Answers

Chapter 1 Number recall

1.1 Number: Recall and extension

Exercise 1A page 5

1 **a** $6\frac{5}{6}$ **b** $8\frac{4}{15}$ **c** $3\frac{19}{42}$ **d** $4\frac{7}{8}$

e $3\frac{13}{120}$ **f** $2\frac{25}{36}$

2 **a** $10\frac{1}{2}$ **b** $2\frac{68}{77}$ **c** $3\frac{1}{19}$ **d** $3\frac{37}{120}$ **e** $6\frac{7}{90}$

f $2\frac{1}{6}$ **g** $41\frac{7}{10}$ **h** 2 **i** $\frac{4}{27}$

3 15 : 28

4 32%

5 36%

6 4.464×10^4 minutes

7 480

8 5.12×10^{21}

9 80 litres

10 More money, e.g. 100 loaves at £1 would change to 88 loaves at £1.14 = £100.32

11 **a** 4.617×10^{12} **b** 1.311×10^{14}

12 70 weeks

1.2 Manipulation of surds

Exercise 1B

1 **a** $\sqrt{6}$ **b** $\sqrt{15}$ **c** 2 **d** 4 **e** $\sqrt{14}$

f 6 **g** 6 **h** $\sqrt{30}$

2 **a** 2 **b** $\sqrt{5}$ **c** $\sqrt{6}$ **d** $\sqrt{3}$ **e** 2

f $\sqrt{6}$ **g** 1 **h** 3

3 **a** $2\sqrt{3}$ **b** $4\sqrt{2}$ **c** $8\sqrt{5}$ **d** 24 **e** $2\sqrt{7}$

f $6\sqrt{3}$

4 **a** $\sqrt{3}$ **b** 1 **c** $2\sqrt{2}$ **d** $\sqrt{5}$ **e** $\sqrt{3}$

f $\sqrt{2}$ **g** $\sqrt{7}$ **h** $2\sqrt{3}$ **i** 1

5 **a** a **b** 1 **c** $\sqrt{a}$

6 **a** $3\sqrt{2}$ **b** $2\sqrt{6}$ **c** $2\sqrt{3}$ **d** $5\sqrt{2}$ **e** $2\sqrt{2}$

f $3\sqrt{3}$ g $4\sqrt{2}$ **h** $10\sqrt{2}$ **i** $10\sqrt{10}$ **j** $5\sqrt{10}$

k $7\sqrt{2}$ **l** $9\sqrt{3}$

7 **a** 36 **b** $16\sqrt{30}$ **c** 54 **d** 32 **e** $48\sqrt{6}$

f $48\sqrt{6}$ **g** $18\sqrt{15}$ **h** 84

8 **a** $20\sqrt{6}$ **b** 24 **c** 16 **d** 18 **e** $10\sqrt{21}$

f $6\sqrt{14}$ **g** 36 **h** $12\sqrt{30}$

9 **a** 6 **b** $3\sqrt{5}$ **c** $6\sqrt{6}$ **d** $2\sqrt{3}$ **e** 6

f $2\sqrt{7}$ **g** 5 **h** 24

10 **a** $2\sqrt{3}$ **b** 4 **c** $6\sqrt{2}$ **d** 4 **e** $3\sqrt{2}$

f $\sqrt{7}$ **g** $8\sqrt{3}$ **h** $10\sqrt{3}$ **i** 6

11 **a** abc **b** $\frac{a}{c}$ **c** $c\sqrt{b}$

12 **a** 20 **b** 24 **c** 10 **d** 24 **e** 3 **f** 6

13 **a** $\frac{3}{4}$ **b** $8\frac{1}{3}$ **c** $\frac{5}{16}$ **d** 12 **e** 2

14 **a** False **b** False

15 **a** $5\sqrt{2}$ **b** $2\sqrt{3}$ **c** $14\sqrt{2}$ **d** $6\sqrt{3}$ **e** $3\sqrt{5}$

f $23\sqrt{2}$

Exercise 1C

1 Expand the brackets each time.

2 **a** $2\sqrt{3} - 3$ **b** $3\sqrt{2} - 8$ **c** $10 + 4\sqrt{5}$

d $12\sqrt{7} - 42$ **e** $15\sqrt{2} - 24$ **f** $9 - \sqrt{3}$

3 **a** $2\sqrt{3}$ **b** $1 + \sqrt{5}$ **c** $-1 - \sqrt{2}$

d $\sqrt{7} - 30$ **e** -41 **f** $7 + 3\sqrt{6}$

g $9 + 4\sqrt{5}$ **h** $3 - 2\sqrt{2}$ **i** $11 + 6\sqrt{2}$

4 **a** $3\sqrt{2}$ cm **b** $2\sqrt{3}$ cm **c** $2\sqrt{10}$ cm

5 **a** $\sqrt{3} - 1$ cm^2 **b** $2\sqrt{5} + 5\sqrt{2}$ cm^2 **c** $2\sqrt{3} + 18$ cm^2

6 **a** $\frac{\sqrt{3}}{3}$ **b** $\frac{\sqrt{2}}{2}$ **c** $\frac{\sqrt{5}}{5}$ **d** $\frac{\sqrt{3}}{6}$

e $\sqrt{3}$ **f** $\frac{5\sqrt{2}}{2}$ **g** $\frac{3}{2}$ **h** $\frac{5\sqrt{2}}{2}$

i $\frac{\sqrt{21}}{3}$ **j** $\frac{\sqrt{2}+2}{2}$ **k** $\frac{2\sqrt{3}-3}{3}$ **l** $\frac{5\sqrt{3}+6}{3}$

7 **a** **i** 1 **ii** –4 **iii** 2 **iv** 17 **v** –44

b They become whole numbers. Difference of two squares makes the ‘middle terms’ (and surds) disappear.

8 Possible answer: $80^2 = 6400$, so $80 = \sqrt{6400}$ and $10\sqrt{70} = \sqrt{7000}$

Since $6400 < 7000$, there is not enough cable.

9 $9 + 6\sqrt{2} + 2 - (1 - 2\sqrt{8} + 8) = 11 - 9 + 6\sqrt{2} + 4\sqrt{2} = 2 + 10\sqrt{2}$

10 $x^2 - y^2 = (1 + \sqrt{2})^2 - (1 - \sqrt{8})^2 = 1 + 2\sqrt{2} + 2 - (1 - 2\sqrt{8} + 8) =$

$3 - 9 + 2\sqrt{2} + 4\sqrt{2} = -6 + 6\sqrt{2}$ $(x + y)(x - y) = (2 - \sqrt{2})(3\sqrt{2}) = 6\sqrt{2} - 6$

11 **a** $\frac{3\sqrt{2}-1}{17}$ **b** $\frac{5(3+\sqrt{3})}{6}$ **c** $\frac{7\sqrt{2}-4}{41}$ **d** $\frac{11+6\sqrt{2}}{7}$

e $13-5\sqrt{5}$ **f** $\frac{13+7\sqrt{3}}{22}$ **g** $-5-2\sqrt{6}$ **h** $\frac{32+7\sqrt{6}}{10}$

12 **a** $x = 23, y = 9$ **b** $x = 128, y = 648$ **c** $x = \frac{1}{2}, y = -\frac{1}{2}$

d $x = 2, y = -6$

Exam-style questions

1 $1\frac{1}{9}$

2 **a** 1.08×10^9 **b** 3.33×10^{-9} seconds

3 $y = \frac{2}{3}x$

4 2 : 3

5 139.5p

6 **a** $13 - 6\sqrt{5}$ **b** $1\frac{2}{41} + \frac{6}{41}\sqrt{5}$

2 Algebra recall

2.1 Recall of basic algebra

Exercise 2A

1 **a** All of them **b** $\frac{1}{2}$

2 **a** $15 - 5m$ **b** $6x + 21$ **c** $x^2 + 2x$

d $10m - 2m^2$ **e** $3nm - 3np$

3 **a** $3(6 - m)$ **b** $x(x + 5)$ **c** $m(10 - m)$

d $3(5s^2 + 1)$ **e** $n(3 - p)$

4 **a** $-3x - 8y$ **b** $-2a + 4b$

5 **a** Side AF – side DE = $4x - 1 - x = 3x - 1$

b $14x$ **c** 84 cm

6 Darren has added 2 and 3 instead of multiplying, and has added 2 and –5 instead of multiplying. The correct answer is $6x - 10$.

7 **a** $4(2y + 4)$ **b** $3(2z + 1)$

8 4 cm × 12 cm

9 **a** 4.1 **b** 8 **c** 4.525

10 Any values that work, e.g. $x = 8$, $b = 4$, $h = 32$

11 **a** x must be 2, y can be any other prime number

b x must be an odd prime, y can be any other prime number

12 **a** $6 + 3 \times 9 - 5 \times 3 = 18$ **b** $2 \times 6 - 9 + 3 \times 3 = 12$

13 **a** $\frac{450}{5n}$ **b** £390

14 $6(3x + 5) - 2(x - 2) = 18x + 30 - 2x + 4 = 16x + 34$

15 **a** Both calculations give the cost of 5 meals and 5 desserts

b Easier to work out as bracketed term evaluates to 10 **c** £50

16 No common factors

17 **a** $3 \times (5 + 1) = 3 \times 6 = 18$, $3 \times 5 + 3 = 15 + 3 = 18$

b $3 \times (n + 2 + 1) = 3 \times (n + 3) = 3n + 9$, $3 \times (n + 2) + 3 = 3n + 6 + 3 = 3n + 9$

18 **a** $12p^3 - 4p^2q$ **b** $10t^4 + 35t^2$ **c** $10x^2 + 35xy$

d $10m^2 - 2m^5$ **e** $8s^4 + 24s^3t$ **f** $6nm^3 - 6n^2m^2$

19 **a** $23x + 11$ **b** $9y + 7$ **c** $2x - 8$

d $22x + 9$ **e** $14x^2 - 10x$ **f** $2x^3 + 17x^2 - 9$

20 **a** $3p(3p + 2t$ **b** $4m(3p - 2m)$ **c** $4ab(4a + 1)$

d $2(2a^2 - 3a + 1) = 2(2a - 1)(a - 1)$ **e** $5xy(4y + 2x + 1)$

f $4mt(2t - m)$

2.2 Expanding brackets and collecting like terms

Exercise 2B

1 **a** $2b + 4de$ **b** $2t$ **c** $4y^2$ **d** $3a^2d$

2 **a** $2x$ and $2y$ **b** a and $7b$

3 **a** $3x - 1 - x$ **b** $10x$ **c** 25 cm

4 **a** $22+ 5t$ **b** $14 + 3g$

5 **a** $2 + 2h$ **b** $6e + 20$

6 **a** $4m + 3p + 2mp$ **b** $3k + 4h + 5hk$ **c** $12r + 24p + 13pr$

d $19km + 20k - 6m$

7 **a** $9t^2 + 13t$ **b** $13y^2 + 5y$ **c** $10e^2 - 6e$ **d** $14k^2 - 3kp$

8 **a** $17ab + 12ac + 6bc$ **b** $18wy + 6ty - 8tw$

c $14mn - 15mp - 6np$ **d** $8r^3 - 6r^2$

9 **a** $5(f + 2s) + 2(2f + 3s) = 9f + 16s$ **b** £$(270f + 480s)$

c £42 450 – £30 000 = £12 450

10 For x-coefficients, 3 and 1 or 1 and 4; for y-coefficients, 5 and 1 or 3 and 4 or 1 and 7

11 $5(3x + 2) - 3(2x - 1) = 9x + 13$

Exercise 2C

1 $x^2 + 5x + 6$

2 $w^2 + 4w + 3$

3 $a^2 + 5a + 4$

4 $x^2 + 2x - 8$

5 $w^2 + 2w - 3$

6 $f^2 - f - 6$

7 $y^2 + y - 12$

8 $x^2 + x - 12$

9 $p^2 - p - 2$

10 $k^2 - 2k - 8$

11 $y^2 + 3y - 10$

12 $t^2 - 25$

13 $t^2 - 4$

14 $y^2 - 64$

15 $p^2 - 1$

16 $x^2 - 36$

17 $(x + 2)$ and $(x + 3)$

18 **a** B: $1 \times (x - 2)$
C: 1×2
D: $2 \times (x - 1)$

b $(x - 2) + 2 + 2(x - 1)$
$= 3x - 2$

c Area A $= (x - 1)(x - 2)$
$=$ area of square minus areas (B + C + D)
$= x^2 - (3x - 2)$
$= x^2 - 3x + 2$

19 **a** $x^2 - 9$ **b** **i** 9991 **ii** 39991

Exercise 2D

1 $6x^2 + 11x + 3$ **2** $8t^2 + 2t - 3$ **3** $12k^2 - 11k - 15$

4 $6a^2 - 7a - 3$ **5** $15g^2 - 16g + 4$ **6** $12d^2 + 5d - 2$

7 $8p^2 + 26p + 15$ **8** $6 - 7t - 10t^2$ **9** $6f^2 - 5f - 6$

10 $4 + 10t - 6t^2$

11 **a** $x^2 + 2x + 1$ **b** $x^2 - 2x + 1$ **c** $x^2 - 1$

d $p + q = (x + 1 + x - 1) = 2x$
$(p + q)^2 = (2x)^2 = 4x^2$
$p^2 + 2pq + q^2 = x^2 + 2x + 1 + 2(x^2 - 1) + x^2 - 2x + 1 = 4x^2 + 2x - 2x + 2 - 2 = 4x^2$

12 **a** $(3x - 2)(2x + 1) = 6x^2 - x - 2$, $(2x - 1)(2x - 1) = 4x^2 - 4x + 1$, $(6x - 3)(x + 1)$
$= 6x^2 + 3x - 3$, $(3x + 2)(2x + 1) = 6x^2 + 7x + 2$

b Multiply the x terms to match the x^2 term and/or multiply the constant terms to get the constant term in the answer.

Exercise 2E

1 $4x^2 - 1$

2 $25y^2 - 9$

3 $16m^2 - 9$

4 $16h^2 - 1$

5 $4 - 9x^2$

6 $36 - 25y^2$

7 $a^2 - b^2$

8 $4m^2 - 9p^2$

9 $a^2b^2 - c^2d^2$

10 $a^4 - b^4$

11 **a** $a^2 - b^2$

b Dimensions: $a + b$ by $a - b$; Area: $a^2 - b^2$

c Areas are the same, so $a^2 - b^2 = (a + b) \times (a - b)$

12 First shaded area is $(2k)^2 - 1^2 = 4k^2 - 1$. Second shaded area is $(2k + 1)(2k - 1) = 4k^2 - 1$

Exercise 2F

1 $x^2 + 10x + 25$

2 $t^2 + 12t + 36$

3 $m^2 - 6m + 9$

4 $k^2 - 14k + 49$

5 $9x^2 + 6x + 1$

6 $25y^2 + 20y + 4$

7 $9x^2 - 12x + 4$

8 $x^2 + 2xy + y^2$

9 $m^2 - 2mn + n^2$

10 $m^2 - 6mn + 9n^2$

11 $x^2 - 10x$

12 $x^2 - 4x$

13 **a** Bernice has just squared the first term and the second term.
She hasn't written down the brackets twice.

b Pete has written down the brackets twice but has worked out $(3x)^2$ as $3x^2$ and not $9x^2$.

c $9x^2 + 6x + 1$

14 Whole square is $(2x)^2 = 4x^2$.
Three areas are $2x - 1$, $2x - 1$ and 1.
$4x^2 - (2x - 1 + 2x - 1 + 1) = 4x^2 - (4x - 1) = 4x^2 - 4x + 1$

Exercise 2G

1 **a** $x^3 + 4x^2 + 2x - 1$ **b** $8x^3 + 22x^2 + 9x + 1$

c $2x^4 - 9x^3 - 14x^2 + 9$ **d** $x^6 + x^5 - 6x^4 + 4x^3 - x$

2 **a** $x^3 + 3x^2 + 3x + 1$ **b** $8x^3 - 12x^2 + 6x - 1$

c $27x^3 + 54x^2 + 36x + 8$ **d** $64x^3 - 144x^2 + 108x - 27$

3 **a** $2x - 2$ **b** $x - 1$ **c** $6x - 13 + \frac{6}{x}$ **d** $16x - x^{-\frac{2}{3}}$

2.3 Factorising

Exercise 2H

1 **a** $4(2m + 3k)$ **b** $m(n + 3)$ **c** $g(5g + 3)$

d $y(3y + 2)$ **e** $t(4t - 3)$ **f** $3m(m - p)$

g $3p(2p + 3t)$ **h** $2p(4t + 3m)$ **i** $2(2a^2 + 3a + 4)$

j $3b(2a + 3c + d)$ **k** $t(5t + 4 + a)$ **l** $3mt(2t - 1 + 3m)$

m $2ab(4b + 1 - 2a)$ **n** $5pt(2t + 3 + p)$

2 **c** and **d** do not factorise.

a $m(5 + 2p)$ **b** $t(t - 7)$

e $a(4a - 5b)$ **f** $b(5a - 3bc)$

3 **a** Bernice

b Aidan has not taken out the largest possible common factor. Craig has taken m out of both terms but there isn't an m in the second term.

4 There are no common factors.

5 $4x^3 - 12x$, $2x - 6$

Exercise 2I

1 $(x + 2)(x + 3)$

2 $(m + 2)(m + 5)$

3 $(p + 2)(p + 12)$

4 $(w + 2)(w + 9)$

5 $(a + 2)(a + 6)$

6 $(b + 8)(b + 12)$

7 $(t - 2)(t - 3)$

8 $(d - 4)(d - 1)$

9 $(x - 3)(x - 12)$

10 $(t - 4)(t - 9)$

11 $(y - 4)(y - 12)$

12 $(j - 6)(j - 8)$

13 $(y + 6)(y - 1)$

14 $(m + 2)(m - 6)$

15 $(n + 3)(n - 6)$

16 $(m + 4)(m - 11)$

17 $(t + 9)(t - 10)$

18 $(h + 8)(h - 9)$

19 $(t + 7)(t - 9)$

20 $(y + 10)^2$

21 $(m - 9)^2$

22 $(x - 12)^2$

23 $(d + 3)(d - 4)$

24 $(q + 7)(q - 8)$

25 $(x + 2)(x + 3)$, giving areas of $2x$ and $3x$, or $(x + 1)(x + 6)$, giving areas of x and $6x$.

26 **a** $x^2 + (a + b)x + ab$ **b** **i** $p + q = 7$ **ii** $pq = 12$

c 7 can only be 1×7 and $1 + 7 \neq 12$

Exercise 2J

1 $(x + 3)(x - 3)$

2 $(t + 5)(t - 5)$

3 $(m + 4)(m - 4)$

4 $(3 + x)(3 - x)$

5 $(7 + t)(7 - t)$

6 $(k + 10)(k - 10)$

7 $(2 + y)(2 - y)$

8 $(x + 8)(x - 8)$

9 $(t + 9)(t - 9)$

10 **a** $x^2 + 4x + 4 - (x^2 + 2x + 1) = 2x + 3$ **b** $(a + b)(a - b)$

c $(x + 2 + x + 1)(x + 2 - x - 1) = (2x + 3)(1) = 2x + 3$

d The answers are the same.

e $(x + 1 + x - 1)(x + 1 - x + 1) = (2x)(2) = 4x$

11 $(x + y)(x - y)$

12 $(x + 2y)(x - 2y)$

13 $(x + 3y)(x - 3y)$

14 $(3x + 1)(3x - 1)$

15 $(4x + 3)(4x - 3)$

16 $(5x + 8)(5x - 8)$

17 $(2x + 3y)(2x - 3y)$

18 $(3t + 2w)(3t - 2w)$

19 $(4y + 5x)(4y - 5x)$

Exercise 2K

1 $(2x + 1)(x + 2)$

2 $(7x + 1)(x + 1)$

3 $(4x + 7)(x - 1)$

4 $(3t + 2)(8t + 1)$

5 $(3t + 1)(5t - 1)$

6 $(4x - 1)^2$

7 $3(y + 7)(2y - 3)$

8 $4(y + 6)(y - 4)$

9 $(2x + 3)(4x - 1)$

10 $(2t + 1)(3t + 5)$

11 $(x - 6)(3x + 2)$

12 $(x - 5)(7x - 2)$

13 $4x + 1$ and $3x + 2$

14 **a** All the terms in the quadratic have a common factor of 6.

b $6(x + 2)(x + 3)$. This has the highest common factor taken out.

Exercise 2L

1 $(x + 2y)(x + 3y)$

2 $(x + 7y)(x + 3y)$

3 $(x – y)(x – 4y)$

4 $(x + y)(x – 7y)$

5 $(x – 9y)(x + 8y)$

6 $(2x + y)(x + 2y)$

7 $(3x + 2y)(x – 6y)$

8 $(5x + 2y)(x – 3y)$

9 $(3x + 5y)(2x + y)$

10 $(5x – y)(3x + y)$

11 $(x^2 + 5y^2)\ (x^2 – 5y^2)$

12 $2x(x + 5)(x – 5)$

13 $(4x^2 + 5y^2)\ (4x^2 – 5y^2)$

14 $16(x + 1)$

15 $8x$

16 $5(x + 1)(x – 1)$

17 $–(4x + 1)$ or $–4x – 1$

18 $(8x + 1)\ (2x + 1)$

Exam-style questions

1 $a = 0.1$

2 $4x^2 – 3$

3 $26y$

4 **a** $(3x + 2)(3x – 2)$ **b** $(3x – 2)(2x + 1)$ **c** $\frac{2x+1}{3x+2}$

5 $7(d – 2)$

6 **a** $x^3 – 6x^2 + 3x + 18$ **b** $64 – 48x + 12x^2 – x^3$ **c** $x + 1$

7 $16(x + 1)(x – 1)$

8 $6x^2(2x – 3)(x + 4)$

Chapter 3 Geometry recall 1

3.1 Perimeter of compound shapes

Exercise 3A

a 10 cm **b** 12 cm **c** 12 cm **d** 14 cm **e** 12 cm **f** 12 cm

3.2 Area of basic shapes

Exercise 3B

1 **a** 21 cm^2 **b** 12 cm^2 **c** 140 cm^2

d 40 cm^2 **e** 65 m^2 **f** 80 cm^2

2 **a** 65 cm^2 **b** 50 m^2

3 **a** 96 cm^2 **b** 70 cm^2 **c** 10 cm^2

4 **a** 27.5 cm, 36.25 cm^2 **b** 33.4 cm, 61.2 cm^2 **c** 38.5 m, 90 m^2

5 **a** 57 m^2 **b** 7.2.5 cm^2 **c** 84 m^2

Exercise 3C

1 **a** 8 cm, 25.1 cm, 50.3 cm^2 **b** 5.2 m, 16.3 m, 21.2 m^2

c 6 cm, 37.7 cm, 113 cm^2 **d** 1.6 m, 10.1 m, 8.04 m^2

2 **a** 5π cm **b** 8π cm **c** 18π m **d** 12π cm

3 **a** 25π cm^2 **b** 36π cm^2 **c** 100π cm^2 **d** 0.25π m^2

4 8.80 m

5 4 complete revolutions

6 1p : 3.1 cm^2, 2p : 5.3 cm^2, 5p : 2.3 cm^2, 10p : 4.5 cm^2

7 0.83 m

8 38.6 cm

9 Claim is correct (ratio of the areas is just over 1.5 : 1)

10 **a** 18 cm^2 **b** 4π cm^2

11 9π cm^2

12 28.3 m^2

13 Diameter of tree is 9.96 m

14 45 complete revolutions

Exercise 3D

1 **a** **i** 21 cm^2 **ii** 63 cm^3 **b** **i** 48 cm^2 **ii** 432 cm^3

c **i** 36 m^2 **ii** 324 m^3

2 **a** A cross-section parallel to the side of the pool always has the same shape

b About 3 hours

3 **a** 21 cm^3, 210 cm^3 **b** 54 cm^2, 270 cm^2

4 146 cm^3

5 327 litres

6 1.02 tonnes

Exercise 3E

1 **a** **i** 226 cm^3 **ii** 207 cm^2 **b** **i** 14.9 cm^3 **ii** 61.3 cm^2

c **i** 346 cm^3 **ii** 275 cm^2 **d** **i** 1060 cm^3 **ii** 636 cm^2

2 **a** **i** 72π cm^3 **ii** 48π cm^2 **b** **i** 112π cm^3 **ii** 56π cm^2

3 £80

4 1.23 tonnes

5 332 litres

Exercise 3F

1 **a** 56 cm^3 **b** 1040 cm^3 **c** 160 cm^3

2 **a** Put the apexes of the pyramids together. The 6 square bases will then form a cube

b If the side of the base is a then the height will be $\frac{1}{2}a$. Total volume of the 6 pyramids is a^3. Volume of one pyramid is $\frac{1}{6}a^3 = \frac{1}{3} \times \frac{1}{2} \times a \times a^2 = \frac{1}{3}$ height × base area

3 **a** 73.3 m^3 **b** 45 m^3 **c** 3250 cm^3

4 6.0 cm

5 260 cm^3

Exercise 3G

1 **a** **i** 3560 cm^3 **ii** 1430 cm^2 **b** **i** 314 cm^3 **ii** 283 cm^2

c **i** 1020 cm^3 **ii** 679 cm^2

2 24π cm^2

3 **a** 816π cm^3 **b** 720π mm^3

4 **a** 4 cm **b** 6 cm

c Various answers, e.g. 60° gives 2 cm, 240° gives 8 cm

5 24π cm^2

6 If radius of base is r, slant height is $2r$.
Area of curved surface = $\pi r \times 2r = 2\pi r^2$, area of base = πr^2

7 2.81 cm

Exercise 3H

1 **a** 36π cm^3 **b** 288π cm^3 **c** 1330π cm^3

2 **a** 36π cm^2 **b** 100π cm^2 **c** 196π cm^2

3 65 400 cm^3, 7850 cm^2

4 **a** 1960 cm^2 **b** 8180 cm^3

5 125 cm

6 7.8 cm

7 48%

8 Radius of sphere = base radius of cylinder = r, height of cylinder = $2r$
Curved surface area of cylinder = circumference × height = $2\pi r \times 2r = 4\pi r^2$
= surface area of sphere

Exam-style questions

1 8.0 cm

2 **a** $24a^3$ cm³ **b** $52a^2$ cm²

3 $r < 3$

4 **a** $x(x + 1)$ **b** $\sqrt{\frac{x(x+1)}{p}}$

5 270 cm^3

6 17 cm

7 265

Chapter 4 Geometry recall 2

4.1 Special triangles and quadrilaterals

Exercise 4A

1 **a** $a = b = 70°$ **b** $e = 55°$, $f = 70°$ **c** $a = 110°$, $b = 55°$

d $c = e = 105°$, $d = 75°$ **e** h = i = 94° **f** $m = o = 49°$, $n = 131°$

2 40°, 40°, 100°

3 $a = b = 65°$, $c = d = 115°$, $e = f = 65°$, $g = 80°$, $h = 60°$, $i = 60°$, $j = 60°$, $k = 20°$

4 **a** $x = 25°$, $y = 15°$ **b** $x = 7°$, $y = 31°$ **c** $x = 60°$, $y = 30°$

5 **a** $x = 50°$; 60°, 70°, 120°, 110° – possibly trapezium

b $x = 60°$; 50°, 130°, 50°, 130° – parallelogram or isosceles trapezium

c $x = 30°$; 20°, 60°, 140°, 140° – possibly kite

d $x = 20°$; 90°, 90°, 90°, 90° – square or rectangle

6 52°

7 Both 129°

8 $y = 360° - 4x$

9 **a** 65°

b Trapezium, angle A + angle D = 180° and angle B + angle C = 180°

4.2 Angles in polygons

Exercise 4B

1	**a**	1440°	**b**	2340°	**c**	17 640°	**d**	7740°
2	**a**	150°	**b**	162°	**c**	140°	**d**	174°
3	**a**	9	**b**	15	**c**	102	**d**	50
4	**a**	15	**b**	36	**c**	24	**d**	72
5	**a**	12	**b**	9	**c**	20	**d**	40
6	**a**	130°	**b**	95°	**c**	130°		
7	**a**	50°	**b**	40°	**c**	95°		

8 Hexagon

9 **a** Octagon **b** 89°

10 **a** **i** 71° **ii** 109° **iii** Equal

b If S = sum of the two opposite interior angles, then $S + I = 180°$ (angles in a triangle), and we know $E + I = 180°$ (angles on a straight line), so $S + I = E + I$, therefore $S = E$

11 $a = 144°$

12 Three angles are 135° and two angles are 67.5°.

13 88°; $\frac{1440° - 5 \times 200}{5}$

14 **a** 36° **b** 10

4.3 Circle theorems

Exercise 4C

1 **a** 56° **b** 62° **c** 105° **d** 45° **e** 55°

f 52° **g** 24° **h** 80°

2 **a** 41° **b** 49° **c** 41°

3 **a** 72° **b** 37° **c** 72°

4 $\angle AZY = 40°$ (angles in a triangle), $a = 50°$ (angle in a semicircle = 90°)

5 68°

6 $\angle ABC = 180° - x$ (angles on a line), $\angle AOC = 360° - 2x$ (angle at centre is twice angle at circumference), reflex $\angle AOC = 360° - (360° - 2x) = 2x$ (angles at a point)

7 **a** x **b** $2x$ **c** $\angle ABC = (x + y)$ and $\angle AOC = 2(x + y)$

4.4 Cyclic quadrilaterals

Exercise 4D

1 **a** $a = 50°, b = 95°$ **b** $d = 110°, e = 110°, f = 70°$

c $g = 105°, h = 99°$ **d** $x = 40°, y = 34°$

2 **a** $x = 48°, y = 78°$ **b** $x = 36°, y = 72°$

c $x = 55°, y = 125°$ **d** $x = 35°$

3 **a** $x = 49°, y = 49°$ **b** $x = 80°, y = 100°$

c $x = 100°, y = 75°$ **d** $x = 92°, y = 88°$

e $x = 55°, y = 75°$ **f** $x = 95°, y = 138°$

g $x = 32°, y = 48°$ **h** $x = 52°$

4 **a** 71° **b** 125.5° **c** 54.5°

5 **a** $x + 2x - 30° = 180°$ (opposite angles in a cyclic quadrilateral), so $3x - 30° = 180°$

b $x = 70°$, so $2x - 30° = 110°$
$\angle DOB = 140°$ (angle at centre equals twice angle at circumference),
$y = 60°$ (angles in a quadrilateral)

6 **a** x

b $360° - 2x$

c $\angle ADC = \frac{1}{2}$reflex $\angle AOC = 180° - x$, so $\angle ADC + \angle ABC = 180°$

7 Let $\angle AED = x$, then $\angle ABC = x$ (opposite angles are equal in a parallelogram),
$\angle ADC = 180° - x$ (opposite angles in a cyclic quadrilateral), so $\angle ADE = x$ (angles on a line)

4.5 Tangents and chords

Exercise 4E

1 **a** 38° **b** 110° **c** 45°

2 **a** 6 cm **b** 3.21 cm **c** 8 cm

3 **a** $x = 12°$, $y = 156°$ **b** $x = 62°$, $y = 28°$ **c** $x = 30°$, $y = 60°$

4 **a** 62° **b** 66° **c** 19° **d** 20°

5 19.5 cm

6 5.77 cm

7 $\angle OCD = 58°$ (triangle OCD is isosceles), $\angle OCB = 90°$ (tangent/radius theorem), so
$\angle DCB = 32°$, hence triangle BCD is isosceles (2 equal angles)

8 **a** OAB and OAC are congruent by RHS: A and C are right angles,
OB is a hypotenuse for both, and OA and OC are equal (radii). The results follow.

b As $\angle AOB = \angle COB$, so $\angle ABO = \angle CBO$, so OB bisects $\angle ABC$

4.6 Alternate segment theorem

Exercise 4F

1 **a** $a = 65°$, $b = 75°$, $c = 40°$ **b** $d = 79°$, $e = 58°$, $f = 43°$

c $g = 41°$, $h = 76°$, $i = 76°$ **d** $k = 80°$, $m = 52°$, $n = 80°$

2 **a** $a = 75°$, $b = 75°$, $c = 75°$, $d = 30°$ **b** $a = 47°$, $b = 86°$, $c = 86°$, $d = 47°$

c $a = 53°$, $b = 53°$ **d** $a = 55°$

3 **a** 36° **b** 70°

4 **a** $x = 25°$ **b** $x = 46°$, $y = 69°$, $z = 65°$

c $x = 38°$, $y = 70°$, $z = 20°$ **d** $x = 48°$, $y = 42°$

5 $\angle ACB = 64°$ (angle in alternate segment), $\angle ACX = 116°$ (angles on a line), $\angle CAX = 32°$ (angles in a triangle), so triangle ACX is isosceles (two equal angles)

6 $\angle AXY = 69°$ (tangents equal and so triangle AXY is isosceles), $\angle XZY = 69°$ (alternate segment), $\angle XYZ = 55°$ (angles in a triangle)

Exam-style questions

1 Angle AED = the angle of a regular pentagon = 108°. Triangle AEB is isosceles so angle AEB = half of (180 – 108) = 36°. Angle DEB = 108 – 36 = 72°. The result follows.

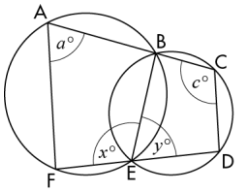
2 $3x + 4x + 5x + 6x = 180 \Rightarrow 18x = 180 \Rightarrow x = 10$. So angle A = 40° and B = 50°. These add up to 90° which means that AB and DC are parallel.

3 240°

4 60 cm²

5 65°

6



$a + x = 180$ and $y + c = 180$ (opposite angles of a cyclic quadrilateral).
So $a + x + y + c = 360$. But $x + y = 180$ so $a + b = 180$ and the result follows.

Chapter 5 Functions

5.1 Function notation

Exercise 5A

1	**a**	12	**b**	26	**c**	7	**d**	–2	**e**	3
2	**a**	0.5	**b**	5	**c**	50.5	**d**	2.5	**e**	0.625 or $\frac{5}{8}$
3	**a**	5	**b**	–3	**c**	999 801	**d**	1	**e**	$\frac{1}{8}$
4	**a**	4	**b**	32	**c**	1	**d**	$\frac{1}{2}$	**e**	$\frac{1}{8}$
5	**a**	3	**b**	2	**c**	0	**d**	–1	**e**	5
6	**a**	7.5	**b**	–2.5	**c**	–5				
7	**a**	6	**b**	97	**c**	3.25				
8	**a**	6	**b**	Check students' graphs. The functions intersect at (6, 4).						
9	**a**	3	**b**	$-\frac{1}{2}$	**c**	$x = 2$	**d**	$x = 0$		
10	**a**	$4x + 7$	**b**	$12x - 1$	**c**	$12x + 2$	**d**	4		
11	**a**	4 and –4	**b**	2 and –2	**c**	$\frac{4}{3}$ and $-\frac{4}{3}$				
12	**a**	0.19 and –5.19	**b**	0.10 and –2.60						
13	**a**	$-\frac{5}{2}$ and 4	**b**	$\frac{3}{2}$ and 0	**c**	$-\frac{20}{3}$				

5.2 Domain and range of a function

Exercise 5B

1 **a** $x < 0$ **b** -1 **c** $x < -1$

d $-\frac{1}{2}$ **e** $x = 1$ and $x = 2$

2 **a** {10, 17, 26} **b** {1, 2, 5} **c** $\{y : 2 \leq y \leq 5\}$

d $\{y : y \geq 101\}$ **e** Same as **d**

3 **a** {0, 1, 4} **b** $\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\}$ **c** {5, 7, 9, 11}

d {5, 4, 3, 2} **e** {0, –2}

4 −2 can be squared so it could be in the domain.

$x^2 = -2$ has no solution so –2 cannot be in the range.

5 **a** Yes **b** No **c** Yes

6 5

7 **a** $f(x) > 16$ **b** Domain $x > 5$, range $f(x) > 61$

8 **a** $f(x) > 3$ **b** Domain $x > 0$, range $f(2x) > 3$

9 **a** 7 **b** 11

10 $a = 2$, $b = 8$

11 $-4 \leq f(x) \leq 5$

5.3 Sketching graphs of linear and quadratic functions

Exercise 5C

1 **a** Values of y: 27, 12, 3, 0, 3, 12, 27 **b** 6.8 **c** 1.8 or –1.8

2 **a** Values of y: 27, 18, 11, 6, 3, 2, 3, 6, 11, 18, 27

b 8.3 **c** 3.5 or –3.5

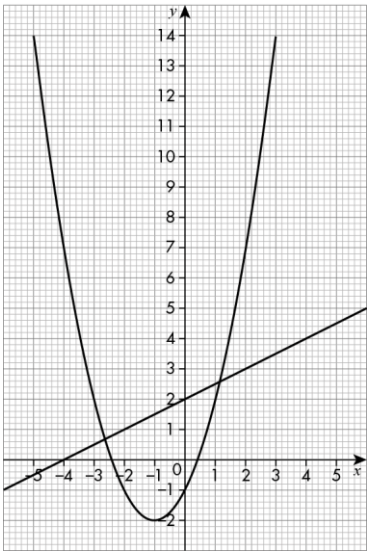
3 **a** Values of y: 27, 16, 7, 0, –5, –8, –9, –8, –5, 0, 7

b –8.8 **c** 3.4 or –1.4

4 **a** Values of y: 2, –1, –2, –1, 2, 7, 14

b 0.25 **c** 0.7 or –2.7

d



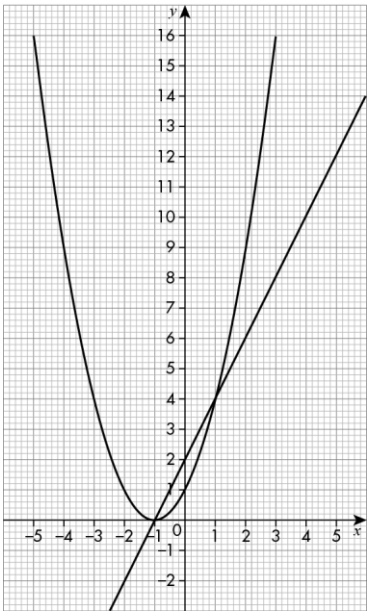
e (1.1, 2.6) and (–2.6, 0.7)

5 **a** Values of y: 18, 12, 8, 6, 6, 8, 12 **b** 9.75 **c** 2 or –1

d Values of y: 14, 9, 6, 5, 6, 9, 14 **e** (1, 6)

6 **a** Values of y: 4, 1, 0, 1, 4, 9, 16 **b** 7.3 **c** 0.4 or –2.4

d



e (1, 4) and (–1, 0)

7 **a** Values of y: 15, 9, 4, 0, –3, –5, –6, –6, –5, –3, 0, 4, 9 **b** –0.5 and 3

5.4 The significant points of a quadratic graph

Exercise 5D

1 **a** Values of y: 12, 5, 0, –3, –4, –3, 0, 5, 12

b 2 and –2

2 **a** The roots are positive and negative square roots of the constant term.

b Check predictions.

c Values of y: 15, 8, 3, 0, –1, 0, 3, 8, 15

d Values of y: 11, 4, –1, –4, –5, –4, –1, 4, 11

e 1 and –1, 2.2 and –2.2

3 **a** Values of y: 5, 0, –3, –4, –3, 0, 5, 12

b –4 and 0

4 **a** Values of y: 10, 4, 0, –2, –2, 0, 4, 10, 18

b –3 and 0

5 **a** The roots are 0 and the negative of the coefficient of x.

b Check predictions.

c Values of y: 10, 4, 0, –2, –2, 0, 4, 10

d Values of y: 6, 0, –4, –6, –6, –4, 0, 6, 14

e 0 and 3, –5 and 0

6 **a** Values of y: 9, 4, 1, 0, 1, 4, 9

b –2

c Only 1 root

7 **a** Values of y: 10, 3, –2, –5, –6, –5, –2, 3, 10

b 0.6 and 5.4

8 **a** Values of y: 19, 6, –3, –8, –9, –6, 1, 12

b 0.9 and –3.4

9 **a** Q1:(0, –4); Q2: (0, –1), (0, –5); Q3: (0, 0); Q4: (0, 0); Q5: (0, 0), (0, 0)

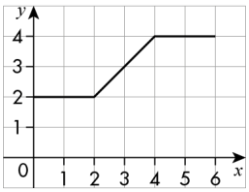
b Q1: (0, –4); Q2: (0, –1), (0, –5); Q3: (–2, –4): Q4: (3, –9), (–1.5, –2.25); Q5: (1.5, –2.25), (–2.5, –6.25)

c The y-intercept; the point where the x-value is the mean of the roots.

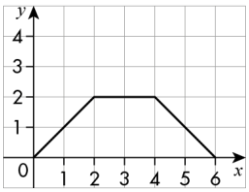
10 $y = (x - 3)^2 - 7$, $y = x^2 - 6x + 9 - 7$, $y = x^2 - 6x + 2$

Exercise 5E

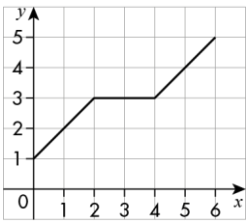
1 **a**



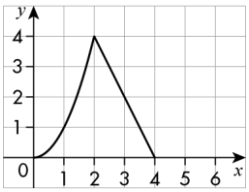
b



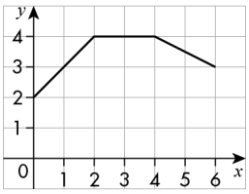
c



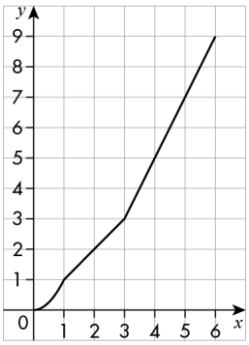
d



e



f



2 **a** $2 \le f(x) \le 4$ **b** $0 \le f(x) \le 2$ **c** $1 \le f(x) \le 5$

d $0 \le f(x) \le 4$ **e** $2 \le f(x) \le 4$ **f** $0 \le f(x) \le 9$

3 **a**

$f(x) = 2$	$-2 \le x < 0$
$= x + 2$	$0 \le x < 2$
$= 4$	$2 \le x \le 4$

b

$f(x) = 6$	$0 \le x < 2$
$= 8 - x$	$2 \le x < 4$
$= 4$	$4 \le x \le 6$

c

$f(x) = x^2$	$0 \le x < 3$
$= 9$	$3 \le x \le 5$

d $f(x) = 1$ $-4 \le x < -1$

$= -x$ $\quad -1 \leq x < 1$

$= -1$ $\quad 1 \leq x \leq 4$

Exam-style questions

1 **a** 110 and –90 **b** $n = 0$ or -1

2 **a** $x = 19.25$ **b** $f(x) \geq 15$

3 **a** $x = -4$ **b** $x = -6$

Chapter 6 Matrices

6.1 Introduction to matrices

Exercise 6A

1 **a** $\begin{pmatrix} 15 \\ -30 \end{pmatrix}$ **b** $\begin{pmatrix} -64 \\ 16 \end{pmatrix}$ **c** $\begin{pmatrix} 20 & -8 \\ -18 & 14 \end{pmatrix}$ **d** $\begin{pmatrix} 32 & 12 \\ -48 & -36 \end{pmatrix}$

e $\begin{pmatrix} -9 \\ 18 \end{pmatrix}$ **f** $\begin{pmatrix} -60 & 24 \\ 54 & -42 \end{pmatrix}$ **g** $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ **h** $\begin{pmatrix} -2 \\ 0.5 \end{pmatrix}$

2 **a** $\begin{pmatrix} 10 & 5 \\ 5 & 5 \end{pmatrix}$ **b** $\begin{pmatrix} 2 & 10 \\ 5 & 27 \end{pmatrix}$

c $\mathbf{C} = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$; $\mathbf{C}^2 = \begin{pmatrix} 5 & 4 \\ 4 & 5 \end{pmatrix}$; $\mathbf{C}^3 = \begin{pmatrix} 13 & 14 \\ 14 & 13 \end{pmatrix}$

3 **a** $\begin{pmatrix} 7 & -7 \\ 8 & 5 \end{pmatrix}$ **b** $\begin{pmatrix} 1 & -5 \\ 16 & 11 \end{pmatrix}$ **c** $\begin{pmatrix} 19 \\ 5 \end{pmatrix}$ **d** $\begin{pmatrix} 6 \\ 5 \end{pmatrix}$

e $\begin{pmatrix} 27 & 8 \\ 16 & 11 \end{pmatrix}$ **f** $\begin{pmatrix} -3 & -8 \\ 8 & 5 \end{pmatrix}$

4 **a** $\begin{pmatrix} 2 & 4 \\ 13 & 5 \end{pmatrix}$ **b** $\begin{pmatrix} 4 & 6 \\ 9 & 10 \end{pmatrix}$

5 **a** **(i)** $\mathbf{AB} = \begin{pmatrix} 7 & 3 \\ 6 & 2 \end{pmatrix}$ **(ii)** $\mathbf{BA} = \begin{pmatrix} 4 & 4 \\ 6 & 5 \end{pmatrix}$ **b** No

6 $x = -2$ and $y = 2$.

7 Sometimes true. **7f** is an example where it is true. **3c** is an example where it is not true.

8 $x = 1.5$, $y = -0.5$

9 $x = 2$, $y = 1$

6.2 The zero matrix and the identity matrix

Exercise 6B

1 **a** $\begin{pmatrix} -5 & 9 \\ 7 & -3 \end{pmatrix}$ **b** $\begin{pmatrix} -12 & 0 \\ 10 & 4 \end{pmatrix}$ **c** $\begin{pmatrix} 14 & 5 \\ -10 & -1 \end{pmatrix}$

d $\begin{pmatrix} 10 & -2 \\ -3 & 7 \end{pmatrix}$

2 **a** **Z** **b** **Q** **c** **Q** **d** **Z**

3 **a** **Z** **b** **I**

4 **a** $\begin{pmatrix} 8 & 2 \\ 2 & 4 \end{pmatrix}$ **b** $\begin{pmatrix} 17 & 6 \\ 6 & 5 \end{pmatrix}$

5 **a** $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ **b** $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ **c** Both equal I

6 **a** $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ **b** $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$ **c** Both equal **Z**

7 Check students' calculations, both parts should equal **I.**

8 Check students' calculations.

6.3 Transformations

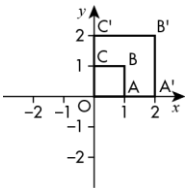
Exercise 6C

1 (0, 1)

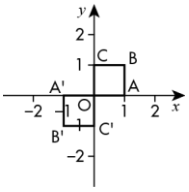
2 (–1, –1)

3 $x = 1$, $y = -3$

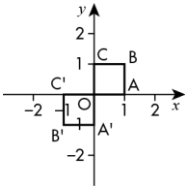
4 **a**



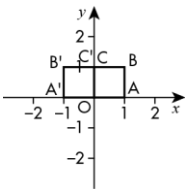
b



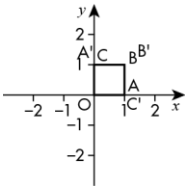
c



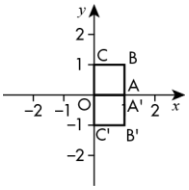
d



e



f



5 Enlargement scale factor 4, centre O.

6 $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

7 $4\sqrt{5}$

6.4 Combinations of transformations

Exercise 6D

1 **a** Reflection in the *x*-axis **b** Reflection in the *y*-axis

c Rotation 180° about O **d** Rotation 180° about O

e Does not move (identity matrix) **f** Does not move (identity matrix)

g Does not move (identity matrix)

2 **a** **i** Reflection in the *x*-axis **ii** $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

b **i** Reflection in the *y*-axis **ii** $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

c **i** Reflection in the *x*-axis **ii** $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

d **i** Reflection in the *y*-axis **ii** $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

3 Rotating 90° clockwise four times takes shape back to starting point (identity matrix).

4 Check students' calculations.

Exam-style questions

1 **a** $\begin{pmatrix} 8 & 12 \\ -8 & 4 \end{pmatrix}$ **b** $\begin{pmatrix} 10 & 4 \\ 6 & -4 \end{pmatrix}$ **c** $\begin{pmatrix} 9 & -2 \\ -4 & 8 \end{pmatrix}$

2 $x = 4$ and $y = 2$

3 (28, –17)

4 $\mathbf{MN} = \begin{pmatrix} 4-7 & -14+14 \\ 2-2 & -7+4 \end{pmatrix} = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix} = -3\mathbf{I}$ and $k = -3$

5 **a** **A**'(0, 1), **B**'(–1, 1) and **C**'(–1, 0)

b A 90° anticlockwise rotation about the origin

c A 180° rotation about the origin

6 Enlargement, centre the origin, scale factor –2

7 **a** **R** a reflection in the y-axis; **S** a reflection in the line $y = x$

b $\mathbf{SR} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ which represents a rotation of 90° clockwise about the origin

c **SR** represents first **R**, then **S**. **RS** represents first **S**, then **R** and is equivalent to a rotation of 90° anticlockwise about the origin. This is different to the **SR** transformation so the matrices must be different.