Chapter 13 Coordinate geometry

Exercise 13A

1 **a** **i** $y = 2$ **ii** $x = –2$ **iii** $y = –3$ **b** B **c** C

2 **a** (1, 2) **b** **i** (–2, –1) **ii** (0, –2) **iii** (3, 1) **c** R

3 **a** $(3\frac{1}{2}, -\frac{1}{2})$ and $(-2\frac{1}{2}, -\frac{1}{2})$ **b** $y = -\frac{1}{2}$

Exercise 13B

1 Extreme points are (0, –3), (10, 2)

2 Extreme points are (–6, 2), (6, 6)

3 **a** Extreme points are (0, –2), (5, 13) and (0, 1), (5, 11) **b** (3, 7)

4 **a** Extreme points are (0, –5), (5, 15) and (0, 3), (5, 13) **b** (4, 11)

5 **a** Extreme points are (0, –1), (12, 3) and (0, –2), (12, 4) **b** (6, 1)

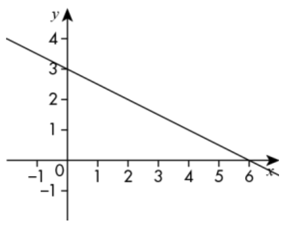
6 **a** Extreme points are (0, 1), (4, 13) and (0, –2), (4, 10)

b Do not cross because they are parallel

Exercise 13C

1 **a** At $x = 6$ **b** At $y = 3$

c



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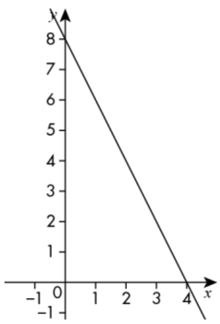

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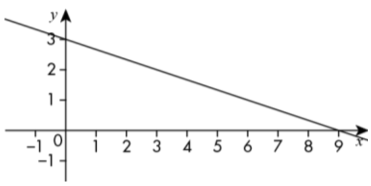

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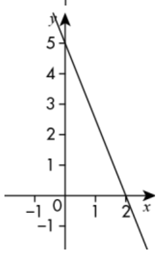

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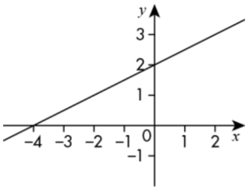
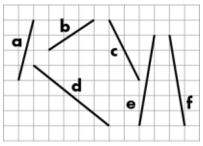


Exercise 13D

1 **a** $\frac{1}{3}$ **b** –3 **c** $-\frac{1}{4}$

2



3 **a** 1

b –1 They are perpendicular and symmetrical about the axes.

4 **a** 0.5 **b** 0.4 **c** 0.2 **d** 0.1 **e** 0

5 **a** $1\frac{2}{3}$ **b** 2 **c** $3\frac{1}{3}$ **d** 10 **e** ∞

6 **a** 4 **b** $\frac{1}{4}$ **c** $2\frac{1}{2}$ **d** 10 **e** –2

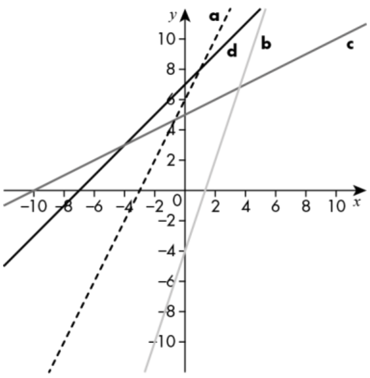
f $-\frac{1}{5}$ **g** 0 **h** $-1\frac{1}{2}$

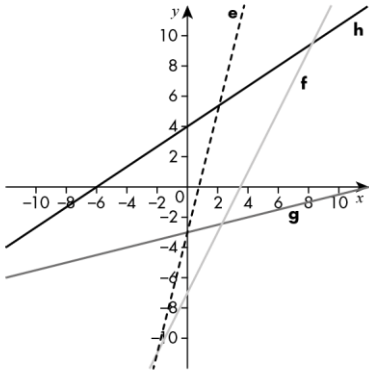
7 **a** 4 **b** $\frac{1}{4}$ **c** 2.5 **d** 10 **e** –2

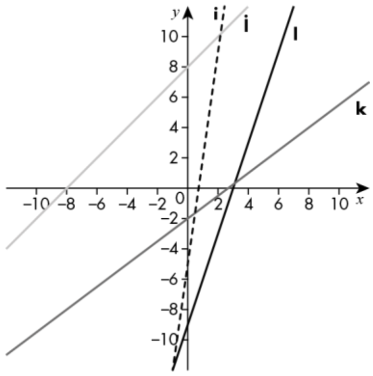
f $-\frac{1}{5}$ **g** 0 **h** –1.5

Exercise 13E

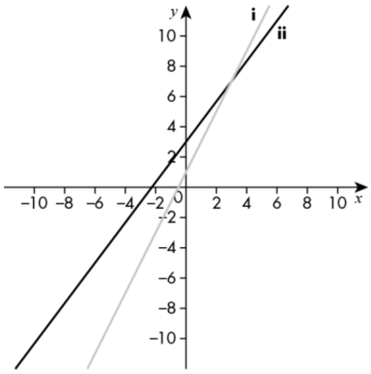
1





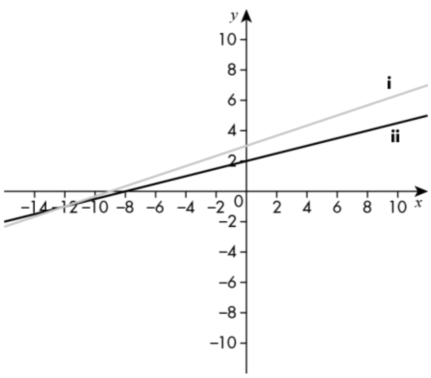


2 **a**



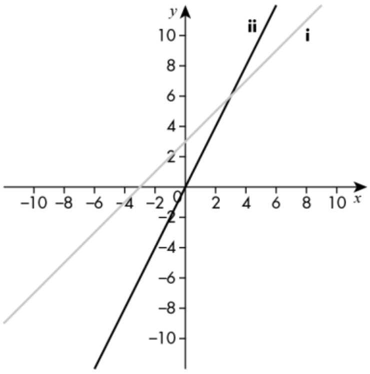
b (2, 7)

3 **a**



b (–12, –1)

4 **a**



b (3, 6)

5 **a** They have the same gradient (3).

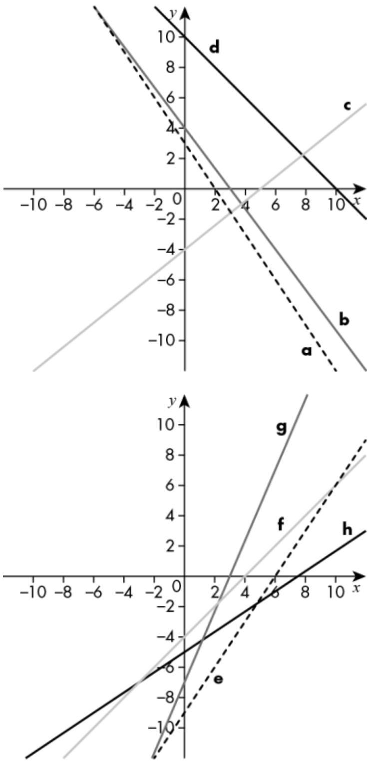
b They intercept the y-axis at the same point (0, –2).

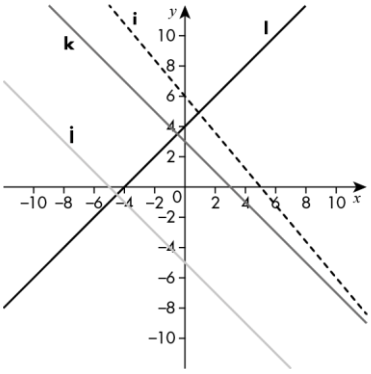
c (–1, –4)

6 **a** –2 **b** $\frac{1}{2}$ **c** 90° **d** Negative reciprocal **e** $-\frac{1}{3}$

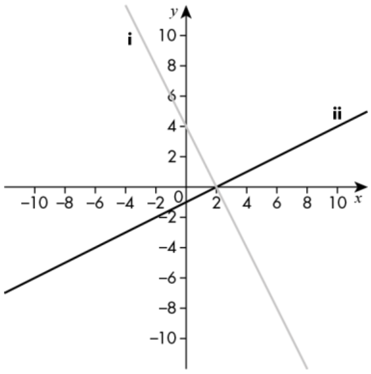
Exercise 13F

1



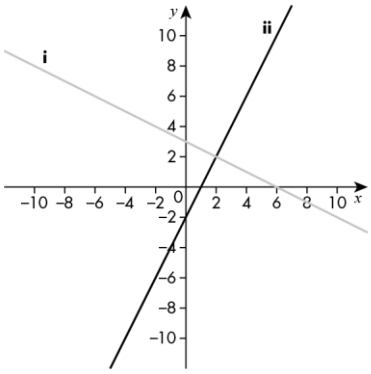


2 **a**



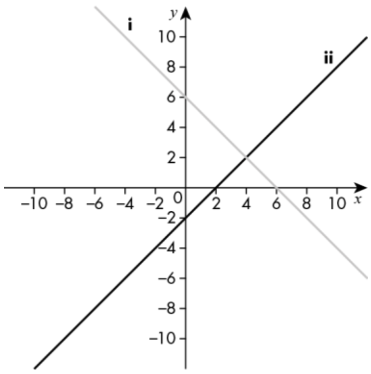
b (2, 0)

3 **a**



b (2, 2)

4 **a**



b (4, 2)

5 **a** Intersect at (6, 0) **b** Intersect at (0, –3)

c Parallel **d** $-2x + 9y = 18$

6 **a** **i** $x = 3$ **ii** $x - y = 4$ **iii** $y = -3$

iv $x + y = -4$ **v** $x = -3$ **vi** $y = x + 4$

b **i** -3 **ii** $\frac{1}{3}$ **iii** $-\frac{1}{3}$

Exercise 13G

1 **a** $y = \frac{4}{3}x - 2$ or $3y = 4x - 6$ **b** $y = x + 1$ **c** $y = 2x - 3$

2 **a** **i** $y = 2x + 1$, $y = -2x + 1$

ii Reflection in y-axis (and $y = 1$)

iii Different sign

b **i** $5y = 2x - 5$, $5y = -2x - 5$

ii Reflection in y-axis (and $y = -1$)

iii Different sign

c **i** $y = x + 1$, $y = -x + 1$

ii Reflection in y-axis (and $y = 1$)

iii Different sign

3 **a** x-coordinates go from $2 \rightarrow 1 \rightarrow 0$ and y-coordinates go from $5 \rightarrow 3 \rightarrow 1$.

b x-step between the points is 1 and y-step is 2.

c $y = 3x + 2$

4 **a** $y = -2x + 1$ **b** $2y = -x$ **c** $y = -x + 1$ **d** $5y = -2x - 5$

e $y = -\frac{3}{2}x - 3$ or $2y = -3x - 6$

5 **a** **i** $2y = -x + 1$, $y = -2x + 1$

ii Reflection in $x = y$

iii Reciprocal of each other

b **i** $2y = 5x + 5$, $5y = 2x - 5$

ii Reflection in $x = y$

iii Reciprocal of each other

c **i** $y = 2$, $x = 2$

ii Reflection in $x = y$

iii Reciprocal of each other (reciprocal of zero is infinity)

6 **a** 3 **b** $\frac{1}{2}$ **c** 4 **d** -1 **e** $-\frac{1}{2}$ **f** $\frac{2}{3}$

7 **a** $y = 2x - 3$ **b** $y = \frac{1}{2}x + 4$ **c** $y = 4x - 2$ **d** $y = -3x + 8$

8 **a** (5, 3) **b** (4, 5) **c** (3, 2) **d** (3, 3)

e (1, 3.5) **f** (–0.5, 0)

9 **a** student's graph **b** $y = 0.5x + 6.5$

c (–1,3) **d** $y = -x + 8$

10 **a** $y = \frac{3}{4}x + \frac{1}{2}$ **b** $y = \frac{1}{2}x + 3\frac{1}{2}$

c $y = -2x + 7$ **d** $y = x + 2$

Exercise 13H

1 **a** Line A does not pass through (0, 1).

b Line C is perpendicular to the other two.

c (i)

2 **a** $-\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $-\frac{1}{5}$ **d** 1 **e** –2 **f** –4

g 3 **h** $\frac{3}{2}$ **i** $-\frac{2}{3}$ **j** $-\frac{1}{10}$ **k** $\frac{1}{6}$ **l** $-\frac{3}{4}$

3 **a** $y = -\frac{1}{2}x - 1$ **b** $y = \frac{1}{3}x + 1$ **c** $y = -x + 2$ **d** $y = x + 2$

e $y = -2x + 3$ **f** $y = -4x - 3$ **g** $y = 3x$ **h** $y = 1.5x - 5$

4 **a** $y = 4x + 1$ **b** $y = \frac{1}{2}x - 2$ **c** $y = -x + 3$

5 **a** $y = -\frac{1}{3}x - 1$ **b** $y = 3x + 5$ **c** $y = -x + 1$

6 **a** $y = -x + 14$ **b** $y = x + 2$

7 $y = 2x + 6$

8 $y = -\frac{1}{4}x + 2$

9 **a** (0, –20) **b** $y = -\frac{1}{5}x + 6$ **c** $y = 5x - 20$

10 $y = -\frac{1}{2}x + 5$

11 **a** $y = 3x - 6$

b Bisector of AB is $y = -2x + 9$, bisector of AC is $y = \frac{1}{2}x + \frac{3}{2}$, solving these equations shows the lines intersect at (3, 3).

c (3, 3) lies on $y = 3x - 6$ because $(3 \times 3) - 6 = 3$

12 $y = 2x + \frac{9}{2}$

Exercise 13I

1 **a** 5 units **b** 13 units **c** 10 units

d 15 units **e** 17 units **f** 25 units

2 **a** $3\sqrt{2}$ units **b** $2\sqrt{5}$ units **c** $2\sqrt{13}$ units

d $5\sqrt{2}$ units **e** $4\sqrt{5}$ units **f** $2\sqrt{17}$ units

3 **a** (2, 3) **b** (3.5, 2) **c** (–3, 3)

d (5, 2) **e** (0, 0) **f** (2, 3)

4 C(7, –12)

5 F(–11, 11)

6 C(0, 15)

7 Z(15, –5)

8 Q(13, 15)

9 Gradient of AB = gradient DC = 3. Gradient BC = gradient AD = $-\frac{3}{2}$.

So ABCD is a parallelogram as AB ≠ BC.

10 19.5 square units

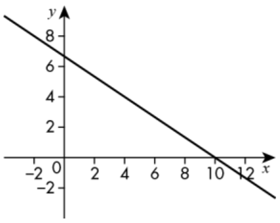
Exam-style questions

1 12.5

2 (0, –20)

3 $x + 2y = -1$

4



5 $y = 3x - 2$

6 AB and DC both have gradient –4. DA and CB both have gradient $\frac{1}{4}$ so they are parallel and they are perpendicular to the other two sides because $-4 \times \frac{1}{4} = -1$.

7 (4.2, 2.8)

8 **a** CA = CB = $\sqrt{40}$ **b** $x + y = 3$ **c** 16

Chapter 14 The equation of a circle

Exercise 14A

1 **a** $x^2 + y^2 = 49$ **b** $x^2 + y^2 = 81$ **c** $x^2 + y^2 = 36$

d $x^2 + y^2 = 2$ **e** $x^2 + y^2 = 5$ **f** $x^2 + y^2 = 12$

2 **a** 10 **b** 12 **c** 15

d $\sqrt{5}$ **e** $\sqrt{28}$ or $2\sqrt{7}$ **f** $3\sqrt{5}$

3 **a** (4, 2) and (–2, –4) **b** $\sqrt{72}$ or $6\sqrt{2}$

4 10

5 **a** (4, –3), (–3, 4) **b** (0.8, 1.8), (–1.8, –0.8) **c** (2.6, 1.6), (–1.6, –2.6)

Exercise 14B

1 **a** $(x - 1)^2 + (y - 2)^2 = 9$ **b** $(x + 1)^2 + (y - 3)^2 = 16$

c $(x - 4)^2 + (y + 2)^2 = 25$ **d** $(x - 3)^2 + (y - 3)^2 = 5$

e $(x - 8)^2 + (y + 3)^2 = 18$ **f** $(x - 4)^2 + (y + 4)^2 = 12$

2 **a** Centre (1, 4), radius 5 units **b** Centre (–2, 3), radius 3 units

c Centre (5, 5), radius 6 units **d** Centre (1, –3), radius $\sqrt{6}$ units

e Centre (2, –3), radius $\sqrt{20}$ units or $2\sqrt{5}$ units

f Centre (1, 4), radius $\sqrt{24}$ units or $2\sqrt{6}$ units

3 $(x - 3)^2 + (y + 1)^2 = 13$

4 **a** (5, -1) and (4, 0) **b** AB = $\sqrt{2}$ units

5 PQ = $9\sqrt{2}$ units

6 $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$

7 $(x + 1)^2 + (y - 2)^2 = 25$

Exam-style questions

1 **a** (0, 0) **b** 5 **c** At (4, 3) and (–4, –3)

2 **a** $(x - 6)^2 + (y + 4)^2 = 52$

b It is inside. The distance from the origin to (5, 5) is $\sqrt{50}$ and this is less than the radius of $\sqrt{52}$.

3 9

4 **a** $(x - 3)^2 + y^2 = 9$ **b** $y = 0.5x - 3$ **c** (1.2, –2.4)

5 **a** $(x - 8)^2 + (y + 7)^2 = 50$

b The radius is $\sqrt{50}$ which is between 7 and 8. The centre is 8 units from the *x*-axis so the circle does not cross that. The centre is 7 units from the *y*-axis so the circle does cross that.

6 $(x - 3)^2 + (y - 5)^2 = 34$

Chapter 15 Indices

Exercise 15A

1 **a** 2^4 **b** 3^5 **c** 7^2 **d** 5^3 **e** 10^7

f 6^4 **g** 4^1 **h** 1^7 **i** 0.5^4 **j** 100^3

2 **a** $3 \times 3 \times 3 \times 3$

b $9 \times 9 \times 9$

c 6×6

d $10 \times 10 \times 10 \times 10 \times 10$

e $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

f 8

g $0.1 \times 0.1 \times 0.1$

h 2.5×2.5

i $0.7 \times 0.7 \times 0.7$

j 1000×1000

3 **a** 16 **b** 243 **c** 49 **d** 125 **e** 10000000 **f** 1296

g 4 **h** 1 **i** 0.0625 **j** 1000000

4 **a** 81 **b** 729 **c** 36 **d** 100000 **e** 1024 **f** 8

g 0.001 **h** 6.25 **i** 0.343 **j** 1000000

5 125 m^3

6 **b** 10^2 **c** 2^3 **d** 5^2

7 **a** 1 **b** 4 **c** 1 **d** 1 **e** 1

8 Any power of 1 is equal to 1.

9 10^6

10 **a** $2x$ **b** xy **c** $\frac{x}{y}$ **d** $32x$ **e** $16xy$ **f** $x\sqrt{2}$

Exercise 15B

1 **a** 5^4 **b** 5^3 **c** 5^6 **d** 5^9 **e** 5^5

f 6^3 **g** $6^4 \div 6$ **h** 6^2 **i** 6^3 **j** 6^1

2 **a** a^3 **b** a^5 **c** a^7 **d** a^4 **e** a^2 **f** a^1

3 **a** Any two values such that $x + y = 10$

b Any two values such that $x - y = 10$

4 **a** 4^6 **b** 4^{15} **c** 4^6

d 4^6 **e** 4^8 **f** 1

5 **a** $\frac{1}{a}$ **b** $\frac{1}{a}^{2}$ **c** a^3 **d** $\frac{1}{a}^{3}$ **e** a

6 **a** 7 **b** 4

7 **a** a^4 **b** a^8 **c** a^3 **d** a^5

8 **a** $3a$ **b** $4a^3$ **c** $3a^4$

d $\frac{6}{a}$ **e** $4a^3$ **f** $\frac{5}{a^4}$

9 12 ($a = 2$, $b = 1$, $c = 3$)

Exercise 15C

1 **a** $\frac{1}{5^3}$ **b** $\frac{1}{6}$ **c** $\frac{1}{10^5}$ **d** $\frac{1}{3^2}$ **e** $\frac{1}{8^2}$ **f** $\frac{1}{9}$ **g** $\frac{1}{w^2}$

h $\frac{1}{t}$ **i** $\frac{1}{x^m}$ **j** $\frac{4}{m^3}$

2 **a** 3^{-2} **b** 5^{-1} **c** 10^{-3} **d** m^{-1} **e** t^{-n}

3 **a** **i** 2^4 **ii** 2^{-1} **iii** 2^{-4} **iv** -2^3

b **i** 10^3 **ii** 10^{-1} **iii** 10^{-2} **iv** 10^6

c **i** 5^3 **ii** 5^{-1} **iii** 5^{-2} **iv** 5^0

d **i** 3^2 **ii** 3^{-3} **iii** 3^0 **iv** -3^5

4 **a** $\frac{5}{x^3}$ **b** $\frac{6}{t}$ **c** $\frac{7}{m^2}$ **d** $\frac{4}{q^4}$ **e** $\frac{10}{y^5}$ **f** $\frac{1}{2x^3}$

g $\frac{1}{2^m}$ **h** $\frac{3}{4r^4}$ **i** $\frac{4}{5y^3}$ **j** $\frac{7}{8x^5}$

5 **a** $7x^{-3}$ **b** $10p^{-1}$ **c** $5t^{-2}$ **d** $8m^{-5}$ **e** $3y^{-1}$

6 **a** **i** 25 **ii** $\frac{1}{125}$ **iii** $\frac{4}{5}$

b **i** 64 **ii** $\frac{1}{16}$ **iii** $\frac{5}{256}$

c **i** 8 **ii** $\frac{1}{32}$ **iii** $\frac{9}{2}$ or $4\frac{1}{2}$

d **i** 1000000 **ii** $\frac{1}{1000}$ **iii** $\frac{1}{4}$

7 **a** a^{-7} **b** a^2 **c** a^4 **d** a^{-5} **e** a^{-6} **f** a^6

Exercise 15D

1 **a** 5 **b** 10 **c** 8 **d** 9 **e** 25

f 3 **g** 4 **h** 10 **i** 5 **j** 8

k 12 **l** 20 **m** 5 **n** 3 **o** 10

p 3 **q** 2 **r** 2 **s** 6 **t** 6

u $\frac{1}{4}$ **v** $\frac{1}{2}$ **w** $\frac{1}{3}$ **x** $\frac{1}{5}$ **y** $\frac{1}{10}$

2 **a** $\frac{5}{6}$ **b** $1\frac{2}{3}$ **c** $\frac{8}{9}$ **d** $1\frac{4}{5}$ **e** $\frac{5}{8}$

f $\frac{3}{5}$ **g** $\frac{1}{4}$ **h** $2\frac{1}{2}$ **i** $\frac{4}{5}$ **j** $1\frac{1}{7}$

3 $\left(x^{\frac{1}{n}}\right)^n = x^{\frac{1}{n}\times n} = x^1 = x$, but $\left(\sqrt[n]{x}\right)^n = \sqrt[n]{x} \times \sqrt[n]{x} \ldots$ n times = x, so $x^{\frac{1}{n}} = \sqrt[n]{x}$

4 $64^{-\frac{1}{2}} = \frac{1}{8}$, others are both $\frac{1}{2}$

5 Possible answer: The negative power gives the reciprocal, so $27^{-\frac{1}{3}} = \frac{1}{27}^{\frac{1}{3}}$. The power one-third means cube root, so you need the cube root of 27 which is 3, so $27^{\frac{1}{3}} = 3$ and $\frac{1}{27}^{\frac{1}{3}} = \frac{1}{3}$

6 Possible answer: $x = 1$ and $y = -1$, $x = 8$ and $y = \frac{1}{64}$.

7 **a** 3 **b** $\frac{1}{3}$ **c** 2 **d** $\frac{1}{2}$ **e** $\frac{1}{2}$ **f** $\frac{1}{4}$

g $\frac{1}{4}$ **h** $\frac{1}{3}$ **i** $\frac{1}{3}$ **j** $\frac{1}{2}$ **k** $\frac{1}{3}$ **l** $\frac{1}{7}$

Exercise 15E

1 **a** 16 **b** 25 **c** 216 **d** 81

2 **a** $t^{\frac{2}{3}}$ **b** $m^{\frac{3}{4}}$ **c** $k^{\frac{2}{5}}$ **d** $x^{\frac{3}{2}}$

3 **a** $\frac{1}{5}$ **b** $\frac{1}{6}$ **c** $\frac{1}{2}$ **d** $\frac{1}{3}$

e $\frac{1}{4}$ **f** $\frac{1}{2}$ **g** $\frac{1}{2}$ **h** $\frac{1}{3}$

4 **a** $\frac{1}{125}$ **b** $\frac{1}{216}$ **c** $\frac{1}{8}$ **d** $\frac{1}{27}$

e $\frac{1}{256}$ **f** $\frac{1}{4}$ **g** $\frac{1}{4}$ **h** $\frac{1}{9}$

5 **a** $\frac{1}{100000}$ **b** $\frac{1}{12}$ **c** $\frac{1}{25}$ **d** $\frac{1}{27}$

e $\frac{1}{32}$ **f** $\frac{1}{32}$ **g** $\frac{1}{81}$ **h** $\frac{1}{13}$

6 $8^{-\frac{2}{3}} = \frac{1}{4}$, others are both $\frac{1}{8}$

7 **a** $\frac{27}{8}$ **b** $\frac{9}{25}$ **c** $\frac{1024}{243}$ **d** $\frac{8}{343}$ **e** $\frac{16}{9}$ **f** $\frac{8}{27}$

g $\frac{625}{256}$ **h** $\frac{32}{243}$

8 **a** $\frac{25}{9}$ **b** $\frac{27}{64}$ **c** $\frac{125}{729}$ **d** $\frac{243}{32}$ **e** $\frac{8}{27}$ **f** $\frac{243}{32}$

g $\frac{9}{4}$ **h** $\frac{125}{343}$ **i** $\frac{16}{25}$ **j** $\frac{512}{125}$ **k** $\frac{243}{32}$ **l** $\frac{32}{243}$

9	**a**	x^4	**b**	x^{-1}	**c**	$4y^2$	**d**	$10x^2$	**e**	$20x^{-1}$	**f**	$\frac{1}{3}y$
10	**a**	x	**b**	d^{-1}	**c**	$t^{\frac{3}{2}}$	**d**	x^2	**e**	$y^{\frac{1}{2}}$	**f**	a^4
11	**a**	$x^{\frac{1}{2}}$	**b**	y^{-1}	**c**	$a^{\frac{5}{3}}$	**d**	t^{-2}	**e**	d^2	**f**	1

12 a x^3 **b** x^{-2} or $\dfrac{1}{x^2}$ **c** x **d** x **e** $x^{\frac{3}{2}}$ **f** x

Exercise 15F

1	**a**	$x = 64$	**b**	$x = 25$	**c**	$x = 8$		
	d	$x = 1000$	**e**	$x = \dfrac{1}{8}$	**f**	$x = \dfrac{4}{5}$		
2	**a**	$x = 2\sqrt{2}$	**b**	$x = 2\sqrt{5}$	**c**	$x = 3\sqrt{5}$		
	d	$x = \sqrt{2}$	**e**	$x = 2\sqrt{3}$	**f**	$x = \dfrac{1}{\sqrt{2}}$ or $x = \dfrac{\sqrt{2}}{2}$		
3	**a**	$x = 81$	**b**	$x = 49$	**c**	$x = 3$		
	d	$x = 18$	**e**	$x = 20$	**f**	$x = 125$		
4	**a**	$x = 8$	**b**	$x = 5\sqrt{5}$	**c**	$x = 81$		
	d	$x = 375\sqrt{3}$	**e**	$x = 4\sqrt{2}$	**f**	$x = 64$		
5	**a**	$x = 27$	**b**	$x = 8$	**c**	$x = 16$		
	d	$x = 1$	**e**	$x = 32$	**f**	$x = 100\ 000$		
6	**a**	$x = \dfrac{1}{4}$	**b**	$x = \dfrac{1}{3}$	**c**	$x = \dfrac{1}{\sqrt{2}}$ or $x = \dfrac{\sqrt{2}}{2}$		
	d	$x = \dfrac{1}{5}$	**e**	$x = \dfrac{1}{3}$	**f**	$x = 2$		
7	**a**	$x = \dfrac{1}{49}$	**b**	$x = \dfrac{1}{64}$	**c**	$x = \dfrac{1}{81}$		
	d	$x = \dfrac{1}{32}$	**e**	$x = \dfrac{1}{6}$	**f**	$x = \dfrac{1}{2\sqrt{2}}$ or $x = \dfrac{\sqrt{2}}{4}$		
8	**a**	$x = \dfrac{1}{4}$	**b**	$x = \dfrac{1}{27}$	**c**	$x = \dfrac{1}{81}$		

d $x = \frac{1}{3125}$ **e** $x = 9$ **f** $x = 3\sqrt{3}$

Exam-style questions

1 **a** $\frac{1}{216}$ **b** 4

2 x

3 $x^{\frac{1}{6}}$

4 **a** $x = 512$ **b** $x = \frac{1}{9}$

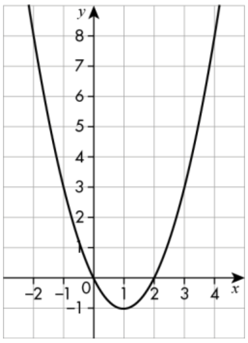
5 **a** z^{-2} **b** $z = \frac{1}{5}$

Chapter 16 Calculus

Exercise 16A

1 **a** The missing numbers are 0, –1, 0

b



c $2x – 2$ **d** 4 **e** 6 **f** Student's choice

g (1, –1) **h** Student's check

2 **a** $2x – 6$ **b** –6 **c** 4 **d** (4, 7)

3 **a** $4x$ **b** 8 **c** –4 **d** (3, 8)

4 **a** $4 – 2x$ **b** 4 and –4 **c** (1, 3) **d** (1.5, 3.75)

5 **a** $2x + 1$ **b** $2x – 7$ **c** $8x – 1$ **d** $0.6x − 1.5$

e $–2 + 2x$ **f** $3 – 2x$ **g** 2 **h** 0

6 $2x + 2$

7 **a** $4x + 2$ **b** $2x + 7$ **c** $2x$

8 **a** (0, –5) **b** 2

Exercise 16B

1 **a** $6x^2$ **b** 6 and 24

2 **a** $3x^2 - 12x + 8$ **b** If $x = 0$ or 2 or 4, $y = 0$ **c** 8; –4; 8

3 **a** $8x^3$ **b** $15x^2 - 2$ **c** $9x^2 + 5$ **d** $-3x^2$ **e** $4x^3 - 1$

4 16 at (2, 0); –16 at (–2, 0); 0 at (0, 0)

5 $x^2 - 5 = 4$ has two solutions, $x = 3$ or -3. Points are (3, −2) and (–3, 10)

Exercise 16C

1 **a** $2x - 4$ **b** $2x - 4 = 0 \Rightarrow x = 2$ (2, –1) **c** Minimum **d** x □ 2

2 **a** (–3, –12) **b** Minimum

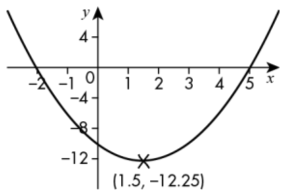
3 **a** $5 - 2x$ **b** (2.5, 7.25) **c** Maximum **d** x □ 2.5

4 **a** $3x^2 - 6x$ **b** $x = 0$ or 2 **c** (0, 0) and (2, –4)

d 0 □ x □ 2

5 **a** If $x = -2$ or 5, $y = 0$ **b** $2x - 3$ **c** (1.5, –12.25); Minimum

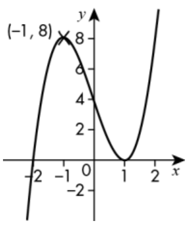
d



e $x = 1.5$

6 **a** $6x^2 - 6$ **b** (1, 0) minimum, (–1, 8) maximum

c



7 **a** $\dfrac{dy}{dx} = 3x^2 + 12x + 12$ **b** (–2, –8), a point of inflection

8 **a** $\dfrac{dy}{dx} = 3x^2 - 30x + 75$, $\dfrac{dy}{dx} = 3(x - 5)^2$. The only stationary point is at $x = 5$, the point (5, 131).

b When $x = 4.9$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0.03$; when $x = 5$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$; when $x = 5.1$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 0.03$, so it is a point of inflection.

Exercise 16D

1 **a** $\frac{\mathrm{d}y}{\mathrm{d}x} = 6x$ **b** $y = 12x – 36$ **c** $12y + x = 146$

2 **a** $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x – 4$ **b** $y = –2x + 2$ **c** $4y + x = 16$

3 **a** $\frac{\mathrm{d}y}{\mathrm{d}x} = 3x^2 + 2x – 1$ **b** $y = 4x + 2$ **c** $x + 7y – 19 = 0$

4 $y = 5x – 15$ and $y = – 5x – 10$

5 **a** $y = –x + 5$ **b** (0, 5)

6 **a** $y = –4x + 9$ **b** $2y + x = 7$ **c** –

Exam-style questions

1 **a** $8x^3 – 12x$ **b** $10x – 3x^3$ **c** $2x – 7$

2 **a** –9 **b** 2 and –2

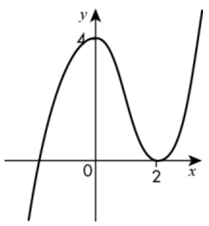
3 **a** 10 **b** (–1, 6) **c** ¼

4 **a** If $x = 2$ then $y = 8 – 8 + 5 = 5$ **b** 8 **c** (0, 11)

5 **a** $\frac{\mathrm{d}y}{\mathrm{d}x} = x^2 – 2x$. If $x = 0$ then $y = 4$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$, so (0, –4) is a turning point.

b (2, 0)

c



6 **a** If $x = 2$ then $y = 16 – 16 + 5 = 5$ **b** $y = 16x – 11$

c $\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow 3x^3 – 8x = 0 \Rightarrow x(x^2 – 2) = 0 \Rightarrow x = 0$ or $–\sqrt{2}$.

Three values give three turning points.

7 **a** $y = x – 8$ **b** (4, -4)

8 10

Chapter 17 Ratios of angles and their graphs

Exercise 17A

1 **a** 36.9°, 143.1° **b** 53.1°, 126.9° **c** 48.6°, 131.4° **d** 224.4°, 315.6°

e 194.5°, 345.5° **f** 198.7°, 341.3° **g** 190.1°, 349.9°

h 234.5°, 305.5° **i** 28.1°, 151.9° **j** 185.6°, 354.4°

2 Sin 234°, as the others all have the same numerical value

3 **a** 438° or 78° + 360*n*° **b** –282° or 78° – 360*n*°

c Line symmetry about ±90*n*° where *n* is an odd integer. Rotational symmetry about ±180*n*° where *n* is an integer

Exercise 17B

1 **a** 53.1°, 306.9° **b** 54.5°, 305.5° **c** 62.7°, 297.3° **d** 54.9°, 305.1°

e 79.3°, 280.7° **f** 143.1°, 216.9° **g** 104.5°, 255.5°

h 100.1°, 259.9° **i** 111.2°, 248.8° **j** 166.9°, 193.1°

2 Cos 58°, as the others are negative

3 **a** 492° or 132° + 360*n*° **b** –228° or 132° – 360*n*°

c Line symmetry about ±180*n*° where *n* is an integer. Rotational symmetry about ±90*n*° where *n* is an odd integer

Exercise 17C

1 **a** 0.707 **b** –1 (–0.9998) **c** –0.819 **d** 0.731

2 **a** –0.629 **b** –0.875 **c** –0.087 **d** 0.999

3 **a** 21.2°, 158.8° **b** 209.1°, 330.9° **c** 50.1°, 309.9° **d** 150.0°, 210.0°

e 60.9°, 119.1° **f** 29.1°, 330.9°

4 30°, 150°

5 –0.755

6 **a** 1.41 **b** –1.37 **c** –0.0367 **d** –0.138

e 1.41 **f** –0.492

7 True

8 **a** Cos 65° **b** Cos 40°

9 **a** 10°, 130° **b** 12.7°, 59.3°

10 38.2°, 141.8°

Exercise 17D

1	**a**	14.5°, 194.5°	**b**	38.1°, 218.1°	**c**	50.0°, 230.0°
	d	61.9°, 241.9°	**e**	68.6°, 248.6°	**f**	160.3°, 340.3°
	g	147.6°, 327.6°	**h**	135.4°, 315.4°	**i**	120.9°, 300.9°
	j	105.2°, 285.2°				

2 Tan 235°, as the others have a numerical value of 1

3 **a** 425° or 65° + 180*n*°, $n \geq 2$ **b** –115° or 65° – 180n°

c No line symmetry. Rotational symmetry about ±180*n*° where *n* is an integer

Exercise 17E

1 115°

2 327°

3 324°

4 195°

5 216°

6 331°

7 210°, 330°

8 135°, 225°

9 120° and 300°

10 a Say 32°, sin 32° = 0.53, cos 58° = 0.53

b Say 70°, sin 70° = 0.94, cos 20° = 0.94

c Sin *x* = cos (90 – *x*)°

d Cos *x* = sin (90 – *x*)°

11 a 64° **b** 206°, 334° **c** 116°, 244°

12 a –0.384 **b** 113°

13 a 0.822 **b** 55.3 **c** No

d The calculator has given the value of the acute angle but the angle 124.7° has the same positive sign

14 a 1.1307

b Error

c If you tried to draw this triangle accurately then you would see that the line that is 12 long does not intersect with the base

15 **a** to **f** All true **g** False

Exercise 17F

1 **a** 3 cm **b** $4\sqrt{3}$ cm **c** 1.5 cm **d** $\frac{5}{\sqrt{3}}$ cm or $\frac{5\sqrt{3}}{3}$ cm

e $3\sqrt{2}$ cm **f** 2 cm **g** 14 cm **h** $\frac{4}{\sqrt{3}}$ cm or $\frac{4\sqrt{3}}{3}$ cm

2 **a** $3\sqrt{2}$ cm **b** $\frac{16\sqrt{3}}{3}$ cm **c** $(5\sqrt{3} - 5)$ cm

d $(\frac{5\sqrt{3}}{3} + 5)$ cm **e** $\sqrt{2}$ cm

Exercise 17G

1 **a** 17.5° and 162.5° **b** 78.5° and 281.5° **c** 50.2° and 230.2°

d 36.9° and 143.1° **e** 0° and 360° **f** 35.0° and 215.0°

2 **a** 197.5° and 322.5° **b** 101.5° and 281.5° **c** 129.8° and 309.8°

d 216.9° and 323.1° **e** 180° **f** 145.0° and 325.0°

3 **a** 194.5° and 345.5° **b** 113.6° and 246.4° **c** 45° and 225°

d 45° and 135° **e** 70.5° and 250.5° **f** 0°, 180° and 360°

4 **a** 63.4° **b** 76.0° **c** 161.6°

d 158.2° **e** 135° **f** 104.0°

5 **a** $\sin^3 x + \sin x \cos^2 x \equiv \sin x (\sin^2 x + \cos^2 x) = \sin x$

b $\cos x \tan x \equiv \cos x \, \frac{\sin x}{\cos x} = \sin x$

c $\frac{1 - \cos^2 x}{\cos^2 x} = \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$

d $\frac{(\sin^2 x + \cos^2 x)\sin x}{\tan x} \equiv \frac{\sin x \cos x}{\sin x} = \cos x$

e $\frac{1}{1 + \tan^2 x} \equiv \frac{1}{1 + \frac{\sin^2 x}{\cos^2 x}} = \frac{\cos^2 x}{\cos^2 x + \sin^2 x} = \cos^2 x$

f $\sin^4 x - \cos^4 x \equiv (\sin^2 x - \cos^2 x)(\sin^2 x + \cos^2 x)$

$= \sin^2 x - \cos^2 x = \sin^2 x - (1 - \sin^2 x)$

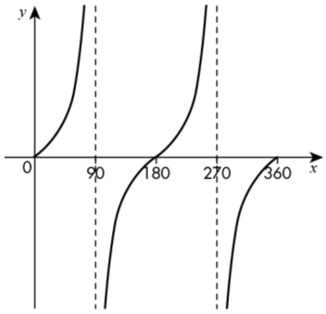
$= \sin^2 x - 1 + \sin^2 x = 2\sin^2 x - 1$

6 **a** 0°, 180°, 360° **b** 45°, 135°, 225°, 315° **c** 0°, 60°, 180°, 240°, 360°

d 120°, 180°, 240° **e** 63.4°, 135°, 243.4°, 315° **f** 30°, 150°

Exam-style questions

1 **a**



b 256°

2 **a** 153° **b** 207° and 233°

3 **a** –0.17 **b** –0.94 **c** 0.17

4 $6 + 2\sqrt{3}$ cm

5 The base is $\frac{1}{2}a$ so the area = $\frac{1}{2} \times \frac{1}{2}a \times a \sin 60° = \frac{1}{4}a^2 \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}a^2}{8}$ cm^2

6 **a** $\frac{4}{5}$ **b** $\frac{3}{4}$ **c** $-\frac{3}{5}$

7 $(1 + \cos\theta)(1 - \cos\theta) = 1 - \cos^2\theta = \sin^2\theta$ and $\sin\theta\cos\theta\tan\theta = \sin\theta\cos\theta \times \frac{\sin\theta}{\cos\theta} = \sin^2\theta$, which shows that the expressions are identical.

8 $x = 18.4°$

9 $x = 30°, 150°, 210°$ or $330°$

Chapter 18 Proof

Exercise 18A

1 **a** Odd, yes

b $(2n + 1) + 2m = 2(n + m) + 1$, which is odd.

2 Check students' proofs.

3 **a** 3, 5, 8, 13, 21, 34, 55

b $3a + 5b, 5a + 8b, 8a + 13b, 13a + 21b, 21a + 34b$

c $(8a + 13b) - (2a + 3b) = 6a + 10b = 2(3a + 5b)$

4 Check students' answers.

5 Check students' answers.

6 Check students’ answers.

7 Check students’ answers.

8 Check students’ answers.

9 Check students’ answers.

10 $(2x + 1)^2 - (x - 1)^2 = 4x^2 + 4x + 1 - (x^2 - 2x + 1)$

$= 4x^2 + 4x + 1 - x^2 + 2x - 1 = 3x^2 + 6x = 3x(x + 2)$

$= (2x + 1 + x - 1)(2x - 1 - (x - 1))$

Exercise 18B

1 $\angle DFE = 180° - (90° + \frac{x}{2}°) = 90° - \frac{x}{2}°$

$\angle DEF = 180° - x° - (90° - \frac{x}{2}°) = 90° - \frac{x}{2}°$

$\angle DFE = \angle DEF$ hence triangle DEF is isosceles.

2 The exterior angle of a triangle is equal to the sum of the opposite 2 interior angles.

$x° = \frac{x}{2}° + \frac{x}{2}°$, hence the triangle is isosceles

3 $\angle AOC = 2x°$, hence $\angle ADC = x°$, reflex angle AOC = $2y°$, hence $\angle ADC = y°$

But $2x° + 2y° = 360°$ (angles around a point) hence $2(x° + y°) = 360°$ giving $x° + y° = 180°$

4 $\angle CED + \angle AEC = 180°$ (angles on a straight line)

$\angle ABC + \angle AEC = 180°$ (cyclic quadrilateral)

But $\angle ABC = \angle ACB$ (isosceles triangle)

Hence $\angle ACB = \angle CED$

5 PS = QR, RS = PQ, both triangles share side QS hence by SSS triangles are congruent

6 **a** Check students’ proofs

b By the alternate segment theorem $\angle TXA = \angle TYB$ hence AX is parallel to BY

7 **a** $\overrightarrow{YW} = \overrightarrow{YZ} + \overrightarrow{ZW} = 2\mathbf{a} + \mathbf{b} + \mathbf{a} + 2\mathbf{b} = 3\mathbf{a} + 3\mathbf{b} = 3(\mathbf{a} + \mathbf{b}) = 3\overrightarrow{XY}$

b 3 : 1

c They lie on a straight line.

d Points are A(6, 2), B(1, 1) and C(2, –4). Using Pythagoras’ theorem, $AB^2 = 26$, $BC^2 = 26$ and $AC^2 = 52$ so $AB^2 + BC^2 = AC^2$ hence $\angle ABC$ must be a right angle

Exam-style questions

1 Check students' answers.

2 Check students' answers.

3 Check students' answers.

4 Check students' answers.

5 Check students' answers.

6 Check students' answers.

7 **a** $10^2 = 100$, $1 + 8 + 27 + 64 = 100$

b $(\frac{1}{2}n(n + 1))^2 = n^2(n + 1)^2$

8 $\angle$QAT = $\angle$QTA (isosceles triangle)

$\angle$PTB = $\angle$QTA (vertically opposite angles)

$\angle$PTB = $\angle$PBT (isosceles triangle)

Hence $\angle$PBT = $\angle$QAT and PB is parallel to AQ