<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Method mark</td>
</tr>
<tr>
<td>A</td>
<td>Accuracy mark</td>
</tr>
<tr>
<td>B</td>
<td>Working mark</td>
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<tr>
<td>C</td>
<td>Communication mark</td>
</tr>
<tr>
<td>P</td>
<td>Proof, process or justification mark</td>
</tr>
<tr>
<td>cao</td>
<td>Correct answer only</td>
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<tr>
<td>oe</td>
<td>Or equivalent</td>
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<td>ft</td>
<td>Follow through</td>
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<tr>
<td>Question</td>
<td>Working</td>
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<td>ii</td>
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<td>iii</td>
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<td>iv</td>
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</tbody>
</table>

**Total:** 12
### Sometimes. Here are examples, one of when it is not true and on of when it is true.

<table>
<thead>
<tr>
<th>Shape A</th>
<th>Perimeter of 14 cm</th>
<th>Area 10 cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shape B</td>
<td>Perimeter of 16 cm</td>
<td>Area 12 cm²</td>
</tr>
<tr>
<td>Shape C</td>
<td>Perimeter of 18 cm</td>
<td>Area 8 cm²</td>
</tr>
</tbody>
</table>

Statement is true for A and B, but false for B and C.

### True. Demonstration of proof of area triangle equal to half area of rectangle true also for non-right angled triangle.

![Diagram of triangle and rectangle](image)

Area of ABT = \( \frac{1}{2} \) of AEBT

\[ \frac{1}{2} \times 4 \times 12 = 24 \text{ cm}^2 \]

Area of CTB = \( \frac{1}{2} \) of CTBF

\[ \frac{1}{2} \times 3 \times 12 = 18 \text{ cm}^2 \]

Area of triangle ABC = 24 + 6 = 30 cm²

\[ \frac{1}{2} \times \text{area AEFC} \]
A rotation of 90° anticlockwise around point (2, 2).

Area of front and back = 2 \times 12 \times 25 = 600 \text{ m}^2
Area of sides = 2 \times 12 \times 12 = 288 \text{ m}^2
Area of openings = 40 \times 2 \times 1 = 80 \text{ m}^2

Total area to be painted = 600 + 288 – 80 = 808 \text{ m}^2
For 2 coats of paint, area = 2 \times 808 = 1616 \text{ m}^2
Number of litres of paint needed = 1616 \div 16 = 101 \text{ litres}
Number of cans of paint = 101 \div 10 = 10.1
11 cans are needed.
Cost of paint = 11 \times £25 = £275

Assume painters work 5 days per week.
Number of days = 2 \times 5 = 10
Cost of painters = 10 \times 3 \times 120 = £3600
Total cost = £275 + £3600 + £500 = £4375
Add 10\%: £4375 \times 1.1 = £4812.50
Add 20\% VAT: £4812.50 \times 1.2 = £5775

The builder should charge the council £5775.
### 6 a
Area of face = $4^2 = 16 \text{ m}^2$
Area of circle = $\pi r^2$
Using $r = 3.142$
area of circle = $\pi \times 1.2^2$
= $4.524 \text{ 48 m}^2$
Remaining SA of front face = $16 - 4.524 \text{ 48 m}^2$
= $11.475 \text{ 52 m}^2$
Total remaining surface area:
front and back = $2 \times 11.47552$
= $22.95104 \text{ m}^2$
Area of other four sides = $4 \times 16 = 64 \text{ m}^2$
Total = $64 + 22.95104 = 86.95104 \text{ m}^2$

### b
Volume of original cuboid = $4^3 = 64 \text{ m}^3$
Volume of cylinder = $\pi r^2h$
= $\pi r^24$
= $4.524 \text{ 48 m}^2 \times 4$
= $18.097 \text{ 92 m}^2$
Remaining volume = $64 - 18.09792$
= $45.902 \text{ 08 m}^3$

### c
Amount of light blue paint
= outside area + coverage of 1 litre of paint
= $87 \div 9 = 9.666$
Surface area inside cylinder = $2\pi rh$
= $2 \times 3.142 \times 1.2 \times 4$
= $30.1632 \text{ m}^2$
Amount of light blue = $9.7$ litres
Amount of dark blue = $3.4$ litres

<table>
<thead>
<tr>
<th>M1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>M1 for the correct method of finding area of a rectangle</td>
<td></td>
</tr>
<tr>
<td>M1 for correct method of finding area of a circle</td>
<td></td>
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<tr>
<td>A1</td>
<td>3</td>
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<tr>
<td>A1 for correct area of circle</td>
<td></td>
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<tr>
<td>A1</td>
<td>3</td>
</tr>
<tr>
<td>A1 for correct area of face with circle.</td>
<td></td>
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<tr>
<td>A1</td>
<td>3</td>
</tr>
<tr>
<td>A1 for correctly combining front and back</td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>3</td>
</tr>
<tr>
<td>A1 for correct area of the other 4 sides</td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>3</td>
</tr>
<tr>
<td>A1 for correct total area, rounded to 2,3 or 4 sf</td>
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</tr>
<tr>
<td>M1</td>
<td>3</td>
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<tr>
<td>M1 for correct method for finding volume of cube</td>
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<tr>
<td>A1</td>
<td>3</td>
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<tr>
<td>A1 for 64</td>
<td></td>
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<tr>
<td>M1</td>
<td>3</td>
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<tr>
<td>M1 for correct method for finding volume of cylinder</td>
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<tr>
<td>A1</td>
<td>3</td>
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<tr>
<td>A1 for a correct volume of cylinder (any rounding)</td>
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<tr>
<td>A1</td>
<td>3</td>
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<tr>
<td>A1 for correct total volume, rounded to 2,3 or 4 sf</td>
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<tr>
<td>M1</td>
<td>3</td>
</tr>
<tr>
<td>M1 for dividing total outside surface by 9</td>
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<tr>
<td>A1</td>
<td>3</td>
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<tr>
<td>A1 for correct answer rounded to 1,2,3 or 4sf</td>
<td></td>
</tr>
<tr>
<td>M1</td>
<td>3</td>
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<tr>
<td>M1 for correct method of finding curved surface area</td>
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<tr>
<td>A1</td>
<td>3</td>
</tr>
<tr>
<td>A1 for a correct surface area (any rounding)</td>
<td></td>
</tr>
<tr>
<td>A1</td>
<td>3</td>
</tr>
<tr>
<td>A1 for correct answer to 2,3 or 4 sf</td>
<td></td>
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</tbody>
</table>

### 7

<table>
<thead>
<tr>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
</tr>
<tr>
<td>C1</td>
</tr>
<tr>
<td>C1 for clear communication that he is correct</td>
</tr>
<tr>
<td>B1</td>
</tr>
<tr>
<td>B1 for each different possible triangle shown and clearly labelled</td>
</tr>
<tr>
<td>C1</td>
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<tr>
<td>C1 for stating d is the only correct option</td>
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<tr>
<td>C1</td>
</tr>
<tr>
<td>C1 for a clear explanation of why</td>
</tr>
<tr>
<td>B1</td>
</tr>
<tr>
<td>B1 for stating d is the only correct option</td>
</tr>
<tr>
<td>C1</td>
</tr>
<tr>
<td>C1 for clear communication, using diagrams to illustrate answer</td>
</tr>
</tbody>
</table>

### 9

| A | 5 cm |
| B | 5 cm |

The locus is none of these as it is a point, so d.
### 10a

Angles in a triangle add up to 180°. You can split any quadrilateral into two triangles.

Therefore, the sum of the interior angles of any quadrilateral = 2 × 180°.

![Diagram of a quadrilateral and two triangles]

For the two outside triangles, use the sum of the angles in triangle = 180° and the interior angle of a regular pentagon = 108°.

\[108° + 2x = 180°\]
\[2x = 180° - 108°\]
\[2x = 72°\]
\[x = 36°\]

For the middle triangle, use interior angle of regular pentagon = 108°.

\[y = 108° - x\]
\[y = 108° - 36°\]
\[y = 72°\]

Using the sum of angles in triangle = 180°:

\[z = 180° - 2y\]
\[z = 180° - 144°\]
\[z = 36°\]

Or \[2x + z = 108°\]
\[x = 36°\]

so \[2 \times 36° + z = 108°\]
\[z + 72° = 108°\]
\[z = 36°\]

Two triangles with one angle = 108° and two other equal angles of 36°.

One triangle with one angle = 36° and two angles = 72°.

### 10b

![Diagram of a quadrilateral with angles labeled]

For the two outside triangles, use the sum of the angles in triangle = 180° and the interior angle of a regular pentagon = 108°.

\[108° + 2x = 180°\]
\[2x = 180° - 108°\]
\[2x = 72°\]
\[x = 36°\]

For the middle triangle, use interior angle of regular pentagon = 108°.

\[y = 108° - x\]
\[y = 108° - 36°\]
\[y = 72°\]

Using the sum of angles in triangle = 180°:

\[z = 180° - 2y\]
\[z = 180° - 144°\]
\[z = 36°\]

Or \[2x + z = 108°\]
\[x = 36°\]

so \[2 \times 36° + z = 108°\]
\[z + 72° = 108°\]
\[z = 36°\]

Two triangles with one angle = 108° and two other equal angles of 36°.

One triangle with one angle = 36° and two angles = 72°.
| 11 | A line of symmetry has the same number of vertices on each side of the line, so there is an even number of vertices and therefore an even number of sides. | C2 | 2 | C1 for line of symmetry and number of vertices link C1 for reference to even number of vertices oe C1 for use of diagram to illustrate the answer | M |
| 12 a | Suitable diagram, e.g. | B1 | 2 | B1 for a correct diagram | M |
|  | Suitable diagram, e.g. as part a | 3 |  |  |  |
|  | In a parallelogram opposite sides are equal. In a trapezium at least one set of opposite sides are parallel. Therefore every parallelogram is also a trapezium. | B1 | 1 | B1 for a correct diagram |  |
|  | | C1 |  | C1 for a correct explanation to support the diagram |  |
| 13 | Always true. To follow a path around the perimeter of any polygon, you must turn through a total of 360° to get back to where you started. Therefore the external angles of every polygon sum to 360°. | B1 | 2 | B1 for always true C1 for a satisfactory explanation | M |
|  | | C1 |  | P1 for use of diagram to illustrate answer |  |
|  | | P1 |  |  |  |
| 14 | Ratio = 6 : 5 : 7 6 + 5 + 7 = 18 6 x 10° = 60° 5 x 10° = 50° 7 x 10° = 70° 60°, 50°, 70° | M1 | 2 | M1 for summing parts of ratio C1 for clear statement regarding angle sum of triangle M1 for dividing 180° by 18 | M |
|  | 60, 50, 70° | C1 |  |  |
|  | | M1 |  |  |
|  | Check: 60 + 50 + 70 = 180 | B3 |  | B1 for each correct angle found |  |
|  | | P1 |  | P1 for showing the check that the answers sum to 180° |  |
Assume the height of one large triangle is equal to the radius of the large circle, 6 cm, and its base is equal to the diameter of the small circle, 3 cm.

Consider one shaded triangle. Its height is \((6 - 1.5) = 4.5\) cm.

The shaded triangle and the large triangle shown are similar triangles, where

\[
\frac{\text{base of shaded triangle}}{3} = \frac{4.5}{6}
\]

Hence shaded base = \(3 \times \frac{4.5}{6} = 2.25\)

The area of one shaded triangle will be

\[
\text{area} = \frac{1}{2} \times 2.5 \times 4.5 = 5.0625 \text{ cm}^2.
\]

As an estimate, call this 5 cm².

So a reasonable estimate for the area of the five shaded triangles could be:

\(5 \times 5 = 25 \text{ cm}^2\)

<table>
<thead>
<tr>
<th>15</th>
<th>25 cm²</th>
<th>C1 2</th>
<th>C1 for stating assumptions clearly</th>
<th>M</th>
</tr>
</thead>
</table>
16 a

b

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>16 a</td>
<td>The interior angle of an equilateral triangle is 60°. The interior angle of a square is 90°. The interior angle of a regular hexagon is 60°. All three are factors of 360° so these shapes will tessellate around a point. This is not true for other regular polygons as their interior angles are not factors of 360°. The interior angle of a regular octagon is 135°. The interior angle of a square is 90°. Using a similar argument to part a: (2 \times 135° + 90° = 270° + 90° = 360°)</td>
<td>C1</td>
<td>2</td>
<td>C1 for clear explanation of all three shapes C1 for use of clear diagrams to support the explanation</td>
<td>M</td>
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<tr>
<td>17</td>
<td>All three sides (SSS). Two sides and the included angle (SAS). Two sides and a non-included angle (SSA). Two angles and a side (ASA or AAS).</td>
<td>B4</td>
<td>3</td>
<td>B1 for each correct statement</td>
<td>M</td>
<td></td>
</tr>
<tr>
<td>18 a</td>
<td>True. In a parallelogram opposite sides are parallel. In a rhombus, opposite sides are parallel and all sides are the same length. So a rhombus is a type of parallelogram. In a square all sides are the same length. So a rhombus with right angles must be a square.</td>
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<tr>
<td>b</td>
<td>True. A rhombus must be a parallelogram (part a) but a parallelogram does not all sides the same length so it does not have to be a rhombus.</td>
<td></td>
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<tr>
<td>c</td>
<td>True. Using the diagram of a trapezium above, you see each pair of angles between the parallel sides are allied angles, adding up to 180°, so each pair can only have a maximum of one acute angle, hence the whole shape can have no more than two acute angles.</td>
<td></td>
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<tr>
<td>d</td>
<td>True. A quadrilateral can have three acute angles, e.g. 80°, 80°, 80° and 120°.</td>
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<tr>
<td>19</td>
<td>Look at the sides and/or angles you have been given and what you need to calculate. When the triangle has a right angle, use Pythagoras' theorem when you need to work out one side length and you know the other two side lengths. Otherwise, when the triangle does not have a right angle, use sine, cosine or tangent when you need to work out an angle or a side. Use the cosine rule to find angles when all sides of any triangle are known or to find the third side when two sides and the included angle are known. Use the sine rule when two sides and one angle other than the included angle are known, or two angles and one side are known.</td>
<td>C1 3</td>
<td>C1 for clear Pythagoras explanation C1 C1 for clear right angled trig explanation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 a i</td>
<td>A suitable simple reflection. A suitable reflection with a mirror line that is parallel to one of the sides of the shape.</td>
<td>B1 2</td>
<td>B1 for a diagram of a simple reflection B1 C1 for a clear explanation</td>
<td></td>
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<td></td>
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<tr>
<td>20 a ii</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 b i</td>
<td>A suitable simple rotation.</td>
<td>B1</td>
<td>B1 for a diagram of a simple rotation</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 b ii</td>
<td>A suitable rotation with centre not on an extension of one of the sides of the shape.</td>
<td>C1</td>
<td>C1 for a clear explanation</td>
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</tbody>
</table>
The lengths of the sides change by the scale factor. Angles in the shape stay the same.

The scale factor and centre of enlargement.

Suitable explanation of enlargements with use of diagram to help explanation. For example: Draw lines connecting corresponding vertices on the shape and its enlargement. The centre of enlargement is where these lines cross.

Centre of enlargement outside the shape: the shape will move up and down a line that passes through the shape.
Centre inside the original shape: the enlargement is either inside or around the shape depending on whether the scale factor is whole or fractional.
When the centre is on a vertex the shape and enlargement share part of two sides.
When the centre is on a side, the shape and enlargement share part of the side. The image is smaller than the object.
### 22 a

<table>
<thead>
<tr>
<th>Method</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>When a shape has been translated the orientation is the same. When it has been reflected its orientation is different.</td>
<td>C2</td>
</tr>
<tr>
<td>Rotating a rectangle about its centre: all the vertices move and the image is superimposed over the object.</td>
<td>P1</td>
</tr>
<tr>
<td>Rotating the rectangle about one of its vertices: all the other vertices move and, as the angle increases, the image is no longer superimposed over the object.</td>
<td>C1</td>
</tr>
</tbody>
</table>

### 22 b

<table>
<thead>
<tr>
<th>Method</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1 for comment about orientation staying the same in translation C1 for comment about orientation being different in rotation P1 for a clear diagram to support the explanation</td>
<td>M</td>
</tr>
<tr>
<td>C1 for clear explanation P1 for good diagram to support explanation</td>
<td></td>
</tr>
</tbody>
</table>

### 23

<table>
<thead>
<tr>
<th>Method</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cross-sectional area is a quarter of circle with radius 1.5 cm and a rectangle 1.5 cm by 6.5 cm. area of quarter circle $= \frac{1}{4}\pi(1.5)^2$ Area of rectangle $1.5 \times 6.5 = 9.75 \text{ cm}^2$ Total area $= 1.7671459 + 9.75 = 11.517146 \text{ cm}^2$ Total volume of wood is $11.517146 \times 12000 = 138205.75 \text{ cm}^2$. Convert this to m$^2$ by dividing by 1000000. Total volume $= 0.13820575 \text{ m}^2$</td>
<td>M</td>
</tr>
<tr>
<td>M1 A1 B1</td>
<td></td>
</tr>
<tr>
<td>M1 for method of finding area of the quadrant A1 for any rounding to 4 or more sf B1 for 9.75 B1 for any rounding to 4 or more sf</td>
<td></td>
</tr>
<tr>
<td>M1 A1</td>
<td></td>
</tr>
<tr>
<td>M1 for method of finding volume A1 for correct answer rounded to either 2 or 3 sf Accept alternative answer in cubic metres given correctly to 2 or 3 sf</td>
<td>M</td>
</tr>
</tbody>
</table>
Area of the sector = \( \frac{\theta}{360} \pi r^2 \)

Area of segment:
= area of sector – area of triangle
= \( \frac{\theta}{360} \pi r^2 - \frac{1}{2} ab \sin \theta \)

As \( a \) and \( b \) are both equal to \( r \), this becomes:
\( \frac{\theta}{360} \pi r^2 - \frac{1}{2} r^2 \sin \theta \)

Factorising gives:
\( r^2 \left( \frac{\theta}{360} \pi - \frac{1}{2} \sin \theta \right) \)

as required.
Using trigonometry:

\[ \cos \alpha = \frac{5}{6} \]
\[ \alpha = \cos^{-1} \frac{5}{6} \]
\[ \alpha = 33.557 \text{ } 31^\circ \]

Area of segment of one circle
\[ = \frac{\theta}{360} \pi r^2 - \frac{1}{2} r^2 \sin \theta \]
\[ = r^2 \left( \frac{\theta}{360} \pi - \frac{1}{2} \sin \theta \right) \]
where \( \theta = 2 \times 33.6^\circ = 67.2^\circ \) and \( r = 6 \text{ cm} \).

Area = \( 6^2 \left( \frac{67.2}{360} \pi - \frac{1}{2} \sin 67.2^\circ \right) \)
\[ = 36(0.586431 - \frac{1}{2} \sin 67.2^\circ) \]
\[ = 36(0.586431 - 0.4609316) \]
\[ = 4.517979 \text{ cm}^2 \]

Area of overlap = \( 2 \times 4.517979 \text{ cm}^2 \)
\[ = 9.035959 \text{ cm}^2 \]

9.0 cm²
### 26 a

| b | Triangle 11 will move round to sit next to face 13, square 4 will move round to be next to face 12, leaving face 2 opposite to face 13. |
| c | I would create the shape first then draw what I see from above as the plan and from the side as the elevation. Once I have created the shape, I can measure the lengths and angles concerned. |

- **B1** for clear explanation
- **C1** for use of diagrams to support the explanation
- **C1** for an explanation of the plan
- **C1** for explanation of elevation

### 27

- **M1** for method of calculating circumference of wheel
- **A1** for full unrounded answer

Circumference of wheel = \(\pi D\)  
\[ \pi \times 68 = 213.6283 \text{ cm} \]  
10 km = 10 × 1000 × 100 cm  
\[ = 1,000,000 \text{ cm} \]  
Number of revolutions in 10 km  
\[ = 1,000,000 \text{ cm} \div 213.6283 \text{ cm} = 4681.028 \text{ complete rotations} \]

- **B1** for use of 1,000,000 as a conversion factor either way round
- **A1** for correct division with common units
- **A1** for cao

### 28

- **C1** for use of a correct diagram
- **C1** for explanation of how and why using Pythagoras

[Diagram of triangle with sides labeled and equations: \(x^2 = 5^2 + 3^2\), \(x^2 = 34\), \(x = \sqrt{34}\), \(x = 5.830 951 9\), \(x = 5.8\) km]

- **M1** for correct application of Pythagoras’ theorem
- **M1** for correct method of finding hypotenuse

- **A1** for correct rounding to 2 or 3 sf
Let \( c \) = the height of the chimney.

\[
\frac{30}{c} = \tan 53^\circ \\
\frac{x}{c} = \tan 53^\circ \\
\frac{30 + x}{c} = \tan 62^\circ \\
x = c \tan 62^\circ - 30
\]

Combining equations 1 and 2 to eliminate \( x \):

\[
c \tan 53^\circ = c \tan 62^\circ - 30
\]

Rearrange to get \( c \) on one side of the equation.

\[
30 = c \tan 62^\circ - c \tan 53^\circ \\
30 = c(\tan 62^\circ - \tan 53^\circ) \\
c = \frac{30}{(\tan 62^\circ - \tan 53^\circ)} \\
\approx \frac{30}{5.85} \\
\approx 5.131 \\
\approx 54.182 \text{ m}
\]

54.2 m

C1 2 C1 for clear correct diagram used

M1 M1 for correct use of trigonometry with \( x, c \) and angle

A1 53° or 37° A1 for correct equation having \( x \) as subject

M1 M1 for correct use of trigonometry with \( x, c \) and angle

A1 62° or 28° A1 for correct equation in format to combine with

equation 1

M1 M1 for correctly eliminating \( x \)

M1 M1 for correct equation with \( c \) as subject

A1 A1 for correct rounding to 2 or 3 sf
**Angle at A is** $90^\circ + (360^\circ - 330^\circ) = 120^\circ$

**Angle at B is** $290^\circ - 270^\circ = 20^\circ$

**Angle at S is** $180^\circ - (120^\circ + 20^\circ) = 40^\circ$

Use the sine rule.

$$\frac{x}{\sin 20^\circ} = \frac{15}{\sin 40^\circ}$$

$$x = 15 \times \frac{\sin 20^\circ}{\sin 40^\circ} = 7.98133$$

8.0 km

**Sometimes true.**

It is not true if the number of individual cubes has fewer than 3 factors, including 1 and itself, for example, you cannot do it with 7 cubes (factors 1 and 7).

You can only make one cuboid if the number of factors, including 1 is equal to 3, for example, with 21 cubes (factors 1, 3 and 7).

You can make more than one cuboid if the number of cubes has more than 3 factors not including itself, for example, 30 (factors 1, 2, 3 and 5).
32 a  

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<td>b</td>
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</table>

Assume the cuboid has dimensions $x$, $y$ and $t$. The surface area = $2(xy + xt + yt)$
Volume = $xyt$
Doubling the lengths gives dimensions as $2x$, $2y$ and $2t$.
So surface area
$= 2(2x \times 2y + 2x \times 2t + 2y \times 2t)$
$= 2(4xy + 4xt + 4yt)$
$= 8(xy + xt + yt)$
which is 4 times the first area and
$V = 2x \times 2y \times 2t$
$= 8x^2y$ which is 8 times the first volume.

33  

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</table>

False. 

B1  

B1 for false
C1 for surface area with either specific lengths or a generalisation
C1 for volume with either specific lengths or a generalisation
P1 for showing correct follow through of double the lengths
C1 for a correct statement of SA with their data
B1 for 4 times area
C1 for a correct statement of volume with their data
B1 for 8 times volume
### 34

Consider just half the shape, where \( x \) is the length of the string.

Use Pythagoras' theorem.

\[
x^2 = 10^2 + 22.5^2
\]

\[
x = \sqrt{606.25}
\]

\[
x = 24.622\ 145
\]

Two lengths of string will be 49.244 289 cm

Subtract the original 45 cm

Gives extension as 4.2 cm

### 35

Let \( AC = x \), the length of the new road.

Use Pythagoras' theorem.

\[
x^2 = 4.9^2 + 6.3^2
\]

\[
x = \sqrt{63.7}
\]

\[
x = 7.981\ 228
\]

Current distance = 4.9 + 6.3 = 11.2 km

Saving = 11.2 – 7.981 228

= 3.218 772 km

### 36

Yes. \( \theta = \sin^{-1} \frac{12}{15} = 53.13^\circ \)

= 53° to the nearest degree

= 50° to 1 sf

12 cm has range of 11.5 to 12.5 cm

15 cm has range of 14.5 to 15.5 cm

The smallest value for \( \sin \theta \) is \( \sin 11.5 \) \( \frac{11.5}{15.5} \)

which gives \( \theta = \sin^{-1} 0.7419 \)

\( \theta = 47.9^\circ \)

So there are values that round to 12 cm and 15 cm which will give an angle that rounds to 50°.
| 37 | Use Pythagoras’ theorem.  
\[ AC^2 = 4^2 - (2\sqrt{2})^2 \]  
\[ = 16 - 8 = 8 \]  
\[ BC^2 = 8 = (2\sqrt{2})^2 \]  
Hence \( BC = AC \), an isosceles triangle. | M1 | 2 | M1 for correct Pythagoras statement  
A1 for correct value of \( AC^2 \)  
M1 for finding \( BC^2 \)  
A1 for correct value of \( BC^2 \)  
C1 for clear explanation of sides being the same length | M |
| 38 | \[ AB^2 = 2^2 - 1^2 \]  
\[ = 4 - 1 = 3 \]  
\[ AB = \sqrt{3} \]  
\[ \sqrt{3} \text{ cm} \] | M1 | 2 | M1 for correct Pythagoras statement  
A1 for \( 3 \)  
C1 for a clear communication of the method used | M |
| 39 | cos 68° = −cos 112° = −cos 248° = 0.3746  
cos 338° = 0.9271  
\[ \cos 338° \text{ is the odd one out.} \]  
All the others have the same numerical value (ignoring signs). | B1 | 2 | B1 for \( \cos 338° \)  
C1 for a clear explanation | M |
| 40 a i | \[ \sin x + 1 = 2 \]  
\[ \sin x = 1 \]  
\[ x = \sin^{-1}1 = 90^\circ \] | M1 | 2 | M1 for \( \sin x = 1 \)  
A1 for \( 90^\circ \) | M |
| ii | \[ 2 + 3\cos x = 1 \]  
\[ 3\cos x = 1 - 2 = -1 \]  
\[ \cos x = -\frac{1}{3} \]  
\[ x = 109.5^\circ \]  
and 360° − 109.5° = 250.5° | M1 | 2 | M1 for first step of solving equation  
A1 for correct statement of \( \cos x \)  
A1 for correct angle to 1 dp  
A1 for correct angle to 1 dp  
M1 for method of getting to \( \sin^{-1} \) |  
| b | \[ \cos 320° = 0.766 044 4 \]  
\[ \sin^{-1}0.766 044 4 = 50^\circ \]  
and 180° − 50° = 130°  
\[ x = 50^\circ \text{ and } 130^\circ \] | A1 | 9 | A1 for \( 50^\circ \)  
A1 for \( 130^\circ \) |  

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41 \( \text{a} \)

\[ \tan x = \sin x + \cos x \]

\[ \frac{\sqrt{8}}{4} \div \frac{3}{\sqrt{18}} \]

\[ = \frac{\sqrt{8}}{4} \cdot \frac{\sqrt{18}}{3} \]

\[ = \frac{\sqrt{144}}{12} \]

\[ = \frac{12}{12} = 1 \]

\( \tan^{-1} 1 = 45^\circ \)

Alternatively, use Pythagoras’ theorem to work out the length of the third side, e.g.

\((\text{side})^2 = 16 - 8 = 8\)

\(\text{side} = \sqrt{8}\)

\(\tan x = \frac{\sqrt{8}}{\sqrt{8}} = 1\)

\(\tan^{-1} 1 = 45^\circ\)

\(45^\circ\)

\(\text{C1} \quad 2\)

\(\text{C1 for communicating effectively how the sine and cosine can be used to find } \tan \text{ (This could be } \tan x = \frac{\sin x}{\cos x} \text{)}\)

\(\text{M1} \quad \text{A1}\)

\(\text{M1 for correct use of } \tan\)

\(\text{A1 for correct combination of surds}\)

\(\text{A1 for } \tan x = 1\)

\(\text{M1 for } \tan^{-1} 1 = 45^\circ\)

\(\text{A1 for } 45^\circ\)

\(\text{M6}\)

42

Using Pythagoras’ theorem, the hypotenuse = \(\sqrt{6+10} = \sqrt{16} = 4\)

Then \(\sin x = \frac{\sqrt{6}}{4}\)

and \(\cos x = \frac{\sqrt{10}}{4}\)

Hence \((\sin x)^2 + (\cos x)^2 = \frac{6}{16} \div \frac{10}{16} = \frac{16}{16} = 1\)

\(\text{M1} \quad \text{A1} \quad 2\)

\(\text{M1 for correct Pythagoras statement}\)

\(\text{A1 for } 4\)

\(\text{B1}\)

\(\text{B1 for correct } \sin x\)

\(\text{B1 for correct } \cos x\)

\(\text{C1}\)

\(\text{C1 for correct explanation}\)

\(\text{5}\)
Use the cosine rule.
\[ c^2 = a^2 + b^2 - 2ab \cos C \]
\[ = 9 + 16 - 2 \times 3 \times 4 \times \cos 60^\circ \]
\[ = 25 - 24 \times \frac{1}{2} \]
\[ = 13 \]
\[ c = \sqrt{13} \]
\[ \sqrt{13} \text{ cm} \]

No. To work out the return vector, multiply each component by \(-1\).
The return vector is \( \begin{pmatrix} 3 \\ 2 \end{pmatrix} \).
<table>
<thead>
<tr>
<th>Problem</th>
<th>Diagram</th>
<th>Mark Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>46 a</td>
<td><img src="image1.png" alt="Diagram" /></td>
<td>C1 for showing construction of $60^\circ$&lt;br&gt;C1 for showing bisection of $60^\circ$&lt;br&gt;C1 for showing bisection of $30^\circ$ to show $15^\circ$</td>
</tr>
<tr>
<td>b</td>
<td><img src="image2.png" alt="Diagram" /></td>
<td>C1 for showing construction of $90^\circ$&lt;br&gt;C1 for showing construction of $60^\circ$ to leave a $30^\circ$ angle&lt;br&gt;C1 for bisecting that $30^\circ$ angle to leave $75^\circ$</td>
</tr>
<tr>
<td>47</td>
<td><img src="image3.png" alt="Diagram" /></td>
<td>C1 for explanation referring to the bisector being equidistant&lt;br&gt;M1 for the interpretation of the intersections</td>
</tr>
</tbody>
</table>

Each angle bisector is the locus of points equidistant from the two arms or sides of the angle.<br>Hence, where they all meet will be the only point that is equidistant from each of the three sides.<br>Hence a circle can be drawn inside the triangle, with this centre, that just touches each side of the triangle.
48 a–d

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<tr>
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<tbody>
<tr>
<td>e</td>
<td>A circle, centre P, should pass through each of the nine labelled points.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>A circle, centre P, should pass through each of the nine labelled points.</td>
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49

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<tbody>
<tr>
<td></td>
<td>$AB = CD$ (given)</td>
<td></td>
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<tr>
<td></td>
<td>$\angle ABD = \angle CDB$ (alternate angles)</td>
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<tr>
<td></td>
<td>$\angle BAC = \angle DCA$ (alternate angles)</td>
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<tr>
<td></td>
<td>so $\triangle ABX \equiv \triangle CDX$ (ASA)</td>
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50

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<tbody>
<tr>
<td></td>
<td>$AB$ and $PQ$ are the corresponding sides opposite the 50° angle but they are not equal in length.</td>
<td></td>
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</tbody>
</table>
### 51 a

**b**

Area of original rectangle = \( x^2 \)

If the length of the side of the inscribed square is \( h \), then:

\[
\begin{align*}
h^2 &= \left( \frac{x}{2} \right)^2 + \left( \frac{x}{2} \right)^2 \\
h^2 &= \frac{x^2}{4} + \frac{x^2}{4} \\
h^2 &= \frac{x^2}{2}
\end{align*}
\]

The area of the inscribed square is half the area of the original square.

### 52

Each internal angle of an octagon is 135°.

Each internal angle of a hexagon is 120°.

The sum of these two angles is 255°.

The sum of the angles in a quadrilateral is 360° so the sum of the remaining angles is:

\[
360° - 255° = 105°.
\]

The two remaining angles are equal, as the line joining through the vertices C, J, G and L (and thus through the obtuse angles) is a line of symmetry.

\( \text{JFG} = 52.5° \)
<table>
<thead>
<tr>
<th>Use the sine rule in both triangles.</th>
</tr>
</thead>
</table>
| \[
sin \frac{P}{b} = \sin \frac{C}{p}
\]
| \[
sin \frac{A}{a} = \sin \frac{C}{c}
\]
| \[
sin \frac{A}{c} = \frac{a \sin \frac{C}{c}}{c}
\]

As \( A = P \), then \( \sin P = \sin A \)

So \[
\frac{bsin c}{p} = \frac{asin C}{c}
\]

Hence \[
\frac{b}{p} = \frac{a}{c}
\]

Hence \[
\frac{p}{b} = \frac{c}{a}
\]

So \[
p = \frac{bc}{a}
\]
Use Pythagoras' theorem to write expressions for $z^2$.

Triangle ABC: $z^2 = r^2 + (s + y)^2$
Triangle ABD: $z^2 = w^2 + (x + t)^2$

So $r^2 + (s + y)^2 = w^2 + (x + t)^2$

Now $r^2 = x^2 - s^2$ and $w^2 = y^2 - t^2$

Substitute for $r^2$ and $w^2$ in the equation:

$x^2 - s^2 + (s + y)^2 = y^2 - t^2 + (x + t)^2$

Multiply out the brackets:

$x^2 - s^2 + s^2 + 2sy + y^2 = y^2 - t^2 + x^2 + 2xt + t^2$

Simplify:

$x^2 + 2sy + y^2 = y^2 + x^2 + 2xt$

$2sy = 2xt$

$xy = xt$

Alternatively:

Assume $T$ is the intersection of AD and BC. ACT is similar to BDT as both contain the same angles. (Angles ATC and BTD are vertically opposite and so are equal.)

Hence by similarity $\frac{x}{y} = \frac{s}{t}$

So $xt = sy$

| 54 | Use Pythagoras’ theorem to write expressions for $z^2$. Triangle ABC: $z^2 = r^2 + (s + y)^2$ Triangle ABD: $z^2 = w^2 + (x + t)^2$ So $r^2 + (s + y)^2 = w^2 + (x + t)^2$ Now $r^2 = x^2 - s^2$ and $w^2 = y^2 - t^2$ Substitute for $r^2$ and $w^2$ in the equation: $x^2 - s^2 + (s + y)^2 = y^2 - t^2 + (x + t)^2$ Multiply out the brackets: $x^2 - s^2 + s^2 + 2sy + y^2 = y^2 - t^2 + x^2 + 2xt + t^2$ Simplify: $x^2 + 2sy + y^2 = y^2 + x^2 + 2xt$ $2sy = 2xt$ $xy = xt$ Alternatively: Assume $T$ is the intersection of AD and BC. ACT is similar to BDT as both contain the same angles. (Angles ATC and BTD are vertically opposite and so are equal.) Hence by similarity $\frac{x}{y} = \frac{s}{t}$ So $xt = sy$ | 2 | P1 | P1 for combining expressions |
| C1 | P1 for substitution |
| C1 | P1 for substitution |
| C1 | P1 for simplifying |
| C1 | P1 for clear explanation |

C1 for $\frac{x}{y} = \frac{s}{t}$
C1 for clear communication of proof
Take the smallest two sides, square each of them, add the squares together then find the square root of the sum.

- If this root is equal to the length of the longest side, then the triangle is right angled.
- If the root is smaller than the length of the longest side, then the angle is greater than 90°.
- If the root is larger than the length of the longest side then there is no angle greater than 90°.

In the first triangle the hypotenuse is \( \sqrt{5^2 + 12^2} = \sqrt{139} = 13 \)

In the second triangle, the hypotenuse is 12 with the unknown side \( \sqrt{12^2 - 5^2} = \sqrt{119} \), which is between 10 and 11 cm.

So they both have a short side of 5 cm.

But the lengths of the other short side and the hypotenuse are different.

\[
\begin{align*}
55 \text{ a} & \quad \text{C1} & \quad 2 & \quad \text{C1 for correct Pythagoras statement} \\
& \quad \text{C1} & \quad \text{C1 for clear explanation} \\
& \quad \text{C1} & \quad \text{C1 for clear explanation} \\
& \quad \text{C1} & \quad \text{C1 for clear explanation} \\
& \quad \text{B1} & \quad \text{B1 for 13} \\
& \quad \text{B1} & \quad \text{B1 for side length between 10 and 11 or more accurate.} \\
& \quad \text{C1} & \quad \text{C1 for explanation of what is the same} \\
& \quad \text{C1} & \quad \text{C1 for clear explanation of what is different} \\
\end{align*}
\]

56

\[
\begin{align*}
& \quad \text{C1} & \quad 3 & \quad \text{C1 for clear diagram} \\
& \quad \text{M1} & \quad \text{A1} & \quad \text{M1 for using a large triangle to identify parts} \\
& \quad \text{A1} & \quad \text{A1 for correct equation} \\
& \quad \text{M1} & \quad \text{A1} & \quad \text{M1 for method of adding P and Q} \\
\end{align*}
\]
area = \( (m + n)t \)

So the area, \( A \), of the whole shape is:

- top triangle ABM + bottom triangle DCN
- \((P + Q) + t^2\)

Area of ABM = \( \frac{1}{2} \times 2x \times x = x^2 \)

Area of DCN = \( \frac{1}{2} \times 2x \times x = x^2 \)

So total area = \( x^2 + x^2 + (m + n)t + t^2 \)

\[ = 2x^2 + t(m + n + t) \]

But \( m + n + t = \frac{x}{5} \)

So \( A = 2x^2 + \frac{tx}{5} \)

But the sides are of length \( 2x \), so \( A = 4x^2 \)

Then \( 2x^2 + \frac{tx}{\sqrt{5}} = 4x^2 \)

and \( \frac{tx}{\sqrt{5}} = 2x^2 \)

\( t \sqrt{5} = 2x \)

Squaring each side gives:

\[ 5r^2 = 4x^2 \]

\[ r^2 = \frac{4x^2}{5} \]

Whole area \( A = 4x^2 \)

Hence the middle square, \( r^2 \), is \( \frac{1}{5} \) of \( A \).
<table>
<thead>
<tr>
<th>57 a</th>
<th>Not true. For example these are both right-angled triangles but their sides are not in proportion.</th>
<th>B1 2</th>
<th>B1 for not true C1 for clear explanation with an example drawn to illustrate</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>True. With an enlargement the final shape will be in the same proportion as the original so it will be similar.</td>
<td>B1  C1</td>
<td>B1 for true C1 for clear explanation</td>
</tr>
<tr>
<td>c</td>
<td>True. All circles are in proportion to each other and so will be similar.</td>
<td>B1  C1</td>
<td>B1 for true C1 for clear explanation</td>
</tr>
<tr>
<td>58 a</td>
<td>Any two regular polygons with the same number of sides will have the same angles so the ratio of lengths of the sides will be the same. This means the shapes will be similar.</td>
<td>C1 2</td>
<td>C1 for clear example P1 for use of diagrams to support the explanation</td>
</tr>
<tr>
<td>b</td>
<td>As the corresponding sides of triangle A and triangle B are the same, the two triangles are congruent, SSS. Therefore equivalent angles are the same. This demonstrates the corresponding angles between parallel lines are the same.</td>
<td>C1  P1</td>
<td>C1 for clear explanation P1 for use of diagram to clarify explanation</td>
</tr>
<tr>
<td>59 a</td>
<td>b</td>
<td>c</td>
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<tr>
<td></td>
<td><img src="image" alt="Diagram" /></td>
<td>Enlargement with scale factor (-\frac{1}{2}) about ((-6, 2)).</td>
<td></td>
</tr>
<tr>
<td></td>
<td><img src="image" alt="Diagram" /></td>
<td>B1</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>B2</td>
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### 60

Enlargement with sf \(-3\) about \((1, 2)\). Drawing lines from the two vertices at the base of triangle A to their image points in B. The intersection shows the centre of enlargement. The base of A is 2 squares wide, the base of B is 6 squares wide, and 6 is 3 times 2, so the sf is 3.

|      | ![Diagram](image) | B1 | 2 | B1 for correct statement |
|      | | C1 | | C1 for explanation of how centre of enlargement is found |
|      | | C1 | | C1 for explanation how sf is found |
### 61 a

Examine the four possible starting points for the stamp. These are at the top right and bottom left of each side, allowing for 180° rotation of each side. Four rotations mean that each of these points is covered once.

No, the machine would not detect the stamp placed on the top left-hand corner because none of the rotations will put the stamp in the top right-hand corner.

Four corners on each side could possibly be the ‘top right’.

One way is to rotate about H and then rotate about one of the diagonals (call it D). Keep repeating the sequence H, D, H, D, … to check all eight corners.

### 62

The hypotenuses (OA and OB) are the same, as each is a radius of the circle. OM is common to both triangles. OMA and OMB are both right angles. Triangles OAM and OBM are congruent, therefore AM = MB. Therefore, M is the midpoint of AB and the chord has been bisected, as required.
Circle Theorem 1 states that the angle at the centre of a circle is twice the angle at the circumference when they are both subtended by the same arc.

Theorem 1 tells you that any angle subtended from arc AB at a point C on the circumference is always half the angle subtended from arc AB at the centre.

Therefore, every possible angle subtended from arc AB at the circumference will correspond to the same angle at the centre.

Hence all the angles subtended at the circumference will be equal.

<table>
<thead>
<tr>
<th>P2</th>
<th>2</th>
<th>C1</th>
<th>C1 for clear overall explanation and clarity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>C1 for use of diagram and mathematical language</td>
</tr>
</tbody>
</table>
Draw triangle BCP, where P is on the circumference in the opposite segment. Then angle BPC is $180° - x$ as BAC and BPC are opposite angles in a cyclic quadrilateral. Angle BOC is double BPC, which is $360° - 2x$. Angle BCO will also be $y$, as BCO is an isosceles triangle. Angles in triangle OBC sum to $180°$, so $2y + 360° - 2x = 180°$ and $2y = 2x - 180°$. Divide both sides by 2 to give: $y = x - 90°$.

As ABCD is a cyclic quadrilateral, the opposite angles will sum to $180°$. So $2x - 5° + 5y - 20° = 180°$
$2x + 5y = 205°$ ........(1)
And $3y + 5 + 2x + 20° = 180°$
$2x + 3y = 155°$ ........(2)
Subtract (2) from (1): $2y = 50°$
$y = 25°$
Substitute $y$ into (2): $2x + 75° = 155°$
$2x = 80°$
$x = 40°$

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$2x = 80°$
$x = 40°$

Subtract (2) from (1): $2y = 50°$
$y = 25°$
Substitute $y$ into (2): $2x + 75° = 155°$
$2x = 80°$
$x = 40°$
Drop perpendiculars down to Y and X and complete the trapezium, with the top triangle being ABT, as shown.

Length TB will be the same as YZ.

Use Pythagoras’ theorem on triangle ABT.

\[ TB^2 = 9^2 - 3^2 \]
\[ = 81 - 9 \]
\[ = 72 \]

\[ TB = \sqrt{72} \]
\[ = 8.4852814 \]

8.49 cm

Circle Theorem 1 states that the angle at the centre of a circle is twice the angle at the circumference when they are both subtended by the same arc. Theorem 1 tells you that any angle subtended from arc AB at a point C on the circumference is always half the angle subtended from arc AB at the centre.

Therefore, every possible angle subtended from arc AB at the circumference will correspond to the same angle at the centre.

Hence angles subtended at the circumference are equal.
The angle PQS is the same as PRS, $3x$, as they are angles in the same quadrant (subtended by the same arc). The angle sum of triangle PQT is $180^\circ$. Hence $2x + 3x + 5x = 180^\circ$

$$10x = 180^\circ$$

$$x = 18^\circ$$

Complete the diagram with all the known angles.

You can now see that: angle $P = 54^\circ$, angle $Q = 72^\circ$, angle $R = 126^\circ$, angle $S = 108^\circ$, and angle STP = $90^\circ$. The numbers 54, 72, 90, 108 and 126 form a sequence with a difference of 18.
As it is shown in the diagram, the proportion of the height of the container that is filled is \( \frac{80}{120} = \frac{2}{3} \).

The current volume of water is \( 90 \times 40 \times 80 = 288000 \text{ cm}^3 \).

Assume you move the container so that the base is \( 40 \times 120 \). The volume of water must stay the same and take its height as \( C \) cm. Then:

\[
40 \times 120 \times C = 288000
\]

so \( 4800 \times C = 288000 \)

\[
C = 60
\]

Comparing this with the height, 90 cm:

\[
\frac{60}{90} = \frac{2}{3}
\]

Now assume you move the container so the base is \( 90 \times 120 \) then

\[
90 \times 120 \times C = 288000
\]

\[
10800C = 288000
\]

\[
C = 80
\]

Comparing this with the height, 40 cm:

\[
\frac{80}{40} = \frac{80}{3 \times 40} = \frac{80}{120} = \frac{2}{3}
\]

So the proportion of the height of the container that is filled with water remains constant.

M1 for method of finding proportion of height

A1 for \( \frac{2}{3} \)

B1 for 288 000

C2 for clear explanation of changing to a different base

C1 for structure and clarity of explanation

C1 for clear explanation

C1 for structure and clarity and explanation

C1 for clear explanation
### 70 a

Assume the sides all have integer values.
The formula for the volume of the prism will be:
\[ a \times b \times 10 = 160 \text{ cm}^3 \]
but if the base is a square, then \( a = b \).
\[ a^2 = 16 \text{ cm}^3 \]
This has only one solution: \( a = 4 \).

### 71 a

The formula for the circumference of a circle is \( 2\pi r \). This is just a length, \( r \), multiplied by a number, so the answer will still be a length.
The formula for the area of a circle is \( \pi r^2 \). This is a number multiplied by a length squared so this will be an area.

A formula for a surface area will include the product of two lengths, but a formula for a volume will include the product of three lengths.

- \( \frac{1}{3} b^2 h \) includes the product of three lengths, \( b \times b \times h \) and so must be the volume.
- \( 4\pi r^2 \) includes the product of two lengths, \( r \times r \), and so must be the area.
Identify the third arrangement.
The single row of 8 cubes shows the cuboid must have a volume of 8 cubes. The missing arrangement is a $1 \times 2 \times 4$ cuboid.

Find the amount of string for each of the arrangements.

For the cuboid above, $1 \times 2 \times 4$:

$L = 4 \times 15$, $W = 2 \times 15$, $H = 1 \times 15$ and $S = 2 \times 60 + 2 \times 30 + 4 \times 15 + 20 = 260$ cm

For the $1 \times 1 \times 8$ cuboid:

$L = 8 \times 15$, $W = 1 \times 15$, $H = 1 \times 15$ and $S = 2 \times 120 + 2 \times 15 + 4 \times 15 + 20 = 350$ cm

For the $2 \times 2 \times 2$ cuboid:

$L = 2 \times 15$, $W = 2 \times 15$, $H = 2 \times 15$ and $S = 2 \times 30 + 2 \times 30 + 4 \times 30 + 20 = 260$ cm

So the least amount of string is used in the $1 \times 2 \times 4$ and the $2 \times 2 \times 2$ arrangements.
Surface area of end = 1000 ÷ 20
= 50 cm$^2$

Consider a cuboid with a square end.
Side length of square = $\sqrt{50} = 7.0711$
Surface area (SA)
= $50 + 50 + 4 \times 7.0711 \times 20$
= 665.6 cm$^2$

Consider a triangular prism with a triangular end.
Area of triangular end = $\frac{1}{4}\sqrt{a^2} = 50$
So $a^2 = \frac{200}{\sqrt{3}} = 115.470\,054$
$a = 10.745\,70$ cm
SA = $50 + 50 + 3 \times 10.745\,70 \times 20$
= 745 cm$^2$

Consider a cylinder with a circular end.
Then $\pi r^2 = 50$
\[ r = \sqrt{\frac{50}{\pi}} = 3.989\,42 \]
SA = $50 + 50 + \pi \times 2 \times 3.989\,42 \times 20$
= 601.4 cm$^2$

Differences between the three surface areas:
- surface area of the triangular prism is 79.4 cm$^2$ larger than the cuboid and 143.6 cm$^2$ larger than the cylinder
- surface area of cuboid is 64.2 cm$^2$ larger than the cylinder.

The larger the surface area, the more packaging material is required therefore the higher the production costs.
**74**

Use the cosine rule.

\[ AB^2 = 2.1^2 + 1.8^2 - 2 \times 2.1 \times 1.8 \times \cos 70° \]

\[ = 5.064 \text{ 328} \]

\[ AB = \sqrt{5.064 \text{ 328}} = 2.250 \text{ 406} \]

Extra distance = \((2.1 + 1.8) - 2.25\) km

\[ = 1.65 \text{ km} \]

**75**

Use Pythagoras’ theorem to find BY.

\[ BY = \sqrt{9 + 4} = 3.605 \text{ 551 3} \]

Use the right-angled triangle XYB to find XY.

\[ XY = \tan 6° \]

\[ \frac{BY}{XY} = \tan 6° \]

\[ XY = BY \times \tan 6° \]

\[ = 0.378 \text{ 958 7 km} \]

\[ = 378.958 \text{ m} \]

**76 a**

Use Pythagoras’ theorem.

\[ PM = \sqrt{8^2 + 4^2} = \sqrt{80} \]

\[ PM = 8.944 \text{ 271 9 cm} \]

\[ = 8.9 \text{ cm} \]
c

\[ VM = \sqrt{10^2 + 4^2} = \sqrt{116} \]
\[ VM = 10.77033 \]

\[ \cos P = \frac{\frac{\sqrt{80}}{2}}{\sqrt{116}} = 0.4152274 \]
Angle VPM = 65.466362°

Vertical height of V above face PRQ is given by VT in diagram above.

\[ VT = \sqrt{VM^2 + \left(\frac{PM}{2}\right)^2} = \sqrt{116 + \frac{80}{4}} \]
\[ = \sqrt{136} \]
\[ = 11.661904 \]
Add the 15 cm of the base to give 26.661904 cm.

10.8 cm

M1
A1
C1

M1 for use of Pythagoras' theorem
A1 for 10.8 rounded to 2 or 3 sf
C1 for use of clear diagram

M1

M1 for use of correct cosine

A1
A1 for 65.5 rounded to 2 or 3 sf
C1
C1 for explanation of how the height is to be found
M1
M1 for use of Pythagoras' theorem
A1
A1 for 11.61904 and any rounding 3 or more sf
A1
A1 for 26.7 rounded to 2 or 3 sf

He needs to find half of AC (not AC) to make a right-angled triangle.

\[ \cos x = \frac{\sqrt{208}}{10} = 0.721110 \]
\[ = \cos^{-1} 0.721110 \]
\[ = 43.853779° \]

43.9

C1
M1
A1

C1 for clear explanation
M1 for correct cosine method
A1 for 43.9 rounded to either 2 or 3 sf
Using angles on a straight line.
\[ \theta = 180^\circ - 58^\circ = 122^\circ \]
Angle ADB = \( \alpha = 180^\circ - (32^\circ + 122^\circ) \)
\[ = 180^\circ - 154^\circ = 26^\circ \]
Use the sine rule.
\[ \frac{e}{\sin 32^\circ} = \frac{28}{\sin 26^\circ} \]
\[ e = \sin 32^\circ \times \frac{28}{\sin 26^\circ} \]
\[ e = \sin 32^\circ \times 63.872 \, 816 \, 9 \]
\[ = 63.847 \, 436 \]
Using trigonometry:
\[ \sin 58^\circ = \frac{c}{e} \]
\[ c = e \sin 58^\circ \]
\[ = 28.704 \, 253 \, 7 \]
So the height of the tower is 29 m to the nearest metre.
\[ \sin 24^\circ = \frac{t}{4.7} \]

\[ t = 4.7 \sin 24^\circ \]

\[ \cos 24^\circ = \frac{x}{4.7} \]

\[ x = 4.7 \cos 24^\circ \]

\[ \sin 52^\circ = \frac{y}{8.6} \]

\[ y = 8.6 \sin 52^\circ \]

\[ \cos 52^\circ = \frac{s}{8.6} \]

\[ s = 8.6 \cos 52^\circ \]

\[ e = x + y = 4.7 \cos 24^\circ + 8.6 \sin 52^\circ \]

\[ = 11.070 \ 556 \ \text{km} \]

\[ f = s - t = 8.6 \cos 52^\circ - 4.7 \sin 24^\circ \]

\[ = 3.383 \ 0264 \ \text{km} \]

Use Pythagoras' theorem to find \( d \).

\[ d = \sqrt{e^2 + f^2} \]

\[ = \sqrt{134.002} \]

\[ = 11.575 \ 93 \ \text{km} \]

\[ = 11.6 \ \text{km} \]

Use trigonometry to find \( \beta \).

\[ \tan \beta = \frac{e}{f} = \frac{11.070556}{3.3830264} \]

\[ = 3.272382 \]

\[ \beta = \tan^{-1} 3.272 \ 382 = 73.007 \ 489^\circ \]

C1 for a clear diagram with all relevant lengths and angles marked

M1 for method of finding \( t \)

A1 for correct expression of \( t \)

M1 for method of finding \( x \)

A1 for correct expression of \( x \)

M1 for method of finding \( y \)

A1 for correct expression of \( y \)

M1 for method of finding \( s \)

A1 for correct expression of \( s \)

M1 for method of finding \( e \)

A1 for correct \( e \), rounded to 4 or more sf

M1 for method of finding \( f \)

A1 for correct \( f \) rounded to 4 or more sf

M1 for using Pythagoras' theorem

A1 for correct \( d \) any value 3 sf or more

A1 for correct \( d \) rounded to 3sf

M1 for method of finding \( \beta \)

A1 for \( \beta \) to 4 or more sf
The required bearing = $360^\circ - \beta$
$= 286.992\, 510\, 8^\circ$
$= 287^\circ$

<table>
<thead>
<tr>
<th>ABC</th>
<th>M1</th>
<th>20</th>
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</thead>
<tbody>
<tr>
<td>80</td>
<td>Use the sine rule to work out angle B.</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\sin B = \frac{\sin 32^\circ}{10}$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$\sin B = \frac{10 \sin 32^\circ}{6}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$B = \sin^{-1} \left( \frac{10 \sin 32^\circ}{6} \right) = 62^\circ$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>But we know ABC is obtuse, so:</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$B = 180^\circ - 62^\circ = 118^\circ$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Then $C = 180^\circ - (118^\circ + 32^\circ) = 30^\circ$</td>
<td>B1</td>
</tr>
<tr>
<td></td>
<td>Use the sine area rule.</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>Area = $\frac{1}{2} absin C$</td>
<td>M1</td>
</tr>
<tr>
<td></td>
<td>$= \frac{1}{2} \times 10 \times 6 \times \sin 30^\circ$</td>
<td>A1</td>
</tr>
<tr>
<td></td>
<td>$= 15, \text{cm}^2$</td>
<td>C1</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>81 a</th>
<th>C1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use Pythagoras’ theorem to work out $H$.</td>
<td>C1</td>
<td>for use of clear diagram</td>
</tr>
<tr>
<td>$H = \sqrt{10^2 + 24^2} = 26$</td>
<td></td>
<td>H</td>
</tr>
<tr>
<td>Use the cosine rule to work out angle D.</td>
<td>M1</td>
<td>for use of Pythagoras’ theorem</td>
</tr>
<tr>
<td>$\cos D = \frac{20^2 + 23^2 - 26^2}{2 \times 20 \times 23} = 0.275$</td>
<td>A1</td>
<td>A1 for 26 cao</td>
</tr>
<tr>
<td>$D = 74.037, 986^\circ$</td>
<td>M1</td>
<td>for use of cosine rule</td>
</tr>
<tr>
<td></td>
<td>A1</td>
<td>A1 for $D$ and any rounding to 2 or more sf</td>
</tr>
</tbody>
</table>
b

Use the area sine rule to work out the area of triangle ADC.

Area of triangle ADC = \( \frac{1}{2} \times 20 \times 23 \times \sin 74.037986^\circ \)

= 221.13217 m²

Area of triangle ABC = \( \frac{1}{2} \times 24 \times 10 \)

= 120 m²

Total area = 341.13217 m²

341.13217 ÷ 5 = 68.226434

68 trees

M1

Use area sine rule
A1

Area correct to 3 or more SF.

M1

Triangle area rule
A1

120 CAO

C1

341 correct to 2 or 3 SF

M1

Dividing by 5
A1

68 CAO

A1

Complete clear solution with correct mathematical notation.

\[ \text{Let the vertical height of the parallelogram be } H. \]

\[ \frac{H}{a} = \sin \theta \]

\[ H = a \sin \theta \]

Area of parallelogram = base \times height

\[ = b \times a \sin \theta \]

\[ = ab \sin \theta \]

M1

Correct use of trigonometry to find the height
A1

Correct height expression
M1

Correct method of finding area of a parallelogram
A1

Correct expression that will simplify to \( ab \sin \theta \)

C1

Complete, clear explanation with good mathematical notation.

C1

Clear diagram showing vertical height.

C1

M1

For use of area sine rule
A1

Area correct to 3 or more SF.

M1

Triangle area rule
A1

120 CAO

C1

341 correct to 2 or 3 SF

M1

Dividing by 5
A1

68 CAO

A1

Complete clear solution with correct mathematical notation.

C1
Use Pythagoras' theorem to work out DB.  
\[ DB = \sqrt{6^2 + 8^2} \]
\[ = 10 \text{ cm} \]

Use the cosine rule to work out angle A.  
\[ \cos A = \frac{9^2 + 14^2 - 10^2}{2 \times 9 \times 14} = 0.7023809 \]
\[ A = 45.381658^\circ \]

Use the area sine rule to work out the area of triangle ADB.  
\[ \text{Area} = \frac{1}{2} \times 9 \times 14 \times \sin 45.381658^\circ \]
\[ = 44.843478 \text{ cm}^2 \]

Area of triangle BDC = \[ \frac{1}{2} \times 6 \times 8 \]
\[ = 24 \text{ cm}^2 \]

Total area = 68.843478 cm²

\[ 69 \text{ cm}^2 \]

<table>
<thead>
<tr>
<th>83</th>
<th><strong>M1</strong></th>
<th><strong>A1</strong></th>
<th><strong>M1</strong></th>
<th><strong>A1</strong></th>
<th><strong>M1</strong></th>
<th><strong>A1</strong></th>
<th><strong>M1</strong></th>
<th><strong>A1</strong></th>
<th><strong>C1</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>M1 for use of Pythagoras' theorem</td>
<td>A1 for 10 cao</td>
<td>M1 for use of cosine rule</td>
<td>A1 for A with any rounding to 2 or more sf</td>
<td>M1 for use of area sine rule</td>
<td>A1 for area correct to 3 or more sf</td>
<td>M1 for triangle area rule</td>
<td>A1 for 24 cao</td>
<td>A1 for 69 correct to 2 or 3 sf</td>
</tr>
<tr>
<td>10</td>
<td>69 cm²</td>
<td><strong>C1</strong> for clear, complete solution with mathematical language</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
A possible triangle is one with sides 3, 4 and 5 (check, using Pythagoras’ theorem).
Then \( s = 6 \) and the area is \( \sqrt{6 \times 3 \times 2 \times 1} = \sqrt{36} = 6 \)
Also, area = \( \frac{1}{2} \times \text{base} \times \text{height} \)
= \( \frac{1}{2} \times 3 \times 4 = 6 \)
which gives the same answer.

Suppose the triangle has a side of 10 cm (you could use any number you like).
The formula gives:
\( \sqrt{15 \times 5 \times 5 \times 5} = \sqrt{3125} = 43.3 \) cm\(^2\)
You could also use the area sine rule:
\[ \text{area} = \frac{1}{2} \times \text{base} \times \text{height} \]
\[ = \frac{1}{2} \times 10 \times 10 \times \sin 60^\circ \]
\[ = 43.3 \text{ cm}\(^2\) \]
which gives the same answer.

The sides of the triangle are 18 metres, 22 metres and 24 metres. So:
\( s = 32 \)
area = \( \sqrt{32 \times 14 \times 10 \times 8} \)
\[ = \frac{1}{2} \times 10 \times 10 \times \sin 60^\circ \]
\[ = 189.314 \text{ } \text{ m}^2 \]

A diagonal will divide the field into two triangles. Measure the four sides and a diagonal. Use the formula to find the area of each triangle separately and then add the answers together.

One example is:
Start at coordinates (1, 1), then move through translation \( \left( \begin{array}{l} 1 \\ 3 \end{array} \right) \)
then through translation \( \left( \begin{array}{l} 3 \\ 3 \end{array} \right) \)

Note there are a few different correct answers. Check all vectors sum to \( \left( \begin{array}{l} 0 \\ 0 \end{array} \right) \)

B1 for having just 3 translations
B1 for the describing first translation
C1 for the describing second translation
C1 for the describing final translation
and finish with \[
\begin{pmatrix}
4 \\
1
\end{pmatrix}
\]
to get back to \((1, 1)\).

| 86 ai | \( \mathbf{b} \) | \( \mathbf{b} - \mathbf{a} \) | \( -2\mathbf{a} \) | \( 2\mathbf{b} - \mathbf{a} \) | \( 2\mathbf{b} - \mathbf{a} \) | Parallel and equal in length. | 4 | B1 | B1 | B1 | B1 | B1 | B1 | B1 | B1 cao | H |
|--------|----------------|------------------|----------------|----------------|----------------|-------------------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| ii     |                |                  |                |                |                |                         |     |     |     |     |     |     |     |     |     |
| iii    |                |                  |                |                |                |                         |     |     |     |     |     |     |     |     |     |
| iv     |                |                  |                |                |                |                         |     |     |     |     |     |     |     |     |     |
| b      |                |                  |                |                |                |                         |     |     |     |     |     |     |     |     |     |

| 87 a   | \( \mathbf{b} \) | \( \mathbf{ABC} = \mathbf{AC} - \mathbf{AB} \) | \( 9\mathbf{a} + 6\mathbf{b} - (6\mathbf{a} + 4\mathbf{b}) \) | \( = 3\mathbf{a} + 2\mathbf{b} \) | \( \mathbf{AB} \) is \( 2 \times \mathbf{BC} \) | So \( \mathbf{AB} : \mathbf{BC} = 2 : 1 \) | 6 | C1 | C1 | M1 | A1 | B1 | B1 | B1 cao | H |
|        |                |                  |                |                |                |                         |     |     |     |     |     |     |     |     |     |

<p>| 88 ai  | ( \mathbf{b} ) | ( \mathbf{BC} = \mathbf{c} - \mathbf{b} ) | ( \mathbf{NQ} = \frac{1}{2} \mathbf{b} + \frac{1}{2} (\mathbf{c} - \mathbf{b}) ) | ( = \frac{1}{2} \mathbf{c} ) | ( \mathbf{MP} = \frac{1}{2} \mathbf{a} + \frac{1}{2} \mathbf{AC} ) | ( \mathbf{AC} = \mathbf{c} - \mathbf{a} ) | ( \mathbf{MP} = \frac{1}{2} \mathbf{a} + \frac{1}{2} (\mathbf{c} - \mathbf{a}) ) | ( = \frac{1}{2} \mathbf{c} ) | It is a parallelogram, because ( \mathbf{NQ} = \mathbf{MP} = \frac{1}{2} \mathbf{c} ) and hence they are parallel and equal in length. | 9 | B1 | B1 | M1 | A1 | M1 | B1 | A1 | B1 | B1 for parallelogram | C2 | C1 for stating vectors are parallel | C1 for stating vectors will be same length | H |
| ii     |                |                  |                |                |                |                         |     |     |     |     |     |     |     |     |     |
| iii    |                |                  |                |                |                |                         |     |     |     |     |     |     |     |     |     |
| b      |                |                  |                |                |                |                         |     |     |     |     |     |     |     |     |     |</p>
<table>
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</table>
| 89   | \[ \begin{align*} AB &= b - a, \\
|     | & AC = AO + OC \\
|     | &= -a + 3b - 2a \\
|     | &= 3b - 3a \\
|     | \text{so } AC = 3 \ AB \\
|     | \text{and hence } ABC \text{ is a straight line.} \end{align*} |
|      | B1 cao \\
|      | M1 for adding vectors |
|      | A1 cao \\
|      | C1 for explaining the two vectors are multiples of each other |
|      | C1 cao \\
|      | C1 for explaining this gives a straight line |
| 90   | The knight can get to all the squares shown. \\
|      | Do not forget that you can use \(-a\) and \(-b\) as well as \(a\) and \(b\). \\
|      | The starting position must match the question (bottom left white square).  \\
|      | The lines show all the possible paths of the Knight, using \(a, b, -a\) and \(-b\). \\
|      | There are many ways to reach the king. However, there are three ways to get to the king in the minimum of five moves. |
|      | C1 cao \\
|      | C1 for explaining there are numerous ways to get to the king |
|      | C1 cao \\
|      | C1 for explaining there is a minimum of five moves to get to the king |
|      | C1 cao \\
|      | C1 for explaining there are only three ways to get to the king with these 5 minimum moves |
|      | C1 cao \\
|      | C1 for a clear cohesive explanation |