29 Stretch lesson: Constructions

Stretch objectives

Before you start this chapter, mark how confident you feel about each of the statements below:	▶	▶▶	▶▶▶
I can construct the perpendicular bisector of a given line.			
I can construct a triangle.			
I can construct the perpendicular from a point to a line.			
I can construct the bisector of a given angle.			
I can construct angles of 90° and 45°.			
I can find and describe regions which satisfy a combination of loci.			
I can solve a variety of locus problems.			

Check-in questions

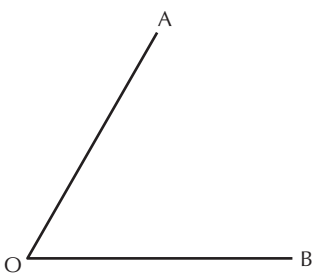
- Complete these questions to assess how much you remember about each topic. Then mark your work using the answers at the end of the lesson.
- If you score well on all sections, you can go straight to the Revision Checklist and Exam-style Questions at the end of the lesson. If you don't score well, go to the lesson section indicated and work through the examples and practice questions there.

1 Draw the perpendicular bisector of an 8 cm line. Go to 29.1

2 Using only a ruler, a pencil and a pair of compasses, construct the perpendicular from the point P. Show all construction lines.

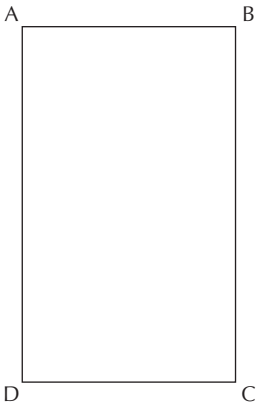
Go to 29.1

3 Using only a ruler and a pair of compasses, bisect a copy of this angle. Show all construction lines. Go to 29.1



4 ABCD is a rectangle. Make an accurate copy of the rectangle and shade the set of points inside the rectangle that are more than 2 cm from point B and more than 1.5 cm from the line AD.

Go to 29.2



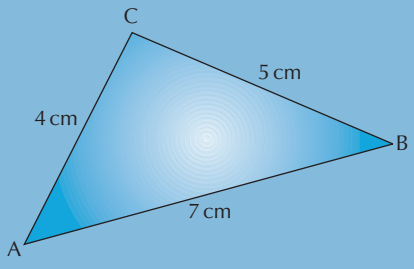
29.1 Constructions

Constructions are accurate drawings of shapes, angles or lines. They should be made using a ruler, a sharp pencil and a pair of compasses.

Constructing a triangle

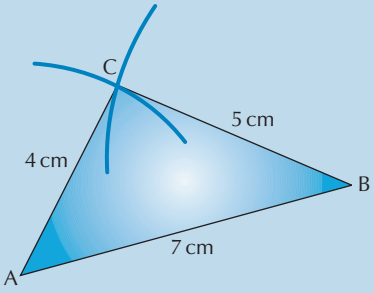
Example 1

Q Use ruler and compasses to construct this triangle accurately. You must show all construction lines.



A

- Draw the longest side AB.
- With the compass point at A, draw an arc of radius 4 cm.
- With the compass point at B, draw an arc of radius 5 cm.
- Join A and B to point C where the two arcs intersect.



Exam tips Do not rub out the arcs; these are your construction lines. Remember: no arcs, no marks!

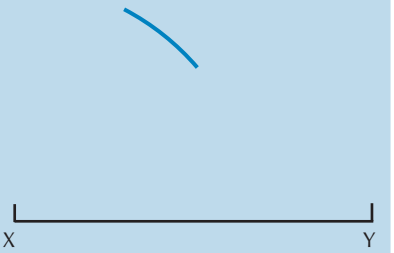
Constructing the perpendicular bisector of a line

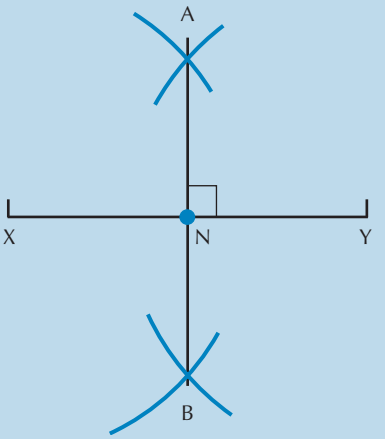
Example 2

Q Draw the perpendicular bisector of a line XY.

A

- Draw two arcs on opposite sides of XY with the pair of compasses, using X as the centre. The compasses must be set at a radius greater than half the distance of XY.
- Keeping the compasses the same distance, move the centre to Y and draw two more arcs to intersect the two already drawn.
- Join the two points where the arcs cross.
- AB is the perpendicular bisector of XY. N is the midpoint of XY.





Exam tips A perpendicular bisector cuts a line in half at right angles. To construct an angle of 90°, construct a perpendicular bisector.

Constructing the perpendicular from a point to a line

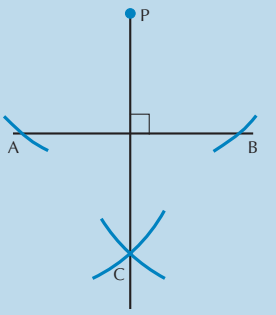
The perpendicular distance from a point to a line is the shortest distance between the point and the line.

Example 3

Q Draw the perpendicular from a point P to a line.

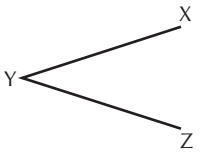
A

- Draw arcs from P with the same radius to cut the line at A and B.
- Open your compasses to a radius larger than half the distance of AB. From A and B draw arcs with the same radius to intersect at C.
- Join P to C. PC is perpendicular to AB.

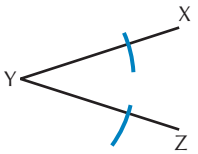


Bisecting an angle

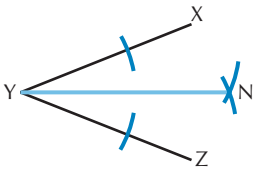
Follow these steps to bisect an angle.



- With your compasses on Y, draw an arc on XY and an arc on YZ.



- Keep the compasses at the same length. Place the compass point at the two arcs on XY and YZ in turn and draw arcs to cross at N.
- Join Y and N. YN is the bisector of angle XYZ.



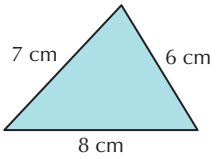
Exam tips

To construct an angle of 45°, construct an angle of 90° and then bisect the angle. Check any angles you bisect with a protractor.
The construction of a 90° angle at a given point is also in the specification but is not covered here.

Practice questions

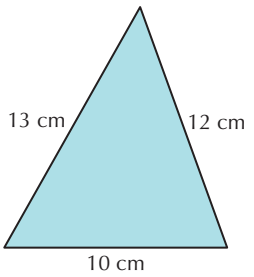
In the following questions, use a ruler and compasses for your constructions. You must show all your construction lines.

1 Construct this triangle.



2 **a** Construct this triangle.

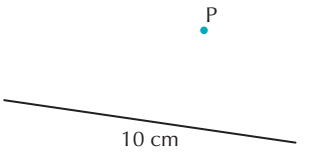
b Construct the perpendicular bisector of each side. (They should all meet at a point.)



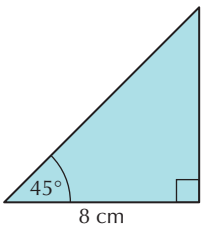
3 Draw a line about 10 cm long.

Mark a point, P, about 5 cm away from the line, as shown in the diagram.

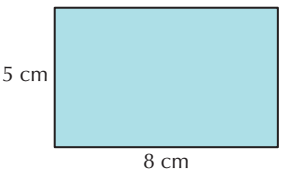
Construct a perpendicular from P to the line.



4 Construct this triangle accurately. Measure the hypotenuse of your triangle.



5 Construct this rectangle.

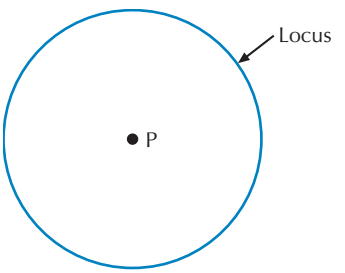


29.2 Loci

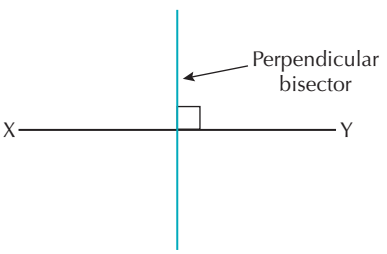
A **locus** is a set of points that satisfy a given condition or rule. The plural of locus is loci.

Types of loci

- The locus of the points that are a constant distance, *d*, from a fixed point, P, is the circumference of a circle, centre P and radius *d*.

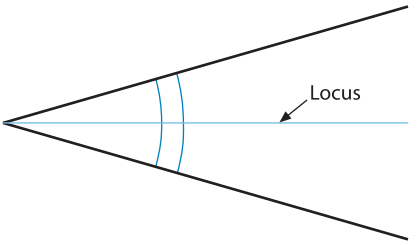


- The locus of the points that are equidistant from two points X and Y is the perpendicular bisector of the line XY.

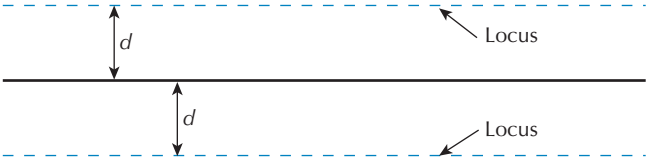


Remember that two lines are perpendicular when they meet at 90°. The perpendicular distance from a point to a line is the shortest distance to the line.

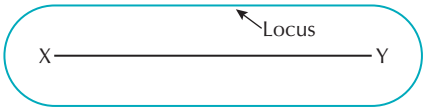
- The locus of the points that are equidistant from two intersecting lines is the line that bisects the angle between the lines.



- The locus of the points that are a constant distance, d, from a line is a pair of lines parallel to the given line, one on either side of it.



- The locus of the points that are a constant distance, d, from a line XY is two parallel lines at distance d from XY and a semicircle of radius d at each end.

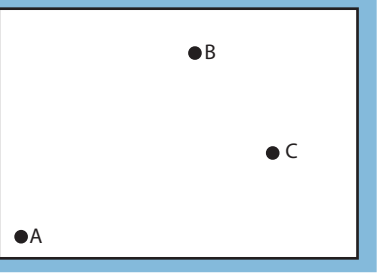


Example 4

Q The diagram shows a scale drawing of a garden with a scale of 1 cm : 2 m. A and B are bushes and C is a pond.

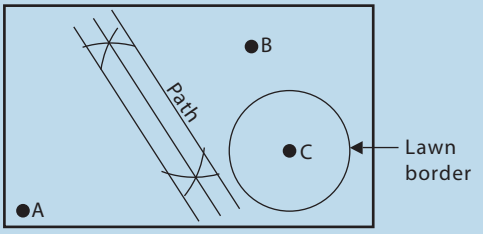
A landscape gardener has decided:

- to lay a path 1 m wide across the garden that is equidistant from the bushes, A and B.
- to lay a lawn around the pond which covers a distance of 2 metres from the centre of the pond. Construct these features on a copy of the plan.



A Draw the perpendicular bisector of the 'line' between the two bushes, and two parallel lines 0.25 cm either side.

Draw a circle of radius 1 cm around C.



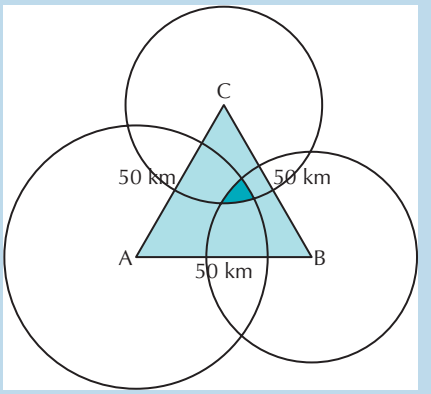
Note this diagram is not drawn to scale.

Example 5

Q Three radio transmitters form an equilateral triangle ABC with sides of 50 km. The ranges of the transmitters are: A 37.5 km, B 30 km and C 28 km. Draw a scale diagram showing the positions of the transmitters. Use a scale of 1 cm to 10 km.

On the scale diagram show, by shading, the region where signals from all three transmitters can be received.

A The area where signals from all three transmitters can be received is shaded dark blue.



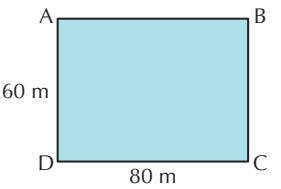
Note: a region is an area bounded by loci.

Practice questions

1 A path crosses a small rectangular field ABCD so that it is always equidistant from the sides AB and AD.

Make a scale drawing to show the field and construct the position of the path.

Use a scale of 1 cm to represent 10 m.



2 David has a vegetable patch.

He uses two sprinklers to water his vegetable patch during the growing season.

Each sprinkler can spray water up to 3 m.

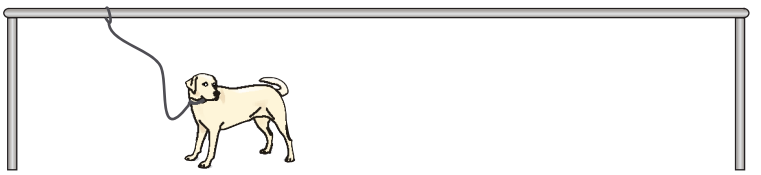
The vegetable patch is a rectangle 12 m by 10 m.

He wants to place his sprinklers so that the maximum possible area is watered.

Make a scale drawing to show where David could place the sprinklers.

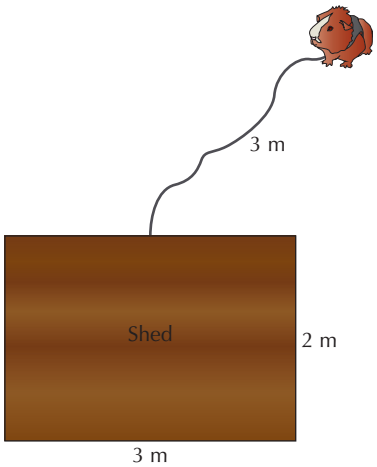
3 Emma ties her dog to a fence rail 10 m long, as shown. The 2-metre lead can slide along the length of the horizontal part of the rail.

Make a scale drawing to show the region in which the dog can move.



4 Dan ties his guinea pig to the centre of one side of his shed with a string 3 m in length.

Make a scale drawing to show the region in which the guinea pig can move.

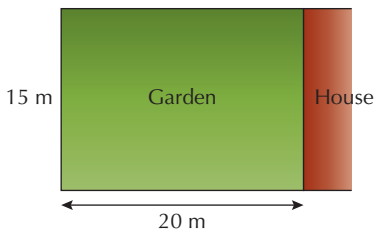


5 The diagram shows the back garden of a house.

Harry wants to plant a tree in the garden.

The tree must be more than 5 m from the back of the house and more than 8 m from the back corners of the garden.

Make a scale drawing of the garden and shade the region in which the tree could be planted.



REVISION CHECKLIST

- *Perpendicular* means at right angles to.
- *To bisect* means to cut in half.
- If you are asked to construct a shape or angle, use only a pencil, a ruler and a pair of compasses. Do not rub out your construction arcs.
- There are four main types of loci:
  - A constant distance from a fixed point is a circle.
  - Equidistant from two fixed points is the perpendicular bisector of the line segment joining the points.
  - Equidistant from two lines that intersect is the bisector of the angle between the lines.
  - A constant distance from a line is a pair of parallel lines above and below.

Exam tips If the line has a fixed length, remember to include the semi-circular ends.

Exam tips Unless a question states specifically that you must use ruler and compasses only for a construction, you may also use a protractor. These questions, which could also include constructing angles such as 38°, will include wording such as 'Make an accurate drawing of'.

Exam-style questions

Use a ruler and compasses for all constructions.

1 Construct a triangle with sides 7 cm, 8 cm and 9 cm.

2 Construct an equilateral triangle of side 6 cm.

3 Construct an angle of 30°. Do not use a protractor.

4 Draw a line, AB, 8 cm long. Then construct the perpendicular bisector of AB.

Hint

Construct two sides of an equilateral triangle and then bisect the angle.

5 The diagram shows the positions of two ships, A and B, 30 km apart.

A third ship, C, sails between them such that it is always equidistant from A and B.

On a copy of the diagram, draw accurately the path of ship C.

A •

• B

6 The side of a rhombus is 8 cm.

The shorter diagonal is 6 cm.

Construct the rhombus.

7 An isosceles triangle has two equal sides of 8 cm. The angle between the two equal sides is 45°.

Construct the triangle.

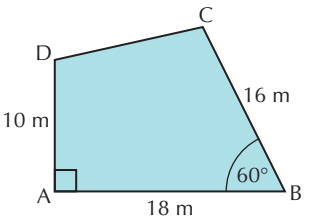
8 The diagram shows the plan of a garden.

AB = 18 m, BC = 16 m and AD = 10m.

Angle A = 90° and angle B = 60°.

a Make a scale drawing of the garden using 1 cm to represent 2 m.

b Use your diagram to find the actual length of DC.

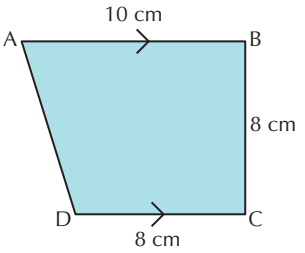


9 Construct an angle of 75°. Do not use a protractor.

Hint

75 = 45 + 30

10 ABCD is a trapezium with AB parallel to DC.



Draw the diagram accurately on squared paper.

Point P is: equidistant from AB and AD

less than 8 cm from C.

Mark the locus of where point P could be placed.

11 The diagram shows two orienteering checkpoints, C and D, 12 km apart.

Rachel is closer to checkpoint D than checkpoint C.

She is less than 7 km from checkpoint C.

Make a scale drawing to show the positions of C and D. Use a scale of 1 cm to 1 km.

Shade the region where Rachel could be.

C

D

12 The diagram shows the position of Peter, P, and a road.

Make a copy of the diagram and construct the shortest route that Peter can take to reach the road.

Road

P

13 The diagram shows the positions of three points, A, B and C.

Make a copy of the diagram with AB ≈ 10 cm, AC ≈ 7 cm and CB ≈ 5 cm.

a Construct the perpendicular bisector of AB.

b Shade the region that is less than 4 cm from C and closer to A than B.

A

B

C

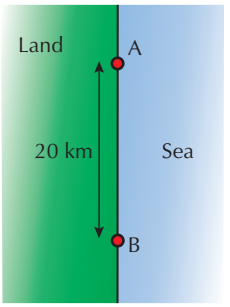
14 The diagram shows a section of coast with two rescue points, A and B, 20 km apart. A is due north of B.

a Make a scale drawing with a scale of 1 cm to 2 km.

The crew at rescue point A can rescue anyone within 10 km of A.

The crew at rescue point B can rescue anyone within 16 km of B.

b Shade the region where someone can be rescued by both crews.



15 **a** Construct a triangle with sides 10 cm, 8 cm and 6 cm.

b Construct the perpendicular bisector of the longest side.

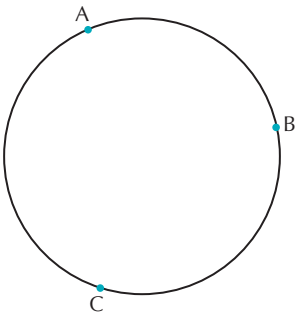
c Construct the perpendicular bisector of one of the other sides.

d The point of intersection of the perpendicular bisectors is the centre of a circle passing through all three vertices of the triangle. Draw this circle.

16 **a** Draw a circle with radius 5 cm. Mark three points on its circumference, A, B and C.

b Construct the perpendicular bisector of AB.

c Shade in the region inside the circle that is closer to B than A and less than 5 cm from C.



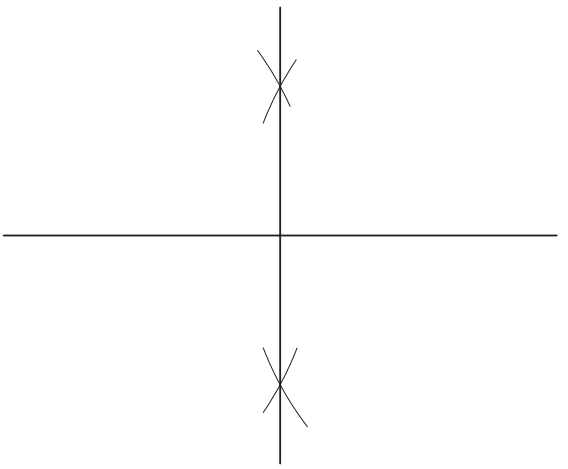
Now go back to the list of objectives at the start of this chapter.

How confident do you now feel about each of them?

Chapter 29 Stretch lesson: Answers

Check-in questions

1



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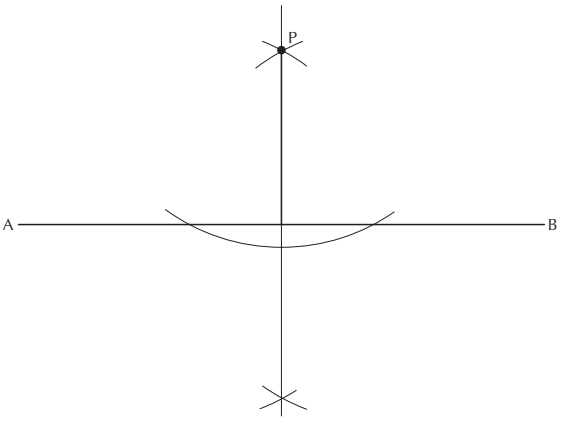

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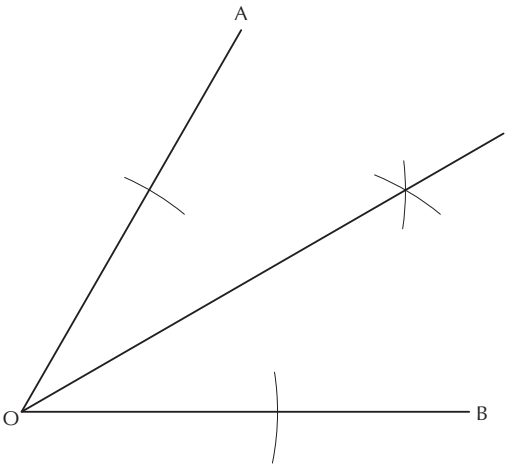

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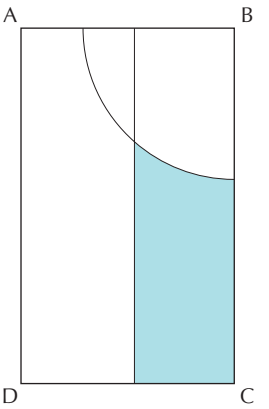

29.1 Constructions

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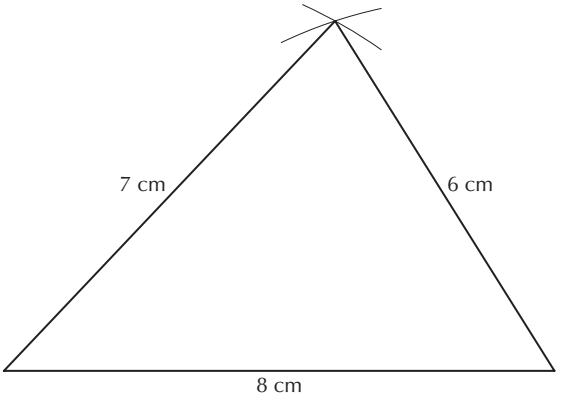

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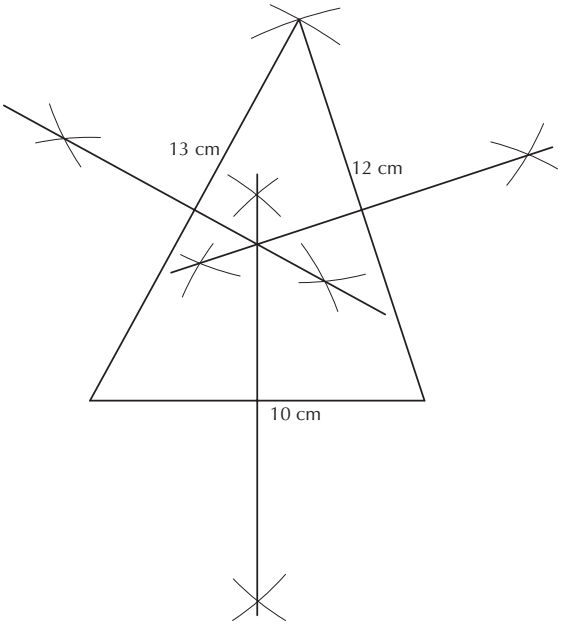

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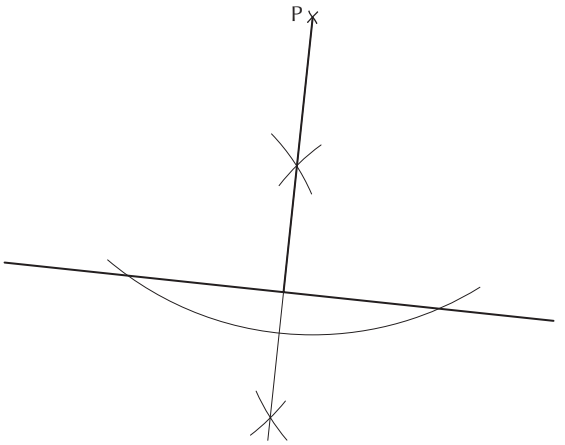
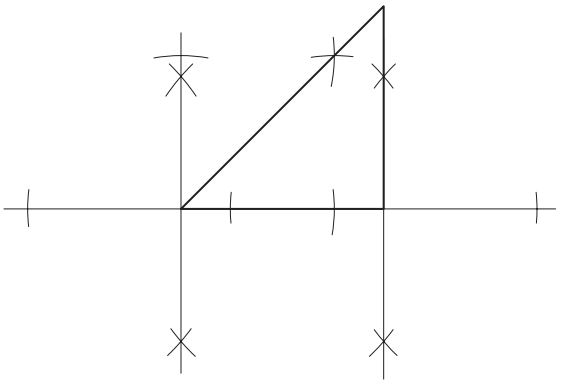


4



11.3 cm

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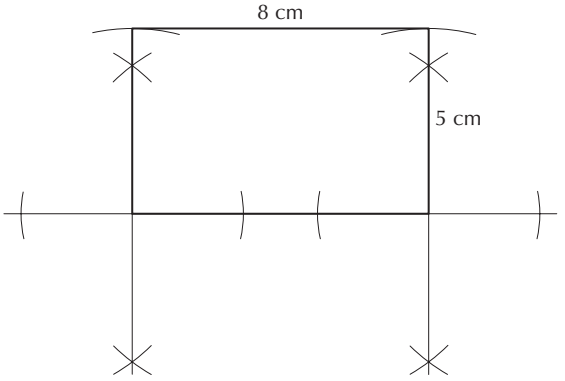

29.2 Loci

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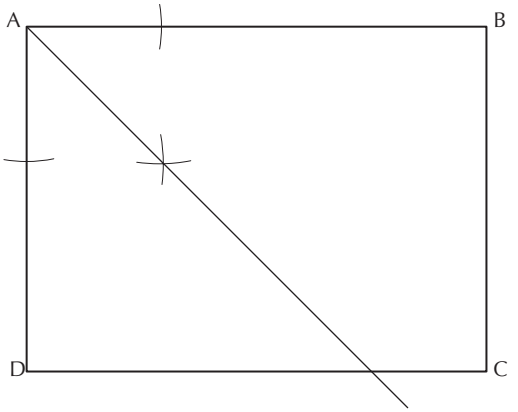

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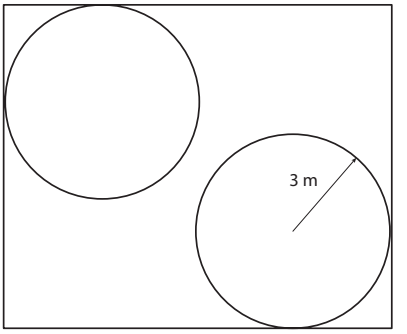

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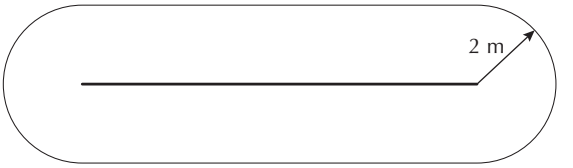

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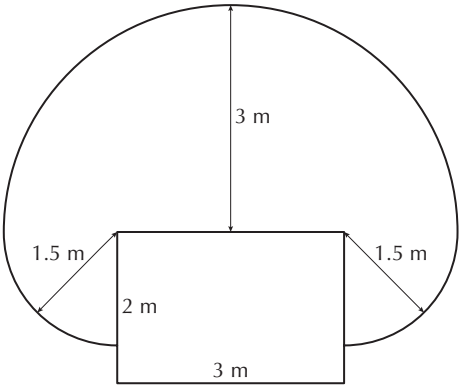

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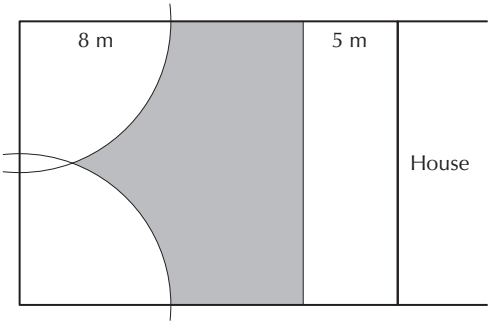

Exam-style questions

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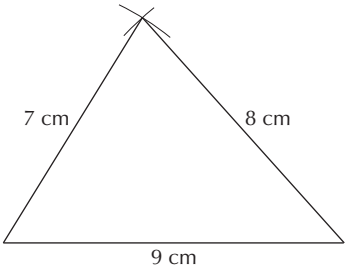

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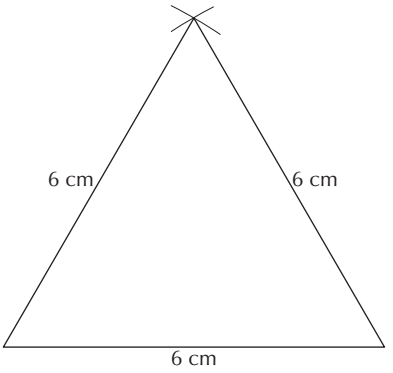

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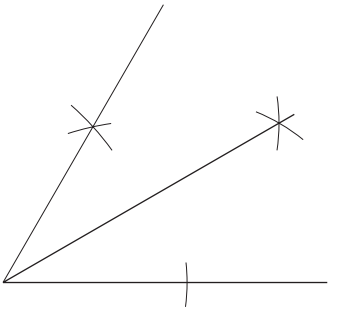

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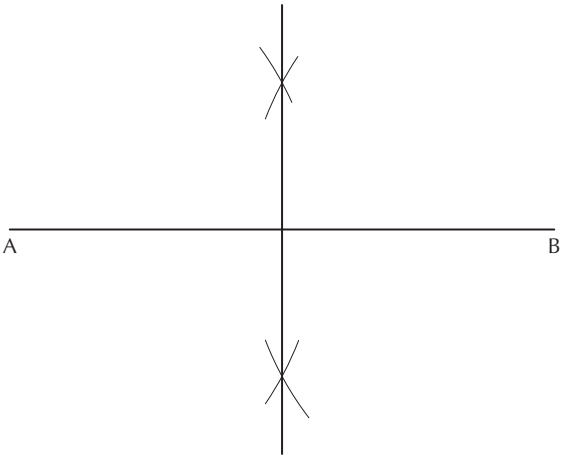

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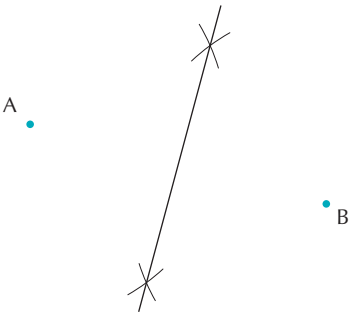

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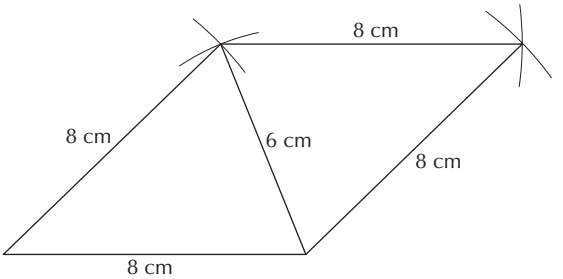

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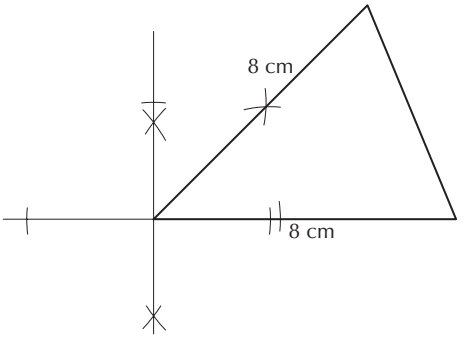

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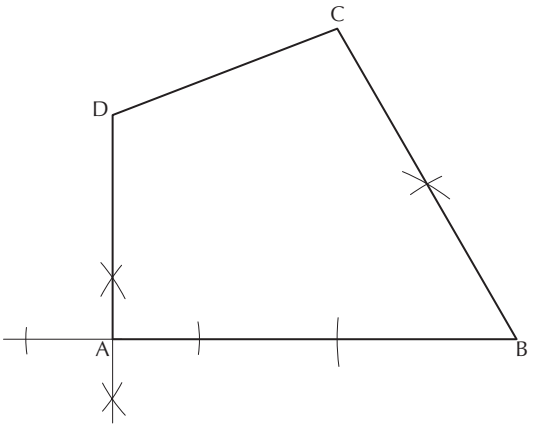

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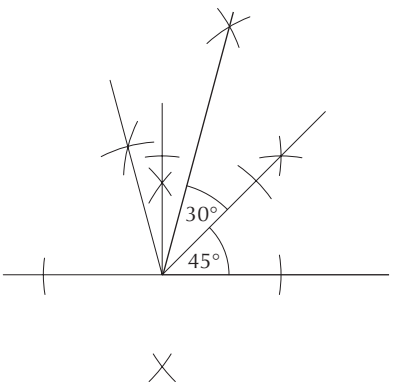

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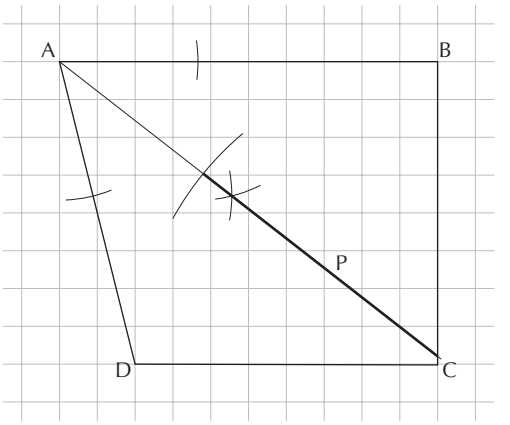

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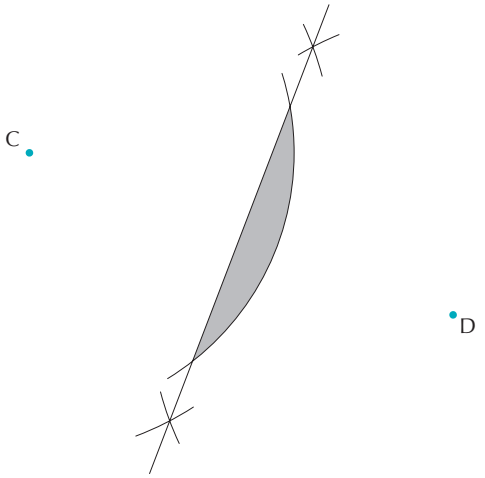

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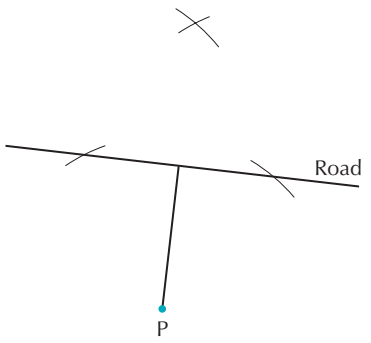

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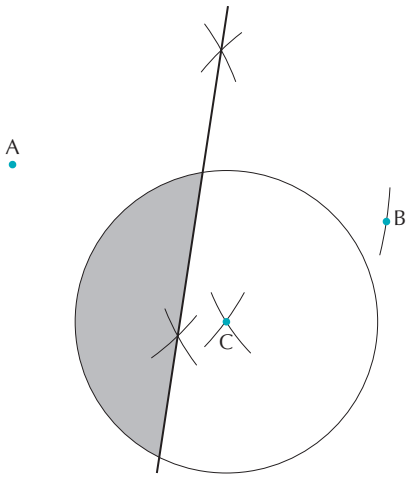

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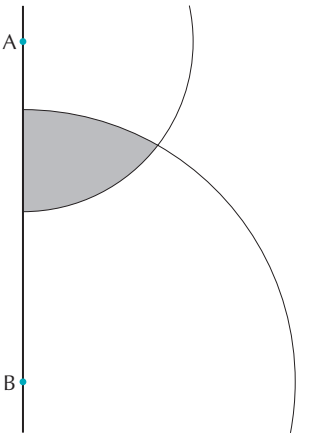

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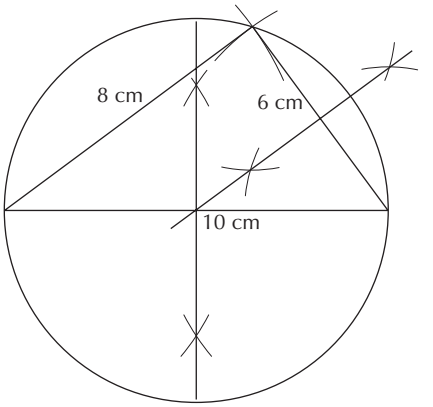

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