

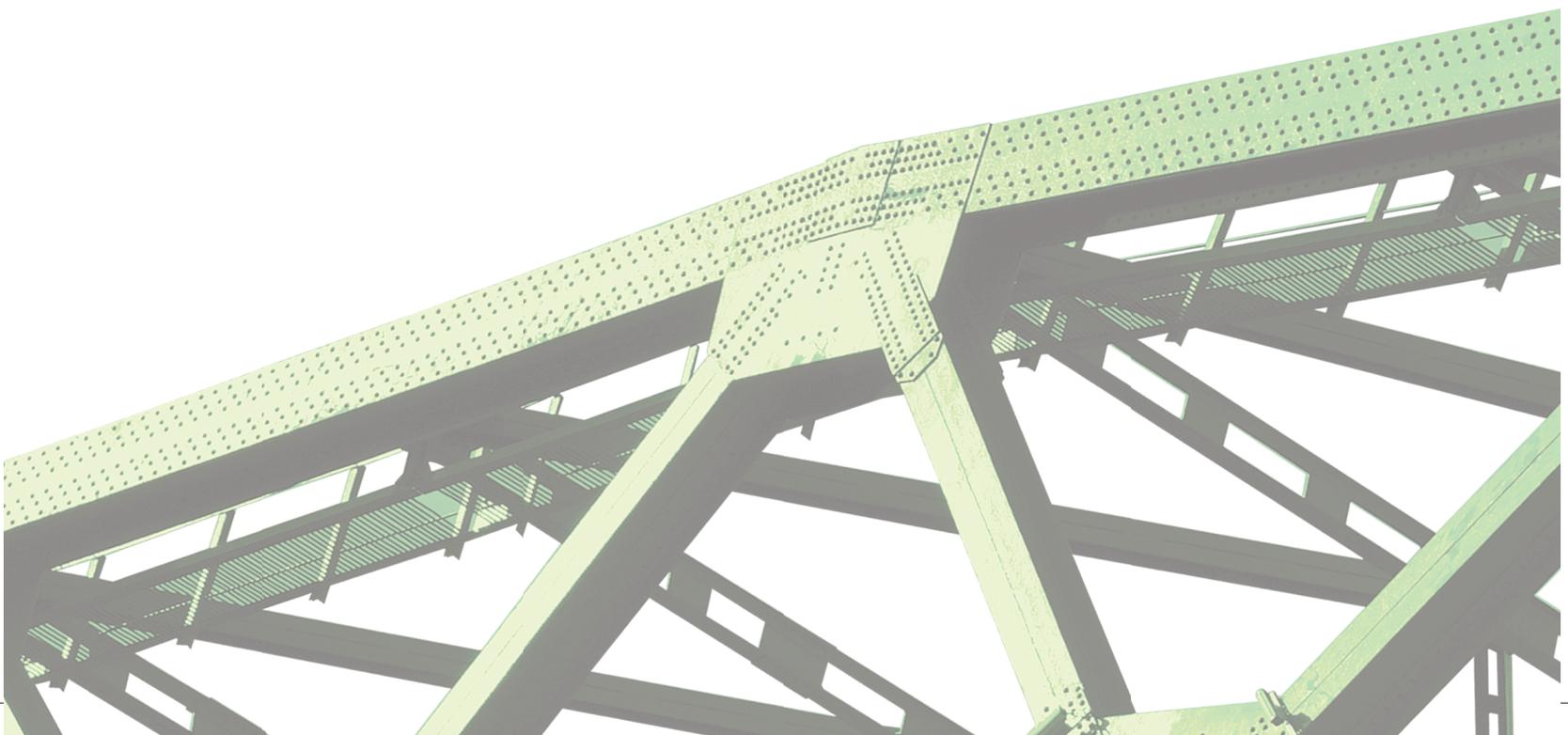
Collins

Teacher Pack 1.2

Maths Frameworking

3rd edition

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17 Ratio

Learning objectives

- How to use ratio notations
- How to compare quantities with ratios
- How to simplify ratios
- How to use ratios to find totals or missing quantities
- The connection between ratios and fractions
- How to use ratios to solve problems in everyday life

Prior knowledge

- How to simplify fractions. In KS2, pupils will have divided numerators and denominators by common factors.
- How to find a fraction of a quantity. Pupils will be familiar with percentage notation.
- The equivalence between simple fractions and percentages. Pupils should be familiar with simple fractions such as three-quarters or one-tenth, and the corresponding percentages.
- How to interpret bar charts and pie charts

Context

- Ratios are a very useful way to compare quantities without the distraction of the actual values. For example, saying that the diameter of Saturn is 10 times the diameter of the Earth (or the ratio is 10 : 1) provides an immediate mental image. This would not be as obvious just by quoting the diameters.
- Many pupils will have experience of riding a bicycle with gears. It may come as a surprise to them to learn that ratios are used to compare one gear with another. If you know the gear ratio, you know if it will be easy or difficult to pedal in that gear.

Discussion points

- Questions to activate prior knowledge and exercise mathematical 'fluency', that is, the ability to manipulate mathematical language and concepts and apply them in different contexts.

Associated Collins ICT resources

- Chapter 17 interactive activities on Collins Connect online platform
- *Zodiac map* Wonder of Maths on Collins Connect online platform
- *Lines of the form $x + y = a$* Worked solution on Collins Connect online platform
- Ratio and proportion tool on Collins Connect platform

Curriculum references

Develop fluency

- Move freely between different numerical, algebraic, graphical and diagrammatic representations (for example, equivalent fractions, fractions and decimals, and equations and graphs)

Ratio, proportion and rates of change

- Use ratio notation, including reduction to simplest form
- Divide a given quantity into two parts in a given part to part ratio
- Express the division of a quantity into two parts as a ratio
- Relate the language of ratios and the associated calculations to the arithmetic of fractions

Fast track for classes following a 2 year scheme of work

- Pupils should have worked with ratio in KS2, when comparing quantities and in problems involving unequal sharing. Pupils may have been introduced to the $a : b$ notation. If pupils can show understanding by answering one or more of the later questions in Exercise 17A of the Pupil Book, they can move on to simplifying ratios in Exercise 17B. Similarly, if pupils are confident about simple sharing problems, as provided in Exercise 17C, then they can move on to concentrate on the mixed questions in Exercise 17D.

Lesson 17.1 Introduction to ratios

Learning objectives

- To use ratio notation
- To use ratio to compare quantities

Resources and homework

- Pupil Book 1.2, pages 315–318
- Intervention Workbook 2, pages 43–45
- Intervention Workbook 3, pages 40–42
- Homework Book 1, pages 111–112
- Online homework Chapter 17.1, exercises 1–10

Links to other subjects

- **Science** – to compare quantities in experiments or compounds; comparing distances in astronomy
- **Geography** – to compare populations, areas, heights or other physical measures

Keywords

- ratio
- quantity

Problem solving and reasoning help

- **MR** question 4 of Exercise 17A has an application of algebra in creating a simple formula. The number in the ratio becomes the coefficient of N .
- **PS** questions 7 and 8 require pupils to work from a ratio to a value. They will look in more detail in a future section at ways to do this. In these simple cases the earlier part of the question should help them to find the missing value.
- The investigation at the end of the exercise encourages pupils to choose their own ratios to evaluate.

Common misconceptions and remediation

- Pupils sometimes have difficulty remembering how to pronounce 'ratio'. They may pronounce it as rat-i-o, with a 't' sound rather than with an 'sh' sound. Help pupils by saying the word often and asking them to use it. Point out that ratio is related to the word 'ration', and that sometimes spell checkers change 'ratio' to 'ration'.
- The order of the numbers in a ratio is important and relates to the order of the quantities being compared. Emphasise this by asking for the ratio the other way round and what it means when looking at examples.

Probing questions

- The ratio of two distances is 10 : 1. Do you know either of the distances? Do you know if they are both a whole number of centimetres? What do you know about the distances? (One distance is 10 times greater than the other distance.)

Part 1

- Write or place these numbers on the board, randomly scattered:
15, 4, 27, 24, 9, 20, 45, 60, 8, 12
- Ask pupils to arrange the numbers in two sets in any way they choose.
- Ask one or two pupils to give their sets, with reasons.
Try to find a pupil who has put 4, 8, 9, 15, 20 in one group and 12, 24, 27, 45, 60 in the other. If no one has done this, do it yourself and ask pupils what is the logic behind your choice. (In these sets, the numbers in one set are three times the numbers in the other set.)

- Write these numbers on the board and ask pupils to divide them into two sets in a similar way: 2, 5, 10, 20, 25, 40, 50, 100, 200, 250.
- We are looking for: 2, 5, 20, 40, 50 and 10, 25, 100, 200, 250. In this case the multiple is 5.
- If pupils need extra multiplication practice, give them a number to multiply by (such as 4 or 20) and then some quick-fire numbers to multiply by that. So if the number is 4, you might ask them to multiply by, for example: 5, 7, 20, 11...

Part 2

- Look at the numbers in the first set in Part 1. They can be paired as 4 and 12, 8 and 24... Explain that we would say: 'The ratio for each pair is 1 to 3 or 1 : 3'. Explain this notation.
- Ask for some other pairs of numbers in the same ratio.
- **Less able** pupils may find a visual representation helpful. Draw two columns of equal width, one three times the height of the other. Say that the ratio of the heights is 1 : 3 and divide the second column into three parts to demonstrate this.
- Pupils should be able to say that numbers in the second example give pairs in the ratio 1 : 5.
- Look at the example of the giraffe and the antelope at the start of section 17.1. The ratio of the height of the giraffe to the height of the antelope is 4 : 1. Point out that we do not know the height of either. The ratio tells us that one animal is four times the height of the other.
- Explain that the ratio can be written the other way around, so that the ratio of the antelope's height to that of the giraffe is 1 : 4.
- Have two bottles. The capacity of one bottle should be one-and-a-half times the capacity of the other bottle. If you do not have bottles, use Example 2 on page 315 of the Pupil Book.
- Explain how we can write the ratio as 3 : 2 by dividing each bottle into sections of equal size.
- **Pupils can now do Exercise 17A from Pupil Book 1.2.**

Part 3

- Once pupils have finished the investigation at the end of Exercise 17A, ask them to share their findings and compare the ratios of the number of coins with the ratios of the values.
- If **less able** pupils are unable to do the investigation, ask individuals to draw a diagram to illustrate a particular ratio such as 1 : 6 or 5 : 2.

Answers

Exercise 17A

1 a 4 b 1 : 4 c 4 : 1

2 a 2 : 1 b 1 : 2 c 4 : 1 d 1 : 2

3 a 2 : 1 b 4 : 1 c 1 : 8

4 a i 3 : 1 ii 6 : 1 iii 5 : 1 iv 1 : 2

b $1\frac{1}{2}$ c 3N buckets of gravel, $2\frac{1}{2}N$ buckets of sand, $\frac{1}{2}N$ buckets of water

5 The ratio of maths pages to science pages is 1 : 4

6 a The ratio of Cara's age to Dani's age is 1 : 2

b The ratio of Elfine's age to Frank's age is 2 : 3

7 a 5 times b i 1 : 5 ii 2 : 1 iii 3 : 2 c 4.2 litres

8 a i 3 : 1 ii 1 : 3 iii 2 : 1 iv 8 : 1 b 1200 million c 9P, 8P, 4P, 3P, 1.5P

9 a 4 : 1 b 1 : 3 c 3 : 2

10 3 : 1

Investigation: Bagging coins

A 50 B 100

C a 1 : 2 b No. The values are in the ratio 2 : 1

D Pupils' own answers

Lesson 17.2 Simplifying ratios

Learning objective

- To write a ratio as simply as possible

Resources and homework

- Pupil Book 1.2, pages 318–321
- Homework Book 1, pages 112–114
- Online homework Chapter 17.2, exercises 1–10

Links to other subjects

- **Science** – to compare the amounts of two elements in a chemical formula
- **Geography** – to compare the area of a country to the area of another, for example, comparing the size of other countries to the size of Wales

Keywords

- simplify
- fraction

Problem solving and reasoning help

- In **PS** question 7 of Exercise 17B in the Pupil Book, percentages are given rather than quantities. This is normal practice on clothing labels. The ratio can be written using the percentages in the same way as if quantities had been given. You could ask pupils why percentages rather than quantities are used here. (Food labelling provides quantities.)
- In **PS** question 9b, pupils should realise that they are looking for numbers where one number is double the other number.
- In the investigation at the end, pupils should realise that the ratios of any corresponding lengths (width, length or perimeter) will be the same. They will not be the same as the area ratios. Pupils may need a hint to recognise square numbers occurring in the area ratios.

Common misconceptions and remediation

- Pupils do not have units. In this section of the Pupil Book, pupils are comparing lengths, masses and so on, but the units of measurement do not appear in the ratio. Emphasise this point when calculating or working with ratios.
- The quantities being compared must be in the same units. Part 3 addresses this point.

Probing questions

- When you are given a ratio, how do you know if it is in its simplest form?

Part 1

- Write some fractions on the board and ask pupils to simplify them. Start with fractions that simplify to 1 in the numerator; then progress to more complicated fractions.
- Emphasise the fact that you want the simplest possible form. For example, the simplest form of $\frac{12}{20}$ is $\frac{3}{5}$, not $\frac{6}{10}$.
- Match the difficulty to the ability of your pupils. For **more able** pupils, choose examples where the common factor is more or less obvious, or where there are multiple factors.

Part 2

- Draw two lines, one 24 cm long and one 40 cm long. Do not reveal the lengths yet.
- Ask pupils to estimate the ratios of the lengths. Do not comment on whether they are correct or not. If you get several suggestions, ask pupils to say which they think is best, and why.

- Now write the lengths on the lines and say that we could use these to write the ratio of the shorter length to the longer length as 24 : 40. Imagine each line marked in centimetre sections. There will be 24 on one line and 40 in the other.
- Say that we can simplify ratios in the same way that we simplify fractions – by dividing by a common factor.
- Ask pupils to simplify $\frac{24}{40}$ to its simplest form. The answer is $\frac{3}{5}$.
- In the same way, the ratio 24 : 40 simplifies to 3 : 5. Did anyone have this as their estimate?
- What does this mean? Divide the 24-cm line into three equal sections and the 40 cm line into five equal sections. This provides a visual representation of the ratio.
- For **more able** pupils, you could explain that we could also write the ratio as $1:1\frac{2}{3}$.
- This is because $40 \div 24 = 1\frac{2}{3}$ and the length of the longer line is $1\frac{2}{3}$ times the length of the smaller line. We tend to use the whole number representation of a ratio when the numbers are reasonably small.
- Give an example of two masses. Two books of different sizes would be suitable or any two objects you have in the classroom. Ask how we could find the ratio of these masses.
- Tell pupils the masses (make them up), as long as the masses seem reasonable and simplify to small numbers. Ask pupils to find the ratio of the masses in its simplest form. Ask for it both ways: lighter to heavier and heavier to lighter.
- Point out that ratios can be used in this way to compare any measurable quantity. You have looked at length and mass. Ask for other examples. Area or volume are possible answers.
- **Pupils can now do Exercise 17B from Pupil Book 1.2.**

Part 3

- Write on the board: The lengths of two lines are 25 cm and 100 mm.
- Say that the ratio of the lengths is *not* 1 : 4 and ask a pupil to explain why this is the case.
- Ask for the correct ratio in its simplest form. The answer is 5 : 2. Ensure that all pupils understand this. Emphasise that units must be the same when comparing two quantities.
- Give more examples using mixed units and ask for the ratio. Possible examples are 50 cm and 2 m; 200 g and 1 kg. **Less able** pupils will benefit if you use very simple examples.
- At the end of the lesson, ask pupils to share their findings from the investigation at the end of the exercise. Check that pupils can work out all the ratios required.

Answers

Exercise 17B

- 1 a $\frac{2}{3}$ b $\frac{1}{5}$ c $\frac{3}{2}$ d $\frac{3}{5}$ e $\frac{1}{30}$
 2 a 1 : 4 b 2 : 3 c 2 : 1 d 8 : 1
 e 2 : 3
 3 a 2 : 5 b 5 : 2
 4 4 : 3
 5 2 : 7
 6 a 5 : 2 b 2 : 3 c 3 : 5
 7 a 6 : 1

Investigation: Ratios in patterns

A a 1 : 3 b 1 : 3 c 1 : 3 d 1 : 9 B a 3 : 2 b 3 : 2 c 3 : 2 d 9 : 4

C For a, b and c it is 2 : 1; for d it is 4 : 1

D a, b and c are the same and in d the numbers are the numbers in a squared.

b Some possible answers are: viscose to polyamide = 4 : 3; polyamide to angora = 9 : 2; angora to cashmere = 1 : 1

8 a 3 : 2 b 13 : 7

9 a i 40 : 1 ii 5 : 2 iii 5 : 4

b sulphur and oxygen

10 a 2 : 1 b 1 : 2 c 2 : 3 d 5 : 6

11 a 2 : 1 b 4 : 3 c 3 : 5

Lesson 17.3 Ratios and sharing

Learning objective

- To use ratios to find totals or missing quantities

Resources and homework

- Pupil Book 1.2, pages 321–323
- Intervention Workbook 3, pages 43–44
- Homework Book 1, pages 114–115
- Online homework Chapter 17.3, exercises 1–10

Links to other subjects

- Science** – to compare quantities in different types of materials

Keywords

- No new keywords for this topic

Problem solving and reasoning help

- In **MR** question 5 of Exercise 17C in the Pupil Book, pupils need to realise that the information given implies that the fractions are $\frac{1}{3}$ and $\frac{2}{3}$.
- In **MR** question 6, the crucial step is finding the fraction of the crowd who are women.
- In the challenge at the end of the exercise, the link between carat and percentage of gold can be illustrated by a horizontal line with a scale from 0 to 24 carats at the top and 0 to 100% on the bottom. This way, pupils can see easily that 24 carats is 100% gold, 12 carats is 50% gold, 18 carats is 25% gold, and so on.

Common misconceptions and remediation

- Pupils sometimes get confused about the exact relationship between ratios and fractions. Emphasise the link between the ratio numbers and the numerators of the fractions, as outlined in the lesson plan.

Probing questions

- A liquid is poured into two glasses. Three-fifths is in one glass. What is the ratio of the contents of the two glasses? Justify your answer.

Part 1

- Pupils can work in pairs on this activity.
- Write £20 on the board. Ask pupils to find what fractions of this will be a whole number of pounds. If they need clarification, show that $\frac{1}{2}$ is £10, which is a whole number of pounds; $\frac{1}{3}$ is £3.33 with 1p left over, which is not a whole number.
- After a few minutes check answers.
- Check that pupils remember how to find a fraction:
For example, $\frac{1}{5}$ of £20 = $20 \div 5 = £4$; $\frac{3}{5}$ of £20 = $3 \times £4 = £12$
- Less able pupils may need more practice with finding fractions. If so, repeat the activity with £18 and/or £12.

Part 2

- Draw a 4 by 3 grid of 12 squares. Colour three of the squares. Ask what fraction is coloured, and then what fraction is uncoloured. The answers should be one-quarter and three-quarters.
- Say that so far they have used ratios to compare two quantities. They can also use ratios to compare the shares when dividing one quantity.
- In the example we have just done, the ratio of shaded to unshaded parts is 1 : 3. Make sure that pupils are happy with this.
- Repeat this activity but shade different numbers of squares each time. For example, 4 or 2, or 1 or 6. Each time, ask for the fraction shaded, the fraction unshaded and the ratio of the two. Pupils should be able to see that the numerators of the fractions give the numbers in the ratio.
- Now work through Example 4 and Example 5 on pages 321 and 322 of the Pupil Book. In Example 4, £20 000 is divided into shares in a given ratio. This is done by working out the fraction required, and reinforcing the link between fractions and ratios. In Example 5 one share is given and the whole is found. Again, fractions are used to tackle this problem.
- **Pupils can now do Exercise 17C from Pupil Book 1.2.**

Part 3

- Say that Eve has 20 apples to share with Adam. Can they share them in the ratio 2 : 1 and have an equal number each? What about sharing them in the ratio 2 : 3?
- What is the smallest number that can be divided equally in the ratio 2 : 1 or 2 : 3? (The answer is 15; the lowest whole number is divisible by both 3 and 5.)
- A question for more able pupils is to find smallest number that can be divided equally in the ratio 2 : 1 or 2 : 5.

Answers

Exercise 17C

1 a $\frac{5}{6}$ b 25

2 a $\frac{4}{5}$ b 500

3 a $\frac{2}{5}$ b £18

4 a $\frac{1}{9}$ b 1080

5 44

6 60

7 20

8 48

9 600

Challenge: Mixing gold

A a 3.0 g b 3.5 g c 1.5 g

B a 3 : 1 b 7 : 1 c 3 : 5

C The ratio is 1 : 2

D The price of gold varies from day to day. At the time of writing it was about £29 per gram. You can find the price on the internet.

Lesson 17.4 Solving problems

Learning objectives

- To understand the connections between fractions and ratios
- To understand how ratios can be useful in everyday life

Links to other subjects

- **Science** – to find unknown quantities of substances
- **Geography** – to compare the resources of different countries

Resources and homework

- Pupil Book 1.2, pages 324–325
- Homework Book 1, pages 115–118

Keywords

- No new keywords for this topic

Problem solving and reasoning help

- Note that all questions in this lesson are Level 6 of the previous curriculum.
- In PS question 4 of Exercise 17D in the Pupil Book, pupils need to be clear about the ratio of the number of coins and the ratio of their values.
- In PS question 5, pupils first need to work out the number of green and brown bottles and then remember to note the change in the total number of bottles. A possible extension for more able pupils is to ask for the ratio if another green bottle falls (1 : 12).
- In the challenge, the fraction received by each child changes each time, but the total remains the same (£60). It is interesting to note that Jack's share increases each year, nearing half (£30), and Jill's share decreases.

Common misconceptions and remediation

- Pupils can make mistakes by not being clear about whether they know the whole or only a part when working out missing values. Encourage pupils always to ask themselves if they know the whole or a part before they try to answer a question.

Probing questions

Ann and Baz share some money in the ratio 3 : 2.

- If you know the total amount, how can you work out what Ann has?
- If you know what Ann has, how can you work out what Baz has?
- If you know what Baz has, how can you work out what Ann has?

Part 1

- Write down the following sentences: 'Tim and Claire bake 60 biscuits in total. Tim bakes 24 of them.'
- Ask pupils to work in pairs and find different ways to write this information. They can use fractions, percentages or ratios.
- After a few minutes take answers from selected pupils. Possible statements are:
- Tim bakes $\frac{2}{5}$ and Claire bakes $\frac{3}{5}$.
- Tim bakes 40% and Claire bakes 60%.
- Tim and Claire bake biscuits in the ratio 2 : 3.
- Make sure that pupils can remember the equivalences between simple fractions and percentages. If not, ask more questions about this, for example, writing tenths, fifths or quarters as percentages.
- This should remind pupils of the work covered in the last lesson.

Part 2

- This lesson is about using the ideas about ratios that pupils have met so far in questions, in a realistic context.
- Tell pupils that the ratio of teachers to pupils in a school is 1 : 12 (or use your own figure if you know it, but round it to a whole number).
- Ask a volunteer to explain what this means. A possible answer is that there is one teacher for every 12 pupils.
- Ask questions that require pupils to use this information in different ways, for example:
 - If there are 30 teachers, how many pupils are there?
 - If there are 60 teachers, how many pupils are there?
 - If there are 1300 pupils and teachers, how many of each are there?
- Ask pupils to explain how they work out the answers each time and check that their reasoning is correct.
- Set the following problem for pupils to tackle in pairs: 100 pupils are going on a trip and the ratio of teachers to pupils must be at least 1 : 9. What is the smallest possible number of teachers?
- The answer is 12. Discuss with pupils different ways to work out the answer. A suitable method for less able pupils is to say that one teacher can take nine pupils, two teachers can take 18 pupils, three teachers can take 27 pupils, and so on.
- More able pupils may prefer a method based on fractions. $\frac{9}{10}$ of the total number is 100, so $\frac{1}{10}$ of the total number is 9, remainder 1. Nine teachers is not quite enough.
- Example 6 in section 17.4 has a similar question for comparison.
- **Pupils can now do Exercise 17D in Pupil Book 1.2.**

Part 3

- Pupils can work in twos or threes on this task.
- Ask pupils to produce a concise statement that will help them to remember the key points about ratios. This could be an example, a diagram or a sentence or two.
- Ask one or two groups to share what they have done with the rest of the class.
- As an extension, pupils could use these ideas to produce posters for a wall display.

Answers

Exercise 17D

1 210 boys and 140 girls

2 3 : 1

3 2 : 1

4 £5.75

5 1 : 6

6 a 2 : 3 b 36 years

7 a 10 : 1 b 3 : 2 c $\frac{5}{6}$

8 a 2 : 3 b $\frac{2}{5}$ c 1 : 2 d $\frac{1}{3}$

Challenge: Presents from Uncle Fred

A Jack £15 and Jill £45

B Jack £20 and Jill £40

C Jack £106.50 and Jill £193.50

Review Questions

(Pupil Book pages 326–327)

- The review questions will help to determine pupils' abilities with regard to the material within Chapter 17.
- These questions also draw on the maths covered in earlier chapters of the book to encourage pupils to make links between different topics.

Problem solving – Smoothie bar

(Pupil Book pages 328–329)

- This activity is designed to reinforce the use of ratios by putting them in a realistic context.
- Pupils will find the information they need in various places on the double page spread of the Pupil Book. They will need to use the appropriate information in each question.
- Pupils should be familiar with grams and millilitres as measures of mass and capacity respectively.

Answers to Review Questions

- 1 a 2 : 1 b $\frac{1}{6}$ c $33\frac{1}{3}\%$ d 600 : 300 is 2 : 1
- 2 a $\frac{2}{5}$ b 120 ml c 3.75 litres
- 3 a 2 : 1 b 2 : 3
- 4 a Number of dry days = $365 - 146 = 219$ and $146 : 219 = 2 : 3$
b 60%
- 5 280 m
- 6 36 years old
- 7 a 5 : 2 b 24000
- 8 a 5 : 2 b $2\frac{1}{2}$ c i 3 : 1 ii 3 : 2
- 9 a 30 b 5
c For example, Jupiter and Saturn, or Saturn and Uranus, or Venus and Mars
d For example, Venus and Earth, or Uranus and Neptune

Answers to Problem Solving – Smoothie bar

- 1 75 g mango, 37.5 g strawberries, 56.25 g bananas, 187.5 ml orange juice
- 2 100 g bananas, 50 g chocolate spread, 400 ml cranberry juice
- 3 5.625 litres
- 4 Six can be made. An extra 2 litres of milk would be required to make 20.
- 5 7.5 kg
- 6 52.6%
- 7 6 Tropical Fruit, 9 Breakfast Boost
- 8 £1.25
- 9 75p
- 10 £10.25