8 Stretch lesson: Geometric sequences

Stretch objectives

Before you start this chapter, mark how confident you feel about each of the statements below:	▶	▶▶	▶▶▶
I can continue a geometric progression and find the term-to-term rule.			

Check-in questions

- Complete these questions to assess how much you remember about each topic. Then mark your work using the answers at the end of the lesson.
- If you score well on all sections, you can go straight to the Revision Checklist and Exam-style Questions at the end of the lesson. If you don't score well, go to the lesson section indicated and work through the examples and practice questions there.

1 Write down the next two terms in each sequence.

Go to 8.1

a $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \_\_\_, \_\_\_$

b $99, 33, 11, \_\_\_, \_\_\_$

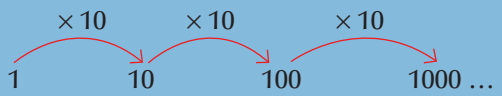
c $5, 50, 500, 5000, \_\_\_, \_\_\_$

8.1 Geometric sequences

1, 2, 4, 8, 16 … is an example of a **geometric sequence**. To get from one term to the next, you multiply by a constant number (**common ratio**), r, each time. In the sequence above, the common ratio is 2.

Example 1

Q Look at this sequence.



Work out:

a the common ratio, r **b** the term-to-term formula, u_{n+1}.

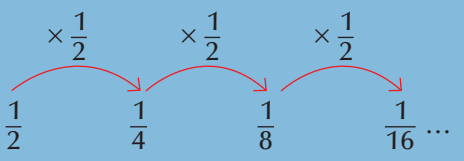
A **a** Each term is multiplied by 10 to produce the next consecutive term. The common ratio r is 10.

b The term-to-term formula is $u_{n+1} = 10u_n$ with $u_1 = 1$.

In other words, the next number in the sequence, u_{n+1}, is equal to 10 times the number before it, that is, $10u_n$.

Example 2

Q Look at this sequence.



Work out:

a the common ratio, r **b** the term-to-term formula, u_{n+1}.

A

a Each term is multiplied by $\frac{1}{2}$ to produce the next consecutive term. So the common ratio r is $\frac{1}{2}$.

b The term-to-term formula is $u_{n+1} = \frac{1}{2}u_n$ with $u_1 = \frac{1}{2}$.

Example 3

Q Look at this sequence.

5 10 20 40 80 …

Work out:

a the common ratio, r

b the term-to-term formula, u_{n+1}

c the position-to-term formula, u_n.

A

a Each term is multiplied by 2 to produce the next consecutive term. So the common ratio r is 2.

b The term-to-term formula is $u_{n+1} = 2u_n$ with $u_1 = 5$.

c The position-to-term formula is $u_n = 5 \times 2^{n-1}$.

Note that when $n = 1$, $u_1 = 5 \times 2^0 = 5 \times 1 = 5$.

Example 4

Q Look at this sequence.

2 6 18 54 162 …

Work out:

a the common ratio, r

b the term-to-term formula, u_{n+1}

c the position-to-term formula, u_n.

A

a Each term is multiplied by 3 to produce the next consecutive term. So the common ratio r is 3.

b The term-to-term formula is $u_{n+1} = 3u_n$ with $u_1 = 2$.

c The position-to-term formula is $u_n = 2 \times 3^{n-1}$.

Practice questions

1. Work out the next term in each sequence.
   a 1, 3, 9, 27 … **b** 0.5, 1, 2, 4 … **c** –1, 2, –4, 8 … **d** 4, 20, 100, 500 …
2. Write down the common ratio for each sequence.
   a 10, 30, 90 … **b** 4, 2, 1 … **c** 2, 6, 18 … **d** 5, –25, 125 …
3. Write down the first three terms of the sequence with nth term 6×2^n.
4. Find the seventh term of the sequence with nth term 0.5×4^n.
5. Is 100 a term in the sequence with nth term 4×0.5^n? You must explain your answer.

REVISION CHECKLIST

- For a geometric sequence such as 3, 9, 27…, you multiply by a constant number (common ratio), r, to find the value of the next term. The common ratio in this example is 3.

Exam-style questions

1. **a** The first four terms in a sequence are 2, 20, 200 and 2000.
   What is the tenth number in the sequence?
   b What is the common ratio of this sequence?
   $\frac{1}{2}$ 5 50 500 5000
2. Work out the next number in this sequence.
   200, 150, 112.5, 84.375 …
3. Find the next term in this sequence.
   2, 8, 18, 32, 50 …
   Explain how you found your answer.
4. There is a famous problem about the number of grains of rice on a chessboard. A chessboard has 64 squares. One grain of rice is put on the first square, two on the second square, four on the third square, eight on the next, then sixteen etc.
   What kind of sequence is this?

Chapter 8 Stretch lesson: Answers

Check-in questions

1 **a** $\frac{1}{16}, \frac{1}{32}$ **b** $\frac{11}{3}, \frac{11}{9}$ **c** 50 000, 500 000

8.1 Geometric sequences

1 **a** 81 **b** 8 **c** –16 **d** 2500

2 **a** 3 **b** 0.5 **c** 3 **d** –5

3 12, 24, 48

4 8192

5 No, because the first term is 2 and the common ratio is < 1, so the numbers in the sequence get progressively smaller.

Exam-style questions

1 **a** 2 000 000 000 **b** 10

2 63.28125

3 72, using the nth term $2n^2$

4 geometric