13 Stretch lesson: Proportionality

Stretch objectives

Before you start this chapter, mark how confident you feel about each of the statements below:	▶	▶▶	▶▶▶
I can set up and use equations to solve problems involving direct and inverse proportion.			

Check-in questions

- Complete these questions to assess how much you remember about each topic. Then mark your work using the answers at the end of the lesson.
- If you score well on all sections, you can go straight to the Revision Checklist and Exam-style Questions at the end of the lesson. If you don't score well, go to the lesson section indicated and work through the examples and practice questions there.

1 Match each statement with the correct equation. Go to 13.1

a y is proportional to x

b y is inversely proportional to x

$y = kx \qquad y = \frac{k}{x}$

2 a is directly proportional to x. When $x = 4$, $a = 8$.

a What is the constant of proportionality?

b What is the value of x when $a = 64$? Go to 13.1

3 y is inversely proportional to x. When $x = 3$, $y = 10$.

a Calculate the value of y when $x = 2$.

b Calculate the value of x when $y = 6$. Go to 13.1

13.1 Proportionality

Direct proportion

Two quantites are in **direct proportion** to each other when they increase at the same rate. The symbol $\propto$ is used to denote direct proportion.

For example, the cost of a bag of apples (C pence) is directly proportional to the mass (M kg) of the apples. The symbol $\propto$ means 'is directly proportional to', so we can write this as $C \propto M$.

If the apples cost k pence per kilogram, then $C = kM$.

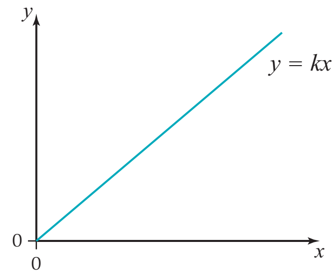
In general, if y is directly proportional to x:

$y \propto x$ and $y = kx$, where k is known as the **constant of proportionality**.

Since $y = kx$, the graph of y against x is a straight line passing through the origin. (See Chapter 10.)

The constant of proportionality, k, is the gradient of this straight line.

You can use given values of the variables to work out the constant of proportionality.



Example 1

Q a is proportional to b and $a = 5$ when $b = 4$.

a Work out the value of k (the constant of proportionality).

b Work out the value of a when $b = 8$.

A **a** $a \propto b$ — Write out a proportion statement using the symbol $\propto$.

$a = kb$ — Write an equation using k as the constant of proportionality.

$5 = k \times 4$ — Substitute the values given to find the value of k.

$k = \frac{5}{4}$

b $a = \frac{5}{4}b$ — Substitute $k = \frac{5}{4}$ into the equation.

$a = \frac{5}{4} \times 8$ — When $b = 8$

$a = 10$

Example 2

Q The voltage, V volts, across an electrical circuit is directly proportional to the current, I amps, flowing through the circuit.

When $I = 2.4$, $V = 156$.

a Work out the formula connecting V and I.

b Work out the value of V when $I = 4$.

c Work out the value of I when $V = 357.5$ volts.

A **a** $V \propto I$

$V = kI$

$156 = k \times 2.4$

$\frac{156}{2.4} = k$

$k = 65$, so $V = 65I$

b $V = 65 \times 4$

$V = 260$ volts

c $V = 65I$

$I = \frac{V}{65}$

$I = \frac{357.5}{65}$

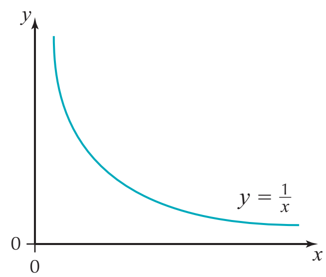
$I = 5.5$ amps

Inverse proportion

Two quantities are in **inverse proportion** when as one variable increases, the other decreases at the same rate and vice versa.

You write y is inversely proportional to x as $y \propto \frac{1}{x}$ or $y = \frac{k}{x}$

When k is positive, the graph $y = \frac{k}{x}$ has a similar shape to $y = \frac{1}{x}$.



Example 3

Q p is inversely proportional to w. When $p = 5$, $w = 2$.
What is the value of w when $p = 20$?

A

$p \propto \frac{1}{w}$ — Write the information with the proportionality sign.

$p = \frac{k}{w}$ — Replace with an equation using the constant of proportionality.

$5 = \frac{k}{2}$ — Substitute known values to find the value of k.

$5 \times 2 = k$

$k = 10$

$p = \frac{10}{w}$ — Rewrite the formula with $k = 10$.

$20 = \frac{10}{w}$ — Find the value of w when $p = 20$.

$w = \frac{10}{20}$

$w = \frac{1}{2}$ or 0.5

Exam tips Check carefully whether you are dealing with direct or inverse proportion.
Find the value of k then write down the equation connecting the variables.

Practice questions

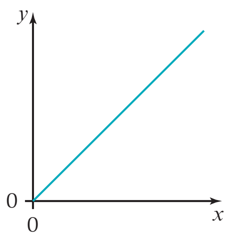
1. T is directly proportional to P. When $P = 4$, $T = 24$.
   a Calculate the value of T when $P = 3$.
   b Calculate the value of P when $T = 9$.

2. R is inversely proportional to S. When $R = 1$, $S = 2$.
   a Calculate the value of R when $S = 4$.
   b Calculate the value of S when $R = 6$.

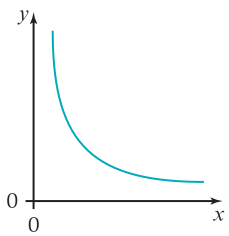
3. A is directly proportional to the square of D. When $D = 3$, $A = 27$.
   a Calculate the value of A when $D = 5$.
   b Calculate the value of D when $A = 108$.

4. Here are four graphs.

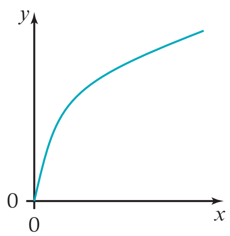
Graph A



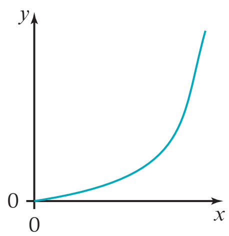
Graph B



Graph C



Graph D



Write down the letter of the graph that represents these relationships.

a y is proportional to x^2

b y is proportional to x

c y is proportional to $\frac{1}{x}$

5 R is inversely proportional to the square of S. When $S = 2$, $R = 2$.

a Calculate the value of R when $S = 6$.

b Calculate the value of S when $R = 0.5$.

REVISION CHECKLIST

- The notation $\propto$ means 'is directly proportional to'. This is often abbreviated to 'is proportional to' or 'varies as'.

Exam-style questions

1 T is directly proportional to P. When $T = 1.5$, $P = 15$.

a Calculate the value of T when $P = 3$.

b Calculate the value of P when $T = 9.5$.

2 R is inversely proportional to S. When $R = 0.5$, $S = 16$.

a Calculate the value of R when $S = 4$.

b Calculate the value of S when $R = 16$.

3 A is directly proportional to the square of D. When $D = 4$, $A = 80$.

a Calculate the value of A when $D = 3$.

b Calculate the value of D when $A = 125$.

4 R is proportional to the square of S. When $S = 2$, $R = 25.2$.

a Calculate the value of R when $S = 3$.

b Calculate the value of S when $R = 157.5$.

Chapter 13 Stretch lesson: Answers

Check-in questions

1 **a** $y = kx$ **b** $y = \frac{k}{x}$

2 **a** 2 ($a \propto x$, $a = kx$, $8 = 4k$, $k = 2$, $a = 2x$)

b $64 = 2x$, $x = 32$

3 **a** 15 $\left(y = \frac{k}{x},\ 10 = \frac{k}{3},\ k = 30,\ y = \frac{30}{x}\right)$

b 5 $\left(6 = \frac{30}{x}\right)$

13.1 Proportionality

1 **a** 18 **b** 1.5

2 **a** 0.5 **b** $\frac{1}{3}$

3 **a** 75 **b** 6

4 **a** D **b** A **c** B

5 **a** $\frac{2}{9}$ **b** 4

Exam-style questions

1 **a** 30 **b** 0.95

2 **a** 2 **b** $\frac{1}{2}$

3 **a** 45 **b** 5

4 **a** 56.7 **b** 5