than 5: 1, 2, 3 and 4.

There are six different possible outcomes altogether, when you roll a dice: 1, 2, 3, 4, 5 and 6.

So P(rolling a dice and getting a number less than 5) is $\frac{4}{6} = \frac{2}{3}$.

Probabilities can be written as either fractions or decimals. They always take values from 0 to 1. The probability of an event happening can be shown on the **probability scale**.



Probabilities of events not occurring

If one outcome is the absolute opposite of another outcome, such as 'raining' and 'not raining', then the probabilities of the two outcomes add up to 1.

Example 2

What is the probability of *not* scoring a number under 5 when you roll a dice?

From example 1, you know that P(number less than 5) is $\frac{4}{6}$.

Then, P(number not less than 5) $= 1 - \frac{4}{6}$

$$= \frac{6-4}{6} = \frac{2}{6} = \frac{1}{3}$$

Example 3

The probability that a woman washes her car on Sunday is 0.7. What is the probability that she does not wash her car on Sunday?

These two outcomes are the opposite of each other, so the probabilities add up to 1.

The probability that she does not wash her car is:

$1 - 0.7 = 0.3$

Example 4

A girl plays a game of tennis. The probability that she wins is $\frac{2}{3}$. What is the probability that she loses?

The probability of her not winning, or P(losing), is:

$1 - \frac{2}{3} = \frac{1}{3}$

Exercise 3A

1 A set of cards is numbered from 1 to 50.

One card is picked at random. Give the probability that the number on it:

a is even **b** has a 7 in it **c** has at least one 3 in it

d is a prime number **e** is a multiple of 6 **f** is a square number

g is less than 10 **h** is a factor of 18 **i** is a factor of 50.

2 A bag contains 32 counters. Some are black and the others are white. The probability of picking a black counter is $\frac{1}{4}$.

How many white counters are there in the bag? Explain how you worked it out.

3 Joe has 1000 tracks on his phone. He has:

250 tracks of White rock

200 tracks of Blues

400 tracks of Country & western

100 tracks of Heavy rock

50 tracks of Quiet romantic.

He sets the player to play tracks at random.

What is the probability that the next track to play is:

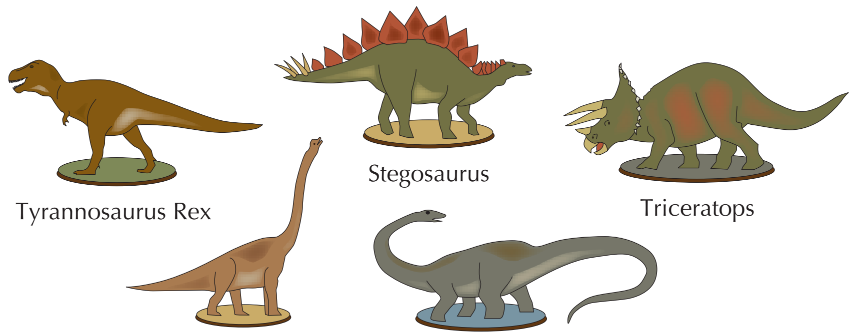
a White rock **b** Blues **c** Country & western

d Heavy rock **e** Quiet romantic **f** not Heavy rock?

4 In each box of cereal there is a free gift of a model dinosaur.

There are five animals to make up the set.

You cannot tell which animal will be in a box. Each one is equally likely.



a Liam needs a diplodocus to complete his set.

His sister Kiera needs a stegosaurus and a triceratops.

They buy one box of cereal.

i What is the probability that the animal is a diplodocus?

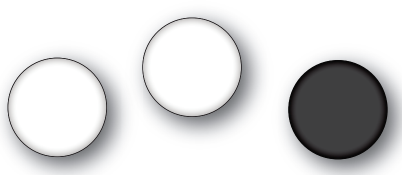
ii What is the probability that the animal is a stegosaurus or a triceratops?

b Their mother opens the box. She tells them the animal is not a brachiosaurus.

i Now what is the probability that the animal is a diplodocus?

ii Now what is the probability that the animal is a stegosaurus or a triceratops?

5 **a** Aidan puts two white counters and one black counter in a bag.



He is going to take one counter without looking.

What is the probability that he will pick a black counter?

PS

b Aidan puts the counter back in the bag and then puts more black counters in the bag.

Again, he is going to take one counter without looking.

The probability that he will pick a black counter is now $\frac{2}{3}$.

How many more black counters did Aidan put in the bag?

6 Fred has a bag of sweets that contains:

3 yellow sweets

5 green sweets

7 red sweets

4 purple sweets

1 black sweet

He takes a sweet from the bag at random.

a What is the probability that Fred will take a black sweet?

b Copy and complete this sentence, writing in the correct colour of sweet.

The probability that Fred will not take a … sweet is $\frac{3}{4}$.

7 Look at this probability scale.

The probability of outcomes A, B, C and D are shown on the scale. Copy the scale and mark underneath it the probabilities of A, B, C and D *not* happening.

8 Copy and complete the table.

Outcome	Probability of outcome occurring (p)	Probability of event not occurring ($1 - p$)
A	$\frac{1}{4}$	
B	$\frac{1}{3}$	
C	$\frac{3}{4}$	
D	$\frac{1}{10}$	
E	$\frac{2}{15}$	
F	$\frac{7}{8}$	
G	$\frac{7}{9}$	

9 In a normal pack of playing cards there are 52 cards divided into four equal-sized sets: clubs (black), spades (black), diamonds (red) and hearts (red). The 13 cards within each set are numbered 1 (called the ace) to 10, plus a jack, a queen and a king.

A card is chosen at random from a pack of 52 playing cards. Calculate the probability that it is:

a a black card **b** an ace **c** not an ace **d** a diamond

e not a diamond **f** not a 2 **g** not a picture **h** not a king

i not a red card **j** not an even number **k** not the ace of spades.

10 In a bus station there are 24 red buses, 6 blue buses and 10 green buses. Calculate the probability that the next bus to leave is:

a green **b** red **c** red or blue **d** yellow

e not green **f** not red **g** neither red nor blue **h** not yellow.

Activity: Will it happen? No it won't!

Design a spreadsheet to convert the probabilities of outcomes happening into the probabilities that they do not happen.

Use some of your answers to Exercise 3A, where you calculated some 'reverse probabilities', to help you to set up the spreadsheet and to test it to see if it works.

3.2 Mutually exclusive outcomes

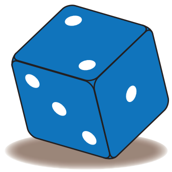
Learning objective

- To recognise mutually exclusive outcomes

Key words
intersection
mutually exclusive
set
union
Venn diagram

Mutually exclusive outcomes are those that cannot occur together. Each excludes the possibility of the other happening. For example, when you roll a dice, throwing a 1 or a 6 are mutually exclusive as you cannot get both results on the same throw of one dice.

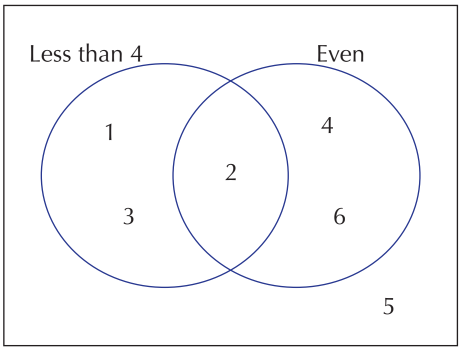
However, suppose you have a dice and are trying to throw numbers less than 4, but you also want to score an even number. Which number is common to both outcomes?



- The numbers less than 4 are 1, 2 and 3.
- The even numbers are 2, 4 and 6.

Because the number 2 is in both groups, the outcomes are not mutually exclusive. This means it is possible to achieve both outcomes at the same time if you throw a 2.

You can use a **Venn diagram** to illustrate this.



Hint A set is just a collection of objects or numbers.

The two circles show the **sets** of the possible outcomes: 'less than 4' and 'even'.

In this diagram:

- the set 'less than 4' has 1, 2 and 3 in it
- the set 'even numbers' has 2, 4 and 6 in it.

The numbers that satisfy either or both outcomes together are in the **union** of the sets.

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The numbers that can satisfy either outcome are in the area where the sets overlap. This is the **intersection** of the sets. Because there are some numbers in the intersection, this shows that the outcomes are not mutually exclusive.

Notice that all of the possible outcomes of the dice roll are included in the Venn diagram. The number 5 does not form part of either outcome, so it is positioned outside the circles.

Once you have drawn a diagram like this, it shows you the probability of both outcomes happening together. This is when the outcomes in the intersection occur.

In this example you can see that only one number (2) belongs in both sets. That means the chance of rolling an even number that is also less than 4 is one of the six possible outcomes, so the probability is $\frac{1}{6}$.

Example 5

Liz is buying fruit. Here is a list of possible outcomes.

A: She chooses strawberries.

B: She chooses red fruit.

C: She chooses green apples.

D: She chooses red apples.

E: She chooses oranges.

She chooses one item only. State which pairs of outcomes are mutually exclusive.

a A and B **b** A and E **c** B and C **d** B and D

a Strawberries are red fruit, so they are not mutually exclusive.

b Strawberries are not oranges, so they are mutually exclusive.

c Green apples are not red fruit, so they are mutually exclusive.

d Red apples are red fruit, so they are not mutually exclusive.

Exercise 3B

1 In a game you need to roll a dice and score an odd number larger than 2.

a Draw a Venn diagram showing the two sets 'odd numbers' and 'numbers larger than 2'.

b Are the outcomes 'scoring an odd number' and 'scoring a number larger than 2' mutually exclusive? Explain your answer.

c Use your Venn diagram to state the probability of rolling an odd number larger than 2.

2 Soolin has a bag containing ten cards, each showing one of the integers 1 to 10. She is playing a game and needs to select a card at random that shows a prime number smaller than 4.

a Draw a Venn diagram showing the two sets 'prime numbers' and 'numbers smaller than 4'.

b Are the outcomes 'selecting a prime number' and 'selecting a number smaller than 4' mutually exclusive? Explain your answer.

c Use your Venn diagram to state the probability of selecting a card showing a prime number smaller than 4.

3 A number square contains the numbers from 1 to 100.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Numbers are chosen from the number square. Here is a list of outcomes.

A: The number chosen is greater than 50.

B: The number chosen is less than 10.

C: The number chosen is a square number (1, 4, 9, 16, …).

D: The number chosen is a multiple of 5 (5, 10, 15, 20, …).

E: The number chosen has at least one 6 in it.

F: The number chosen is a factor of 100 (1, 2, 5, 10, …).

G: The number chosen is a prime number (2, 3, 5, 7, …).

State whether the outcomes in each pair are mutually exclusive or not.

a A and B **b** A and C **c** B and C **d** C and D
e B and F **f** C and F **g** C and G **h** D and E
i D and G **j** E and F **k** E and G **l** F and G

4 A sampling bottle contains 40 different coloured beads.

a After 20 trials Dan has seen 12 black beads and 8 white beads. Does this mean that there are only black and white beads in the bottle? Explain your answer.

Hint **A sampling bottle is a plastic bottle in which only one bead can be seen at a time.**

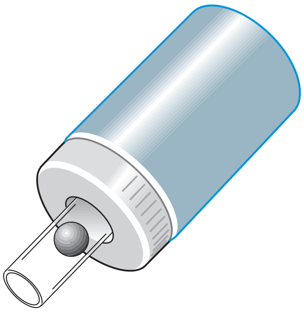
b You are told that there are 20 black beads, 15 white beads and 5 red beads in the bottle. State which of these pairs of outcomes are mutually exclusive.

i Seeing a black bead and seeing a white bead

ii Seeing a black bead and seeing a bead that is not white

iii Seeing a black bead and seeing a bead that is not black

iv Seeing any colour bead and seeing a red bead



MR **5** **a** Tom has three coins in his pocket: a 1p, a 2p and a 5p.

If he takes out one or more coins, the possible outcomes are:

1 coin	a 1p or a 2p or a 5p
2 coins	a 1p and a 2p
	a 1p and a 5p
	a 2p and a 5p
3 coins	a 1p and a 2p and a 5p.

Write down the different totals of each of the seven different outcomes.

b Some money falls out of Tom's pocket. Which is more likely, that he loses more than 4p or less than 4p?

Explain your answer.

c Elle has 1p, 2p, 5p and 10p in her pocket.

As in part **a**, find the different totals that Elle could take out of her pocket.

d Some money accidently falls out of her pocket. Which is more likely, that she loses more than 10p or less than 10p?

Explain your answer.

MR **6** The British coins currently are 1p, 2p, 5p, 10p, 20p, 50p, £1 and £2 coins.

Ilya has two different coins in his pocket.

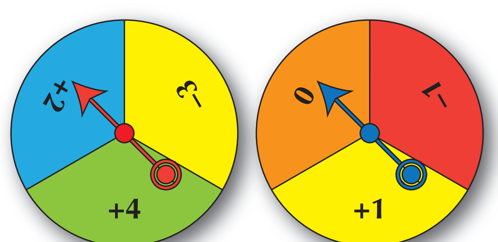
a List all the possible different amounts of money that he could have in his pocket.

b Which is more likely, that he has more than 60p or that he has less than 60p?

Explain your answer.

MR **7** **a** Copy and complete the table to show all the possible pairs of scores if you spin these two spinners.

Spinner 1	Spinner 2	Total score
+2	0	2
+2	−1	1



b Gary spins both spinners. Is he:

i more likely to get a positive total than a negative total

ii more likely to get an even total than an odd total?

Explain both your answers.

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Challenge: Four men run a race

Imagine a race between two men called Arran and Benji. They could finish the race in two different ways:

- Arran first and Benji second (AB)
- Benji first and Arran second (BA).

A Now think about a race with Arran, Benji and Callum. How many ways can they finish the race?

B Extend this problem to four men, and so on. Put your results into a table. See if you can work out a pattern to predict how many different ways a race with ten men could finish.

C When you have finished this, explore what the factorial ! button does on a calculator. This may help you to solve problems like this more quickly.

3.3 Using a sample space to calculate probabilities

Learning objective

- To use sample spaces to calculate probabilities

Key word

sample space

To help you work out the probabilities of events happening together you can use tables or diagrams called **sample spaces**.

A sample space is the set of all possible outcomes from a specific event.

Some events are simple, such as rolling a dice.

The sample space is {1, 2, 3, 4, 5, 6}.

Some are more complicated, such as rolling two dice.

The sample space could be written as {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12}.

However, this is not helpful as the outcomes are not equally likely.

A better way to show this sample space is in a table that combines the equally likely outcomes of each dice.

		Dice 1					
		1	2	3	4	5	6
Dice 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

This sample space shows the result of combining the outcomes from both dice. You can now see that the chances of rolling a total of 7 are far greater than the chance of rolling a total of 12.

Example 6

Find the probability of getting a head and a six when you roll a dice and toss a coin at the same time.

This sample space shows all the possible outcomes of throwing a coin and a dice together.

	1	2	3	4	5	6
Head	H, 1	H, 2	H, 3	H, 4	H, 5	H, 6
Tail	T, 1	T, 2	T, 3	T, 4	T, 5	T, 6

You can now work out the probability of getting both a head and a 6.

$$\text{P(outcome)} = \frac{\text{the number of ways the outcome could occur}}{\text{the total number of possible outcomes}}$$

P(head and a six) = $\frac{1}{12}$

You can also use a Venn diagram or a table to illustrate a sample space.

Example 7

In a class of 32 pupils, there are 15 boys. Out of eight left-handed pupils in the class, six are girls.

What is the probability of choosing a right-handed boy from the class?

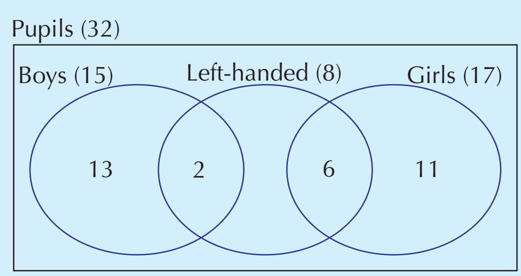
You could put this information into a table.

	Boy	Girl	Total
Left-handed	2	6	8
Right-handed	13	11	24
Total	15	17	32

You can now see that:

P(right-handed boy) = $\frac{13}{32}$

You can also show this in a Venn diagram, like this one.



Exercise 3C

1 **a** Draw a sample space to show the results of rolling a coin and tossing a dice together.

b Use your sample space to find the probability of scoring:

i a 3 and a tail

ii a head and an even number

iii a number less than 5 and a tail.

MR **2** A class has equal numbers of boys and girls. First one pupil is chosen, then another.

a Write down the four possible combinations of pupils that could be chosen.

b Jo says that the probability of choosing two boys is $\frac{1}{3}$. Explain why he is wrong.

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MR **3** A bag contains apples, bananas and pears. Liam chooses two fruits at random.

a List all the possible outcomes.

b You are told that you have more chance of choosing an apple and pear than an apple and a banana. Explain how that could happen.

4 A market trader sells jacket potatoes plain, with cheese or with beans. Clyde and Delroy each buy a jacket potato.

a Copy and complete the sample space table.

Clyde	Delroy
plain	plain
plain	cheese

b Give the probability of:

i Clyde choosing plain

ii Delroy choosing plain

iii both choosing plain

iv Clyde choosing plain and Delroy choosing beans

v Clyde choosing beans and Delroy choosing cheese

vi both choosing the same

vii neither choosing plain

viii each choosing a different flavour.

5 Bret rolls two dice and adds the scores together. Copy and complete the sample space of his scores.

	1	2	3	4	5	6
1	2	3				
2	3					

a What is the most likely total?

b Give the probability that the total is:

i 4 **ii** 5 **iii** 1 **iv** 12 **v** less than 7

vi less than or equal to 7 **vii** greater than or equal to 10 **viii** even

ix 6 or 8 **x** greater than 5.

6 Bart rolls two dice and then multiplies the scores together.

a Draw the sample space of his scores.

b Which total is the most likely to occur?

c What would be the probability of rolling a score greater than 17?

PS 7 Bella spins two spinners together. Each spinner has the numbers 0, 1, 2 and 3 on it. What is the probability that the sum of the numbers she scores is less than 2?

PS 8 A café makes 100 paninis. All the paninis are either meat or cheese, but 75 of them have salad in them, as well.

30 paninis have cheese with salad.

15 paninis have meat with no salad.

a Show this information in a Venn diagram.

b Use your Venn diagram to help you calculate the probability of selecting at random a cheese panini without salad.

Problem solving: Odd socks

In Andrew's sock drawer, all the socks are mixed up.

He knows that he has three pairs of blue socks, two pairs of white socks and a pair of green socks.

He takes two socks out without looking at their colour.

What is the probability that these socks are both the same colour?

Hint The answer is not $\frac{1}{3}$.

3.4 Experimental probability

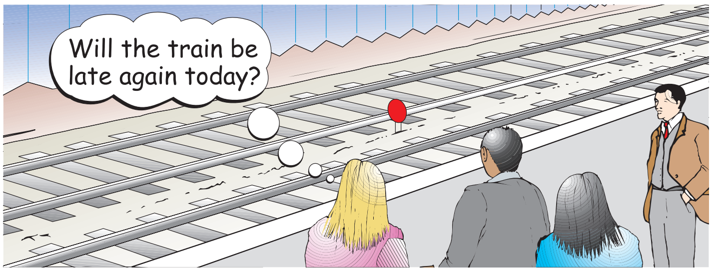
Learning objective

- To calculate probabilities from experiments

Key word

experimental probability

How could you estimate the probability that a train will be late?



You could keep a record of the number of times that the train arrives late over a period of 10 days, and then use these results to estimate the probability that it will be late in future. The experiment here is observing the train, and the outcome you record is the train being late. The results enable you to find the **experimental probability** of the train being late.

$$\text{Experimental probability} = \frac{\text{number of trials that produce the outcome}}{\text{total number of trials carried out}}$$

Example 8

An electrician wants to estimate the probability that a new light bulb lasts for less than one month.

He fits 20 new bulbs and three of them fail within one month.

What is his estimate of the probability that a new light bulb fails within the month?

3 out of 20 bulbs fail within 1 month, so his experimental probability is $\frac{3}{20}$.

Example 9

A dentist keeps a record of the number of fillings she gives her patients over two weeks.

Here are her results.

Number of fillings	None	1	More than 1
Number of patients	80	54	16

Estimate the probability that a patient does not need a filling.

She saw 150 patients in total and 80 did not need fillings.

$$\text{Experimental probability} = \frac{80}{150} = \frac{8}{15}$$

Example 10

A company manufactures items for computers. The number of faulty items is recorded in this table.

Number of items produced	Number of faulty items	Experimental probability
100	8	0.08
200	20	
500	45	
1000	82	

a Copy and complete the table.

b Which is the best estimate of the probability of an item being faulty? Explain your answer.

a

Number of items produced	Number of faulty items	Experimental probability
100	8	0.08
200	20	0.1
500	45	0.09
1000	82	0.082

b The best estimate is the last result (0.082), as the experiment is based on more results.

Exercise 3D

PS **1** Jacob had an old computer that kept crashing. He kept a record of the days it crashed. This table shows his results.

Number of days	Number of days the computer crashed
50	32
100	72
150	106
200	139
250	175

a From the results, would you say that on any one day there is a greater chance of Jacob's computer crashing or not? Explain your answer.

b Which part of the table shows the most reliable data to use? Why?

c From his data, how could Jacob estimate the probability of his computer crashing on any given day?

d What would his answer be?

PS **2** Amanda wants to test her octagonal dice to see if it is biased. She rolls the dice 100 times. Her results are shown in this table.

Score	1	2	3	4	5	6	7	8
Frequency	8	14	12	11	19	11	12	13

a Do you think the dice is biased? Give a reason for your answer.

b How could Amanda improve the experiment?

c From the results, estimate the probability of her rolling a 7.

d From the results, estimate the probability of her rolling a 3 or a 4.

e From the results, estimate the chance of her *not* rolling a 5.

3 Faye started an experiment to find the probabilities of spinning a coin and getting a head or a tail. These are her results.

	Number of trials	Heads	Tails	P(H)	P(T)
First 20	20	8	12	$\frac{8}{20} = 0.4$	$\frac{12}{20} = 0.6$
Next 20	40	11	9	$\frac{19}{40} = 0.475$	$\frac{21}{40} = 0.525$

a Use your own coin to spin the next 20, creating the next part of Faye's chart.

b Complete the chart, writing down P(H) and P(T) from all 60 trials.

c Repeat the above for the next 20 spins (giving a total of 80).

d What do you notice about P(H)?

4 Lewis said to his dad: 'We always have chips and peas with our school lunch.'

His dad asked him to keep a record each day for a month of when chips and peas were on the menu.

He presented his results to his dad like this.

Week 1	Chips	No chips
Peas	2	1
No peas	1	1

Week 2	Chips	No chips
Peas	3	1
No peas	1	0

Week 3	Chips	No chips
Peas	2	0
No peas	1	2

Week 4	Chips	No chips
Peas	3	0
No peas	1	1

a Create a summary table to show Lewis's results.

b What is the probability of the school lunch including chips and peas?

c What is the probability of the school lunch including neither chips nor peas?

d What would make this a better method of sampling the school meals?

e Add another question:

5 Lightco have created a new halogen bulb. The inventors claimed that it would last for over 3000 hours. The customer research team wanted to test this theory out, so they ran a test, keeping some bulbs on continuously until they failed to light.

a How many weeks and days is 3000 hours?

MR

b It was suggested that they test just 10 bulbs. Why is this not a good sample?

Finally, they decided to test 100 bulbs. These are the results.

Time, T (hours)	$0 \leqslant T < 500$	$500 \leqslant T < 1000$	$1000 \leqslant T < 1500$	$1500 \leqslant T < 2000$	$2000 \leqslant T < 2500$	$2500 \leqslant T < 3000$	3000+
Frequency	1	1	1	3	6	8	80

c What is the probability of one of the new halogen bulbs lasting over 3000 hours?

d Is the claim of the inventors true?

Problem solving: Roll the dice!

A Roll a pair of dice 50 times. Record the total of the two numbers shown, each time.

B What are the probabilities of each total being rolled?

C Repeat the experiment for another 50 rolls and record the results.

D Do you get the same results?

E Explain why your results are the same as – or different from – the first experiment.

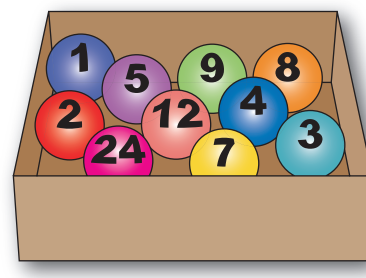
Ready to progress?

I can calculate probabilities involving equally likely outcomes.
I can calculate probability from experimental data.

I know what mutually exclusive outcomes are.
I can use the probability of an outcome to calculate the probability that the outcome does not happen.
I can use sample spaces and Venn diagrams to help calculate probabilities.

Review questions

1 Helen chooses a numbered ball, at random, from the box. What is the probability that the number on the ball she chooses:



a is a factor of 12 **b** is a square number

c is the HCF of 24 and 36 **d** is a prime number?

PS **2** Matt draws all the triangles he can, so that:

- one of the angles is always 70° smaller than another angle
- all the angles are multiples of 10°.

a Sketch all the triangles Matt could have drawn, showing the angles.

He drew each of his triangles in a different colour. One of the colours was red.

b What is the probability that his red triangle has:

i a right angle in it **ii** an angle of 30°?

c One of the other triangles was blue. What is the probability that this triangle was obtuse-angled?

PS **3** Kirsty drew all the rectangles she could with an area of 36 cm^2. The side lengths were all integer values.

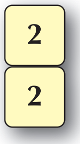
When she had finished she spilt coffee on just one of the rectangles. What is the probability that the rectangle she spilt coffee on had a perimeter longer than 36 cm?

4 Hannah had a set of cards.

She used them to make fractions less than 1, like this.

a Create the sample space showing all the possible fractions Hannah could have made.

Hint: She cannot use the same card twice in one fraction, for example, she cannot make:



b Hannah's little brother, Darren, picks up two of the cards at random and gives them to Hannah. What is the probability she can make a fraction equivalent to $\frac{1}{2}$ with them?

5 Lynne wanted to find out how many people used Facebook. She surveyed 100 people aged under 60 and 100 aged 60 or over.

Here are her survey results.

		Do you use Facebook?	
		Yes	No
Is your age	Under 60?	80	20
	60 or over?	10	90

a What percentage of people over 60 said they used Facebook?

b What is the probability of meeting someone under 60 who does not use Facebook?

6 Mr Speed teaches mathematics. He had a box that contained a cylinder, a cuboid, a cube, a square-based pyramid, a hexagonal prism, a cone and a sphere.

Katrina chose a shape at random out of the box.

Each shape was equally likely to have been chosen.

What is the probability that the shape Katrina chose:

a had six faces **b** had more than six faces?

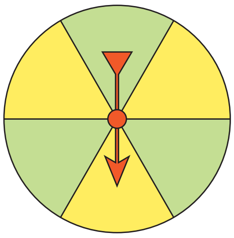
PS **7** Abbas was putting numbers on his spinner.

He wanted to arrange the numbers so that:

a he had more chance of spinning a negative number than not

b the probability of getting an even number was 0.5.

Draw a spinner showing numbers Abbas could have used.



PS **8** Brian regularly travelled down from Newcastle to London by train.

Over three weeks he counted how many times the drinks trolley passed him on each journey. His survey over three weeks showed these results.

		Monday	Tuesday	Wednesday	Thursday	Friday
Week 1	Newcastle to London	3	1	2	4	2
	London to Newcastle	2	2	3	1	0
Week 2	Newcastle to London	2	3	3	2	2
	London to Newcastle	3	4	4	1	1
Week 3	Newcastle to London	0	1	4	2	1
	London to Newcastle	2	2	3	4	2

a What is the modal number of times the drinks trolley passes?

b What is the experimental probability that on any journey the drinks trolley passes him four times?

Financial skills

Fun in the fairground

The fair has come to town.

Hoopla

You can buy five hoops for £1.25 and there are three types of prize.

You can win a prize by throwing a hoop over that prize and the base it is standing on!

Ben watched people at this stall. He counted how many tries they had and how many times someone won.

This table shows his results.

Prize	Number of throws	Number of wins
Watch	320	1
£10 note	240	4
£1 coin	80	2

Use the information about **Hoopla** to answer these questions.

1 What income would the throws that Ben recorded have made for the stall?

2 From the results shown, what is the probability of someone aiming for and winning:

a a £1 coin **b** a £10 note **c** a watch?

3 What would you say is the chance of someone winning a prize with:

a one hoop **b** five hoops?

4 After watching, Ben decided to try for a £10 note. He bought 25 hoops and all his throws were aimed at the £10 note.

a How much did this cost him?

b What is the probability of his winning a £10 note?

5 On a Saturday afternoon, the stall would expect about 500 people each to buy a set of hoops. Assume that the throws would have been aimed at the various prizes in the same proportion as Ben observed.

a How many of each prize would the stall expect to have to give away?

b How much income would be generated from the 500 people?

c If the watches cost £18 each, how much profit would the stall expect to make on a Saturday afternoon?

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Hook a duck

On this stall, plastic ducks float in a moat around a central stall. Each duck has a number written on its underside, which cannot be seen until the duck is caught, by means of a hook on a stick. The number is checked by the stall holder.

If the number on the duck is:

1 you win a lollipop 2 you win a yo-yo 5 you win a cuddly toy.

Each time a duck is hooked, it is replaced in the water. Cindy, the stall holder, set up the stall one week with 45 plastic ducks.

- Only one had the number 5 on it.
- Nine had the number 2 on them.
- All the rest had the number 1 on them.

Cindy charged 40p for one stick, to hook up just one duck.

Use the information about **Hook a duck** to answer these questions.

6 What is the probability of winning:

a a cuddly toy **b** a yo-yo **c** a lollipop?

7 What is the probability of winning anything other than a lollipop?

8 Tom wanted his sister, Julie, to win a yo-yo.

a How many ducks should Julie hook to expect to pick up at least one with a number 2 on it?

b How much will it cost Tom to pay for the number of ducks he expects Julie to need, to win a yo-yo?

9 Before lunch on Sunday, Cindy took £100 from the stall.

a How many ducks had been hooked that morning?

b How many cuddly toys would you expect Cindy to have given away that morning?

c How many yo-yos would you expect Cindy to have given away that morning?

10 Cindy bought the cuddly toys for £4 each and the yo-yos for 50p each. She gets the lollipops in a jar of 100 for £4. Cindy expects to take £250 on a Friday night.

a How many ducks will she expect to be hooked that night?

b How many lollipops will she expect to give away that evening?

c How many yo-yos will she expect to give away that evening?

d How many cuddly toys will she expect to give away that evening?

e What will be the value of all the prizes she expects to give away that night?