22 Stretch lesson: Vectors

Stretch objectives

Before you start this chapter, mark how confident you feel about each of the statements below:	▶	▶▶	▶▶▶
I can calculate using column vectors and represent the sum and difference of two vectors graphically.			
I can identify column vectors that are parallel.			

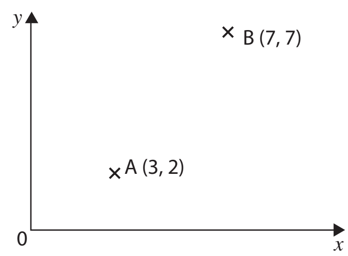
Check-in questions

- Complete these questions to assess how much you remember about each topic. Then mark your work using the answers at the end of the lesson.
- If you score well on all sections, you can go straight to the Revision Checklist and Exam-style Questions at the end of the lesson. If you don't score well, go to the lesson section indicated and work through the examples and practice questions there.

1 In the diagram, A is the point (3, 2) and B is the point (7, 7).

a Write down the vector $\overrightarrow{AB}$ as a column vector $\begin{pmatrix} x \\ y \end{pmatrix}$.

b Write down the coordinates of point C such that $\overrightarrow{BC} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$.



Go to 22.1

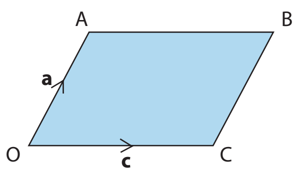
2 OABC is a parallelogram.

AB is parallel to OC.

OA is parallel to CB.

$\overrightarrow{OA} = \mathbf{a}$

$\overrightarrow{OC} = \mathbf{c}$



a Express each vector in terms of **a** and **c**.

i $\overrightarrow{AC}$ **ii** $\overrightarrow{BO}$

N is the midpoint of $\overrightarrow{AC}$.

b Express $\overrightarrow{ON}$ in terms of **a** and **c**.

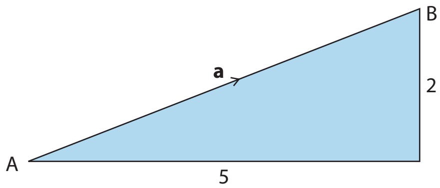
Go to 22.1

22.1 Vectors

- A **scalar** quantity can be fully described by its magnitude or size, for example, temperature. A **vector** quantity is fully described by its magnitude and direction, for example, velocity has a magnitude (called speed) and a direction.
- Four types of notation can be used to represent vectors. The vector indicated by an arrow in the shape below can be written as:

  $\begin{pmatrix} 5 \\ 2 \end{pmatrix}$ *or* $\mathbf{a}$ *or* $\overrightarrow{AB}$ *or* $\underline{a}$

- The direction of the vector is usually shown by an arrow.

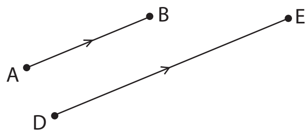


- If $\overrightarrow{DE} = k\overrightarrow{AB}$, then $\overrightarrow{AB}$ and $\overrightarrow{DE}$ are parallel and the length of $\overrightarrow{DE}$ is k times the length of $\overrightarrow{AB}$.

  $\overrightarrow{AB} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$

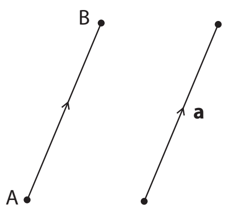
  $\overrightarrow{DE} = \begin{pmatrix} 10 \\ 4 \end{pmatrix} = 2\begin{pmatrix} 5 \\ 2 \end{pmatrix}$

  $\overrightarrow{DE} = 2\overrightarrow{AB}$

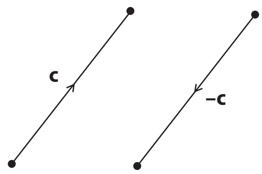


- If two vectors are equal, they are parallel and equal in length.

  $\overrightarrow{AB}$ is equal to **a**.

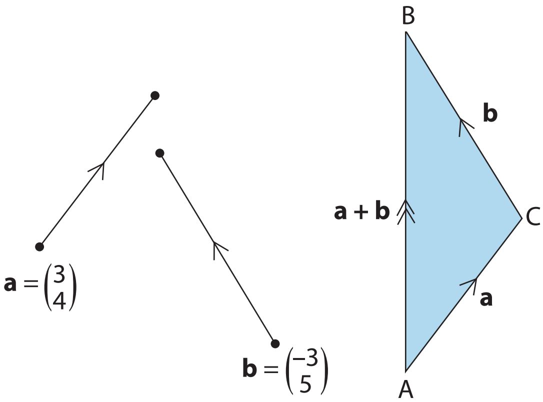


- If the vector $\mathbf{c} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$, then the vector $-\mathbf{c}$ is in the opposite direction to $\mathbf{c}$, so $-\mathbf{c} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$. The column vector $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$ means 3 units to the left and 4 units down.



Adding vectors

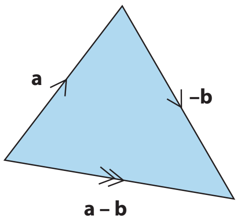
The **resultant** of two vectors is found by adding them. Vectors must always be added end to end so that the arrows follow on from each other. A resultant is usually labelled with a double arrow.



To take the route directly from A to B is equivalent to travelling from A to C and then from C to B. The starting points and finishing points are the same. This means $\overrightarrow{AB} = \mathbf{a} + \mathbf{b}$.

$$\mathbf{a} + \mathbf{b} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} -3 \\ 5 \end{pmatrix} = \begin{pmatrix} 0 \\ 9 \end{pmatrix}$$

Subtracting vectors



$\mathbf{a} - \mathbf{b}$ can be interpreted as $\mathbf{a} + (-\mathbf{b})$.

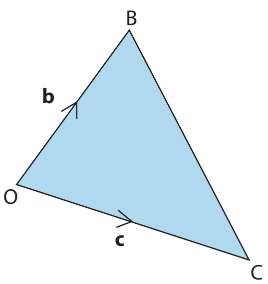
$$\mathbf{a} + (-\mathbf{b}) = \begin{pmatrix} 3 \\ 4 \end{pmatrix} + \begin{pmatrix} 3 \\ -5 \end{pmatrix}$$

Therefore $\mathbf{a} - \mathbf{b} = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$

Position vectors

The position vector of a point B is the vector $\overrightarrow{OB}$, where O is the origin.

In the diagram, the position vectors of B and C are **b** and **c** respectively.



Using this notation, $\overrightarrow{BC} = -\mathbf{b} + \mathbf{c}$

$= \mathbf{c} - \mathbf{b}$

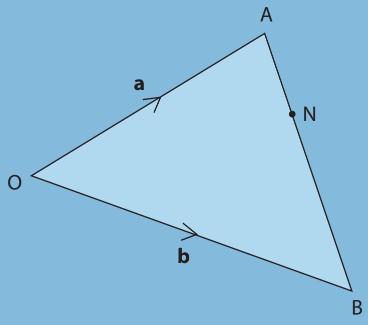
Exam tips

On an exam paper, vectors represented by lower case letters will be shown in bold. In your working, you are expected to underline lower case letters representing vectors, for example, a̲.
Always put arrows to show direction on lines that represent vectors.

Example 1

Q OAB is a triangle. N splits $\overrightarrow{AB}$ in the ratio 1 : 2. $\overrightarrow{OA}$ = **a** and $\overrightarrow{OB}$ = **b**.

Express, in terms of **a** and **b**, the vectors **i** $\overrightarrow{AB}$ **ii** $\overrightarrow{ON}$.



A **i** $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$ — Go from A to B via O.

$= -\mathbf{a} + \mathbf{b}$

ii $\overrightarrow{ON} = \overrightarrow{OA} + \overrightarrow{AN}$ — $\overrightarrow{AN} = \frac{1}{3}\overrightarrow{AB}$

$= \mathbf{a} + \frac{1}{3}(-\mathbf{a} + \mathbf{b})$

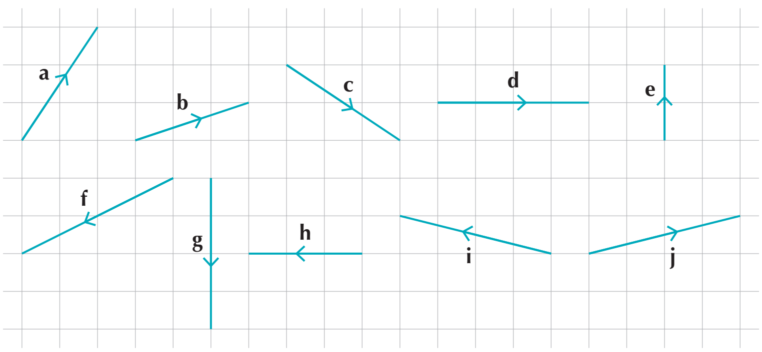
$= \mathbf{a} - \frac{1}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

$= \frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$

$= \frac{1}{3}(2\mathbf{a} + \mathbf{b})$ — Factorise the expression.

Practice questions

1 Express each of these as a column vector.

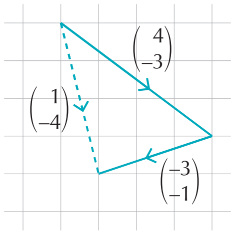


2 Draw a diagram on squared paper to represent each vector.

$\mathbf{k} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\mathbf{l} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$ $\mathbf{m} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ $\mathbf{n} = \begin{pmatrix} -2 \\ 0 \end{pmatrix}$ $\mathbf{p} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$ $\mathbf{r} = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

3 Add the column vectors and draw a triangle on a grid to show the resultant.
The first one is done as an example.

a $\begin{pmatrix} 4 \\ -3 \end{pmatrix} + \begin{pmatrix} -3 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ -4 \end{pmatrix}$



b $\begin{pmatrix} -3 \\ -1 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

c $\begin{pmatrix} 3 \\ 2 \end{pmatrix} + \begin{pmatrix} -1 \\ 4 \end{pmatrix}$

d $\begin{pmatrix} -3 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

4 $\mathbf{p} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} -1 \\ 3 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$

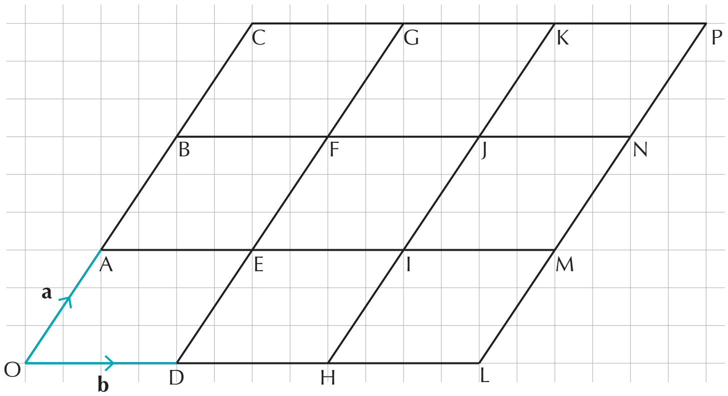
a Draw a vector diagram on a grid to represent each of these.

i $\mathbf{p}$ **ii** $\mathbf{q}$ **iii** $\mathbf{r}$ **iv** $\mathbf{p} + \mathbf{q}$ **v** $2\mathbf{r}$ **vi** $2\mathbf{r} + \mathbf{p}$

b Write each resultant as a single column vector.

i $\mathbf{p} + \mathbf{r}$ **ii** $\mathbf{q} + \mathbf{r}$ **iii** $\mathbf{p} - \mathbf{q}$ **iv** $\mathbf{q} - \mathbf{r}$
v $2\mathbf{p}$ **vi** $3\mathbf{q}$ **vii** $-\mathbf{r}$ **viii** $3\mathbf{q} - \mathbf{r}$

5



$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OD} = \mathbf{b}$

a Write down the resultant of each of these.
The first one is done as an example.

i $\mathbf{a} + \mathbf{b} = \overrightarrow{OE}$ **ii** $2\mathbf{a} + \mathbf{b}$ **iii** $\mathbf{a} + 2\mathbf{b}$ **iv** $3\mathbf{a} + \mathbf{b}$ **v** $\mathbf{a} + 3\mathbf{b}$

b Write each of these vectors in terms of **a** and/or **b**.

i $\overrightarrow{OH}$ **ii** $\overrightarrow{OC}$ **iii** $\overrightarrow{OI}$ **iv** $\overrightarrow{OJ}$ **v** $\overrightarrow{ON}$

REVISION CHECKLIST

- The direction of a vector is shown with an arrow.
- The resultant of two vectors is found by vector addition or subtraction. Vectors must always be combined end to end.

Exam-style questions

1 $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Work out each of these:

i $\mathbf{a} - \mathbf{b}$ **ii** $2\mathbf{a} - 3\mathbf{b}$ **iii** $\mathbf{a} + 2\mathbf{b}$

2 $\mathbf{a} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

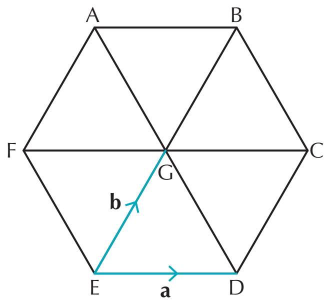
Work out:

i $\mathbf{a} + \mathbf{b}$ **ii** $\mathbf{a} - \mathbf{b}$ **iii** $2\mathbf{a} - 3\mathbf{b}$

3 The diagram shows a regular hexagon, ABCEFG with centre D.

Express these vectors in terms of **a** and **b**.

a $\overrightarrow{FB}$ **b** $\overrightarrow{FE}$ **c** $\overrightarrow{GD}$ **d** $\overrightarrow{FA}$



Chapter 22 Stretch lesson: Answers

Check-in questions

1 **a** $\begin{pmatrix}4\\5\end{pmatrix}$ **b** C(5, 4)

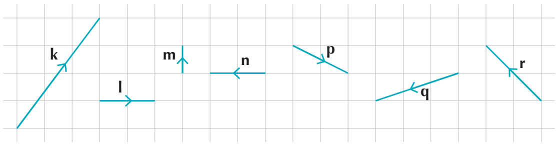
2 **a** **i** $\mathbf{c} - \mathbf{a}$ **ii** $-\mathbf{a} - \mathbf{c}$ **b** $\frac{1}{2}(\mathbf{a} + \mathbf{c})$

22.1 Vectors

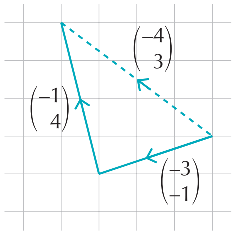
1 $\mathbf{a} = \begin{pmatrix}2\\3\end{pmatrix}$ $\mathbf{b} = \begin{pmatrix}3\\1\end{pmatrix}$ $\mathbf{c} = \begin{pmatrix}3\\-2\end{pmatrix}$ $\mathbf{d} = \begin{pmatrix}4\\0\end{pmatrix}$ $\mathbf{e} = \begin{pmatrix}0\\2\end{pmatrix}$

$\mathbf{f} = \begin{pmatrix}-4\\-2\end{pmatrix}$ $\mathbf{g} = \begin{pmatrix}0\\-4\end{pmatrix}$ $\mathbf{h} = \begin{pmatrix}-3\\0\end{pmatrix}$ $\mathbf{i} = \begin{pmatrix}-4\\1\end{pmatrix}$ $\mathbf{j} = \begin{pmatrix}4\\1\end{pmatrix}$

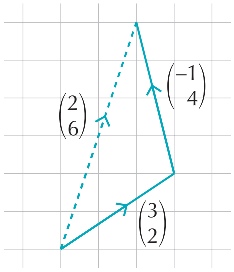
2



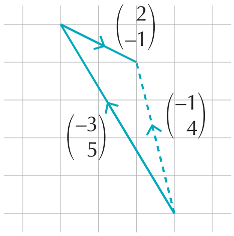
3 **b** $\begin{pmatrix}-3\\-1\end{pmatrix} + \begin{pmatrix}-1\\4\end{pmatrix} = \begin{pmatrix}-4\\3\end{pmatrix}$



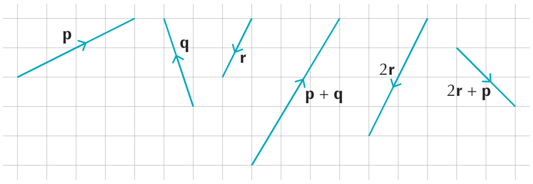
c $\begin{pmatrix}3\\2\end{pmatrix} + \begin{pmatrix}-1\\4\end{pmatrix} = \begin{pmatrix}2\\6\end{pmatrix}$



d $\begin{pmatrix}-3\\5\end{pmatrix} + \begin{pmatrix}2\\-1\end{pmatrix} = \begin{pmatrix}-1\\4\end{pmatrix}$



4 **a**



b **i** $\begin{pmatrix}3\\0\end{pmatrix}$ **ii** $\begin{pmatrix}-2\\1\end{pmatrix}$ **iii** $\begin{pmatrix}5\\-1\end{pmatrix}$ **iv** $\begin{pmatrix}0\\5\end{pmatrix}$

v $\begin{pmatrix}8\\4\end{pmatrix}$ **vi** $\begin{pmatrix}-3\\9\end{pmatrix}$ **vii** $\begin{pmatrix}1\\2\end{pmatrix}$ **viii** $\begin{pmatrix}-2\\11\end{pmatrix}$

5 **a** **ii** $\overrightarrow{OF}$ **iii** $\overrightarrow{OI}$

iv $\overrightarrow{OG}$ **v** $\overrightarrow{OM}$

b **i** 2**b** **ii** 3**a**

iii **a** + 2**b** or 2**b** + **a**

iv 2**a** + 2**b** or 2**b** + 2**a**

v 2**a** + 3**b** or 3**b** + 2**a**

Exam-style questions

1 **i** $\begin{pmatrix}2\\2\end{pmatrix}$ **ii** $\begin{pmatrix}3\\2\end{pmatrix}$ **iii** $\begin{pmatrix}5\\8\end{pmatrix}$

2 **i** $\begin{pmatrix}5\\0\end{pmatrix}$ **ii** $\begin{pmatrix}3\\4\end{pmatrix}$ **iii** $\begin{pmatrix}5\\10\end{pmatrix}$

3 **a** 2**b** **b** **a** + **b** **c** **b** – **a** **d** 2**b** – **a**