

# 28

## Stretch lesson: Simultaneous linear equations

### Stretch objectives

Before you start this chapter, mark how confident you feel about each of the statements below:

I can write simultaneous equations to represent a situation.

I can solve two linear simultaneous equations algebraically.

I can solve two linear simultaneous equations graphically.

I can solve simultaneous linear equations representing a real-life situation and interpret the solution.

	▶	▶▶	▶▶▶
I can write simultaneous equations to represent a situation.			
I can solve two linear simultaneous equations algebraically.			
I can solve two linear simultaneous equations graphically.			
I can solve simultaneous linear equations representing a real-life situation and interpret the solution.			

### Check-in questions

- Complete these questions to assess how much you remember about each topic. Then mark your work using the answers at the end of the lesson.
- If you score well on all sections, you can go straight to the Revision Checklist and Exam-style Questions at the end of the lesson. If you don't score well, go to the lesson section indicated and work through the examples and practice questions there.

1 Solve these simultaneous equations.

a  $4b + 7a = 10$

b  $5a - 2b = 19$

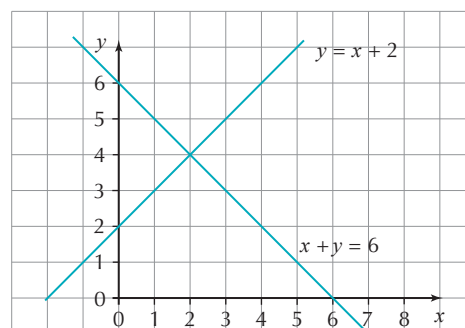
$2b + 3a = 3$

$3a + 4b = 1$

Go to 28.1

2 The graphs of  $x + y = 6$  and  $y = x + 2$  are plotted on the same axes.

Use the graphs to solve this pair of simultaneous equations. Go to 28.2



3 Sahid has lots of sports equipment in different bags. One bag contains two footballs and three cricket balls, which have a total mass of 1360 g. A second bag contains three footballs and two cricket balls, which have a total mass of 1640 g. Work out the mass of one cricket ball.

Go to 28.3

# 28.1 Solving simultaneous equations algebraically

**Simultaneous equations** in two variables are equations that are both true for the same pair of variables. You can solve simultaneous equations using algebraic methods or by using a graph.

In straightforward examples, the coefficients of one of the variables will be the same in both equations so you can eliminate one of the variables by adding or subtracting the equations. Otherwise you will need to multiply one or both of the original equations by a constant before you can use the elimination method.

The example shows how to use the elimination method when there are no matching coefficients.

**Example 1** Q Solve this pair of simultaneous equations.

$$3x + 2y = 8$$

$$2x - 3y = 14$$

**A**  $3x + 2y = 8$  (1)

$$2x - 3y = 14$$
 (2)

$$6x + 4y = 16$$
 (3)

$$6x - 9y = 42$$
 (4)

First, label the equations.

Since no coefficients match, multiply equation (1) by 2 and equation (2) by 3 and rename them (3) and (4).

The coefficient of  $x$  in equations (3) and (4) is the same, so you can now subtract them and solve to find the value of  $y$ .

$$0x + 13y = -26$$

$$4y - (-9y) = 4y + 9y$$

$$y = -26 \div 13$$

$$= -2$$

Substitute the value of  $y$  ( $= -2$ ) into equation (1) to find the value of  $x$ .

$$3x + 2 \times (-2) = 8$$

You could also substitute into equation (2).

$$3x + (-4) = 8$$

$$3x = 8 + 4$$

$$3x = 12$$

$$x = 4$$

Check in equation (2):  $(2 \times 4) - (3 \times -2) = 14$  ✓

So, the solution is  $x = 4, y = -2$ .

## Exam tips

Always check that your solutions are correct by substituting them into the other equation.

## Practice questions

**1** Solve these simultaneous equations using algebra. Remember to check your answers.

**a**  $x + y = 5$

$$x - y = 1$$

**b**  $4x + 3y = 16$

$$x + 3y = 13$$

**c**  $x + 2y = 8$

$$x - y = -1$$

- 2 Solve the simultaneous equations algebraically.
- |                |                |                 |
|----------------|----------------|-----------------|
| a $2x + y = 2$ | b $x + 2y = 4$ | c $3x + 2y = 5$ |
| $4x + y = 3$   | $2x + 2y = 7$  | $9x - y = 1$    |
- 3 Solve the pairs of simultaneous equations using the elimination method.
- |                   |               |                |
|-------------------|---------------|----------------|
| a $6x + 12y = 36$ | b $x + y = 2$ | c $4x - y = 2$ |
| $x + y = 2$       | $2x + 3y = 9$ | $x + y = 3$    |
- 4 Solve these simultaneous equations.
- |                 |                  |                |
|-----------------|------------------|----------------|
| a $2x - 3y = 5$ | b $4x - 5y = 16$ | c $4x - y = 0$ |
| $x + y = -5$    | $x + y = -5$     | $2x + y = 1$   |
- 5 Solve each pair of simultaneous equations.
- |                 |                   |                   |
|-----------------|-------------------|-------------------|
| a $5p + 2q = 0$ | b $2x + 2y = 0.4$ | c $2x - 3y = 2.1$ |
| $10p - 3q = 7$  | $3x - 5y = -3.4$  | $5x + 7y = 2.35$  |

## 28.2 Solving simultaneous equations graphically

When you plot two graphs on the same pair of axes, the point of **intersection** (where they cross) represents the solution of the simultaneous equations.

**Example 2** Q Solve these simultaneous equations graphically.

$$2x + 3y = 6$$

$$x + y = 1$$

**A** Draw the graph of  $2x + 3y = 6$ .

When  $x = 0$ ,  $3y = 6$  so  $y = 2$ : (0, 2)

When  $y = 0$ ,  $2x = 6$  so  $x = 3$ : (3, 0)

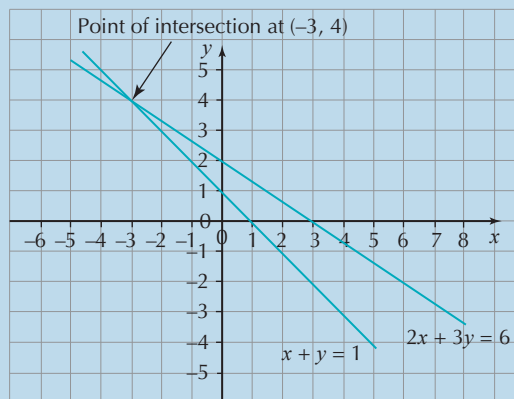
Draw the graph of  $x + y = 1$ .

When  $x = 0$ ,  $y = 1$ : (0, 1)

When  $y = 0$ ,  $x = 1$ : (1, 0)

The lines intersect at  $(-3, 4)$ .

The simultaneous solution of the equations is  $x = -3, y = 4$ .

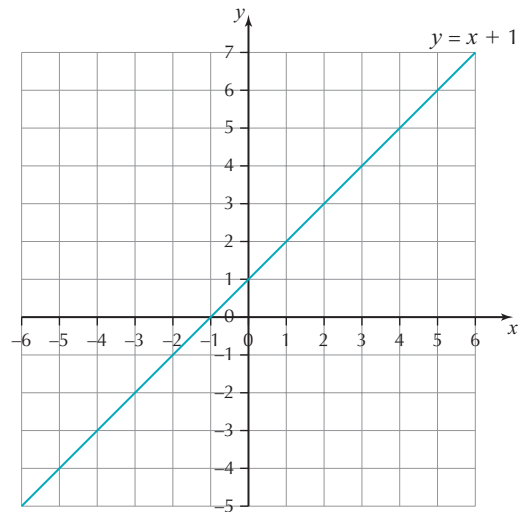


## Practice questions

- 1 The graph of  $y = x + 1$  is shown on the axes.

Set up a table of values for the equation  $y = 3x - 1$ , then draw the graph on a copy of the axes.

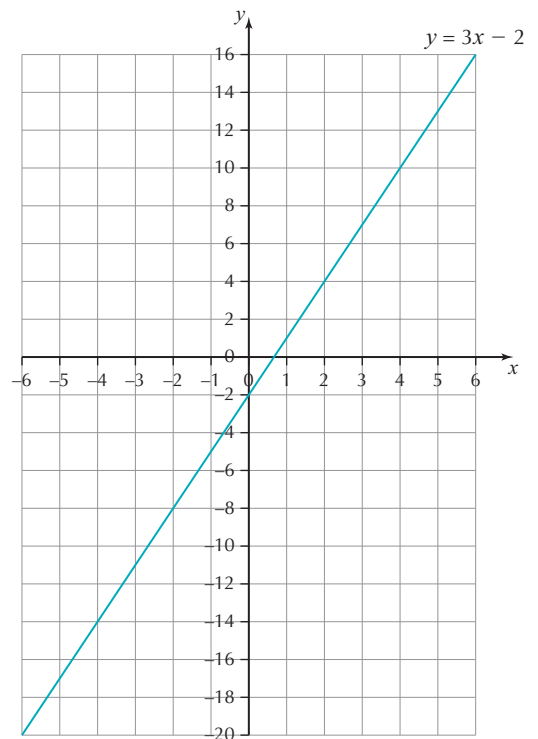
Using your graph, find the solution of the simultaneous equations.



- 2 The graph of  $y = 3x - 2$  is shown on the axes.

Draw the graph of  $y = 2x + 1$  on a copy of the grid.

Using your graph, find the simultaneous solution of the equations.



- 3 Draw the graphs of  $y - 4x = 1$  and  $y - x = 4$  on the same axes. Solve the equations simultaneously by reading the coordinates of the point of intersection from your graph.

### Hint

Rearrange each equation before you set up a table of values.

4 Solve each pair of simultaneous equations graphically.

a  $y = 2x + 1$

b  $y = -2x + 7$

$y = -x + 7$

$2y = x + 4$

5 Solve these simultaneous equations graphically.

a  $x + y = 5$

b  $2x + y = 6$

c  $2x - y = 0$

$x - y = 1$

$5x - y = 1$

$2x + 5y = 7$

## 28.3 Solving problems using simultaneous equations

Simple real-life problems can be solved by forming and solving a pair of simultaneous equations.

Example  
3

**Q** Tracy and Ian are organising a party. Tracy paid £22 for five hats and four balloons from the toy shop. She found that she didn't have enough, so Ian went back and bought three of the same hats and five of the same balloons. He paid £21.

Work out the cost of one hat and the cost of one balloon.

Use  $h$  to represent the cost of a hat and  $b$  to represent the cost of a balloon.

**A** First set up two equations and label them.

$$5h + 4b = 22 \quad (1)$$

$$3h + 5b = 21 \quad (2)$$

Multiply, since none of the coefficients are the same.

$$15h + 12b = 66 \quad (3) \quad \text{---} \quad (1) \times 3$$

$$15h + 25b = 105 \quad (4) \quad \text{---} \quad (2) \times 5$$

Subtract to eliminate the  $15h$ .

$$13b = 39 \quad \text{---} \quad (4) - (3)$$

$$b = 3$$

Substitute  $b = 3$  into (1).

$$5h + (4 \times 3) = 22$$

$$5h = 10$$

$$h = 2$$

Check in (2).

$$3 \times 2 + 5 \times 3 = 21$$

So one hat costs £2 and one balloon costs £3.

**Exam tips**

When setting up your own simultaneous equations, use appropriate letters for each of the items, for example,  $c$  for cakes and  $m$  for muffins. Remember to interpret your results at the end of the question.

## Practice questions

- 1 A café sells a total of 300 cakes and muffins for £870. The cost of a cake is £3.50 and the cost of a muffin is £2.50. How many cakes did they sell?
- 2 Mandy is selling hot dogs and burgers at a village fair.  
Tom pays £8.50 for three hot dogs and two burgers.  
Mick pays £8 for four hot dogs and one burger.
  - a Find the cost of one hot dog.
  - b Find the cost of one burger.
- 3 Anne sells homemade shawls and scarves. She sells one shawl and two scarves for £17. She sells two shawls and six scarves for £43.  
How much does one scarf cost?
- 4 Elliot does MOTs and fits brake pads. He is writing his schedule for the week.

<u>Day</u>	<u>MOTs</u>	<u>Brake pads</u>	<u>Time allowed</u>
Monday	3	4	5 hours 40 minutes
Tuesday	5	2	6 hours 20 minutes

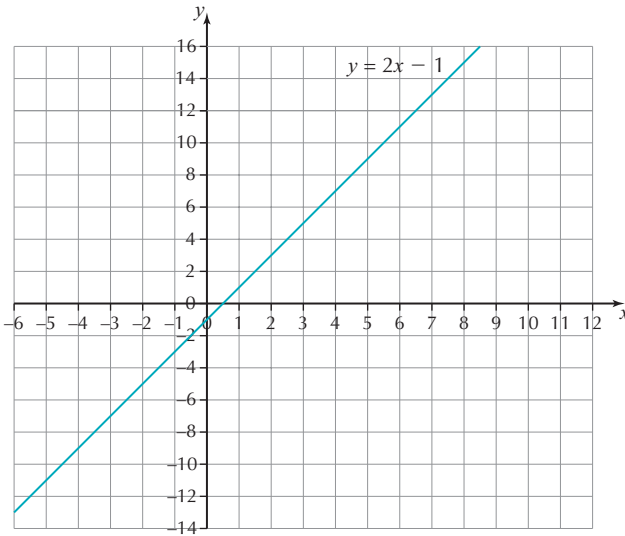
- a How much time does Elliot allow to fit one set of brake pads?
  - b How long will Mrs Thomson have to wait for Elliot to fit one set of brake pads and MOT her car?
- 5 Liz runs a dog walking service. She offers individual short walks or long walks. Two short walks and three long walks take her 2 hours 30 minutes. Three short walks and five long walks take her 4 hours and 5 minutes.  
How long will she need for one short walk and four long walks?

## REVISION CHECKLIST

- When you solve a pair of linear simultaneous equations, you need to find the values of both unknown variables.
- You can use the elimination method to eliminate one of the variables in a pair of simultaneous equations.
- The coordinates of the point of intersection of two graphs give the solution of the simultaneous equations.

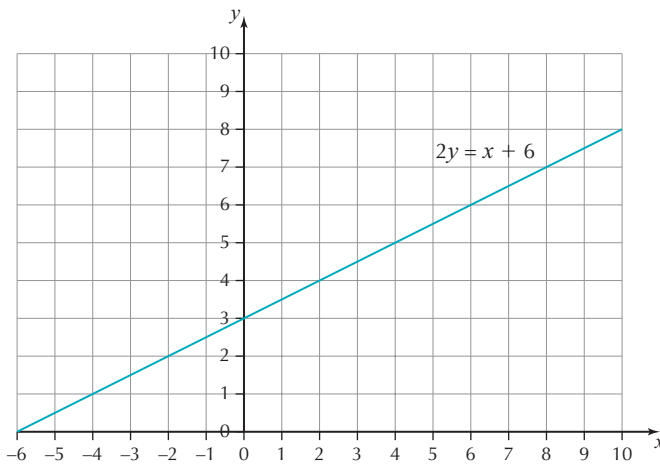
# Exam-style questions

- 1 Draw graphs to solve the simultaneous equations  $2x + y = 7$  and  $y - x = 1$ .
- 2 The graph of  $y = 2x - 1$  is plotted on the axes below.



Use the graph to help you solve the simultaneous equations  $2x - y = 1$  and  $x + y = 14$ .

- 3 The graph of  $2y = x + 6$  is shown on the axes.



Use the graph to help you solve the simultaneous equations  $x - 2y = -6$  and  $x + y = 9$ .

- 4 Solve the simultaneous equations  $y = x + 1$  and  $x + y = 7$  graphically.
- 5 Solve the simultaneous equations  $x + 4y = 8$  and  $x + y = 5$  graphically.
- 6 Solve these simultaneous equations.  
 $x + y = 5$   
 $2x + 3y = 12$
- 7 The sum of two numbers is 13. The difference between the same two numbers is 3. What are the two numbers?
- 8 Solve these simultaneous equations.  
 $x + 3y = 9$   
 $x + y = 6$

- 9 The sum of two numbers is 23. The difference between the same two numbers is 7. What are the two numbers?
- 10 Georgia buys two cans of cola and a chocolate bar for £1.60.  
Marcus buys three cans of cola and two chocolate bars for £2.55.  
How much will Jen pay for one can of cola and four chocolate bars?
- 11 Solve these simultaneous equations.  $x + y = 2$   
 $y - x = 6$
- 12 Sami buys 3 DVDs and 1 CD for £6.80.  
Dougie buys 5 DVDs and 3 CDs for £12.40.  
How much will Angie pay for 2 DVDs and 2 CDs?
- 13 Solve these simultaneous equations.  $2x + y = 2$   
 $3y - 2x = 2$
- 14 The mean of two numbers is 8. The difference between the same two numbers is 4. Find the two numbers.
- 15 Harriet is selling second-hand books. She sells four paperback books and three hardback books for £2.60. She sells seven paperback books and eight hardback books for £6.20. How much will she charge for five paperback books and four hardback books?
- 16 Iannis teaches music. On Wednesday he teaches five guitar lessons and two theory lessons; this takes 1 hour 35 minutes. On Thursday he teaches three guitar lessons and ten theory lessons; this takes 2 hours 25 minutes. On Friday he expects to teach four guitar lessons and six theory lessons. How much time will this take?
- 17 The mean of two numbers is 9. The difference between the same two numbers is 2. Find the two numbers.
- 18 Bernice paid £20.10 for three large candles and eight small candles.  
Pauline paid £20.20 for two large candles and twelve small candles.  
How much will Claire pay for five large candles and two small candles?
- 19 Gunna works in a nursing home. She styles hair and paints nails for the female residents.  
On Monday, she styles three lots of hair and paints five sets of nails. This takes her 4 hours 20 minutes.  
On Tuesday, she styles five lots of hair and paints three sets of nails. This takes her 5 hours.  
She has been asked to style two lots of hair and paint four sets of nails on Thursday. If she arrives at the nursing home at 9 a.m., at what time can she expect to leave?
- 20 Alan works in a print shop. It takes 136 minutes 40 seconds to print 400 flyers and 1500 leaflets. It takes 90 minutes 20 seconds to print 240 flyers and 1000 leaflets.  
How much time will it take to print 500 flyers and 270 leaflets?

Now go back to the list of objectives at the start of this chapter.  
How confident do you now feel about each of them?



# Chapter 28 Stretch lesson: Answers

## Check-in questions

- 1 **a**  $a = 4, b = -4.5$       **b**  $a = 3, b = -2$   
2  $x = 2, y = 4$   
3 160 g

## 28.1 Solving simultaneous equations algebraically

- 1 **a**  $x = 3, y = 2$       **b**  $x = 1, y = 4$   
**c**  $x = 2, y = 3$   
2 **a**  $x = 0.5, y = 1$       **b**  $x = 3, y = 0.5$   
**c**  $x = \frac{1}{3}, y = 2$   
3 **a**  $x = -2, y = 4$       **b**  $x = -3, y = 5$   
**c**  $x = 1, y = 2$   
4 **a**  $x = -2, y = -3$       **b**  $x = -1, y = -4$   
**c**  $x = \frac{1}{6}, y = \frac{2}{3}$   
5 **a**  $p = \frac{2}{5}, q = -1$       **b**  $x = -0.3, y = 0.5$   
**c**  $x = 0.75, y = -0.2$

## 28.2 Solving simultaneous equations graphically

- 1  $x = 1, y = 2$   
2  $x = 3, y = 7$   
3  $x = 1, y = 5$   
4 **a**  $x = 2, y = 5$       **b**  $x = 2, y = 3$   
5 **a**  $x = 3, y = 2$       **b**  $x = 1, y = 4$   
**c**  $x = \frac{7}{12}, y = \frac{7}{6}$

## 28.3 Solving problems using simultaneous equations

- 1 120 cakes  
2 Cost of one hot dog £1.50, cost of one burger £2  
3 £4.50  
4 **a** 40 minutes      **b** 1 hour 40 minutes  
5 2 hours 55 minutes

## Exam-style questions

- 1  $x = 2, y = 3$   
2  $x = 5, y = 9$   
3  $x = 4, y = 5$   
4  $x = 3, y = 4$   
5  $x = 4, y = 1$   
6  $x = 3, y = 2$   
7 5 and 8  
8  $x = 4.5, y = 1.5$   
9 8 and 15  
10 £1.85  
11  $x = -2, y = 4$   
12 £5.60  
13  $x = 0.5, y = 1$   
14 6 and 10  
15 £3.40  
16 2 hours  
17 8 and 10  
18 £21.94  
19 12:10 p.m.  
20 37 minutes 5 seconds