Bridging GCSE and A-level Maths

Exam paper: Mark scheme

1 a $27^n = \frac{1}{9} \Rightarrow (3^3)^n = 3^{-2}$		
$3^{3n} = 3^{-2}$	M1	
$3n = -2$		
$n = -\frac{2}{3}$	A1	**[2]**
b $\frac{5-2\sqrt{3}}{4+\sqrt{3}} = \frac{(5-2\sqrt{3})(4-\sqrt{3})}{(4+\sqrt{3})(4-\sqrt{3})}$	M1	
$= \frac{20-5\sqrt{3}-8\sqrt{3}+6}{16-3}$		
$= \frac{26-13\sqrt{3}}{13}$	A1	
$= 2-\sqrt{3}$		
$a = 2, b = -1$	A1	**[3]**

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2 Discriminant $b^2 - 4ac = 0$	M1	
$(-12)^2 - 4(4)c = 0$		
$144 - 16c = 0$	A1	
leading to $c = 9$	A1	**[3]**

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3 a The aeroplane is descending at a rate of 0.12 km per minute.	B1	**[1]**
b i Solve $2.7 - 0.12t = 0$	M1	
leading to $t = 22.5$		
Coordinates are (22.5, 0)	A1	**[2]**
ii It takes 22.5 minutes from the start of its descent for the aeroplane to land.	B1	**[1]**

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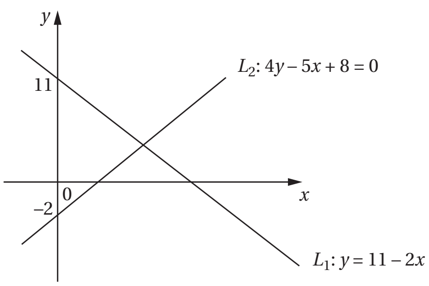
4 a Solve $4(11 - 2x) - 5x + 8 = 0$	M1	
leading to $52 - 13x = 0$		
$x = 4$		
$y = 11 - 2(4)$	M1	
$= 3$		
Intersection point has coordinates (4, 3)	A1	**[3]**
[Alt: Solve $11 - 2x = \frac{1}{4}(5x - 8)$]		

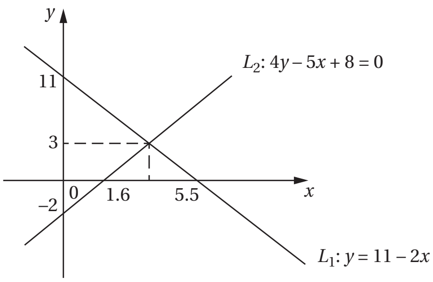
b



Correct y-intercepts and direction of slopes B1

Intersection point in 1st quadrant B1 **[2]**

c



x-intercepts: $L_1 : x = \frac{11}{2} = 5.5$

$L_2 : x = \frac{8}{5} = 1.6$ M1

Area of triangle $= \frac{1}{2} \times (5.5 - 1.6) \times 3$ M1

$= 5.85$ square units. A1 **[3]**

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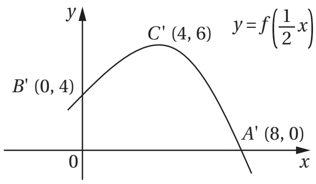
5 $A(4,0) \longrightarrow A'(8,0)$

$B(0,4) \longrightarrow B'(0,4)$

$C(2,6) \longrightarrow C'(4,6)$



Correct shape B1

Correct coordinates for B' B1

Correct coordinates for A' and C' B1 **[3]**

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6 a $AC = 8$

Rearranged cosine rule: $\cos\theta = \frac{3^2 + 8^2 - 7^2}{2(3)(8)}$ M1

$= \frac{1}{2}$

$\theta = \cos^{-1}\left(\frac{1}{2}\right)$ M1

$= 60°$ A1 **[3]**

b Area of triangle $= \frac{1}{2}(3)(8)\sin 60°$

$= 10.392\ldots$ M1

Angle CAD $= 180° - 60°$ A1

$= 120°$

Sector area $= \left(\frac{120°}{360°}\right) \times \pi \times (8)^2$ M1

$= 67.020\ldots$

	Area of shape	$= 10.392\ldots + 67.020\ldots$		
		$= 77.412\ldots$		
		$= 77.4\text{ cm}^2$ (1 decimal place)	A1	**[4]**

7	**a**	$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$		
		$= (0\mathbf{i} + 6\mathbf{j}) - (4\mathbf{i} + \mathbf{j})$		
		$= -4\mathbf{i} + 5\mathbf{j}$	B1	**[1]**
	b	$\overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB}$		
		$= (p\mathbf{i} + 10\mathbf{j}) - (0\mathbf{i} + 6\mathbf{j})$		
		$= p\mathbf{i} + 4\mathbf{j}$		
		$\lvert\overrightarrow{BC}\rvert = \sqrt{p^2 + 16}$	M1, A1	**[2]**
	c	*ABCD* a square so $\lvert\overrightarrow{BC}\rvert = \lvert\overrightarrow{AB}\rvert$		
		$\sqrt{p^2 + 16} = \sqrt{(-4)^2 + 5^2}$	M1	
		leading to $p^2 + 16 = 41$		
		$p^2 = 25$	A1	
		$p = 5$ (reject $p = -5$ as $p > 4$)	A1	**[3]**
	d	$\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD}$	M1	
		$= \overrightarrow{OA} + \overrightarrow{BC}$	M1	
		$= (4\mathbf{i} + \mathbf{j}) + (5\mathbf{i} + 4\mathbf{j})$		
		$= 9\mathbf{i} + 5\mathbf{j}$	A1	**[3]**

8	**a**	$y = x^2 - 8x + 19$		
		$\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - 8$	M1	
		$x = 5, \frac{\mathrm{d}y}{\mathrm{d}x} = 2(5) - 8 = 2$ $x = 5, y = (5)^2 - 8(5) + 19 = 4$	A1, A1	
		Equation of *T*: $y - 4 = 2(x - 5)$		
		leading to $y = 2x - 6$	A1	**[4]**
	b	*L* is parallel to *T* so $\text{grad}_L = 2$	M1	
		L passes through *A*(2, 7)		
		Equation of *L*: $y - 7 = 2(x - 2)$		
		leading to $y = 2x + 3$	A1	**[2]**
	c	*x*-coordinate of *B*		
		Solve: $x^2 - 8x + 19 = 2x + 3$		
		leading to: $(x - 2)(x - 8) = 0$		
		$x = 8$ ($x = 2$ corresponds to point *A*)	M1	
		$\int_2^8 x^2 - 8x + 19\,dx = \left[\frac{1}{3}x^3 - 4x^2 + 19x\right]_2^8$		
		$= \left(\frac{200}{3}\right) - \left(\frac{74}{3}\right)$		
		$= 42$	A1	

$\int_2^8 2x + 3\,dx = [x^2 + 3x]_2^8$

$= (88) - (10)$

$= 78$ A1

Area of bounded region $= 78 - 42$ M1

$= 36$ square units. A1 **[5]**

Alt1. Area $= \int_2^8 (2x+3) - (x^2 - 8x + 19)\,dx$ M1

$= \int_2^8 -x^2 + 10x - 16\,dx$ A1

$= \left[-\frac{1}{3}x^3 + 5x^2 - 16x\right]_2^8$ M1, A1

$= \left(\frac{64}{3}\right) - \left(-\frac{44}{3}\right)$

$= 36$ square units. A1 **[5]**

Alt2. Finds y-coordinate of B

$x = 8, y = 2(8) + 3$

$= 19$ M1

Area of trapezium $= \frac{1}{2} \times (7 + 19) \times 6$

$= 78$ A1

Area under curve: $\int_2^8 x^2 - 8x + 19\,dx = \left[\frac{1}{3}x^3 - 4x^2 + 19x\right]_2^8$ M1, A1

$= \left(\frac{200}{3}\right) - \left(\frac{74}{3}\right)$

$= 42$

Answer: $78 - 42 = 36$ square units. A1 **[5]**