

6

## Edexcel GCSE (9-1) Maths for post-16

B

 $x = \sqrt{\frac{b^2}{c}}$ 

 $E = \frac{b(6,2)}{xy=ab^2}$ 

F\* 4: 92b

xy=ab

+1=2

#### Fiona Mapp with Su Nicholson

## Contents

	Introduction How to use this book	5 13
1	Numbers	14
1.1	Positive and negative numbers	15
1.2	Square, cube and triangular numbers	18
1.3	Factors, multiples and primes	20
1.4	Indices	23
1.5	Standard index form	24
2	Fractions and decimals	28
2.1	Fractions	29
2.2	Calculating with fractions	31
2.3	Fraction problems	33
2.4	Decimals and fractions	34
3	Calculations	38
3.1	Multiplying and dividing numbers	
	by power of 10	39
3.2	Written methods	41
3.3	Order of operations	44
3.4	Using a calculator	46
4	Accuracy	50
4.1	Rounding	51
4.2	Estimating	54
5	Formulae and	
	expressions	58
5.1	Algebraic terms and conventions	59
5.2	Simplifying expressions	60
5.3	Laws of indices	62
5.4	Substituting into formulae	63
5.5	Writing formulae	65
6	Algebraic manipulation	69
6.1	Expanding single brackets	70
6.2	Expanding two pairs of brackets	71
6.3	Factorisation	72
6.4	Rearranging formulae	73
7	Equations and inequalities	76
7.1	Linear equations of the form	
	ax + b = c	77
7.2	Linear equations of the form	
	ax + b = cx + d	79

7.3 7.4	Linear equations with brackets Equation problems	80 81
7.5	Inequalities	82
8	Sequences	86
8.1 8.2	Sequences Finding the <i>n</i> th term of an	87
	arithmetic sequence	88
9	Straight-line graphs	92
9.1	Coordinates	93
9.2 9 3	Calculating the gradient of a	94
5.5	straight-line graph	96
10	Curved graphs	102
10.1	Quadratic graphs	103
10.2	Using graphs to solve quadratic	105
10.3	Graph shapes	105
11	Interpreting graphs	113
11.1	Conversion graphs	115
11.2	Real-life graphs	116
11.3	graphs	119
12	Percentages	130
12.1	Percentage of a quantity	131
12.2	Increasing and decreasing	400
123	One quantity as a percentage	132
12.0	of another	133
12.4	Percentage change	134
12.5	Repeated percentage change	136
12.6	Interest and tax	137
12.7	Reverse percentage problems	139
13	Ratio and proportion	143
13.1	Simplifying ratios	144
13.2	Sharing a quantity in a given ratio	145
13.3	Problem solving	147
13.4	given ratio	149
14	Measurement	153
14.1	Units of measurement	154
14.2	Maps and diagrams	157

14.2Maps and diagrams15/14.3Compound measures159

15	Similarity and	
	congruence	165
15.1	Similar shapes	166
15.2	Congruent shapes	169
16	2D and 3D shapes	175
16.1	Properties of 2D shapes	177
16.2	Symmetry	180
16.3	Properties of 3D shapes	181
16.4	Plans and elevations	184
17	Angles	189
17.1	Classifying angles	191
17.2	Angle facts	192
17.3	Angles in parallel lines	195
17.4	Angles in polygons	198
17.5	Scale drawings	201
17.0	Scale urawings	205
18	Pythagoras' theorem	211
18.1	Calculating an unknown length	212
18.2	Calculating the length of a line	
	segment on a coordinate grid	214
18.3	Solving problems	216
19	Trigonometry	223
19 19.1	Trigonometry Trigonometric ratios	<b>223</b> 224
19 19.1 19.2	Trigonometry Trigonometric ratios Calculating a length	223 224 226
19 19.1 19.2 19.3	Trigonometry Trigonometric ratios Calculating a length Calculating an angle	223 224 226 228
19 19.1 19.2 19.3 20	Trigonometry Trigonometric ratios Calculating a length Calculating an angle Perimeter and area	223 224 226 228 235
19 19.1 19.2 19.3 20 20.1	Trigonometry Trigonometric ratios Calculating a length Calculating an angle Perimeter and area Plane shapes	223 224 226 228 235 236
19 19.1 19.2 19.3 20 20.1 20.2	Trigonometry Trigonometric ratios Calculating a length Calculating an angle Perimeter and area Plane shapes Compound shapes and problem	223 224 226 228 235 236
19   19.1   19.2   19.3   20   20.1   20.2	Trigonometry Trigonometric ratios Calculating a length Calculating an angle Perimeter and area Plane shapes Compound shapes and problem solving	223 224 226 228 235 236 240
19   19.1   19.2   19.3   20   20.1   20.2   20.3	Trigonometry Trigonometric ratios Calculating a length Calculating an angle Perimeter and area Plane shapes Compound shapes and problem solving Circles	223 224 226 228 235 236 240 242
19   19.1   19.2   19.3   20   20.1   20.2   20.3   20.4	Trigonometry Trigonometric ratios Calculating a length Calculating an angle Perimeter and area Plane shapes Compound shapes and problem solving Circles Arc length and sector area	223 224 226 228 235 236 240 242 244
19   19.1   19.2   19.3   20   20.1   20.2   20.3   20.4	Trigonometry Trigonometric ratios Calculating a length Calculating an angle Perimeter and area Plane shapes Compound shapes and problem solving Circles Arc length and sector area Surface area and	223 224 226 228 235 236 240 242 244
19   19.1   19.2   19.3   20   20.1   20.2   20.3   20.4	Trigonometry Trigonometric ratios Calculating a length Calculating an angle Perimeter and area Plane shapes Compound shapes and problem solving Circles Arc length and sector area Surface area and volume	223 224 226 228 235 236 240 242 244 244
19   19.1   19.2   19.3   20   20.1   20.2   20.3   20.4   21   21.1	Trigonometry Trigonometric ratios Calculating a length Calculating an angle Perimeter and area Plane shapes Compound shapes and problem solving Circles Arc length and sector area Surface area and volume Simple 3D shapes	223 224 226 228 235 236 240 242 244 242 244 251 252
19   19.1   19.2   19.3   20   20.1   20.2   20.3   20.4   21   21.1   21.2	Trigonometry Trigonometric ratios Calculating a length Calculating an angle Perimeter and area Plane shapes Compound shapes and problem solving Circles Arc length and sector area Surface area and volume Simple 3D shapes More complex solids	223 224 226 228 235 236 240 242 244 244 251 252 254
19   19.1   19.2   19.3   20   20.1   20.2   20.3   20.4   21   21.1   21.2	Trigonometry Trigonometric ratios Calculating a length Calculating an angle Perimeter and area Plane shapes Compound shapes and problem solving Circles Arc length and sector area Surface area and volume Simple 3D shapes More complex solids Converting units of area and	223 224 226 228 235 236 240 242 244 244 251 252 254
19   19.1   19.2   19.3   20   20.1   20.2   20.3   20.4   21   21.1   21.2   21.3	TrigonometryTrigonometric ratios Calculating a length Calculating an anglePerimeter and areaPlane shapes Compound shapes and problem solving Circles Arc length and sector areaSurface area and volumeSimple 3D shapes More complex solids Converting units of area and volume	223 224 226 228 235 236 240 242 244 242 244 251 252 254 258
19   19.1   19.2   19.3   20   20.1   20.2   20.3   20.4   21   21.2   21.3   22	Trigonometry Trigonometric ratios Calculating a length Calculating an angle Perimeter and area Plane shapes Compound shapes and problem solving Circles Arc length and sector area Surface area and volume Simple 3D shapes More complex solids Converting units of area and volume Vectors and	223 224 226 228 235 236 240 242 244 251 252 254 258
19   19.1   19.2   19.3   20   20.1   20.2   20.3   20.4   21   21.2   21.3   22	TrigonometryTrigonometric ratiosCalculating a lengthCalculating an anglePerimeter and areaPlane shapesCompound shapes and problemsolvingCirclesArc length and sector areaSurface area and volumeSimple 3D shapes More complex solids Converting units of area and volumeVectors and transformations	223 224 226 228 235 236 240 242 244 244 251 252 254 258 258
19   19.1   19.2   19.3   20   20.1   20.2   20.3   20.4   21   21.1   21.2   21.3   22   22.1	Trigonometry Trigonometric ratios Calculating a length Calculating an angle Perimeter and area Plane shapes Compound shapes and problem solving Circles Arc length and sector area Surface area and volume Simple 3D shapes More complex solids Converting units of area and volume Vectors and transformations	223 224 226 228 235 236 240 242 244 252 254 252 254 258 258 264 266
19   19.1   19.2   19.3   20   20.1   20.2   20.3   20.4   21   21.1   21.2   21.3   22   22.1   22.1   22.1	TrigonometryTrigonometric ratiosCalculating a lengthCalculating an anglePerimeter and areaPlane shapesCompound shapes and problemsolvingCirclesArc length and sector areaSurface area and volumeSimple 3D shapes More complex solids Converting units of area and volumeVectors and transformationsTranslations Reflections	223 224 226 228 235 236 240 242 244 251 252 254 258 258 258 264 266 268

22.3 22.4	Rotations Enlargements	270 273
23	Probability	282
23.1 23.2	The probability scale Calculating probabilities	284
23.3	associated with single events Probability that an event will	285
22.4	not happen Experimental probability	28/
23.4	Possible outcomes of two or	200
	more events	290
23.6	Probabilities of multiple events and tree diagrams	292
24	Sets and Venn	
	diagrams	299
24.1	Set notation	300
24.2	Venn diagrams	301
24.3	Using Venn diagrams to solve probability questions	303
25	Statistics	310
25.1	Collecting data	313
25.2	Organising data	315
25.3	Displaying data	316
23.4	Fie Charts	521
26	Averages	329
26.1	Comparing data: averages and	220
26.2	Spread Finding averages from a	330
20.2	frequency table	332
26.3	Averages of grouped data	334
26.4	Stem and leaf diagrams	335
27	Time series and	
	scatter graphs	342
27.1	Scatter graphs and correlation	343
27.2	Time-series graphs	347
Onlin	e stretch lessons	353
Answe	ers	355
Formu	Ilae sheet	397
Glossa	ary	398
Index		402

# Algebraic manipulation

#### Objectives

Before you start this chapter, mark how confident you feel about each of the statements below: I can expand a single pair of brackets.

I can expand two pairs of brackets.

I can square a linear expression, e.g.  $(x + 1)^2$ .

I can factorise algebraic expressions by taking out common factors.

I can factorise quadratic expressions of the form  $x^2 + bx + c$ .

I can factorise a quadratic expression of the form  $x^2 - a^2$  using the difference of two squares.

I can change the subject of a formula.

#### Check-in questions

- Complete these questions to assess how much you remember about each topic. Then mark your work using the answers at the back of the book.
- If you score well on all sections, you can go straight to the Revision checklist and Exam-style questions at the end of the chapter. If you don't score well, go to the chapter section indicated and work through the examples and practice questions there.



# 6.1 Expanding single brackets

Multiplying out brackets helps to simplify algebraic expressions. The term outside the bracket is multiplied by each of the terms inside the bracket.

**Exam tips** When you multiply out brackets, check that the signs are correct for each term. If the signs are the same, it is +. If the signs are different, it is –.

#### **Practice questions**



## 6.2 Expanding two pairs of brackets

Every term in the second bracket must be multiplied by every term in the first bracket. Often, but not always, the two middle terms are like terms and can be collected together.

$$(x + 4)(x + 2) = x^{2} + 2x + 4x + 8$$
$$= x^{2} + 6x + 8$$



**Exam tips** Remember to check that the signs are correct for each term.

#### **Practice questions**

1	Expand and simplify. <b>a</b> $(y + 2)(y + 3)$	<b>b</b> $(x+4)(x+2)$	<b>c</b> $(m+3)(m+5)$	
2	Expand and simplify. <b>a</b> $(x + 4)(x - 1)$	<b>b</b> $(y+6)(y-2)$	<b>c</b> $(m-2)(m+8)$	<b>d</b> $(x+5)(x-5)$
3	Expand and simplify. <b>a</b> $(3x + 2)(x + 4)$	<b>b</b> (5 <i>y</i> – 2)(5 <i>y</i> + 4)	<b>c</b> $(7-2y)(3-8y)$	



Match the expression on the left with the correct expansion on the right.

(x-2)(x+2)	$2x^2 - x - 1$
(x+3)(x-1)	$x^2 - 4$
(x+4)(x-2)	$x^2 + 2x - 3$
(2x + 1)(x - 1)	$x^2 + 2x - 8$

Exam tips

Questions 1 to 3 ask you to 'Expand and simplify'. After expanding remember to collect like terms.



Tash expands and simplifies (x + 2)(x - 3). Her result is  $x^2 - 1$ . Identify any errors she has made.

2

## 6.3 Factorisation

**Factorisation** is the opposite of expansion. It involves looking for common factors and putting them outside brackets.

#### One pair of brackets

To factorise 4x + 6:

- Identify the HCF of the terms.
- Divide the expression by the HCF. 2x + 3
- Write the HCF outside the brackets and the rest of the expression inside the brackets. 2(2x + 3)When it is expanded or multiplied, it will equal the original expression.

Example	<b>Q</b> Factorise fully	<b>Q</b> Factorise fully.				
3	<b>a</b> 8 <i>x</i> – 16	<b>b</b> 3 <i>x</i> + 18	<b>c</b> $5x^2 + x$	<b>d</b> $4x^2 + 8x$		
	<b>A a</b> 8( <i>x</i> −2)	<b>b</b> 3( <i>x</i> + 6)	<b>c</b> $x(5x + 1)$	<b>d</b> $4x(x+2)$		

**Exam tips** Remember to check that you have put all the common factors outside the bracket, otherwise the expression is not fully factorised. Check there is no common factor inside the bracket for your final answer.

#### Two pairs of brackets

Two pairs of brackets are obtained when a quadratic expression of the type  $ax^2 + bx + c$  is factorised.

Example	<b>Q</b> Factorise these expressions.
4	<b>a</b> $x^2 + 4x + 3$ <b>b</b> $x^2 - 7x + 12$ <b>c</b> $x^2 + 3x - 10$
	<b>A</b> a Find two numbers with a product of 3 (the constant) and a sum of 4 (the coefficient of <i>x</i> ): 3 and 1 (3 × 1 = 3 and 3 + 1 = 4) ( $x$ + 1)( $x$ + 3)
	<b>b</b> Find two numbers with a product of 12 (the constant) and a sum of $-7$ (the coefficient of <i>x</i> ): $-4$ and $-3$ ( $-4 \times -3 = 12$ and $-4 + -3 = -7$ ) ( $x - 3$ )( $x - 4$ )
	<b>c</b> Find two numbers with a product of $-10$ (the constant) and a sum of 3 (the coefficient of <i>x</i> ): $-2$ and 5 ( $-2 \times 5 = -10$ and $-2 + 5 = 3$ ) ( $x + 5$ )( $x - 2$ ) • Note that although $-5 \times 2 = -10, -5 + 2 = -3$

Example	<b>Q</b> Factorise these expressions.				
5		a	$x^2 - 64$	<b>b</b> $81x^2 - 25y^2$	
	A	a b	$x^{2} - 64 = (x + 8)(x - 8)$ 81x <sup>2</sup> - 25y <sup>2</sup> = (9x + 5y)(x - 8)	9x - 5y	This is known as the 'difference of two squares'. In general, $x^2 - a^2 =$
		N,	01x - 23y - (3x + 3y)(	<i>3x 3yy</i>	(x+a)(x-a).

#### **Practice questions**

1	Factorise these expre	essions.		
	<b>a</b> 4 <i>x</i> + 10	<b>b</b> 15 <i>x</i> + 35	<b>c</b> 14 <i>x</i> – 120	<b>d</b> 6 <i>x</i> – 30
2	Factorise fully these	expressions.		
	<b>a</b> $5y + 10x + 25z$	<b>b</b> $16x - 48x^2 + 56xz$	<b>c</b> $14x - 42y + 63z$	<b>d</b> $-9m + 12n - 21p$
3	Factorise these quad	ratic expressions.		
	<b>a</b> $x^2 + 5x + 6$	<b>b</b> $x^2 + 10x + 16$	<b>c</b> $x^2 - 10x + 21$	<b>d</b> $x^2 + 6x - 16$
4	Factorise these expre	essions.		
	<b>a</b> $x^2 - 9$	<b>b</b> $y^2 - 49$	<b>c</b> $m^2 - 1$	<b>d</b> $9t^2 - 121y^2$
5	In an exam, Ravjeet is	s asked to fully factorise	$25x^2 - 40x + 15xy.$	
	As her answer, she w	rites $5(5x^2 - 8x + 3xy)$ .		
	She does not score f	ull marks. Explain why.		

## 6.4 Rearranging formulae

The subject of a formula is the letter that appears on its own on one side of the formula.

Any letter in a formula can become the subject if you rearrange the formula. It is important to do the same things to both sides of the formula as you rearrange it.



When the subject occurs on both sides of the equals sign, they need to be collected on one side. It is easier to collect the terms involving the new subject on the side where there are more of them.



## **REVISION CHECKLIST**

- To expand single brackets, multiply the term outside the brackets by everything inside the brackets.
- To expand two pairs of brackets, multiply each term in the second bracket by each term in the first bracket.
- Factorising is the opposite of expanding brackets. Look for the highest common factor (HCF).
- You can rearrange a formula to make any letter the subject. When rearranging a formula, you must do the same thing to both sides.

## Exam-style questions

- 1 Which of these are true?
  - **a** 5(a+3) = 5a+3
  - **c**  $5c(c-2) = 5c^2 2c$

**b** 3(b+2) = 3b+6**d**  $2d(d+4) = 2d^2 + 8d$ 

2 Factoris

Factorise this expression.  $3pr + 12qr - 21r^2$ 

6 Algebraic manipulation



Now go back to the list of objectives at the start of this chapter. How confident do you now feel about each of them?

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