Algebra and Functions

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$f(x) = 3x^4 - 8,\ x \in \mathbb{R}: x \geqslant 0$

Find $f^{-1}(x)$, identifying the domain and the range.

1

Algebra and Functions

Let $y = 3x^4 - 8$

Then $\sqrt[4]{\frac{y+8}{3}} = x$

So $f^{-1}(x) = \sqrt[4]{\frac{x+8}{3}}$

Domain $\{x \in \mathbb{R}: x \geqslant -8\}$

Range $\{y \in \mathbb{R}: y \geqslant 0\}$

1

Algebra and Functions

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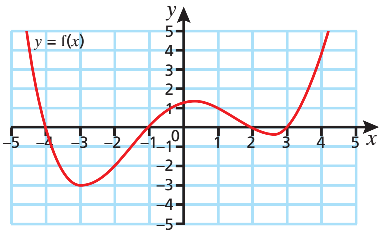
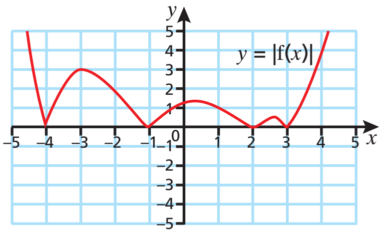
Given the graph $y = f(x)$, sketch the graph of $y = |f(x)|$.



2

Algebra and Functions

Reflect all points that are below the x-axis above the x-axis:



2

Algebra and Functions

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Decompose $\frac{3x+2}{(x-4)(x+3)}$ into partial fractions.

3

Algebra and Functions

$\frac{3x+2}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x+3}$

$3x + 2 = A(x + 3) + B(x - 4)$

$3x + 2 = x(A + B) + 3A - 4B$

Equating coefficients:

$3 = A + B \quad 2 = 3A - 4B$

Using simultaneous equations:

$A = 2,\ B = 1$

$\therefore \frac{3x+2}{(x-4)(x+3)} = \frac{2}{x-4} + \frac{1}{x+3}$

3

Coordinate Geometry

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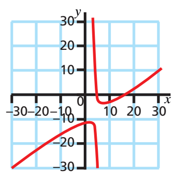
The curve C has parametric equations $x = 3t + 4$, $y = 2t + \frac{1}{t} - 8,\ t \neq 0$.

Show that the Cartesian equation of the curve C can be written in the form $y = \frac{2x^2 - ax + b}{3(x-4)},\ x \neq 4$ and state the values of a and b.



4

Coordinate Geometry

Substitute $t = \frac{x-4}{3}$ into $y = 2t + \frac{1}{t} - 8$

$y = \frac{2(x-4)}{3} + \frac{3}{x-4} - 8$

Rearrange to $y = \frac{2x^2 - 40x + 137}{3(x-4)}$

$a = -40,\ b = 137$

4

Coordinate Geometry

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A ball is kicked from the ground with an initial speed of A ms^{-1} at an angle of $\theta°$. The path of the ball can be modelled using parametric equations $x = Bt$, $y = -4.9t^2 + Ct$.

Outline the steps required to find the time taken and the horizontal distance travelled by the ball when it hits the ground.

5

Coordinate Geometry

To find the time taken for the ball to reach the ground, set $y = 0$ and solve for t.

To find the distance, substitute the value of t into $x = Bt$.

5

Sequences and Series

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Write the series $\frac{1}{2}+\frac{3}{8}+\frac{3}{10}+\frac{1}{4}+\cdots\frac{1}{10}$ in the form $\sum_{1}^{k} u_n$

6

Sequences and Series

The nth term formula is $u_n = \frac{3}{2n+4}$

The term $\frac{1}{10}$ is the 13th term in the sequence.

$\sum_{1}^{13} \frac{3}{2n+4}$

6

Sequences and Series

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An arithmetic sequence has fifth term –7 and tenth term –17.

Find the nth term rule for the sequence and list the first five terms.

7

Sequences and Series

$u_n = a + d(n-1)$

$u_5 \Rightarrow -7 = a + 4d$

$u_{10} \Rightarrow -17 = a + 9d$

Using simultaneous equations, $d = -2$, $a = 1$

$u_n = 1 + -2(n-1) = 3 - 2n$

7

Sequences and Series

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A geometric sequence is defined as $u_n = 100\times\left(\frac{1}{2}\right)^{n-1}$.

Find the first term that is less than 1.

8

Sequences and Series

$u_n = 100\times\left(\frac{1}{2}\right)^{n-1}$

$100\times\left(\frac{1}{2}\right)^{n-1} < 1$

$\log_{10}\left(\left(\frac{1}{2}\right)^{n-1}\right) < \log_{10}\left(\frac{1}{100}\right)$

$(n-1) > \frac{\log_{10}\left(\frac{1}{100}\right)}{\log_{10}\left(\frac{1}{2}\right)}$

$n > 7.64 \Rightarrow n = 8$

8

Sequences and Series

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Outline the steps required to find the first four terms of the expansion of $\sqrt[3]{8-x}$.

9

Sequences and Series

Rearrange $\sqrt[3]{8-x} = (8-x)^{\frac{1}{3}}$

Factor out the 8,

$(8-x)^{\frac{1}{3}} = \sqrt[3]{8}\left(1-\frac{1}{8}x\right)^{\frac{1}{3}} = 2\left(1-\frac{1}{8}x\right)^{\frac{1}{3}}$

Use the expansion of $(1+bx)^n$ to find the first four terms of the expansion of $\left(1-\frac{1}{8}x\right)^{\frac{1}{3}}$ (substitute $b=-\frac{1}{8}$ and $n=\frac{1}{3}$), then multiply through by 2.

9

Trigonometry

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Convert $\frac{5\pi}{3}$ radians to degrees.

10

Trigonometry

$\frac{5\pi}{3}$ radians $= \frac{5}{3}\times 180° = 300°$

10

Trigonometry

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What is the magnitude of the vector 2**i** + 2**j** + 2**k** in simplified surd form?

11

Trigonometry

$\sqrt{2^2+2^2+2^2}=\sqrt{12}=2\sqrt{3}$

11

Trigonometry

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What is the exact value of $\sec\frac{2\pi}{3}$?

12

Trigonometry

$\sec\frac{2\pi}{3}=\frac{1}{\cos\frac{2\pi}{3}}=\frac{1}{-0.5}=-2$

12

Trigonometry

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What are the solutions to $\cot x = 1$ in the range $0 \leqslant x \leqslant 2\pi$?

13

Trigonometry

$\cot x = 1 \Rightarrow \tan x = 1$

$x=\frac{\pi}{4}$ or $\pi+\frac{\pi}{4}=\frac{5\pi}{4}$

13

Trigonometry

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Express $\sin^2 x\cos^2 x$ in terms of $\sin 2x$.

14

Trigonometry

$\sin 2x = 2\sin x\cos x$

$\therefore \sin x\cos x = \frac{1}{2}\sin 2x$

$\therefore \sin^2 x\cos^2 x = \frac{1}{4}\sin^2 2x$

14

Trigonometry

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State the amplitude of the graph of $y = 6\cos x - 6\sin x$.

15

Trigonometry

$\sqrt{6^2+(-6)^2}$

$=\sqrt{72}$

$=6\sqrt{2}$

15

Differentiation

Write down $\frac{dy}{dx}$ when:

a) $y = \ln x$

b) $y = e^x$

c) $y = \cos x$

d) $y = \sin x$

e) $y = \tan x$

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16

Differentiation

a) $\frac{dy}{dx} = \frac{1}{x}$

b) $\frac{dy}{dx} = e^x$

c) $\frac{dy}{dx} = -\sin x$

d) $\frac{dy}{dx} = \cos x$

e) $\frac{dy}{dx} = \sec^2 x$

16

Differentiation

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Use implicit differentiation to find the derivative of $x^4 + 3xy - 2y^2 = 12$.

17

Differentiation

Differentiate each term with respect to x.

By the product rule,

$$\frac{d}{dx}(3xy) = 3\left(x\frac{dy}{dx} + y\right) = 3x\frac{dy}{dx} + 3y$$

By the chain rule,

$$\frac{d}{dx}(-2y^2) = -2\left(2y\frac{dy}{dx}\right) = -4y\frac{dy}{dx}$$

$$4x^3 + 3y + 3x\frac{dy}{dx} - 4y\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = -\frac{4x^3 + 3y}{4y - 3x}$$

17

Differentiation

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Find $\frac{dy}{dx}$ for the parametric equations $x = \sin(t)$, $y = 2 - \cos(3t)$.

18

Differentiation

$$\frac{dx}{dt} = \cos(t)$$

$$\frac{dy}{dt} = \frac{d}{dt}(2) - \frac{d}{dt}(\cos(3t))$$

$$= 0 - \left(-\frac{d}{dt}(3t)\right)\sin(3t)$$

$$= 3\sin(3t)$$

$$\therefore \frac{dy}{dx} = \frac{3\sin(3t)}{\cos(t)}$$

18

Integration

Find the area between the curve with the equation $y = -2x^2 + 3$ and the line $y = 1$.

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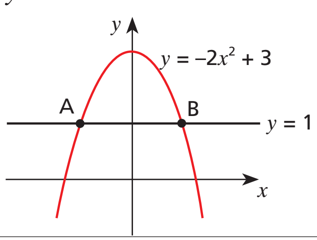

19

Integration

$$-2x^2 + 3 = 1 \Rightarrow x = -1,\ x = 1$$

$$\int_{-1}^{1} (-2x^2 + 3) - 1\, dx = \int_{-1}^{1} -2x^2 + 2\, dx$$

$$= \left[-\frac{2x^3}{3} + 2x\right]_{-1}^{1}$$

$$= \left(-\frac{2 \times 1^3}{3} + 2 \times 1\right) - \left(-\frac{2 \times (-1)^3}{3} + 2 \times (-1)\right) = \frac{8}{3}$$

19

Integration

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Find $\int x \cos x \, dx$.

20

Integration

Let $u = x$ and $\frac{dv}{dx} = \cos x$

$\frac{du}{dx} = 1$ and $v = \sin x$

$\int x \cos x \, dx = x \sin x - \int \sin x \, dx$

$= x \sin x + \cos x + c$

20

Integration

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Find $\int \frac{3x-1}{(x+1)(x-3)}\,dx$

21

Integration

Using partial fractions,

$\int \frac{3x-1}{(x+1)(x-3)}\,dx = \int \frac{1}{x+1} + \frac{2}{x-3}\,dx$

$= \int \frac{1}{x+1}\,dx + 2\int \frac{1}{x-3}\,dx$

$= \ln(x+1) + 2(\ln(x-3)) + c$

21

Integration

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Find the general solution to the differential equation $\frac{dy}{dx} = 2xe^{-y}$

22

Integration

$\frac{dy}{dx} = 2xe^{-y}$

$e^{y}\frac{dy}{dx} = 2x$

$\int e^{y}\frac{dy}{dx}\,dx = \int 2x\,dx$

$e^{y} = x^2 + c$

$y = \ln(x^2 + c)$

22

Numerical Methods

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Considering the function $f(x) = x - 2 - \sin 2x$, explain why a solution to $f(x) = 0$ lies between $x = 0$ and $x = \pi$.

23

Numerical Methods

$f(0) < 0$ and $f(\pi) > 0$, so there is a sign change in the interval $(0, \pi)$, and $f(x)$ is continuous in this interval.

23

Numerical Methods

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Explain how the Newton-Raphson process could be used to find a rational approximation to $\sqrt{2}$.

24

Numerical Methods

Consider the function $f(x) = x^2 - 2$ and use the iteration $x_{n+1} = x_n - \left(\frac{x^2 - 2}{2x}\right)$ as many times as required.

24

Proof

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Prove by contradiction that $2^{22} - 1$ is not a prime number.

25

Proof

If $2^{22} - 1$ was prime, then no factors would exist, except 1 and $2^{22} - 1$.

Note that $2^{22} - 1 = (2^{11} + 1)(2^{11} - 1)$ and neither of these factors are 1 or $2^{22} - 1$.

Therefore $2^{22} - 1$ cannot be prime.

25

Probability

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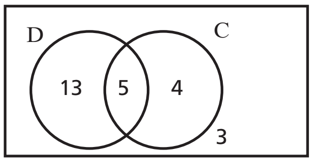
In a survey of 25 people, 18 people said they owned dogs, 9 people owned cats and 3 people owned neither a cat nor a dog. Find the probability that someone owned a cat given that they owned a dog.

26

Probability

Let D be the event that someone owns a dog and C be the event that someone owns a cat.



$P(C|D) = \frac{5}{13}$

26

Probability

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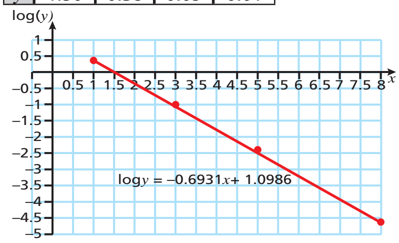
The graph shows the relationship between log(y) and x of the data given.

x	1	3	5	8
y	1.50	0.38	0.09	0.01



Show that the data follows the model $y = ka^x$ and estimate the values of k and a.

27

Probability

The graph of log(y) against x is a straight line.

$y = ka^x \Rightarrow \log y = x \log a + \log k$

$\ln k = 1.0986 \Rightarrow k = e^{1.0986} \approx 3$

$\ln a = -0.6931 \Rightarrow a = e^{1.0986} \approx 0.5$

$\therefore\ y = 3 \times 0.5^x$

27

Statistical Distributions

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A random variable $Y \sim N(12, 4)$.

Find $P(13 < Y < 14)$.

28

Statistical Distributions

$Z_1 = \frac{13-12}{2} = 0.5,\ Z_2 = \frac{14-12}{2} = 1$

$P(13 < Y < 14) = P(0.5 < Z < 1)$

$= P(Z < 1) - P(Z < 0.5)$

$= 0.841 - 0.691 = 0.15$

28

Statistical Distributions

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A random variable $X \sim B(90, 0.4)$.

Use a normal approximation to estimate $P(X > 40)$.

29

Statistical Distributions

$np = 90 \times 0.4$

$= 36,\ np(1-p)$

$= 36 \times 0.6$

$= 21.6$

$X \sim B(90, 0.4) \Rightarrow Y \sim N(36, 21.6)$

$P(X \geqslant 40) \approx P(Y > 39.5) = 0.227$

29

Statistical Hypothesis Testing

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A random sample is taken from a population with mean 160 and variance 36. A sample of 25 is taken from the population and the sample mean is calculated to be 162. Test, at the 5% significance level, whether or not there is enough evidence to support that the mean of the population is higher than 160.

30

Statistical Hypothesis Testing

$H_0: \mu = 160,\ H_1: \mu > 160$

$Z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{162 - 160}{\frac{6}{\sqrt{25}}} = 1.67$

The critical value is 1.6449 (one-tailed test)

1.67 > 1.6449; there is sufficient evidence to reject the null hypothesis.

30

Kinematics

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After t seconds, a particle P has position vector $\mathbf{r} = t^2\mathbf{i} - t\mathbf{j}$.

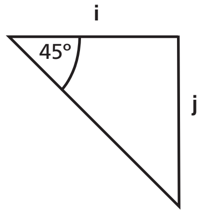
State its direction of motion after 0.5 seconds.

31

Kinematics

$v = \frac{\mathrm{d}r}{\mathrm{d}t} = 2t\mathbf{i} - \mathbf{j}$

$t = 0.5 \Rightarrow v = \mathbf{i} - \mathbf{j}$



Direction south east (bearing 135°)

31

Kinematics

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A particle P is projected at an angle of 30° to the horizontal, and another particle, Q, is projected at an angle of 60° to the horizontal, with the same speed.

They both travel the same horizontal distance. True or false?

32

Kinematics

True. Horizontal range is $\frac{u^2 \sin 2\alpha}{g}$ and $\sin 60° = \sin 120°$

32

Forces

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A 10 kg mass rests in equilibrium on a plane inclined at 60° to the horizontal.

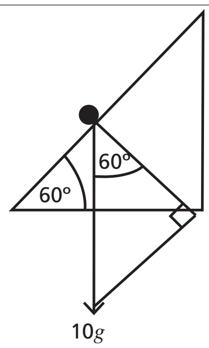
What is the component of the weight acting down the plane?

33

Forces

Weight $10g$ vertically resolved parallel to plane

$10g \sin 60° = 5\sqrt{3}g$ N



33

Forces

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A 2 kg mass is released from rest and slides down a smooth plane inclined at 30° to the horizontal. Find how far it travels in two seconds.

34

Forces

Force parallel to plane

$2g \sin 30 = ma$

$\Rightarrow a = g \sin 30 = \frac{9.8}{2}$

$s = ut + \frac{1}{2}at^2 = 0 \times 2 + \frac{1}{2} \times \frac{9.8}{2} \times 2^2$

$= 9.8$m

34

Moments

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A ladder rests against a vertical wall, with the foot of the ladder resting on horizontal ground. Explain, using mechanics, why the ground **must** be rough in order for the ladder to remain in equilibrium.

35

Moments

There is a horizontal reaction force from the wall on the ladder, therefore there must be a horizontal force in the opposite direction to balance it. This can only be due to friction from the ground.

35