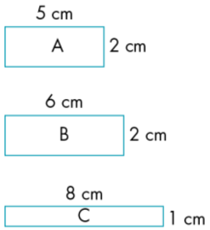
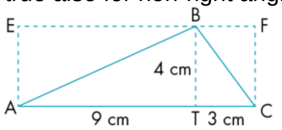
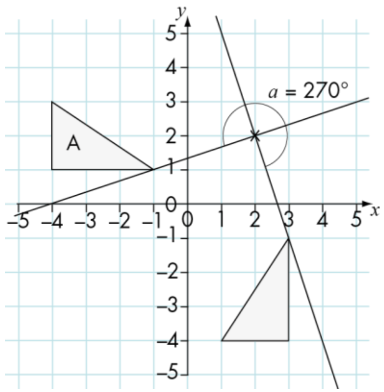
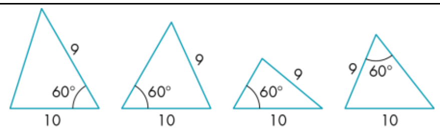
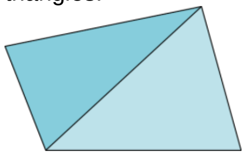
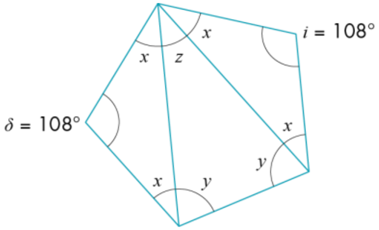
Guidance on the use of codes for this mark scheme	
M	Method mark
A	Accuracy mark
B	Working mark
C	Communication mark
P	Proof, process or justification mark
cao	Correct answer only
oe	Or equivalent
ft	Follow through

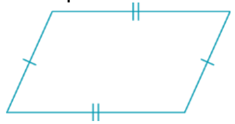
Question	Working	Answer	Mark	AO	Notes	Grade
1 a i ii iii iv		No Yes Yes No	B1 B1 B1 B1	2		B
b i		Assume third angle is 90°. 180° – 90° = 90°. So both the remaining angles must be acute. If the third angle is bigger than 90° both remaining must also be acute. If the third angle is acute you would need to make one of the other angles at least 90°.	C1 C1		C1 for full explanation C1 for clarity of explanation	
ii		Example with 2 acute angles.	B1		B1 for giving an example that works	
iii		Example with 1 obtuse angle.	B1		B1 for giving an example that works	
iv		The obtuse angles will be (90° + x) and (90° + y), adding these together you get 180° + x + y, which is more than the sum of the angles in a triangle, so it's impossible.	C1 C1		C1 for a correct explanation C1 for clear communication	
c		If you draw a line between two parallel lines, the two allied angles formed add up to 180°, which leaves nothing for a third angle.	C1 C1		C1 for clear explanation C1 for clarity of the communication	
			12			

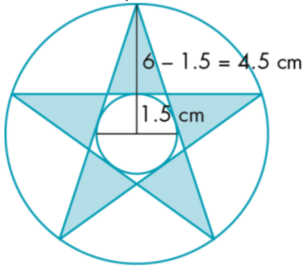
2		Sometimes. Here are examples, one of when it is not true and on of when it is true.  Shape A: perimeter of 14 cm area 10 cm^2 Shape B; perimeter of 16 cm area 12 cm^2 Shape C; perimeter of 18 cm area 8 cm^2 Statement is true for A and B, but false for B and C.	B1 B2 C1 **4**	2	B1 for sometimes B1 for example that shows it can be true B1 for example that shown it can be false C1 for clear communication of both	B
3		True. Demonstration of proof of area triangle equal to half area of rectangle true also for non-right angled triangle.  Area of ABT = $\frac{1}{2}$ of AEBT $= \frac{1}{2}$ of 36 cm^2 = 18 cm^2 Area of CTB = $\frac{1}{2}$ of CTBF $= \frac{1}{2}$ of 12 cm^2 = 6 cm^2 Area of triangle ABC = 18 + 6 = 24 cm^2 $= \frac{1}{2} \times 4 \times 12$ $= \frac{1}{2}$ × area AEFC	B1 C1 C1 **3**	2 3	B1 for true C1 for clear explanation C1 for concise communication with clear diagrams	B

4	 A rotation of 90°anticlockwise around point (2, 2).		P1 A1 A1 C1 **4**	3	P1 for a process of finding the centre of rotation A1 for indicating 90° anticlockwise (or 270°clockwise) A1 for indicating centre of rotation as (2, 2) C1 for full, clear description	B
5	Area of front and back = 2 × 12 × 25 = 600 m^2 Area of sides = 2 × 12 × 12 = 288 m^2 Area of openings = 40 × 2 × 1 = 80 m^2 Total area to be painted = 600 + 288 – 80 = 808 m^2 For 2 coats of paint, area = 2 × 808 = 1616 m^2 Number of litres of paint needed = 1616 ÷ 16 = 101 litres Number of cans of paint = 101 ÷ 10 = 10.1 11 cans are needed. Cost of paint = 11 × £25 = £275 Assume painters work 5 days per week. Number of days = 2 × 5 = 10 Cost of painters = 10 × 3 × 120 = £3600 Total cost = £275 + £3600 + £500 = £4375 Add 10%: £4375 × 1.1 = £4812.50 Add 20% VAT: £4812.50 × 1.2 = £5775	The builder should charge the council £5775.	M1 M1 A1 B1 M1 A1 M1 A1 M1 A1 A1 M1 M1 A1 C2 **16**	2 3	M1 for correct formula for area of rectangle M1 for correct method of finding total surface area A1 for 808 cao M1 for correct method of finding number of tins A1 for correct number of tins used M1 for method of finding cost of tins A1 for 275 cao M1 for method of calculating cost for two days A1 for 3600 cao A1 for 4375 cao M1 for correct calculation of 10% M1 for correct calculation of 20% A1 for correct total cost £5775 C1 for clear explanation marks with structure and technical use of language in explanation and C1 for stating any necessary assumptions	B

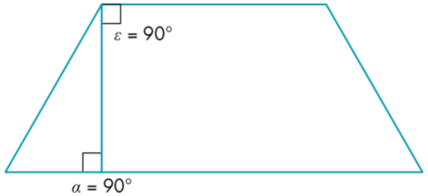
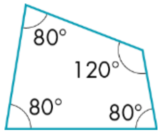
6 **a**	Area of face = 4^2 = 16 m^2 Area of circle = πr^2 Using π = 3.142 area of circle = $\pi 1.2^2$ = 4.524 48 m^2 Remaining SA of front face = 16 – 4.524 48 = 11.475 52 m^2 Total remaining surface area: front and back = 2 × 11.47552 = 22.95104 m^2 Area of other four sides = 4 × 16 = 64 m^2 Total = 64 + 22.951 04 = 86.951 04 m^2	87.0 m^2	M1 M1 A1 A1 A1 A1 A1	3	M1 for the correct method of finding area of a rectangle M1 for correct method of finding area of a circle A1 for correct area of circle A1 for correct area of face with circle. A1 for correctly combining front and back A1 for correct area of the other 4 sides A1 for correct total area, rounded to 2,3 or 4 sf	B
b	Volume of original cuboid = 4^3 = 64 m^3 Volume of cylinder = $\pi r^2 h$ = $\pi r^2 4$ = 4.524 48 × 4 = 18.097 92 m^3 Remaining volume = 64 – 18.097 92 = 45.902 08 m^3	45.9m^3	M1 A1 M1 A1 A1		M1 for correct method for finding volume of cube A1 for 64 M1 for correct method for finding volume of cylinder A1 for a correct volume of cylinder (any rounding) A1 for correct total volume, rounded to 2,3 or 4 sf	
c	Amount of light blue paint = outside area ÷ coverage of 1 litre of paint = 87 ÷ 9 = 9.666 Surface area inside cylinder = $2\pi rh$ = 2 × 3.142 × 1.2 × 4 = 30.1632 m^2 30.1632 ÷ 9 = 3.3515	Amount of light blue = 9.7 litres Amount of dark blue = 3.4 litres	M1 A1 M1 A1 A1 **17**		M1 for dividing total outside surface by 9 A1 for correct answer rounded to 1,2,3 or 4sf M1 for correct method of finding curved surface area A1 for a correct surface area (any rounding) A1 for correct answer to 2,3 or 4 sf	
7		Yes, he is correct. This is one of the conditions for being able to draw a triangle (SAS).	C1 **1**	3	C1 for clear communication that he is correct	M
8			B4 **4**	3	B1 for each different possible triangle shown and clearly labelled	M
9		The locus is none of these as it is a point, so d.	B1 C1 C1 **3**	3	B1 for stating d is the only correct option C1 for a clear explanation of why C1 for clear communication, using diagrams to illustrate answer	M

10 a	Angles in a triangle add up to 180°. You can split any quadrilateral into two triangles.  Therefore, the sum of the interior angles of any quadrilateral = 2 × 180º.		C1 C1 B1	2	C1 for clear explanation C1 for communication with clear diagram B1 for showing inter angles of quadrilateral = 2 × 180	M
b	 For the two outside triangles, use the sum of the angles in triangle = 180° and the interior angle of a regular pentagon = 108°. 108° + 2x = 180° 2x = 180° – 108° 2x = 72° x = 36° For the middle triangle, use interior angle of regular pentagon = 108°. y = 108° – x y = 108° – 36° y = 72° Using the sum of angles in triangle = 180°: z = 180 – 2y z = 180 – 144 z = 36º Or 2x + z = 108° x = 36° so 2 × 36° + z = 108° z + 72° = 108° z = 36°	Two triangles with one angle = 108° and two other equal angles of 36°. One triangle with one angle = 36° and two angles = 72°.	C1 C1 C1 M1 A1 M1 A1 M1 A1 C1 C1		C1 for a clear diagram correctly showing the three triangles C1 for identifying each angle of the triangles with symbols showing which angles are equal C1 for clear explanation M1 for correct method of finding angle x A1 for 36° M1 for correct method of finding angle y A1 for 72° M1 for correct method of finding angle z A1 for 36° C1 for clear argument and stating assumptions used C1 for use of diagram with clarity of explanation	
			14			

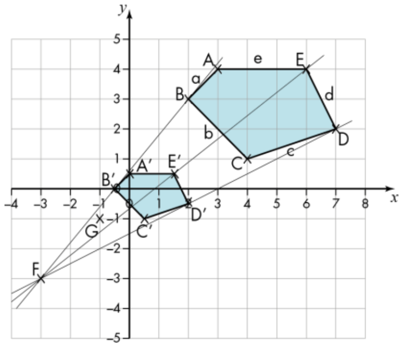
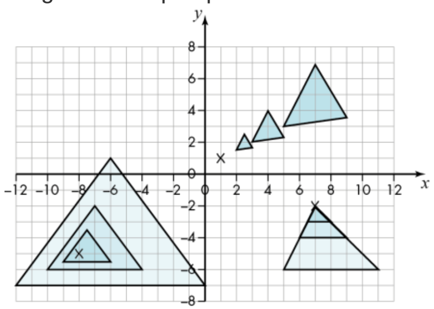
11		A line of symmetry has the same number of vertices on each side of the line, so there is an even number of vertices and therefore an even number of sides.	C2 C1 **3**	2	C1 for line of symmetry and number of vertices link C1 for reference to even number of vertices oe C1 for use of diagram to illustrate the answer	M
12 a **b** **c**		Suitable diagram, e.g.  Suitable diagram, e.g. as part a In a parallelogram opposite sides are equal. In a trapezium at least one set of opposite sides are parallel. Therefore every parallelogram is also a trapezium. 	B1 B1 B1 C1 **4**	2 3	B1 for a correct diagram B1 for a correct diagram B1 for a correct diagram of a parallelogram C1 for a correct explanation to support the diagram	M
13		Always true. To follow a path around the perimeter of any polygon, you must turn through a total of 360° to get back to where you started. Therefore the external angles of every polygon sum to 360º.	B1 C1 P1 **3**	2	B1 for always true C1 for a satisfactory explanation P1 for use of diagram to illustrate answer	M
14	Ratio = 6 : 5 : 7 6 + 5 + 7 = 18 Sum of the angles in a triangle = 180° and 180° ÷ 18 = 10° Therefore the angles are: 6 x 10° = 60° 5 x 10° = 50° 7 x 10° = 70° Check: 60 + 50 + 70 = 180	 60°, 50°, 70°	 M1 C1 M1 B3 P1 **7**	2	 M1 for summing parts of ratio C1 for clear statement regarding angle sum of triangle M1 for dividing 180° by 18 B1 for each correct angle found P1 for showing the check that the answers sum to 180°	M

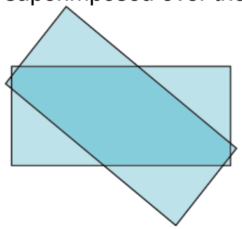
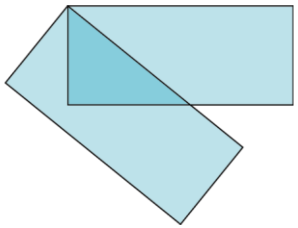
15	Assume the height of one large triangle is equal to the radius of the large circle, 6 cm, and its base is equal to the diameter of the small circle, 3 cm.	25 cm^2	C1	2	C1 for stating assumptions clearly	M
						
	Consider one shaded triangle. Its height is (6 – 1.5) = 4.5 cm		B1		B1 for correct triangle height	
	The shaded triangle and the large triangle shown are similar triangles, where		M1		M1 for a sensible way to estimate triangle base length	
	$\frac{\text{base of shaded triangle}}{3} = \frac{4.5}{6}$		A1		A1 for an accurate calculation at this point	
	Hence shaded base = $3 \times \frac{4.5}{6} = 2.25$					
	The area of one shaded triangle will be area = $\frac{1}{2} \times 2.5 \times 4.5 = 5.0625$ cm^2.		M1		M1 for correct method of finding area of triangle	
	As an estimate, call this 5 cm^2.		A1		A1 for an suitably estimated area (1 or 2 sf)	
	So a reasonable estimate for the area of the five shaded triangles could be: $5 \times 5 = 25$ cm^2		M1 A1 C1 P1		M1 for multiplying one area by 5 A1 for correct final estimation (integer value) C1 for clear explanation supporting the working P1 for clear diagrams illustrating the approach	
			10			

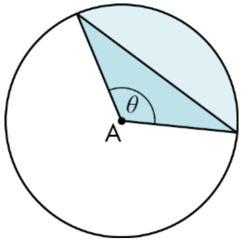
16 a **b**		The interior angle of an equilateral triangle is 60°. The interior angle of a square is 90°. The interior angle of a regular hexagon is 60°. All three are factors of 360° so these shapes will tessellate around a point. This is not true for other regular polygons as their interior angles are not factors of 360°. The interior angle of a regular octagon is 135°. The interior angle of a square is 90°. Using a similar argument to part a: 2 × 135° + 90° = 270° + 90° = 360°	C1 C1 C1 C1 P1 **8**	2	C1 for clear explanation of all three shapes C1 for use of clear diagrams to support the explanation C1 for clear explanation. C1 for clear explanation P1 for use of clear diagrams to support the explanation	M
17		All three sides (SSS). Two sides and the included angle (SAS). Two sides and a non-included angle (SSA). Two angles and a side (ASA or AAS).	B4 **4**	3	B1 for each correct statement	M

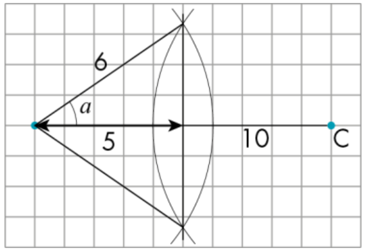
18 a		True. In a parallelogram opposite sides are parallel. In a rhombus, opposite sides are parallel and all sides are the same length. So a rhombus is a type of parallelogram. In a square all sides are the same length. So a rhombus with right angles must be a square.	B1 C1 C1	3	B1 for true C1 for clear explanation C1 for clear explanation	M
b		True. A rhombus must be a parallelogram (part a) but a parallelogram does not all sides the same length so it does not have to be a rhombus.	B1 C1		B1 for true C1 for clear explanation	
c			P1		P1 for clear use of diagrams to support explanation	
		True. Using the diagram of a trapezium above, you see each pair of angles between the parallel sides are allied angles, adding up to 180°, so each pair can only have a maximum of one acute angle, hence the whole shape can have no more than two acute angles.	B1 C1 P1		B1 for true C1 for clear explanation P1 for clear use of diagrams to support the explanation	
d		True. A quadrilateral can have three acute angles, e.g. 80°, 80°, 80° and 120°. 	B1 B1 M1		B1 for true B1 for clear explanation supported by a clear diagram M1 for clear use of a correct diagram: a diagram is essential	
			12			

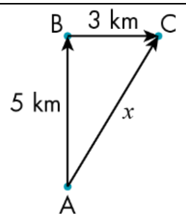
19		Look at the sides and/or angles you have been given and what you need to calculate. When the triangle has a right angle, use Pythagoras' theorem when you need to work out one side length and you know the other two side lengths. Otherwise, when the triangle does not have a right angle, use sine, cosine or tangent when you need to work out an angle or a side. Use the cosine rule to find angles when all sides of any triangle are known or to find the third side when two sides and the included angle are known. Use the sine rule when two sides and one angle other than the included angle are known, or two angles and one side are known.	C1 C1 C1 C1 **4**	3	C1 for clear Pythagoras explanation C1 for clear right angled trig explanation C1 for clear cosine rule explanation C1 for clear sine rule explanation	M
20 a i **ii** **b i** **ii**		A suitable simple reflection. A suitable reflection with a mirror line that is parallel to one of the sides of the shape. A suitable simple rotation. A suitable rotation with centre not on an extension of one of the sides of the shape.	B1 C1 B1 C1 **4**	2 3	B1 for a diagram of a simple reflection C1 for a clear explanation B1 for a diagram of a simple rotation C1 for a clear explanation	M

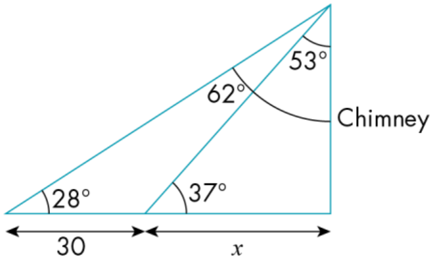
21 a		The lengths of the sides change by the scale factor. Angles in the shape stay the same.	C2	2 3	C1 for correct statement about the lengths C1 for correct statement about the angles	M
b		The scale factor and centre of enlargement.	B2		B1 for scale factor B1 for centre of enlargement	
c		Suitable explanation of enlargements with use of diagram to help explanation. For example: Draw lines connecting corresponding vertices on the shape and its enlargement. The centre of enlargement is where these lines cross. 	C1 C1		C1 for a clear explanation C1 for a good accurate diagram to support the explanation	
d i	Diagram to help explanation: 	Centre of enlargement outside the shape: the shape will move up and down a line that passes through the shape. Centre inside the original shape: the enlargement is either inside or around the shape depending on whether the scale factor is whole or fractional. When the centre is on a vertex the shape and enlargement share part of two sides. When the centre is on a side, the shape and enlargement shape part of the side. The image is smaller than the object.	C2 P1 C1		C1 for correct explanation of centre outside the shape C1 for correct explanation of centre within the shape P1 for clear diagrams used in both explanations C1 for clear explanation.	
ii	The image is smaller than the object.		C1		C1 for clear explanation.	
			11			

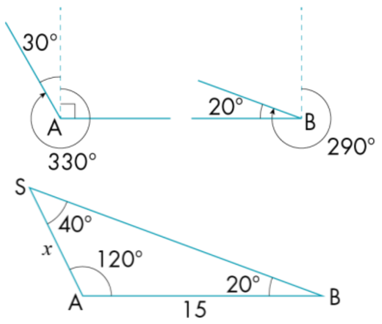
22 a		When a shape has been translated the orientation is the same. When it has been reflected its orientation is different.	C2 P1	2	C1 for comment about orientation staying the same in translation C1 for comment about orientation being different in rotation P1 for a clear diagram to support the explanation	M
b		Rotating a rectangle about its centre: all the vertices move and the image is superimposed over the object.  Rotating the rectangle about one of its vertices: all the other vertices move and, as the angle increases, the image is no longer superimposed over the object. 	C1 P1 C1 P1 **7**		C1 for clear explanation P1 for good diagram to support explanation C1 for clear explanation P1 for use of diagram to illustrate explanation	
23	Cross-sectional area is a quarter of circle with radius 1.5 cm and a rectangle 1.5 cm by 6.5 cm. area of quarter circle= $\frac{1}{4}\pi 1.5^2$ = 1.767 145 9 cm^2 Area of rectangle 1.5 × 6.5 = 9.75 cm^2 Total area = 1.767 145 9 + 9.75 = 11.517 146 cm^2 Total volume of wood is 11.517 146 × 12 000 = 138 205.75 cm^2. Convert this to m^2 by dividing by 1 000 000. Total volume = 0.138 205 75 m^2	138 000 cm^2 or 0.14 m^2	M1 A1 B1 B1 M1 A1 6	2 3	M1 for method of finding area of the quadrant A1 for any rounding to 4 or more sf B1 for 9.75 B1 for any rounding to 4 or more sf M1 for method of finding volume A1 for correct answer rounded to either 2 or 3 sf Accept alternative answer in cubic metres given correctly to 2 or 3 sf	M

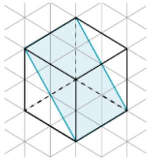
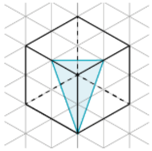
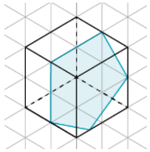
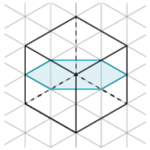
24		 Area of the sector = $\frac{\theta}{360}\pi r^2$ Area of segment = area of sector – area of triangle = $\frac{\theta}{360}\pi r^2 - \frac{1}{2}ab\sin\theta$ As a and b are both equal to r, this becomes: $\frac{\theta}{360}\pi r^2 - \frac{1}{2}r^2\sin\theta$ Factorising gives: $r^2(\frac{\theta}{360}\pi - \frac{1}{2}\sin\theta)$ as required.	B1 B1 M1 M1 M1 A1 C1 **7**	3	B1 for correct formula of area of sector B1 for correct formula of area of triangle M1 for correctly stating the combined equation for segment area M1 for correct use of r in triangle formula M1 for factorising A1 for correct factorisation C1 for clear use of diagram to support explanation	M

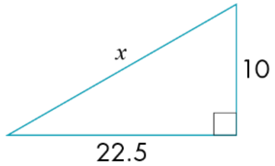
25				2		M
	Using trigonometry: $\cos \alpha = \frac{5}{6}$					
	$\alpha = \cos^{-1} \frac{5}{6}$		M1		M1 for correct trigonometric statement for angle	
	$\alpha = 33.557\ 31°$		A1		A1 for any rounded answer to 2 or more sf	
	Area of segment of one circle $= \frac{\theta}{360} \pi r^2 - \frac{1}{2} r^2 \sin\theta$		M1		M1 for correct segment formula	
	$= r^2(\frac{\theta}{360} \pi - \frac{1}{2} \sin \theta)$ where $\theta = 2 \times 33.6° = 67.2°$ and $r = 6$ cm.		M1		M1 for correct factorisation	
	Area $= 6^2(\frac{67.2}{360} \pi - \frac{1}{2} \sin 67.2°)$		A1		A1 for correct substitution of radius and a correct angle	
	$= 36(0.586\ 431 - \frac{1}{2} \sin 67.2°)$ $= 36(0.586\ 431 - 0.460\ 931\ 6)$ $= 4.517\ 979\ \text{cm}^2$		A1		A1 for correct answer to 2 or more sf	
	Area of overlap $= 2 \times 4.517\ 979\ \text{cm}^2$ $= 9.035\ 959\ \text{cm}^2$	9.0 cm^2	M1 A1 C1		M1 for multiplying by 2 A1 for correct answer to either 1 or 2 sf C1 for use of mathematical language and diagrams to support solution	
			9			

26 a		14 faces: the same as the number of polygons in the net.	B1 C1	2	B1 for the 14 faces C1 for clear explanation	M
b	Triangle 11 will move round to sit next to face 13, square 4 will move round to be next to face 12, leaving face 2 opposite to face 13.	13	B1		B1 for face 13	
c		I would create the shape first then draw what I see from above as the plan and from the side as the elevation. Once I have created the shape, I can measure the lengths and angles concerned.	C1 P1 C2		C1 for clear explanation P1 for use of diagrams to support the explanation C1 for an explanation of the plan C1 for explanation of elevation	
			7			
27	Circumference of wheel = πD $= \pi \times 68$ = 213.6283 cm		M1 A1	2	M1 for method of calculating circumference of wheel A1 for full unrounded answer	M
	10 km = 10 × 1000 × 100 cm = 1 000 000 cm		B1		B1 for use of 1 000 000 as a conversion factor either way round	
	Number of revolutions in 10 km = 1 000 000 cm ÷ 213.6283 cm = 4681.028	4681 complete rotations	M1 A1		M1 for correct division with common units A1 for cao	
			5			
28			C1 C1	2	C1 for use of a correct diagram C1 for explanation of how and why using Pythagoras	M
	$x^2 = 5^2 + 3^2$ $x^2 = 34$		M1 M1		M1 for correct application of Pythagoras' theorem M1 for correct method of finding hypotenuse	
	$x = \sqrt{34}$ = 5.830 951 9	x = 5.8 km	A1		A1 for correct rounding to 2 or 3 sf	
			5			

29	 Let c = the height of the chimney. $\frac{x}{c}$ = tan 53° $x = c$ tan 53° $\frac{(30 + x)}{c}$ = tan 62° 30 + x = c tan 62° x = c tan 62° – 30 Combining equations 1 and 2 to eliminate x: c tan 53° = c tan 62° – 30 Rearrange to get c on one side of the equation. 30 = c tan 62° – c tan 53° 30 = c(tan 62° – tan 53°) $c = \frac{30}{(\tan 62° - \tan 53°)}$ = 54.182 761 m	 54.2 m	C1 M1 A1 M1 A1 M1 M1 A1 **8**	2	C1 for clear correct diagram used M1 for correct use of trigonometry with x, c and angle 53° or 37° A1 for correct equation having x as subject M1 for correct use of trigonometry with x, c and angle 62° or 28° A1 for correct equation in format to combine with equation 1 M1 for correctly eliminating x M1 for correct equation with c as subject A1 for correct rounding to 2 or 3 sf	M

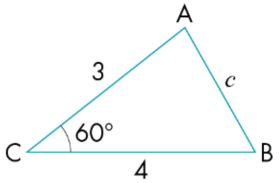
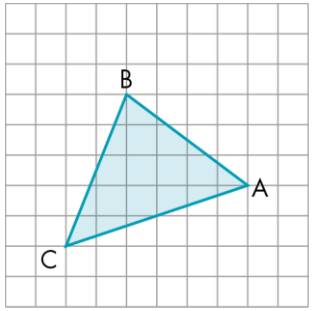
30	 Angle at A is 90° + (360° – 330°) = 120° Angle at B is 290° – 270° = 20° Angle at S is 180° – (120° + 20°) = 40° Use the sine rule. $\frac{x}{\sin 20°} = \frac{15}{\sin 40°}$ $x = 15 \times \frac{\sin 20°}{\sin 40°}$ $= 7.981\ 33$	8.0 km	C2 B1 B1 B1 M1 A1 **7**	2	C1 for diagram illustrating how angles found at A and B C1 for complete triangle drawn, showing all relevant data B1 for 120° B1 for 20° B1 for 40° M1 for use of sine rule A1 for correct answer rounded to 1, 2 or 3 sf	M
31 a **b** **c**		Sometimes true. It is not true if the number of individual cubes has fewer than 3 factors, including 1 and itself, for example, you cannot do it with 7 cubes (factors 1 and 7) You can only make one cuboid if the number of factors, including 1 is equal to 3, for example, with 21 cubes (factors 1, 3 and 7). You can make more than one cuboid if the number of cubes has more than 3 factors not including itself, for example, 30 (factors 1, 2, 3 and 5).	C1 C1 C1 C1 C1 **5**	2	C1 for sometimes true C1 for clear explanation for when not true C1 for clear explanation for when only 1 cuboid could be made C1 for clear explanation for when more than 1 cuboid could be made C1 for use of examples to illustrate the explanations	M

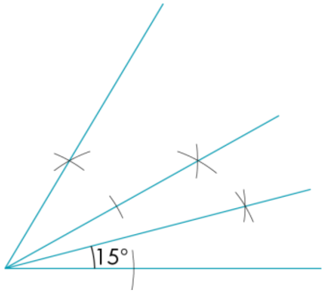
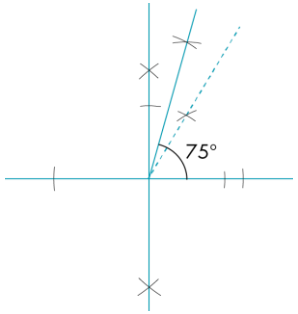
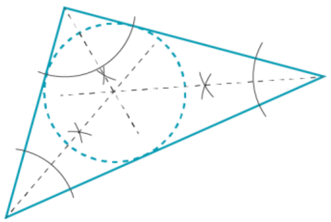
32 a		Yes. 	B1 P1	2	B1 for yes P1 for a clear diagram or explanation	M
b		Yes. 	B1 P1		B1 for yes P1 for a clear diagram or explanation	
c		Yes. 	B1 P1		B1 for yes P1 for a clear diagram or explanation	
d		Yes. 	B1 P1 **8**		B1 for yes P1 for a clear diagram or explanation	
33	Assume the cuboid has dimensions x, y and t. The surface area = $2(xy + xt + yt)$ Volume = xyt Doubling the lengths gives dimensions as $2x$, $2y$ and $2t$. So surface area $= 2(2x \times 2y + 2x \times 2t + 2y \times 2t)$ $= 2(4xy + 4xt + 4yt)$ $= 8(xy + xt + yt)$ which is 4 times the first area. and $V = 2x \times 2y \times 2t$ $= 8xyt$ which is 8 times the first volume.	False.	B1 C1 C1 P1 C1 B1 C1 B1 **8**	2	B1 for false C1 for surface area with either specific lengths or a generalisation C1 for volume with either specific lengths or a generalisation P1 for showing correct follow through of double the lengths C1 for a correct statement of SA with their data B1 for 4 times area C1 for a correct statement of volume with their data B1 for 8 times volume	M

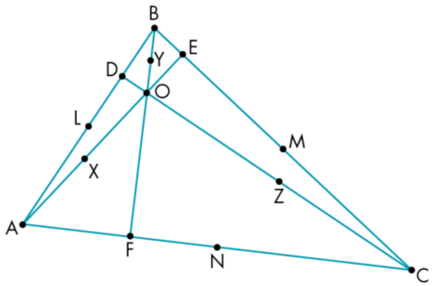
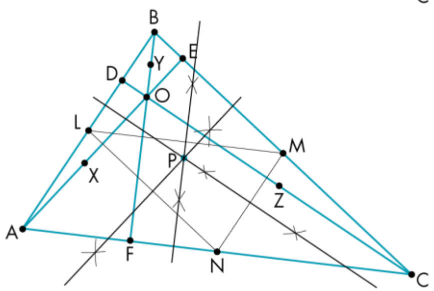
34	 Consider just half the shape, where x is the length of the string. Use Pythagoras' theorem. $x^2 = 10^2 + 22.5^2$ $= 606.25$ $x = \sqrt{606.25}$ $= 24.622\,145$ Two lengths of string will be 49.244 289 cm Subtract the original 45 cm Gives extension as 4.244 289	4.2 cm	C1 M1 M1 A1 A1 A1 **6**	2	C1 for clear diagram M1 for correct Pythagoras statement M1 for correct method of applying Pythagoras' theorem A1 for full answer A1 for double the initial x A1 for rounded answer to either 2 or 3 sf	M
35	Let AC = x, the length of the new road. Use Pythagoras' theorem. $x^2 = 4.9^2 + 6.3^2$ $= 63.7$ $x = \sqrt{63.7}$ $= 7.981\,228$ Current distance = 4.9 + 6.3 = 11.2 km Saving = 11.2 – 7.981 228 = 3.218 772 km	3.22 km	C1 M1 M1 A1 B1 M1 A1 **7**	2	C1 for use of a diagram to assist the explanation M1 for clear statement of Pythagoras M1 for correctly applying Pythagoras' theorem A1 for full answer B1 for 11.2 M1 for subtracting lengths A1 for correct rounding to 2 or 3 sf	M
36		Yes. $\theta = \sin^{-1}\frac{12}{15} = 53.13°$ = 53° to the nearest degree = 50° to 1 sf 12 cm has range of 11.5 to 12.5 cm 15 cm has range of 14.5 to 15.5 cm The smallest value for sin θ is $\frac{11.5}{15.5}$ which gives $\theta = \sin^{-1} 0.7419$ $\theta = 47.9°$ So there are values that round to 12 cm and 15 cm which will give an angle that rounds to 50°.	B1 P1 P1 B1 C1 **5**	2	B1 for yes P1 for showing that using trigonometry and rounding can give 50° P1 for showing the ranges of lengths of the sides B1 for showing the least possible value of the angle given the ranges C1 for final summary explaining that it is possible	M

37		Use Pythagoras' theorem. $AC^2 = 4^2 – (2\sqrt{2})^2$ $= 16 – 8 = 8$ $BC^2 = 8 = (2\sqrt{2})^2$ $= 8$ Hence BC = AC, an isosceles triangle.	M1 A1 M1 A1 C1 **5**	2	M1 for correct Pythagoras statement A1 for correct value of AC^2 M1 for finding BC^2 A1 for correct value of BC^2 C1 for clear explanation of sides being the same length	M
38	$AB^2 = 2^2 – 1^2$ $= 4 – 1 = 3$ $AB = \sqrt{3}$	$\sqrt{3}$ cm	M1 A1 C1 **3**	2	M1 for correct Pythagoras statement A1 for 3 C1 for a clear communication of the method used	M
39	cos 68° = –cos 112° = –cos 248° = 0.3746 cos 338° = 0.9271	cos 338° is the odd one out. All the others have the same numerical value (ignoring signs).	B1 C1 **2**	2	B1 for cos 338° C1 for a clear explanation	M
40 a i **ii** **b**	$\sin x + 1 = 2$ $\sin x = 1$ $x = \sin^{-1}1 = 90°$ $2 + 3\cos x = 1$ $3\cos x = 1 – 2 = –1$ $\cos x = -\frac{1}{3}$ $x = 109.5°$ and 360° – 109.5° = 250.5° cos 320° = 0.766 044 4 $\sin^{-1}$ 0.766 044 4 = 50° and 180° – 50° = 130°	$x = 90°$ $x = 109.5°, 250.5°$ $x = 50°$ and 130°	M1 A1 M1 A1 A1 A1 M1 A1 A1 **9**	2	M1 for $\sin x = 1$ A1 for 90° M1 for first step of solving equation A1 for correct statement of cos x A1 for correct angle to 1 dp A1 for correct angle to 1dp M1 for method of getting to $\sin^{-1}$ A1 for 50° A1 for 130°	M

41 a **b**	tan x = sin x ÷ cos x $\frac{\sqrt{8}}{4} \div \frac{3}{\sqrt{18}}$ $= \frac{\sqrt{8}}{4} \times \frac{\sqrt{18}}{3}$ $= \frac{\sqrt{144}}{12}$ $= \frac{12}{12} = 1$ $\tan^{-1} 1 = 45°$ Alternatively, use Pythagoras' theorem to work out the length of the third side, e.g. $(\text{side})^2 = 16 - 8 = 8$ side = $\sqrt{8}$ tan x = $\frac{\sqrt{8}}{\sqrt{8}} = 1$ $\tan^{-1} 1 = 45°$	tan x = 1 45°	C1 M1 A1 A1 M1 A1 **6**	2	C1 for communicating effectively how the sine and cos can be used to find tan (This could be tan $x = \frac{\sin x}{\cos x}$) M1 for correct use of tan A1 for correct combination of surds A1 for tan x = 1 M1 for use of inverse tan or recognising an isosceles triangle A1 for 45°	M
42		Using Pythagoras' theorem, the hypotenuse = $\sqrt{6+10} = \sqrt{16} = 4$ Then sin $x = \frac{\sqrt{6}}{4}$ and cos $x = \frac{\sqrt{10}}{4}$ Hence $(\sin x)^2 + (\cos x)^2 =$ $\frac{6}{16} + \frac{10}{16} = \frac{16}{16} = 1$	M1 A1 B1 B1 C1 **5**	2	M1 for correct Pythagoras statement A1 for 4 B1 for correct sin x B1 for correct cos x C1 for correct explanation	M

43	 Use the cosine rule. $c^2 = a^2 + b^2 - 2ab \cos C$ $= 9 + 16 - 2 \times 3 \times 4 \times \cos 60°$ $= 25 - 24 \times \frac{1}{2}$ $= 13$ $c = \sqrt{13}$	$\sqrt{13}$ cm	C1 M1 A1 B1 M1 A1 **6**	2	C1 for clear use of diagram M1 for correct cosine rule statement A1 for correct substitution B1 for $\cos 60° = \frac{1}{2}$ M1 for taking square root A1 for correct surd form	M
44	 	$\begin{pmatrix} 6 \\ 2 \end{pmatrix}$	B1 B1 **2**	2	B1 for correct diagram B1 for correct vector	M
45		No. To work out the return vector, multiply each component by –1. The return vector is $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$.	B1 B1 C1 **3**	2	B1 for No B1 for correct vector C1 for a clear explanation of what Joel should have done	M

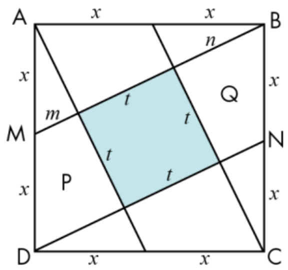
46 a			C3	2	C1 for showing construction of 60° C1 for showing bisection of 60° C1 for showing bisection of 30° to show 15°	H
b			C3		C1 for showing construction of 90° C1 for showing construction of 60° to leave a 30° angle C1 for bisecting that 30° angle to leave 75°	
			6			
47		Each angle bisector is the locus of points equidistant from the two arms or sides of the angle. Hence, where they all meet will be the only point that is equidistant from each of the three sides.  Hence a circle can be drawn inside the triangle, with this centre, that just touches each side of the triangle.	C2 P1	2	M1 for explanation referring to the bisector being equidistant M1 for the interpretation of the intersections P1 for the use of a diagram to help interpret the explanation	H
			3			

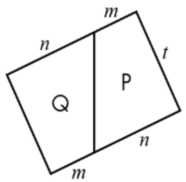
48 a–d			M1 A1 M1 A1 A1 M1 A1	2	M1 for method of constructing any midpoint A1 for all three midpoints correct and labelled M1 for constructing any perpendicular from a vertex to opposite face A1 for all three correct A1 all 3 feet correctly labelled M1 for constructing a midpoint of AO or BO or CO A1 for all midpoints correct and labelled	H
e			M1 A1		M1 for constructing a bisector of LM or LN or MN A1 for all three bisectors correct and point of intersection labelled P	
f		A circle, centre P, should pass through each of the nine labelled points.	C1 **10**		C1 for correct explanation	
49		AB = CD (given) ∠ABD = ∠CDB (alternate angles) ∠BAC = ∠DCA (alternate angles) so ΔABX ≡ ΔCDX (ASA)	C3 C1 **4**	2	C1 for correct statement with justification C1 for correct statement with justification C1 for correct statement with justification C1 for stating ASA within correct explanation	H
50		AB and PQ are the corresponding sides opposite the 50° angle but they are not equal in length.	C2 **2**	2	C1 for stating the corresponding side link C1 for complete clear statement	H

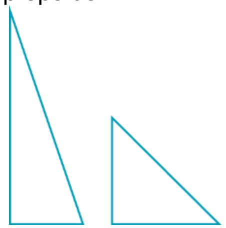
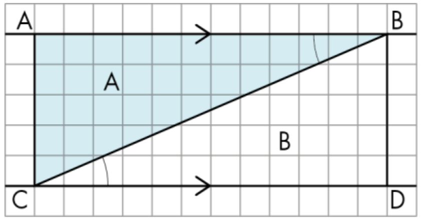
51 a		Joining adjacent midpoints forms a right-angled isosceles triangle, with the perpendicular sides each half the length of the side of the square. All four triangles are congruent. At each midpoint, there are two angles of 45°, leaving the vertex of the new shape being 90°. Since all the sides are equal and all the angles are 90°< the new shape is a square.	C2	2	C1 for explaining all sides are the same length C1 for explaining how all angles are 90°	H
b	Area of original rectangle = x^2 If the length of the side of the inscribed square is h, then: $h^2 = \left(\frac{x}{2}\right)^2 + \left(\frac{x}{2}\right)^2$ $h^2 = \frac{2x^2}{4}$ $h^2 = \frac{x^2}{2}$ Area of inscribed square is h^2 $= \frac{x^2}{2}$	The area of the inscribed square is half the area of the original square.	C1 M1 A1 A1 C1 **7**		C1 for communicating a given original side length and finding the length of a side of the inscribed square M1 for correctly using Pythagoras' theorem to find the length of the side of the new square A1 for correct expression of h^2 A1 for correctly stating area C1 for clear communication of complete solution	
52		Each internal angle of an octagon is 135°. Each internal angle of a hexagon is 120°. The sum of these two angles is 255°. The sum of the angles in a quadrilateral is 360° so the sum of the remaining angles is: 360° – 255° = 105°. The two remaining angles are equal, as the line joining through the vertices C, J, G and L (and thus through the obtuse angles) is a line of symmetry. JFG = 52.5°	C2 B1 M1 A1 C2 B1 **8**	2	C1 for explanation of 135° C1 for explanation of 120° B1 for 255° M1 for subtraction from 360° A1 for 105° C1 for explaining two angles are equal C1 for clear reasons given as to why B1 for correct 52.5°	H

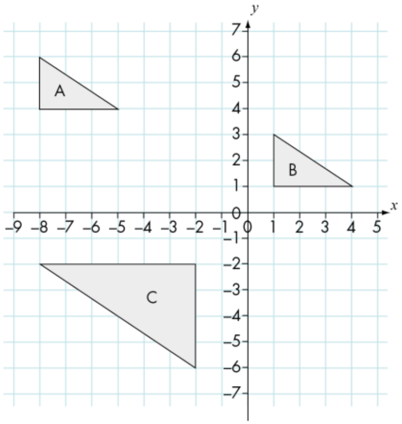
53		Use the sine rule in both triangles.	M1	2	M1 for use of sine rule	H
		$\frac{\sin P}{b} = \frac{\sin C}{p}$	A1		A1 for correct equation	
		$\frac{\sin A}{a} = \frac{\sin C}{c}$	A1		A1 for correct equation	
		$\sin A = \frac{a \sin C}{c}$				
		As $A = P$, then $\sin P = \sin A$	M1		M1 for equating both known angles	
		So $\frac{b \sin c}{p} = \frac{a \sin C}{c}$				
		Hence $\frac{b}{p} = \frac{a}{c}$	B1		B1 for correct statement linking p, a, b and c	
		Hence $\frac{p}{b} = \frac{c}{a}$	P1		P1 for clear communication of the full solution	
		So $p = \frac{bc}{a}$				
			6			

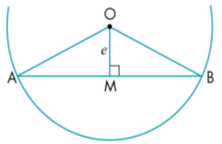
54		Use Pythagoras' theorem to write expressions for z^2. Triangle ABC: $z^2 = r^2 + (s + y)^2$ Triangle ABD: $z^2 = w^2 + (x + t)^2$ So $r^2 + (s + y)^2 = w^2 + (x + t)^2$ Now $r^2 = x^2 - s^2$ and $w^2 = y^2 - t^2$ Substitute for r^2 and w^2 in the equation: $x^2 - s^2 + (s + y)^2 = y^2 - t^2 + (x + t)^2$ Multiply out the brackets: $x^2 - s^2 + s^2 + 2sy + y^2$ $= y^2 - t^2 + x^2 + 2xt + t^2$ Simplify: $x^2 + 2sy + y^2 = y^2 + x^2 + 2xt$ $2sy = 2xt$ $sy = xt$ Alternatively: Assume T is the intersection of AD and BC. ACT is similar to BDT as both contain the same angles. (Angles ATC and BTD are vertically opposite and so are equal.) Hence by similarity $\frac{x}{y} = \frac{s}{t}$ So $xt = sy$	P1 P1 P1 P1 P1 C1 C1 **7**	2	P1 for combining expressions P1 for substitution P1 for multiplying out to get individual terms P1 for simplifying P1 for clear explanation C1 for $\frac{x}{y} = \frac{s}{t}$ C1 for clear communication of proof	H

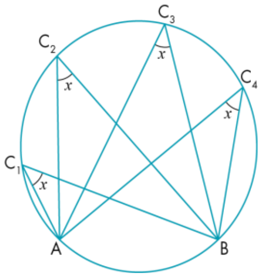
55 a		Take the smallest two sides, square each of them, add the squares together then find the square root of the sum. If this root is equal to the length of the longest side, then the triangle is right angled. If the root is smaller than the length of the longest side, then the angle is greater than 90°. If the root is larger than the length of the longest side then there is no angle greater than 90°.	C1 C1 C1 C1	2	C1 for correct Pythagoras statement C1 for clear explanation C1 for clear explanation C1 for clear explanation	H
b		In the first triangle the hypotenuse is $\sqrt{5^2+12^2}=\sqrt{139}=13$ In the second triangle, the hypotenuse is 12 with the unknown side $\sqrt{12^2-5^2}=\sqrt{119}$, which is between 10 and 11 cm. So they both have a short side of 5 cm. But the lengths of the other short side and the hypotenuse are different.	B1 B1 C1 C1 **8**		B1 for 13 B1 for side length between 10 and 11 or more accurate. C1 for explanation of what is the same C1 for clear explanation of what is different	
56		 In the top triangle ABM: $(m+t+n)^2=(2x)^2+x^2=5x^2$ So $m+t+n=x\sqrt{5}$ Putting together shape P and Q:	C1 M1 A1 M1 A1	3	C1 for clear diagram M1 for using a large triangle to identify parts A1 for correct equation M1 for method of adding P and Q	H

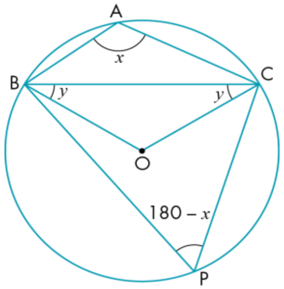
		area = $(m + n)t$			A1 for $(m + n)t$	
			M1		M1 for adding all separate components	
		So the area, A, of the whole shape is: top triangle ABM + bottom triangle DCN + (P + Q) + t^2				
		Area of ABM = $\frac{1}{2} \times 2x \times x = x^2$				
		Area of DCN = $\frac{1}{2} \times 2x \times x = x^2$	A1		A1 for correct statement of A	
		So total area = $x^2 + x^2 + (m + n)t + t^2$ $= 2x^2 + t(m + n + t)$	M1		M1 for substituting $(m + n + t)$	
		But $m + n + t = x\sqrt{5}$ So $A = 2x^2 + tx\sqrt{5}$				
		But the sides are of length $2x$, so $A = 4x^2$	B1		B1 for whole area $4x^2$	
		Then $2x^2 + tx\sqrt{5} = 4x^2$	M1		M1 for equating the two equations	
		and $tx\sqrt{5} = 2x^2$	A1		A1 for equation enabling t to be identified	
		$t\sqrt{5} = 2x$ Squaring each side gives:	M1		M1 for squaring	
		$5t^2 = 4x^2$ $t^2 = \frac{4x^2}{5}$	A1		A1 for correct expression for t^2	
		Whole area $A = 4x^2$	C1		C1 for clear explanation showing $\frac{1}{5}$ idea	
		Hence the middle square, t^2, is $\frac{1}{5}$ of A.	C1		C1 for complete clear solution	
			15			

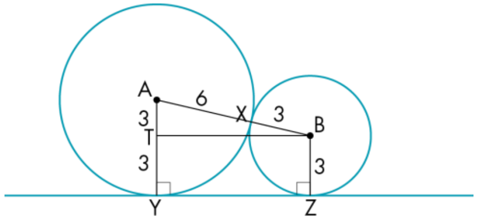
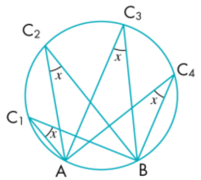
57 a		Not true. For example these are both right-angled triangles but their sides are not in proportion. 	B1 C1	2	B1 for not true C1 for clear explanation with an example drawn to illustrate	H
b		True. With an enlargement the final shape will be in the same *proportion* as the original so it will be similar.	B1 C1		B1 for true C1 for clear explanation	
c		True. All circles are in proportion to each other and so will be similar.	B1 C1		B1 for true C1 for clear explanation	
			6			
58 a		Any two regular polygons with the same number of sides will have the same angles so the ratio of lengths of the sides will be the same. This means the shapes will be similar.	C1 P1	2	C1 for clear example P1 for use of diagrams to support the explanation	H
b		 As the corresponding sides of triangle A and triangle B are the same, the two triangles are congruent, SSS. Therefore equivalent angles are the same. This demonstrates the corresponding angles between parallel lines are the same.	C1 P1		C1 for clear explanation P1 for use of diagram to clarify explanation	
			4			

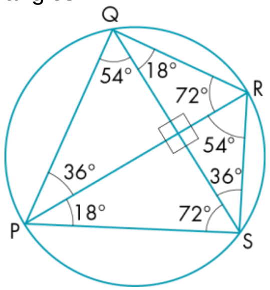
59 a			B1	2	B1 for accurate image B drawn	H
b			B2		B1 for enlargement of scale factor –2 B1 for correct enlargement to image C	
c		Enlargement with scale factor $-\frac{1}{2}$ about (–6, 2).	B2		B1 for enlargement scale factor $-\frac{1}{2}$ B1 for centre of enlargement (–6, 2)	
			5			
60		Enlargement with sf –3 about (1, 2).	B1	2	B1 for correct statement	H
		Drawing lines from the two vertices at the base of triangle A to their image points in B. The intersection shows the centre of enlargement.	C1		C1 for explanation of how centre of enlargement is found	
		The base of A is 2 squares wide, the base of B is 6 squares wide, and 6 is 3 times 2, so the sf is 3.	C1		C1 for explanation how sf is found	
			3			

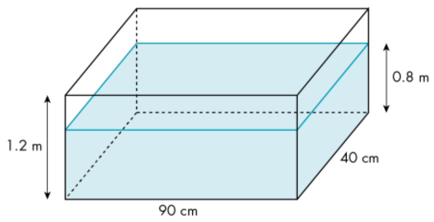
61 a		Examine the four possible starting points for the stamp. These are at the top right and bottom left of each side, allowing for 180° rotation of each side. Four rotations mean that each of these points is covered once.	C1	2	C1 for clear explanation	H
b		No, the machine would not detect the stamp placed on the top left-hand corner because none of the rotations will put the stamp in the top right-hand corner.	B1		B1 for No	
c		Four corners on each side could possibly be the 'top right'.	C1		C1 for clear explanation	
d		One way is to rotate about H and then rotate about one of the diagonals (call it D). Keep repeating the sequence H, D, H, D, … to check all eight corners.	C1		C1 for clear explanation	
			4			
62		 The hypotenuses (OA and OB) are the same, as each is a radius of the circle. OM is common to both triangles. OMA and OMB are both right angles. Triangles OAM and OBM are congruent, therefore AM = MB. Therefore, M is the midpoint of AB and the chord has been bisected, as required.	C4 C2	2	C1 for hypotenuse same C1 for OM common B1 for right angles C1 for congruency C1 for clear explanation and good use of mathematical language. C1 for use of diagram to support proof	H
			6			

63		Circle Theorem 1 states that the angle at the centre of a circle is twice the angle at the circumference when they are both subtended by the same arc. Theorem 1 tells you that any angle subtended from arc AB at a point C on the circumference is always half the angle subtended from arc AB at the centre.  Therefore, every possible angle subtended from arc AB at the circumference will correspond to the same angle at the centre. Hence all the angles subtended at the circumference will be equal.	P2 C1 C1 **4**	2	P1 for stating Theorem 1 P1 for extending this to this proof C1 for clear overall explanation and clarity C1 for use of diagram and mathematical language	H

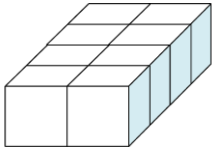
64		 Draw triangle BCP, where P is on the circumference in the opposite segment . Then angle BPC is 180° – x as BAC and BPC are opposite angles in a cyclic quadrilateral. Angle BOC is double BPC, which is 360° – 2x. Angle BCO will also be y, as BCO is an isosceles triangle. Angles in triangle OBC sum to 180°, so 2y + 360° – 2x = 180° and 2y = 2x – 180° Divide both sides by 2 to give: y = x – 90°.	C1 M1 A1 A1 B1 M1 A1 C1 **8**	2	C1 for clear use of diagram M1 for using point P A1 for 180 – x A1 for identifying 360° – 2x B1 for identifying BCO M1 for adding known angles together A1 for simplifying C1 for a clear, well presented solution	H
65	As ABCD is a cyclic quadrilateral, the opposite angles will sum to 180°. So 2x – 5° + 5y – 20° = 180° 2x + 5y = 205° ………(1) And 3y + 5 + 2x + 20° = 180° 2x + 3y = 155° ……..(2) Subtract (2) from (1): 2y = 50° y = 25° Substitute y into (2): 2x + 75° = 155° 2x = 80° x = 40°	y = 25° x = 40°	C1 M1 A1 A1 M1 A1 M1 A1 **8**	2	C1 for explanation of cyclic quadrilateral M1 for adding opposite angles A1 for first correct equation A1 for second correct equation M1 for method of eliminating one variable A1 for 25° M1 for method of substitution A1 for 40°	H

66	  Drop perpendiculars down to Y and X and complete the trapezium, with the top triangle being ABT, as shown. Length TB will be the same as YZ. Use Pythagoras' theorem on triangle ABT. $TB^2 = 9^2 - 3^2$ $= 81 - 9$ $= 72$ $TB = \sqrt{72}$ $= 8.485\ 281\ 4$	8.49 cm	C1 C1 C1 M1 A1 A1 **6**	3	C1 for good clear diagram showing perpendiculars to Y and Z C1 for clear diagram of ABT C1 for clear explanation of how the triangle ABT has been formed M1 for use of Pythagoras' theorem A1 for correct statement A1 for correct answer to 2 or 3 sf	H
67	Circle Theorem 1 states The angle at the centre of a circle is twice the angle at the circumference when they are both subtended by the same arc.Theorem 1 tells you that any angle subtended from arc AB at a point C on the circumference is always half the angle subtended from arc AB at the centre.  Therefore, every possible angle subtended from arc AB at the circumference will correspond to the same angle at the centre. Hence angles subtended at the circumference are equal.		P2 C1 C1 **4**	3	P1 for stating Theorem 1 P1 for extending this to this proof C1 for clear overall explanation and clarity C1 for use of diagram and mathematical language.	H

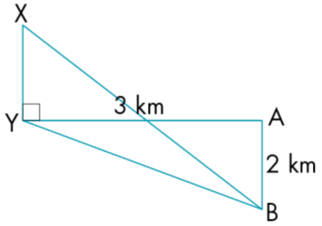
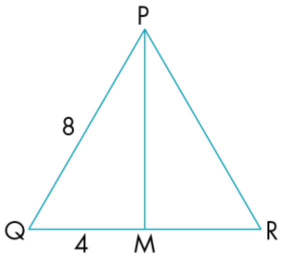
68 a	The angle PQS is the same as PRS, $3x$, as they are angles in the same quadrant (subtended by the same arc).		C1	2	C1 for clear explanation	H
	The angle sum of triangle PQT is 180°.		M1		M1 for using angles in a triangle	
	Hence $2x + 3x + 5x = 180°$ $10x = 180°$ $x = 18°$	$x = 18$	A1		A1 for 18	
b	Complete the diagram with all the known angles. 		C1		C1 for clear use of a diagram marked with all the angles	
	You can now see that: angle P = 54°, angle Q = 72°, angle R = 126°, angle S = 108° and angle STP = 90°.		C1		C1 for clear explanation	
			C1		C1 for finding all the angles and showing them clearly	
	The numbers 54, 72, 90, 108 and 126 form a sequence with a difference of 18.		C1		C1 for clear explanation of the number sequence found	
			7			

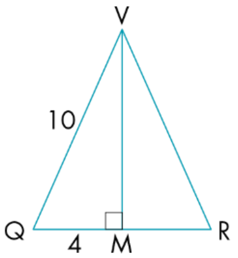
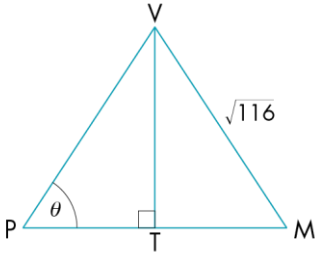
69		 As it is shown in the diagram, the proportion of the height of the container that is filled is $\frac{80}{120} = \frac{2}{3}$. The current volume of water is $90 \times 40 \times 80 = 288\,000$ cm^3. Assume you move the container so that the base is 40×120. The volume of water must stay the same and take its height as C cm. Then: $40 \times 120 \times C = 288\,000$ so $4800 \times C = 288\,000$ $C = 60$ Comparing this with the height, 90 cm: $\frac{60}{90} = \frac{2}{3}$ Now assume you move the container so the base is 90×120 then $90 \times 120 \times C = 288\,000$ $10\,800C = 288\,000$ $C = \frac{80}{3}$ Comparing this with the height, 40 cm: $\frac{80/3}{40} = \frac{80}{3 \times 40} = \frac{80}{120} = \frac{2}{3}$ So the proportion of the height of the container that is filled with water remains constant.	M1 A1 B1 C2 C2 C1 **8**	3	M1 for method of finding proportion of height A1 for $\frac{2}{3}$ B1 for 288 000 C1 for clear explanation of changing to a different base C1 for structure and clarity of explanation C1 for clear explanation of changing to a different base C1 for structure and clarity of explanation C1 for clear explanation	H

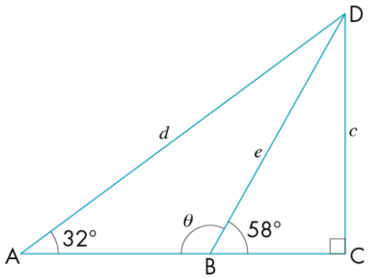
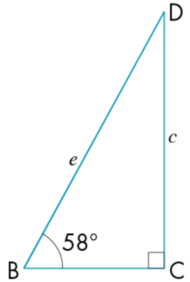
70 a		The end face or cross-section of a right prism is a polygon and is perpendicular to the vertical sides all through its length. The end faces (polygons) are the same shape and size. The volume is equal to the area of cross-section × the height of the prism. So if you know the height and the volume you know the area of the cross-section but not the dimensions or the shape of the base of the prism.	C3	2	C1 for perpendicular sides C1 for same base as top C1 for statement about base area	H
b	Assume the sides all have integer values. The formula for the volume of the prism will be: $a \times b \times 10 = 160$ cm^3 but if the base is a square, then $a = b$. $a^2 = 16$ cm^3 This has only one solution: $a = 4$.	Only 1 4 × 4 × 10	B1 B1 C1		B1 for answer 1 B1 for identifying the 4 × 4 × 10 C1 for clarity and detail of explanation	
			6			
71 a		The formula for the circumference of circle is $2\pi r$. This is just a length, r, multiplied by a number, so the answer will still be a length. The formula for the area of a circle is πr^2. This is a number multiplied by a length squared so this will be an area.	C2	2	C1 for explanation about circumference C1 for explanation about area	H
b		A formula for a surface area will include the product of two lengths, but a formula for a volume will include the product of three lengths.	C1		C1 for clear of explanation	
c		$\frac{1}{3}b^2h$ includes the produce of three lengths, $b \times b \times h$ and so must be the volume. $4\pi r^2$ includes the product of two lengths, $r \times r$, and so must be the area.	C1 C1 P1		C1 for clear explanation C1 for clear explanation P1 for demonstrating an understanding of how to help other people to understand this concept.	
			6			

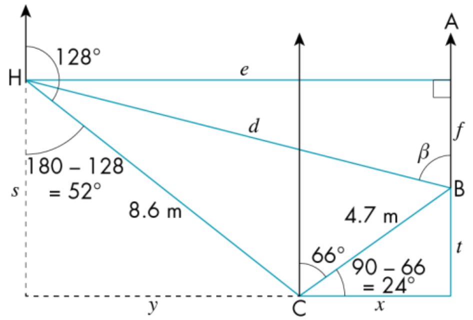
72		Identify the third arrangement.		2		H
		The single row of 8 cubes shows the cuboid must have a volume of 8 cubes.	B1		B1 for volume being 8 cubes	
		The missing arrangement is a 1 × 2 × 4 cuboid.	B1		B1 for missing dimensions	
			P1		P1 for clearly showing how to find the missing arrangement	
		Find the amount of string for each of the arrangements.				
		For the cuboid above, 1 × 2 × 4: L = 4 × 15, W = 2 ×15, H = 1 × 15 and S = 2 × 60 + 2 × 30 + 4 × 15 + 20	M1		M1 for correct method of using formula	
		= 260 cm	A1		A1 for 260	
		For the 1 × 1 × 8 cuboid: L = 8 × 15, W = 1×15, H = 1×15 and S = 2 × 120 + 2 × 15 + 4 × 15 + 20 = 350 cm	A1		A1 for 350	
		For the 2 × 2 × 2 cuboid: L = 2 × 15, W = 2 × 15, H = 2 × 15 and S = 2 × 30 + 2 × 30 + 4 × 30 + 20 = 260 cm	A1		A1 for 260	
		So the least amount of string is used in the 1 × 2 × 4 and the 2 × 2 × 2 arrangements.	B1 C1		B1 for identifying which uses least string C1 for clear complete solution.	
			9			

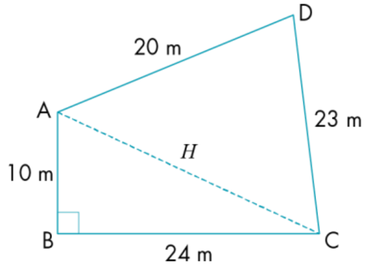
73		Surface area of end = 1000 ÷ 20 = 50 cm^2	 B1	3	 B1 for 50	H
		Consider a cuboid with a square end. Side length of square = $\sqrt{50}$ = 7.0711 Surface area (SA) = 50 + 50 + 4 × 7.0711 × 20 = 665.6 cm^2	 B1 B1		 B1 for 7.07 any rounding B1 for 665.6 any rounding 2sf or more	
		Consider a triangular prism with a triangular end. Area of triangular end = $\frac{1}{4}\sqrt{3}a^2 = 50$	 M1		 M1 for use of given formula	
		So $a^2 = \frac{200}{\sqrt{3}}$ = 115.470 054 a = 10.745 70 cm SA = 50 + 50 + 3 × 10.745 70 × 20 = 745 cm^2	A1 A1		A1 for 10.75 any rounding 2sf or more A1 for 745 any rounding 2sf or more	
		Consider a cylinder with a circular end. Then $\pi r^2 = 50$ r = $\sqrt{\frac{50}{\pi}}$ = 3.989 42 SA = 50 + 50 + π × 2 × 3.989 42 × 20 = 601.4 cm^2	M1 A1 A1		M1 for use of formula A1 for correct answer to any rounding A1 for 601.4 any rounding	
		Differences between the three surface areas:	C1		C1 for clear explanation	
		• surface area of the triangular prism is 79.4 cm^2 larger than the cuboid and 143.6 cm^2 larger than the cylinder	C1		C1 for clear explanation	
		• surface area of cuboid is 64.2 cm^2 larger than the cylinder.	C1		C1 for clear explanation	
		The larger the surface area, the more packaging material is required therefore the higher the production costs.	C1 **13**		C1 for a clear complete solution	

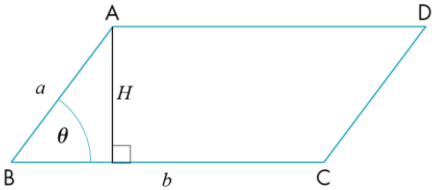
74	Use the cosine rule. $AB^2 = 2.1^2 + 1.8^2 - 2 \times 2.1 \times 1.8 \times \cos 70°$ $= 5.064\ 328$ $AB = \sqrt{5.06} = 2.250\ 406$ Extra distance = (2.1 + 1.8) – 2.25 km	1.65 km	M1 A1 A1 A1 A1 **5**	3	M1 for use of cosine rule A1 for correct substitution into cosine rule A1 for 5.06 rounded to 3 or more sf A1 for 2.25 rounded to 2 or 3 sf A1 1.65 rounded to 2 or 3 sf	H
75	 Use Pythagoras' theorem to find BY. $BY = \sqrt{9+4}$ $= 3.605\ 551\ 3$ Use the right-angled triangle XYB to find XY. $\frac{XY}{BY} = \tan 6°$ $XY = BY \times \tan 6°$ $= 0.378\ 958\ 7$ km $= 378.9587$ m	379 m	C1 M1 A1 M1 A1 A1 **6**	3	C1 for clear diagram showing all data M1 for use of Pythagoras' theorem A1 for BY unrounded M1 for use of tangent A1 for XY unrounded A1 for 379 in metres and rounded to 2 or 3 sf	H
76 a	 $PM = \sqrt{8^2 + 4^2} = \sqrt{80}$ $PM = 8.944\ 271\ 9$ cm	8.9 cm	C1 M1 A1	3	C1 for use of clear diagram M1 for use of Pythagoras' theorem A1 for 8.9 rounded to 2 or 3 sf	H
b			C1		C1 for clear diagram	

c	 VM = $\sqrt{10^2 + 4^2}$ = $\sqrt{116}$ VM = 10.770 33	10.8 cm	M1 A1 C1		M1 for use of Pythagoras' theorem A1 for 10.8 rounded to 2 or 3 sf C1 for use of clear diagram	
d	 cos P = $\dfrac{\frac{1}{2}\sqrt{80}}{\sqrt{116}}$ = 0.415 227 4 Angle VPM = 65.466 362° Vertical height of V above face PRQ is given by VT in diagram above. VT = $\sqrt{VM^2 + \left(\frac{1}{2}PM\right)^2}$ = $\sqrt{116 + \frac{1}{4} \times 80}$ = $\sqrt{136}$ = 11.661 904 Add the 15 cm of the base to give 26.661 904 cm.	65.5° 26.7 cm	M1 A1 C1 M1 A1 A1 **13**		M1 for use of correct cosine A1 for 65.5 rounded to 2 or 3 sf C1 for explanation of how the height is to be found M1 for use of Pythagoras' theorem A1 for 11.619 04 and any rounding 3 or more sf A1 for 26.7 rounded to 2 or 3 sf	
77	He needs to find half of AC (not AC) to make a right-angled triangle. i.e. cos x = $\dfrac{\frac{1}{2}\sqrt{208}}{10}$ = 0.721 110 = $\cos^{-1}$ 0.721 110 = 43.853 779°	43.9	C1 M1 A1 **3**	2	C1 for clear explanation M1 for correct cosine method A1 for 43.9 rounded to either 2 or 3 sf	H

78	 Using angles on a straight line. $\theta = 180° - 58° = 122°$ Angle ADB = $\alpha = 180° - (32° + 122°)$ $= 180° - 154° = 26°$ Use the sine rule. $\frac{e}{\sin 32°} = \frac{28}{\sin 26°}$ $e = \sin 32° \times \frac{28}{\sin 26°}$ $e = \sin 32° \times 63.8728169$ $= 63.847436$  Using trigonometry: $\sin 58° = \frac{c}{e}$ $c = e \sin 58°$ $= 28.7042537$ So the height of the tower is 29 m to the nearest metre.		M1 B1 M1 B1 M1 M1 A1 C1 **8**	3	M1 for method for finding angle ABD B1 for 26° M1 for use of sine rule B1 for e and any rounding M1 for correct use of trigonometry M1 for correct expression for c A1 for 29 answer rounded to 2 or 3 sf A1 for complete accurate solution well laid out.	H

79	$\sin 24° = \frac{t}{4.7}$			3		H
			C1		C1 for a clear diagram with all relevant lengths and angles marked	
	$t = 4.7 \sin 24°$		M1 A1		M1 for method of finding t A1 for correct expression of t	
	$\cos 24° = \frac{x}{4.7}$ $x = 4.7 \cos 24°$		M1 A1		M1 for method of finding x A1 for correct expression of x	
	$\sin 52° = \frac{y}{8.6}$ $y = 8.6 \sin 52°$		M1 A1		M1 for method of finding y A1 for correct expression of y	
	$\cos 52° = \frac{s}{8.6}$		M1		M1 for method of finding s	
	$s = 8.6 \cos 52°$		A1		A1 for correct expression of s	
	$e = x + y = 4.7\cos 24° + 8.6\sin 52°$ $= 11.070\,556$ km		M1 A1		M1 for method of finding e A1 for correct e, rounded to 4 or more sf	
	and $f = s - t = 8.6 \cos 52° - 4.7\sin 24°$ $= 3.383\,0264$ km		M1 A1		M1 for method of finding f A1 for correct f rounded to 4 or more sf	
		Use Pythagoras' theorem to find d. $d = \sqrt{e^2 + f^2}$	M1		M1 for using Pythagoras' theorem	
		$= \sqrt{134.002}$ $= 11.575\,93$ km $= 11.6$ km	A1 A1		A1 for correct d any value 3 sf or more A1 for correct d rounded to 3sf	
		Use trigonometry to find β. $\tan \beta = \frac{e}{f} = \frac{11.070556}{3.3830264}$	M1		M1 for method of finding β	
		$= 3.272382$ $\beta = \tan^{-1} 3.272\,382 = 73.007\,489°$	A1		A1 for β to 4 or more sf	

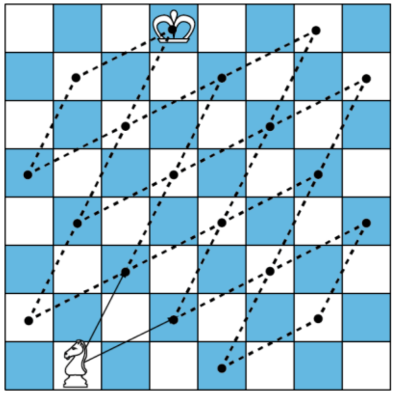
		The required bearing = 360° – β = 286.992 510 8° = 287°	A1 C1 **20**		A1 for final three-figure bearing C1 for effective use of diagrams and good use of mathematical language	
80	Use the sine rule to work out angle B. $\frac{\sin B}{10} = \frac{\sin 32°}{6}$ $\sin B = \frac{10\sin 32°}{6}$ $B = \sin^{-1} \frac{10\sin 32°}{6} = 62°$ But we know ABC is obtuse, so: B = 180° – 62° = 118° Then C = 180° – (118° + 32°) = 30° Use the sine area rule. Area = $\frac{1}{2}ab\sin C$ = $\frac{1}{2}$ × 10 × 6 × sin 30° = 15 cm^2	15 cm^2	M1 A1 A1 B1 M1 M1 A1 C1 **8**	3	M1 for use of sine rule A1 for 62°, accept any rounding to 2sf or more A1 for 118° and any rounding to 3sf or more B1 for 30° with any rounding to 2sf or more M1 for use of area sine rule M1 for correct substitution A1 for 15 rounded to 2 or 3 sf C1 for a complete solution clearly set out with correct mathematical language and symbols	H
81 a	 Use Pythagoras' theorem to work out H. $H = \sqrt{10^2 + 24^2}$ = 26 Use the cosine rule to work out angle D. $\cos D = \frac{20^2 + 23^2 - 26^2}{2 ´ 20 ´ 23} = 0.275$ D = 74.037 986°		C1 M1 A1 M1 A1	3	C1 for use of clear diagram M1 for use of Pythagoras' theorem A1 for 26 cao M1 for use of cosine rule A1 for D and any rounding to 2 or more sf	H

b	Use the area sine rule to work out the area of triangle ADC. Area = $\frac{1}{2}$ × 20 × 23 × sin 74.037 986° = 221.13217 m^2 Area of triangle ABC = $\frac{1}{2}$ × 24 × 10 = 120 m^2 Total area = 341.132 17 m^2 341.1321 ÷ 5 = 68.226 434	341 m^2 68 trees	M1 A1 M1 A1 C1 M1 A1 A1 **13**		M1 for use of area sine rule A1 for area correct to 3 or more sf M1 for triangle area rule A1 for 120 cao C1 for 341 correct to 2 or 3 sf M1 for dividing by 5 A1 for 68 cao A1 for complete clear solution with correct mathematical notation.	
82		 Let the vertical height of the parallelogram be H. $\frac{H}{a} = \sin\theta$ $H = a\sin\theta$ Area of parallelogram = base × height $= b \times a\sin\theta$ $= ab\sin\theta$	C1 M1 A1 M1 A1 C1 **6**	2	C1 for clear diagram showing vertical height. M1 for correct use of trigonometry to find the height A1 for correct height expression M1 for correct method of finding area of a parallelogram A1 for correct expression that will simplify to $ab\sin\theta$ C1 for complete, clear explanation with good mathematical notation	H

83	Use Pythagoras' theorem to work out DB. DB = $\sqrt{6^2+8^2}$ = 10 cm Use the cosine rule to work out angle A. $\cos A = \dfrac{9^2 + 14^2 - 10^2}{2 \times 9 \times 14}$ = 0.702 380 9 A = 45.381 658° Use the area sine rule to work out the area of triangle ADB. Area = $\frac{1}{2}$ × 9 × 14 × sin 45.381 658° = 44.843 478 cm^2 Area of triangle BDC = $\frac{1}{2}$ × 6 × 8 = 24 cm^2 Total area = 68.843 478 cm^2	69 cm^2	M1 A1 M1 A1 M1 A1 M1 A1 A1 C1 **10**	3	M1 for use of Pythagoras' theorem A1 for 10 cao M1 for use of cosine rule A1 for A with any rounding to 2 or more sf M1 for use of area sine rule A1 for area correct to 3 or more sf M1 for triangle area rule A1 for 24 cao A1 for 69 correct to 2 or 3 sf C1 for clear, complete solution with mathematical language	H

84 a	A possible triangle is one with sides 3, 4 and 5 (check, using Pythagoras' theorem). Then s = 6 and the area is $\sqrt{6 \times 3 \times 2 \times 1}$ = $\sqrt{36}$ = 6 Also, area = $\frac{1}{2}$ × base × height = $\frac{1}{2}$ × 3 × 4 = 6 which gives the same answer.		C2	2 3	C1 for showing an example that works C1 for clarity of explanation to support the example	H
b	Suppose the triangle has a side of 10 cm (you could use any number you like). The formula gives: $\sqrt{15 \times 5 \times 5 \times 5} = \sqrt{1875}$ = 43.3 cm^2 You could also use the area sine rule: area = $\frac{1}{2}ab\sin C$ = $\frac{1}{2}$ × 10 ×10 × sin 60° = 43.3 cm^2 which gives the same answer.		C2		C1 for showing an example that works C1 for clarity of explanation to support the example	
c	The sides of the triangle are 18 metres, 22 metres and 24 metres. So: s = 32 area = $\sqrt{32 \times 14 \times 10 \times 8}$ = $\sqrt{35\,480}$ = 189.314 55	189 m^2	B1 M1 A1		B1 for 32 M1 for correct substitution A1 for 189 to 2 or 3 sf	
d	A diagonal will divide the field into two triangles. Measure the four sides and a diagonal. Use the formula to find the area of each triangle separately and then add the answers together.		C1		C1 for clear explanation	
			8			
85	One example is: Start at coordinates (1, 1), then move through translation $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$ then through translation $\begin{pmatrix} 3 \\ -3 \end{pmatrix}$		B1 C3	3	Note there are a few different correct answers. Check all vectors sum to $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ B1 for having just 3 translations C1 for the describing first translation C1 for the describing second translation C1 for the describing final translation	H

	and finish with $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$ to get back to (1, 1).		**4**			
86 a i **ii** **iii** **iv** **b**		**b** – **a** –2**a** 2**b** – **a** 2**b – a** Parallel and equal in length.	B1 B1 B1 B1 B1 B1 **6**	2	B1 cao B1 cao B1 cao B1 cao B1 for parallel B1 for equal in length	H
87 a **b**		They lie in a straight line, AC = $1\frac{1}{2} \times \overrightarrow{AB}$ $\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB}$ 9**a** + 6**b** – (6**a** + 4**b**) = 3**a** + 2**b** $\overrightarrow{AB}$ is 2 × $\overrightarrow{BC}$ So AB : BC = 2 : 1	C1 C1 M1 A1 B1 **5**	2	C1 for clearly stating they lie in a straight line C1 for explaining one is a multiple of the other M1 for finding $\overrightarrow{BC}$ A1 for $\overrightarrow{BC}$ cao B1 for correct ratio	H
88 a i **ii** **iii** **b**		$\overrightarrow{BC}$ = **c** – **b** $\overrightarrow{NQ} = \frac{1}{2}\mathbf{b} + \frac{1}{2}(\mathbf{c} - \mathbf{b})$ $= \frac{1}{2}\mathbf{c}$ $\overrightarrow{MP} = \frac{1}{2}\mathbf{a} + \frac{1}{2}$AC $\overrightarrow{AC}$ = **c** – **a** $\overrightarrow{MP} = \frac{1}{2}\mathbf{a} + \frac{1}{2}(\mathbf{c} - \mathbf{a})$ $= \frac{1}{2}\mathbf{c}$ It is a parallelogram, because $\overrightarrow{NQ}$ = MP = $\frac{1}{2}\mathbf{c}$ and hence they are parallel and equal in length.	B1 M1 A1 M1 B1 A1 B1 C2 **9**	2 3	B1 cao M1 method of finding NQ A1 cao M1 method of finding MP B1 cao A1 cao B1 for parallelogram C1 for stating vectors are parallel C1 for stating vectors will be same length	H

89		$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$, $\overrightarrow{AC} = \overrightarrow{AO} + \overrightarrow{OC}$ $= -\mathbf{a} + 3\mathbf{b} - 2\mathbf{a}$ $\mathbf{= 3b - 3a}$ so $\overrightarrow{AC} = 3\ \overrightarrow{AB}$ and hence ABC is a straight line.	B1 M1 A1 C1 C1 **5**	2	B1 cao M1 for adding vectors A1 cao C1 for explaining the two vectors are multiples of each other C1 for explaining this gives a straight line	H
90		The knight can get to all the squares shown. Do not forget that you can use –**a** and –**b** as well as **a** and **b**. The starting position must match the question (bottom left white square). The lines show all the possible paths of the Knight, using **a**, **b**, **–a** and **–b**. There are many ways to reach the king. However, there are three ways to get to the king in the minimum of five moves. 	C1 C4 **5**	3	C1 is for a clear diagram to support the explanation, showing all the possible places the knight can move to C1 for explaining there are numerous ways to get to the king C1 for explaining there is a minimum of five moves to get to the king C1 for explaining there are only three ways to get to the king with these 5 minimum moves C1 for a clear cohesive explanation	H