7 Algebra

7.1 Manipulation of rational expressions

Exercise 7A

1 **a** $\frac{x^2y+8}{4x}$ **b** $\frac{7x+3}{4}$ **c** $\frac{13x+5}{15}$ **d** $\frac{5x-10}{4}$

e $\frac{xy^2-8}{4y}$ **f** $\frac{x+1}{4}$ **g** $\frac{-7x-5}{4}$ **h** $\frac{2-3x}{4}$

i $\frac{2xy}{3}$ **j** $\frac{x^2-2x}{10}$ **k** $\frac{1}{6}$ **l** $\frac{1}{2x}$

m 1 **n** 3 **o** $\frac{13x+7}{10}$ **p** $\frac{x+3}{2}$

2 **a** $\frac{7x+9}{(x+1)(x+2)}$ **b** $\frac{11x-10}{(x-2)(x+1)}$ **c** $\frac{2-13x}{(4x+1)(x+2)}$

d $\frac{8-10x}{(2x-1)(x+1)}$ **e** $\frac{x+1}{(2x-1)(3x-1)}$

3 $\frac{2x^2+x-3}{4x^2-9}$

4 **a** $\frac{9x+13}{(x+1)(x+2)}$ **b** $\frac{14x+19}{(4x-1)(x+1)}$ **c** $\frac{2x^2+x-13}{2(x+1)}$ **d** $\frac{x+1}{(2x-1)(3x-1)}$

5 **a** $\frac{x-1}{2x+1}$ **b** $\frac{2x+1}{x+3}$ **c** $\frac{2x-1}{3x-2}$

d $\frac{x+1}{x-1}$ **e** $\frac{2x+5}{4x-1}$

6 **a** $\frac{x(x-1)}{x+2}$ **b** $\frac{x(x-5)}{x-4}$ **c** $\frac{x(x-4)}{x-5}$

d $\frac{x(x-6)}{x-7}$ **e** $\frac{x(x-5)}{x+3}$ **f** $\frac{x(x+3)}{x+8}$

7 **a** $x+2$ **b** $x+3$ **c** $4x+5$

d $2x-7$ **e** $3(x+1)$ **f** $x+2$

8 **a** $\frac{4x(3x+1)}{2x-3}$ **b** $\frac{x(9x-8)}{3x-7}$ **c** $\frac{2x(x+4)}{4x-3}$

d $\frac{6(x-1)}{5x+6}$ **e** $\frac{6x(x+1)}{8x-3}$ **d** $\frac{x(4x-5)}{3x+4}$

7.2 Use and manipulation of formulae and expressions

Exercise 7B

1 $m = gv$ **2** $m = \sqrt{t}$ **3** $r = \frac{C}{2\pi}$ **4** $b = \frac{A}{h}$

5 $l = \frac{P-2w}{2}$ **6** $p = \sqrt{m-2}$ **7** $a = \frac{v-u}{t}$ **8** $d = \sqrt{\frac{4A}{\pi}}$

9 **a** $n = \frac{W-3}{t}$ **b** $t = W - 3n$

10 $p = \sqrt{\frac{k}{2}}$

11 **a** $t = u^2 - v$ **b** $u = \sqrt{v+t}$

12 **a** $m = k - n^2$ **b** $n = \sqrt{k-m}$

13 $r = \sqrt{\frac{T}{5}}$

14 **a** $w = K - 5n^2$ **b** $n = \sqrt{\frac{K-w}{5}}$

Exercise 7C

1 **a** 2.5 **b** $a = \sqrt{c^2 - b^2}$

2 **a** 60 **b** $a = \frac{2(s-ut)}{t^2}$

3 **a** $b = ac - 2$ **b** $c = \frac{b+2}{a}$

4 $t = \frac{r}{p} + 3$

5 $e = (\frac{12}{d} - 1)^2$

6 **a** 5 **b** $u = \sqrt{v^2 - 2as}$ **c** $s = \frac{v^2 - u^2}{2a}$

7 **a** $L = \left(\frac{t}{2\pi}\right)^2 G$ **b** Check students' proof

8 **a** $R = \sqrt{\frac{D + \pi r^2}{\pi}}$ **b** $r = \sqrt{\frac{\pi R^2 - D}{\pi}}$ **c** $\pi = \frac{D}{R^2 - r^2}$

9 **a** $x = 5$ or -5 **b** $x = \sqrt{\frac{11 + 4y^2}{3}}$ **c** $y = \sqrt{\frac{3x^2 - 11}{4}}$

10 **a** $a = \left(\frac{T}{2}\right)^2 (c + 3)$ **b** $c = a\left(\frac{2}{T}\right)^2 - 3$

11 $T = \frac{b^2 + c^2 - a^2}{2bc}$

12 **a** 12 **b** $f = \frac{uv}{u + v}$ **c** $u = \frac{fv}{v - f}$ **d** $v = \frac{fu}{u - f}$

13 $x = \frac{yz}{y + z}$

14 $s = \frac{2r}{t + 1}$

15 $h = \frac{2gi}{5i - 1}$

16 $k = \frac{j}{t}$

17 $c = \frac{81b}{4ad}$

18 $h = 9gi\left(\frac{12e}{5} - \frac{1}{8f}\right)$

7.3 The factor theorem

Exercise 7D

2 **a** $(x + 1)(x - 3)(x + 5)$ **b** $(x + 4)(x - 1)(x + 2)$

c $(x + 1)(x + 2)(x + 3)$ **d** $(x - 2)(x - 3)(x - 5)$

e $(2x + 1)(x - 4)(x - 3)$ **f** $(3x - 1)(2x + 3)(x + 2)$

3 a $x = 1, x = -2, x = 5$ **b** $x = 1$

c $x = 4, x = -4, x = -2$ **d** $x = 3, x = -1, x = -5$

e $x = -2, x = -6, x = -7$ **f** $x = -4, x = 10, x = 5$

4 –2

5 **b** –1 and 3

6 $a = 3$, $b = 1$, $c = 1$

7 $x - 4$, $x + 4$, $x + 3$

8 $x + 2$ and $x - 4$

Exam-style questions

1 $\dfrac{3}{(x+1)(x+2)}$

2 $\dfrac{(2x-1)(x+3)}{2x+1}$

3 $\dfrac{x}{(x+2)^2}$

4 $x = \dfrac{4y-2}{9}$

5 $v = \dfrac{uf}{u-f}$

6 $C^2 = (2\pi r)^2 = 4\pi^2 r^2 \Rightarrow \dfrac{C^2}{4\pi} = \dfrac{4\pi^2 r^2}{4\pi} = \pi r^2 = A$

7 **a** f(–4) = –64 – 16 + 56 + 24 = 0

b $(x - 2)(x - 3)(x + 4)$

8 **a** f(2) = 8 – 24 + 24 – 8 = 0 **b** $(x - 2)^3$

9 $(x - 4)(x + 5)(x - 5)$

10 $x = -1, -2$ or -3

11 **a** $x = -1$ or 4 **b** $x = 0, -1$ or 4 **c** $x = -2$, 2 or 3

8 Sequences

8.1 Number sequences

Exercise 8A

1 **a** 28, 34, 40: add 6 **b** 23, 28, 33:add 5

c 20 000, 200 000, 2 000 000: multiply by 10

d 19, 22, 25: add 3 **e** 46, 55, 64: add 9

f 405, 1215, 3645: multiply by 3 **g** 18, 22, 26: add 4

h 625, 3125, 15 625: multiply by 5

2 **a** 16, 22 **b** 26. 37 **c** 31, 43 **d** 46, 64
e 121, 169 **f** 11, 13 **g** 33, 65 **h** 78, 108

3 **a** 48, 96, 192 **b** 33, 39, 45 **c** 4, 2, 1 **d** 38, 35, 32
e 26, 33, 41 **f** 19, 22, 25 **g** 28, 36, 45
h 0.0625, 0.031 25, 0.015 625

4 **a** 21, 34: add previous 2 terms **b** 49, 64: next square number
c 47, 76: add previous 2 terms **d** 216, 343: cube numbers

8.2 The *n*th term of a sequence

Exercise 8B

1 **a** 4, 5, 6, 7, 8 **b** 2, 5, 8, 11, 14 **c** 3, 8, 13, 18, 23 **d** 9, 13, 17, 21, 25

2 **a** £305 **b** £600 **c** 3 **d** 5

8.3 The *n*th term of a linear sequence

Exercise 8C

1 **a** 13, 15, $2n + 1$ **b** 25, 29, $4n + 1$ **c** 33, 38, $5n + 3$
d 32, 38, $6n - 4$ **e** 20, 23, $3n + 2$ **f** 37, 44, $7n - 5$
g 21, 25, $4n - 3$ **h** 23, 27, $4n - 1$ **i** 17, 20, $3n - 1$

2 **a** $3n + 1$, 151 **b** $2n + 5$, 105 **c** $5n - 2$, 248
d $4n - 3$, 197 **e** $8n - 6$, 394 **f** $n + 4$, 54

3 **a** 33rd **b** 30th **c** 100th = 499

4 **a** **i** $4n + 1$ **ii** 401 **iii** 101, 25th
b **i** $2n + 1$ **ii** 201 **iii** 99 or 101, 49th and 50th
c **i** $3n + 1$ **ii** 301 **iii** 100, 33rd
d **i** $2n + 6$ **ii** 206 **iii** 100, 47th
e **i** $4n + 5$ **ii** 405 **iii** 101, 24th
f **i** $5n + 1$ **ii** 501 **iii** 101, 20th

5 **a** $\frac{2n+1}{3n+1}$ **b** Getting closer to $\frac{2}{3}$ $(0.\dot{6})$
c **i** 0.667 774 (6 d.p.) **ii** 0.666 778 (6 d.p.)
d 0.666 678 (6 d.p.), 0.666 667 (6 d.p.)

6 **a** $\frac{3}{4}, \frac{5}{7}, \frac{7}{10}$
b **i** 0.666 666 777 8 **ii** $\frac{2}{3}$
c For n, $\frac{2n - 1}{3n - 1} \approx \frac{2n}{3n} = \frac{2}{3}$

7 **a** Sequence goes up in 2s; first term is 2 + 29

b $n + 108$ **c** Because it ends up as $2n \div n$ **d** 79th

8.4 The *n*th term of a quadratic sequence

Exercise 8D

1 **a** 36, 49 **b** 38, 51 **c** 42, 56 **d** 74, 100

e 78, 105 **f** 109, 148 **g** 43, 57 **h** 178, 243

i 114, 154 **j** 66, 91

2 **a** n^2 **b** $n^2 + 2$ **c** $n^2 + n$ **d** $2n^2 + 2$

e $2n^2 + n$ **f** $3n^2 + 1$ **g** $n^2 + n + 1$ **h** $5n^2 - 2$

i $3n^2 + n$ **j** $2n^2 - n$

3 **a** $0, \frac{1}{3}, \frac{8}{13}$ **b** 6th term

4 $4n^2$ is positive since n is always positive and n^2 is positive. $3n - 2$ is always positive since when $n = 1$, $3n - 2 = 1$, so first term = 1 and terms are increasing by 3 each time, so 3n – 2 is always positive. Positive divided by positive is always positive.

5 $\frac{n^2}{2n+1}$

6 **a** $3n - 2$ **b** $4n - 2$ **c** $(3n - 2)(4n - 2)$ or $12n^2 - 14n + 4$

8.5 The limiting value of a sequence as $n \to \infty$

Exercise 8E

1 **a** 2 **b** $\frac{1}{3}$ **c** 4

d –1 **e** 0 **f** 2

2 **a** 1 **b** 0 **c** 0

d $\frac{2}{5}$ **e** $\frac{1}{5}$ **f** 0

Exam-style questions

1 29, 41; the numbers go up in 12s.

2 No, one sequence the numbers are always a multiple of three ($3n$); in the other the numbers are always one less that a multiple of three ($3n - 1$). Hence no term in common.

3 **a** $4, 5\frac{1}{3}$ and 6 **b** The eighth **c** 8

4 $n^2 + 2n$ or $n(n + 2)$

5 Four

9 Pythagoras' theorem and trigonometry

9.1 Pythagoras' theorem

Exercise 9A

1 10.3 cm

2 8.5 cm

3 20.6 cm

4 The square in the first diagram and the two squares in the second have the same area.

5 **a** 15 cm **b** 14.7 cm **c** 6.3 cm **d** 18.3 cm

6 **a** 5 m **b** 6 m **c** 50 cm

7 There are infinite possibilities, e.g. any multiple of 3, 4, 5 such as 6, 8, 10; 9, 12, 15; 12, 16, 20; multiples of 5, 12, 13 and of 8, 15, 17.

8 42.6 cm

Exercise 9B

1 No. The foot of the ladder is about 6.6 m from the wall.

2 About 17 minutes, assuming it travels at the same speed.

3 13 units

4 **a** 4.85 m **b** 4.83 m (There is only a small difference.)

5 Yes, because $24^2 + 7^2 = 25^2$

6 6 cm

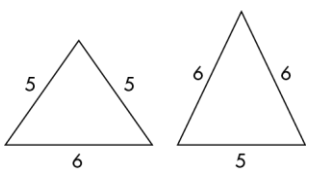
Exercise 9C

1 **a** 32.2 cm^2 **b** 2.83 cm^2 **c** 50.0 cm^2

2 22.2 cm^2

3 15.6 cm^2

4 **a**



b The areas are 12 cm^2 and 13.6 cm^2 respectively, so triangle with 6 cm, 6 cm, 5 cm sides has the greater area.

5 259.8 cm^2

6 19.8 or 20 m^2

7 **a** 10 cm **b** 26 cm **c** 9.6 cm

Exercise 9D

1 **a** **i** 14.4 cm **ii** 13 cm **iii** 9.4 cm **b** 15.2 cm

2 No, 6.6 m is longest length

3 **a** 20.6 cm **b** 15.0 cm

4 21.3 cm

5 a 8.49 m b 9 m

6 17.3 cm

7 20.6 cm

8 **a** 11.3 cm **b** 7 cm **c** 8.06 cm

9 **a** 50.0 cm **b** 54.8 cm **c** 48.3 cm **d** 27.0 cm

9.2 Trigonometry in right-angled triangles

Exercise 9E

1 **a** 0.682 **b** 0.829 **c** 0.922 **d** 1

2 **a** **i** 0.574 **ii** 0.574 **b** **i** 0.208 **ii** 0.208

c **i** 0.391 **ii** 0.391 **d** Same

e **i** sin 15° is the same as cos 75°.

ii cos 82° is the same as sin 8°

iii sin x is the same as cos $(90° - x)$

3 **a** 0.933 **b** 1.48 **c** 2.38 **d** infinite

4 Has values > 1

5 **a** 3.56 **b** 8.96 **c** 5.61 **d** 7.08

6 **a** $\frac{4}{5}, \frac{3}{5}, \frac{4}{3}$ **b** $\frac{5}{13}, \frac{12}{13}, \frac{5}{12}$ **c** $\frac{7}{25}, \frac{24}{25}, \frac{7}{24}$

7 **a** 30° **b** 51.7° **c** 39.8° **d** 61.3° **e** 87.4° **f** 45.0°

8 **a** 60° **b** 50.2° **c** 2.6° **d** 45.0 **e** 78.5° **f** 45.6°

9 **a** 31.0° **b** 20.8° **c** 41.8° **d** 46.4° **e** 69.5° **f** 77.1°

10 Error message, largest value 1, smallest value –1

Exercise 9F

1 **a** 17.5° **b** 22.0° **c** 32.2°

2 **a** 5.29 cm **b** 5.75 cm **c** 13.2 cm

3 **a** 4.57 cm **b** 6.86 cm **c** 100 cm

4 **a** 5.12 cm **b** 9.77 cm **c** 11.7 cm **d** 15.5 cm

5 **a** 51.3° **b** 75.5° **c** 51.3°

6 **a** 5.35 cm **b** 14.8 cm **c** 12.0 cm **d** 8.62 cm

7 **a** 5.59 cm **b** 46.6° **c** 9.91 cm **d** 40.1°

8 **a** 33.7° **b** 36.9° **c** 52.1°

9 **a** 9.02 cm **b** 7.51 cm **c** 7.14 cm **d** 8.90 cm

10 **a** 13.7 cm **b** 48.4° **c** 7.03 cm **d** 41.2°

Exercise 9G

1 **a** 12.6 **b** 59.6 **c** 74.7 **d** 16.0 **e** 67.9 **f** 20.1

2 **a** 44.4° **b** 39.8° **c** 44.4° **d** 49.5° **e** 58.7° **f** 38.7°

3 **a** 67.4° **b** 11.3 **c** 134 **d** 28.1° **e** 39.7

f 263 **g** 50.2° **h** 51.3° **i** 138 **j** 22.8

4 **a** Sides of right-hand triangle are sine θ and cosine θ

b Pythagoras' theorem

c Students should check the formulae.

Exercise 9H

1 65°

2 The safe limits are between 1.04 m and 2.05 m. The ladder will reach between 5.63 m and 5.90 m up the wall.

3 31°

4 **a** 338 km **b** 725 km

5 43 km

6 170 km

7 One way is stand opposite a feature, such as a tree, on the opposite bank, move a measured distance, x, along your bank and measure the angle, θ, between your bank and the feature. Width of river is $x \tan \theta$. This of course requires measuring equipment! An alternative is to walk along the bank until the angle is 45° (if that is possible). This angle is easily found by folding a sheet of paper. This way an angle measurer is not required.

Exercise 9I

1 10.1 km

2 22°

3 429 m

4 **a** 156 m

b No. The new angle of depression is $\tan^{-1}\frac{200}{312}$ = 33° and half of 52° is 26°.

5 **a** 222 m **b** 42°

6 **a** 21.5 m **b** 17.8 m

7 13.4 m

8 19°

9 The angle is 16° so Cara is not quite correct.

Exercise 9J

1 25.1°

2 **a** 25 cm **b** 58.6° **c** 20.5 cm

3 **a** 3.46 m **b** 75.5° **c** 73.2°

4 **a** 24.0° **b** 48.0° **c** 13.5 cm **d** 16.7°

5 **a** It is 44.6°; use triangle XDM where M is the midpoint of BD; triangle DXB is isosceles, as X is over the point where the diagonals of the base cross; the length of DB is $\sqrt{656}$ and the cosine of the required angle is $0.5\sqrt{656} \div 18$.

b 61.1°

9.3 The sine rule and the cosine rule

Exercise 9K

1 **a** 3.64 m **b** 8.05 cm **c** 19.4 cm

2 **a** 46.6° **b** 112.0° **c** 36.2°

3 3.47 m

4 **a** **i** 30° **ii** 40° **b** 19.4 m

5 36.5 m

6 22.2 m

7 64.6 km

8 134°

Exercise 9L

1 **a** 7.71 m **b** 29.1 cm **c** 27.4 cm

2 **a** **i** 76.2° **ii** 125.1° **iii** 90° **b** Right-angled triangle

3 **a** 10.7 cm **b** 41.7° **c** 38.3° **d** 6.69 cm

4 58.4 km at 092.5°

5 21.8°

6 42.5 km

7 111°; the largest angle is opposite the longest side.

Exercise 9M

1 **a** 8.60 m **b** 90° **c** 27.2 cm **d** 26.9°

e 27.5° **f** 62.4 cm

2 7 cm

3 11.1 km

4 **a** $A = 90°$; this is Pythagoras' theorem **b** A is acute **c** A is obtuse

5 142 m

Exam-style questions

1 84 cm^2

2 8 cm

3 $\frac{7}{12}$

4 73°

5 **a** 39° **b** 11.1 cm or 11.2 cm

6 5.16 cm

7 65.5 cm

8 **a** 12.3 cm **b** 29° **c** 31°

9 58 km

10 17.8 m

11 17.3 cm