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PHYSICS

Astrophysics
AQA A-level
Year 2

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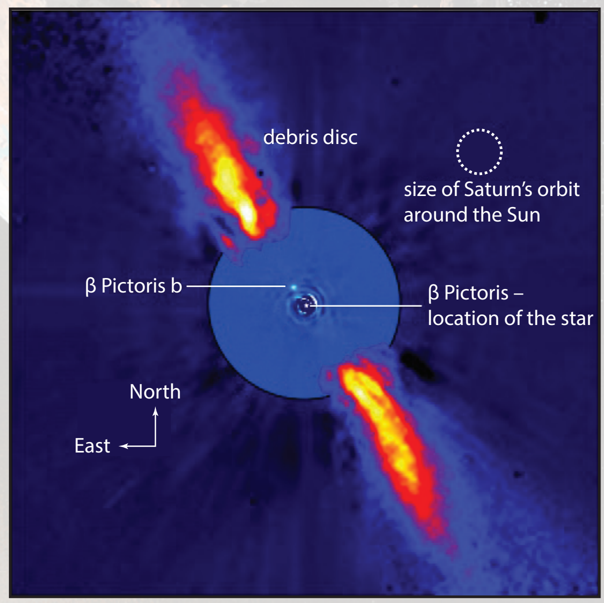
ASTROPHYSICS

Astrophysics is a branch of astronomy concerned with the physics of the Universe, particularly the physics of stars and galaxies. Astrophysicists use many other areas of physics, including mechanics, thermodynamics, quantum mechanics, optics, electromagnetism, atomic and nuclear physics, and special and general relativity, to describe and model astronomical phenomena. The study of exoplanets – planets orbiting other stars – is just one area where astrophysics can be applied to understand the mysteries of our Universe.

Exoplanets are some of the most important discoveries of late 20th century science, made possible by advances in astronomical image processing and measurement (see photo and caption). Nearly 2000 exoplanets are now known to exist, some of which are Earth-like, leading to speculation that life may exist elsewhere in our Galaxy and in the wider Universe.

Observations of exoplanetary systems help us to understand how our own solar system formed and why we have the planets distributed as we see them today. They help astrophysicists to develop theories as to why we have four rocky planets close to the Sun and large gas giants much further away, and whether or not it is unique that on Earth we have the conditions necessary for liquid water to exist and the formation of molecular compounds needed to support life.

The first discoveries of exoplanets were bodies completely unlike those in our own solar system. 'Hot Jupiters' are exoplanets as massive as Jupiter that orbit so close to their parent star that they are roasted to high temperatures. Some exoplanets follow highly elliptical orbits, unlike the nearly circular ones in our solar system. However, evidence is now accumulating that Earth-type exoplanets may exist. The *Kepler* space observatory was launched in 2009 and has surveyed a section of our own Galaxy, looking at the dimming of a star as an exoplanet crossed in front of it. Data from *Kepler* have allowed scientists to deduce the existence of at least three Earth-like exoplanets in habitable zones (where liquid water may exist) around other stars. Our studies of exoplanets are just beginning, and our ability to image and catalogue them continually improves. Very soon we may be able answer the question: 'Is our own solar system unique in supporting life?'



A direct image of exoplanet beta Pictoris b next to the star beta Pictoris, 63 light years from our solar system in the constellation Pictor in the southern hemisphere. The image was taken by the Very Large Telescope (VLT) at the European Southern Observatory (ESO) in Chile. The planet has been imaged using special techniques that suppress the brightness of the star, allowing the planet to be seen.

1 TELESCOPES

PRIOR KNOWLEDGE

From your previous studies at GCSE and in Year 12, you will have learnt about lenses and mirrors and how they can reflect and refract light. You may wish to refer back to the sections in *Chapter 5 of Year 1 Student Book* on waves, in particular electromagnetic waves, to Chapter 6 on diffraction and Chapter 7 on reflection and refraction, to refresh your ideas of wave properties and their measurement. You will also need to be familiar with the use of the radian for angular measure – see Year 2 Student Book Chapter 1.

LEARNING OBJECTIVES

In this chapter you will learn how lens telescopes, reflecting telescopes and radio telescopes are used to image the Universe, as well as those imaging in infrared, ultraviolet and X-rays. You will consider the relative advantages and limitations of different types of telescope, including the importance of collecting power and resolution. You will learn about the use of very sensitive electronic detectors called charge-coupled devices to store astronomical images for processing, enhancement and distribution.

(Specification 3.9.1.1 to 3.9.1.4)

Figure 1 *One of Galileo's original telescopes, consisting of two lenses – a primary convex lens and an eyepiece with a single concave lens. The best telescope that Galileo made had a magnification of about × 30.*

1.1 EARLY TELESCOPES AND THE USE OF LENSES

Galileo was the first astronomer known to use a telescope to study the night sky some 400 years ago. He made a number of telescopes and used them to observe the moons of Jupiter and the rings of Saturn. This revolutionised our understanding not only of our solar system but subsequently of the Universe as a whole. The telescope he used (Figure 1) was based on lenses made of glass, which alter the direction of light rays by refracting them.

To understand how an astronomical telescope works, you need to know how lenses form images. There are two basic types of optical lens. A **concave lens**, also called a diverging lens, spreads an incident beam of light into a diverging emergent beam. A **convex lens**, also called a converging lens, can focus an incident beam. As well as in telescopes, lenses are used in optical instruments, such as binoculars, slide projectors, cameras, spectacles and magnifying glasses, to produce an image.

For a single converging lens, the line that passes through the centre of the lens at right angles to it is called the **principal axis** or optical axis. Light rays from a distant object that are essentially parallel to the principal axis of the lens converge to a point called the **principal focus**, F (Figure 2). The distance between the principal focus and the centre of the lens is called the **focal length**, *f*. The shorter the focal length of a converging lens, the more strongly it converges light rays.

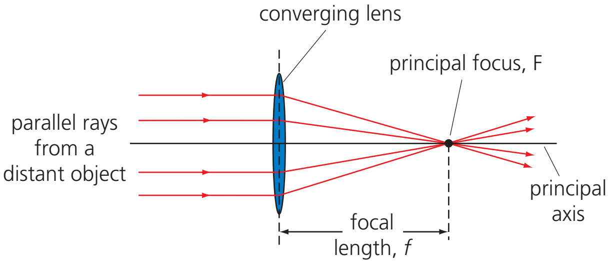
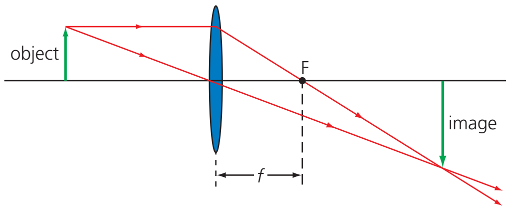


Figure 2 *A ray diagram showing the action of a converging lens on a beam of light*

The construction of a ray diagram (Figure 2) is the best method to gain a good visual understanding of the way an incident light beam behaves on passing through a lens system.

A converging lens can produce both a **real image** and a **virtual image**. When an object is further away from the lens than the focal length, a real image is formed, inverted, on the far side of the lens (Figure 3). A real image is one that can be formed on a screen; a virtual image cannot be.



- Light rays that pass through the centre of the lens are undeviated.
- Light rays parallel to the principal axis converge to the focal point.
- By convention the rays are shown changing direction just once on passing through the lens.

Figure 3 *A ray diagram for a converging lens, showing how a real image is formed*

When an object is closer to the lens than the focal length, the lens acts as a magnifying glass. A magnified virtual image is formed, the right way up, on the same side of the lens (Figure 4).

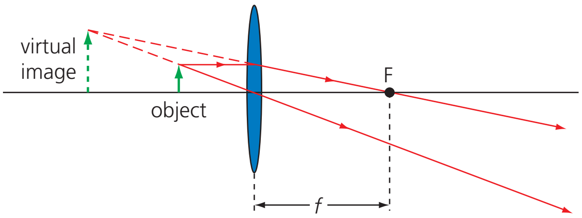


Figure 4 *A ray diagram showing a converging lens producing a virtual image*

1.2 ASTRONOMICAL TELESCOPE CONSISTING OF TWO CONVERGING LENSES

Astronomical telescopes that receive light in the visible part of the electromagnetic spectrum are collectively termed **optical telescopes**. Those that focus the incident light by refraction through lenses, just as in Galileo's instrument (Figure 1), are called **refracting telescopes**. They are now much bigger than in Galileo's time (Figure 5), allowing a much greater **magnification**.

Figure 5 *The Yerkes telescope at the Yerkes Observatory in Wisconsin, USA. Although its construction was completed in 1895, it is still the largest refracting telescope currently in use. It has an objective lens of 1 m diameter and a focal length of 19.4 m.*

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A simple refracting telescope (Figure 6) has a converging **objective lens**, which produces a real image of a very distant object, and a converging **eyepiece lens**, which acts as a magnifying glass. The light rays leaving the eyepiece are parallel, and so the final image appears at infinity. This means that the observer's eye does not have to keep refocusing between looking at a distant object and looking through the eyepiece at the image, and this reduces eye strain. This setting is called **normal adjustment** for an astronomical telescope.

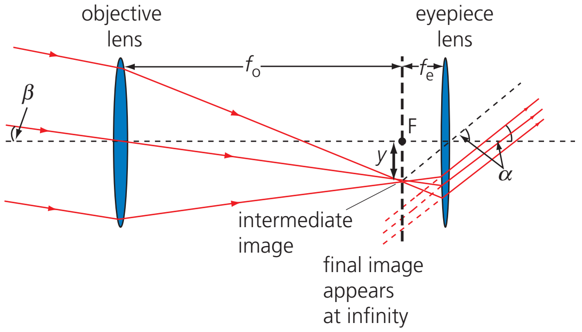


Figure 6 *The lens arrangement for a refracting telescope. In normal adjustment, the final magnified image appears to be at infinity.*

Light from the edge of the object enters the objective lens at an angle β to the optical axis and forms an intermediate real image between the lenses. The angle β is the angle subtended by the object to the unaided eye and is a very small angle. An object's **angular size** is the angle between the lines of sight to its two opposite ends and is a measure of how big the object appears to the unaided eye (Figure 7). In normal adjustment, parallel light emerges from the eyepiece lens. This occurs when the focus of the eyepiece lens (focal length f_e) is coincident with that of the objective lens (focal length f_o). When looking through the eyepiece, the angle subtended by the image to the eye, α, has now increased.

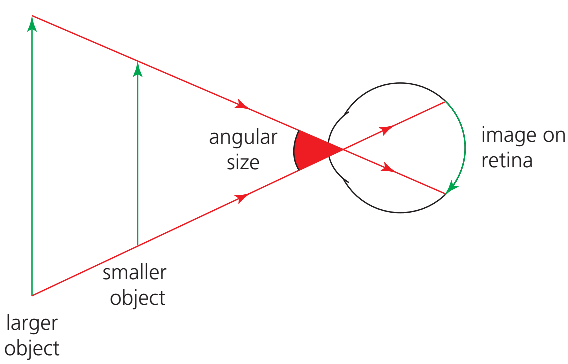


Figure 7 *The angular size of an object depends on both its actual size and its distance away.*

The **angular magnification**, M, or magnifying power of a refracting telescope is given by

$$M = \frac{\text{angle subtended by image at eye}}{\text{angle subtended by object at unaided eye}} = \frac{\alpha}{\beta}$$

In normal adjustment, the magnification can be expressed in terms of the focal lengths of the lenses. From Figure 6, we have, by simple geometry (properties of vertical angles), that

$$\tan\alpha = \frac{y}{f_e}$$

and that

$$\tan\beta = \frac{y}{f_o}$$

As the angles α and β are very small, the tangent approximation is used (this can be used for angles less than about 6°) – for angles α and β in radians, $\tan\alpha \approx \alpha$ and $\tan\beta \approx \beta$. This then gives the angular magnification as

$$M = \frac{\alpha}{\beta} = \frac{(y / f_e)}{(y / f_o)}$$

So

$$M = \frac{f_o}{f_e}$$

The angular magnification (in normal adjustment) is given by the ratio of the focal length of the objective lens to the focal length of the eyepiece lens. Note that you can see from Figure 6 that the length of a refracting telescope has to be at least the sum of f_o and f_e, which explains their long length.

Worked example

The James Lick telescope at the Lick Observatory in California, USA, was built in 1888 and is still in use. It has a primary convex lens of 36 inches (0.91 m) and a focal length of 57.8 feet (17.6 m). What is its magnification if used with an eyepiece of focal length 55 mm? What is the sum of the focal lengths f_o and f_e?

How would these calculations change for an eyepiece of focal length 35 mm?

For $f_e = 55$ mm:

$$M = \frac{f_o}{f_e} = \frac{17.6}{55 \times 10^{-3}} = 320$$

$$f_o + f_e \approx 17.6\text{ m}$$

For $f_e = 35\,\text{mm}$:

$$M = \frac{f_o}{f_e} = \frac{17.6}{35 \times 10^{-3}} = 503$$

$$f_o + f_e \approx 17.6\,\text{m}$$

It is clear that the length of the telescope is dominated by the large focal length of the objective lens.

QUESTIONS

1. Calculate the angular magnification of a telescope with an objective lens of focal length 1200 mm using an eyepiece of focal length
   a. 25 mm
   b. 10 mm.
2. The Moon has an angular size of 0.5° in the sky when viewed with the naked eye. Suppose the Moon is viewed through a telescope with an objective of focal length 100 cm and an eyepiece of focal length 20 mm. What is the angular size of the Moon as seen through this telescope?
3. The 1 m diameter objective lens of the Yerkes refractor (Figure 5) weighs 225 kg. Suggest two factors that limit the size of refractors for astronomy.

1.3 CHROMATIC AND SPHERICAL ABERRATION

Refracting telescopes suffer from fundamental aberrations (faults), of which there are two main ones. The first is due to the fact that refraction by a lens causes white light to separate into its component colours (wavelengths). This is an effect called **dispersion**, which occurs because the refractive index of the lens material is different for different wavelengths of light (*see Chapter 7 in Year 1 Student Book*).

An objective lens focuses the different colours over a range of focal lengths, a deficiency known as **chromatic aberration** (Figure 8). This produces coloured edges to the image, which may be corrected by careful design and choice of high-quality optical materials. Figure 9 shows chromatic aberration in an objective lens forming an image of the words 'chromatic aberration' producing a coloured fringe effect due the lens focusing colours to different focal lengths.

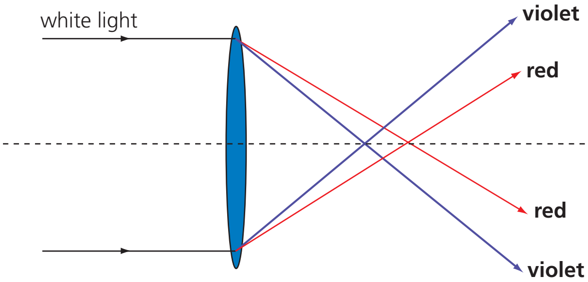
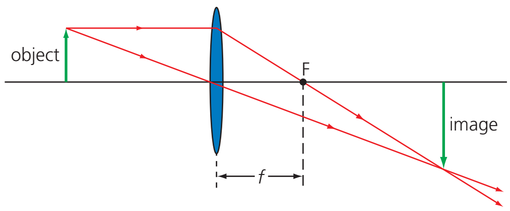


Figure 8 *Chromatic aberration causes light of different wavelengths (colours) to focus at different positions along the optical axis.*



- Light rays that pass through the centre of the lens are undeviated.
- Light rays parallel to the principal axis converge to the focal point.
- By convention the rays are shown changing direction just once on passing through the lens.

Figure 9 *Chromatic aberration through a lens*

Because of the curvature of the lens, **spherical aberration** results in light rays in a parallel beam being focused at slightly different positions (Figure 10). Light rays near the edge of the lens are deviated more than those near the optical axis. The effect is most pronounced in lenses of large diameter, resulting in a blurring of the image. The effect can be minimised by making both surfaces of the lens contribute equally to the ray deviation.

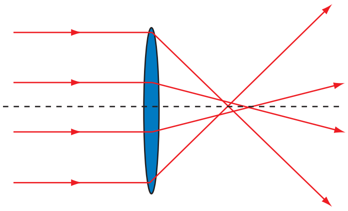


Figure 10 *Spherical aberration causes rays to focus at different positions, causing image blurring.*

The objective lens in a refractor is often an **achromatic doublet** (Figure 11). This is made up of two individual lens elements cemented together and corrected to bring light of two wavelengths (two of red, green and blue) into focus in the same plane. Each lens is made from glass with different dispersion. The convex lens in the doublet is made of crown glass and has a low dispersion; the concave lens, made of flint glass, has a higher dispersion and is shaped in such a way that the chromatic aberration of one lens is compensated by that of the other. The doublet is also designed to keep spherical aberration to a minimum.

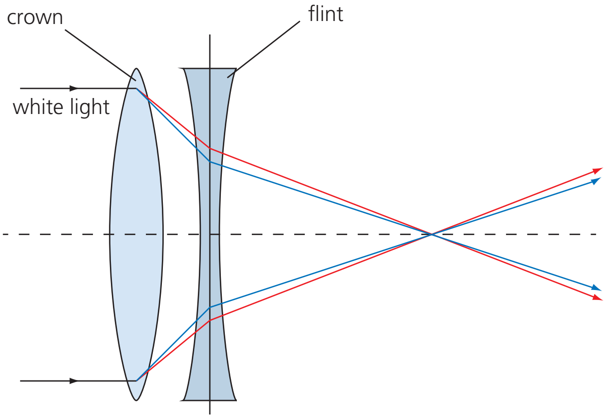


Figure 11 *An achromatic doublet that brings red and blue light to the same focus*

KEY IDEAS

- A simple astronomical refracting telescope is made of two converging lenses, the objective lens and the eyepiece lens.
- The final image is magnified, inverted and (in normal adjustment) at infinity.
- The angular magnification is defined as

$$M = \frac{\text{angle subtended by image at eye}}{\text{angle subtended by object at unaided eye}}$$

- For a refracting telescope in normal adjustment

$$M = \frac{f_o}{f_e}$$

where f_o is the focal length of the objective lens and f_e is the focal length of the eyepiece lens.

- Refracting telescopes suffer from chromatic and spherical aberrations, which respectively produce coloured edges and blurring in the image.

1.4 REFLECTING TELESCOPES

There are several different designs of **reflecting telescope**, but all use a curved objective mirror, or 'primary mirror', to collect light from a distant object and direct it onto a secondary mirror.

The diameter of the primary mirror determines the ability of the telescope to collect light. The light-gathering power is proportional to the mirror area, and so to the square of the diameter of the mirror. Modern reflecting telescopes use primary mirrors up to 10 m in diameter.

Spherical aberration (see Astrophysics section 1.3) occurs for mirrors as well as for lenses, if the mirror has spherical curvature. The ideal objective mirror is parabolic in shape because this focuses parallel light rays from a distant object to a single focal point (Figure 12), so eliminating spherical aberration. The mirror itself consists of a very thin coating of silver or aluminium atoms that have been deposited onto a backing material. The thickness of this coating is often less than 25 nm (2.5×10^{-8} m). This provides as smooth a surface as possible and so minimises distortions.

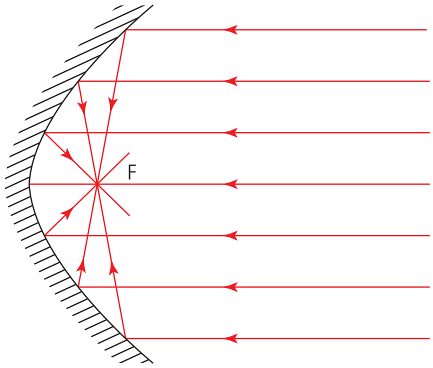


Figure 12 *A perfectly parabolic reflector eliminates spherical aberration.*

The magnification of a reflecting telescope is found by using the same formula as for a refracting telescope – it is the ratio of the focal length of the objective (mirror) to the focal length of the eyepiece.

Worked example

A reflecting telescope with a parabolic mirror has a diameter of 80 cm and a focal length of 1 m. It is used with an eyepiece of 15 mm. What is the magnification of the telescope?

$$M = \frac{f_o}{f_e} = \frac{1}{15 \times 10^{-3}} = 67$$

One important advantage of reflecting telescopes over refracting telescopes is that they allow the large focal length of the objective mirror to be 'folded up' to produce an instrument with large magnification in a compact space. A common design for reflecting telescopes is the **Cassegrain arrangement** (Figure 13). The large primary mirror has a parabolic shape. A convex secondary mirror with a hyperbolic shape is used, which sends the rays down an opening in the primary mirror (usually in the centre), where the image is brought to a focus using an eyepiece or an imaging camera. The *Hubble* space telescope uses a variation of the Cassegrain design.

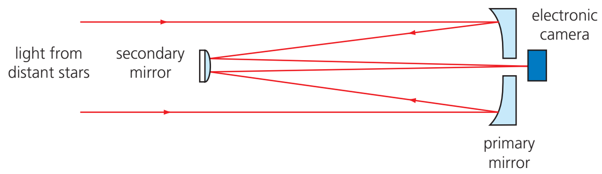


Figure 13 *The Cassegrain arrangement for a reflecting telescope*

Mirrors are unaffected by chromatic aberration. However, if an eyepiece is used in the focal plane of a reflecting telescope, there may be some chromatic aberration by the lenses in the eyepiece. This can be minimised by using an achromatic doublet (see Astrophysics section 1.3).

Table 1 shows the key differences between refracting and reflecting telescopes.

Disadvantages of refracting telescopes	Advantages of reflecting telescopes
• Mounting of the lens and support can only be made using the edge of the lens • Using glass of sufficient clarity and purity and free from defects to make large-diameter telescopes is extremely difficult • Large-diameter lenses are heavy and tend to distort under their own weight • Suffer from chromatic aberration and spherical aberration • Heavy and difficult to manoeuvre quickly • Difficult to mount heavy observing equipment and associated electronics • Large magnifications require large objective lenses and very long focal lengths	• Large single mirrors can be made, which are light and easily supportable from behind • Mirror surfaces can be made just a few nanometres thick, giving excellent image properties • Mirrors use only the front surface for reflection, so removing many of the problems associated with lenses • No chromatic aberration, and no spherical aberration when using parabolic mirrors • Relatively light mirrors allow rapid response to astronomical events • Smaller segmented mirrors can be used to form a large composite objective mirror

Table 1 *Comparing refracting and reflecting telescopes*

QUESTIONS

4. What is the magnification of a Cassegrain reflecting telescope whose mirror has a focal length of 2800 mm and is used with eyepieces of focal length

a. 5 mm

b. 15 mm

c. 25 mm?

KEY IDEAS

- Reflecting telescopes use curved mirrors to reflect light from a distant object and form an image.
- A common design of reflecting telescopes is the Cassegrain arrangement, made up of a combination of a primary concave (parabolic) mirror and a secondary convex (hyperbolic) mirror.
- The mirrors in a Cassegrain telescope do not suffer from chromatic aberration but can be affected by spherical aberration if they are not perfectly parabolic (or hyperbolic).

1.5 LIMITATIONS OF GROUND-BASED OPTICAL TELESCOPES

For ground-based optical telescopes, atmospheric absorption and distortion in the visible region of the electromagnetic spectrum are limiting factors in image quality. Ozone, oxygen, water vapour and carbon dioxide all contribute to the absorption of light, from the ultraviolet through visible to infrared. Dust within the atmosphere also absorbs and scatters light on its way to the telescope, and atmospheric turbulence (due to convection currents) reduces image quality. Such problems are avoided by building observatories in dry, pollution-free areas at high altitude, or, better, by putting telescopes in orbit around the Earth beyond the atmosphere. The *Hubble* space telescope is due to be succeeded in 2018 by the *James Webb* space telescope.

Visible light is not the only part of the electromagnetic spectrum through which we can explore the Universe. Many astronomical formations and events can only be detected, or can be imaged much more clearly, by detecting emissions of electromagnetic waves with wavelengths beyond the visible spectrum (see Figure 19 in Astrophysics section 1.8). Optical telescopes are no good for this – we need special non-optical telescopes, for example, radio or X-ray telescopes.

Large ranges of non-visible wavelengths are also absorbed by our atmosphere (Figure 14). **Atmospheric opacity** is a measure of the absorption of electromagnetic radiation by the atmosphere, as a function of wavelength. You can see from Figure 14 that the atmosphere is in fact relatively transparent at optical (visible) wavelengths, and is transparent for a range of radio wavelengths, which means that they can be detected from the ground (see Astrophysics section 1.8).

Gamma rays, X-rays, most ultraviolet and some infrared are strongly absorbed, so to probe the Universe at these wavelengths we need space-based observatories (see Astrophysics section 1.9). There are some exceptions. While much infrared is absorbed, there are infrared windows where observation can be made at ground level or at high altitudes. Highly energetic gamma rays can be detected by the large air showers of ionised particles and electromagnetic radiation they produce, which can be detected by instruments mounted in balloons and even at ground level.

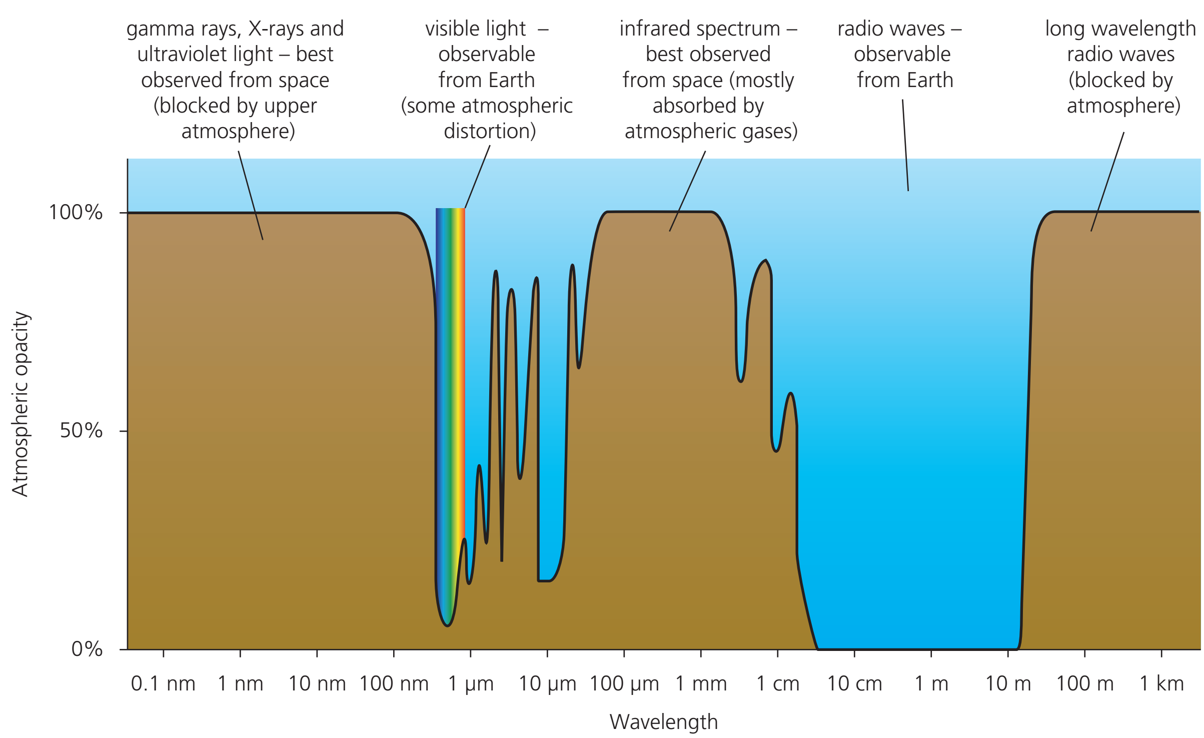


Figure 14 *The opacity of the atmosphere to electromagnetic radiation*

1.6 RESOLVING POWER OF TELESCOPES

A very important performance parameter for any kind of telescope is its **resolving power**. This is its ability to produce separate images of closely spaced objects. Electromagnetic radiation travels in the form of waves. When the waves pass through an opening or **aperture** of a telescope, they will diffract and interfere constructively or destructively to produce a diffraction pattern (see section 6.2 in *Chapter 6 in Year 1 Student Book*). It is for this reason that an imaging system like a telescope will not focus a star to a perfect point but to a disc instead, called an **Airy disc** (Figure 15).

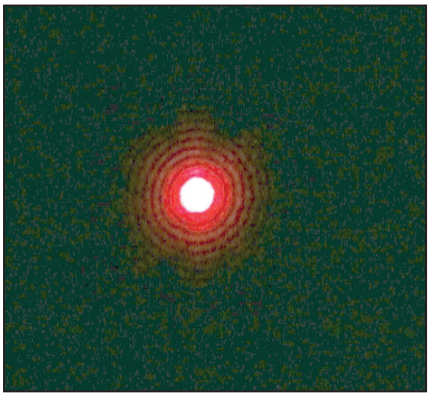


Figure 15 *Airy disc diffraction pattern of the star Betelgeuse*

The size of the central maximum determines how much blurring of the image there is: the smaller the width of the disc, the less blurring and so the more detail will be seen.

The angular location of the first dark fringe in the Airy disc is given (approximately) by the formula

$$\sin\theta = \frac{\lambda}{D}$$

where λ is the wavelength of light in metres, D is the diameter of the mirror or lens in metres and θ is the angular position in radians. As the angles involved are exceedingly small, we can make the approximation that $\sin\theta \approx \theta$ in radians. Therefore we have

$$\theta \approx \frac{\lambda}{D}$$

This gives us the important result that the width of the central maximum can be reduced by making the diameter, D, of the mirror or lens as large as possible, for a fixed value of the wavelength. Also, the shorter the wavelength, the smaller the width of the disc.

If two point objects (for example, two stars at a distance L away) are very close together (distance of separation x), their Airy discs will overlap. The degree of overlap will dictate whether or not the two stars can be resolved as two separate light sources (Figure 16).

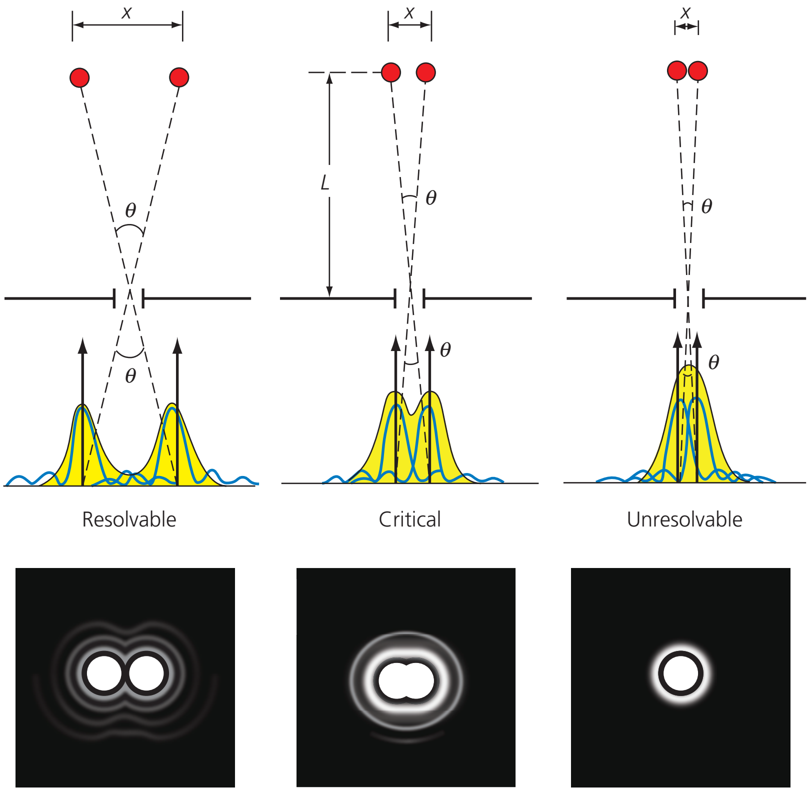


Figure 16 *Effect of overlapping two Airy discs from two distinct objects, illustrating the criteria for resolving astronomical objects. The middle image is just resolvable.*

The critical stage is reached when the central maximum of one of the Airy discs is over the first minimum of the other Airy disc (the middle intensity graph and image in Figure 16). The imaging process is said to be 'diffraction-limited' at this separation. The two objects will just be resolved, and the following holds:

$$\theta = \frac{x}{L} \approx \frac{\lambda}{D}$$

The **Rayleigh criterion** states that two point objects can be resolved if their angular separation is at least

$$\theta \approx \frac{\lambda}{D}$$

The angle θ is known as the **minimum angular resolution** of the instrument at a particular wavelength λ.

Angular measure and the size of astronomical objects

Angular measurements are used describe the apparent size (angular size, see Astrophysics section 1.2) of an object in space. The angular size of an object is expressed in degrees, **arcminutes** and/or **arcseconds**. Just as an hour is divided into 60 minutes, and a minute into 60 seconds, a degree is divided into 60 arcminutes (or 'minutes of arc'), and an arcminute is divided into 60 arcseconds (or 'seconds of arc'):

1 degree = 1° = 1/360 of a circle

1 arcminute = 1' = 1/60 of a degree

1 arcsecond = 1" = 1/60 of an arcminute
= 1/3600 of a degree

To get a rough estimate of the angular size of objects in space, you can go out on a clear night when the full Moon is visible. Extend your arm towards the sky. Your fist, at arm's length, covers about 10° of the sky, your thumb covers about 2°, and your little finger covers about 1°. The full Moon is about 0.5°, or 30' (30 arcminutes) across. Coincidentally, so is the Sun.

The face of Jupiter is about 50" (50 arcseconds) across. A good optical telescope in steady skies can resolve down to about 1" (1 arcsecond).

QUESTIONS

5. The diameter of the objective mirror of the Hale telescope is 5.1 m, and it is observing a star emitting light of wavelength 510 nm.

a. What is the minimum angular resolution of the telescope in radians?

b. What is the smallest detail the telescope can detect on the surface of the Moon?

[Distance to Moon = 3.8×10^8 m; take the wavelength of light detected to be 510 nm]

6. Planets have been detected around other stars (see Astrophysics section 4.8). Estimate the diameter of a telescope objective lens required to resolve a Jupiter-sized planet orbiting the nearest star, which is about 4×10^{16} m away. Assume you are observing at a wavelength of 550 nm. The diameter of Jupiter is about 1.5×10^8 m.

1.7 COLLECTING POWER OF TELESCOPES

The **collecting power** of an imaging system such as a telescope is another important parameter. It is a measure of its ability to collect incident electromagnetic radiation. It is directly proportional to the square of the diameter of its objective. This is because the surface area of a circular object of diameter a is equal to $\pi \times \left(\frac{1}{2}a\right)^2 = \frac{1}{4}\pi a^2$. So

$$\text{collecting power} \propto (\text{objective diameter})^2$$

An optical telescope, for example, has a collecting power specifically called its **light-gathering power (LGP)**, measured in m^2. The LGP is a relative measure for comparing the ability of different telescopes to 'grasp light' – the larger the LGP, the brighter the image.

It is clear that there are direct advantages, in improved resolving power and collecting power, with large-diameter telescopes. Limitations to the size of the objective diameters will be discussed for the various types of telescopes in Astrophysics section 1.10.

KEY IDEAS

- The resolving power of a telescope is its ability to produce separate images of closely spaced objects.
- The resolving power is limited by diffraction at the circular aperture (objective). A point object becomes a disc.
- The Rayleigh criterion states that two point objects can be resolved if their angular separation is $\theta \approx \frac{\lambda}{D}$ or greater, where λ is the wavelength of the light and D is the diameter of the objective.
- The value $\theta \approx \frac{\lambda}{D}$ is the minimum angular resolution of the telescope.
- The collecting power of a telescope is proportional to (objective diameter)^2.

ASSIGNMENT 1: USING A SIMPLE HOMEMADE ASTRONOMICAL REFRACTOR

(PS1.1, PS1.2, PS2.1, PS3.2, PS4.1)

This assignment concerns a simple astronomical refractor, similar to Galileo's, which can be easily made in the school laboratory.

It is constructed of:

- Two convex lenses – one with a long focal length, of 200 mm, and diameter 50 mm (this is the lens furthest away from the eye, the objective), and a second lens with a short focal length, of 25 mm, and diameter 30 mm (this is the eyepiece lens).
- Tubing – one tube for the objective and one for the eyepiece, which slide inside each other. The diameter of each tube is only very slightly larger than the diameter of its lens. They can be constructed from mailing tubes, plastic piping or thick cardboard. The sum of the lengths of the two tubes is greater than the sum of the focal lengths of the lenses. The outside of the tubes can be greased (for example, with Vaseline) where they slide inside one another, to ensure smooth operation.

The arrangement is shown in Figure A1.

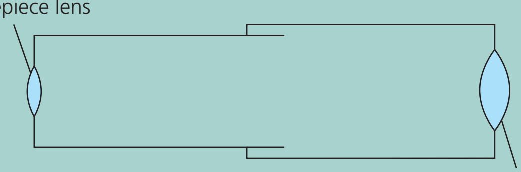


Figure A1 *Sliding tube telescope arrangement*

The lenses are attached to either end of each tube using sticky tape or a thin layer of glue around the edge of the lens, taking care not to obscure the view through the tube.

The telescope is lined up to view a distant (but not astronomical) object – such as a bright white lamp – and the eyepiece tube is moved in and out until the object comes into focus. An inverted image is seen.

Questions

A1 What is the magnification of the constructed telescope?

A2 What is the telescope's collecting power compared to that of the human eye, which has a lens diameter of about 10 mm?

A3 **a.** Calculate the theoretical angular resolution of your telescope for the following wavelengths: **i.** red (685 nm), **ii.** green (550 nm) and **iii.** blue (445 nm). Which wavelength gives the best angular resolution?

b. Explain what is meant by a telescope being *diffraction-limited*. Why is the home-constructed telescope unlikely to be diffraction-limited?

c. Epsilon Lyrae is a star known as 'the Double Double'. When seen through a telescope with high magnification, two stars can be seen, but on closer inspection each of those is also a double star. The first double is 2.8 arcseconds apart, and the other is 2.2 arcseconds apart. Comment on whether the telescope would be able to resolve this double star system.

A4 The image of a white lamp seen through the telescope may have colours and may be slightly distorted in shape. Explain why this is.

A5 Suggest ways in which you can improve the telescope by

a. giving a brighter image

b. increasing the magnification

c. improving the angular resolution.

A6 What practical problems might be encountered in viewing an image, as the magnification of the telescope is increased?

1.8 RADIO TELESCOPES

The science of radio astronomy was born when a telephone engineer, Karl Jansky, was looking for sources of static noise affecting radiotelephony communication circuits, in the 1930s. Using a radio antenna, Jansky discovered that some of this 'noise' was coming from radio sources in space – from the central region of the Milky Way. Radio telescopes were then developed to study these signals. Radio telescopes can be ground-based because the atmosphere is transparent to a large range of radio wavelengths (see Astrophysics section 1.5).

The simplest radio telescope consists of a single parabolic 'dish' antenna (the 'objective') by which radio energy is collected and brought to a focus in a receiver where it is amplified and displayed as an intensity trace (Figure 17). Radio astronomy uses radio frequencies allocated by international agreement in the megahertz (MHz) and gigahertz (GHz) bands, although there is some overlap with domestic communication channels.

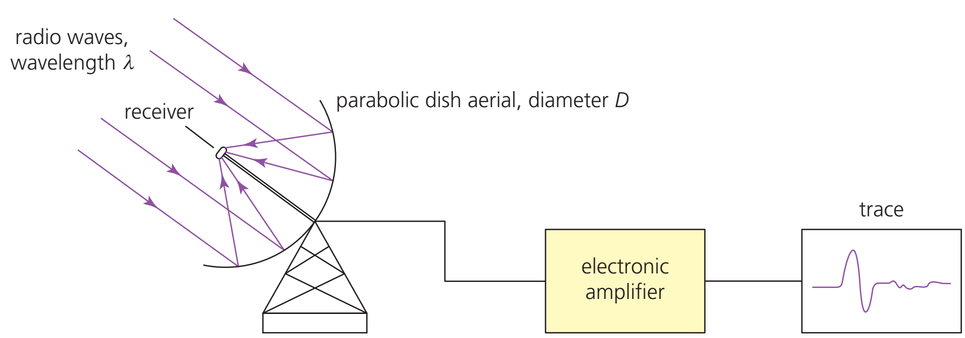


Figure 17 *Single-dish radio telescope*

Compared to an optical telescope, a radio telescope has a low angular resolution (see Astrophysics section 1.6) because of the dependence on wavelength in the Rayleigh criterion,

$$\theta \approx \frac{\lambda}{D}$$

This is why radio telescopes have very large dishes. Radio astronomy wavelengths are on the scale of metres, as opposed to nanometres in optical astronomy, and so to resolve objects with small angular sizes it is necessary to use a much larger-diameter aperture (the aperture being the parabolic dish). The largest single-dish radio telescope in the world is the 305 m diameter spherical-shaped fixed dish located at Arecibo, Puerto Rico (Figure 18a). Other radio telescopes need large mechanical structures to support them and most are steerable rather than fixed (Figure 18b).

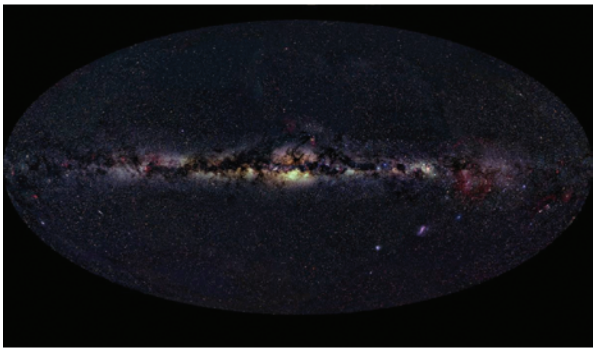
(a)

(b)

Figure 18 (a) The Arecibo radio telescope, Puerto Rico, was constructed inside a natural geologic depression. (b) The Parkes radio telescope in New South Wales, Australia, has a fully steerable 64 m diameter dish.

Unlike optical telescopes, radio telescopes can operate during the day as well as at night. They are usually situated away from radio transmitters and other sources of Earth-based radio emissions, which can drown out the signals from astronomical objects. The images formed by radio telescope look quite different from those formed by optical telescopes. Figure 19a shows an optical image of the sky at visible wavelengths, with the Milky Way dominating the centre of the image. Figure 19b shows the same part of the sky but produced by a radio telescope operating at a wavelength of 21 cm, and shows a map of the radio intensity of the Galaxy. This 21 cm radio emission is produced by changes in the energy state of neutral hydrogen atoms from hydrogen gas in the Milky Way and can penetrate thick dust clouds. This allows us to see the distribution of the gas in our Galaxy that is obscured at optical wavelengths.

(a)



(b)

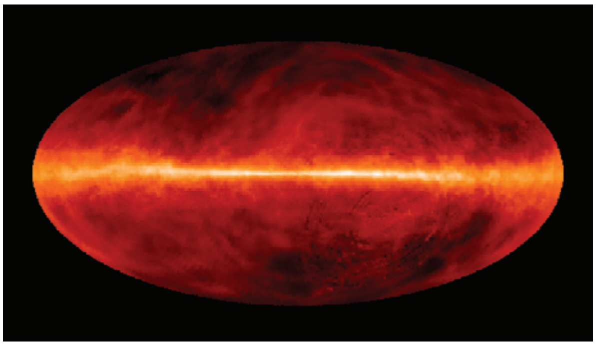


Figure 19 (a) Optical image of the Milky Way. (b) A false-colour radio intensity map of the Milky Way at a wavelength of 21 cm

Radio telescopes are not immune from interference. Below the lower band limit of 30 MHz, the ionosphere itself strongly absorbs the signal, while above 60 GHz absorption by water vapour in the atmosphere is a significant problem. Between these frequencies, artificial interferences, such as those produced from mobile phones, radio telephones and radar scanners, can pose serious problems with sensitive instrumentation, and so radio telescopes tend to be located in isolated areas.

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QUESTIONS

7. a. The Arecibo radio telescope in Figure 18 has a dish diameter of 305 m and has been used for observations using radio wavelengths as low as 4 cm. What is its theoretical angular resolution at this wavelength? How does this compare with the resolution of the Hale optical telescope calculated in question **5a**?

   b. Compare the collecting powers of the Lovell radio telescope at Jodrell Bank in Cheshire, UK, which has a diameter of 76.2 m, the Arecibo radio telescope, with a nominal diameter of 305 m, and the Hale optical telescope, which has a mirror diameter of 5.1 m.

8. Give one advantage and one disadvantage that radio telescopes have compared with optical telescopes.

KEY IDEAS

- Radio telescopes can be ground-based because the atmosphere is transparent to a large range of radio wavelengths.
- Radio telescopes focus radio energy by means of a parabolic (or spherical) 'objective' dish antenna.
- The diameter of the dish antenna needs to be large in order to obtain good angular resolution, because of the long wavelength of radio waves.
- Radio dish antennae need large structures to support them and to steer them.
- Radio telescopes can operate during the day and night and are situated away from artificial sources of radio interference.

1.9 INFRARED, ULTRAVIOLET AND X-RAY TELESCOPES

Infrared telescopes

Infrared astronomy is used to make observations of cool regions (temperatures between a few tens and a hundred kelvin), such as interstellar gas, cooler stars, star formation regions and active galaxies, and of the large-scale structure of the Universe. An infrared (IR) telescope is designed to observe astronomical objects at IR wavelengths. These range from about 0.7 to 450 μm (one micrometre, $1\ \mu m = 10^{-6} m$, and is sometimes called a 'micron'). Since most IR radiation is strongly absorbed in the atmosphere by water vapour, carbon dioxide and other gases (see Figure 14), the surface of the Earth is not ideal to observe objects at IR wavelengths. Space-based observatories are used that can observe high above the atmosphere. However, there are some infrared 'spectral windows' where the atmosphere is transparent to IR wavelengths and objects can be observed from the ground or at altitude with little absorption. These windows lie approximately at 3–5 μm and 7–14 μm.

An infrared telescope has the same components and follows the same principles as visible light telescopes. A combination of lenses and mirrors gathers and focuses radiation onto an infrared detector for analysis. The detector itself is designed to detect very small changes in temperature caused by absorption of IR radiation. It is usually a collection of specialised metallic semiconductor devices and commonly the superconductor alloy mercury cadmium telluride is used. It is important that the IR detector is kept very cold – it must be cooled by a cryogenic fluid such as liquid nitrogen or helium to temperatures approaching absolute zero. It must be well shielded in order to avoid 'thermal contamination' from its own IR emissions and those of surrounding heat sources.

The *Spitzer* space telescope, launched in 2003, was the largest ever space-based infrared telescope. It used a Cassegrain optical assembly, similar to that of the *Hubble* telescope. It was designed to observe the sky at wavelengths between 3 and 180 μm and has given valuable information as to how stars form. Its detector was cooled to −268 °C, and although the coolant has now run out, it is still able to make measurements over a reduced wavelength range.

QUESTIONS

9. a. Explain why most IR observations have to be carried out above the atmosphere.
   b. What is meant by *infrared windows*?

10. SOFIA is a flying IR observatory housed in an aircraft (Figure 20), with a telescope objective dish of diameter 2.4 m. If it is observing an object at a wavelength of 24 μm, what is its minimum angular resolution? How would this compare with that of an optical telescope of the same diameter observing at 510 nm?

Figure 20 *SOFIA's reflector dish in situ*

Ultraviolet telescopes

Ultraviolet (UV) telescopes are used to examine objects in the UV part of the electromagnetic spectrum, with wavelengths from about 400 nm down to about 10 nm. The ozone layer in the Earth's atmosphere blocks all UV wavelengths shorter than 300 nm from reaching the ground, so rocket-launched satellites are needed for UV astronomy. Like optical and IR reflecting telescopes, a UV telescope uses a Cassegrain mirror system, which brings the UV radiation to a focus, where it is detected by special solid-state devices. These detectors use the **photoelectric effect** to convert UV photons to electrons (*see section 8.4 in Year 1 Student Book*).

From 1978 to 1996 a space-based UV observatory called the *International Ultraviolet Explorer* (*IUE*) observed astronomical objects at UV wavelengths from 120 to 340 nm. It collected data on the composition of cometary tails and on the energy profiles of exploding stars. Another UV space observatory, the *Extreme Ultraviolet Explorer* (*EUVE*), which operated from 1992 to 2001, observed the Universe in the extreme-UV wavelength range between 7 and 76 nm. Other more recent UV observatories are the *Far Ultraviolet Spectroscopic Explorer* (*FUSE*) launched in 1999, which looked at UV wavelengths between 95.5 and 119.5 nm, and the *Galaxy Evolution Explorer* (*GALEX*) launched in 2003, which observed in the range 140 to 280 nm. The *Hubble* space telescope has also been able to serve as an ultraviolet telescope, since a detector sensitive to UV wavelengths between 115 and 320 nm was installed by space shuttle astronauts in 2009.

The detection and analysis of ultraviolet radiation can tell us a great deal about astrophysical processes. Ultraviolet spectral measurements are used to determine the chemical composition and temperature of the interstellar medium and also the temperature and composition of hot young stars. Young massive stars, very old stars, white dwarf stars, active galaxies and quasars shine very brightly at ultraviolet wavelengths. UV telescopes have revealed the existence of a hot gaseous halo surrounding our own Galaxy, and, even closer to home, UV emissions from the Sun help us to understand the solar corona.

QUESTIONS

11. a. The *IUE* had a parabolic mirror of diameter 45 cm and focused UV radiation. For a wavelength of 120 nm, compare its minimum angular resolution and its collecting power with that of an optical telescope of the same diameter operating at 510 nm.
    b. The detector on a UV telescope uses the photoelectric effect to convert UV photons to electrons. Calculate the energy of a UV photon with a wavelength of 120 nm. [Planck constant = 6.63×10^{-34} J s]

X-ray and gamma ray telescopes

X-ray astronomy is the study of astronomical objects that emit in the X-ray part of the electromagnetic spectrum – wavelengths from 0.01 to 10 nm. As X-rays are absorbed by the Earth's atmosphere, they can only be observed from space, using X-ray counters on rockets or space-based observatories. The first X-ray source, Scorpius X-1, was discovered in 1962, and since then dedicated X-ray observatories such as *XMM-Newton* have been used to undertake whole-sky surveys, revealing hundreds of thousands of cosmic X-ray sources. X-rays come from extremely hot gas, in the temperature range 10^6–10^8 K, associated with highly energetic processes, and as such provide a rich array of objects to study, including interacting binary stars, active galaxies, galaxy clusters and supernova remnants (see Astrophysics section 3.3). Interest has also focused on pulsars, neutron stars and black holes.

An X-ray telescope is an instrument that can form an image by bringing X-rays to a focus. X-rays have such high energies that reflecting mirrors such as those in an optical or infrared telescope cannot be used because the X-rays would penetrate into the mirror. Instead, the mirror for an X-ray telescope has to be extremely smooth and be specially shaped as a combination of parabolic and hyperbolic surfaces (Figure 21). For lower X-ray energies (up to 10 keV), this causes the X-rays coming into the telescope to just skim off the surface of the mirror instead of penetrating it, in what is called 'grazing incidence', rather like a stone skipping on water. They are then brought into focus at the focal plane and detected using charge-coupled devices (CCDs) (see Astrophysics section 1.11) that are optimised to detect X-ray energies.

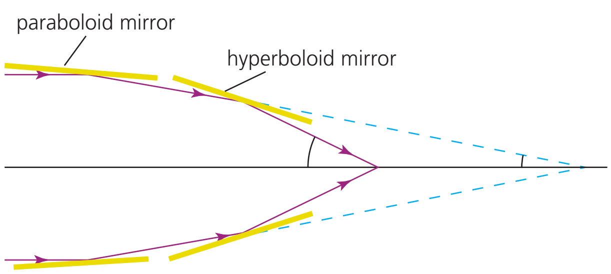

Figure 21 *X-rays enter the XMM-Newton telescope at grazing incidence and are doubly reflected off first a highly polished paraboloid mirror, and then a highly polished hyperboloid mirror. The XMM-Newton X-ray telescope was launched in 1999. It has an angular resolution of 5–14 arcseconds and a collecting power of 4425 cm² at an X-ray energy of 1.5 keV and 1740 cm² at 8 keV.*

Figure 22 shows an X-ray image taken by *XMM-Newton* of a supernova remnant in the Large Magellanic Cloud. These remains of a supernova explosion appear as a complete ring of more than 100 light years in diameter. A central point X-ray source was also found from the *XMM-Newton* image.

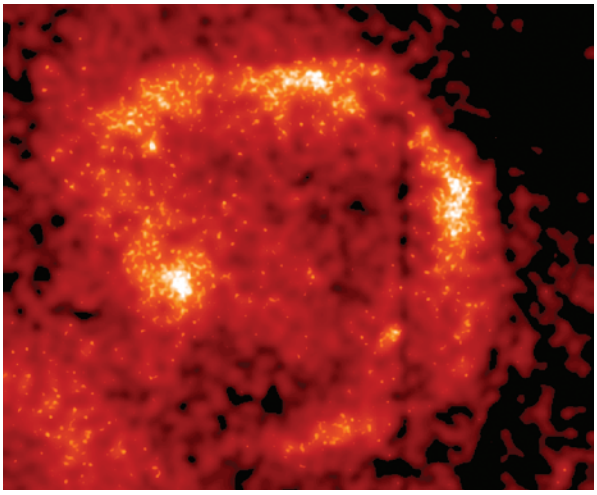

Figure 22 *A supernova remnant imaged by XMM-Newton*

Gamma ray astronomy is the study of astronomical objects in the gamma ray part of the electromagnetic spectrum, down to wavelengths shorter than 0.01 nm. Gamma ray telescopes, such as the orbiting *Fermi Gamma-ray Space Telescope*, do not use mirrors at all; instead, they have special detectors to measure the energy and direction of gamma rays.

The major sources of such high-energy radiation include solar flares, pulsars, quasars, active galaxies and supernova remnants. Sudden bursts of gamma radiation that last from 0.01 s to 1000 s have also been detected in all parts of the sky. The origin of these gamma ray bursts (GRBs) is unknown (see Astrophysics section 3.3) and there is a very active research programme to study these.

QUESTIONS

12. Explain why it is necessary for most IR and all UV and X-ray telescopes to be positioned in space.

KEY IDEAS

- Telescopes can be built that operate in the IR, UV and X-ray parts of the electromagnetic spectrum.
- Infrared telescopes are similar in construction to reflecting optical telescopes, but their detectors have to be kept very cold and shielded from other heat sources.
- Most IR observations are made from space-based observatories or at high altitude to avoid absorption by the atmosphere.
- There are some infrared windows (wavelength ranges) for which infrared observations from the ground are possible with minimal atmospheric absorption.
- UV and X-ray telescopes are positioned in space because of the atmospheric absorption at these wavelengths.
- By observing at different wavelengths using detectors in the focal plane matched to these wavelengths, we can learn different information about different astrophysical processes.

1.10 PRODUCING LARGER-DIAMETER TELESCOPES

We saw in section 1.6 that the resolving power of any telescope is diffraction-limited and depends on the wavelength λ of the light and the diameter D of the telescope. The Rayleigh criterion states that the minimum angular resolution is $\theta \approx \lambda/D$. This is the main reason why radio telescopes are so large compared to optical ones. The large-diameter dish also gives a large collecting power to maximise the signal strength of the low-energy radio waves. However, the weight and size of the large dish pose engineering challenges.

For a large optical reflecting telescope, there is a limit to the maximum size of a primary mirror that is made out of a single piece of glass with a reflective coating. The mass of a large primary mirror can cause it to deform under its own weight. Other considerations, such as cost and mechanical strength, mean that the largest single primary mirror for a reflecting telescope is limited to few metres in diameter. The largest is the Subaru Cassegrain telescope on Mauna Kea Observatory in Hawaii, which has a primary mirror diameter of 8.2 m.

However, telescope designers have come up with ways of increasing the diameter of telescope objectives to increase both the resolving power and the collecting power.

Segmented mirror telescopes

A segmented mirror telescope is an optical telescope whose objective is an array of smaller mirrors, which act as segments of an equivalent single large curved mirror. The segments themselves are curved and ground to a precise shape and mechanically positioned by a computer-controlled system using actuators that accurately align them. This 'active optics' system allows objective mirrors to be built with very large diameters. One of the largest segmented mirror telescopes is the Gran Telescopio Canarias (GTC) on the Canary Islands, which has 36 separate mirror segments, giving an effective aperture diameter of 10.4 m (Figure 23).

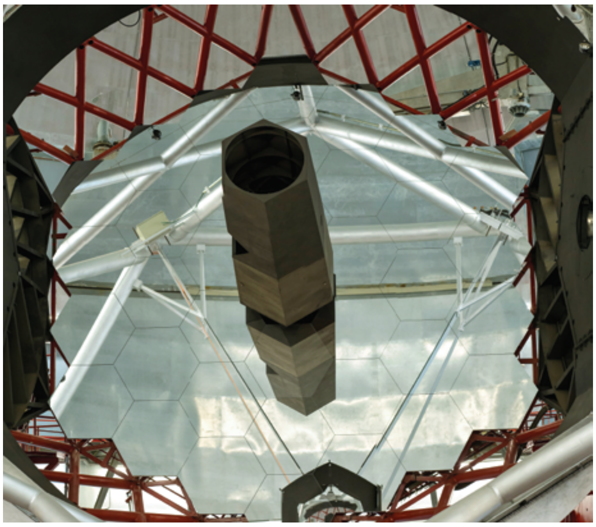

Figure 23 *Primary mirror of the GTC telescope, showing its segmented mirrors*

Radio telescope interferometers

To improve the resolution of a radio telescope, a **radio interferometer** can be used. To see how this works, look at Figure 24.

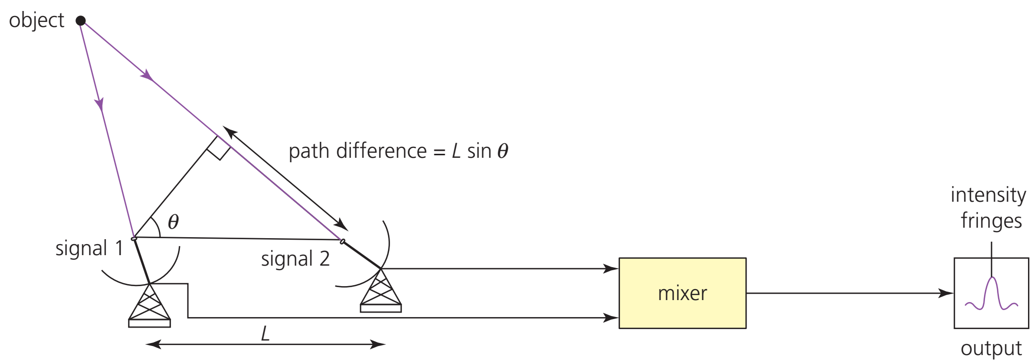

Figure 24 *The principle of a radio interferometer. If the path difference of the radio signal from the object is a whole number of wavelengths, then the two received signals constructively interfere. The angular resolution is approximately λ/L.*

Two identical parabolic dish antennas are placed a distance L apart, called the baseline, and their signals, including their phase and amplitude, are fed into a receiver, which mixes them together. If an astronomical radio source is directly overhead, then the signals will arrive at the antennas in phase and constructively interfere, giving a strong signal. Conversely, when the signals are 180° out of phase, the signals destructively interfere. As the source moves across the sky, an interference pattern of maxima and minima is recorded exactly like that for light passing through a double slit (*see Chapter 6 in Year 1 Student Book*).

The angular distance between successive maxima is the angular resolution of the radio interferometer, and it can be shown that this is approximately λ/L – equivalent to that of a single-dish antenna of diameter L. So if the baseline between the individual antennas can be made very large, the image resolution can be hugely improved. The simplest interferometer consists of just two radio telescopes, but more can be added to further improve the resolution and overall collecting power.

Even better resolution can be obtained by using very large baseline interferometry (VLBI). The signals from a common radio source are received by radio telescopes that are very long distances apart, and may even be on different continents. The signals are recorded and stored in a computer, and if the time of observation and the locations are accurately known, then the signal can be combined to give a detailed image.

It is possible to connect optical telescopes together in a similar manner to increase their resolving power. The Very Large Telescope (VLT) interferometer in the Atacama Desert in Chile and the Keck interferometer on Mauna Kea, Hawaii, are two examples of optical interferometers.

Worked example

A radio interferometer has an angular resolution of one milliradian (1 mrad) and is observing the 21 cm wavelength from hydrogen gas in the Milky Way. What would the diameter of a single-dish radio telescope have to be with the same resolving power?

Minimum angular resolution

$$\theta = \frac{\lambda}{D}$$

so

$$D = \frac{\lambda}{\theta} = \frac{0.21}{1.0 \times 10^{-3}} = 210\,\text{m}$$

QUESTIONS

13. If the twin Keck telescopes on Mauna Kea, Hawaii, are operating as an optical interferometer with a baseline of 85 m, what is the theoretical angular resolution at a wavelength of 2.2 μm?

KEY IDEAS

- The advantages of large-diameter telescopes are improved angular resolution and greater collecting power.
- Mass and mechanical strength of the primary mirror limit how large the primary objective in a single reflecting telescope can be.
- Interferometers allow large-aperture objectives to be realised by combining signals from two or more separate telescopes in phase and amplitude.

ASSIGNMENT 2: ASSESSING RADIO INTERFEROMETERS

(MS 0.1, MS 0.2, MS 0.3, MS 1.4, MS 2.3)

The Multi-Element Radio Linked Interferometer Network (MERLIN) is a radio interferometer made up of seven radio telescopes spread across England (Figure A1). MERLIN has a minimum angular resolution of 40 milliarcseconds, which is similar to that of the *Hubble* space telescope at optical wavelengths, and is equivalent to measuring the diameter of a one-pound coin from a distance of 100 km.

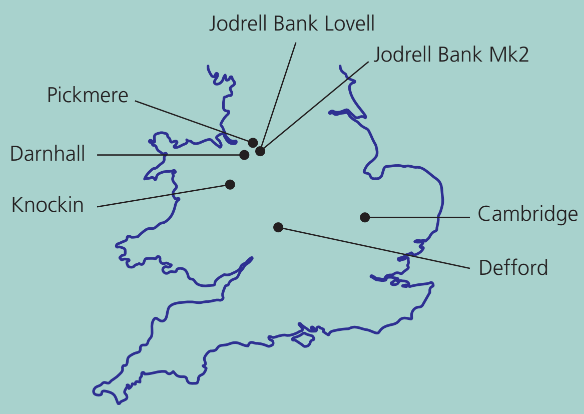

Figure A1 *Locations of the telescopes in the MERLIN radio interferometer array*

Questions

A1 **a.** One of MERLIN's antennas, at Jodrell Bank, has dish diameter of 28 m. What is its theoretical angular resolution in arcseconds when used alone to observe 1.5 cm radio waves?

b. The longest baseline currently possible with MERLIN's antennas is 217 km. If they are observing a radio source at a wavelength of 6 cm, what is the theoretical resolution in arcseconds?

c. Compare the resolution of MERLIN in your answer to **b** with the minimum optical angular resolution of the *Hubble* space telescope, which is 0.05 arcsecond.

d. Suppose that two of MERLIN's antennas were placed on the equator at opposite ends of the Earth's diameter. What would the theoretical angular resolution at this wavelength then be? [Radius of Earth = 6400 km]

e. Explain the advantage of having seven dishes across England linked up as a network.

A2 ALMA (Atacama Large Millimeter/submillimeter Array) is a large radio interferometer sited at an altitude of 5000 m in northern Chile, at the European Southern Observatory (*see the introductory page of Chapter 8 in Year 1 Student Book*). You can find more information on ALMA at www.eso.org. Use your research to answer these questions.

a. Why is ALMA situated at high altitudes?

b. What is the theoretical resolution at a radio frequency of 1 THz of

i. a single dish

ii. the whole array?

iii. By what order of magnitude is the resolution improved?

c. What kind of objects is ALMA designed to investigate?

1.11 CHARGE-COUPLED DEVICES IN ASTRONOMY

A **charge-coupled device (CCD)** is a semiconductor device in which light is converted directly into digital information. CCDs are divided into small regions called **pixels**. A typical CCD array used in astronomy may have several million pixels extending over an area of a few square centimetres arranged in rows and columns (Figure 25).

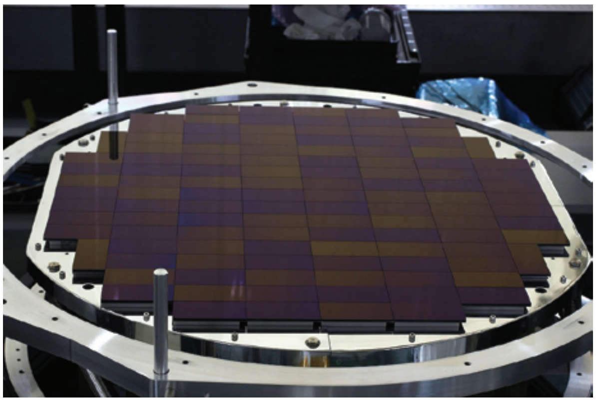

Figure 25 *CCD array for the 8.3 m Subaru telescope at Mauna Kea, Hawaii*

When light strikes the CCD, electric charge is accumulated in the pixels. The amount of charge is proportional to the brightness at a particular pixel location. This makes the response of the CCD linear, which means that it is easy to calculate the number of photons that hit the detector from the object, and then to measure the object's brightness.

One huge advantage of CCDs over other types of light detector, such as photographic film, is that the image is produced and stored digitally as a file that can be image-processed, transmitted to research centres around the world and archived for easy retrieval. This is particularly important for space-based telescopes, where the entire image acquisition is automated.

Quantum efficiency

An important measure of a photon detector's sensitivity is its **quantum efficiency (QE)**. This is defined as

$$\text{quantum efficiency (QE)} = \frac{\text{number of photons detected}}{\text{number of photons incident}} \times 100\%$$

The quantum efficiency tells us how well a detector can capture photons and make them available for further amplification and imaging. An ideal detector would have a QE of 100%. The human eye, which, of course, is also a light detector, has a low QE of about 4–5%, whereas CCDs can have QEs in excess of 80%, making them very efficient light detectors.

A high QE means that the time needed to acquire an image of the same intensity relative to other imaging devices is much smaller, so CCDs require shorter exposure times. The collecting power of a smaller telescope equipped with a CCD as a detector gives comparable performance to a much larger telescope using a detector with a lower QE, such as photographic film, which is typically less than 10%. Additionally, a CCD has a wider spectral range able to detect wavelengths from 200 nm to over 1100 nm.

Resolution

The resolving power of a CCD is defined differently from that of an optical system and is dependent on the number of pixels and their size (typically a few micrometres) relative to the size of the image projected on it. The smaller the size of the pixel, the better the resolution will be and the clearer the image. CCDs can work over a large wavelength range and can be optimised in sensitivity for particular wavelength bands.

In comparison, the theoretical angular resolution of the human eye may be found using the Rayleigh criterion (Astrophysics section 1.6), but, in practice, the spacing between the light-sensitive cells on the eye's retina determines the usable resolution. The retina contains two types of light-sensitive cells called **rods** and **cones**. Cones are responsible for colour vision and rods, which have higher sensitivity than cones, for black and white. The cones are concentrated towards the centre of the retina and are fewer in number, whereas the rods are situated further out to the retina's periphery. Most astronomical observations are due to rod vision, since they are more numerous and more sensitive to low levels of illumination, with about 10^8 rods and about 6×10^6 cones in total. The actual resolution of the eye is about 1–2 arcminutes.

Worked example

A CCD detector looking at a very faint object detects 3500 of the 4000 photons incident on it during a given time period. What is the quantum efficiency of the CCD?

$$\text{QE} = \frac{3500}{4000} \times 100\% = 87.5\%$$

QUESTIONS

14. If the quantum efficiency of the human eye is about 4%, and 10 000 photons fall onto a light-sensitive cell on the back of the retina, how many photons will the eye detect?

15. Summarise the main reasons why CCDs are the detector of choice for modern astronomy.

Stretch and challenge

16. Light of intensity 5.3×10^{-3} W m^{-2} and frequency 3.9×10^{14} Hz falls onto a pixel of area 4.2×10^{-12} m^2 in a CCD detector in the focal plane of a telescope. The quantum efficiency of the CCD is 85%. How many photons of light are incident on the pixel per second? How many of these photons are actually detected and produce a signal?

KEY IDEAS

- Charge-coupled devices (CCDs) are used for astronomical imaging because they have much greater quantum efficiency than other types of light detector.
- Quantum efficiency (QE)

$$= \frac{\text{number of photons detected}}{\text{number of photons incident}} \times 100\%$$

- CCDs have a linear response, which means that they can be used for making accurate measurements of the brightness of objects.
- The resolution of a CCD depends on the size of the individual pixels and their number.
- The information from a CCD can be stored digitally and transmitted for remote image processing and analysis.

PRACTICE QUESTIONS

1. **a.** Explain, with the help of a diagram, what is meant by a refracting telescope being in *normal adjustment*. Your diagram should include the paths of two rays and show the position of the focus of the objective and the eyepiece.

   b. A refracting telescope used by an amateur astronomer is in normal adjustment when looking at the Moon. The telescope is 1 m long and has a maximum useful magnification of 200. State the focal lengths of

   i. the objective lens

   ii. the eyepiece lens.

   c. A spacecraft is in orbit around the Moon at an altitude of 50 km. It is trying to find the landing site of one of the Apollo moon missions of the 1970s. The descent stage of the Apollo Lunar Module that was left on the Moon is 4.3 m wide. The orbiting spacecraft is equipped with a telescope with an angular magnification of 200. What is the angle subtended by the image of the Apollo descent stage by the spacecraft above the Moon's surface?

2. **a.** Draw the ray diagram for a Cassegrain telescope. Your diagram should show the paths of two rays, initially parallel to the principal axis, as far as the eyepiece.

   A telescope design very similar to the Cassegrain was first proposed by James Gregory in 1663. His telescope design was also the first to include a parabolic primary reflector. The use of a parabolic reflector overcomes the problem of spherical aberration.

   b. i. Draw a ray diagram to show how spherical aberration is caused by a concave spherical mirror.

   ii. The first telescope constructed to this design had a primary mirror of diameter 0.15 m. Calculate the minimum angular separation that could be resolved by this telescope when observing point sources of light of wavelength 630 nm. State an appropriate unit.

iii. The astronomer Edmund Halley claimed to have used this telescope to observe the Cassini division, a dark band in the rings of Saturn. Calculate the angle subtended by the width of this band at the Earth, and comment on whether Halley's claim is likely to be valid.

[Width of Cassini division = 4.8×10^3 km, distance from Earth to Saturn = 1.4×10^9 km]

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3. Astronomical objects emit the full range of electromagnetic wavelengths. Observations in different wavelength ranges can provide a huge amount of information about the nature of the objects. Telescopes of different designs are needed to collect this information. Discuss the factors that need to be taken into account when deciding
   a. the size of telescopes
   b. the siting of different types of telescope.

4. There is a supermassive black hole at the centre of the Milky Way galaxy. It is difficult to resolve images of the region around this black hole directly. Astronomers investigating the supermassive black hole detect radio waves at a frequency of 230 GHz. By correlating the information from several radio telescopes, they can obtain images with the same resolution as a single radio telescope with a diameter of 5000 km.

   Calculate the minimum angular separation (in rad) which could be resolved by a radio telescope of diameter 5000 km detecting waves of frequency 230 GHz.

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5. Explain what is meant by *chromatic aberration*, and how it may be corrected in a convex lens.

2 CLASSIFICATION OF STARS

PRIOR KNOWLEDGE

You will need knowledge of how light is emitted from atoms, the concept of atomic energy levels and characteristic spectral lines – you may wish to refer back to *Chapter 8 of Year 1 Student Book*. You will need to be familiar with the use of logarithms and their manipulation.

LEARNING OBJECTIVES

In this chapter you will learn how we can classify the many different stars into types by their physical properties, such as temperature and spectral characteristics. You will learn how we can measure their brightness on the magnitude scale and how the physics of thermal radiation allows us to estimate how large and how hot they are. We will also introduce the common units used in astronomical distance measurements.

(Specification 3.9.2.1 to 3.9.2.4)

2.1 STELLAR LUMINOSITY, BRIGHTNESS AND APPARENT MAGNITUDE

When you look up at the stars at night (Figure 1), it is obvious that some stars are brighter than others. However, this is deceptive, because the observed brightness of an object clearly depends on how far away you are from it. A 60 W light bulb at a distance of 7.5 m and a 100 W bulb at a distance of 9.7 m have the same *apparent* brightness, but put side by side, the 100 W bulb is clearly the more luminous of the two. As it spreads out from distant objects, light obeys an inverse square law – so, for example, a bulb appears 1/100 times dimmer at a distance 10 times further away.

Figure 1 *Some stars in the night sky appear brighter than others.*

The **luminosity** L of a star is the amount of energy in joules it actually radiates per second (that is, its power) and is measured in watts, W. If we imagine a star as a 'point source' centred on a sphere of radius r, the energy passing through each square metre every second is the luminosity divided by the surface area of the sphere. This is the intensity of the radiation and we define it as the **brightness** b of a star:

$$b = \frac{L}{4\pi r^2}$$

The unit of brightness is $W\,m^{-2}$. The radiation from a star illuminates an ever-increasing area of a sphere as the distance from the star increases, so the brightness decreases as the square of the distance (Figure 2).

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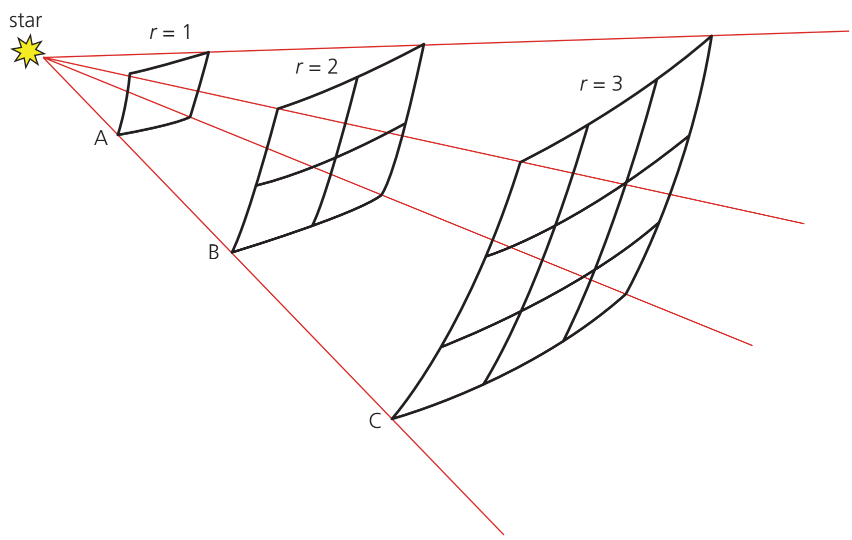

Figure 2 *The brightness of the star is nine times less at C than it is at A.*

Stellar magnitude

Astrophysicists prefer to talk about a star's brightness, as seen from Earth, rather than its luminosity. Relative brightness is expressed on the **Hipparchus scale**, based on a convention first devised by Hipparchus of Nicaea (190–120 BC). On this scale, stars are classified by their **apparent magnitude**, *m*, with the brightest stars that can be seen with the naked eye as magnitude 1.0 and the faintest as magnitude 6.0. Subsequently, with the invention of the telescope, the scale was extended to classify stars with magnitudes greater than 6.0. It has also been extended to values less than 1.0 for very bright objects such as the Sun. The star Vega is assigned a magnitude of 0, the Sun has a magnitude of −26.74, and the full Moon has a magnitude of −12.6.

It is important to understand that the *more negative* the value of apparent magnitude, the *brighter* the star appears. Conversely, the larger (more positive) the magnitude, the fainter the star appears.

Brightness measured in this way is a subjective scale, as we need to know how far away a star is from us in order to know its true luminosity. Also, note that apparent magnitude (and absolute magnitude, see Astrophysics section 2.3) refers to the brightness of a star in the *visible* part of the spectrum (what we can see). Very hot stars radiate much of their power outside the visible spectrum, so their luminosity may be greater than that we can detect only in the visible region.

The human eye perceives equal ratios of brightness at equal intervals. So, on the Hipparchus scale, the brightness coming from stars of magnitude 1.0 was about 100 times greater than from stars of magnitude 6.0. Therefore, a difference of 6 – 1 = 5 magnitudes corresponds to a brightness ratio of 100. A magnitude difference of 1 therefore corresponds to a brightness ratio of $(100)^{1/5}$ or 2.51. The magnitude scale is therefore a *logarithmic* scale. An English astronomer, Norman Pogson, in 1856 formulated the Hipparchus scale into a precise mathematical law, called **Pogson's law**, expressed as

$$m_2 - m_1 = -2.5 \log_{10}\left(\frac{b_2}{b_1}\right)$$

where m_1 = apparent magnitude of star 1, b_1 = received brightness of star 1, m_2 = apparent magnitude of star 2, b_2 = received brightness of star 2, and log means log to the base 10, that is, $\log_{10}$.

We can see that Pogson's law is consistent if we note that the minus sign ensures that magnitudes are a measure of faintness. If b_2 is less than b_1 (star 2 is fainter than star 1), then the brightness ratio is less than 1 and the log of the ratio is negative. So $m_2 - m_1$ is positive and the fainter star has the larger magnitude, as expected. The multiplier 2.5 is simply a scaling factor that ensures that a brightness ratio of 100 corresponds to a magnitude difference of 5.0. (Note that the factor of 2.5 in the equation above is not the value 2.51 rounded down. It is an exact value, due to the fact that $\log_{10}(2.51) = \log_{10}(100^{1/5})$ $= 0.2 \log_{10}(100) = 0.2 \times 2 = 0.4 = 1/2.5$, exactly.)

Worked example

The apparent magnitude m_{Sun} of the Sun is – 26.8 and the apparent magnitude m_{Moon} of the full Moon is –12.6. By what factor is the Sun brighter than the Moon?

Using Pogson's law:

$$m_{Sun} - m_{Moon} = -2.5\log_{10}\left(\frac{b_{Sun}}{b_{Moon}}\right)$$

$$-26.8 - (-12.6) = -2.5\log_{10}\left(\frac{b_{Sun}}{b_{Moon}}\right)$$

$$-14.2 = -2.5\log_{10}\left(\frac{b_{Sun}}{b_{Moon}}\right)$$

$$5.68 = \log_{10}\left(\frac{b_{Sun}}{b_{Moon}}\right)$$

Therefore

$$\left(\frac{b_{Sun}}{b_{Moon}}\right) = 10^{5.68} = 478\,000$$

The Sun is about 480 000 times brighter than the full Moon.

QUESTIONS

1. By what factor is a star of apparent magnitude 1 brighter than one of apparent magnitude 3?

2. a. Table 1 shows a number of stars and their apparent magnitudes. Rank them in increasing order of brightness.

Star	Apparent magnitude
Aldebaran	1.0
Arcturus	−0.1
Sirius	−1.5
Deneb	1.3
Rigel	0.2
Altair	0.9
Canopus	−0.9
Mizar	2.2

Table 1

   b. By how much is the star Canopus brighter than Altair?

3. The Sun has a luminosity of 3.90×10^{26} W. What is its brightness, in W m^{-2}, at
   a. the top of the Earth's atmosphere (mean distance from Sun = 1.50×10^{11} m)
   b. the surface of Pluto (mean distance from Sun = 5.93×10^{12} m)?

KEY IDEAS

- The luminosity L of a star is the amount of energy in joules that it radiates per second. It is measured in watts, W.
- The brightness b of a star at a distance r is

$$b = \frac{L}{4\pi r^2}$$

in W m^{-2}.

- The Hipparchus scale of apparent magnitude, m, assigns a perceived brightness to stars seen from Earth. The value of m is a number with no unit. The more negative the value of m, the brighter the star appears.
- Pogson's law relates a difference in magnitude to a ratio of brightness:

$$m_2 - m_1 = -2.5\log_{10}\left(\frac{b_2}{b_1}\right)$$

A difference of 1 magnitude corresponds to a brightness ratio of 2.51.

2.2 ASTRONOMICAL DISTANCE

The astronomical unit

A natural starting point as a unit for astronomical distances is the mean distance from the Earth to the Sun. This distance is called the **astronomical unit** (**AU**) and 1 AU is equal to 1.50×10^{11} m. However, this unit is only appropriate on interplanetary scales, as the distances to other stars are so great as to render it too small to be useful.

The parsec

For interstellar distances, astronomers use a unit called the **parsec**. In order to understand how the parsec is defined, we need to look at some trigonometry and what is meant by **parallax**.

Imagine looking at a candle held at arm's length. If you alternately open one eye and close the other, several times, then the candle will appear to jump back and forth relative to a fixed point in the background. The angle that the candle makes with your eye as it shifts to and fro is called the **parallax angle** of the candle. We can see parallax happening on an astronomical scale, as the Earth orbits the Sun. This time the candle

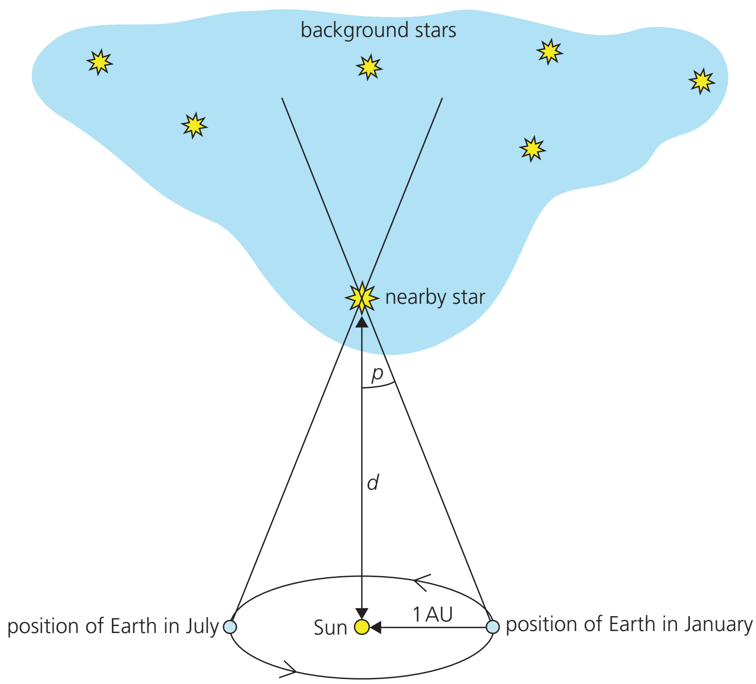

Figure 3 *As the Earth orbits the Sun, a nearby star appears to shift its position with respect to the background of distant stars.*

is a nearby star, and the fixed point corresponds to the background of distant stars that do not appear to change their positions as the Earth orbits the Sun (Figure 3). Suppose we record the position of a nearby star at two points on the Earth's orbit separated by a time interval of six months. These positions of the Earth are separated by a distance of 2 AU.

Owing to parallax, the nearby star appears to shift in position relative to the background stars. By using simple trigonometry, we can show that the **parallax angle** p (measured in radians) is related to the distance d of the star by

$$d = \frac{1\,\text{AU}}{\tan p} = \frac{1\,\text{AU}}{p}$$

using the fact that, for small angles, $\tan p = p$.

One radian = 57° 17′ 45″ and if we now measure all angular sizes in the unit of arcsecond (″), then

$$1\text{ rad} = (57 \times 3600)'' + (17 \times 60)'' + 45 = 206265''$$

So

$$p(\text{rad}) = \frac{p\text{ (arcsecond)}}{206265}$$

and substituting for p (rad) in the equation above, we get

$$d = \frac{206\,265}{p\text{ (arcsecond)}}$$

We are now in a position to define the distance unit **parsec** (**pc**). The word is an abbreviation of parallax and arcsecond and

$$1\text{ parsec (1 pc)} = 206\,265\text{ AU}$$

We can then write

$$d = \frac{1}{p}$$

where the unit of d is parsec and the unit of p is arcsecond. Thus

1 parsec is the distance at which the observed parallax angle of the star is equal to 1 arcsecond (1 second of arc).

It is now apparent why this unit is so useful. Once the parallax of a star in arcsecond is known, then its distance in parsec is found simply by taking the reciprocal. Since huge distances are dealt with in astronomy, the following units are common:

$$1 \text{ kiloparsec (1 kpc)} = 10^3 \text{ pc}$$

$$1 \text{ megaparsec (1 Mpc)} = 10^6 \text{ pc}$$

The light year

One **light year (ly)** is the distance that a photon of light travels through space in one year. Since light travels at $3 \times 10^8 \text{ m s}^{-1}$, we calculate this value as

$$3.00 \times 10^8 \text{ m s}^{-1} \times 365 \text{ day} \times 24 \text{ hour} \times 3600 \text{ s} = 9.46 \times 10^{15} \text{ m}$$

Sub-units are:

$$1 \text{ light minute} = 3.00 \times 10^8 \text{ m s}^{-1} \times 60 \text{ s} = 1.80 \times 10^{10} \text{ m}$$

$$1 \text{ light second} = 3.00 \times 10^8 \text{ m s}^{-1} \times 1 \text{ s} = 3.00 \times 10^8 \text{ m}$$

Also:

$$1 \text{ pc} = 3.26 \text{ ly}$$

QUESTIONS

4. a. Explain what is meant in astronomy by a *parallax angle*.
   b. A measurement of the parallax of the star 61 Cygni in the constellation of Cygnus (the Swan) is found to be 0.316″. What is its distance from the Earth in the following units:
      i. parsec
      ii. astronomical unit
      iii. light year?
5. Figure 4 shows Proxima Centauri.
   a. What is its distance from Earth in parsec?
   b. What would its parallax angle be?

Figure 4 *Proxima Centauri, a red dwarf, imaged by the Hubble telescope. It is 4.2 ly from Earth and is our nearest star (other than the Sun).*

KEY IDEAS

- One astronomical unit (1 AU) is the distance between the Earth and the Sun.
- One light year (1 ly) is the distance travelled by light in one year.
- One parsec (1 pc) is the distance at which the observed parallax angle is equal to 1 arcsecond (1 second of arc).

2.3 ABSOLUTE MAGNITUDE

Suppose you could place all the stars at a fixed distance from the Earth. Differing distances would not then be a factor in how bright the stars appeared. Instead, the differences in magnitude would be due only to differences in luminosity, and as such these values would be *absolute*. Astronomers use a standard distance of 10 parsecs for absolute magnitude comparison. We therefore define the magnitude that a star would have if it was placed 10 pc from the Earth as its **absolute magnitude**, M.

What is the relationship between a star's apparent magnitude m (which we observe), and its absolute magnitude M? Consider a star with luminosity L, absolute magnitude M, apparent magnitude m and brightness received at Earth, a distance d from the star, b_d. Let b_{10} be the brightness the star would have at a distance of 10 pc. Then using Pogson's law we find that

$$m - M = -2.5 \log_{10}\left(\frac{b_d}{b_{10}}\right)$$

but the brightness is equal to $\frac{L}{4\pi r^2}$, assuming that the luminosity is radiated uniformly over the area of a sphere of radius r, so that

$$\frac{b_d}{b_{10}} = \frac{L}{4\pi d^2} \div \frac{L}{4\pi(10)^2} = \frac{L}{4\pi d^2} \times \frac{4\pi(10)^2}{L} = \left(\frac{10}{d}\right)^2$$

where d is expressed in parsecs. Therefore

$$m - M = -2.5 \log_{10}\left(\frac{10}{d}\right)^2$$

Or, using the properties of logarithms,

$$\begin{aligned} m - M &= -5 \log_{10}\left(\frac{10}{d}\right) \\ &= -5(\log_{10} 10 - \log_{10} d) \\ &= 5(\log_{10} d - \log_{10} 10) \end{aligned}$$

So we finally obtain

$$m - M = 5 \log_{10}\left(\frac{d}{10}\right)$$

The quantity $(m - M)$ is called the **distance modulus** since it is directly related to the star's distance d from the Earth. For example, the absolute magnitude M of Capella is 0.40 and its apparent magnitude m is 0.08; therefore, its distance modulus is

$$0.08 - 0.40 = -0.32$$

The above equation shows that if a star's distance is known and its apparent magnitude is measured, then we can determine its absolute magnitude. Conversely, if we know the absolute magnitude of a star and its apparent magnitude, we can determine the distance. Rearranging the above equation gives

$$\begin{aligned} \frac{m - M}{5} &= \log_{10}\left(\frac{d}{10}\right) \\ 10^{(m-M)/5} &= \frac{d}{10} \\ 10^{[(m-M)/5]+1} &= d \\ d &= 10^{(m-M+5)/5} \end{aligned}$$

It may seem unlikely that we would know the absolute magnitude of a star. But **Cepheid variable** stars have a remarkable property. They have a periodic variation in luminosity that has a constant known relationship with their maximum luminosity. From measuring the period of their variation in luminosity, their absolute magnitude can be calculated. Such stars have been used to determine distances to star clusters or galaxies well beyond what is possible from parallax measurements (because the angular displacements would be too small to measure). Astronomical objects such as these, for which the luminosity can be calculated directly, are called **standard candles**.

Worked example

A Cepheid variable star is observed in another galaxy that is close to the Milky Way. From its periodic variation in luminosity, its absolute magnitude is determined as being 15.56. It is observed to have an apparent magnitude of −3.60. Estimate how far the galaxy is from Earth in parsecs.

The distance modulus is $m - M = 15.56 - (-3.60)$
$= 19.20$

So

$$d = 10^{(19.2+5)/5} = 10^{4.84} = 69\,000\,\text{pc}$$

QUESTIONS

6. **a.** Explain the difference in meaning between *apparent magnitude* and *absolute magnitude*.
   b. The star Procyon A has an apparent magnitude of +0.34 and is at a distance of 3.5 pc. What is its absolute magnitude?
   c. The star Regulus has an apparent magnitude of +1.35 and an absolute magnitude of −0.30. What is its distance modulus?
   d. How far away from us is Regulus in parsecs?
7. Why is there a limit to distances that can be measured using parallax?

KEY IDEAS

- The absolute magnitude of a star is its apparent magnitude if it were located at a distance of 10 parsecs from the Earth.
- Apparent magnitude m and absolute magnitude M are related by

$$m - M = 5 \log_{10}\left(\frac{d}{10}\right)$$

where d is the distance in parsecs. The quantity $(m - M)$ is called the distance modulus.

ASSIGNMENT 1: USING CEPHEID VARIABLES AS STANDARD CANDLES

(PS 2.2, PS 2.3, PS 2.4, PS 3.1, PS 3.2, MS 1.2, MS 2.5, MS 3.2, MS 3.10)

A standard candle is an astronomical object that has a known luminosity and so allows distance to be calculated. In 1908, the American astronomer Henrietta Swan Leavitt discovered Cepheid variable stars, whose luminosity varies with a regular period of a few hours, days or weeks, dependent on their maximum luminosity. The stars were so-called because the first to be identified was observed in the constellation Cepheus.

If the period of the variability of a Cepheid is measured, then its absolute magnitude can be predicted. In this assignment you will see how Cepheid variables can be used to calculate the distance of galaxies.

Figure A1 shows the brightness variation of four Cepheid variable stars HV 837, HV 1967, HV 843 and HV 2063 in the Large Magellanic Cloud (LMC), a galaxy close to the Milky Way, plotted as apparent magnitude against time in days.

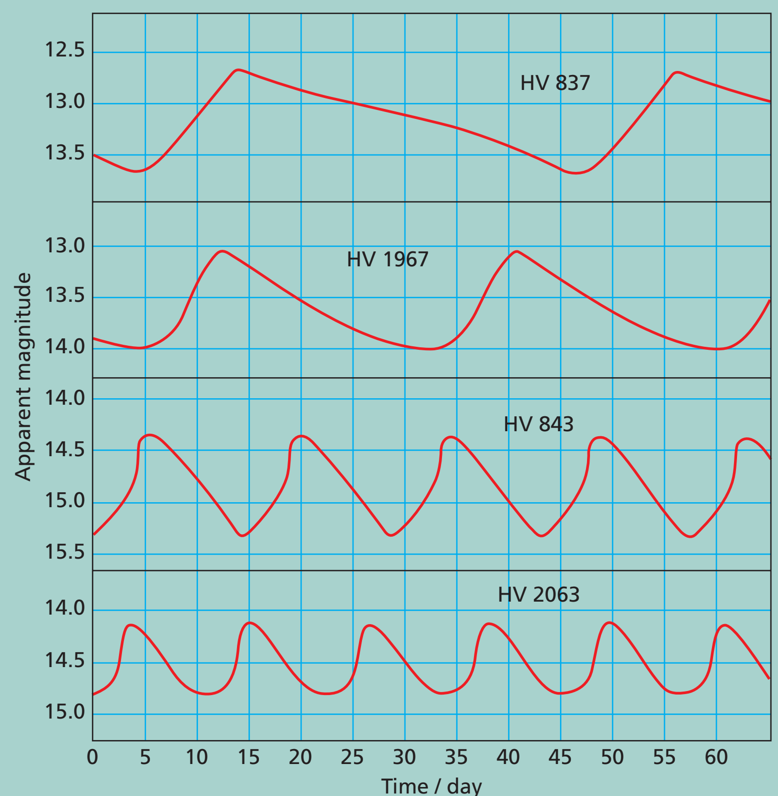

Figure A1 *Apparent magnitude against time for four Cepheid variables*

Questions

A1 For each of the stars, read off from Figure A1 their maximum and minimum apparent magnitude values (m_{Max} and m_{Min}) to the nearest 0.1 magnitude. Take the mean of these two values. For each star find its period *T* in days and take the logarithm of the period to two decimal places. Enter your values in a table like Table A1.

Star	m_{Max}	m_{Min}	Mean *m*	*p* / day	$\log_{10} p$
HV 837	12.60	13.65	13.13	42	1.62
HV 1967	13.00	14.00	13.50	26	1.41
HV 843	14.35	15.30	14.83	15	1.81
HV 2063	14.10	14.80	14.45	11	1.04

Table A1

Using a spreadsheet such as Excel (or other graph plotting tool), plot for each star the mean apparent magnitude against $\log_{10} T$. Draw a straight line to fit the four data points as well as possible.

Your plot gives a relation between the apparent magnitude and the variability period for the Cepheid variable stars in the LMC.

A2 Suggest why you have plotted apparent magnitude against $\log_{10} T$ and not against *T*.

A3 How could the accuracy of the plot be improved?

A4 Why can all the Cepheids in the LMC be regarded has being the same distance from us?

A5 Why can the plot you have made not be used to determine the distance of the Cepheids?

The astronomer Harlow Shapley used a parallax method to work out the distance to a group of Cepheids in our own galaxy. Shapley then provided a table of absolute magnitude *M* and period *T* for nearby Cepheids, which is reproduced in Table A2.

$\log_{10} P$	Absolute magnitude *M*	$\log_{10} P$	Absolute magnitude *M*
0.0	−0.4	1.0	−2.9
0.2	−0.8	1.2	−3.6
0.4	−1.2	1.4	−4.4
0.6	−1.6	1.6	−5.1
0.8	−2.2	1.8	−5.8

Table A2 *Shapley's data*

Questions

A6 Using a spreadsheet such as Excel (or other graph plotting tool), plot the data from Table A2, with *M* against $\log_{10} T$, and draw the best straight line though the points. You now have a similar plot to the one you drew before but with absolute magnitude plotted against $\log_{10} T$.

A7 Explain how both of your graphs can be used to work out the distance to a galaxy in which Cepheids are observed.

A8 What important assumption is made about Cepheid variables in galaxies?

A9 Suggest a reason why we cannot use this method to find the distance to very distant galaxies in the Universe.

2.4 CLASSIFICATION OF STARS BY THEIR TEMPERATURE

Stefan's law

We have seen that stars can be classified by their luminosity. They emit **thermal radiation**, which is electromagnetic radiation generated by the thermal motion of charged particles in matter. The luminosity, the rate of thermal energy radiated, depends on the temperature of the star and its size.

In the late 19th century, the Austrian physicist Josef Stefan carried out a series of experiments which showed the relationship between the rate of thermal energy emitted by a hot object and its temperature. This empirical relationship was also derived theoretically by another physicist, Ludwig Boltzmann, using thermodynamic assumptions about atoms and molecules. They were both led to the following conclusion:

A body, when heated, will emit electromagnetic radiation over a range of wavelengths with a total intensity that is proportional to the fourth power of its absolute temperature.

This can be written as

$$I \propto T^4$$

where I is the intensity, or radiated power per unit area, and T is the absolute temperature in kelvin. We then obtain the **Stefan–Boltzmann law**, or **Stefan's law** as it is more commonly known,

$$P = \sigma A T^4$$

where P is the total power in watts radiated by an object of surface area A, and σ is a constant of proportionality called the **Stefan–Boltzmann constant**, which is equal to $5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$.

Stefan's law holds true for an object that is in thermal equilibrium. This means that it is at a steady temperature, so the rate of energy absorbed by the body from its surroundings must equal the rate of energy flowing out from it. **Kirchhoff's law of thermal radiation** states:

For any given temperature, the ratio of the capacity of a body to emit radiation to its capacity to absorb it (at a particular wavelength) is constant and is independent of the composition of the body.

This statement implies that, if an object is an efficient absorber of radiation at a given wavelength, then it will also be an efficient radiator at that wavelength. It was shown by Boltzmann that Stefan's law is valid only for a body that is a perfect absorber of energy. Such an object is known as a **black body**, because it does not reflect any light. Its radiated energy flux depends *only* on its temperature and not on its surface composition, in accordance with Stefan's law.

The Sun and other stars emit radiation very much like an ideal black body. Intuitively, this seems rather strange. Why are they called 'black' when they most obviously are not? We have to understand that stars are black bodies because they absorb light at any wavelength but do not reflect any back – if you were to shine a beam of light at the Sun, it would not be reflected back to you.

For a spherical star of radius R, the surface area is $4\pi R^2$. The total power radiated, P, defines the luminosity of the star, L:

$$L = \sigma A T^4 = 4\pi R^2 \sigma T^4$$

So stellar luminosity is proportional to R^2 and to T^4. In the case of the Sun, taking the Sun's surface temperature to be 5800 K and its radius to be 7×10^8 m, the luminosity of the Sun is

$$\begin{aligned} L_{\mathrm{Sun}} &= \sigma A T^4 = 4\pi R^2 \sigma T^4 \\ &= 4\pi \times (7 \times 10^8)^2 \times 5.67 \times 10^{-8} \times (5800)^4 \\ &= 3.97 \times 10^{26}\,\mathrm{W} \end{aligned}$$

Stefan's law forms the basis for all estimates of stellar size. But in order to determine R, we need to know the temperature, T. Fortunately, the colour of a star is a good guide to its approximate surface temperature, as we will see in the next subsection.

Wien's displacement law

Suppose we take a metal bar and heat it with a blowtorch. At first the bar will glow a dull red. As it grows hotter, the bar will change colour from red through to orange and then yellow; and if it gets extremely hot (and could be prevented from melting), to a brilliant bluish white. This suggests that, as an object is heated further, it emits radiation of shorter wavelengths (*see section 8.1 in Chapter 8 in Year 1 Student Book*).

To understand why the bar changes colour when its temperature increases, we need to consider the properties of thermal radiation. A black body emits electromagnetic radiation over a wide range of wavelengths, but there will be one wavelength, called the peak wavelength, for which the emission of radiation has its maximum intensity. In 1894, the German physicist Wilhelm Wien discovered a simple relationship between the absolute temperature T of a black body and the peak wavelength λ_{max} at which the radiated energy reaches its maximum intensity.

The wavelength of the peak emission intensity is inversely proportional to the absolute temperature of the object.

This can be written as

$$\lambda_{max} T = \text{constant} = 2.90 \times 10^{-3}\,\text{m K}$$

This relationship is known as **Wien's displacement law** or sometimes simply **Wien's law**. Note that 'm K' is metre kelvin, *not* millikelvin. It shows that the dominant wavelength of a black-body radiator decreases as its gets hotter, just as we observe when we heat the metal bar. An object at room temperature (300 K), for example, emits mainly infrared radiation. A very cold object of temperature a few kelvin above absolute zero emits primarily microwaves, whereas an object of a few million kelvin would emit at X-ray wavelengths.

The intensity distribution of black-body radiation always has a characteristic shape, and a graph showing the intensity with wavelength is a continuous one. Figure 5 shows the intensity distribution of black-body radiators at different temperatures. Notice that the higher the temperature, the shorter the wavelength of maximum intensity, just as we would expect from Wien's law.

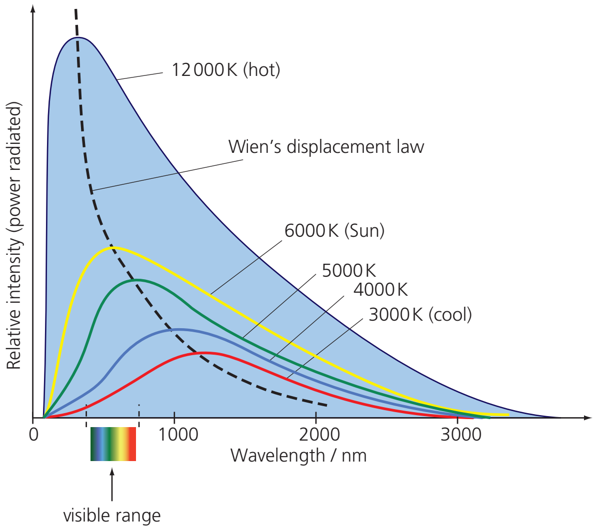

Figure 5 *The intensity–wavelength curves of black-body radiators at different temperatures*

These curves are called **black-body curves**. It is important to realise that when you see a black-body curve you know that the processes that give rise to the emission of radiation depend only on temperature and not on any other property, such as the chemical composition of the object.

Measurements made above the Earth's atmosphere of the intensity distribution of sunlight over a broad range of wavelengths show that the Sun is a good approximation to a black body when compared with the theoretical black-body curve at a temperature of 5800 K, with its peak in the yellow region of the visible spectrum. While the entire star is not in thermal equilibrium and has a temperature gradient towards its centre, the photosphere of the star, where the emitted light is generated, is close to thermal equilibrium and maintains a common temperature over a long period of time.

It is because stars are so much like black bodies that astrophysicists are able to deduce their surface temperatures. Hotter stars emit most of their radiation at shorter wavelengths and will appear to be bluer, whereas cooler stars emit at longer wavelengths and will appear to be redder (Figure 6).

(a)

(b)

Figure 6 *The colour of stars depends on their temperature. (a) Vega is a hot bluish star with a surface temperature of 9600 K. (b) Aldebaran is a cooler red giant star with a surface temperature of 3900 K.*

QUESTIONS

8. The star Arcturus has a radius of 25 times the radius of the Sun and a surface temperature of about 4300 K. Estimate the luminosity of Arcturus. [Radius of Sun = 6.96×10^8 km]

9. a. The star Rigel has a luminosity 66 000 times that of the Sun. If its surface temperature is 11 000 K, estimate the radius of Rigel. [Luminosity of Sun = 3.9×10^{26} W]

   b. What is the peak wavelength of emission of Rigel?

KEY IDEAS

- The luminosity of a star depends on its size and its temperature.
- Stars may be regarded as black bodies with a characteristic black-body curve depending on their surface temperature.
- Stefan's law states that

$$P = \sigma A T^4$$

for a black-body radiator, where P is the total power radiated by an object of surface area A, T is the absolute temperature of the object, and σ is a constant of proportionality called the Stefan–Boltzmann constant.

- Wien's displacement law relates the peak wavelength λ_{max} of a star's black-body spectrum to its temperature:

$$\lambda_{max} T = \text{constant} = 2.90 \times 10^{-3}\,\text{m K}$$

2.5 STELLAR SPECTRAL CLASSES

The intensity–wavelength distribution of a star – a black-body curve like those in Figure 5 – is its **emission spectrum**, which is a continuous spectrum. The distribution depends only on the star's surface temperature. Dull red stars are cool and bluish white stars are very hot.

Stars are classified by their temperature using letters of the alphabet. There are seven main types of stars denoted, in order of decreasing temperature, O, B, A, F, G, K and M (Table 2). This order may be remembered by the mnemonic: Only Bright Astrophysicists Fight Green Killer Martians (but you can think up your own).

Spectral class	Intrinsic colour	Temperature / K
O	Blue	25 000–50 000
B	Blue	11 000–25 000
A	Blue-white	7500–11 000
F	White	6000–7500
G	Yellow-white	5000–6000
K	Orange	3500–5000
M	Red	< 3500

Table 2 *The alphabetic classification of stars by their surface temperatures*

However, the radiation detected on Earth from a star can tell us much more than this, and we can further classify stars by their spectral characteristics. **Stellar spectroscopy** is a method of analysing the spectrum of stars and is a powerful tool in determining not only the precise surface temperature but also the composition of and physical conditions within stars.

Spectroscopy gives rise to three types of spectra:

- an emission line spectrum
- an emission continuous spectrum
- an absorption spectrum.

Each of these gives different information about its source (*see Chapter 8 in Year 1 Student Book*).

In a gas at low temperature and pressure, almost all the atomic electrons are in the lowest energy level (ground state). As the temperature increases, more atomic collisions take place and electrons are raised to excited states. These electrons eventually return to lower energy levels, emitting photons at precise characteristic energies corresponding exactly to the spacing of the energy levels within the atoms of the gas. The spectrum recorded is of bright lines on a dark background, an emission **line spectrum** (Figure 7), with the intensity and position of these lines corresponding to particular electronic transitions in the atoms of the gas.

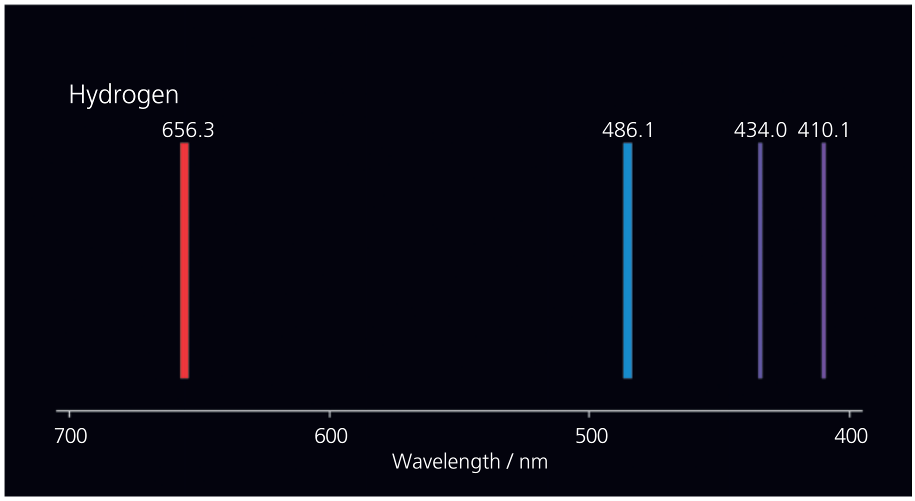

Figure 7 *An emission line spectrum for hydrogen, showing bright Balmer lines on a dark continuous background*

In a hot star, the gas is at high pressure. Atoms have considerable kinetic energy and undergo multiple collisions and their electrons are in excited states. By the time the excited electrons fall back into one of the discrete energy levels, further atomic collisions have occurred. This results in a blurring of the emission spectrum and the loss of any detail about the atoms in the gas, giving rise to a **continuous spectrum** (Figure 8). This is typical of the emission spectrum obtained from the region of a star, the **photosphere**, from where the light is radiated.

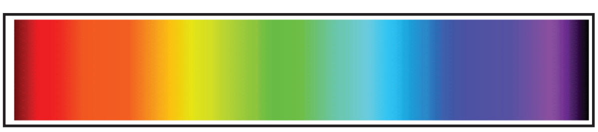

Figure 8 *The continuous spectrum of the photosphere of the Sun*

The photosphere acts as a source of visible light. This light then passes through the outer layers of the Sun, which are much cooler and composed mainly of hydrogen gas. Photons of the characteristic energies of the transitions in the gas will be absorbed and atomic electrons raised to an excited state (perhaps to the second level or shell, $n = 2$, or even higher shells, $n = 3, 4, 5, 6$ and so on). As electrons fall back to the first level, $n = 1$ (the ground state), or intermediate levels (Figure 9), photons are emitted, but in random directions. The resulting spectrum comprises dark lines (undetected photons) characteristic of an **absorption spectrum** (Figure 10).

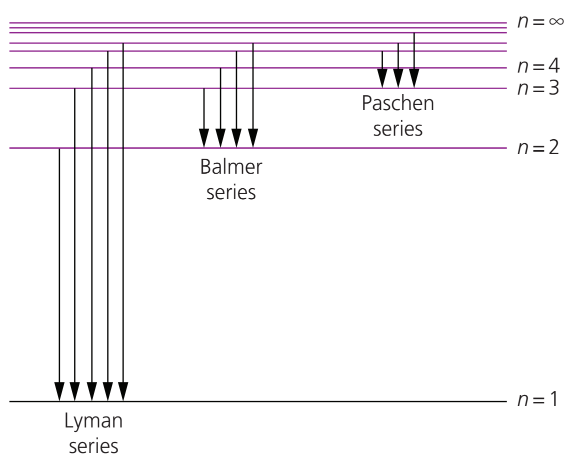

Figure 9 *Transitions between energy levels giving rise to line series in the hydrogen spectrum*

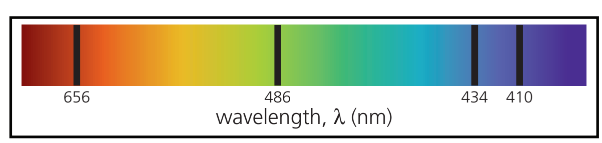

Figure 10 *An absorption spectrum for hydrogen, showing dark Balmer lines on a continuous background. The dark lines in an absorption spectrum correspond exactly to the bright lines in an emission line spectrum produced by the same gas. Compare this spectrum with that in Figure 7.*

The absorption lines for hydrogen in the visible part of the spectrum result from electrons moving from the first excitation level ($n = 2$) to higher energy levels (*see sections 8.2 and 8.3 in Chapter 8 in Year 1 Student Book*). This leads to a series of dark lines shown in Figure 10 called the **Balmer series**. The intensity of the absorption lines depends on the particular temperature of the star's photosphere.

Other dark lines in a star's absorption spectrum are characteristic of other particular elements within the gas in the outer layers (Figure 11). A full analysis of the absorption lines also reveals the state of the atoms, that is, whether they are neutral or ionised, which also depends on the temperature. The absorption spectrum therefore not only enables identification of the elements present in the star but also allows the temperature of the star to be determined accurately.

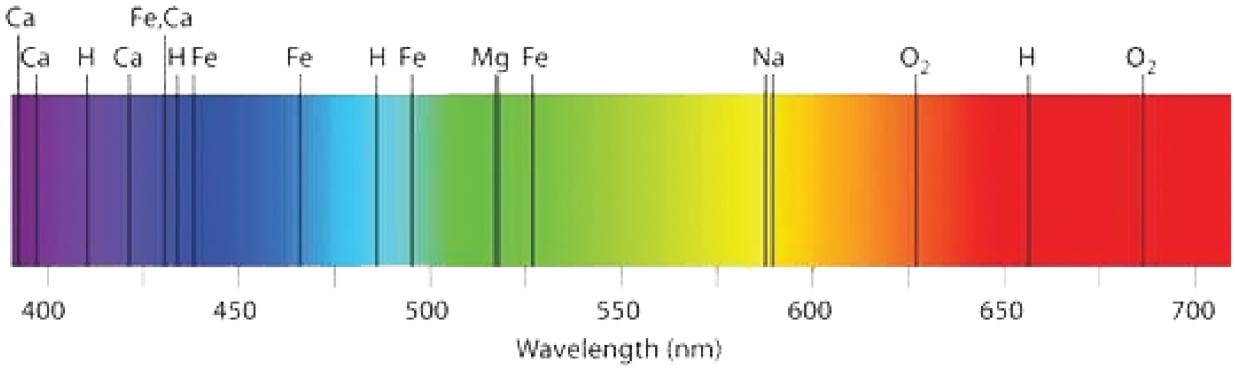

Figure 11 *Absorption spectrum of the Sun showing the absorption of light by elements in the cooler outer part of the Sun's atmosphere (although the O_2 absorption lines at 628 and 687 nm are actually due to the absorption of light in the Earth's atmosphere, before reaching the ground-based telescope).*

The relative strength of particular absorption lines (see Figure 12), and hence temperature, gives the **spectral class** of a star. We can further define the classification by temperature (summarised in Table 2) with a description of the prominent spectral absorption lines, as shown in Table 3.

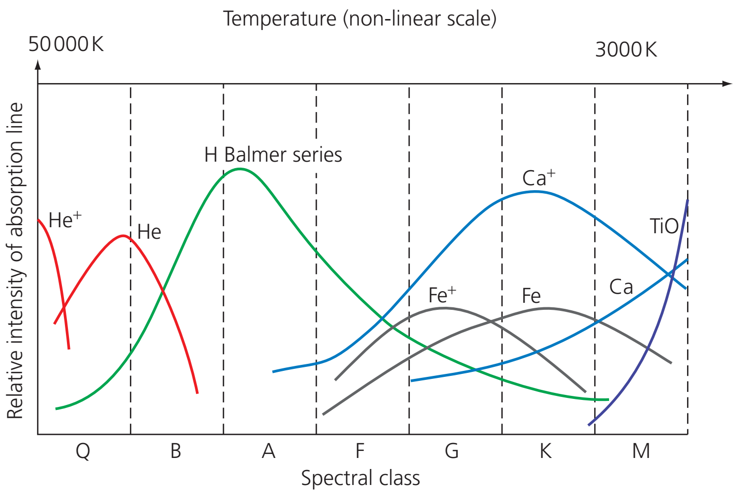

Figure 12 *The intensity of particular absorption lines depends on the temperature, and hence can be used to determine the star's spectral class. The Balmer series is particularly useful for this classification.*

Spectral class	Colour	Temperature range / K	Prominent absorption lines	Example star
O	Blue	25 000–50 000	He^+, He, H	10 Lacertae
B	Blue	11 000–25 000	He, H	Rigel, Spica
A	Bluish white	7500–11 000	H (strongest), ionised metals	Sirius, Vega
F	White	6000–7500	Ionised metals	Procyon
G	Yellow-white	5000–6000	Ionised and neutral metals	Sun, Capella
K	Orange	3500–5000	Neutral metals	Aldebaran
M	Red	< 3500	Neutral atoms, TiO	Betelgeuse, Antares

Table 3 *The classification of stars by spectral class, including the prominent absorption lines in each class*

Figure 13 shows the spectrum of the star Vega, spectral class A, in which the prominent hydrogen absorption lines can be clearly seen.

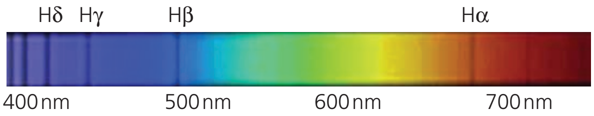

Figure 13 *The spectrum of Vega*

QUESTIONS

10. Barnard's star, the fourth nearest star to the Earth, has a surface temperature of 3134 K. Which spectral class does it belong to?

11. When you look up at the night sky on a clear night and observe some of the brightest stars with the naked eye, is there any way you can tell which are hot and which are cooler?

12. The helium atom has a transition between electronic states that produces an emission line at 587.56 nm. Using a diffraction grating, a German telescope maker, Joseph von Fraunhofer (1787–1826), discovered 784 dark lines in the spectrum of the Sun, now called Fraunhofer lines. One of these was an absorption line at a wavelength of 587.56 nm that did not correspond to any known element on Earth. Explain how this led to the discovery of helium.

KEY IDEAS

- A star has an absorption spectrum – a series of dark lines – superimposed on its black-body continuous emission spectrum, characteristic of the elemental composition of its outer layers.
- The position and intensity of the absorption lines allow an accurate determination of the star's temperature.
- Stars are classified into spectral classes based on their temperature. There are seven main classes, O, B, A, F, G, K and M in order of decreasing temperature, from over 50 000 K to below 3500 K.

PRACTICE QUESTIONS

1. Sirius is a binary system consisting of two stars, Sirius A and Sirius B, the properties of which are summarised in Table Q1.

	Sirius A	Sirius B
Absolute magnitude	1.4	11.2
Apparent magnitude	−1.4	8.4
Diameter / 10^3 km	2400	12
Black-body temperature / K	10 000	25 000

Table Q1

a. Calculate the distance to Sirius A, giving an appropriate unit.

b. i. Calculate the ratio

$$\frac{\text{power output of Sirius A}}{\text{power output of Sirius B}}$$

ii. Show that the data in Table Q1 suggest that one star is about 8000 times brighter than the other.

iii. With reference to the spectra of the two stars, explain why the value in part **b ii** is much greater than the answer to part **b i**.

AQA Unit 5A June 2010 Q2

2. Hydrogen Balmer absorption lines are seen in the spectra of many stars. Explain how these arise. The quality of your written communication will be assessed in your answer.

3. Deneb is the brightest star in the constellation Cygnus.

a. The black-body radiation curve for Deneb shows a peak at a wavelength of 3.4×10^{-7} m. Calculate the black-body temperature of Deneb. Give your answer to an appropriate number of significant figures.

b. The power output of Deneb is 70 000 times greater than the Sun. Calculate the radius of Deneb.
[Surface temperature of the Sun = 5700 K, radius of Sun = 6.96×10^5 km]

AQA Unit 5A June 2011 Q2 part b

4. a. Bellatrix and Betelgeuse are stars in the constellation of Orion. Some of their properties are summarised in Table Q2.

	Bellatrix	Betelgeuse
Absolute magnitude	−6.0	−2.7
Apparent magnitude	0.4	1.6
Black-body temperature / K	22 000	2400

Table Q2

i. Explain what is meant by *absolute magnitude*.

ii. Which of the two stars is closer to the Earth? Explain your answer.

b. i. Calculate the wavelength of the peak intensity in the black-body radiation curve of Bellatrix.

ii. Sketch a relative intensity versus wavelength black-body radiation curve for Bellatrix. Label the wavelength axis with a suitable scale.

c. Detailed analysis of the light from both stars reveals the presence of prominent absorption lines in the spectra.

i. To which spectral class does Bellatrix belong?

ii. Prominent features in the Bellatrix spectrum are the Balmer absorption lines due to hydrogen. State the other element responsible for the prominent absorption lines in the spectrum of Bellatrix.

iii. Why does the spectrum of Betelgeuse not contain prominent hydrogen Balmer absorption lines?

AQA Unit 5A June 2012 Q3

5. Table Q3 summarises some of the properties of two stars in the constellation of Ursa Minor.

Name	Apparent magnitude	$\frac{\text{radius of star}}{\text{radius of the sun}}$	Spectral class
Polaris	2.0	50	F
Kocab	2.0	50	K

Table Q3

a. Using these data, describe and explain *one similarity* and *one difference* in the appearance of the two stars as seen with the unaided eye by an observer on the Earth.

b. Deduce which of the two stars is further from the Earth.

AQA June 2013 Q4 part a

3 STELLAR EVOLUTION

PRIOR KNOWLEDGE

You will need to be familiar with concepts from Astrophysics Chapter 2 about the classification of stars – absolute and apparent magnitude, spectral class and luminosity – and also with the use of astronomical distance units, such as the light year and parsec. You will need an understanding of nuclear fusion reactions, how they release energy and how to express them as nuclear equations, so you may want to refer back to Chapter 10. You should recall how heat can be transferred from one point to another by convection and radiation. You will need to be able to manipulate the equation for escape velocity (Chapter 4).

LEARNING OBJECTIVES

In this chapter you will learn that stars do not remain constant but change their luminosity and temperature with time. You will find out how this information may be represented graphically using the Hertzsprung–Russell diagram, which is a plot of the evolutionary stages of stars – the main sequence, giants and dwarfs. You will gain an understanding that what happens at the end of a star's life depends on its mass and how it may explode as a supernova, one of the most energetic events in the Universe, leaving behind an exotic object such as a neutron star or a black hole.

(Specification 3.9.2.5, 3.9.2.6)

3.1 THE BIRTH OF A STAR

Protostars

Stars do not shine for ever. **Stellar evolution** is the process by which stars are 'born', start to shine, continue shining in a stable state until eventually (after a time depending on their mass) they change, ending up as a variety of different stellar objects (again depending on their mass). Compared with the age of the Universe, high-mass stars can have relatively short lifetimes – possibly just a few million years – whereas the least massive stars can have extremely long lifetimes – exceeding the current estimated age of the Universe. The changes in stars as they evolve occur too slowly for us to detect. Instead, astrophysicists observe numerous stars at different points in their lifetimes and construct computer models of their structure and evolution.

Figure 1 *The Orion nebula. This is a star-forming region in the constellation of Orion 24 ly across where numerous stars are in the process of being born.*

Stars are born in the 'space' between stars called the **interstellar medium**, which contains **molecular clouds** (Figure 1) that are mostly made up of cold hydrogen gas in the form of atoms, molecules and ions at temperatures of 10–50 K and densities of 10^8–10^{15} molecules per cubic metre. About 1% of this material is 'dust' in the form of silicates and graphite material. Molecular clouds have masses many times greater than the mass of a single star, and contain fragments of varying masses which clump together under gravitational attraction. The irregular clumps tend to rotate, and a combination of the action of gravity and the conservation of angular momentum spins them inwards to form a denser spherical centre, forming a **protostar**. It is surrounded by a rotating flat disc of material called the **circumstellar disc**, where planets may form (Figure 2).

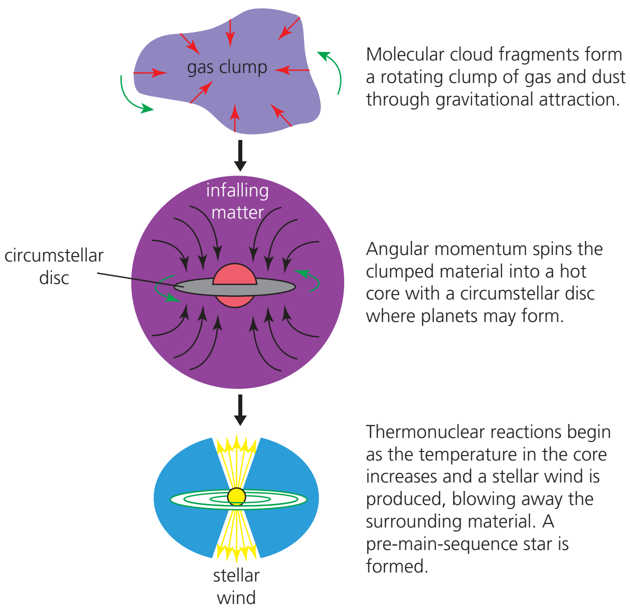

Figure 2 *The formation of a protostar from a molecular cloud clump*

Infalling matter from the cloud fragment causes the protostar to increase in size, and the density and temperature also increase. It begins to shine dimly in the infrared, the energy source being the gravitational energy of the infalling material.

After a time that may be as much as a few million years, the temperature of the star is such that the mutual electrostatic repulsion between hydrogen nuclei can be overcome and **nuclear fusion** reactions begin in its core. A strong outward stellar wind is produced, which opposes the infall of material. It starts to shine in the visible part of the electromagnetic spectrum, and is now known as a **pre-main-sequence star**.

QUESTIONS

1. a. Explain how a protostar is formed from the interstellar medium.
   b. What is the source of energy of a protostar before nuclear fusion reactions begin?
   c. Why are high temperatures needed for nuclear fusion in stars to start?

The stable period of a star's life

Eventually, when nuclear fusion in the star's core has become established, an equilibrium state is reached. The star now has a fixed mass, and its energy comes only from nuclear fusion, not from gravitational contraction. It is now a **main-sequence star**. Its mass will determine its future evolution.

The fusion of hydrogen nuclei with a release of nuclear binding energy, known as **hydrogen burning**, is the primary source of energy generation in main-sequence stars. There are two principal nuclear reaction pathways in which hydrogen burning occurs in a star, determined by the core temperature of the star. These are the **proton–proton chain** (or **p–p chain**) and the **carbon–nitrogen–oxygen cycle** (or **CNO cycle**). In each of these reactions, four protons combine by nuclear fusion to form a single helium nucleus with a small loss of mass, which, by the mass–energy relation $\Delta E = \Delta mc^2$ (see Chapter 10) is released as energy.

For stars that have masses not exceeding that of the Sun, the temperature in the core of the star does not get higher than about 16×10^6 K and hydrogen burning occurs via the proton–proton chain. In stars with masses greater than that of the Sun, the core temperatures exceed this value, and hydrogen burning proceeds through the CNO cycle. (See the following subsection on 'Nuclear reaction pathways' for more details.)

Nuclear fusion reactions like these continue in the core and provide the star's energy source for most of its life. The star is held in *equilibrium* because of a balance between the star's own gravitational force due to the tremendous mass of its outer layers pushing inwards and the internal gas pressure caused by hydrogen burning pushing outwards.

Energy from the fusion reactions is transported from the core to the outermost layers by convection and radiative (photon) diffusion. Convection occurs when hot gases rise towards the star's surface and cooler gases sink back down, setting up circulation currents in which heat energy is transferred to the outer layers of the star from its interior. In the fusion reactions, photons are created that also carry away energy. The photons diffuse outwards from the hot core towards the outer layers of the star. Although the motion of these photons is entirely random (Figure 3), because they are absorbed and re-emitted when they interact with atoms and free electrons in the star's interior, their net motion is towards the cooler, outer layers of the star. This photon migration towards the surface and then their escape into space can take tens of thousands of years. The sunlight that you feel on a sunny day is therefore due to photons that were created in the Sun thousands of years ago!

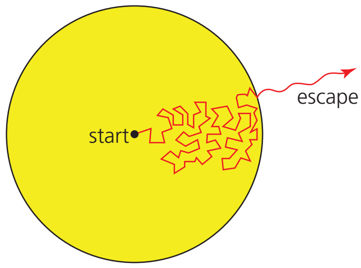

Figure 3 *Radiative diffusion in a star. Photons from the core follow a random path as they travel to the surface, taking thousands of years to do so.*

Stretch and challenge

Nuclear reaction pathways

The proton–proton chain converts hydrogen into helium in three steps by the following nuclear reactions:

$$^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + ^{0}_{1}e + \nu_e$$

$$^{1}_{1}H + ^{2}_{1}H \rightarrow ^{3}_{2}He + \gamma$$

$$^{3}_{2}He + ^{3}_{2}He \rightarrow ^{4}_{2}He + ^{1}_{1}H + ^{1}_{1}H$$

The CNO cycle has six steps:

$$^{12}_{6}C + ^{1}_{1}H \rightarrow ^{13}_{7}N + \gamma$$

$$^{13}_{7}N \rightarrow ^{13}_{6}C + ^{0}_{1}e + \nu_e$$

$$^{13}_{6}C + ^{1}_{1}H \rightarrow ^{14}_{7}N + \gamma$$

$$^{14}_{7}N + ^{1}_{1}H \rightarrow ^{15}_{8}O + \gamma$$

$$^{15}_{8}O \rightarrow ^{15}_{7}N + ^{0}_{1}e + \nu_e$$

$$^{15}_{7}N + ^{1}_{1}H \rightarrow ^{12}_{6}C + ^{4}_{2}He$$

Notice that, as in the p–p chain, the CNO cycle takes four hydrogen nuclei (protons) and converts them into a single helium nucleus together with positrons, neutrinos and some high-energy gamma rays.

The $^{12}_{6}C$ nucleus acts as a catalyst for the reaction. While it is consumed in the first step, it is replaced in the last step, so that, in the CNO reaction chain, carbon is not used up.

KEY IDEAS

- Interstellar molecular clouds of hydrogen gas and dust form clumps that collapse under their own gravity to form protostars.
- As the density and temperature of a protostar increase, it begins to shine, first in the infrared. Then nuclear fusion reactions start in its core and it becomes a pre-main-sequence star.
- When fusion reactions are established, a stable equilibrium state is reached and the star shines visibly as a main-sequence star for most of its life.
- The fusion of hydrogen into helium is the primary source of energy in a main-sequence star.
- The energy from the core is transferred by convection and radiative diffusion to the star's outer layers and escapes into space as photons.

3.2 THE HERTZSPRUNG–RUSSELL DIAGRAM

In Astrophysics section 2.3 the absolute magnitude of a star was introduced as a measure of the actual luminosity of a star, independent of the star's distance from Earth. In Astrophysics section 2.5 we classified stars according to their surface temperature by assigning them a spectral class, O to M in order of decreasing temperature. Suppose we plot a graph of absolute magnitude versus spectral class for all types of star for which these variables can be measured. Then we obtain a diagram like the one illustrated in Figure 4, which is known as a **Hertzsprung–Russell diagram** (or **HR diagram**), after Enjar Hertzsprung and Henry Norris Russell, the two astronomers who first made this kind of plot.

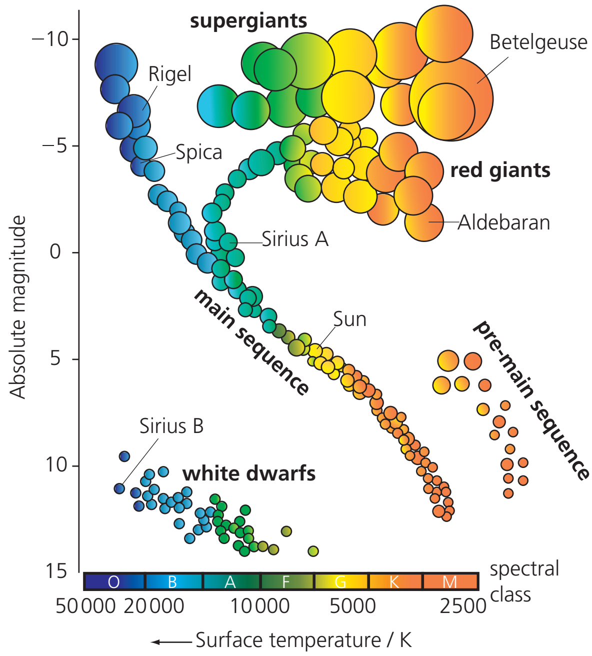

Figure 4 *The Hertzsprung–Russell diagram, showing examples of different types of star*

An HR diagram is essentially a plot of the luminosity of stars against their surface temperature, and a great deal of information about the properties of stars can be obtained from it. First of all, you will notice that the stars on the HR diagram are not randomly scattered. They are divided into four principal groupings.

1. The long diagonal band is called the **main sequence**. The stars with observational properties that place it on this band are what we have called main-sequence stars. This is where stars are stable and long-lived, and where nuclear fusion of hydrogen is the dominant energy-producing mechanism in the star. Approximately 90% of observable stars are on the main sequence. The Sun is a main-sequence star of average mass and luminosity, spectral class G, and its position is shown in Figure 4. At the top of the main sequence are the hot and luminous blue stars, and at the bottom are the cool and dim reddish stars.
2. **Red giants** are similar in mass to our Sun but have an expanded outer shell and hence large size and surface area. They are cooler and hence redder but highly luminous. Nuclear fusion of helium occurs in their cores.
3. **Supergiants** have masses typically 10–100 times that of the Sun and are therefore substantially larger and more luminous even than the red giants. In their cores the temperatures are hot enough for nuclear fusion reactions to produce carbon and heavier elements.
4. **White dwarfs** are old stars that have a high surface temperature but are not very luminous, because they no longer generate energy by nuclear fusion, and because they are small (planet sized). They are extremely dense. Eventually, they cool to the point of emitting no heat or light and become **black dwarfs**, which appears to be the end state of all low-mass stars.

The significance of the HR diagram is that it tells us that there exist fundamentally different kinds of stars. 'Normal stars' like the Sun are those which lie along the main sequence. 'Unusual stars' are the giants and white dwarfs, which seem to have a very different relationship between luminosity and temperature. From the HR diagram, we can see different stages of stellar evolution – how stars are born, grow old and die.

Evolution of a Sun-like star on the Hertzsprung–Russell diagram

The evolutionary life cycle of a star can be tracked on the HR diagram. Its fate depends on its mass at various stages. Figure 5 shows the evolutionary path for an average star like the Sun.

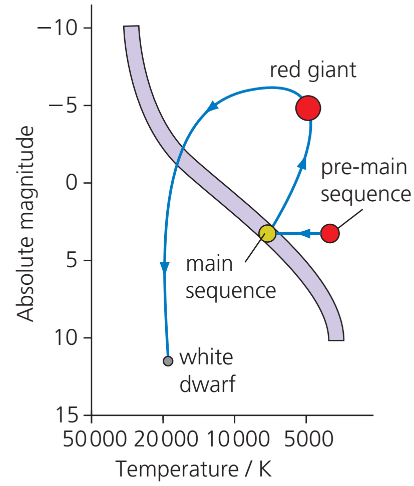

Figure 5 *The evolution of a star like the Sun on the Hertzsprung–Russell diagram*

The star begins as a protostar in an interstellar gas cloud. As nuclear fusion reactions begin, it becomes a pre-main-sequence star, just before moving to a position on the main sequence along the line running from top left to bottom right. It then remains in that position on the main sequence for most of its life (for a star of one solar mass, this is about 10 billion years) – until the hydrogen in its core is used up. The star then starts to burn hydrogen in its outer layers, causing it to expand in size greatly into a **red giant** with a lower surface temperature but higher luminosity. It moves off the main sequence to the right-hand top corner of the HR diagram.

Eventually, when the red giant star has exhausted all of its nuclear fuel, its outer layers are ejected (thrown off), forming a **planetary nebula** (Figure 6), and its core collapses into a dense **white dwarf**. The star has lost its outer layers but its core is still initially very hot, and its position on the HR diagram is now in the bottom left-hand corner.

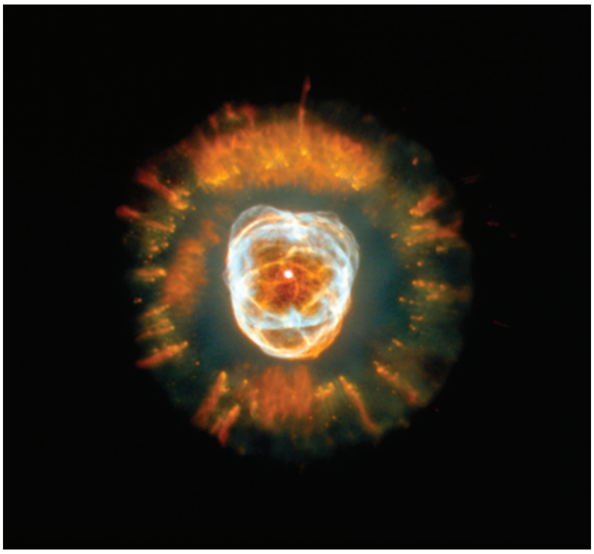

Figure 6 *The Eskimo nebula. This is a planetary nebula formed by the ejected outer layer of a star similar to the Sun.*

Since nuclear burning has ceased, there is no more outward pressure to halt the crushing force of gravity, and the core of a white dwarf is compressed to a size roughly the same as that of the Earth. The density of its matter rises to some 10^8–$10^9\,kg\,m^{-3}$. If you had a teaspoonful of white dwarf matter on Earth, it would weigh several tonnes! The star is eventually prevented from further collapse by a quantum rule (the Pauli exclusion principle), which, loosely stated, means that no two particles can be together in exactly the same quantum state at the same time. For matter at high densities, this prevents electrons occupying the same space, and as a result they exert a powerful outward pressure called 'degeneracy pressure' that opposes any further contraction by gravity. The star is then said to be in a degenerate state and gradually cools to become a **black dwarf** star, which emits no significant amount of heat or light.

QUESTIONS

2. The HR diagram tells us that there exist different types of stars. List the four main categories of stars found on the HR diagram.
3. Two protostars, A and B, form in the same molecular cloud. As pre-main-sequence stars, A is five solar masses and B is one solar mass. Suggest which star would reach the main sequence first. Explain your reasoning.

The lifetimes of stars

The lifetime of a star is determined by its mass – see Table 1. Stars spend roughly 90% of their lives converting hydrogen into helium on the main sequence, and the mass of a star determines the rate of hydrogen burning. In more massive stars fusion reactions proceed at a faster rate than in lower mass stars due to the higher temperature and pressures in their cores. They therefore use up their hydrogen fuel more rapidly. For this reason they spend shorter times on the main sequence before evolving into red giants.

Mass / M_{Sun}	Spectral class	Main-sequence lifetime / 10^6 year
25	O	3
15	B	15
3	A	500
1.5	F	3000
1.0	G	10 000
0.75	K	15 000
0.50	M	> 200 000

Table 1 *The main-sequence lifetimes for stars of spectral classes O to M*

The Sun, a G type star, has a main-sequence lifetime of about 10^{10} years. It is currently about 5×10^9 years old. Stars higher along the main sequence than the Sun (spectral classes O to F) must be younger than the Sun or they would have used up all the hydrogen in their cores and would have moved off the main sequence. At the other end of the scale, given that the age of the Universe is believed to be about 13.7×10^9 years (see Astrophysics Chapter 4), every M type star in existence is still on the main sequence. The oldest stars in the Universe are called **red dwarfs**, which have low mass, low temperature and low luminosity. This means that they burn through their supply of hydrogen very slowly, giving them extremely long lifetimes well in excess of Sun-like stars and longer than the age of the Universe.

It should be understood that the HR diagram is a 'snapshot' of a collection of different types of stars. Stars do not move along the main sequence. Depending on its mass when it is a pre-main-sequence star, the star reaches a point on the main sequence and stays there. Then, at a far future time when it is nearing the end of its life, it moves off the main sequence and evolves into a different type of star.

Figure 7 summarises the evolutionary stages of a star similar to the Sun during its lifetime.

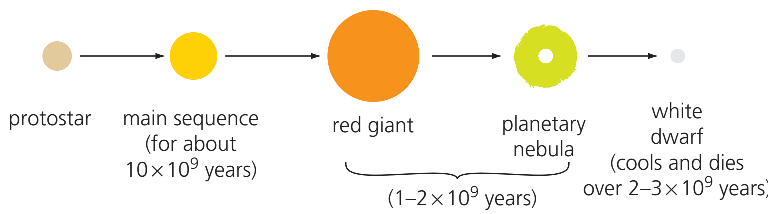

Figure 7 *The evolution of a star like the Sun from protostar to white dwarf*

QUESTIONS

4. Which single property is most important in determining the evolutionary stages of a star?

5. Where would a red dwarf appear on the HR diagram?

6. Why are most of the stars that we see in the sky main-sequence stars?

KEY IDEAS

- Stars evolve with time and change in both temperature and spectral class.
- The Hertzsprung–Russell diagram is a graph of absolute magnitude versus spectral class or temperature, and shows that stars can be grouped together on the basis of their physical similarities.
- Stars spend most of their lives in a stable state represented by a point on a diagonal line on the HR diagram called the main sequence.
- The lifetime of a star on the main sequence depends on its mass. High-mass high-temperature stars (O types) have shorter lifetimes. Low-mass low-temperature stars (M types) have the longest lifetimes.
- The Sun is situated about halfway along the main sequence. It is spectral class G (surface temperature about 6000 K) and has an absolute magnitude just below 5.
- Stars like the Sun will leave the main sequence when the hydrogen in their cores is used up and will expand to become red giants.
- When all the nuclear fuel is exhausted, the star will collapse and become a white dwarf.

ASSIGNMENT 1: USING THE HERTZSPRUNG–RUSSELL DIAGRAM TO FIND THE SIZES OF STARS

(MS 0.1, MS 0.2, MS 3.1, MS 3.2, PS2.2, PS3.1, PS3.2)

In this assignment you will identify stars on the HR diagram and determine their radius.

Stars spend about 90% of their lives on the main sequence. They then evolve, changing both temperature and size. The luminosity L of a star is given by

$$L = (4\pi R^2)\sigma T^4$$

where R is the radius of the star, T is the star's surface temperature and $\sigma = 5.67 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$ is Stefan's constant.

Questions

A1 Rearrange the equation given earlier to give R in terms of the luminosity and surface temperature.

A2 The Sun has a surface temperature of about 5800 K.

a. In what spectral class is the Sun?

b. From the Sun's luminosity of 3.90×10^{26} W, estimate its radius.

A3 **a.** Table A1 gives the luminosity (as a fraction of the Sun's luminosity) of six stars of different types: Betelgeuse, Aldebaran, Sirius A, Spica, Rigel and Sirius B. Referring to the HR diagram shown in Figure 4, copy and complete the table by recording the data for the other columns, estimating the stars' radii as you did in question **A2** for the Sun.

Star name	Spectral class	Type of star	Colour	Absolute magnitude M	L/L_{Sun}	T / K	R/R_{Sun}
Betelgeuse		Supergiant			1.25×10^5		
Aldebaran		Red giant			520		
Sirius A		Main sequence			25.4		
Spica		Main sequence			12 100		
Rigel		Main sequence			1.25×10^5		
Sirius B		White dwarf			0.026		

Table A1

b. Use the internet to research a further six examples of stars of different categories and obtain information about their luminosity, absolute magnitude and temperature. A way to start is to use the star type as a search string, for example 'supergiant', and then look for particular names of stars. Note that you may get a range of values for these parameters, so for this exercise take the average value of those that you find.

Set out your data in a table like Table A1, in descending order of radius.

A4 Do all main-sequence stars have approximately the same radius?

A5 Plot your own version of an HR diagram for the stars you have researched.

3.3 EVOLUTION OF MASSIVE STARS POST-MAIN SEQUENCE

Giant and supergiant stars

The evolution of stars with a mass higher than about $1.4 M_{Sun}$ is different from that described in Astrophysics section 3.2. This is because these stars fuse hydrogen to helium but do so primarily via the CNO cycle (see Astrophysics section 3.1) due to the high pressures and high temperatures in their cores. Stars between $1.4 M_{Sun}$ and $3 M_{Sun}$ also evolve into red giants, but they end their life as **supernovae**, leaving behind a **neutron star**. Stars with a main-sequence mass in excess of $3 M_{Sun}$ evolve into red **supergiants**, and when these explode as supernovae they leave behind a **black hole** (Figure 8).

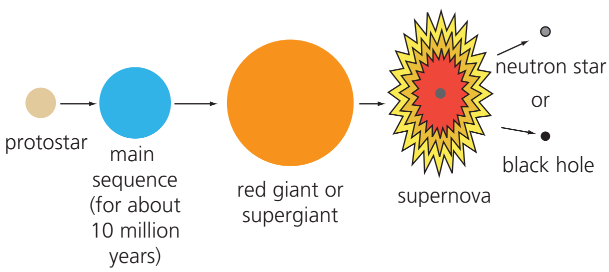

Figure 8 *Evolution of a high-mass star*

A red supergiant is formed when the high-mass star runs out of hydrogen in its core. The core contracts and the star expands in size, burning hydrogen in its outer layers, increasing its luminosity and becoming much redder. The interior temperature gets much higher than in red giants, so elements heavier than hydrogen and helium can be fused, producing elements as heavy as iron, in a series of layers around their core.

Red supergiants burn at a very fast rate, consuming all their hydrogen in just a few million years. In that time, they increase their luminosity to about 100 000 times that of the Sun. Their size can range from 30 to 1000 or more solar radii (Figure 9). See Assignment 1.

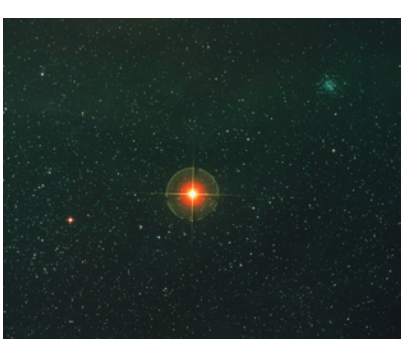

Figure 9 *The red supergiant Antares has a radius in excess of 800 times that of the Sun*

Blue supergiants also exist, which are much hotter than red supergiants but smaller, only about 25 times the size of the Sun. They form when a star of more than 10 solar masses exhausts the nuclear fuel in its core and starts burning its outer layers, increasing in luminosity. Like red supergiants, they have very short lifetimes of only a few million years.

QUESTIONS

7. A main-sequence star with a mass of 10 solar masses becomes a red supergiant. The rate of radiation from its surface increases greatly but its surface cools down (it becomes redder). Explain how this is possible.

Supernovae

A **supernova** is a star that suddenly and very rapidly increases in absolute magnitude because of an explosion that ejects most of its mass. A supernova can become so bright that it can be seen in other galaxies and is one of most energetic events in the Universe.

Supernovae are classified into two types:

- Type I supernova. This is a star that accretes (draws in) matter from another star in a **binary system** until it becomes compressed and runaway nuclear reactions are set off, blasting its matter into space. We will look at these again in Astrophysics section 3.4.
- Type II supernova. This is a single star – a red giant or supergiant – that runs out of nuclear fuel and collapses rapidly under its own gravity, ejecting its outer layers with enormous energy.

In this section we are concerned with Type II supernovae. For a Type II supernova event to happen, the star must be several times more massive than the Sun. The star becomes a red giant (or supergiant) after its main-sequence stage. But when the nuclear fuel is exhausted, the gravitational compression is so strong that the star collapses on itself extremely rapidly – in a matter of a few seconds. The infalling matter produces extremely powerful shock waves, creating a gigantic explosion (Figure 10), and rapidly increasing the absolute magnitude. The outer parts of the star are blown into space in an expanding gas shell at speeds of 5000–10 000 $km\,s^{-1}$ (Figure 11). The energy released by a supernova explosion is stupendous, of the order of 10^{46} J, and can produce

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enough radiation to temporarily outshine a whole galaxy. For comparison, the energy output of the Sun each day is 3.3×10^{31} J. What is left is called a supernova remnant, at the centre of which is an exotic object called a **neutron star**.

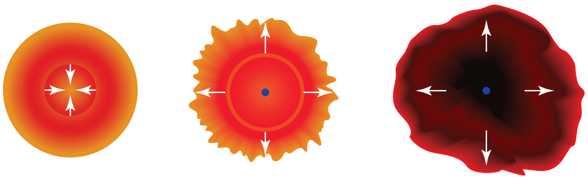

Figure 10 *Stages in a Type II supernova*

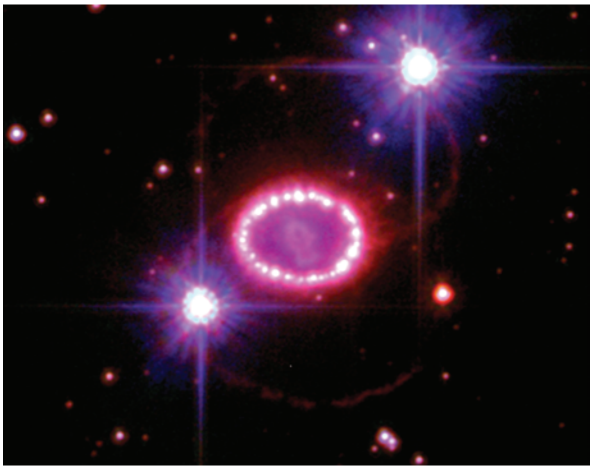

Figure 11 *The expanding gas shell of supernova 1987a. This was a Type II supernova that exploded in the Large Magellanic Cloud, a galaxy about 158000 ly from Earth, first observed in 1987.*

Since heavier elements in the Periodic Table, including iron and nickel, are fused in the interiors of massive stars, supernova explosions eject these into the interstellar medium, where they are dispersed across the Universe, making up planets, including the Earth.

Supernova events are not very common. In a galaxy like the Milky Way, we can expect to see two or three supernova events per century. But because the Universe contains billions of galaxies, we can often observe supernovae in other galaxies.

Neutron stars and pulsars

What is left after a supernova is an extremely dense object called a **neutron star**. The gravitational contraction has become so great that the electrons in the atoms are forced into protons, forming neutrons. A neutron star is thus composed almost entirely of neutrons. It has a rigid core of neutrons and neutron-rich nuclei, surrounded by an iron outer crust. The gravitational field of a neutron star is so strong that to escape from the surface would require an **escape velocity** approaching 0.8 of the speed of light. The escape velocity from the surface of an object of mass M and radius R (see Chapter 4) is given by

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

where G is the gravitational constant

$$G = 6.67 \times 10^{-11}\,\text{N m}^2\,\text{kg}^{-2}$$

Neutron stars contain their mass in a diameter of only about 20 km, with a density of about 2×10^{17} kg m^{-3}. On the surface of a neutron star, gravity would be 2×10^9 times stronger than gravity on Earth, and if you had a teaspoonful (5 ml) of neutron star material on Earth it would weigh 5.5×10^{12} kg!

Worked example

What is the escape velocity from a neutron star of mass two times that of the Sun and radius 20 km? [Mass of Sun = 1.99×10^{30} kg]

$$v_{esc} = \sqrt{\frac{2GM}{R}}$$

$$= \sqrt{\frac{2 \times 6.7 \times 10^{-11} \times 2 \times 1.99 \times 10^{30}}{2 \times 10^{4}}}$$

$$= 160\,000\ \text{km s}^{-1}$$

Neutron stars appear in supernova remnants, either as single objects or in **binary star** systems (see Astrophysics Chapter 4). They may behave as a **pulsar**. A pulsar is a rotating neutron star with a very strong magnetic field (Figure 12). These objects were discovered by Jocelyn Bell, a graduate student in Cambridge, in 1967. The surface of a neutron star has numerous protons and electrons where the gravitational field is not strong enough for them to be pushed into each other to form neutrons. They are accelerated towards the magnetic poles of the neutron star, and in doing so emit electromagnetic radiation over a wide range of wavelengths in a narrow beam in opposite directions. The neutron star can rotate up to 600 times per second, giving a pulsed beam rather like a lighthouse.

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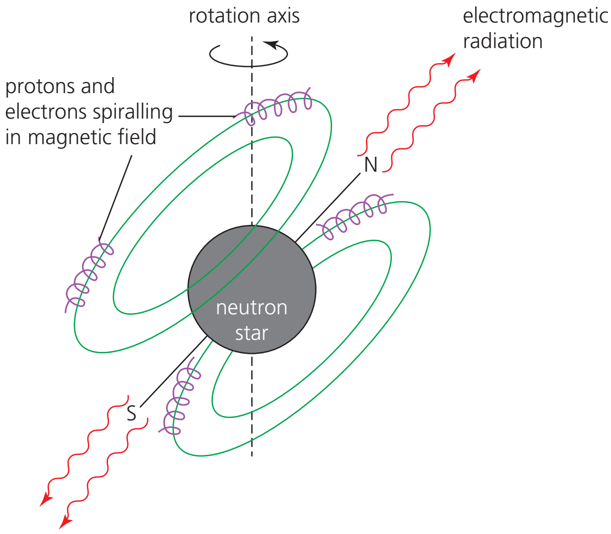

Figure 12 *A rotating neutron star (pulsar)*

QUESTIONS

8. Compare the escape velocity from the neutron star calculated in the Worked example with the escape velocity from the Earth, which you will need to calculate. [Mass of Earth = 5.98×10^{24} kg, radius of Earth = 6370 km]

Black holes

For extremely massive stars, whose core after a supernova is more than three solar masses, gravitational compression in the neutron star continues unabated, inevitably producing a **black hole**. A black hole is a region of space-time that has such a strong gravitational field that no particles or electromagnetic radiation can escape from it. The escape velocity is greater than the speed of light, c, which from Einstein's theory of special relativity is impossible to achieve.

How big is a black hole? To answer this, consider the escape velocity. If we put $v_{esc} = c$ as a minimum, and square the expression

$$v_{esc} = \sqrt{\frac{2GM}{R}} = c$$

we get

$$c^2 = \frac{2GM}{R}$$

Then we can obtain the maximum value of R as

$$R = R_S = \frac{2GM}{c^2}$$

This radius R_S is called the **Schwarzschild radius** after the German astrophysicist Karl Schwarzschild, who first calculated it from Einstein's general theory of relativity. The Schwarzschild radius tells us, for a given mass, how small an object must be for it to trap light around it and therefore appear black. To calculate the Schwarzschild radius of any object – a planet, a galaxy, or even an apple – all you need to know is the mass to be compressed.

This radius effectively forms a boundary that we call the **event horizon** of the black hole. Within this, the escape velocity is greater than or equal to the speed of light. Hence, all information from inside the event horizon is lost. Since black holes cannot be directly seen, information about them can only be inferred from the effects they have on nearby objects.

QUESTIONS

9. a. What is the Schwarzschild radius of a black hole with the mass of the Sun? [Mass of Sun = 1.99×10^{30} kg]

b. Explain the significance of this.

c. Estimate the density of the object within the Schwarzschild radius.

10. A massive star explodes in a supernova, leaving behind a behind a black hole of 50 solar masses. Calculate its Schwarzschild radius. [Mass of Sun = 1.99×10^{30} kg]

Gamma ray bursts

About once a day, intense flashes of gamma rays coming from distant galaxies in random directions, lasting from a few milliseconds to tens of seconds, are observed by gamma ray telescopes. These **gamma ray bursts** (**GRBs**) are thought to originate in supernovae, when supergiant stars collapse to form neutron stars or black holes. Since the bursts are known to come from distant galaxies, they must be extremely energetic. The gamma rays are thought to be emitted as a narrow beam of intense radiation. The total energy radiated by a GRB is estimated to be over 10^{48} J, making a GRB

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one of the brightest electromagnetic events known to occur in the Universe. A GRB may release as much energy in one short burst as the Sun will in its entire lifetime.

GRBs are potentially very hazardous. It has been speculated that a supernova generating a GRB in our own galaxy emitting radiation pointing towards the Earth would kill most life, and might have been responsible for mass extinction events during past geological epochs.

Supermassive black holes

Observations have shown that stars and gas orbiting near the centres of galaxies are being accelerated to very high orbital velocities. This can be explained if a large supermassive object with a strong gravitational field in a small region of space is attracting them. The most likely candidate is a **supermassive black hole**.

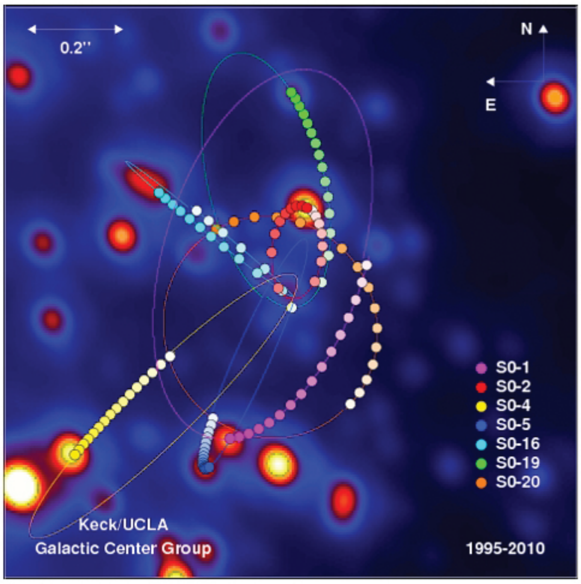

Figure 13 *Orbits of stars near the centre of the Milky Way*

Figure 13 shows the orbits of seven stars within a region of space 1.0×1.0 arcsecond square in the direction of the centre of the Milky Way. This image was processed using the Keck Observatory on Mauna Kea in Hawaii. The motions of these stars, labelled SO-1 to SO-20, have been measured over a period of 15 years. Calculations of their orbital parameters provide the best evidence yet that they are in orbit about a supermassive black hole, which has a mass 4.1 million times the mass of the Sun.

Astrophysicists now think that there is a supermassive black hole at the centre of every galaxy, but they are not certain how it forms. One suggestion is that one could form out of the collapse of massive clouds of gas during the early stages of galaxy formation. Another idea is that an 'ordinary' stellar black hole devours enormous amounts of material over millions of years, increasing its mass to supermassive proportions. A third possible mechanism is that clusters of stellar black holes form and eventually merge into each other, forming a supermassive black hole.

QUESTIONS

11. The supermassive black hole at the centre of the Milky Way galaxy has an estimated mass of 4.1×10^6 solar masses. Calculate its Schwarzschild radius.
[Mass of the Sun = 1.99×10^{30} kg]

12. Some cosmologists think that miniature black holes, called primordial black holes, may have formed in the early stage of the Big Bang when densities were very high. Such objects are thought to have masses in the range 10^{14}–10^{23} kg and may be a candidate for dark matter (see Astrophysics section 4.5). Estimate the Schwarzschild radius of a primordial black hole of mass 1.0×10^{20} kg.

KEY IDEAS

- Stars with main-sequence masses greater than $1.4M_{Sun}$ at the end of their life explode as supernovae, leaving either a neutron star or a black hole.
- A neutron star is a very dense compact object consisting almost entirely of neutrons. Rotating neutron stars are called pulsars and emit electromagnetic radiation in opposite directions.

- A black hole is the end state of a massive star. Gravitational compression produces a volume of space-time with a gravitational field so intense that the escape velocity exceeds that of light.
- The event horizon is a boundary around a black hole beyond which no light or other radiation can escape and its radius is called the Schwarzschild radius, given by

$$R_S = \frac{2GM}{c^2}$$

where M is the mass of the body that forms the black hole.
- Gamma ray bursts are highly energetic flashes of gamma rays associated with supernovae.
- A supermassive black hole may exist at the centre of all galaxies. One at the centre of our own Galaxy has been inferred from the rapid motions of stars near the centre.

3.4 TYPE IA SUPERNOVAE AS STANDARD CANDLES

As we saw in Astrophysics section 3.3, there are two types of supernova. Type I, produced when matter accretes onto one star in a binary system, has a sub-type called Type Ia (or 1a). This is thought to originate from a white dwarf star in a close binary system with a companion star. As the companion nears the end of its life and expands into a red giant, the gravity of the white dwarf accumulates material from the companion, compressing it to a critical mass and setting off a runaway nuclear reaction that leads to a supernova explosion.

Supernovae undergo a rapid increase in brightness. Their absolute magnitude increases rapidly, in less than a day, reaching a peak absolute magnitude and then dimming over a period of several months. A graph of absolute magnitude versus time is called a **light curve**. The light curves for Type Ia and Type II supernovae are different, and shown in Figure 14. Type Ia supernovae exhibit a sharp maximum in their absolute magnitude and then die away smoothly and gradually. All Type Ia supernovae explosions occur at the same critical mass, and thus produce very consistent light curves, with the same peak value of absolute magnitude, −19.3, about 20 days from the beginning of the collapse.

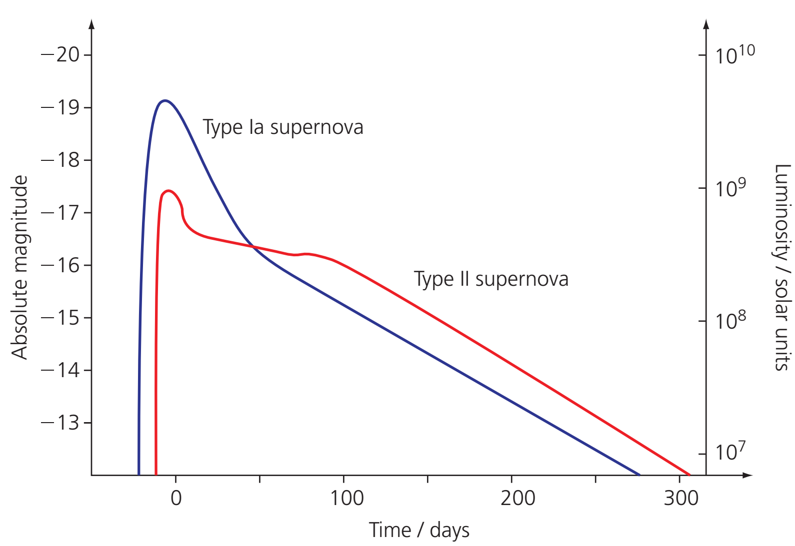

Figure 14 *Typical light curves from supernovae. Type Ia supernovae are significantly brighter, and the rate at which Type Ia and Type II fade away is different. Note that the peak magnitude defines the time t = 0.*

Astronomers measure large astronomical distances using bright objects, with a known luminosity and absolute magnitude, which act as a **standard candle** (see Astrophysics section 2.3). Supernovae Type Ia can therefore be used as standard candles.

We can measure the distance d in parsecs of an object by measuring its apparent magnitude m using the relation $m - M = 5 \log_{10}(d/10)$. So, since we know the absolute magnitude M of a Type Ia supernova, we can calculate how far away it is. At very large distances, we cannot see individual stars in galaxies, so Cepheid variables cannot be used as standard candles for such distances. Supernovae, however, can be seen in other galaxies (Figure 15) – they emit so much energy and are so bright that they can be seen at distances out to 1000 Mpc (3.26 billion light years), which is a significant fraction of the radius of the known Universe. Such distances are known as **cosmological distances**.

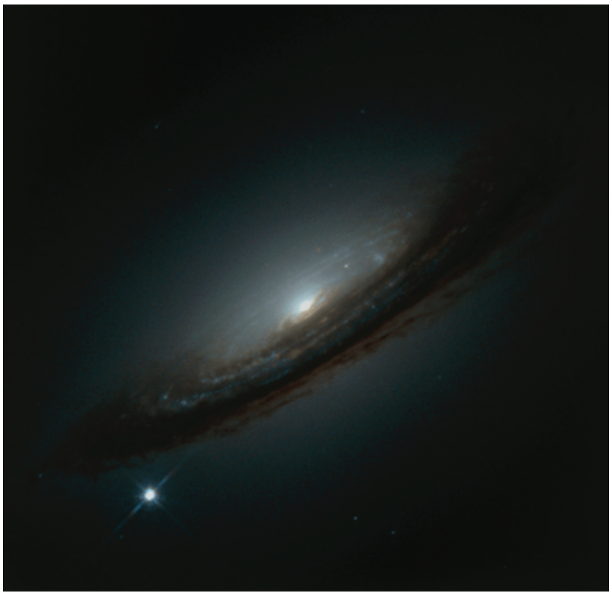

Figure 15 *A Type Ia supernova in a galaxy 55 million light years from Earth, imaged by the Hubble telescope. Since all Type Ia supernovae have the same peak absolute magnitude, measuring its apparent magnitude means that we can calculate its distance and therefore the distance of its parent galaxy.*

Worked example

A Type Ia supernova is observed in another galaxy with a peak apparent magnitude of +10. Estimate the distance of the galaxy from Earth in parsecs.

Using the fact that all Type Ia supernovae can be assumed to have a peak absolute magnitude of −19.3, then using $m - M = 5 \log_{10}(d/10)$ we have

$$10 - (-19.3) = 5 \log_{10}\left(\frac{d}{10}\right)$$

$$29.3 = 5 \log_{10}\left(\frac{d}{10}\right)$$

$$\log_{10}\left(\frac{d}{10}\right) = 5.86$$

$$\log_{10} d - \log_{10} 10 = 5.86$$

$$\log_{10} d - 1 = 5.86$$

$$\log_{10} d = 6.86$$

$$d = 10^{6.86} = 7.2\,\text{Mpc}$$

QUESTIONS

13. A Type Ia supernova in a distant galaxy is observed to have a peak apparent magnitude of 14. Estimate how far away the galaxy is.

14. Explain why Type II supernovae cannot be used as standard candles whereas Type Ia supernovae can.

One of the most surprising findings from using Type Ia supernovae to measure cosmological distances was that the data suggested, controversially, that the expanding Universe is *accelerating* and not slowing down. For this to happen implies that there is some as-yet undetected energy permeating the Universe that acts in opposition to gravity. This has been given the name **dark energy** and its origin is currently a mystery to astrophysicists (see Astrophysics section 4.5).

KEY IDEAS

- Supernovae increase rapidly in absolute magnitude and then dim over a period of days and months as described by their light curves.
- Type Ia supernovae may be used as standard candles to estimate cosmological distances.
- Such distance measurements have provided evidence that the expansion of the Universe is accelerating. Dark energy, an unknown property of space permeating the entire Universe, is thought to be responsible.

PRACTICE QUESTIONS

1. a. The *Chandra X-ray Observatory* was launched into orbit in 1999. It is used to observe hot and turbulent regions. Explain why X-ray telescopes need to be in orbit.

   b. In 2000, the *Chandra* telescope was used to observe a black hole in Ursa Major.

      i. Explain what is meant by a black hole.

      ii. The black hole is believed to have a mass 7 times that of the Sun. Calculate the radius of its event horizon. [Take mass of the Sun = 2.0×10^{30} kg]

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2. a. Define the term *absolute magnitude*.

   b. Sketch the axes of a Hertzsprung–Russell diagram. Mark suitable scales on the absolute magnitude and temperature axes.

   c. Label a possible position of each of the following stars on your HR diagram:

      i. the Sun

      ii. star W, which has the same intrinsic brightness as the Sun, but has a significantly higher temperature

      iii. star X, which has a similar spectrum to the Sun, but is significantly larger

      iv. star Y, which is significantly larger than the Sun and has prominent absorption lines of neutral atoms and titanium oxide (TiO) in its spectrum.

   d. How does the diameter of star W, in part **c ii**, compare with the diameter of the Sun? Explain your answer.

   AQA Unit 5A June 2014 Q3

3. a. State what is meant by a *supernova*.

   b. Type II supernovae play a part in the evolution of some stars. Describe briefly what causes this to occur and what remains of the star following the event.

   c. i. Explain why Type Ia supernovae can be used as standard candles to determine distances.

      ii. Sketch the light curve of a typical Type Ia supernova, on axes of absolute magnitude against time in days.

   d. It is thought that the star 'IK Pegasi' may explode as a Type Ia supernova at some stage in the future. IK Pegasi is 46 pc from Earth. Given its peak value of absolute magnitude, −19.3, calculate its peak apparent magnitude if it explodes. Would we be able to see it in daylight? [The apparent magnitude of the full Moon = −13]

4. Table Q1 shows the spectral class, absolute and apparent magnitudes of five stars.

Star	Absolute magnitude	Apparent magnitude	Spectral class
Wolf 359	+16.7	13.5	M
Formalhault	+2.0	1.2	A
Achernar	−1.0	0.5	B
Procyon	+2.7	0.3	F
Pollux	+0.8	1.2	K

Table Q1

   a. i. Which star appears the brightest?

      ii. Which star appears the most dim?

      iii. Which star is the coolest?

      iv. Which star is the hottest?

   b. i. Sketch the Hertzsprung–Russell (HR) diagram on axes of absolute magnitude against spectral class, with the magnitude scale ranging from +17 to −10. Label the main sequence, giant stars, white dwarf stars, and the position of the Sun.

      ii. Plot the stars in Table Q1 on your HR diagram.

      iii. State the type of the stars you have plotted. Explain what this means about the stage of evolution the stars are at.

   c. Estimate the distance from the Earth to Wolf 359 using the data in Table Q1.

4 COSMOLOGY

PRIOR KNOWLEDGE

You may need to refresh your understanding of wave motion from Chapter 5 of Book 1, including frequency and wavelength. You will need to use what you learnt in Astrophysics Chapter 2 about thermal radiation, stellar spectral lines, luminosity and Pogson's law. You may also want to look back to circular motion in Chapter 1. You will need to be familiar with the use of astronomical units, such as light year and parsec.

LEARNING OBJECTIVES

In this chapter you will learn about the observational evidence and physical principles that underpin cosmology. You will learn how the Doppler effect is used to determine whether an object in space is moving towards or away from us. You will see how measurement of the velocities of galaxies gave rise to Hubble's law and show that the Universe is expanding. You will examine different types of evidence that suggest that the Universe began in a hot dense state that rapidly expanded and formed the stars and galaxies that we see today. You will find out about quasars, which are the most distant measurable objects, and the important recent and ongoing detection of exoplanets, which are planets orbiting other stars.

(Specification 3.9.3.1 to 3.9.3.4)

4.1 WHAT IS COSMOLOGY?

The study of the structure and development of the Universe as a whole is called **cosmology**. The task of the cosmologist is to construct theories of how different phenomena of nature, from small elementary particles and fundamental forces, right up to very large-scale structures in the Universe such as clusters of galaxies, all fit together. Observational data and mathematical theory are both needed – often together with creative inspiration – to try to understand how the Universe formed and what might happen to it in the future.

When we use telescopes to look at distant regions of the Universe, we are looking back in time. This is because, although it is high, the speed of light ($3.00 \times 10^8\,m\,s^{-1}$) is finite. It takes light from the nearest star about four years to reach us across space. Some very distant galaxies are millions or even billions of light years away, and so we see the farthest galaxies as they were in the early Universe – the light that left them then is finally reaching us just now (Figure 1).

Figure 1 *The Hubble Ultra Deep Field. This image, taken by the Hubble space telescope in the direction of the constellation Fornax, shows an estimated 10 000 distant galaxies. The most distant objects in the image are over 13 billion light years away, so we see them when the Universe was just a few million years old.*

4.2 THE DOPPLER EFFECT

A vast amount of astrophysical information is available to us because of a seemingly everyday effect of physics. When a high-speed train is coming towards you while you are standing on a railway station platform, you may have noticed that the note of its sound is higher and then drops in frequency as it passes by and starts to recede. This is an example of the **Doppler effect**, named after the 19th century Austrian physicist Christian Doppler.

The reason why this happens is that, when the train is approaching, more sound waves per second are reaching your ears than if the train is stationary, and the wavelength becomes shortened as a result. Figure 2 shows, for an instant in time, wave fronts (1 to 4) that have emerged from the train as the train moves from right to left. If the train is receding from you, there are fewer sound waves per second reaching your ears, so the wavelength is lengthened.

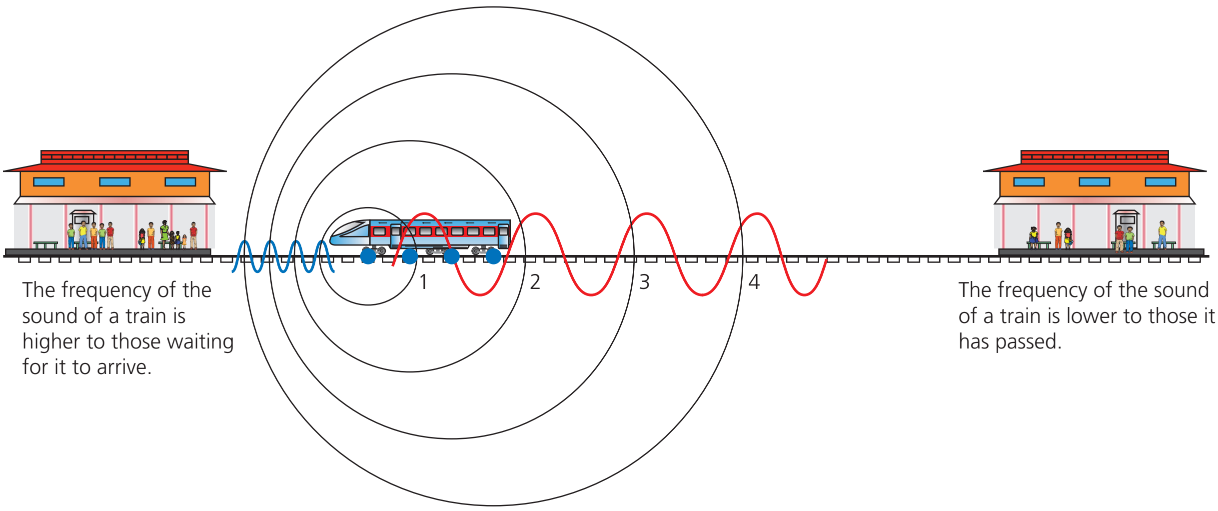
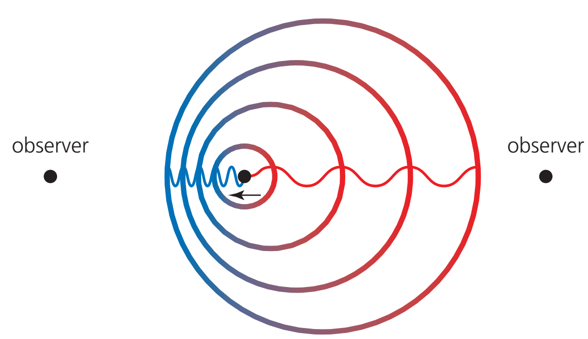


Figure 2 *The Doppler effect for sound waves*

Since, for a given wave speed, the frequency is inversely proportional to the wavelength, the frequency of the note of an approaching train is higher, and for a receding train the frequency is lower.

The same phenomenon occurs with all other types of waves, including electromagnetic radiation. Depending on the motion of the object with respect to the observer, the frequency – and hence colour in the case of light – is affected. The colour of an approaching light source is a shifted to the blue (shorter wavelength) than it would otherwise be, and the colour of one that is moving away is shifted to the red (longer wavelength), as shown in Figure 3.



- Source moving towards observer
- Wavelength decreased; frequency increased
- Observer sees light blue-shifted

- Source moving away from observer
- Wavelength increased; frequency decreased
- Observer sees light red-shifted

Figure 3 *The Doppler effect for light waves*

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The effect depends on the relative motion of the source and the observer. So, if a light source is stationary and the observer is moving towards or away from it, the same shift to the blue or red occurs. Unlike with sound, we do not generally notice this effect with light, because the relative speed of source and observer needs to be very high.

A consequence of the Doppler effect is that the lines in a star's absorption spectrum (see Astrophysics section 2.5) are shifted when compared to the same lines as measured in a laboratory (Figure 4). This is due to the motion of the star relative to the Earth. If the star and the Earth are moving *towards* each other, then the wavelengths of the absorption lines are shortened, that is, shifted towards the blue end of the spectrum (or 'blue-shifted'), and the effect is called **blue-shift**. Conversely, if the star and the Earth are moving *away from* each other, then the wavelengths of the absorption lines are lengthened, that is, moved towards the red end of the spectrum (or 'red-shifted'), and this is called **red-shift**.

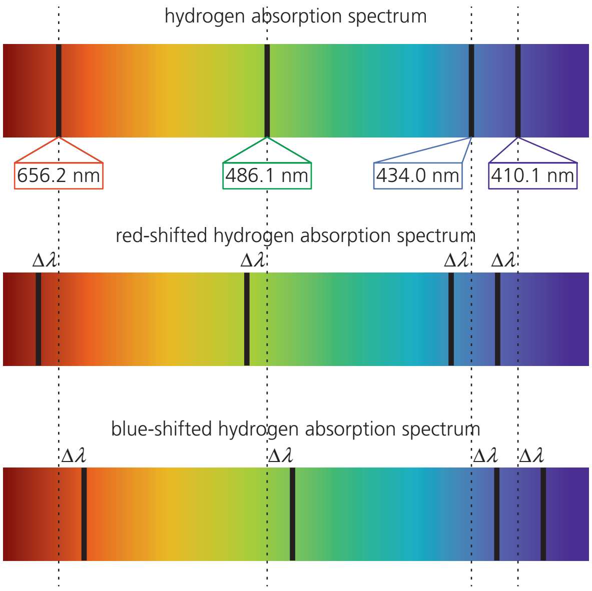

Figure 4 *Doppler shift of absorption lines in the hydrogen spectrum from a star. The top diagram shows the hydrogen spectrum from a source at rest with respect to the observer (that is, the spectrum as observed in a laboratory). The centre diagram shows the observed hydrogen lines from the same star red-shifted by an amount $\Delta\lambda$ (the star is receding from the observer). The bottom diagram shows the observed hydrogen lines from a similar star blue-shifted by an amount $\Delta\lambda$ (the star is approaching the observer).*

The size of the wavelength shift, $\Delta\lambda$, depends on the relative velocity of the star and the observer. The relationship is given by the **Doppler equation**:

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda_{app} - \lambda}{\lambda} = -\frac{v}{c}$$

Here λ is the true wavelength of the absorption line, λ_{app} is the apparent wavelength of the observed absorption line on Earth, v is the relative velocity of the star and the Earth, and c is the velocity of light. The relative velocity v is taken to be the relative velocity of *approach*, so that v is positive when the two objects are approaching one another and negative if they are receding.

The Doppler equation can also be expressed in terms of the change in frequency:

$$\frac{\Delta f}{f} = \frac{v}{c}$$

These expressions are only valid when v is much less than c, since the derivation (see following subsection) ignores the effects of special relativity. Also note that v is the relative velocity *along the line of sight*.

The Doppler equation works for shifts in all parts of the electromagnetic spectrum.

Stretch and challenge

Derivation of the non-relativistic Doppler equation

Suppose an object S emits electromagnetic radiation of wavelength λ, frequency f and speed c, and is moving at a velocity v towards a stationary observer where $v << c$. In a time equal to the wave period T, the radiation has travelled a distance equal to λ, and S has travelled a distance vT.

The wavelength λ_{app} seen by the observer is thus $\lambda_{app} = \lambda - vT$ and the change in wavelength $\Delta\lambda$ is

$$\Delta\lambda = \lambda_{app} - \lambda = -vT$$

But from $T = 1/f$ and $c = f \times \lambda$, we get

$$T = \frac{\lambda}{c}$$

so the change in wavelength can be written as

$$\Delta\lambda = \lambda_{app} - \lambda = -v \times \frac{\lambda}{c}$$

giving finally

$$\frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$$

or

$$\lambda_{app} = \lambda\left(1 - \frac{v}{c}\right)$$

We can express this in terms of the frequency, so that

$$\frac{1}{f_{app}} = \frac{1}{f}\left(1 - \frac{v}{c}\right)$$

$$f = f_{app}\left(1 - \frac{v}{c}\right)$$

$$f_{app} = \frac{f}{\left(1 - \frac{v}{c}\right)}$$

This leads to

$$\Delta f = f_{app} - f$$

$$\Delta f = \frac{f}{\left(1 + \frac{v}{c}\right)} - f$$

Since v < < c the denominator is approximately 1 so we can write this as

$$\Delta f = f - f\left(1 + \frac{v}{c}\right)$$

so that $$\Delta f = f\left(\frac{v}{c}\right)$$

and $$\frac{\Delta f}{f} = \frac{v}{c}$$

Measuring the velocities of stars

The Doppler effect for light can be used to estimate the speed at which a distant star is moving relative to the Earth (along the line of sight), by looking at the change in the wavelengths in the absorption lines of its visible spectrum (Figure 4) compared to their values measured in a laboratory on Earth.

The Doppler equation states that

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda_{app} - \lambda}{\lambda} = -\frac{v}{c}$$

where $\Delta\lambda$ is the change in wavelength, λ is the true wavelength of the absorption line, λ_{app} is the apparent wavelength of the observed absorption line on Earth, v is the relative velocity of approach of the star and the Earth, and c is the velocity of light.

The quantity $\frac{\Delta\lambda}{\lambda}$ is termed the **Doppler shift** and is given the symbol z, so that

$$z = \frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$$

For a receding object, v is negative by convention, so z is positive. Again, these expressions are based on the non-relativistic approach to the Doppler shift, that is, for $v << c$.

In terms of frequency,

$$z = -\frac{\Delta f}{f}$$

Worked example

The hydrogen absorption line from the star Vega is observed to have a wavelength of 656.255 nm (λ_{app}) compared to the same line in the laboratory of 656.285 nm. Determine the velocity of the star Vega relative to the Earth, and state whether it is approaching us or receding from us.

From the information given we obtain

$$\Delta\lambda = \lambda_{app} - \lambda = 656.255 - 656.285 = -0.030\,\text{nm}$$

So rearranging the equation for the Doppler shift

$$\frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$$

$$v = -\frac{c\Delta\lambda}{\lambda}$$

and substituting numerical values gives

$$v = -\frac{(3.00 \times 10^{8}) \times (-0.030 \times 10^{-9})}{656.258 \times 10^{-9}}$$

$$= 1.37 \times 10^{4}\,\text{m s}^{-1}$$

$$= 1.4 \times 10^{4}\,\text{m s}^{-1} \text{ to 2 s.f.}$$

Since v is positive, Vega is approaching the Earth with a speed of $14\,\text{km s}^{-1}$.

QUESTIONS

1. A particular spectral line in the spectrum of a star is found to have a wavelength of 600.80 nm compared to 600.00 nm as measured in the laboratory. What is the velocity of the star? Is it moving towards us or away from us?
2. The H-alpha spectral line in the hydrogen spectrum is at 656.00 nm when measured in the laboratory. Star A is observed to have that line at 656.60 nm, star B at 655.90 nm and star C at 656.40 nm.
   a. Which star is moving the fastest relative to Earth (along the line of sight)?
   b. What is the direction of motion of each of the stars?
3. Neutral, atomic hydrogen gas in the spiral arms of the Milky Way emits a spectral line of wavelength 21 cm, which is in the microwave part of the electromagnetic spectrum. The spectral line when detected by a radio telescope in a certain orientation is observed to be shifted by 0.1 mm less than 21 cm. How fast is this part of the Galaxy moving relative to us along the line of sight? Is it moving towards us or away from us?

KEY IDEAS

- The Doppler effect is a change in observed frequency when a source of waves is moving towards or away from an observer.
- The Doppler shift in a star's spectral lines, compared with the same spectral lines observed in a laboratory on Earth, can be used to measure its velocity v relative to the Earth, along the line of sight.
- In terms of wavelength, the Doppler shift is

$$z = \frac{\Delta\lambda}{\lambda} = -\frac{v}{c}$$

- In terms of frequency, the Doppler shift is

$$z = -\frac{\Delta f}{f} = \frac{v}{c}$$

- If the star is approaching the Earth, the relative velocity v is positive: $\Delta\lambda$ is then negative and there is observed blue-shift.
- If the star is receding from the Earth, the relative velocity v is negative: $\Delta\lambda$ is then positive and there is observed red-shift.

4.3 DOPPLER SHIFT AND THE MOTION OF BINARY STARS

The Doppler effect can be used to determine the rotational velocity and the distance between two stars in a **binary star** system (Figure 5). Roughly half the stars found are in a binary system, in which two companion stars orbit their common centre of mass, with periods ranging from hours to many thousands of years.

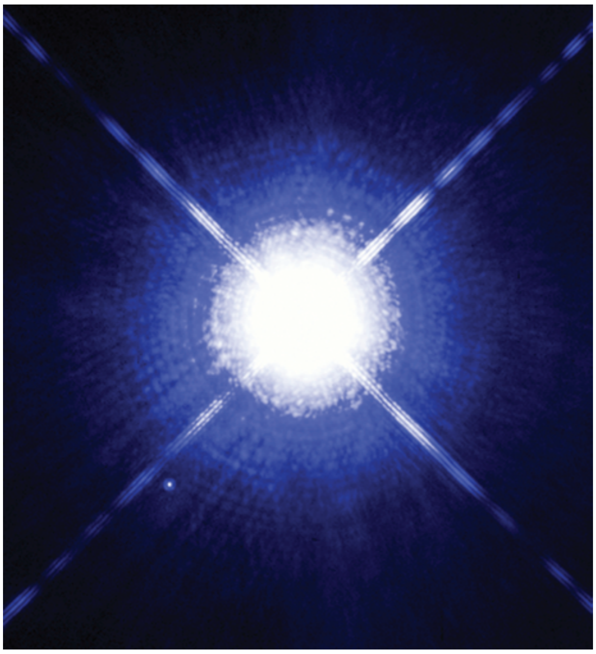

Figure 5 *Sirius, the brightest star in the sky, is in a binary system. Its faint white dwarf companion, Sirius B, is just visible here at the 7 to 8 o'clock position. The two stars revolve around a common centre of mass, and the distance between them varies from 8.2 to 31.5 AU.*

The classification of binary stars is dependent upon the nature of the observation (most binaries are not resolved even with powerful telescopes). Here we will consider spectroscopic binaries, revealed by the Doppler shift of lines in their spectrum. To avoid further complications, only **eclipsing binaries** will be looked at, which means those whose orbit lies in the same plane as the line of sight from Earth. These binary systems can be identified by their combined light curve, because, as one star eclipses the other, the apparent brightness of the combined binary image decreases. Eclipsing binaries may be partial (Figure 6a) or total (Figure 6b).

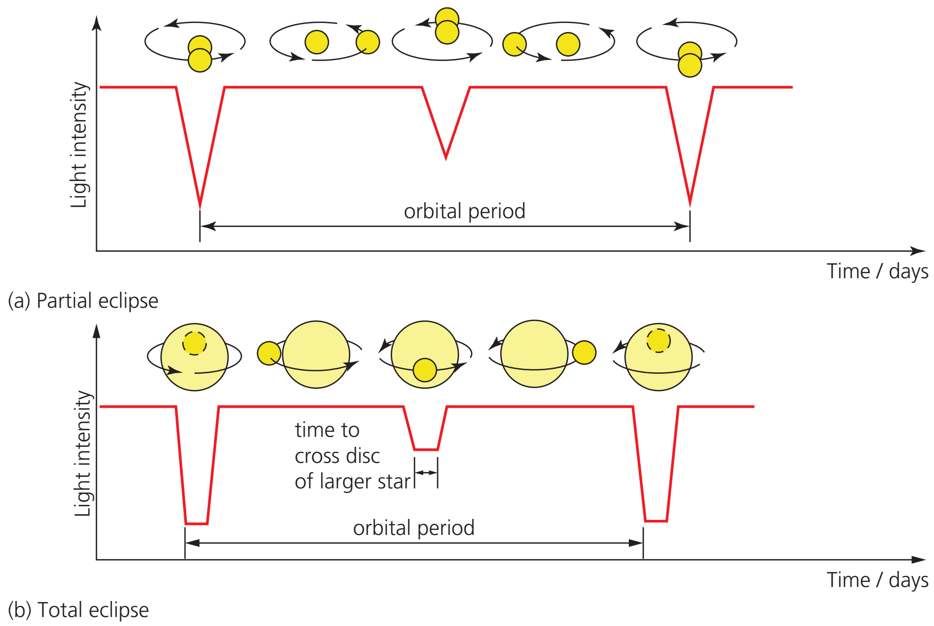

Figure 6 *Light curves of eclipsing binaries*

Consider a binary system with one bright star and one faint star (as shown by Figure 6b). The absorption lines seen from Earth will be Doppler-shifted as the stars rotate about their centre of mass, moving between longer and shorter wavelengths in a periodic motion.

There will be a blue-shift in the star's spectral lines when the star is moving towards the Earth, a red-shift in the spectral lines when the star is moving away from the Earth, and no spectral shift in the lines when the star is moving perpendicularly to the line of sight (Figure 7).

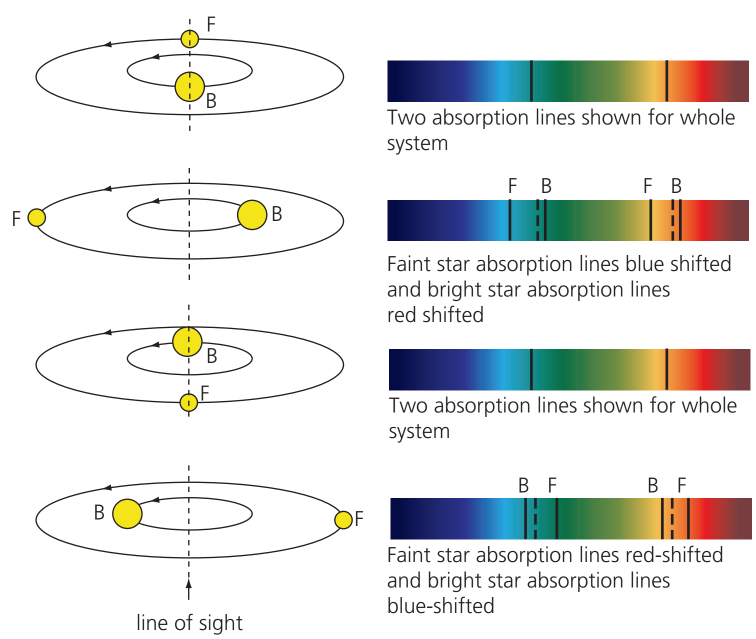

Figure 7 *The sequence of changing positions of spectral lines as two stars rotate about each other in an eclipsing binary system. B is a bright star and F is a faint star. The amount of spectral shift depends on the rotational velocity.*

Analysis of the spectral motion reveals a cyclic movement of a particular absorption line, shifted one way and then the next with a constant period, superimposed on the velocity of the binary system relative to the Earth. It is possible to calculate the linear velocity along the line of sight and the period, hence the distance between the two stars using the mechanics of circular motion (see Chapter 1).

Worked example

Spectroscopic data on a binary star system of two stars S_1 and S_2 show that it has a period $T = 68$ days. The absorption line of calcium is observed to be double, with a periodic variation. When one line is at a maximum of 393.45 nm, the other is at a minimum of 393.39 nm. In a laboratory, the absorption line of calcium appears at 393.40 nm. Assuming that the two stars are in circular orbits around their centre of mass, and are viewed directly along the plane of the orbit, calculate the distance (in AU) between the two stars.

The two components of the spectral line are due to the motions of the individual stars and they will have the same rotational periods.

$$\text{Rotational period } T = (68 \times 24 \times 3600) = 5.9 \times 10^6\,\text{s}$$

$$\text{Speed of } S_1 = c\frac{\Delta\lambda}{\lambda} = 3.00 \times 10^8 \times \frac{393.40 - 393.45}{393.40} = 3.8 \times 10^4\,\text{m s}^{-1}$$

$$\text{Speed of } S_2 = c\frac{\Delta\lambda}{\lambda} = 3.00 \times 10^8 \times \frac{393.40 - 393.39}{393.40} = 7.6 \times 10^3\,\text{m s}^{-1}$$

(Note that we are not concerned about the signs, as we are calculating speeds.)

$$\text{Radius of orbit of } S_1 = R_1 = \frac{\text{circumference}}{2\pi} = \frac{\text{speed of } S_1 \times T}{2\pi}$$

$$R_1 = \frac{3.8 \times 10^4 \times 5.9 \times 10^6}{2\pi} = 3.6 \times 10^{10}\,\text{m} = \frac{3.6 \times 10^{10}\,\text{m}}{1.5 \times 10^{11}\,\text{m}} = 0.24\,\text{AU}$$

$$\text{Radius of orbit of } S_2 = R_2 = \frac{\text{circumference}}{2\pi} = \frac{\text{speed of } S_2 \times T}{2\pi}$$

$$R_2 = \frac{7.6 \times 10^3 \times 5.9 \times 10^6}{2\pi} = 7.1 \times 10^9\,\text{m} = \frac{7.1 \times 10^9\,\text{m}}{1.5 \times 10^{11}\,\text{m}} = 0.05\,\text{AU}$$

Therefore, the distance between the stars is the sum of the radii of their orbits = 0.24 + 0.05 = 0.29 AU.

Stretch and challenge

The masses of binary stars

It is possible to calculate the ratio of the masses of two binary stars that are in circular orbits. We will assume that:

1. The stars are perfect spheres.
2. The centre of mass of the binary system lies on a line joining the centres of the two stars (Figure 8).

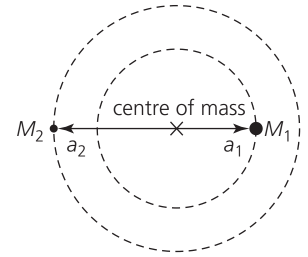

Figure 8 *Two binary stars of mass M_1 and M_2, and their centre of mass*

The centre of mass of the binary system is the point where all of the mass ($M_1 + M_2$) of the system can be considered to be located. From the definition of the centre of mass,

$$M_1 \times a_1 = M_2 \times a_2$$

So

$$\frac{a_1}{a_2} = \frac{M_2}{M_1}$$

If we know the distance between the two stars, $a = a_1 + a_2$, then

$$a_2 = \left(\frac{M_1}{M_1 + M_2}\right)a$$

QUESTIONS

4. An eclipsing binary system consists of star X and star Y, orbiting one another.

   a. Figure 9 is a graph showing how the spectral line at wavelength $\lambda = 477\,\text{nm}$ from the system changes due to the Doppler effect during one full period of revolution of the stars.

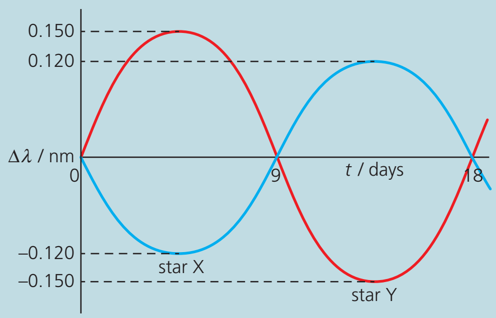

Figure 9

   i. Why are the curves not identical?

   ii. Explain why the two curves are exactly out of phase.

   iii. Calculate the maximum linear speed of star X.

   b. Figure 10 shows a plot of how the brightness of the binary pair, seen as an unresolved single star, varies with time. Use this graph to explain why the shape of the curve of star X in Figure 9 is not the same as that of star Y.

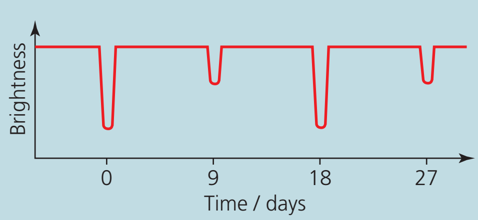

Figure 10

Stretch and challenge

5. What is the ratio of the masses of the two stars S_1 and S_2 in the previous worked example?

KEY IDEAS

- Binary stars are two stars that orbit each other.
- Eclipsing binaries are those that we view in the plane of their orbit.
- The linear velocity, angular velocity and distance apart of eclipsing binaries can be calculated using the periodic Doppler shift in their spectral lines.

4.4 THE RECESSION OF GALAXIES AND QUASARS

Observations of distant galaxies cannot be resolved into individual stars. The light from the whole galaxy is analysed. In the vast majority of cases, the absorption (or emission) spectra from distant galaxies are found to be red-shifted (Figure 11). This indicates that all of these galaxies are moving away from us and so is evidence of an expanding Universe. The red-shift is given by

$$z = -\frac{v}{c}$$

where v is the galaxy's **recession velocity** relative to our own Galaxy, the Milky Way. Note that z is positive for a red-shift, since recession velocity is taken to be negative (see Astrophysics section 4.2). This equation, however, is only valid if $v < 0.1c$.

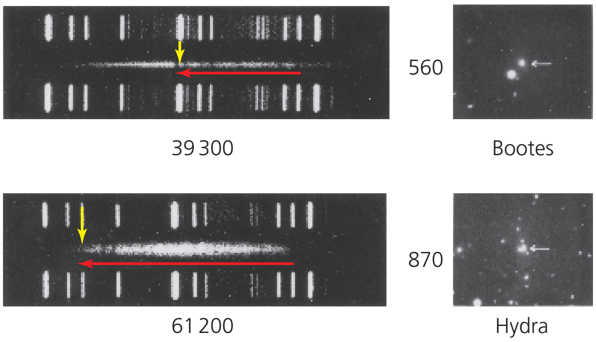

Figure 11 *The optical spectra for two elliptical galaxies. Both have been taken with the same magnification. The yellow arrow indicates a pair of dark absorption lines that are shifted to longer wavelengths (red-shifted). The figures on the right give the distance of the galaxy in Mpc and those below each spectrum give the recession velocity in km s^{-1}.*

Worked example 1

The K absorption line in singly ionised calcium normally has a wavelength of 393.4 nm. In a spectrum from galaxy NGC 4889, the line occurs at 401.8 nm. Determine the red-shift of this galaxy and the recession velocity.

Here we have $\lambda = 393.4\,\text{nm}$ and $\lambda_{\text{app}} = 401.8\,\text{nm}$, and therefore

$$\Delta\lambda = \lambda_{\text{app}} - \lambda = 401.8 - 393.4 = 8.4\,\text{nm}$$

The red-shift z and recession velocity v are

$$z = \frac{\Delta\lambda}{\lambda} = \frac{8.4}{393.4} = 0.0214$$

$$v = -cz = -3.00 \times 10^{8} \times 0.0214 = -6.42 \times 10^{6}\,\text{m}\,\text{s}^{-1}$$

The galaxy Hydra (see Figure 11) shows a recession velocity of 61 000 km s^{-1}, which is 0.2c and above the threshold for an accurate determination of v (which is 0.1c). In this case, an error in excess of 12% is introduced. To derive a more accurate value of v and hence z, a relativistic Doppler equation has to be used – that is, one that takes account of the effects of special relativity. Very distant galaxies are observed to have red-shifts significantly greater than 1, so clearly $z = -\frac{v}{c}$ will not be valid because it would give a speed greater than c.

Quasars are very luminous objects (see Figure 17 in Astrophysics section 4.7) whose spectra show very broad absorption lines and high red-shifts (Figure 12). Values of z have been observed in excess of 7, which means that the recession velocity is a significant fraction of the speed of light. Quasars are thought to be the most distant objects in the Universe. We will consider them in further detail in Astrophysics section 4.7.

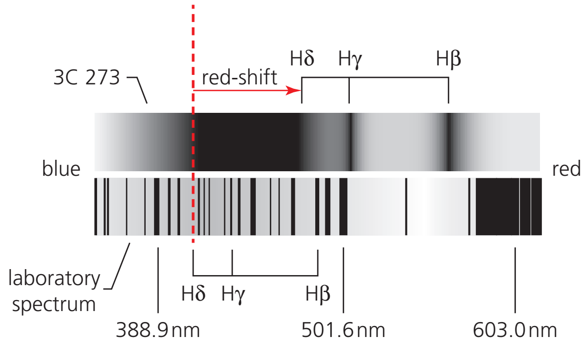

Figure 12 *Spectrum of quasar 3C 273 showing hydrogen lines. Notice the large size of the red-shift and the broad widths of the three hydrogen spectral lines marked as Hδ, Hγ and Hβ.*

QUESTIONS

6. Measurements of the red-shift of the 21 cm H1 line in the spectrum of galaxy M84 suggest that the galaxy is receding from us at a velocity of 900 km s^{-1}. Calculate the value of the red-shift z for galaxy M84.

7. An absorption line of calcium usually has a wavelength of 393.4 nm, but it is observed in a distant galaxy to have a wavelength of 820.9 nm. What is the red-shift? Comment on your answer.

Stretch and challenge

Relativistic red-shift

To account for observed z values greater than 1, a relativistic red-shift equation needs to be used that takes into account the effects of special relativity. The relativistic red-shift equation is

$$z = \sqrt{\frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}} - 1$$

where v is the recession velocity.

Worked example 2

A spectral line observed in the hydrogen spectrum of a very distant galaxy has a wavelength of 1032.0 nm. The value of the line in the laboratory is 91.2 nm. Calculate the red-shift and show that the recession velocity is less than c.

The red-shift is

$$z = \frac{\Delta\lambda}{\lambda} = \frac{1032.0 - 91.2}{91.2} = 10.3$$

Rearranging the relativistic red-shift equation and then squaring both sides gives

$$(z + 1)^2 = \frac{1 + \frac{v}{c}}{1 - \frac{v}{c}}$$

and then rearranging this to give v in terms of z and c results in

$$v = \left[\frac{(z+1)^2 - 1}{(z+1)^2 + 1}\right] \times c \text{ m s}^{-1}$$

Using the calculated value of z in this last expression gives

$$v = \left[\frac{(10.3+1)^2 - 1}{(10.3+1)^2 + 1}\right] \times 3.00 \times 10^8 \text{ m s}^{-1}$$
$$= 2.95 \times 10^8 \text{ m s}^{-1}$$

In fact, cosmologists do not think of distant galaxies or quasars as moving through space away from us at such high speeds, but regard space itself to be expanding, and the light waves being stretched along with it. The wavelength of light will increase as it crosses the expanding Universe, between its point of emission and where it is detected, by the same amount that space has expanded during the crossing time. This gives rise to a ‘cosmological red-shift’, which is governed by general relativity. The Doppler red-shift and the cosmological red-shift cannot be distinguished from one another by observing the spectrum of the light source.

KEY IDEAS

- The light from all observable distant galaxies is red-shifted, and this is evidence that the Universe is expanding.
- The size of the red-shift $z = \Delta\lambda/\lambda$ gives a galaxy’s recession velocity, which is its outward velocity relative to the Milky Way. For $v < 0.1c$

$$z = -\frac{v}{c}$$

- For $v > 0.1c$, a relativistic red-shift equation is needed to calculate the recession velocity.
- Quasars are highly luminous objects that exhibit high values of red-shift, indicating high recession velocities.

4.5 HUBBLE’S LAW

The spectra of all galaxies, apart from a few very near to our own Milky Way, show red-shift. A plot of the recession velocity against distance for galaxies is close to a straight line (Figure 13) and is called a **Hubble diagram**, named after Edwin Hubble, who published the relationship in 1929. Hubble had measured the distances of Cepheid variables (see Astrophysics section 2.3) out to distances of about 20 Mpc (Figure 13a). Recent observational data has extended this to include galaxies as far distant as 5000 Mpc (Figure 13b), where recession velocities are extremely high, so the relativistic expression is used to determine the recession velocity from the red-shift.

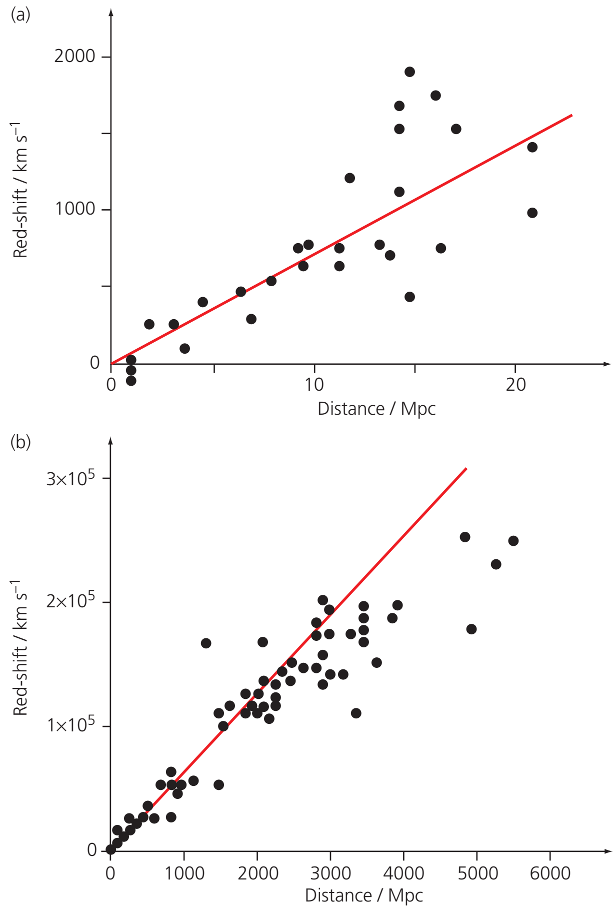

Figure 13 *(a) Hubble’s original data (replotted) showing the recession velocity of 28 nearby galaxies against their distance. The line of best fit indicates a Hubble constant of 68 km s^{-1} Mpc^{-1}. Notice that some galaxies exhibited a small blue-shift. (b) Recent galactic data. Hubble’s original data were confined to distances in the region between 0 and 20 Mpc.*

The data show that the rate at which a galaxy recedes is directly proportional to its distance from us, that is,

$$v = Hd$$

where v is the recession velocity in km s^{-1} and d is the distance of the galaxy in Mpc. This is called **Hubble's law** and the constant of proportionality H (sometimes denoted by H_0) is the **Hubble constant**, which is determined from the gradient of a Hubble diagram. Current best estimates give $H = 67.3$ km s^{-1} Mpc^{-1}. However, this value is constantly under review as more data are collected.

Note that the SI unit for H is s^{-1}. To get H in SI units, v has to be in m s^{-1} and d in m ($1\text{ Mpc} = 3.09 \times 10^{22}$ m).

Once a value of a distant galaxy's recession velocity is known, Hubble's law can be used to estimate its distance.

Worked example 1

a. The size of the recession velocity for the galaxy NGC 4889 has been determined to be $v = 6420$ km s^{-1}. Calculate its distance in Mpc. Take $H = 67.3$ km s^{-1} Mpc^{-1}.

b. How does this compare with a galaxy with a recession velocity of 2.83×10^8 m s^{-1}?

a. $d = \frac{v}{H} = \frac{6420}{67.3} = 95.4\text{ Mpc}$

b. $d = \frac{v}{H} = \frac{2.83 \times 10^5}{67.3} = 4205\text{ Mpc}$

Worked example 2

A source of radio waves is carbon monoxide molecules in the gas clouds of a galaxy. When measured from a laboratory-based source, these waves have a frequency of 120 GHz. What is the frequency of the waves detected from the galaxy if it is 800 million light years away? [Take the Hubble constant H to be 67.3 km s^{-1} Mpc^{-1}]

Distance of galaxy from Earth $d = 800$ million light years $= 800 \times 10^6 \times 9.46 \times 10^{15}\text{ m} = 7.57 \times 10^{24}\text{ m} = (7.57 \times 10^{24})/(3.09 \times 10^{22}) = 245\text{ Mpc}$.

Using Hubble's law, we find that the galaxy has a recession velocity of

$$v = Hd = 67.3 \times 245 = 16\,500\text{ km s}^{-1}$$
$$= 1.65 \times 10^7\text{ m s}^{-1}$$

We can use the non-relativistic Doppler equation, as the speed is about $0.06c$, so

$$\frac{\Delta f}{f} = \frac{v}{c}$$

Rearranging gives

$$\Delta f = f \times \frac{v}{c} = 120 \times \frac{1.65 \times 10^7}{3.0 \times 10^8} = 6.6\text{ GHz}$$

Measured frequency $= f - \Delta f = 120 - 6.6$
$= 113.4$ GHz

Hubble's law is a simple statement but with huge consequences. It states that the Universe is expanding, and is observational evidence in support of Einstein's mathematical predictions. An expanding Universe means that it is cooling down – so the further back in time, the smaller and hotter the Universe was. This implied to theoretical physicists that at a time $t = 0$ the Universe came into being from an infinitely hot, infinitely dense point (called a singularity, a mathematical concept that appeared in Einstein's equations) and has been expanding ever since. This is the **Big Bang theory**, sometimes now called the Hot Big Bang (HBB) model.

QUESTIONS

8. Show that the reciprocal of the Hubble constant has the unit of second.

9. The radial velocity of the Coma cluster of galaxies has been measured at 7200 km s^{-1}. What is the distance to the cluster? [Take $H = 67.3$ km s^{-1} Mpc^{-1}]

10. a. The graph in Figure 14 shows the recession velocity against distance for a number of galaxies. Estimate from it the value of the Hubble constant.

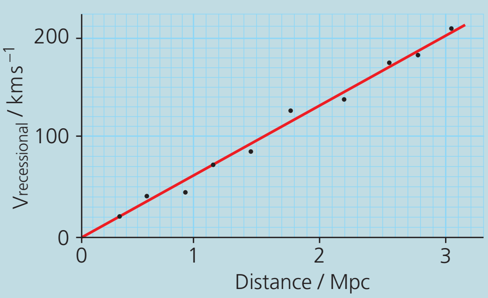

Figure 14

b. What is the distance, in Mpc, of a galaxy with $z = 0.002$? Use your value of H obtained from part **a**.

The age of the Universe

An accurate value of the Hubble constant, and the assumption that this has remained constant through all time, allows an estimate of the age of the Universe. If in time t a galaxy has moved outwards a distance d at velocity v, then

$$t = \frac{d}{v}$$

But from Hubble's law we have

$$v = Hd$$

So, if we assume H has been constant, then

$$\text{time (age of Universe)} = \frac{d}{v} = \frac{1}{H}$$

Here H needs to have unit s^{-1}. Taking $H = 67.3\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$, using 1 Mpc $= 3.09 \times 10^{22}$ m, we obtain

$$H = 2.18 \times 10^{-18}\,\text{s}^{-1}$$

This gives the estimated age of the Universe as

$$\frac{1}{H} = 4.59 \times 10^{17}\,\text{s} = 14.5 \text{ billion years}$$

In the limit that $v = c$, it is possible to determine the distance to the edge of the observable Universe. Using $H = 67.3\,\text{km}\,\text{s}^{-1}\,\text{Mpc}^{-1}$ and the relativistic equation for red-shift gives a distance of approximately 14 600 Mpc.

Rate of expansion of the Universe

The Hubble constant is one of the most fundamental quantities of nature, as it specifies the rate of expansion of the entire Universe. Only if the Universe has been expanding uniformly with time is H constant. There has been considerable controversy in the past over whether the expansion of the Universe is steady or is slowing down. If the rate of expansion of the Universe were decreasing, as might be expected because of the effects of gravity, then there would be some deviations from the predictions of Hubble's law:

- More distant objects would be seen to be receding faster (since the expansion was faster in the past).
- Objects would appear brighter than predicted (since they would be closer than predicted because of the decreasing expansion rate).

Recent systematic observations of Type Ia supernovae (which act as standard candles – see Astrophysics section 3.4) in distant galaxies have shown clearly that they are less bright than expected. This shows that they are further from us than predicted by Hubble's law – the light from them has taken longer to reach us than predicted by a constant rate of expansion. These data indicate that the rate of expansion is not steady and is certainly not slowing, but is *accelerating*. Studies of the cosmological microwave background (see Astrophysics section 4.6) have also shown clear evidence for an accelerating Universe.

The consequence of this acceleration is that the Universe is actually older than predicted by the Hubble law. (Strictly speaking, the Hubble constant is known as the 'Hubble parameter', because its value decreases with time as the Universe's expansion accelerates.)

Cosmologists were puzzled as to what could be driving this acceleration. The cause did not appear to be either matter or radiation, and is still at present unknown. Several possibilities have been put forward, including the notion of **dark energy**. This is a postulated energy that exerts an overall repulsive effect throughout the Universe, causing 'empty' space to expand, and its effect increases as the Universe expands.

Astrophysicists are not sure what dark energy is, but it is likely that it is a quantum field phenomenon and is related to the 'cosmological constant'. This was a mathematical term introduced by Einstein that denotes the value of the energy density of the vacuum of space and was originally postulated to make his equations of general relativity work. While dark energy opposes the force of gravity, it adds to the total mass–energy density within the Universe. Dark matter emits no radiation, so is difficult to measure – its presence is inferred by the movement of galaxies. Current experimental data estimate that the Universe is composed of 27% matter (mostly unobserved **dark matter** – *see the introduction to Chapter 1 of Year 1 Student Book*) and 73% dark energy, resulting in an ever-expanding Universe.

QUESTIONS

11. a. There is some uncertainty in the value of H: (71 ± 10%) km s^{-1} Mpc^{-1}. If a galaxy is moving away from us with a recession velocity of 5500 km s^{-1}, calculate its maximum and minimum distance from us.

b. How is the estimated age of the Universe affected by this range of values for H? [Take 1 Mpc = 3.1×10^{19} km]

12. Suppose that the Universe stopped expanding and started contracting. What feature in the spectra of galaxies would enable us to tell that this had happened?

KEY IDEAS

- Hubble's law results from observational data and states that the recession velocity of a distant galaxy is proportional to its distance:

$$v = Hd$$

  and implies that the Universe is expanding. The constant of proportionality H is the Hubble constant, usually expressed in km s^{-1} Mpc^{-1}.
- The Hubble constant, if assumed constant through time, gives an estimate of the age of the Universe as $t = 1/H$, where H is in s^{-1}.
- There is evidence to suggest that the Universe is actually accelerating and is older than Hubble's law predicts.
- As the expansion of the Universe accelerates, the value of the Hubble constant will decrease with time.

4.6 EVIDENCE FOR THE BIG BANG

We have seen that the Big Bang theory was developed as a result of Einstein's mathematics and Hubble's observational data. More recently, there has been further evidence to support the theory.

Cosmological microwave background

Crucial evidence for the Hot Big Bang (HBB) model includes precise measurements of the remnants of the very early Universe through the so-called **cosmological microwave background** (or cosmic microwave background). The HBB model (see Astrophysics section 4.5) predicts that high-energy (gamma) electromagnetic radiation produced around $t \approx 300\,000$ years should still be observed today, but owing to the expansion of the Universe should be red-shifted down to the millimetre wavelength (microwave) region. This isotropic (coming from all directions equally) 'background' radiation was accidentally picked up by Arno Penzias and Robert Wilson in the 1960s. Penzias and Wilson were working at Bell Telephone Laboratories in New Jersey, USA, using a microwave antenna designed for satellite communications. As they pointed the antenna towards the sky, their receiver detected a faint 'hiss' coming from all directions that was highly isotropic, constant in time and could be detected at any time of day and year. What they had found was a relic of the Big Bang – the thermal radiation from the Big Bang itself! The thermal intensity of the spectrum fitted perfectly to a black-body curve corresponding to a temperature of 2.73 K (see Astrophysics section 2.4).

In 1989 a satellite called the *Cosmic Background Explorer* (*COBE*) was launched, which carried out highly accurate measurements of the cosmological microwave background (CMB) and determined the precise distribution of microwave radiation in the Universe. This confirmed a peak wavelength λ_{max} corresponding to a black-body temperature of 2.725 K and is exactly what is expected if this radiation was emitted in the gamma region of the electromagnetic spectrum soon after the Big Bang when the Universe was very small and very hot. It also showed fluctuations in the temperature of the microwave background. These tiny fluctuations reflect tiny energy-density variations in the early Universe sufficient for gravitational forces to act and to 'seed' the formation of the galaxies we observe today. The successor to *COBE* is the *Microwave Anisotropy Project*, now called the *Wilkinson Microwave Anisotropy Project* (*WMAP*), which permits much more accurate measurements of the temperature differences in the microwave background (Figure 15).

Although the temperature of the CMB is almost completely uniform at 2.7 K, there are very tiny variations in the temperature of the order of 10^{-5} K, which appear on the maps in Figure 15 as cooler blue and warmer red patches. The key findings of *WMAP* were that a more accurate age of the Universe could be established as 13.7 billion years ± 0.2 billion years, a more accurate date for when the first stars formed – only a few million years after the Big Bang, and solid evidence that the Universe will expand for ever.

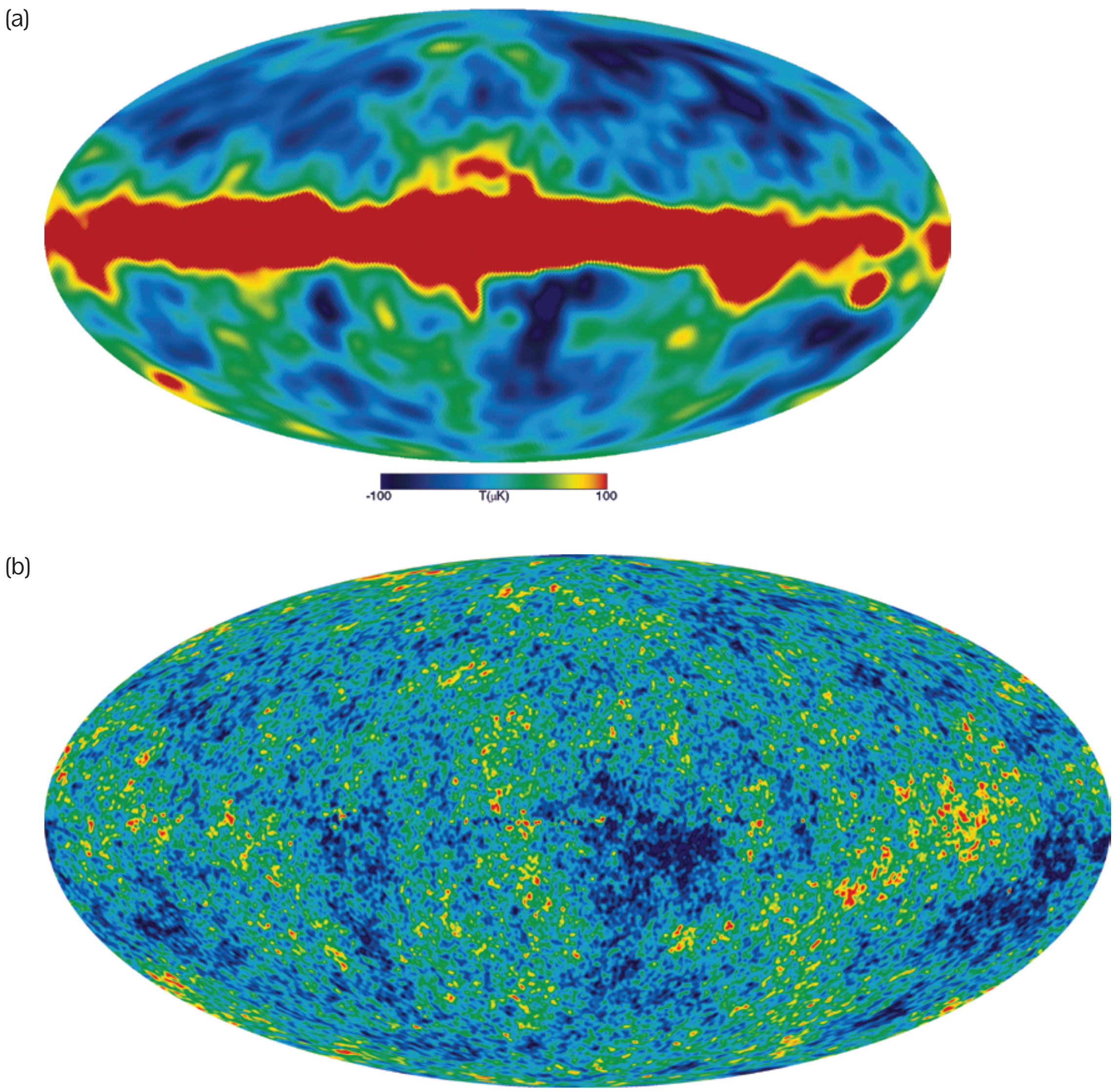

Figure 15 *The variations in the cosmological microwave background as seen by the COBE (a) and WMAP (b) missions. COBE was the first mission to see the small variations in temperature from one region to another in the CMB. WMAP, whose instruments have temperature sensitivity a thousand times greater, made more detailed observations of these temperature variations.*

Hydrogen and helium abundances

Hydrogen and helium account for nearly all the matter in the Universe that we observe today. The **relative abundance**, by mass, of these elements in the Universe is 25% helium and 73% hydrogen, with all the other elements amounting to 2%. These observed values, determined from the spectral characteristics of stars, are consistent with the HBB model of hydrogen formation and the fusion of hydrogen into helium in the very early Universe, and provides very strong evidence to support the Big Bang theory.

The HBB model predicts that **primordial nucleosynthesis**, the process by which the lightest elements such as H and He formed, began approximately 100 s after the Big Bang. Owing to the immense temperatures and pressures, nuclear fusion reactions converted hydrogen into helium, resulting in a ratio of hydrogen to helium of 3 : 1. Then, owing to the rapid expansion of the Universe, temperatures dropped below those required to sustain fusion. As a result, nucleosynthesis lasted only for about three minutes. A quarter of the atomic hydrogen had been converted into helium-4. No elements heavier than lithium could synthesise (Figure 16). All the heavier elements, including those of which the planets and you and I are made, were created later by long-lived fusion processes inside stars and were dispersed across the interstellar medium by supernovae (see Astrophysics section 3.3).

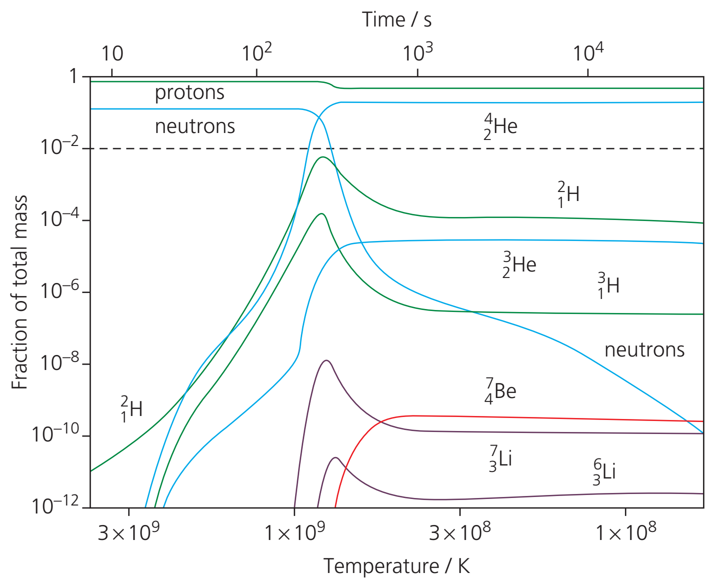

Figure 16 *The abundance of elements up to three hours after the Big Bang. At extremely high temperatures (greater than 1×10^9 K), only free protons and neutrons exist. As the Universe expands and cools, deuterium ^2_1H, an isotope of hydrogen, and helium ^4_2He are formed, resulting in a decrease in the number of free protons and neutrons. Very small amounts of beryllium and lithium are also synthesised. By about 300s, 25% by mass of the matter in the Universe is in the form of helium nuclei, and the synthesis of these light elements is complete, leading to the abundances we observe in the Universe today.*

QUESTIONS

13. The cosmological microwave background has a thermal black-body spectrum at a temperature of 2.725 K. What is its peak emission wavelength? How does this explain the name of the background radiation?

14. **a.** Explain how the observed abundances of hydrogen and helium are seen as evidence for the Big Bang.

b. The early Universe contained only light elements (Figure 16). However, at the present time, there are large amounts of heavier elements. Explain this.

Stretch and challenge

15. The Big Bang theory postulates that, as the Universe cooled after the Big Bang, there were seven protons created for every neutron. Show that this predicts that the helium abundance by mass in the early Universe was 25%.

KEY IDEAS

- The Big Bang theory or Hot Big Bang (HBB) model states that the Universe came into being from an infinitely hot, infinitely dense point called a singularity and has been expanding ever since.
- The cosmological microwave background and the relative abundances of hydrogen and helium are strong evidence for the Big Bang theory.

4.7 QUASARS

The name 'quasar' originated from the term 'quasi-stellar radio source'. These were star-like objects, but with unusually strong radio emission. Only about a quarter of all quasars known today are predominantly radio emitters, so quasars are now also known as 'quasi-stellar objects' (or QSOs). Many emit most of their energy in the infrared. Quasars are distinguished by extremely large red-shifts (see Figure 11) and are therefore believed to be some of the most distant objects in the known Universe. More than 30 000 quasars have been detected, many with red-shifts well in excess of 0.1c, giving recession velocities in excess of $4 \times 10^7\,\mathrm{m\,s^{-1}}$ and hence, from Hubble's law, distances of more than 700 Mpc. The farthest quasar detected is some 9000 Mpc away.

Optically, quasars are very faint and star-like, but application of the inverse square law (see Astrophysics section 2.1) reveals them to be amongst the brightest objects in the Universe. Quasar 3C 273 (Figure 17) has a luminosity of about 10^{40} W, comparable to 20 trillion Suns, and this is typical of many quasars. Overall, quasar luminosities range from 10^{38} to 10^{42} W. One quasar may emit hundreds or even thousands of times the entire power output of our Galaxy.

Quasar 3C 273 (Figure 17) was the first quasar to be identified. The hydrogen Balmer line from the quasar is measured at a wavelength of 760 nm, compared to its value in a laboratory on Earth of 656 nm. This gives a z value of 0.158. The relativistic equation for red-shift needs to be used to calculate its recession velocity. This works out to be $43\,600\,\mathrm{km\,s^{-1}}$. From Hubble's law, its distance from the Earth is then calculated as

$$d = 43\,600/67.3 = 646\,\mathrm{Mpc}$$

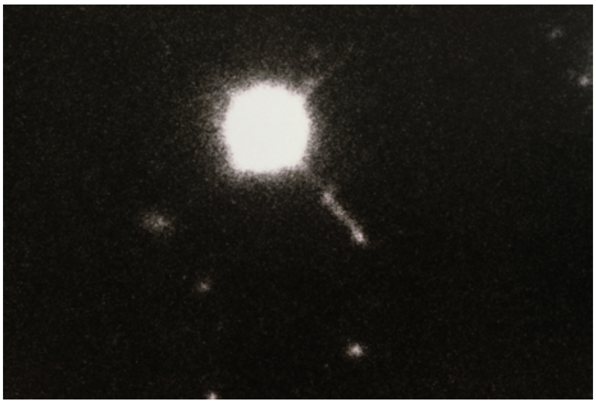

Figure 17 *The quasar 3C 273 imaged by the Hubble space telescope. It lies in an elliptically shaped galaxy in the constellation Virgo, and has a red-shift z of 0.158.*

Quasars are now regarded by astrophysicists to be part of a class of objects known as 'active galactic nuclei' (AGN). These are intensely bright, powerful cores of distant galaxies, powered by a huge disc of particles surrounding and falling into a supermassive black hole. As material from this disc falls inwards, some quasars – including 3C 273 – have been observed to fire off super-fast jets into the surrounding space. In Figure 17 you can see one of these jets streaming away (bottom right) as a cloudy streak, which measures some 200 000 light years in length.

QUESTIONS

You may need to refer back to Astrophysics sections 2.1 and 2.2 to answer these questions.

16. A quasar is found to lie at a distance of 5×10^9 pc from the Earth. To have the same apparent magnitude as the quasar, the Sun would need to be placed a distance of 3×10^3 pc from the Earth. Using the inverse square law, calculate the ratio of luminosity of the quasar to that of the Sun.

17. The absolute magnitude of the Milky Way has been estimated at – 20.5. The apparent magnitude of quasar 3C 273 is 13. A distance measurement of 3C 273 puts it at a distance of 749 Mpc.

a. What is the absolute magnitude of 3C 273?

b. How much brighter is 3C 273 than the Milky Way?

KEY IDEAS

- Quasars are extremely luminous objects with high red-shifts and lie at very great distances.
- They are believed to be the powerful cores of distant galaxies, powered by matter falling into a supermassive black hole.

4.8 EXOPLANETS

An **exoplanet** (or extrasolar planet) is a planet that orbits a star other than the Sun. Exoplanets were first discovered in 1992, when two planets were observed orbiting a pulsar. The discovery of the first planet orbiting a main-sequence star was made in 1995, when a giant planet was found in a four-day orbit around the star 51 Pegasi in the constellation Pegasus. Since then, nearly 2000 exoplanets have been discovered, some of them Earth-like, and many more await confirmation.

As planets only reflect the light of the star around which they orbit, they are much fainter than the star and so are lost in its glare and very difficult to detect directly. A few have been imaged directly – see the introduction to the Astrophysics option unit. Most have been found using indirect methods that involve tiny but measurable effects of the exoplanet on its parent star.

Discovering exoplanets – the radial velocity method

We think of a planet in orbit around a star, but, in fact, because each exerts a gravitational force on the other, they both orbit around the centre of mass of the star–planet system (Figure 18). Since the mass of the star is by far the larger, the centre of mass of the system will be very close to the centre of mass of the star (perhaps even within the star itself) and the star will be observed to 'wobble' as it moves around this point. This wobbling effect will also show up as a Doppler shift in the star's spectral lines.

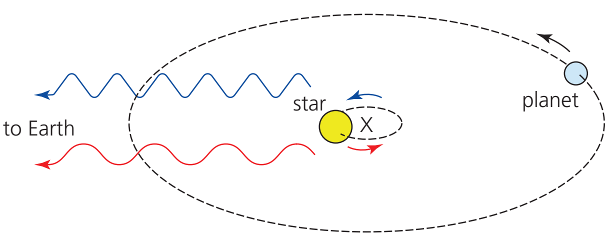

Figure 18 *A star–planet system orbits around its centre of mass indicated by X*

The **radial velocity method** in the search for planets looks for periodic variation in Doppler shift in the star's spectral lines superimposed on its radial velocity either away from or towards the Earth (similar to that observed for binary stars; see Astrophysics section 4.3). The Doppler shift is used to calculate the radial (line-of-sight) velocity of the star as it moves about the centre of mass, and a radial velocity curve can be constructed, as shown in Figure 19.

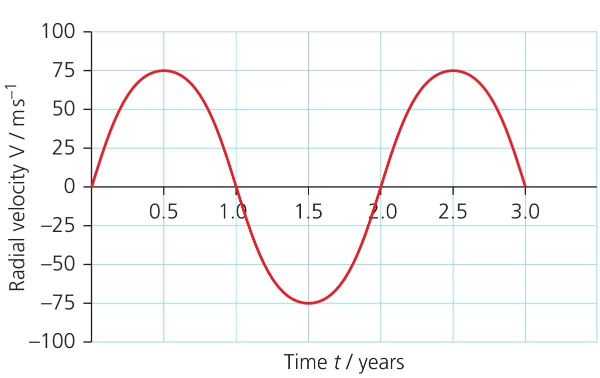

Figure 19 *The radial velocity of a star as it moves around the centre of mass due to the presence of an exoplanet. The period of the curve is equal to the orbital period of the exoplanet.*

Note that a radial velocity curve shows the velocity of the *star*. The period of the exoplanet causing the wobble is equal to the period of the radial velocity curve. Velocity measurements allow determination of the size and shape of the orbits of an extrasolar planet as well as a lower limit of the planet's mass. (They provide only a lower limit on planetary mass, because they measure just the component of the star's motion towards and away from the Earth.)

QUESTIONS

18. Figure 19 shows the radial velocity variation measured using Doppler spectroscopy of a star being orbited by a single imaginary exoplanet moving in a circular orbit.

   a. What is the maximum variation in radial velocity?

   b. Where is the centre of mass of the planet–star system likely to be?

   c. What is the orbital period of the planet?

   d. Estimate the radius of the orbit of the stars wobble. With reference to the orbital period of the plant suggest whether it is close to, or distant from the parent star

Discovering exoplanets – the transit method

The **transit method** for discovering exoplanets works by detecting a dimming in the star's brightness as an exoplanet moves across its disc, perpendicular to our line of sight – called a **transit**. From Earth, both Mercury and Venus occasionally transit the Sun. When they do, they look like tiny black dots passing across the bright surface. The same effect for other stars and an exoplanet gives a very small decrease in brightness – if a distant star was transited by a planet the size of Jupiter, the brightness would be reduced by about 1%, and this can be detected using sensitive instruments. A light curve is produced, as shown in Figure 20.

The decrease in observed brightness allows the radius of the exoplanet to be calculated if the radius of the parent star is known. If the star has a radius r_{star} and the planet has radius r_{planet}, the fractional drop in brightness will be

$$\frac{\pi r_{planet}^2}{\pi r_{star}^2} = \frac{r_{planet}^2}{r_{star}^2} = \left(\frac{r_{planet}}{r_{star}}\right)^2$$

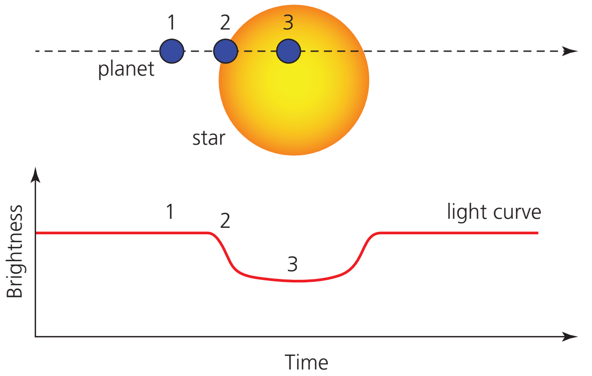

Figure 20 Decrease in observed brightness of a star as an exoplanet moves across its disc

The *Kepler* space observatory (Figure 21) uses the transit method as it scans many thousands of stars in the Milky Way. Observable transits, when the orbital configuration is suitable, occur infrequently, but many exoplanets called 'hot Jupiters' have been found in this way, and a few Earth-like ones. Exoplanets discovered using the radial velocity method have also been confirmed by observing transits.

Figure 21 Within a few years of operating, the Kepler space observatory had discovered three Earth-like planets. Kepler-438b and Kepler-44b are about the size of the Earth and are likely to be rocky. Kepler-440b is termed a 'super-Earth' – it has a mass substantially higher than the Earth but less than that of our solar system's gas giants, and may also be rocky.

Worked example

Figure 22 shows the light curve of an exoplanet in transit across a distant star.

a. Approximately how long does it take the exoplanet to cross the star's disc?

b. What is the orbital period of the exoplanet?

c. By what percentage is the light of the parent star reduced? What size of exoplanet does this suggest?

a. About one-fifth of a day, so about 5 hours

b. About 2.25 days

c. Decrease in brightness is

$$\frac{1-0.9930}{1} \times 100\% = 0.7\%$$

This suggests quite a large exoplanet, but somewhat smaller in size than Jupiter, assuming the star is a similar size to the Sun.

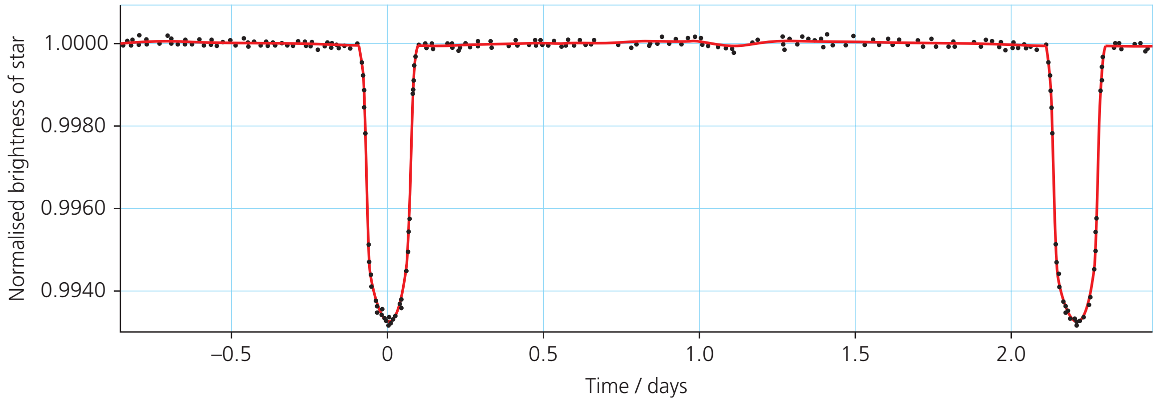

Figure 22

QUESTIONS

19. What factors make the detection of transit exoplanets difficult?

20. Figure 23 shows the reduction in brightness of the exoplanet Kepler-7b as it passes in front of its parent star Kepler 7. The radius of Kepler 7 is 1.8 times the radius of the Sun. [Take the radius of the Sun as 7.0×10^5 km]

a. Estimate the radius of the exoplanet Kepler-7b. How big is it relative to Jupiter? [Take the radius of Jupiter as 70 000 km]

b. Estimate the transit time of Kepler-7b across the surface of Kepler 7.

c. What additional information would be needed to work out the orbital period of Kepler-7b?

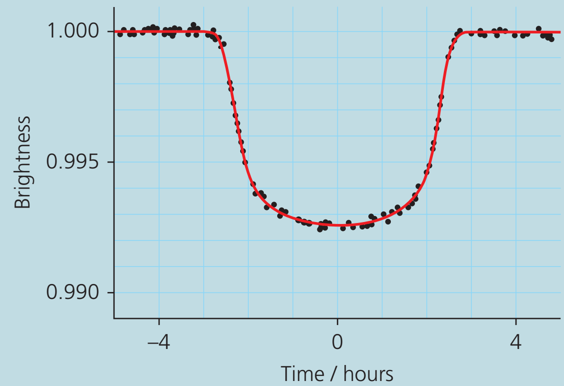

Figure 23

KEY IDEAS

- An exoplanet is a planet orbiting a star other than the Sun. While some can be directly imaged, most methods of detecting them rely on the effect they have on their parent star.
- The radial velocity method allows the detection of an exoplanet by Doppler shifts in the spectral lines of the star, due to the gravitational force the planet exerts on it causing it to move round the centre of mass and hence wobble as seen from Earth.
- The transit method relies on the passage of the planet across the star dimming its brightness.

ASSIGNMENT 1: SEARCHING FOR EXTRA-TERRESTRIAL LIFE

(MS 0.1, MS 0.2, MS 2.3)

The discovery of exoplanets raises the intriguing possibility as to whether any of them may support life. Currently, the only place in the Universe where life is known to exist is on Earth. So, as a logical starting point in the search for life, we can see if there are exoplanets that may be Earth-like. In this assignment, you will consider some of the necessary conditions for life to exist on an exoplanet and how we might look for Earth-type worlds.

Life on Earth needs energy and water. Most living things on Earth contain carbon, and carbon compounds form complex molecules that are essential for the assembly of living organisms. Liquid water is essential, as it acts as a solvent for the mixing of carbon compounds. Water is also involved

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in delivering essential vitamins and nutrients from food to cells so they can metabolise and reproduce. Our bodies are made up of nearly 60% water, and we would not be able to survive for more than a few days without it.

In looking for life on other planets, a fundamental assumption is made that life elsewhere in the Universe is similar to that on Earth inasmuch as it is carbon-based and needs liquid water.

Questions

A1 Carbon has four valence electrons. From your knowledge of GCSE Chemistry, suggest why carbon is very common in living things on Earth.

A2 Where has all the carbon on Earth come from?

On Earth, water exists in liquid form at a temperature T between 0 and 100 °C (between about 273 and 373 K). In the search for life-bearing exoplanets, astronomers define a region around a star called the 'habitable zone', which is the range of distances from a star in which liquid water could exist. To determine this, consider the simplest case of a single planet at a distance d in a circular orbit around a star with luminosity L_{star}.

The intensity radiated or absorbed by a black body is related to its effective temperature by Stefan's law, $I = \sigma T^4$ (see Astrophysics Chapter 2). We can model a planet as being like a black body that is in equilibrium. To maintain its surface temperature, the rate of energy radiated from it must be equal to the rate of energy absorbed. So a planet of surface temperature T_p must receive an intensity from its parent star equal to

$$I = \sigma T_p^4$$

The intensity at distance d from a star with luminosity L_{star} is

$$I = \frac{L_{star}}{4\pi d^2}$$

Equating the two gives

$$d = \sqrt{\frac{L_{star}}{4\pi\sigma T_p^4}}$$

Questions

A3 **a.** Calculate the maximum and minimum distances in AU from the Sun for the range of temperatures for which liquid water can exist on Earth. [Luminosity of Sun = 3.90×10^{26} W; 1 AU = 1.50×10^{11} m]

b. Published values are about d_{max} = 1.5 AU and d_{min} = 0.7 AU. Suggest why your answers are likely to be an overestimate.

c. Using the published values in part **b** for the habitable zone, how close is the Earth to the edges of the zone?

A4 The *Kepler* space observatory was designed to look for exoplanets. In 2014, using the transit method, *Kepler* discovered an exoplanet Kepler-186f orbiting a red dwarf star called Kepler 186. The distance of Kepler-186f from the red dwarf is estimated at 0.37 AU. The luminosity of the red dwarf is about 0.04 times that of the Sun. Explain whether you think liquid water could exist on Kepler-186f.

A5 Why is it important, if planets are to support life, for their orbits to be nearly circular?

A6 Astrobiologists are scientists who study the origin, evolution, distribution and future of life in the Universe. Many astrobiologists think that K-type stars, which have main-sequence lifetimes greater than that of the Sun, may be good candidates for finding life on planets existing within their habitable zones. Suggest a reason why they think this.

PRACTICE QUESTIONS

You will need to refer to the Data section at the end of this book.

1. The Antennae galaxies are a pair of colliding galaxies in the constellation Corvus. Measurements of the red-shift of radio signals from the galaxies suggest they are approximately 25 Mpc from the Earth.
   - **a.** Explain what is meant by *red-shift*.
   - **b.** Calculate the recession velocity of the Antennae galaxies.

   AQA Unit 5A June 2011 Q3 part a

2. Ursa Minor contains the galaxy NGC 6251. Measurements indicate that the light from the galaxy has a red-shift, z, of 0.025 and that the galaxy is 340 million light years from Earth.
   - **a.** Use these data to calculate a value for the Hubble constant.
   - **b.** Use your answer to part **a** to estimate a value for the age of the Universe. State an appropriate unit for your answer.

   AQA Unit 5A June 2013 Q4 part b

3. Measurements of the shift in the 21 cm H1 line in the spectrum of galaxy M84 suggests that it is receding at a velocity of $900\,\mathrm{km\,s^{-1}}$.
   - **a.** Calculate the value of the red-shift, z, for this galaxy.
   - **b.** Calculate the distance to this galaxy.

   AQA Unit 5A June 2010 Q4 part b

4. Explain how observations of Type 1a supernovae led to the conclusion that the Universe is expanding at an accelerating rate. Discuss why this conclusion was controversial. The quality of your written communication will be assessed in your answer.

5. TRAPPIST is a robotic telescope designed to detect exoplanets, which are planets outside our solar system.
   - **a.** The charge coupled device (CCD) attached to TRAPPIST has a quantum efficiency of 96% for light of wavelength 750 nm. Explain what is meant by the *quantum efficiency* of a CCD.
   - **b.** **i.** The optical arrangement of the telescope includes an objective mirror of diameter 0.60 m.

     Calculate the minimum angular separation of two objects which can be resolved by the telescope for light of wavelength 750 nm.

     ii. One of the nearest exoplanets orbits the star Epsilon Eridani, which is 10.5 light years from Earth. The exoplanet has an elliptical orbit, whose orbital radius varies from 1 AU to 5 AU. Calculate the maximum angular separation of the star and the planet when viewed from a distance of 10.5 light years.

     iii. TRAPPIST detects the presence of exoplanets by measuring the reduction in light intensity that occurs as the planet passes in front of the star. Explain why it is unlikely that the telescope could be used to observe such planets

   AQA Unit 5A June 2012 Q2 parts a, b

6. The Big Bang theory describes the formation of the Universe.
   - **a.** State the main proposals of the theory.
   - **b.** State the observational evidence for the theory and explain how each observation supports the theory.

ANSWERS TO IN-TEXT QUESTIONS

1 TELESCOPE

1. **a.** $M = 1200/25 = 48$
   b. $M = 1200/10 = 120$
2. $M = \alpha/\beta = f_o/f_e$, $\alpha = (100/2) \times 0.5 = 25°$
3. One factor is that refractors with high magnifications have large objective lenses, which makes them long, and so they require giant domes to house them. Another limiting factor is that large glass lenses are so heavy that gravity causes them to sag under their own weight, distorting their shape and therefore the image that is formed.
4. **a.** 560
   b. 187
   c. 112
5. **a.** Minimum angular resolution ≈ $\frac{510 \times 10^{-9}}{5.1} = 1 \times 10^{-7}$ rad.
   b. Smallest detail $d = 1 \times 10^{-7} \times 3.8 \times 10^{8}$ = 38 m
6. At a distance of 4×10^{16} m the angular size of the Jupiter-sized planet is
   $$\theta = \frac{1.5 \times 10^{8}}{4 \times 10^{16}} = 3.8 \times 10^{-9} \text{ rad.}$$
   Using the Rayleigh criterion, minimum angular resolution = λ/D, so diameter D of lens needs to be at least $\frac{550 \times 10^{-9}}{3.8 \times 10^{-9}} = 145$ m.
7. **a.** Angular resolution ≈ $\frac{0.04}{305} = 1.3 \times 10^{-4}$ rad.
   Hale telescope resolution is 10^{-7} rad, which is 1000 times better. Despite the much larger size of their dishes, the angular resolutions of radio telescopes are poorer because radio wavelengths are longer than optical ones.
   b. The ratios of their collecting powers are in the ratios of the squares of their objective diameters. So ratios of collecting powers are Hale : Lovell : Arecibo = 26 : 5800 : 93000 = 1 : 223 : 3500.
8. Advantage: Radio telescopes can operate day and night, whereas optical telescopes can only operate at night with clear skies. Disadvantage: Since they operate at longer wavelengths, radio telescopes have poorer angular resolution than optical telescopes.
9. **a.** Because most IR wavelengths are absorbed by the Earth's atmosphere.
   b. Infrared windows are parts of the infrared spectrum that are transparent to the Earth's atmosphere and IR radiation can reach the ground without being absorbed.
10. Minimum angular resolution of SOFIA
   $$= \frac{24 \times 10^{-6}}{2.4} = 1 \times 10^{-5} \text{ m}$$
   Minimum angular resolution of optical telescope
   $$= \frac{510 \times 10^{-9}}{2.4} = 2 \times 10^{-7} \text{ m}$$
   The optical telescope has an angular resolution about 100 times better than the SOFIA IR telescope of the same diameter.

11. **a.** Minimum angular resolution of *IUE*

$$= \frac{120 \times 10^{-9}}{45 \times 10^{-2}} = 2.7 \times 10^{-7}\text{ m}$$

Minimum angular resolution of optical telescope

$$= \frac{510 \times 10^{-9}}{45 \times 10^{-2}\text{m}} = 1.1 \times 10^{-6}\text{ m}$$

The resolving power of the optical telescope is about 10 times better than that of *IUE*. Collecting powers are the same, as they have the same objective mirror diameter.

b. Energy of UV photon

$$= hc/\lambda = \frac{(6.63 \times 10^{-34}) \times (3.00 \times 10^{8})}{120 \times 10^{-9}}$$
$$= 1.66 \times 10^{-18}\text{ J (10.3 eV)}$$

12. IR, UV and X-rays are heavily absorbed by the atmosphere of the Earth, so such telescopes need to be positioned in space above the atmosphere where these wavelengths are not blocked.

13. $\theta = \lambda/D = \frac{2.2 \times 10^{-6}}{85} = 2.6 \times 10^{-8}$ rad

14. Detected photons = 0.04 × 10 000 = 400

15. CCDs have high quantum efficiencies, so can record faint objects during short exposures.

CCD images can be stored electronically and sent over communication links for distribution.

They can be image-processed.

They can operate over a wider spectral range than film and the eye.

They have a linear response.

16. Power = (energy of photon)/time
= intensity × area = $(5.3 \times 10^{-3}) \times (4.2 \times 10^{-12})$
= 2.2×10^{-14} W

Energy of photon = hf
= $(6.6 \times 10^{-34}) \times (3.9 \times 10^{14}) = 2.6 \times 10^{-19}$ J,
which in 1 s is 2.6×10^{-19} W.

So number of photons = $\frac{2.2 \times 10^{-14}}{2.6 \times 10^{-19}} = 8.5 \times 10^{4}$ photon s^{-1}.

The QE is 85%, so $0.85 \times 8.5 \times 10^{4}$
= $7.2 \times 10^{4}\ s^{-1}$ are actually detected.

2 CLASSIFICATION OF STARS

1. A magnitude difference of 1 corresponds to a brightness ratio of $(100)^{1/5}$ or 2.51. So a magnitude difference of (3 – 1) = 2 corresponds to a brightness ratio of $(2.51)^2 = 6.3$.

2. **a.** Mizar, Deneb, Aldebaran, Altair, Rigel, Arcturus, Canopus, Sirius

b. $-0.9 - (0.9) = -2.5 \log_{10}(b_{Canopus}/b_{Altair})$

$-1.8 = -2.5 \log_{10}(b_{Canopus}/b_{Altair})$

$0.72 = \log_{10}(b_{Canopus}/b_{Altair})$

so

$$(b_{Canopus}/b_{Altair}) = 10^{0.72} = 5.2$$

Canopus is about 5 times brighter than Altair.

3. Brightness $b = L/4\pi R^2$

a. $b = (3.90 \times 10^{26})/(4\pi \times (1.50 \times 10^{11})^2)$
$= 1380\text{ W m}^{-2}$

b. $b = (3.90 \times 10^{26})/(4\pi \times (5.93 \times 10^{12})^2)$
$= 0.88\text{ W m}^{-2}$

4. **a.** The parallax angle is half the angle between the direction of a nearby star from the Earth at one time of year, and its direction from the Earth six months later.

b. **i.** 1/0.316 = 3.16 pc

ii. 1 pc = 206 265 AU
3.16 × 206 265 AU
= 650 000 AU

iii. 1 pc = 3.26 ly
3.16 × 3.26 ly = 10.3 ly

5. **a.** 1 ly = (1/3.26) pc.
Proxima Centauri is 4.2/3.26 = 1.3 pc

b. Parallax angle = 1/1.3 pc = 0.77 arcsecond

6. **a.** The apparent brightness of a star observed from the Earth is called the apparent magnitude. Absolute magnitude is defined to be the apparent magnitude an object would have if it were located at a distance of 10 pc. If a star was at 10 pc distance from us, then its apparent magnitude would be equal to its absolute magnitude.

b. $M = m - 5\log_{10}(d/10) = 0.34 - 5\log_{10}(3.5/10)$
$= 0.34 - (-2.3) = 2.64$

c. Distance modulus is 1.35 – (– 0.30) = 1.65

d. Distance $d = 10^{(1.65+5)/5} = 13.5$ pc

7. As the distance to an object increases, the parallax angle becomes smaller to the point where it can be no longer measured.

8. $L = \sigma A T^4 = 4\pi R^2 \sigma T^4$

$$= 4 \times \pi \times (25 \times 6.96 \times 10^8)^2 \times (5.67 \times 10^{-8}) \times (4300)^4 = 7.4 \times 10^{28}\ \text{W}$$

9. a. $R = \sqrt{\dfrac{L}{4\pi\sigma T^4}} =$

$$\sqrt{\frac{6.6 \times 10^4 \times 3.9 \times 10^{26}}{4\pi \times 5.67 \times 10^{-8} \times (11000)^4}} = 5.0 \times 10^7\ \text{km}$$

b. $\lambda_{max} = \dfrac{2.90 \times 10^{-3}}{T} = \dfrac{2.90 \times 10^{-3}}{11000} = 2.6 \times 10^{-7}\ \text{m}$

$= 260\ \text{nm}$

10. Barnard's star has a temperature less than 3500 K and therefore is an M type star.

11. By looking at their colour. With good eyesight and on a clear night Vega, for example, can be seen to be bluish white and a therefore a hotter star than Antares, which appears red and is much cooler. A long exposure photographic image will show the different colours of stars clearly.

12. A photon of wavelength 587.56 nm emitted in the Sun's interior has passed through its outer layers and been absorbed in the solar atmosphere, giving a dark line in the solar continuous spectrum. There must be an element in the outer layers whose atom has an electron transition matching this photon's energy. This was a previously unknown element. (It was named after the Greek word for the Sun, *helios*, and was subsequently discovered on Earth 40 years later.)

3 STELLAR EVOLUTION

1. a. A protostar is formed from molecular clouds by gravitational attraction, as shown in Figure 2.

b. Gravitational potential energy.

c. The temperatures need to be high enough so that hydrogen nuclei have enough kinetic energy to overcome their mutual electrostatic repulsion and fuse together.

2. Supergiants, red giants, main-sequence stars, white dwarfs.

3. Star A would reach the main sequence first, because it has a greater mass. The greater the mass of the star, the stronger is the inward gravitational contraction. This in turn increases the core temperatures to where nuclear fusion reactions can start to be reached sooner. Once hydrogen burning is established, a star is on the main sequence.

4. The main-sequence mass.

5. Red dwarfs are cool dim stars, which means that they would appear in the bottom right-hand corner of the HR diagram.

6. Most stars we observe are main-sequence stars because all stars spend about 90% of their lives on the main sequence.

7. The star has expanded massively in size to a red supergiant, which means that the energy is radiated from a much larger surface area.

8. For the Earth, escape velocity is

$$v_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2 \times 6.67 \times 10^{-11} \times 5.98 \times 10^{24}}{6.37 \times 10^6}}$$

$$= 11.2\ \text{km s}^{-1}$$

The ratio 160 000/11 means that the escape velocity from a neutron star is nearly 15 000 times greater than that from the Earth.

9. a. $R_S = \dfrac{2GM}{c^2} = \dfrac{2 \times 6.67 \times 10^{-11} \times 1.99 \times 10^{30}}{(3.00 \times 10^8)^2}$

$= 2.95 \times 10^3\ \text{m}$

b. The Schwarzschild radius forms an event horizon. Nothing can escape from inside it. If the Sun could be compressed so that its radius was less than 3 km, no light would escape from it.

c. Density = mass/volume, so

$$\text{density} = \frac{1.99 \times 10^{30}}{\frac{4}{3} \times \pi \times (2.95 \times 10^3)^3}$$

$$\approx 3 \times 2 \times 10^{30}/[4 \times 3 \times 3^3 \times 10^9] \approx 2 \times 10^{19}\ \text{kg m}^{-3}$$

10. $R_S = \dfrac{2GM}{c^2} = \dfrac{2 \times 6.67 \times 10^{-11} \times 50 \times 1.99 \times 10^{30}}{(3.00 \times 10^8)^2}$

$= 1.48 \times 10^5\ \text{m}$

(or simply 50 times the value calculated in question 9a)

11. $R_S = \dfrac{2GM}{c^2} = \dfrac{2 \times 6.67 \times 10^{-11} \times 4.1 \times 10^6 \times 1.99 \times 10^{30}}{(3.00 \times 10^8)^2}$

$= 1.2 \times 10^{10}\ \text{m}$

12. $R_S = \dfrac{2GM}{c^2} = \dfrac{2 \times 6.67 \times 10^{-11} \times 1.0 \times 10^{20}}{(3.00 \times 10^8)^2}$

$= 1.5 \times 10^{-7}\ \text{m}$

13. Use $m - M = 5\log_{10}\left(\frac{d}{10}\right)$ so

$$14 - (-19.3) = 5\log_{10}\left(\frac{d}{10}\right)$$

$$\log_{10}\left(\frac{d}{10}\right) = 6.66$$

$$d = 10^{7.66} = 46\,\text{Mpc}$$

14. Astrophysicists believe that all Type Ia supernovae have approximately the same peak absolute magnitude (about −19.3), but Type II supernovae are not consistent in this way.

4 COSMOLOGY

1. $\Delta\lambda = \lambda_{app} - \lambda = 600.80 - 600.00 = 0.80$ nm

$$v = -c\frac{\Delta\lambda}{\lambda} = -\frac{(3.00 \times 10^8) \times (0.80 \times 10^{-9})}{600.00 \times 10^{-9}}$$

$$= -4 \times 10^5 \text{ m s}^{-1}$$

Since λ shows a red-shift, and v is negative, the star is receding from the Earth, at a speed of 400 km s^{-1}.

2. The shifts for the three stars are as follows.

Star A: $\Delta\lambda = 656.60 - 656.00 = 0.60$ nm

Star B: $\Delta\lambda = 655.90 - 656.00 = -0.10$ nm

Star C: $\Delta\lambda = 656.40 - 656.00 = 0.40$ nm

a. Star A shows the greatest shift, so is moving the fastest relative to Earth.

b. $v/c = -\Delta\lambda/\lambda$, so star A is receding from Earth (red-shift, negative velocity), star B is approaching Earth (blue-shift, positive velocity), and star C is receding.

3. $\Delta\lambda = 20.99 - 21 = -0.01$ cm $= -1.0 \times 10^{-4}$ m

$v/c = -\Delta\lambda/\lambda$, so

$$v = -\frac{(3.00 \times 10^8) \times (-1.0 \times 10^{-4})}{21 \times 10^{-2}}$$

$$= 1.4 \times 10^5 \text{ m s}^{-1}$$

This is positive, so it is moving towards us.

4. a. i. The curves are not identical because the different motions of the two stars relative to us in their orbital paths gives rise to different changes in wavelength due to the Doppler effect.

ii. The graphs are in anti-phase, with one star moving towards the observer while the other is moving away.

iii. $$v_X = c \times \frac{\Delta\lambda}{\lambda} = 3 \times 10^8 \times \frac{0.120 \times 10^{-9}}{477 \times 10^{-9}}$$

$$= 7.55 \times 10^4 \text{ m s}^{-1} = 75.5 \text{ km s}^{-1}$$

b. The brightness graph shows that the stars are partially eclipsing binaries of significantly different luminosity and are likely to be of significantly different mass and size. Hence their centre of mass about which they orbit is not half-way between them.
The wavelength shift of star X is less than that of Y because its linear velocity ($v = \omega r$) is less than that of star Y, although their angular velocity ω is the same as they both have the same period. Star X has a greater mass than star Y as its wavelength shift is smaller. It must therefore have a smaller linear velocity, and so is a smaller distance from the centre of mass.

5. In the worked example, the ratio of the two masses M_1/M_2 = radius of S_2/radius of S_1 = 0.05/0.24 = 0.21. So S_1 is approximately one-fifth of the mass of S_2.

6. $$z = \frac{\text{recession velocity}}{\text{speed of light}} = \frac{9 \times 10^6}{3 \times 10^8} = 0.03$$

7. $\Delta\lambda = 820.9 - 393.4 = 427.5$ nm, giving

$$z = \frac{\Delta\lambda}{\lambda} = \frac{427.5}{393.4} = 1.09$$

So $v = c \times z = 3.00 \times 10^8 \times 1.09 = 3.27 \times 10^8$ m s^{-1} which is 9% faster than the speed of light, and that is not possible. The formula $z = -v/c$ is not valid at relativistic speeds.

8. $v = H \times d$ so $1/H = d/v$, which in SI units is km/(km s^{-1}). The km cancel out, leaving the unit of second.

9. $v = H \times d$, so $d = \frac{v}{H} = \frac{7200}{67.3} = 107$ Mpc

10. a. The Hubble constant is found from the slope of the graph, which is about 203/3 = 68 km s^{-1} Mpc^{-1}

b. $v = z \times c = 0.002 \times 3.00 \times 10^8 = 600$ km s^{-1}

So distance = v/H = 600/68 = 8.8 Mpc

11. a. Maximum and minimum values of H for ±10% of 71 km s^{-1} Mpc^{-1} are 78 and 64 km s^{-1} Mpc^{-1}, respectively. Using Hubble's law,

maximum distance of galaxy = 5500/64
= 86 Mpc

minimum distance of galaxy = 5500/78 = 71 Mpc

b. For H = 64 km s^{-1} Mpc^{-1},

$$\text{maximum age} = 1 \times \frac{3.1 \times 10^{19}}{64} = 5 \times 10^{17}\,\text{s}$$

= 15.9 billion years

For H = 71 km s^{-1} Mpc^{-1},

$$\text{minimum age} = 1 \times \frac{3.1 \times 10^{19}}{71} = 4 \times 10^{17}\,\text{s}$$

= 12.7 billion years

12. We would observe blue-shifts in their spectral characteristics.

13. Using Wien's displacement law (see Chapter 12), $\lambda_{max} = \frac{2.9 \times 10^{-3}}{2.725} = 1.1$ mm. This is in the microwave region of the electromagnetic spectrum.

14. a. The Big Bang theory proposes primordial nucleosynthesis – the formation of hydrogen nuclei from free protons (and neutrons), and the formation of helium nuclei from fusion of hydrogen. The theory predicts a hydrogen : helium ratio of 3 : 1, which is very near what we see today.

b. The heavier elements are made by fusion in stars, which, when the star ends its life in a supernova, are dispersed into the interstellar medium.

15. The nucleus of helium has four nucleons made up of two protons and two neutrons. So, if the ratio of protons : neutrons was 7 : 1, to assemble one He nucleus means there must be 14 protons and two neutrons, of which the two neutrons combine with two protons, leaving 12 extra protons. So four of the original nucleons are used up in a helium nucleus and the other 12 nucleons are still protons – hydrogen nuclei. Thus out of 16 nucleons, four nucleons are used to make a helium nucleus, and consequently $\frac{4}{16} = 25\%$ of the total nucleon mass turned into helium.

16. If they have the same apparent magnitude, then they appear equally bright, so that

$$\frac{L_{Sun}}{d_{Sun}^2} = \frac{L_{quasar}}{d_{quasar}^2}$$

and so

$$\frac{L_{quasar}}{L_{Sun}} = \frac{d_{quasar}^2}{d_{Sun}^2} = \left(\frac{d_{quasar}}{d_{Sun}}\right)^2 = \left(\frac{5 \times 10^9}{3 \times 10^3}\right)^2 = 2.8 \times 10^{12}$$

The quasar is about three trillion times more luminous than the Sun!

17. a. Using $m - M = 5 \log_{10}(d/10)$ gives

$$m - M = 5\log_{10}\left(\frac{749 \times 10^6}{10}\right) = 39$$

So the absolute magnitude M of the quasar is $13 - 39 = -26$.

b. The difference in magnitudes between 3C 273 and the Milky Way is about 5.5, so the brightness difference is $(2.51)^{5.5} = 158$. The quasar is about 160 times brighter than the Milky Way.

18. a. From the amplitude of the graph, the maximal variation in radial velocity of the star is ± 1.0 m s^{-1}.

b. The centre of mass of the system will be very close to the centre of mass of the star, as the star is very much more massive than the planet.

c. The orbital period of the planet around the star is the same as the periodic time of the radial velocity of the star, which from the graph is 2 years.

d. $v = r \times \omega$, so radius is

$$r = \frac{vT}{2\pi} = \frac{1.0 \times 2 \times 365 \times 24 \times 3600}{2\pi} = 1.0 \times 10^7 \text{ ms}^{-1}$$

The orbital radius of the star's wobble is therefore similar the radius of the Sun (7×10^8 m). The planet therefore probably orbits very close to its star, so will be very hot.

19. The difficulty in measuring the reduction in brightness accurately. The infrequency of transits with the correct orientation viewed from Earth.

20. a. From the graph, the reduction in brightness is $(1.000 - 0.9925) = 0.0075$. So

$$\frac{r_{Kepler\text{-}7b}^2}{r_{Kepler}^2} = 0.0075$$

$$r_{Kepler\text{-}7b}^2 = 0.0075 \times r_{Kepler}^2$$
$$= 0.0075 \times (1.8 \times 7.0 \times 10^5)^2$$
$$r_{Kepler\text{-}7b} = \sqrt{0.0075} \times (1.8 \times 7.0 \times 10^5)$$
$$= 0.0866 \times 1.8 \times 7.0 \times 10^5$$
$$= 109000 \text{ km}$$

Relative to Jupiter

$$\frac{\text{radius of Kepler-7b}}{\text{radius of Jupiter}} = \frac{109000}{70000} = 1.6$$

So Kepler-7b is about 1.6 times larger than Jupiter.

b. About 5 hours.

c. We would need to know the time between successive transits.

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GLOSSARY

Absolute magnitude *(M)* The apparent magnitude a star would have if it were placed at a standard distance of 10 parsec from the Earth.

Achromatic doublet Two individual lens elements cemented together and corrected to bring light of two wavelengths, such as red and blue, into focus in the same plane.

Airy disc The bright central region in an optical diffraction pattern caused by light entering a circular aperture.

Angular magnification The magnifying power of a refracting telescope, given by the ratio of the objective focal length to the eyepiece focal length.

Angular size The angle between the lines of sight to the two opposite sides of an object.

Aperture The opening to a camera or telescope which admits light.

Apparent magnitude (*m*) The apparent brightness of a star expressed on the magnitude scale.

Arcminute An angle of one sixtieth of a degree.

Arcsecond An angle of one sixtieth of an arcsecond, or 1/3600 of a degree.

Astronomical unit (AU) The average distance between the Earth and the Sun: 1.496×10^8 km.

Atmospheric opacity The measure of the absorption of electromagnetic radiation by the atmosphere, as a function of wavelength.

Big Bang theory The explosion event ~ 14 billion years ago that cosmologists consider the beginning of the Universe.

Binary star system A binary star system, in which two companion stars orbit their common centre of mass.

Black body A body that absorbs all the radiation incident upon it and reflects none, i.e. it is a perfect absorber and also a perfect emitter; the surface temperature determines how much energy it emits at each wavelength.

Black dwarf The end stage of a low-mass star such as the Sun. These are extremely dense and emit little or no heat or light radiation.

Black hole Highly dense matter around which gravity is so strong that the escape velocity exceeds the speed of light.

Black-body curves The intensity of radiation emitted by a black body as a function of wavelength (or frequency) and characteristic of its temperature.

Blue-shift A decrease in wavelength of radiation emitted by an object approaching an observer.

Brightness The amount of energy radiated per second per square metre (also called intensity or radiation flux); unit W m^{-2}.

Carbon–nitrogen–oxygen cycle (CNO cycle) A nuclear fusion cycle occurring in the core of stars of greater mass than the Sun.

Cassegrain arrangement A reflecting telescope where the image is reflected by a secondary mirror through the centre of the primary mirror.

Charge-coupled device (CCD) A semiconductor device in which light is converted directly into digital information, commonly used in cameras and in conjunction with telescopes for digital imaging.

Chromatic aberration An optical defect that causes light of different colours to be focused at different focal points.

Circumstellar disc A rotating flat disc of material surrounding a protostar and from which planets may form.

Collecting power A measure of a telescope's ability to collect incident electromagnetic radiation, and which is directly proportional to the square of the diameter of its objective.

Concave lens A lens that spreads a parallel beam into a divergent emergent beam.

Cones In the context of the eye, light-sensitive cells up the retina, responsible for colour vision.

Continuous spectrum A spectrum showing all frequencies (c.f. line spectrum).

Convex lens A lens that causes a parallel beam to converge to a point called the focus or focal point.

Cosmological distances Distances which are a significant fraction of the radius of the known universe.

Cosmological microwave background (or cosmic microwave background) Isotropic radiation in the microwave region with a black-body temperature of 2.7K; believed to be a remnant of the Big Bang: 2.7 °C.

Cosmology The study of the structure and development of the Universe as a whole.

Dark energy A hypothetical form of energy that permeates all space and tends to increase the rate of expansion of the Universe.

Dark matter Unobserved matter that is believed to be abundant within galaxies throughout the Universe.

Dispersion The separation of polychromatic light into a spectrum by refraction or diffraction grating.

Distance modulus The difference between a star's apparent magnitude, *m*, and its absolute magnitude, *M*, and which is related to the star's distance from the Earth.

Doppler effect The change in frequency and wavelength of radiation due to relative motion of the source and observer.

Doppler equation The formula used to calculate the change in wavelength due to relative motion of the source and observer:

$$\frac{\lambda_{app} - \lambda}{\lambda} = -\frac{v}{c}.$$

Doppler shift The change in frequency of waves emitted by an object as it moves towards or away from an observer, often denoted by the symbol, *z*: $z = \frac{-\Delta f}{f}$.

Eclipsing binaries A binary star system whose orbit lies in the same plane as the line of sight from Earth.

Emission line spectrum A pattern

of bright spectral lines produced by photons emitted as a result of precise energy changes within an atom.

Absorption spectrum In the context of stars, a pattern of dark spectral lines in a continuous spectrum produced by the absorption of photons of precise energy which cause changes within an atom.

Escape velocity The speed necessary for an object to escape the gravitational pull of another object, such as a planet or star.

Event horizon The imaginary spherical boundary around a black hole within which all information is lost.

Exoplanet Planets which orbit stars other than the Sun.

Exponential decay When a quantity reduces in magnitude by a certain factor, e.g. half, in a constant time period it is said to decay exponentially.

Eyepiece lens A converging lens at the observer's end of a telescope or microscope which acts as a magnifying glass for the real image produced by the objective lens.

Focal length (*f*) The distance between the principal focus of a lens and its optical centre.

Gamma ray astronomy The study of astronomical objects in the gamma-ray part of the electromagnetic spectrum.

Gamma ray bursts (GRBs) Flashes of gamma rays lasting from a few milliseconds to tens of seconds coming from distant galaxies and thought to originate in supernovae.

Hertzsprung–Russell diagram (HR diagram) A plot of absolute magnitude (luminosity) of stars against their spectral class (surface temperature) which shows the evolutionary stages of different stars.

Hipparchus scale A scale describing the apparent magnitude (relative brightness) first devised by Hipparchus of Nicaea (190–20BC).

Hubble constant The constant of proportionality in the relation between recession velocity of a distant astronomical object and its distance, often denoted by H or H_0.

Hubble diagram A plot of recession velocities of distant astronomical objects against their distance, which approximates to a straight line.

Hubble's Law Data shows that the rate at which a galaxy recedes is directly proportional to its distance from us, i.e.

$$\upsilon = H_0 d$$

where υ is the recession velocity in $\text{kms}^{-1}$and d is the distance of the galaxy in Mpc.This is called **Hubble's Law**

Hydrogen burning The fusion of hydrogen nuclei with a release of nuclear binding energy, which is the primary source of energy generation in main sequence stars.

Interstellar medium The 'space' between stars which contains molecular clouds where new stars are formed.

Kirchhoff's law of thermal radiation For any given temperature, the ratio of the capacity of a body to emit radiation to its capacity to absorb it (at a particular wavelength) is constant and is independent of the composition of the body. Therefore, objects that are good heat emitters are also good heat absorbers.

Light curve A graph of star brightness against time, used to identify phenomena such as eclipsing binary star systems, exoplanets and Cepheid variable stars.

Light year (ly) The distance light travels in a vacuum in one year: 9.46×10^{15}m.

Light-gathering power (LGP) A relative measure for comparing the ability of different telescopes to collect light.

Luminosity The total energy radiated by a star each second (also called power); units J s^{-1} or W.

Magnification Ratio of image size to object size; for a lens it is equal to the ratio of image distance v to object distance u.

Main sequence The well-defined band on the Herztsprung–Russell diagram in which stable stars are found; their exact location and time spent on the main sequence is governed by their initial mass.

Main-sequence star A star whose energy comes only from nuclear fusion rather from gravitational contraction.

Minimum angular resolution The minimum angle, θ, that an instrument can distinguish between two small objects for a particular wavelength of light or other electromagnetic radiation, as determined by the Rayleigh criterion: $\theta \approx \frac{\lambda}{D}$.

Molecular clouds Low density matter cloud in interstellar space comprising mainly of old hydrogen gas in the form of atoms, molecules and ions at temperatures of 10 to 50K. These clouds are the birthplace of new stars.

Neutron star The highly dense remnant of a star after a supernova explosion, composed mainly of neutrons.

Normal adjustment The setting for a refracting telescope in which the light emerges parallel from the eyepiece lens and the image is viewed at infinity.

Nuclear fusion The process of joining two or more light nuclei together to form new nuclei of heavier elements.

Objective lens The lens of a telescope or microscope nearest the object that produces a real image which the eyepiece lens magnifies.

Optical telescopes A telescope designed to receive light, i.e. radiation in the visible region.

Parallax The effect whereby the position or direction of an object appears to differ when viewed from different positions e.g. the position of a nearby star against more distant stars appears to change as the Earth orbits the Sun.

Parallax angle The angle between the Earth at one time of year, and the Earth six months later, as measured from a nearby star.

Parsec (pc) The astronomical distance at which the angle subtended by the mean distance of the Earth–Sun system, i.e. 1 AU, is one arcsecond; in other words, the distance at which an object lies if its measured parallax angle is 1 arcsecond; 1 pc = 3.262 ly or 2.06×10^5 AU.

Photoelectric effect The liberation of electrons from a metal surface exposed to electromagnetic radiation of frequency above a minimum frequency called the threshold frequency.

Pixels A picture element that makes up a digital image.

Planetary nebula An expanding glowing shell of ionised gas ejected from old red giant stars late in their lives prior to them collapsing to a white dwarf.

Pogson's law A law describing the Hipparchus scale as a mathematical relationship. It is used to calculate the apparent brightness of a star by using a star of known brightness using the equation:

$$m_2 - m_1 = -2.5\log\left(\frac{b_2}{b_1}\right)$$

Pre-main-sequence star A star which has begun nuclear fusion reactions within its core but has not reached an equilibrium state.

Primordial nucleosynthesis The production of nuclei other than hydrogen-1 during the early phases of the universe after the Big Bang.

Principal axis An imaginary line drawn at right angles to a lens passing through the optical centre, used in constructing ray diagrams.

Principal focus (F) A particular point on the optical axis of a lens where ray of light parallel to the principal axis is focused.

Proton–proton chain (p–p chain) A nuclear fusion cycle occurring in the core of stars of mass equal to or less than that of the Sun.

Protostar A star in its earliest stage of formation from a dense cloud of gas, prior to fusion reactions within the core.

Pulsar A rotating neutron star with a very strong magnetic field and strong radio emissions.

Quantum efficiency (QE) In the context of a CCD detector, the ratio of photons detected to photons incident.

Quasar An astronomical object with a very large red-shift and high luminosity, sometimes associated with radio emission; thought to be the bright nucleus of a distant active galaxy.

Radial velocity method Quasars are very luminous objects whose spectra show high red shifts showing their recession velocity is a significant fraction of the speed of light. Quasars are thought to be the most distant objects in the universe.

Radio interferometer An array of two or more radio telescopes used to produce higher resolution images than a single radio telescope.

Rayleigh criterion This states that two point objects can be resolved by an optical instrument if their angular separation is at least λ/D, where λ is the wavelength of the radiation and D is the diameter of the objective mirror or lens.

Real image An image formed by the convergence of rays of light, which can be formed on a screen or viewed virtually using an eyepiece lens.

Recession velocity The rate at which an object such as a star or galaxy is moving away from the Earth.

Red dwarfs The oldest stars in the universe, which have a low mass, temperature and luminosity.

Red giants A large, relatively cool star of high luminosity, similar in mass to our Sun but with a greatly expanded outer shell and hence large size and surface area.

Red-shift The increase in wavelength of radiation emitted by an object that is moving away from the observer.

Reflecting telescope A telescope that uses mirrors to capture and focus the light.

Refracting telescopes A telescope that uses lenses to capture and focus the light; at its most simple, a two-lens arrangement of objective lens and eyepiece lens.

Relative abundance The ratio of amount of one element to another, for example hydrogen to helium in the Universe.

Resolving power A measure of the ability of a telescope to distinguish between adjacent astronomical features or objects (also called the angular resolution).

Rods In the context of the eye, light-sensitive cells in the retina, with greater sensitivity than cone cells, but which cannot distinguish colour.

Schwarzschild radius The radius of an imaginary sphere from the centre of a black hole at which the escape velocity is equal to the speed of light. It defines the event horizon.

Spherical aberration The distortion of an image due to imperfections in the mirror or lens causing differing focal lengths.

Standard candle An astronomical object of known intrinsic brightness, for example a supernova, that is used to determine astronomical distances.

Stefan–Boltzmann law (Stefan's law) The relation that gives the total energy emitted per square metre per second from an object at a given temperature T to be proportional to T^4. The constant of proportionality is σ, the Stefan–Boltzmann constant.

Stellar evolution The process by which a star changes during its lifetime, which depends on the mass of the star

Stellar spectroscopy The analysis of spectra from stars in order to obtain precise information about surface temperature, composition and physical conditions with a star.

Supergiants Highly luminous stars with masses 10–100 times that of the Sun and high core temperatures.

Supermassive black hole A black hole having a mass of 10^6 to 10^9 that of the Sun, usually found at the centres of galaxies.

Supernova The explosive death of a star, caused by the sudden onset of nuclear burning or energetic shock wave; one of the most energetic events in the Universe.

Thermal radiation Heat radiation in the form of electromagnetic waves.

Transit In the context of astronomy, the passage of a planet in front of the star it orbits.

Transit method The method of detecting an exoplanet by detecting the dimming of a star as the planet passes in front of it.

Virtual image An image caused by rays that do not converge; the image can be seen by the eye but cannot be formed on a screen.

White dwarf A low-mass small star (~ Earth size) that has exhausted all its nuclear fuel. They are extremely dense and have a high surface temperature.

Wien's displacement law (Wien's law) For a hot object, the wavelength of the peak emission intensity is inversely proportional to the absolute temperature of the object: $\lambda_{max}T = 0.0029$ m K ·

X-ray astronomy The study of astronomical objects that emit in the X-ray part of the electromagnetic spectrum such as interacting binary stars, active galaxies, galaxy clusters and supernova remnants.

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