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Mathematics

H

SET A – Higher Tier

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Answers

Key to abbreviations used within the answers

M method mark (e.g. M1 means 1 mark for method)

A accuracy mark (e.g. A1 means 1 mark for accuracy)

B independent marks that do not require method to be shown (e.g. B2 means 2 independent marks)

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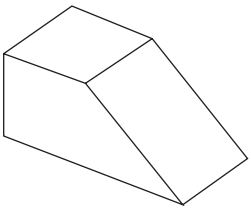
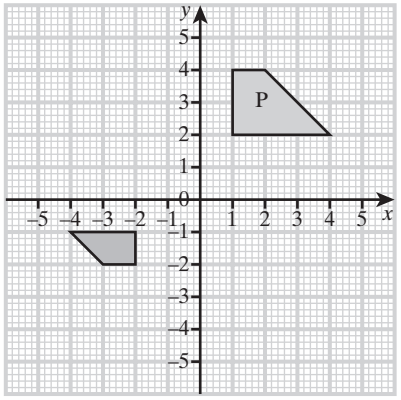
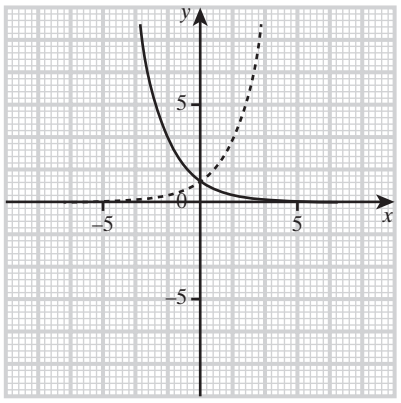
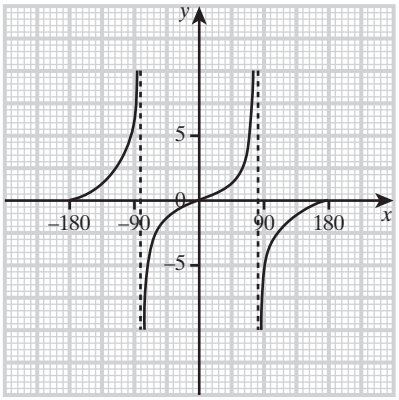
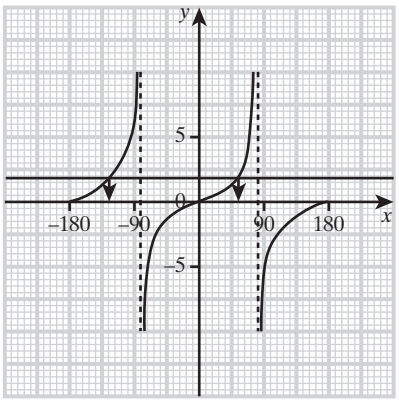
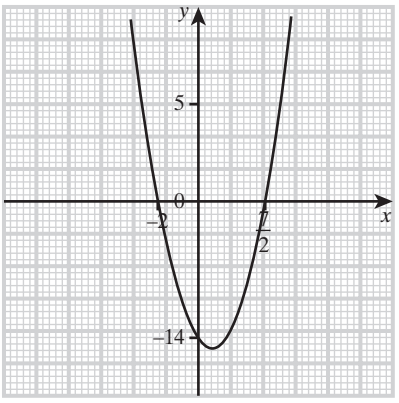
Paper 1

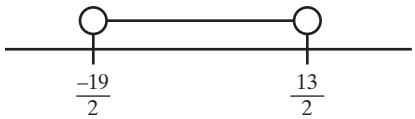
Question	Answer	Mark
1	$6 = 2 \times 3$ $15 = 3 \times 5$ $40 = 2 \times 2 \times 2 \times 5$ LCM is $2 \times 2 \times 2 \times 3 \times 5 = 120$	 M1 M1 A1
2	$3(x-1) = 6(10-x)$ $3x - 3 = 60 - 6x$ $9x = 63$ $x = 7$	M1 A1 A1
3 (a)		B1
(b)	$\frac{1}{2} \times (6+2) \times 2 \times 2 = 16 \text{ cm}^3$	M1 A1
4	$3 \times 3 \times 2 = 18$	M1 A1
5 (a)	100, 95, 90, 85, 80	B1
(b)	Common difference of –5 Sequence is $-5n + c$ $c = 105$ Formula is $-5n + 105$ **or** $105 - 5n$	 M1 A1
6 (a)	3.3×10^4	B1
(b)	8.2×10^{-3}	B1
(c)	2×10^{-7}	B1
7 (a)	$\frac{10}{30} = \frac{1}{3}$	B1
(b)	$\frac{10}{22} = \frac{5}{11}$	B1
(c)	$\frac{17}{20}$	B1
8	$p(3+q) = 3-q$ $3p + pq = 3 - q$ $pq + q = 3 - 3p$ $q(p+1) = 3 - 3p$ $q = \frac{3-3p}{1+p}$	M1 M1 A1
9	$(2x-1)^3 = (2x-1)(2x-1)^2$ $= (2x-1)(4x^2 - 4x + 1)$ $= 2x(4x^2 - 4x + 1) - 1(4x^2 - 4x + 1)$ $= 8x^3 - 8x^2 + 2x - 4x^2 + 4x - 1$ $= 8x^3 - 12x^2 + 6x - 1$	M1 A1 M1 A1

Question	Answer	Mark
10	 Correct sized shape Correctly positioned	 B1 B1
11 (a)	16	B1
(b)	$= \left(\frac{25}{16}\right)^{\frac{3}{2}} = \left(\frac{5}{4}\right)^3$ $= \frac{125}{64}$	M1 A1
12 (a)		B1
(b)	$y = 2^{-x}$	B1
13	$2(x^2 - 16)$ $= 2(x+4)(x-4)$	B1 B1
14	p = 150° radius meets tangent at 90°; OABC quadr so 360 – (90 +90) q = 75° angle subtended at centre is twice the angle subtended at the circumference r = 105° opposite angles in cyclic quad sum to 180°	 B1 B1 B1
15 (a)	Cyprus: 27 – 16 = 11 degrees Majorca: 24 – 17 = 7 degrees	B1 B1
(b)	Choose Cyprus because the median temperature is highest (comparing median temperatures (22 > 19)) OR choose Majorca because the temperatures are more consistent (comparing IQ ranges (7 < 11))	B1 B1 B1 B1

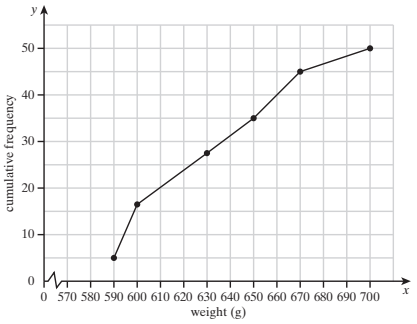
Question	Answer	Mark
16	$u_1 = 2\sqrt{3}$ $u_2 = 12$ $u_3 = 24\sqrt{3}$ $u_4 = 144$ $u_1 + u_2 + u_3 + u_4 = 2\sqrt{3} + 12 + 24\sqrt{3} + 144$ $= 156 + 26\sqrt{3}$	B1 B1 M1 A1
17 (a)	$2 : 5 = 6 : 15$ $3 : 7 = 15 : 35$ Brazil : walnut = $6 : 35$	B1 B1 B1
(b)	$105 \times \frac{6}{35} = 18$ brazil OR $105 \times \frac{3}{7} \times \frac{2}{5} = 18$ brazil	B1 B1
18	Use short division method to evaluate $7.1 \div 9$ (or equivalent) $0.78888\ldots$ $0.7\dot{8}$	M1 A1 A1
19 (a)	Shape Asymptotes at $\pm 90^\circ$  	B1 B1
(b)	Draw line through approximately $y = 1.73$  $x = 60^\circ$ $x = -120^\circ$	M1 A1 A1

Question	Answer	Mark
20	$2x^2 - 3x + 5 = 8 - 2x$ $2x^2 - x - 3 = 0$ $(2x - 3)(x + 1) = 0$ $2x - 3 = 0 \Rightarrow x = \frac{3}{2}$ and $y = 5$ $x + 1 = 0 \Rightarrow x = -1$ and $y = 10$	B1 M1 A1 A1 A1 A1 A1
21	$(3n+1)^2 - (3n-1)^2$ $= 9n^2 + 6n + 1 - (9n^2 - 6n + 1)$ $= 12n$ $= 6 \times 2n$ so is a multiple of 6	M1 A1 A1
22 (a)	$\sqrt{5}(2 - \sqrt{5})^2 = \sqrt{5}(9 - 4\sqrt{5})$ $= -20 + 9\sqrt{5}$	B1 B1
(b)	$\frac{5}{5 - 3\sqrt{5}} = \left(\frac{5}{5 - 3\sqrt{5}}\right)\left(\frac{5 + 3\sqrt{5}}{5 + 3\sqrt{5}}\right)$ $= \frac{25 + 15\sqrt{5}}{25 - 45} = \frac{25 + 15\sqrt{5}}{-20}$ $= -\frac{5}{4} - \frac{3}{4}\sqrt{5}$	M1 A1 A1
23 (a)	 Correct shape and orientation Intersection points marked at: $(-2, 0)\left(\frac{7}{2}, 0\right)(0, -14)$	B1 A1 A1 A1
(b)	Solution is $(-\infty, -2) \cup \left(\frac{7}{2}, \infty\right)$	B1 B1

Paper 2

Question	Answer	Mark
1 (a)	$510 \le x < 520$ (cm)	B1
(b)	$\sum \frac{fx}{f} = \frac{\left(\begin{array}{l}505\times 2+515\times 6+525\\ \times 1+535\times 4+545\times 3\end{array}\right)}{16}$ $= \frac{8400}{16} = 525$ cm	M1 A1
2	$(6y+18)+(x-23)=180$ $x+6y=185$ (1) $2x+(2x-4y)=180$ $x-y=45$ (2) Attempt to solve (1) and (2) simultaneously: $x=65,\ y=20$	M1 A1 A1 M1 A1
3	$-3 < \frac{2x+7}{4} \Rightarrow x > -\frac{19}{2}$ $\frac{2x+7}{4} < 5 \Rightarrow x < \frac{13}{2}$ Solution is $-\frac{19}{2} < x < \frac{13}{2}$  	B1 B1 B1 B1
4	$1000\times 1.02\times 1.0125^4 = £1072$	M1 A1
5 (a)	Paper 1: 0.7, 0.3 Paper 2: 0.8, 0.2, 0.8, 0.2	B1 B1
(b)	$1-(0.3\times 0.2)=0.94$ (or $0.8\times 0.7+0.8\times 0.3+0.2\times 0.7$)	M1 A1
6	$(x+10)(x-9)=0$ $x=-10$ or $x=9$	M1 A1 A1
7	$2\begin{pmatrix}3\\-2\end{pmatrix}-3\begin{pmatrix}-2\\-1\end{pmatrix}=\begin{pmatrix}6\\-4\end{pmatrix}+\begin{pmatrix}6\\3\end{pmatrix}$ $=\begin{pmatrix}12\\-1\end{pmatrix}$	M1 A1
8 (a)	$AC^2 = 20^2+33^2-2\times 20\times 33\times\cos 40^\circ$ $AC^2 = 477.82\ldots$ $AC = 21.9$ cm	M1 A1 A1
(b)	Area $ABD = \frac{1}{2}\times 20\times BD\times\sin 20^\circ$ Area $BDC = \frac{1}{2}\times BD\times 33\times\sin 20^\circ$ So ratio area of triangle ABD : area triangle BCD = 20 : 33	B1 B1 B1

Question	Answer	Mark
9	$y \propto \frac{1}{\sqrt{x}}$ $y = \frac{k}{\sqrt{x}}$ Substitute $x=16, y=12.5 \Rightarrow k=50$ $y = \frac{50}{\sqrt{x}}$ Substitute $x=0.25 \Rightarrow y$ – 100	 B1 M1 A1 A1
10	Bisect angle ABC with construction lines Bisect the angle just constructed (with construction lines)	 B1 B1
11 (a)	$m = \frac{2}{4} = \frac{1}{2}$ $c = -2$ $y = \frac{1}{2}x - 2$	B1 B1 B1
(b)	Gradient of new line is –2 Equation is $y = -2x + c$ Substituting $x=10, y=0$ $0 = -20 + c$ $c = 20$ $y = -2x + 20$	B1 M1 A1
12	$0.1\text{m}^3 = 100000\ \text{cm}^3$ Length scale factor is $\sqrt[3]{27} : \sqrt[3]{125} = 3:5$ Area scale factor is $3^2 : 5^2 = 9:25$ Required surface area is $100\,000 \times \frac{9}{25} = 36\,000\ \text{cm}^3$	B1 B1 B1 M1 A1
13	Using $11.5 < V < 12.5$ And $13.75 < P < 13.85$ $R_{\min} = \frac{11.5^2}{13.85} = 9.55\Omega$ $R_{\max} = \frac{12.5^2}{13.75} = 11.4\Omega$	 M1 A1 M1 A1

Question	Answer	Mark
14 (a)	5, 17, 27, 35, 45, 50	B1
(b)	Plot (590, 15), (610, 17)… etc. and join consecutive points with straight lines or a curve  	B1 B1
(c)	Draw line from 615 on x-axis, up to line, and across (left) to intersect y-axis at approx. 20 20 hedgehogs underweight implied 50 – 20 = 30 healthy hedgehogs Percentage of healthy hedgehogs is $\left(\frac{30}{50}\right)\times 100 = 60\%$	M1 A1 A1
15 (a)	$y = \frac{1}{x-2}$ $(x-2)y = 1$ $xy - 2y = 1$ $xy = 1 + 2y$ $x = \frac{1+2y}{y}$ $f^{-1}(x) = \frac{1+2x}{x}$	M1 A1 A1
(b)	$gf(x) = \frac{1}{(x-2)^2}$	B1
(c)	$fg(x) = \frac{1}{x^2-2}$ $(x-2)^2 = x^2 - 2$ $x^2 - 4x + 4 = x^2 - 2$ $x = \frac{3}{2}$	M1 A1
16	$AB = CD$ since opposite sides of a parallelogram are of equal length Angles $\angle BAE$ and $\angle FCD$ are equal, since opposite angles in a parallelogram are equal By SAS rule, ΔBAE and ΔDCF are congruent Therefore $BE = FD$ as required	B1 B1 B1 B1

Question	Answer	Mark
17	Volume of hemisphere $= \frac{2}{3}\pi r^3 = \frac{2}{3}\pi\left(\sqrt{3}\right)^3 = \frac{2}{3}\pi 3\sqrt{3} = 2\sqrt{3}\pi$ Volume of cone $= \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(3)2\sqrt{3} = 2\sqrt{3}\pi$ Total volume $= 2\sqrt{3}\pi + 2\sqrt{3}\pi = 4\sqrt{3}\pi$	M1 A1 M1 A1 A1
18 (a)	$1.3^4 + 2\times 1.3 - 7 = -1.54 < 0$ $1.5^4 + 2\times 1.5 - 7 = 1.0625 > 0$ There is a sign change, so there is a solution between $x = 1.3$ and $x = 1.5$	B1 B1
(b)	$x_0 = 1.4$ $x_1 = 1.4316$ $x_2 = 1.4262$ $x_3 = 1.427$	B1 B1 B1
19	$y = -\frac{b}{2a}$ $= -\left(\frac{-2}{-8}\right)$ $x = -\frac{1}{4}$ $y = 3 - 2\left(-\frac{1}{4}\right) - 4\left(-\frac{1}{4}\right)^2$ $= 3 + \frac{1}{2} - \frac{1}{4}$ $= \frac{13}{4}$ Maximum point is $\left(-\frac{1}{4}, \frac{13}{4}\right)$	M1 A1 M1 A1

Paper 3

Question	Answer	Mark
1	$\tan 35^\circ = \frac{12}{x}$ $x = \frac{12}{\tan 35^\circ} = 17.1$ cm	M1 A1
2	$\frac{£21120}{0.88} = £24000$	M1 A1
3	Using similar triangles $\frac{x+12}{13} = \frac{12}{10.5}$ Solve to give $x = 2.86$ cm	M1 A1 A1

Question	Answer	Mark
4	Let $x = 0.1\dot{2}\dot{7} = 0.127272727\ldots$	B1
	$10x = 1.27272727\ldots$	B1
	$1000x = 127.272727\ldots$	
	$990x = 126$	M1
	$x = \frac{126}{990}$	
	$= \frac{7}{55}$	A1
5 (a)	Equation – only valid for certain values of x	B1
(b)	Identity – true for all values of x	B1
(c)	Equation – only valid for certain values of x	B1
6	Distance = area under graph	M1
	$= \left(\frac{1}{2}\times 8\times 5\right) + (22\times 5) + \left(\frac{1}{2}\times 4\times 5\right)$	A1
	= 140m	A1
7	Use $A = \frac{\theta}{360}\times \pi r^2$	M1
	$\theta = \frac{250\times 360}{\pi\times 15^2}$	A1
	= 127°	
8 (a)	Summing areas under the bars	M1
	$25 + 14 + 16 + 4h = 70$	A1
	$4h = 15$	
	$h = 3.75$	A1
(b)	Sum of first two bars' area is 39, so median lies in 2nd bar	
	Suppose the median is x	
	Then by considering areas,	
	$25 + 7(x - 10) = 35$	M1
	Solve to give $x = 11.4$	A1
9	$12(x-3) - 2(x-2) = 3(x-2)(x-3)$	M1
	$12x - 36 - 2x + 4 = 3(x^2 - 5x + 6)$	
	$10x - 32 = 3x^2 - 15x + 18$	
	$3x^2 - 25x + 50 = 0$	A1
	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	M1
	$x = \frac{25 \pm \sqrt{25^2 - 4\times 3\times 50}}{6}$	
	$x = 5$ or $x = 3.33$	A1 A1
10 (a)	$\mathbf{b} - \mathbf{a}$	B1
(b)	$\frac{1}{n+1}(\mathbf{b} - \mathbf{a})$	B1

Question	Answer	Mark
(c)	$\mathbf{a} + \frac{1}{n+1}(\mathbf{b} - \mathbf{a})$	M1
	$= \frac{(n+1)\mathbf{a} + (\mathbf{b} - \mathbf{a})}{n+1}$	
	$= \frac{n\mathbf{a} + \mathbf{b}}{n+1}$	A1
(d)	$\overrightarrow{OD} = \lambda\overrightarrow{OC} = \frac{\lambda n}{n+1}\mathbf{a} + \frac{\lambda}{n+1}\mathbf{b}$	B1
	Also $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \mathbf{a} + \frac{2}{5}\overrightarrow{AF}$	
	$= \mathbf{a} + \frac{2}{5}\left(-\mathbf{a} + \frac{1}{2}\mathbf{b}\right) = \frac{3}{5}\mathbf{a} + \frac{1}{5}\mathbf{b}$	B1
	Equating coefficients and solving simultaneously	M1
	$\frac{\lambda n}{n+1} = \frac{3}{5}$ and $\frac{\lambda}{n+1} = \frac{1}{5}$	
	$\Rightarrow \lambda = \frac{3(n+1)}{5n}$ and $\lambda = \frac{n+1}{5}$	
	$\Rightarrow \frac{3(n+1)}{5n} = \frac{n+1}{5}$	
	$\Rightarrow n = 3$	A1
11 (a)	The graph shows a curve and not a straight line	B1
(b)	Draw a tangent line at the point on the curve where $x = 2$	M1
	Select two points on the line and calculate the gradient of the line using	
	$m = \frac{y_2 - y_1}{x_2 - x_1}$	M1
	Calculate $m \approx -0.4$	A1
	Deceleration ≈ 0.4 m/s^2	A1
12 (a)	$x^4 - 5x^2 - 1 = 0$	
	$x^4 - 5x^2 + 3 = 4$	B1
	$x = -2.3$ or $x = 2.3$	B1 B1
(b)	$x^4 - 5x^2 - x + 2 = 0$	
	$x^4 - 5x^2 + 3 = x + 1$	B1
	$x = -2,\ x = -0.8,\ x = 0.6,\ x = 2.25$	B1 B1
13	RGB may be arranged in 3! = 6 ways	B1
	Each arrangement has the probability occurring of	
	$\frac{2}{20}\times\frac{8}{19}\times\frac{10}{18} = \frac{4}{171}$	M1 A1
	Total probability is $6\times\frac{4}{171} = \frac{24}{171}$	M1 A1

Question	Answer	Mark
14 (a)	$2x^2 - 6x + 1$	
	$= 2\left(x^2 - 3x + \frac{1}{2}\right)$	M1
	$= 2\left(\left(x - \frac{3}{2}\right)^2 - \frac{9}{4} + \frac{1}{2}\right)$	M1
	$= 2\left(\left(x - \frac{3}{2}\right)^2 - \frac{7}{4}\right)$	A1
	$= 2\left(x - \frac{3}{2}\right)^2 - \frac{7}{2}$	A1
(b)	Range is $f(x) \geq -\frac{7}{2}, f(x) \in \mathbb{R}$	B1 B1
15	$8 = A \times 5^{-20t}$ (1)	M1
	$1.6 = A \times 5^{-40t}$ (2)	M1
	Equation (1) divide equation (2)	A1
	$5 = 5^{20t}$	
	$20k = 1$	
	$k = \frac{1}{20}$	A1
	Substitute in (1)	
	$8 = A \times 5^{-1}$	
	$A = 140$	A1
16	Total surface area $= \pi r^2 + \pi rl$	M1
	$= \pi r^2 + 25\pi r = 600\pi$	A1
	$r^2 + 25r - 600 = 0$	
	$(r + 40)(r - 15) = 0$	M1
	$r = 15$	A1
	$h = \sqrt{25^2 - 15^2} = 20\,\text{cm}$	M1
	Volume of cone $V = \frac{1}{3}\pi r^2 h$	M1
	$V = \frac{1}{3} \times \pi \times 15^2 \times 20$	
	$= 1500\pi\ \text{cm}^3$	A1

Question	Answer	Mark
17 (a)	$OA^2 = \left(2\sqrt{5}\right)^2 + 6^2 = 56$	M1
	Equation of circle is $OA^2 = \left(2\sqrt{5}\right)^2 + 6^2$	A1
(b)	Gradient of OA is $\frac{6}{2\sqrt{5}} = \frac{3}{\sqrt{5}}$	B1
	Gradient of AB is $-\frac{\sqrt{5}}{3}$	M1
	Equation of AB is $y = -\frac{\sqrt{5}}{3}x + c$	
	Substitute $x = 2\sqrt{5}$, y = 6	M1
	$6 = -\frac{\sqrt{5}}{3}\left(2\sqrt{5}\right) + c$	
	$c = \frac{28}{3}$	
	$y = -\frac{\sqrt{5}}{3}x + \frac{28}{3}$	A1
(c)	At B, $y = 0$, so $x = \frac{28}{\sqrt{5}} = \frac{28\sqrt{5}}{5}$	B1
	Area $OAB = \frac{1}{2}bh$	M1
	$= \frac{1}{2}\left(\frac{28\sqrt{5}}{5}\right)6$	
	$= \frac{84\sqrt{5}}{5}$	A1

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