<table>
<thead>
<tr>
<th>Guidance on the use of codes for this mark scheme</th>
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<tbody>
<tr>
<td>M</td>
</tr>
<tr>
<td>A</td>
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<tr>
<td>B</td>
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<tr>
<td>C</td>
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<tr>
<td>P</td>
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<td>cao</td>
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<td>oe</td>
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<tr>
<td>ft</td>
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<tr>
<td>Question</td>
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<tr>
<td>1 a</td>
</tr>
<tr>
<td>b</td>
</tr>
<tr>
<td>c</td>
</tr>
<tr>
<td>d</td>
</tr>
<tr>
<td>2 a</td>
</tr>
<tr>
<td>b</td>
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</table>

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| 3a | b | Appropriate workings related to their question. | For example: Easy: a shop increased its prices by 10%. If an item costs £100, how much more does it cost after the price increase? £10 Easy to find because original amount is £100. Difficult: A worker’s hourly rate increased by 25%. If the hourly rate was £8 before the increase, how much does the worker get paid per hour after the increase? £10 Difficult to find because the percentage is not a multiple of 10 and context is more complex. | C1 | 2 | C1 for clarity of question C1 for explanation that links complexity of mathematics to context of question | B |
| 4 | | The formula for density is: \( \text{density} = \frac{\text{mass}}{\text{volume}} \) If the objects have the same volume but different masses, this formula indicates that the densities will be different and so suggests the objects are made from different metals. | C1 | 3 | C1 for insight into the effect of changing a variable in a formula | M |
5 a  
1 g/cm³ = 1000 kg/m³
So
2.3 g/cm³ = 2300 kg/m³
Use the formula:
density = \frac{mass}{volume}
Rearrange the formula:
volume = \frac{mass}{density}
1 tonne = 1000 kg
So volume = \frac{30 000}{2300} kg = 2300 kg/m³
2.7 g/cm³ = 2700 kg/m³
They both have the same volume.
Again, use the formula:
mass = density \times volume
13 \times 2700 = 35 100
The granite has a mass of 35.1 tonnes and the sandstone has a mass of 30 tonnes
OR
\frac{35 100}{30 000} = 1.17
5.1 tonnes heavier or 17% heavier.

5 b  
= 13 m³ to nearest m³

P1 2 3 P1 for conversion from g/cm³ to kg/m³

B1 B1 for correct rearrangement of formula

A1 A1 oe

B1 B1 for calculating correct tonnage for granite

M1 M1 for correct method for comparison of mass

C1 C1 for stating correct comparison
The carvings are identical so the volume is the same.

\[
\frac{0.315 \times 630}{550} = \frac{m_0}{p}
\]

Rearranging:

\[
m_0 = 550 \times \frac{0.315}{630} = 0.275 \text{ kg}
\]

- **3** M1 for dividing mass by volume and making correct comparison
- **B1** B1 for rearranging
- **A1** A1 oe

<table>
<thead>
<tr>
<th>7 a</th>
<th>The ratio men : women is 5 : 2. There are 24 women so the total membership is: 5 \times 12 : 2 \times 12 The ratio becomes 60 : 24 Then the total membership = 60 + 24 = 84</th>
</tr>
</thead>
</table>
| 84  | **3** M1 for multiplying by 12 oe
|     | **A1** A1 for 84 members in total

<table>
<thead>
<tr>
<th>7 b</th>
<th>The ratio R : S : J is 2 : 3 : 5. There are 10 shares. £85 ÷ 10 = £8.50 Shaun pays 3 \times £8.50 = £25.50</th>
</tr>
</thead>
</table>
| 84  | **M1** M1 for division of 85 by 10
|     | **A1** A1 for correct multiplication 3 \times £8.50 oe

<table>
<thead>
<tr>
<th>7 c</th>
<th>Own question like the one in part a. For example: In a tennis club, 30 members are men. The ratio of women to men is 6 : 5. How many of the members are female? 36</th>
</tr>
</thead>
</table>
|     | **C1** C1 for correct type of question

- **5**
### 8 a

\[ b_2 = \frac{5}{4} \times b_1 \]

\[ b_2 = \frac{5}{4} \times 8 \]

\[ = \frac{40}{4} = 10 \text{ hours} \]

**b**

\[ b_2 \text{ costs } £198 \]

\[ b_1 \text{ costs } £118 \]

\[ \frac{198}{118} = 1.68 \text{ to 2dp} \]

\[ \frac{5}{4} = 1.25 \]

\[ \frac{b_2}{b_1} = \frac{\frac{5}{4}}{\frac{5}{4}} \]

\[ = \frac{\frac{590}{4}}{\frac{4}{4}} = £147.50 \]

**Reduction is:**

\[ £198 - £147.50 = £50.50 \]

**The increase in cost is proportionally more than the increase in battery life.**

**c**

\[ \frac{b_2}{b_1} = \frac{5}{4} \]

\[ b_2 = \frac{5 \times 118}{4} \]

\[ = \frac{590}{4} = £147.50 \]

**Reduction is:**

\[ £198 - £147.50 = £50.50 \]

**She would need a reduction of £50.50.**

---

### 9 a

**For the first 5-pack:**

\[ 5 \times 90 \text{ minutes} = 450 \text{ minutes} \]

\[ £6.60 = 660p \]

\[ 650p \div 450 = 1.44p \text{ per minute} \]

**For the 10-pack:**

\[ 10 \times 80 = 800 \text{ minutes for } £6.50 \div 800 = 0.8125p \text{ per minute cheapest} \]

**For the second 5-pack:**

\[ 5 \times 80 = 400 \text{ minutes} \]

\[ £4.00 = 400p \]

\[ 400p \div 400 = 1p \text{ per minute} \]

Or

\[ 450 \div 6.50 = 69 \text{ minutes per £1} \]

\[ 800 \div 6.50 = 123 \text{ minutes per £1 best value} \]

**The best buy is the 10-pack of 80 minutes each @ £6.50.**

**P1**

**3**

**P1 for process of setting up equation**

**P1 for process of multiplying up for total minutes and then division to identify either cost per minute or time per £**

**M1**

**6**

**B2 for correct workings in each of the three cases**

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</thead>
<tbody>
<tr>
<td></td>
<td>400 ÷ 4.00 = 100 minutes per £1</td>
<td>There are more CDs than are needed. A recording time of 80 minutes is not long enough. £6.50 is too expensive at time of purchase (prefer just to spend £4).</td>
<td>C1</td>
<td>C1 for explanation of possible reasons not to choose the best buy</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>£800 × 1.19 gives €952 £800 × 1.22 gives €976 €976 – €952 = €24</td>
<td>They will get €24 more.</td>
<td>M1</td>
<td>M1 for multiplications B1 for subtraction ft A1 cao</td>
<td></td>
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<tr>
<td>11 a i</td>
<td>By expressing this as: ‘How many….. in ….’</td>
<td>C2</td>
<td>C1 for correct justification C1 for showing diagram oe</td>
<td></td>
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</tr>
<tr>
<td>ii</td>
<td>How many in Answer 2</td>
<td>C1</td>
<td>C1 for correct justification showing diagram oe</td>
<td></td>
<td></td>
</tr>
<tr>
<td>iii</td>
<td>How many in Answer 3</td>
<td>C1</td>
<td>1 for correct justification showing diagram oe</td>
<td></td>
<td></td>
</tr>
<tr>
<td>b</td>
<td>Use chosen method from part a to explain correctly how to divide, using fractions.</td>
<td>C1 P1</td>
<td>C1 for correct explanation P1 for process showing that dividing by ( \frac{1}{2} ) doubles the number of pieces, so is the same as multiplying by 2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
12 a
8 kg = 8000 g
8000 ÷ 250 = 32
3 kg = 3000 g
3000 ÷ 85 = 35 (to nearest whole number)
2 kg = 2000 g
2000 ÷ 20 = 100
7 kg = 7000 g
7000 ÷ 250 = 28

So the limiting value is the amount of icing sugar. Therefore she can make 24 × 28 = 672 biscuits.

She can make 44 complete packs of 15 biscuits.

b
3 kg = 3000 g
3000 ÷ 85 = 35 (to nearest whole number)
2 kg = 2000 g
2000 ÷ 20 = 100
7 kg = 7000 g
7000 ÷ 250 = 28

So the limiting value is the amount of icing sugar. Therefore she can make 24 × 28 = 672 biscuits.
672 ÷ 15 = 44.8
44 × \frac{3}{4} = 33
33 × £2.99 = £98.67
44 – 33 = 11 discounted
£2.99 × 0.85 = £2.54 to 2 dp
11 × 2.54 = £27.94

Total sales
= £98.67 + £27.94
= £126.61

Total costs
= £59 + £26 = £85

To calculate percentage profit:
profit = \frac{(£126.61 – £85)}{£85}
= 0.489 529 412
and percentage profit
= 0.489 529 412 × 100% = 48.95%

49% profit to the nearest integer.

13
Price including VAT = £595 ÷ 1.20 = £714
With a 20% discount: £714 × 0.8 = £571.20
£571.20 ÷ £595 = £23.80

He is overpaying by £23.80

Price including VAT = £595 
With a 20% discount: £595 × 0.8 = £476
£476 × 1.2 = £571.20

He is overpaying by £23.80

Disagree. He would pay the shop more than he needs to.

He is overpaying by £23.80

Disagree. He would pay the shop more than he needs to.
### 14 a

With a reduction of 15%, the sale price \((B)\) is \(A \times 0.85\).  
\[
A = \frac{B}{0.85}
\]

Yes, the new value will always be the original value multiplied by a percentage, calculated from the percentage change. For a reduction, the multiplier is \((100 - \text{the percentage reduction})\%\), for an increase it is \((100 + \text{the percentage increase})\%\).

Percentage change problem, for example: The cost of a new car was £\(A\). In the new financial year, it increased by 5% to £\(B\). Write a formula to describe the proportional change.

\[
B = A \times 1.05 \quad \text{and} \quad A = \frac{B}{1.05}
\]

### 15 a

\(A \times 1.5 \times 1.5 = A \times 1.5^2\)  
\(= A \times 2.25\)

No, an increase to \(A\) of 50% followed by another increase of 50% gives 2.25\(A\). Doubling would give 2\(A\) and 2\(A\) ≠ 2.25\(A\).

80% discount gives a price of \(A \times 0.20\). 60% followed by 20% gives a price of \(A \times 0.4 \times 0.8 = A \times 0.32\).

An 80% discount off the price of \(A\) gives a new price of 0.2\(A\). A 60% discount off the price of \(A\), followed by a further 20% discount, gives a new price of 0.32\(A\) so the 80% discount is better value.

If the original cost is \(A\), the cost after a discount of 25% is 0.75\(A\) and paying VAT at 20% gives a new price of 0.9\(A\). If VAT is added first, the price is 1.2\(A\). A 25% reduction gives a new price of 0.9\(A\). Because multiplication is commutative, the final prices are the same. It makes no difference.
### 16 a

\[ A \times \frac{6}{7} = £996 \]
\[ A = £996 \times \frac{7}{6} = £1162 \]

### 16 b

\[ A \times 1.04 = £6.50 \]
\[ A = \frac{£6.50}{1.04} = £6.25 \]

### 16 c

\[ A \times 1.07 = £957.65 \]
\[ A = \frac{£957.65}{1.07} = £895 \]

If the original amount is \( A \), the multiplier is \( b \) for a percentage increase or decrease, and the new value is \( C \):

\[ A \times b = C \]

### 16 e

If the multiplier is \( x \):

- \( x > 1 \) means an increase
- \( 0 < x < 1 \) means a decrease.

\[ A = C \times \frac{1}{b} \]

### 17 a

Comparing salary in May and April:

\[ £1568 - £1544 = £24 \]

Comparing sales in May and April:

\[ £24 \text{ earned on £4000 sales.} \]
\[ 24000 \div 4000 = 6 \]
\[ 6 \times £24 = £144 \]
\[ £1544 - £144 = £1400 \]

So the basic salary is £1400.

\[ £1553 - £1400 = £153 \]

\[ \frac{153}{24} = \frac{51}{8} = 6.375 \]
\[ 6.375 \times 4000 = £25500 \]

### 17 b

Own question

\[ £25500 \]

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18. Number on Saturday = 2 × number on Friday
   \[ S \times 1.5 = (2F) \times 1.5 \]
   \[ S = \frac{3F}{1.5} = 2F \]
   There are still twice as many visitors on Saturday as on Friday. There are 100% more visitors on Saturday compared to Friday.

   \[ C1 \]
   \[ 3 \]
   \[ C1 \text{ for an explanation that includes an appreciation that the two sets of visitors increase proportionally and that the original proportion therefore does not change} \]

   \[ M \]

19 a. Number of workers = \( W \)
Number of days = \( t \)

   \[ K = \text{constant} \]
   \[ W = \frac{K}{t} \]
   \[ 2 = \frac{K}{20} \text{ so } k = 40 \]
   \[ W = \frac{40}{t} \]

   With 3 workers:
   \[ 3 = \frac{40}{t} \]
   \[ t = \frac{40}{3} = 13 \frac{1}{3} \text{ days} \]

   They would finish after 13\(\frac{1}{3}\) days.

   \[ M1 \]
   \[ B1 \]
   \[ 3 \]
   \[ M1 \text{ for finding constant of proportionality} \]
   \[ B1 \text{ for division of 40 by 3 and relating this to number of days worked} \]

b. They would probably get in each other’s way and would not be able to complete the job in a very short time.

   Some jobs have to wait until others are finished, for example, they can’t paint until the walls have been plastered.

   \[ C1 \]
   \[ \text{C1 for an appropriate reason} \]

20. Current costs are £1.50 per mile and 20p per minute.

   Competitive pricing structure: answers will vary.

<table>
<thead>
<tr>
<th>Time taken</th>
<th>2 min</th>
<th>5 min</th>
<th>10 min</th>
<th>12 min</th>
<th>15 min</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance</td>
<td>1 mile</td>
<td>2 miles</td>
<td>3 miles</td>
<td>5 miles</td>
<td>6 miles</td>
</tr>
<tr>
<td>Total change (A)</td>
<td>£2.50</td>
<td>£4.00</td>
<td>£6.50</td>
<td>£9.90</td>
<td>£12.00</td>
</tr>
<tr>
<td>Total change (B)</td>
<td>£1.90</td>
<td>£4.00</td>
<td>£6.50</td>
<td>£9.90</td>
<td>£12.00</td>
</tr>
</tbody>
</table>

   \[ P1 \]
   \[ B1 \]
   \[ 2 \]
   \[ P1 \text{ for process of finding charges} \]
   \[ B1 \text{ for working out current price structure} \]

   \[ B1 \text{ for correct calculation of a pricing structure that has an element of competition} \]
   \[ \text{The suggestion (B) competes for short distances, matches for mid distances and is not competitive for longer journeys.} \]

   \[ M \]
### 21 a
Travel 30 miles in 45 minutes.

45 minutes = \( \frac{3}{4} \) hour

\[
\begin{align*}
30 \times \frac{3}{4} &= \frac{120}{3} \\
&= 40 \text{ mph as required}
\end{align*}
\]

Not changing minutes into hours.

Units of speed = \( \frac{\text{units of distance}}{\text{units of time}} \)

Own easy and difficult examples

C1 2
C1 for correct explanation with calculation that indicates 10 miles every 15 minutes implies 40 miles every 60 minutes oe

C1 clear explanation

C1 for stating a common misconception

C1 for correctly stating the relationship between speed, distance and time

B2
B1 for one easy and one difficult example with justification

B1 for multiple different examples

### 22
A rectangle 1 m × 2 m
Area = 2 m²
A rectangle 4 m × 8 m
Area = 32 m²
Length scale factor = 4
Area scale factor = 16 (4²)

32 m²

P1 2
3
P1 for process of trial and improvement

A1
cao

### 23
75 ÷ 30 = 2.5
Length scale factor is 2.5
Volume scale factor is \((2.5)^3 = 15.625\)

5000 ÷ 15.625 = 78,125 cm³ = 78.125 litres

78.125 litres

B1 2
3
B1 for calculation of length scale factor

M1 for calculation of volume scale factor

A1
cao

### 24
Length scale factor = 450 ÷ 15 = 30
Volume scale factor = 30³ = 27 000
450 × 27 000 = 12 150 000 cm³
\((\div 100^3 \text{ for } m^3)\)

= 12.15 m³

12.15 m³

B1 2
3
B1 for calculation of length scale factor

M1 for calculation of volume scale factor

M1 for correct conversion to cubic metres

A1
cao

### 25
Length scale factor = 18 ÷ 12 = 1.5
Volume scale factor = (1.5)³
Volume of paint in big tin = 800 ml × (1.5)³

= 2700 ml

2700 ÷ 800 = 3.375

So he can fill 3 tins.

3 small tins can be filled from one large tin.

B1 2
B1 for calculation of length scale factor

M1 for calculation of volume scale factor

A1
cao
26 a New area is \((a \times 1.15)^2\)
\[ = a^2 \times 1.15^2 \]
\[ = 1.3225a^2 \]
Percentage increase = 
\[ (1.3225 - 1) \times 100\% \]

b \[ a \times 1.15 \]
Area = \(a \times 1.15 \times b \times 0.95\)
\[ = ab \times 1.15 \times 0.95 = 1.0925ab \]
Percentage increase \((1.0925 - 1) \times 100\%\)

Area increases by 32.25%.

Area increases by 9.25%.

27 a
<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>4</td>
<td>17</td>
</tr>
<tr>
<td>4</td>
<td>3.2</td>
<td>13.6</td>
</tr>
<tr>
<td>2</td>
<td>1.6</td>
<td>6.8</td>
</tr>
<tr>
<td>8</td>
<td>6.4</td>
<td>27.2</td>
</tr>
<tr>
<td>12</td>
<td>9.6</td>
<td>40.8</td>
</tr>
<tr>
<td>6.8</td>
<td>5.44</td>
<td>23.12</td>
</tr>
<tr>
<td>2.8</td>
<td>2.24</td>
<td>9.52</td>
</tr>
</tbody>
</table>

\[ \frac{B}{A} = \frac{1.6}{2} = 0.8 \]
\[ \frac{C}{A} = \frac{17}{5} = 3.4 \]

This also means that
\[ \frac{C}{B} = \frac{17}{4} = 4.25 \]

So yes there is enough information.

b

Yes, there is sufficient information.
One variable is isolated from the other two.

9 items e.g.

<p>| | | |</p>
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<td>●</td>
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</tbody>
</table>

There should be at least one value in each row and two rows should have at least two pairs linking a different pair.

Always start in a row where at least 2 quantities are given, to work out the third quantity, so that relationships between all three are known. Then use these to work out other quantities.

In this example there are 2 possible starting points.

No, Sam is incorrect.
She will have £8349.34
See workings as explanation.

In year 1:
£8000 × 0.027 = £216
Interest = £216
Less 20% tax:
£216 × 0.8 = £172.80
So the total at end of year 1 = £8000 + £172.80 =£8172.80
In year 2:
£8172.80 × 0.027 = £220.67
Interest = £220.67
Less 20% tax:
£220.67 × 0.8 = £176.54
At end of year 2:
Amount = £8172.80 + £176.54 = £8349.34

C1 for clear explanation that there should be at least one value in each row and two rows should have at least two values linking a different pair of A, B, C oe

C1 for clear explanation

C1 for explanation of the best starting point and stating how many different starting points there are

P1 for use of correct multipliers
B1 for multistep calculation for year 1

B1 for multistep calculation for Year 2 (ft)
C1 for clarity of explanation through setting out of calculations
| 29 | $B \times 0.8^n < \frac{B}{2}$  
|    | Divide both sides by $B$.  
|    | $0.8^n < \frac{1}{2}$  
|    | $0.8^3 = 0.512$  
|    | $0.8^4 = 0.4096$  
|    | OR  
|    | £100 $\times$ 0.8 = £80  
|    | £80 $\times$ 0.8 = £64  
|    | £64 $\times$ 0.8 = £51.20  
|    | £51.20 $\times$ 0.8 = £40.96  
|    | 4 weeks  
|    | A1  
|    | P1 for choosing a starting a position, either a variable such as $B$ or a specific amount such as £100  
|    | M1 for working through the weeks in some way  
|    | P1 for the process of finding amounts for weeks 3 and 4 to show the point at which the bank account first dips below 50% of the original balance  
| 30 |  
|    | i graph d  
|    | ii graph e  
|    | iii graph b  
|    | iv graph c  
|    | v graph f  
|    | vi graph a  
|    | B6  
|    | B1 for each correctly identified graph with reference to why, for example:  
|    | $f(x) = x^2$ is graph d as points are $(-2, 4)$, $(-1, 1)$, $(0, 0)$, $(1, 1)$, $(2, 4)$ and it is a parabola  
|    | $f(x) = 2x$, $x > 0$ $f(x) = -2x$, $x < 0$ is graph e as it is linear and has no negative $f(x)$ values; the gradient is 2 and –2  
| 31 a |  
|    | b $y = \frac{k}{x}$  
|    | $xy = k$ where $k$ is the constant of proportionality  
|    |  
|    | c Inverse proportion describes the relationship between two variables such that as one increases the other decreases.  
|    | $xy = k$  
|    | or $y = \frac{k}{x}$  
|    | Own problem, for example: It takes 5 men 10 days to dig a hole. The number of men, $y$, is inversely proportional to the number of days, $x$. How long would it take for ten men to dig the same hole? (5 days)  
|    | C1  
|    | C1 for clear explanation of inverse proportion  
|    | M1  
|    | M1 for correct equation  
|    | C1  
|    | C1 for clear question  
|    |  
32 a  

b  

\[ r = 6 \times 10^3 \text{m} \]

\[ F_1 \propto \frac{1}{(6 \times 10^3)^2} \]

\[ = \frac{1}{3.6 \times 10^7} \]

\[ F_2 \propto \frac{1}{(6 \times 10^3 + 12)^2} \]

\[ = \frac{1}{3.6144 \times 10^7} \]

\[ \frac{F_1}{F_2} = 0.996 \]

The difference is too small (reference part b).

<table>
<thead>
<tr>
<th>M1</th>
<th>2</th>
<th>M1 for correct function</th>
<th>H</th>
</tr>
</thead>
</table>

33  

The speed of the faster car is 40 mph.

\[ T = \frac{\text{20 miles}}{40 \text{mph}} = \frac{1}{2} \]

So they meet after 30 minutes.

Speeds are in the ratio 1 : 2 = 20 : 40 = 10 : 20

So the cars meet when the slower car has travelled 10 miles and the faster car has travelled 20 miles. It will take half an hour for a car travelling at 20 mph to go a distance of 10 miles.

<table>
<thead>
<tr>
<th>M1</th>
<th>2</th>
<th>M1 for recognising and using the ratio of the speeds</th>
<th>H</th>
</tr>
</thead>
</table>

| C1 | 3 | C1 for clarity of reasoning and explanation, diagram oe | --- |
### 34

\[4y = 2x^2\]

\[y = \frac{x^2}{2}\]

\[\text{gradient} = \frac{f(x_2) - f(x_1)}{2}\]

\[= \frac{4^2 - 2^2}{2}\]

\[= \frac{8 - 2}{2}\]

\[= 3\]

**M1** 2

**M1 for rearranging and substituting given values of x**

**H**

### 35

\[f(x + h) - f(x) = \frac{(2 + 2)^2 - 2^2}{2}\]

\[= \frac{4^2 - 2^2}{2}\]

\[= \frac{2}{2}\]

As above.

**M1** 2

**M1 for appropriate substitution to enable comparison with Q35**

**H**

### 36 a

\[f(x) = mx + c\]

The gradient = \[\frac{m(x + h) + c - (mx + c)}{h}\]

= \[\frac{mh + c - mx - c}{h}\]

= \[m\]

As working

**A1**

**P1**

2

**A1 for clarity of proof**

**P1 for accuracy with manipulation of function**

**H**

### 36 b

\[f(x) = \frac{x^2}{2}\] at \[x = 2\]

\[\text{The gradient} = \frac{(2 + h)^2 - \frac{x^2}{2}}{h}\]

\[= \frac{1}{2}(2x + h)\]

As working

**A1**

**P1**

2

**A1 for clear reasoning**

**P1 for accuracy with manipulation of function to show a gradient of 2**

**H**
\[
\frac{1}{2} (2x) = x
\]
At \(x = 2\), gradient = 2.
From the graph, points on the tangent are \((1, 0)\) and \((2, 2)\).
The gradient = \(\frac{2 - 0}{2 - 1} = 2\).

<table>
<thead>
<tr>
<th>37 a</th>
<th>(\£28\ 000 \times 1.05^3 = \£32\ 413.50)</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>(\£14\ 500 \times 1.05^n &gt; \£20\ 000)</td>
</tr>
<tr>
<td></td>
<td>(\frac{20000}{15400} = 1.4)</td>
</tr>
<tr>
<td></td>
<td>Try (n = 7) years. (\£14\ 500 \times 1.05^7 = \£20\ 402.96)</td>
</tr>
</tbody>
</table>

| 38   | Sycamore: \(4 \times 1.08^{11} = 9.327\) |
|      | \(4 \times 1.08^{12} = 10.073\) |
|      | Conifer: \(2 \times 1.15^{11} = 9.305\) |
|      | \(2 \times 1.15^{12} = 10.7\) |
|      | After 11 years, the sycamore is 9.326 m tall and the conifer is 9.305 m tall. After 12 years, the sycamore is 10.073 m tall and the conifer is 10.7 m tall. |

| M1   | M1 for gradient from points on the straight line |
| 5    |                                              |
| B1   | B1 for identification and use of multiplier |
| 2    |                                              |
| M1   | M1 for trial and improvement or reasoning to try 7 years |
| A1   | A1 cao |
| 3    |                                              |
| M1   | M1 for correct calculation method to find heights of trees |
| B1   | B1 for clarity of final reasoning |
| P1   | P1 for finding all four heights after 11 and 12 years |
| 3    |                                              |
### 39 a

\[ A \times 1.04^n = 2A \]

Divide both sides by \( A \).

1.04^2 = 2 
1.04^{10} = 1.48 \text{ (2 dp)} 
1.04^{15} = 1.80 \text{ (2 dp)} 
1.04^{20} = 2.19 \text{ (2 dp)} 
1.04^{17} = 1.95 \text{ (2 dp)} 
1.04^{18} = 2.03 \text{ (2 dp)} 

<table>
<thead>
<tr>
<th>( n )</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>18</td>
<td>1.48 (2 dp)</td>
</tr>
<tr>
<td>15</td>
<td>1.80 (2 dp)</td>
</tr>
<tr>
<td>20</td>
<td>2.19 (2 dp)</td>
</tr>
<tr>
<td>17</td>
<td>1.95 (2 dp)</td>
</tr>
<tr>
<td>18</td>
<td>2.03 (2 dp)</td>
</tr>
</tbody>
</table>

### b

\[ 10 \times \left( \frac{3}{5} \right)^n = 1 \]

\[ \left( \frac{3}{5} \right)^n = 0.1 \]

- \( 0.6^2 = 0.36 \)
- \( 0.6^3 = 0.07776 \)
- \( 0.6^4 = 0.1296 \)

#### 18 years

A1

M1

M1 for appropriate iterations to find number of bounces

### c

4 bounces

Own problem

C1

C1 for clarity, relevance and accuracy of own question

### 40 a

\[ f(x) = a(b)^x \]

<table>
<thead>
<tr>
<th>Day</th>
<th>Number of bacteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>5</td>
<td>32</td>
</tr>
<tr>
<td>6</td>
<td>64 = 2^6</td>
</tr>
</tbody>
</table>

\( 2^6 = 64 \)

The population doubles each day.

\[ a \] and \( b \) are constants.
\( a \) is the starting size of the population and so doesn’t change.
\( b \) is the multiplier (by how much the population grows each day) and the value of this doesn’t change.
\( x \) is a variable as it represents the changing number of days.

### b

A1

A1 cao

M1

M1 for correct iterations

A1 cao

C3: one mark for each explanation of \( a, b \) and \( x \)
<table>
<thead>
<tr>
<th>i</th>
<th>ii</th>
<th>iii</th>
<th>41</th>
<th>42 a</th>
<th>b</th>
<th>43</th>
<th>44 a</th>
<th>b</th>
</tr>
</thead>
<tbody>
<tr>
<td>F(x) = a(b)^x</td>
<td>Epidemic started by a single carrier so x_0 = 1.</td>
<td>p = e^{-\frac{7}{7}}</td>
<td>x = 1 + \frac{11}{x - 3}</td>
<td>if x_1 = 5</td>
<td>\begin{align*} F(x) &amp; = a(b)^x \ b &lt; 1 \text{ the population decreases.} \ b = 1 \text{ the population stays the same.} \ b &gt; 1 \text{ the population increases.} \end{align*}</td>
<td>x_{n+1} &amp; = R^n x_0 \ x_{10} &amp; = R^{10}</td>
<td>Show that… as workings.</td>
<td>if x_1 = 5 \ x_2 = 1 + \frac{11}{(5 - 3)} = 1 + \frac{11}{2} = 6.5 \ x_3 = 1 + \frac{11}{6.5 - 3} = 4.14286</td>
</tr>
</tbody>
</table>