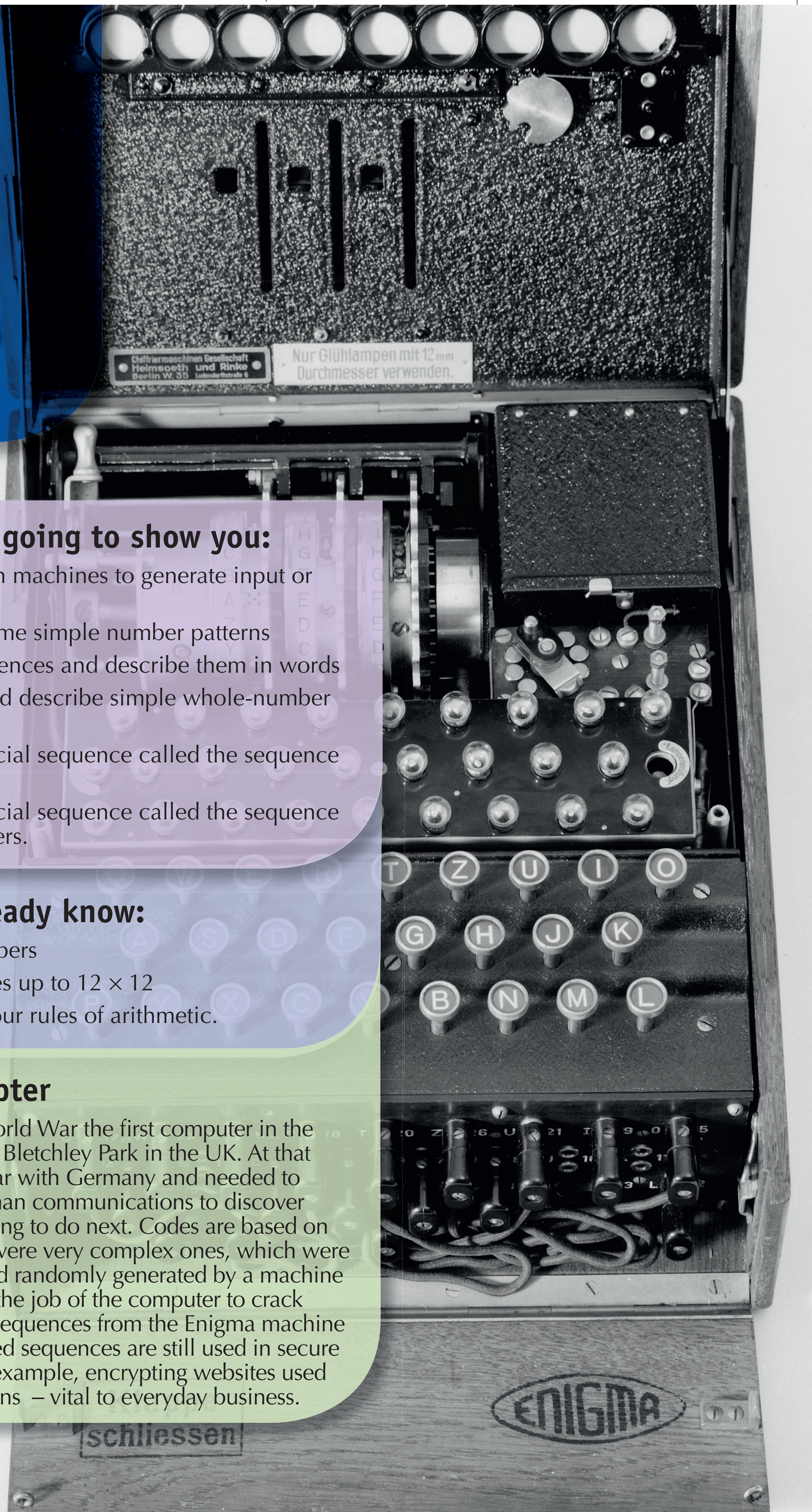
2

Sequences



This chapter is going to show you:

- how to use function machines to generate input or output values
- how to describe some simple number patterns
- how to create sequences and describe them in words
- how to generate and describe simple whole-number sequences
- how to use the special sequence called the sequence of square numbers
- how to use the special sequence called the sequence of triangular numbers.

You should already know:

- odd and even numbers
- multiplication tables up to 12 × 12
- how to apply the four rules of arithmetic.

About this chapter

During the Second World War the first computer in the world was invented at Bletchley Park in the UK. At that time, Britain was at war with Germany and needed to break the coded German communications to discover what they were planning to do next. Codes are based on sequences and these were very complex ones, which were changed every day and randomly generated by a machine called Enigma. It was the job of the computer to crack each day's new code sequences from the Enigma machine – and fast. Today, coded sequences are still used in secure communications, for example, encrypting websites used for financial transactions – vital to everyday business.

2.1 Function machines

Learning objectives

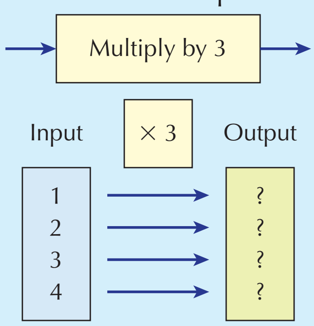
- To use function machines to generate inputs and outputs
- To use given inputs and outputs to work out a rule

Key words	
function machine	input
output	rule

Function machines can help you to understand how sequences are formed. They use mathematical **rules** to change one number into another. The numbers you start with are called the **input**. The numbers you finish with, after using the rules, are called the **output**.

Example 1

Work out the outputs for this function machine.



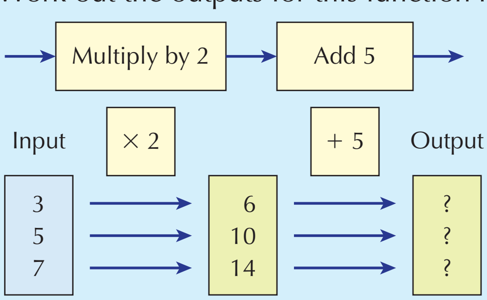
Multiply each input number by 3.

The outputs are:

3
6
9
12

Example 2

Work out the outputs for this function machine.



Multiply each input number by 2 and add 5.

The outputs are:

11
15
19

When working backwards from the output to the input, you reverse the operations.

So in Example 2 the reverse of + 5 is – 5 and the reverse of × 2 is ÷ 2.

Example 3

Work out the inputs for this function machine.

→ Multiply by 2 → Add 5 →

Input	× 2		+ 5	Output
?	→	?	→	23
?	→	?	→	31

Working backwards from an output of 23 gives 23 – 5 = 18 and then 18 ÷ 2 = 9 so the input is 9.

Similarly, when the output is 31 the input is 13.

Exercise 2A

1 Complete the inputs and outputs for each function machine.

a

Input	+ 3	Output
4	→	?
5	→	?
8	→	?
11	→	?

b

Input	– 2	Output
4	→	?
5	→	?
8	→	?
11	→	?

c

Input	+ 4	Output
3	→	?
?	→	9
8	→	?
?	→	15

2 Complete the inputs and outputs for each function machine.

a

Input	÷ 10	Output
100	→	?
80	→	?
60	→	?
50	→	?

b

Input	× 3	Output
4	→	?
?	→	18
8	→	?
?	→	36

c

Input	× 5	Output
4	→	?
5	→	?
8	→	?
11	→	?

3 Draw a function machine for each set of inputs and outputs.

a

Input		Output
2	→	4
3	→	5
4	→	6
5	→	7

b

Input		Output
3	→	9
4	→	12
5	→	15
6	→	18

c

Input		Output
1	→	6
2	→	12
3	→	18
4	→	24

d

Input		Output
24	→	12
12	→	6
8	→	4
6	→	3

e

Input		Output
2	→	9
3	→	10
4	→	11
5	→	12

f

Input		Output
2	→	16
4	→	32
6	→	48
8	→	64

4 Work out the outputs for each function machine.

a

Input	× 4	+ 1 → Output
1	?	?
2	?	?
3	?	?
4	?	?

b

Input	+ 3	× 2 → Output
1	?	?
2	?	?
3	?	?
4	?	?

5 Draw diagrams to illustrate these rules. Choose four input numbers and work out the outputs for them.

a → Multiply by 5 → Subtract 2 →

b → Subtract 2 → Multiply by 6 →

c → Add 4 → Divide by 2 →

6 Fill in the missing numbers and rules in each function machine.

a

Input	× 4	? → Output
3	?	9
5	?	17
8	?	29
10	?	37

b

Input	?	÷ 3 → Output
5	3	?
8	6	?
11	9	?
14	12	?

7 Work out the number I am thinking of, in each case.

a I think of a number. I multiply it by 4. I then add 8 to my answer. My final answer is 20.

b I think of a number. I multiply it by 5. I then subtract this from 15. My final answer is 5.

c I think of a number. I add 4. I then multiply by 6. My final answer is 18.

d I think of a number. I add 3. I then multiply by 2. My final answer is 18.

MR **8** Work out the input for each output.

a

Input	× 3	+ 4 → Output
?	?	7
?	?	13
?	?	16
?	?	25

b

Input	+ 5	× 2 → Output
?	?	14
?	?	16
?	?	20
?	?	26

c

Input	× 4	− 3 → Output
?	?	9
?	?	17
?	?	33
?	?	37

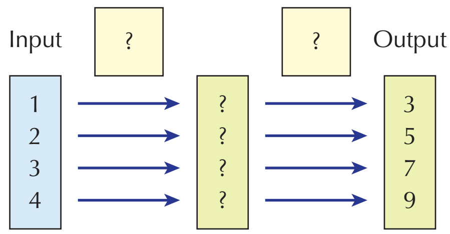
Challenge: Rules in sequences

Below are the inputs and outputs from some function machines.

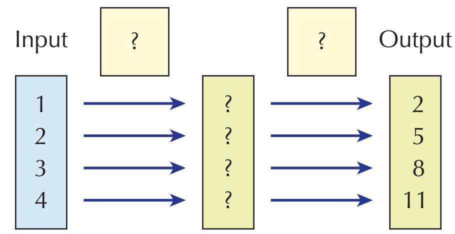
Each function machine uses two rules.

Draw a function machine or describe the rules for each set of inputs and outputs.

A



B



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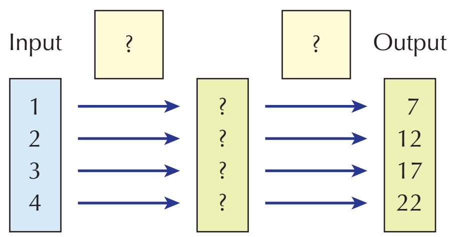


2.2 Sequences and rules

Learning objective

- To recognise, describe and generate sequences that use a simple rule

Key words

first term	geometric sequence
linear sequence	sequence
term	term-to-term rule

A **sequence** is a list of numbers that follow a pattern or rule.

You can use simple rules to make up many different sequences with whole numbers.

Sequences may also have different starting points.

The numbers in a sequence are called **terms** and the starting number is called the **first term**. The rule is often called the **term-to-term rule**.

Sequences that increase or decrease by a fixed amount, from one term to the next, are called **linear sequences**.

Example 4

Rule: add 3 Starting at 1 gives the sequence 1, 4, 7, 10, 13, …

Starting at 2 gives the sequence 2, 5, 8, 11, 14, …

Starting at 6 gives the sequence 6, 9, 12, 15, 18, …

Sequences in which you find each term after the first term by multiplying or dividing by a fixed amount are called **geometric sequences**.

Example 5

Rule: multiply by 2

Starting at 1 gives the sequence 1, 2, 4, 8, 16, …

Starting at 3 gives the sequence 3, 6, 12, 24, 48, …

Starting at 5 gives the sequence 5, 10, 20, 40, 80, …

Exercise 2B

1 Use the term-to-term rule to work out the first five terms of each sequence.
Start from a first term of 1.

a add 3	**b** multiply by 3	**c** add 5	**d** multiply by 10
e add 9	**f** multiply by 5	**g** add 7	**h** multiply by 2
i add 11	**j** multiply by 4	**k** add 8	**l** add 105

2 Use the term-to-term rule to work out the first five terms of each sequence.
Start from a first term of 5.

a add 3	**b** multiply by 3	**c** add 5	**d** multiply by 10
e add 9	**f** multiply by 5	**g** add 7	**h** multiply by 2
i add 11	**j** multiply by 4	**k** add 8	**l** add 105

3 Work out the next two terms in each sequence.
Describe the term-to-term rule you have used.

a 2, 4, 6, …, …	**b** 3, 6, 9, …, …	**c** 1, 10, 100, …, …	**d** 1, 2, 4, …, …
e 2, 10, 50, …, …	**f** 0, 7, 14, …, …	**g** 7, 10, 13, …, …	**h** 4, 9, 14, …, …
i 4, 8, 12, …, …	**j** 9, 18, 27, …, …	**k** 12, 24, 36, …, …	**l** 2, 6, 18, …, …

4 Work out the next two terms in each sequence.
Describe the term-to-term rule you have used.

a 50, 45, 40, 35, 30, …, …	**b** 35, 32, 29, 26, 23, …, …
c 64, 32, 16, 8, 4, …, …	**d** 10, 7, 4, 1, −2, …, …
e 9, 5, 1, −3, −7, …, …	**f** 6.5, 1.5, −3.5, −8.5, −13.5, …, …

5 Work out the first four terms of each sequence.

a Start with a first term of 3. To work out the next term, multiply by 3 and then add 7.

b Start with a first term of 5. To work out the next term, subtract 2 and then multiply by 4.

c Start with a first term of 32. To work out the next term, divide by 2 and then subtract 4.

PS **6** Work out two terms between each pair of numbers, to form a sequence. Describe the term-to-term rule you have used.

a 1, …, …, 8 **b** 3, …, …, 12 **c** 5, …, …, 20

d 4, …, …, 10 **e** 80, …, …, 10 **f** 2, …, …, 54

PS **7** Describe each sequence in words, as in question **5**.

a 7, 15, 31, 63, 127, … **b** 6, 4, 3, 2.5, 2.25, … **c** 3, 2, 0, −4, −12

PS **8** For each pair of numbers, find at least two different sequences and write down the next two terms. Describe the term-to-term rule you have used.

a 1, 4, …, … **b** 3, 9, …, … **c** 2, 6, …, …

d 3, 6, …, … **e** 4, 8, …, … **f** 5, 15, …, …

Investigation: Common terms

Write out the first 20 terms of the sequence that starts 1, 4, 7, 10, 13, … .

Write out the first 20 terms of the sequence that starts 2, 6, 10, 14, 18, … .

10 is a common term in both sequences.

A What are the other common terms?

B Is there a pattern to them?

C Do all linear sequences have common terms?

2.3 Working out missing terms

Learning objective

- To work out missing terms in a sequence

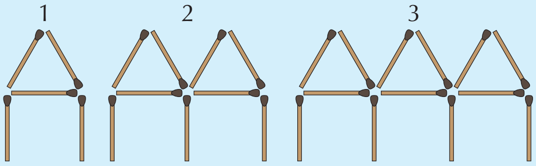
The terms in a sequence are called the first term, second term, third term, fourth term and so on. You need to know how to work out any term in a sequence.

Example 6

Work out the fourth term in the sequence of patterns made with matches, below.

Look at the number sequence shown by this pattern.

Term number 1 2 3

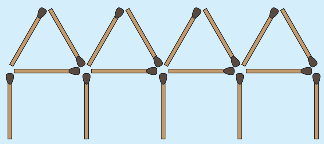


Number of matchsticks 5 9 13

(*Continued*)

You can see that four more matches are added each time.

You can draw the pattern for the fourth term and work out that it has 17 matches.



A better way of showing the patterns in the example above is to put the numbers into a table.

Term	Matches
1	5
2	9
3	13
4	17

You could continue this table to find the number of matches for the patterns for all the terms but there is an easier way.

For example, if you want to work out the number of matches in the pattern for the 10th term, you need to add nine more terms to the first term to get to the 10th term. The first term has 5 matches but the rest have 4 matches each so the number of matches you add to the first term is 36 (9×4).

Then add the number of matches in the first term, which is 5.

$36 + 5 = 41$

So there are 41 matches in the 10th term.

In Example 6, the tenth term would be:

$5 + (10 - 1) \times 4 = 5 + 36 = 41$ matches

Example 7

In the sequence 7, 10, 13, 16, …, what is the fifth term, what is the 25th term and what is the 50th term?

You first need to know what the term-to-term rule is.

You can see that you add 3 from one term to the next.

+ 3   + 3   + 3   + 3

7   10   13   16   …

1st   2nd   3rd   4th   5th … 50th

To get to the fifth term, you add 3 to the fourth term, which gives 19.

To get to the 25th term, you will have to add on 3 a total of 24 times ($25 - 1$) to the first term, 7.

This will give $7 + 3 \times 24 = 7 + 72 = 79$.

To get to the 50th term, you will have to add on 3 a total of 49 times ($50 - 1$) to the first term, 7.

This will give $7 + 3 \times 49 = 7 + 147 = 154$.

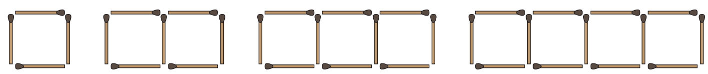
Exercise 2C

In questions **1** to **4**:

a draw the next pattern of matches

b work out the number of matches in the 10th pattern.

1



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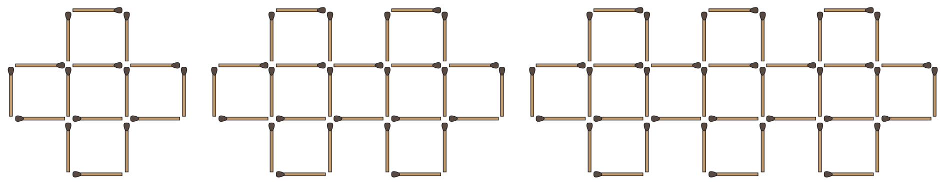

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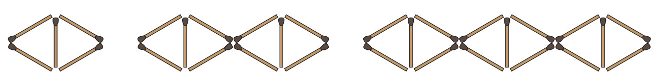

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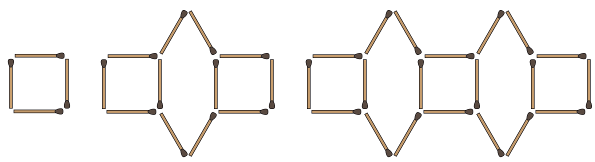


5 Work out the fifth and the 50th term in each sequence.

a 4, 6, 8, 10, … **b** 1, 6, 11, 16, … **c** 3, 10, 17, 24, …

d 5, 8, 11, 14, … **e** 1, 5, 9, 13, … **f** 2, 10, 18, 26, …

g 20, 30, 40, 50, … **h** 10, 19, 28, 37, … **i** 3, 9, 15, 21, …

6 Work out the fifth and the 50th term in each sequence.

a −2, −4, −6, −8, … **b** −2, −7, −12, −17, … **c** −4, −11, −18, −25, …

d 8, 5, 2, −1, … **e** −1, −6, −11, −16, … **f** −12, −20, −28, −36, …

g 9, 3, −3, −9, … **h** −37, −28, −19, −10, … **i** −21, −18, −15, −12, …

PS **7** In each sequence, work out the first term, then work out the 25th term.

In each case, you have been given the fourth, fifth and sixth terms.

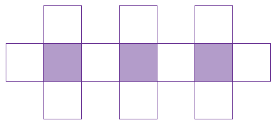
a …, …, …, 13, 15, 17, … **b** …, …, …, 18, 23, 28, …

c …, …, …, 19, 23, 27, … **d** …, …, …, 32, 41, 50, …

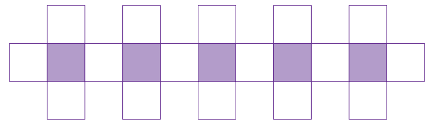
8 Work out the 80th term in the sequence with the term-to-term rule 'add 4' and a first term of 9.

PS **9** This is a sequence of patterns made from mauve and white squares.

The diagrams show the patterns for the third and fifth terms.



3rd pattern



5th pattern

a How many mauve squares are there in the pattern for the fourth term?

b How many white squares are there in the pattern for the fourth term?

c Draw the pattern for the first term.

PS **10** The second and third terms of a sequence are 2 and 4.

…., 2, 4, …, …

There are several different sequences that could have 2 and 4 as the second and third terms.

a Write down a rule for the way the sequence is building up and work out the first and fourth terms.

b Write down a different rule for the way the sequence is building up and work out the first and fourth terms.

MR **11** In a sequence, the 50th term is 254, the 51st is 259 and the 52nd is 264.

Work out the first term and the 100th term.

Problem solving: Missing terms

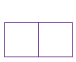
In each of the following sequences, work out the missing terms and the 50th term.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th	50th
Sequence A	…	…	…	…	17	19	21	23	…
Sequence B	…	9	…	19	…	29	…	39	…
Sequence C	…	…	16	23	…	37	44	…	…
Sequence D	…	…	25	…	45	…	…	75	…
Sequence E	…	5	…	11	…	…	20	…	…
Sequence F	…	…	12	…	…	18	…	22	…

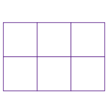
Challenge: Patterns in squares

In this sequence of patterns, how many squares will there be in:

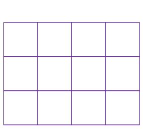
a the fifth pattern **b** the 20th pattern?



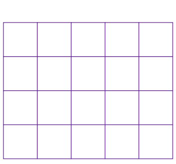
Pattern 1



Pattern 2



Pattern 3



Pattern 4

2.4 Other sequences

Learning objective

- To know and understand the sequences of numbers known as the square numbers and the triangular numbers

Key words

- powers
- square numbers
- triangular numbers

Square numbers and triangular numbers

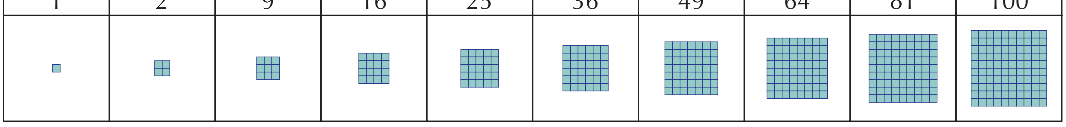
When you multiply a number by itself you are squaring the number. The result is a **square number**. For example:

- 4 is a square number and it is the square of 2 ($2 \times 2 = 4$)
- 9 is a square number and it is the square of 3 ($3 \times 3 = 9$) and so on.

Instead of writing 1×1, 2×2, 3×3 and so on you can write 1^2, 2^2, 3^2. The small 2 is called a **power** and the power of 2 is also called 'square'. You refer to the number it is attached to as 'squared', for example, you say 5^2 as 'five squared'.

This table shows the first 10 square numbers. You can see from the bottom line of the table why they are called square numbers.

1×1	2×2	3×3	4×4	5×5	6×6	7×7	8×8	9×9	10×10
1^2	2^2	3^2	4^2	5^2	6^2	7^2	8^2	9^2	10^2
1	2	9	16	25	36	49	64	81	100



This is the start of the sequence of square numbers. You need to learn the square numbers up to $15^2 = 225$.

Another well-known sequence is 1, 3, 6, 10, 15, 21, ….

This is called the sequence of **triangular numbers**. The sequence builds up like this.

First term: 1

Second term: add 2 to the first term ($1 + 2 = 3$)

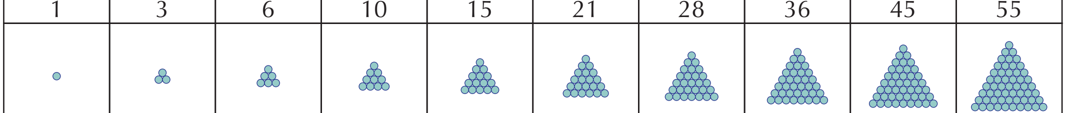
Third term: add 3 to the second term ($3 + 3 = 6$)

Fourth term: add 4 to the third term ($6 + 4 = 10$)

Fifth term: add 5 to the fourth term ($10 + 5 = 15$)

This table shows you the first 10 triangular numbers. You can see from the bottom line of the table why they are called triangular numbers.

1	$1 + 2$	$3 + 3$	$6 + 4$	$10 + 5$	$15 + 6$	$21 + 7$	$28 + 8$	$36 + 9$	$45 + 10$
1	3	6	10	15	21	28	36	45	55



The triangular numbers occur in ten-pin bowling and snooker.

Exercise 2D

1 Copy the first three rows of the table of square numbers, opposite, and continue it, up to 15 × 15.

2 Write each number below as the sum of two square numbers.

The first two have been done for you.

a 5 = 1 + 4	**b** 10 = 1 + 9	**c** 13 = …. + ……
d 17 = …. + ….	**e** 20 = ….. + …..	**f** 25 = ….. + ……
g 26 = …. + ….	**h** 29 = ….. + …..	**i** 34 = ….. + ……
j 37 = …. + ….	**k** 40 = ….. + …..	**l** 41 = ….. + ……
m 45 = …. + ….	**n** 50 = ….. + …..	**o** 52 = ….. + ……

3 Copy the first two rows of the table of triangular numbers, opposite, and continue it, to the 15th triangular number.

4 Write each number as the sum of two triangular numbers.

The first two have been done for you.

a 4 = 1 + 3	**b** 7 = 1 + 6	**c** 9 = … + …
d 11 = … + …	**e** 13 = … + …	**f** 16 = … + …
g 18 = … + …	**h** 21 = … + …	**i** 22 = … + …
j 24 = … + …	**k** 25 = … + …	**l** 27 = … + …
m 29 = … + …	**n** 31 = … + …	**o** 34 = … + …

5 Write down two numbers that are both square numbers and also triangular numbers.

6 Look at this pattern of numbers.

$1 = 1 = 1^2$

$1 + 3 = 4 = 2^2$

$1 + 3 + 5 = 9 = 3^2$

$1 + 3 + 5 + 7 = 16 = 4^2$

$1 + 3 + 5 + 7 + 9 = 25 = 5^2$

a Write down the next two lines of this number pattern.

b What is special about the numbers on the left-hand side?

PS

c Without working them out, write down the answers to these calculations.

i 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 = …

ii 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19 + 21 + 23 + 25 + 27 + 29 = …

7 Some sums of two square numbers are special because they give an answer that is also a square number. For example:

$3^2 + 4^2 = 9 + 16 = 25 = 5^2$

Which of these pairs of squares give a total that is also a square number?

a $5^2 + 12^2$	**b** $2^2 + 5^2$	**c** $6^2 + 8^2$
d $7^2 + 9^2$	**e** $7^2 + 24^2$	**f** $10^2 + 24^2$

8 **a** Add up the first 10 pairs of consecutive square numbers, starting with 1 + 4, 4 + 9, 9 + 16, …

b Is it possible to get an even number if you add any pair of consecutive square numbers? If not, explain why not.

c Work out the differences between the totals.

d Do they form a sequence? If so, describe the sequence.

9 Look at this pattern of numbers.

1 = 1
1 + 2 = 3
1 + 2 + 3 = 6
1 + 2 + 3 + 4 = 10
1 + 2 + 3 + 4 + 5 = 15

a Write down the next two lines of this number pattern.

b What is special about the numbers on the left-hand side?

c What is special about the numbers on the right-hand side?

PS

d Without working them out, write down the answers to these calculations.

i 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 = …

ii 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 + 13 + 14 + 15 = …

10 **a** Add up the first 10 pairs of consecutive triangular numbers, starting with 1 + 3, 3 + 6, 6 + 10, …

b What is special about the answers?

MR

11 This number pattern is called Pascal's triangle, after the famous French mathematician, Blaise Pascal.

Row											
1						1					
2					1		1				
3				1		2		1			
4			1		3		3		1		
5		1		4		6		4		1	
6	1		5		10		10		5		1

Each row starts and ends with 1.

Each of the other numbers is the sum of the two numbers above it, to the left and right.

For example, in row 5:

4 = 1 + 3   6 = 3 + 3   4 = 3 + 1

a Write down the next two rows of Pascal's triangle.

b Copy and complete the table to show the numbers and the total for each row.

Row	Sum of numbers in the row	Total
1	1	1
2	1 + 1	2
3	1 + 2 + 1	
4	1 + 3 + 3 + 1	
5		
6		
7		
8		

c Write down the totals as a sequence and then describe the rule for the sequence.

Challenge: Testing the rule

A Here is a rule.

8 × any triangular number + 1 gives a square number

Test this rule for the first triangular number, 1.

$8 \times 1 + 1 = 8 + 1$

$= 9$

$= 3^2$

Does the rule always work?

Use at least four other triangular numbers to test the rule.

B Now test this rule and say whether it always works.

9 × any triangular number + 1 is also a triangular number

C Now test this rule and say whether it always works.

The sum of the squares of any two consecutive triangular numbers is also a triangular number.

Investigation: Consecutive numbers and Fibonacci numbers

A Take any three consecutive numbers, for example, 7, 8, 9.

Multiply the first and last numbers.

$7 \times 9 = 63$

Square the middle number.

$8^2 = 64$

Subtract the first result from the second.

$64 - 63 = 1$

Try this with at least three different sets of consecutive numbers.

What happens in every case?

B This sequence is called the Fibonacci sequence. It helps to understand some of the patterns in nature.

1 1 2 3 5 8 13 21…

Can you work out what the next term is?

C Now try the process used in part **A** with any three consecutive Fibonacci numbers, for example, 5, 8, 13.

Repeat this with at least three different sets of consecutive Fibonacci numbers subtracting the smaller number from the larger each time.

What happens this time?

Ready to progress?

I can work out the output values for a function machine when I know the input values.
I can work out a sequence, given the first term and a term-to-term rule.
I can work out the term-to-term rule for a sequence.
I can work out the rule for a function machine, when I am given the input and output values.
I know the square numbers up to 15 × 15.

I can work out any term in a sequence, given the first term and the term-to-term rule.
I can recognise and work out the sequence of triangular numbers.
I can investigate the patterns and connections within the square and triangular numbers.
I can work out the inputs to a function machine when given the outputs and the rules.

I can work out the operations in a function machine that uses more than one rule, when I am given the input and output values.
I can investigate a given rule and reach a conclusion about whether it always works.

Review questions

1 **a** Write down the next two numbers in the sequence below.

72, 66, 60, 54, 48, 42, … , …

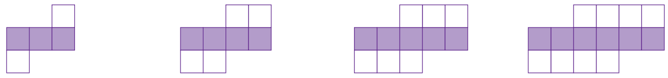
b Write down the next two numbers in the sequence below.

1, 3, 6, 10, 15, 21, … , …

c Continue both sequences.
Which two numbers appear in both sequences?

2 Here is a sequence of shapes made with mauve and white tiles.

Shape number 1 Shape number 2 Shape number 3 Shape number 4



The number of mauve tiles = the shape number + 2

The number of white tiles = 2 × the shape number

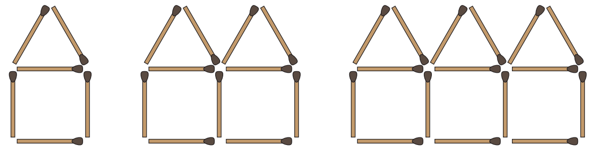
a How many mauve tiles will there be in shape number 7?

b How many white tiles will there be in shape number 25?

c How many tiles altogether will there be in shape number 10?

d Write down the missing numbers from this sentence.
The total number of tiles = ☐ × the shape number + ☐

3 This is a sequence of huts made from matches.



1 hut 6 matches | 2 huts 11 matches | 3 huts 16 matches

The rule for how many matches you need to make the huts is:

number of matches = 5 × number of huts + 1

a Use the rule to find how many matches you need to make 10 huts.

b I use 76 matches to make some huts. How many huts do I make?

c I have 100 matches. What is the largest number of huts I can make? How many matches will be left over?

FS

4 **a** Jeni saves £40 each week for a year.

How much will she have saved after:

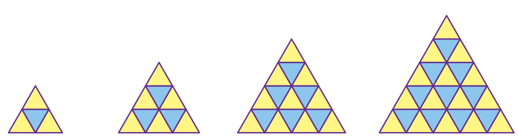
i 23 weeks **ii** a year (52 weeks)?

b Lucie saves £1 the first week, £3 the second week, £5 the third week and so on for a year. Copy and complete this table.

Week	1	2	3	4	5	6
Amount saved	£1	£3	£5	£7	£9	£11
Total amount saved	£1	£4				

Work out who will have more money at the end of the year. Show your working.

5 The patterns in this sequence are made from blue and yellow triangles.



Pattern 1 | Pattern 2 | Pattern 3 | Pattern 4

a Copy and complete this table.

Pattern (term) number	Number of blue triangles	Number of yellow triangles	Total number of triangles
1	1	3	4
2	3		
3			
4			

b Describe the sequence formed by the number of blue triangles.

c Describe the sequence formed by the number of yellow triangles.

d Describe the sequence formed by the total number of triangles.

e Work out the total number of triangles in Pattern 10.

Mathematical reasoning

Valencia Planetarium

This is the planetarium in the city of Valencia in Spain. Many of the features of the building are based on a repeating sequence.

This key explains the diagrams on these pages.

Ladders and grids are made from combinations of:

L links T links X links R rods

Each combination can be expressed as an algebraic rule.

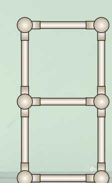
Examples

This ladder is a combination of L links, T links and R rods.

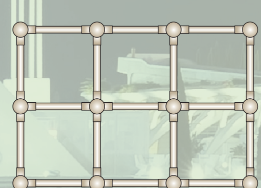
The combination can be written as the rule: $4L + 2T + 7R$

This grid is a combination of L links, T links, X links and R rods.

The combination can be written as the rule: $4L + 6T + 2X + 17R$



$4L + 2T + 7R$



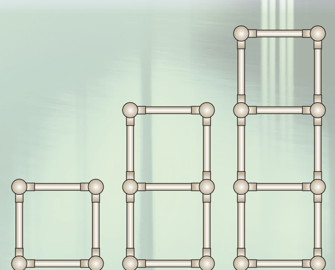
$4L + 6T + 2X + 17R$

1 Look at the ladders on the right.

a Use letters to write down a rule for each of them.

b Copy and complete this table.

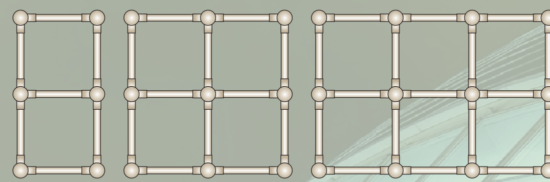
Ladder	L links	T links	R rods
1	4	0	4
2	4	2	7
3			
4			
5			



Ladder 1 Ladder 2 Ladder 3

c Use letters to write down a rule for the links and rods in ladder 10.

2 These rectangles are 2 squares deep.



i ii iii

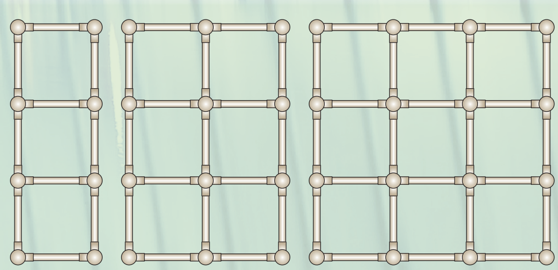
a Use letters to write down a rule for each of them.

b Copy and complete this table.

Rectangle	L links	T links	X links	R rods
2 by 1	4	2	0	7
2 by 2	4	4	1	12
2 by 3				

c Use letters to write down a rule for the links and rods in a 2 by 10 rectangle.

3 These rectangles are 3 squares deep.



i ii iii

a Use letters to write down a rule for each of them.

b Copy and complete this table.

Rectangle	L links	T links	X links	R rods
3 by 1	4	4	0	10
3 by 2	4	6	2	17
3 by 3				
3 by 4				
3 by 5				

c Use letters to write down a rule for the links and rods in a 3 by 10 rectangle.