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| **Guidance on the use of codes for this mark scheme** | |
| M | Method mark |
| A | Accuracy mark |
| B | Working mark |
| C | Communication mark |
| P | Proof, process or justification mark |
| cao | Correct answer only |
| oe | Or equivalent |
| ft | Follow through |

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| **Question** | **Working** | **Answer** | **Mark** | **AO** | **Notes** | **Grade** |
| **1 a**  **b**  **c**  **d** | Example  2 × 3 = 6  2 + 3 = 5  3(c+ 5) = 3c + 15  Example  32 = 3 × 3 = 9  2 × 3 = 6  Example  2n2 = 2 × (32) =  2 × 9 = 18  (2 × 3)2 = 6 × 6 = 36 | 2n means 2 × nwhich is different from  n + 2.  n2 means n*×* n which is different from 2n.  BIDMAS for 2n2 tells you to calculate the power first. BIDMAS for (2n)2 tells you that you do the calculation inside the bracket first. | C1  C1  M1  C1  C1  C1 | 2 | C1 for explanation; an example could be given to support the argument  An additional mark can be given for identifying the exception, which is when n = 2  M1 for multiplying out the brackets to show that the two expressions are not equivalent  C1 for explanation; an example could be given to support the argument  An additional mark can be given for identifying the exception which is when n = 2  C1 for an explanation; an example could be given to support the argument | B |
| **6** |
| **2** |  | A letter, say f, stands for an unknown if it is in an equation such as 3f + 2 = 14. Then f = 4 is the only number that satisfies this equation.  A letter stands for a variable if it is part of an equation that has more than two letters, e.g. A = πr2 , where both A and r are variables that will be different for different values of A or r. | C1  C1  C1  C1 | 2 | C1 for a clear explanation  C1 for an example alongside the explanation  C1 for a clear explanation  C1 for an example alongside the explanation | B |
| **4** |

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| **3 a**  **b**  **c**  **d** | | 5(c+ 4) = 5c+ 20  Feedback: Don’t forget to multiply out both terms in the brackets.  6(t– 2) = 6t– 12  Feedback: Don’t forget 6(…..) means **multiply** both terms by 6.  –3(4 – s) = –12 + 3s  Feedback: Don’t forget –3(……) means **multiply** both terms by 6 and minus × minus = …  15 – (n– 4) = 15 – n+ 4 = 15 + 4 – n  = 19 – n  Feedback: Don’t forget –(n – 4) means multiply each term inside the brackets by –1 and that the – inside the brackets belongs to the 4 to make it – 4. |  | | M1  C1  M1  C1  M1  C1  M1  C1 | 2 | M1 for correctly expanding the brackets  C1 for suitable feedback  M1 for correctly expanding the brackets  C1 for suitable feedback  M1 for correctly expanding the brackets  C1 for suitable feedback  M1 for correctly expanding the brackets  C1 for suitable feedback | B |
| **8** |
| **4** | |  | Start with numbers that work.  = 2.5  So z = will satisfy conditions.  Start with a formula. e.g.  z =   Substitute z = 2.5, s = 6, t = 2 to find x.  5 = 18 – 8 + x  x = –5  so z = satisfies conditions. | | M1  A1  M1  B1  C1 | 2  3 | M1 for first method, e.g. starting with numbers  A1 for an example that works  M1 for second method, e.g. starting with a formula  B1 for an example that works  C1 for a clear, complete solution showing two different methods and two examples | B |
| **5** |
| **5** | |  | =  = n + 3 | | M1  B1 | 2 | M1 for factorising  B1 for any correct expression | B |
| **2** |
| **6** | |  | Let the base length be b, then the height will be 3b.  Area of triangle =  × base × height  =  × b × 3b  = b2  Where A = 6  b2 = 6  b2 = = 4  b = 2  so height is 3 × 2 which is 6 cm. | | C1  C1  B1    M1  A1  A1 | 3 | C1 for stating variables  C1 for stating triangle formula  B1 for correct expression  M1 for equating 6 with found expression  A1 for b = 2  A1 for 6 cm | B |
| **6** |
| **7** | |  | Boys get:  one red egg each from each of 4 girls  = 4 red  one green egg from each other = 2 green  Girls get:  one blue egg from each of the 2 boys  = 8 blue  one yellow egg from each other = 3 yellow eggs each = 12 yellow altogether. | | C1  C1  C1  C1  C1 | 3 | C1 for explanation of 4 red  C1 for explanation of 2 green  C1 for explanation of 8 blue  C1 for explanation of 12 yellow  C1 for complete clear solution | B |
| **5** |
| **8 a**  **b**  **c**  **d**  **e**  **f** | |  | | Abi  Abi stopped  12 minutes  Bryn  By 1.8 km  4.5 km  Another suitable question | B1  B1  B1  B1 B1  B1  C1  C1 | 3 | B1 cao  B1 cao  B1 cao  B1 cao  B1 cao  B1 cao  C1 for suitable question using a linear function  C1 for a suitable graph | B |
| **8** |
| **9 a**  **b**  **c** | Need to find both times when h = 0.  Substitute u = 16 m/s into the equation.  16t– 5t2 = 0  t(16 – 5t) = 0  so t= 0 or (16 – 5t) = 0  t = 0 or 5t = 16  t = 3.2  Maximum height = 16 × 1.6 – 5 × 1.62  = 25.6 – 12.8  = 12.8 m  h = ut + 5t2 + 1 | | | 3.2 s | C1  M1  A1  A1  C1  B1  B1 | 3 | C1 for clear explanation  M1 for setting h = 0  A1 for 0 and 3.2  A1 for 3.2 seconds  C1 for substituting *t* = 1.6  B1 for 12.8 m  B1 cao |  |
| **7** |
| **10 a**  **b**  **c** | Use s = ut – gt2.  Assuming g = 10, given u = 8 sin θ  and assuming suitable value for θ, for example, 30°  sin 30° = 0.5  So s = –5*t*2 + 4*t*  Complete the square to give:  = 0  Comparing this to the equation of y = x2.  Then the horse will reach its maximum at  t = 0.4.  Substitute this into s = –5t2 + 4t  to give s = 0.8 m. | | | Parabola/quadratic equation  s = 0.8 m  Suitable justification, e.g. Yes it does, the horse is in the air for 0.8 s and jumps 0.8 m into the air. | B1  C1  P1  M1  A1  A1 | 3 | B1 for either of these  EC for stating suitable assumptions for the starting point  P1 for a suitable method for finding greatest height, could also be sketch graph  M1 for suitable comparison  A1 for ft from the initial assumption  A1 cao | B |
| **6** |
| **11 a**  **b**  **c**  **d** | This graph shows expected sales for the different prices charged. If he prices his snowboard at more than £257 demand will be 0.  The graph also shows that the cheaper the snowboards, the more he will sell. But he needs to consider his charges to make sense of it all.  Number sold = 450 00 – 175P  Sales = number sold × P  = (45 000 – 175P)P  Costs = set-up fees + manufacturing costs per board | | | Minimum demand = 1 So 1 = 45 000 – 175*P* 175*P* = 44 999 *P* =   = £257  Demand = 45 000 + 95P  Profit = sales – costs  Profit = (45 000 – 175*P*)P   – (45 000 + 95P)  = 45 000P – 175*P*2 – 45 000 – 95P  = 44 905P – 175*P*2 – 45 000  From a graph of this quadratic function, his maximum profit would be approximately £2 850 000 if he sold his boards at £135. | C1  M1  A1  C1  C2  M1  M1  A1  B2  C1 | 3 | C1 for a clearly graph drawn  M1 for method of solving equation when demand = 1  A1 for answer £257  C1 for a clear explanation  C1 for explaining the relationship  C1 for commenting that there will be other costs to take into account  M1 for setting up the sales equation  M1 for setting up the profit equation  A1 for cao  B1 for profit between £2 800 000 and £2 900 000  B1 for cost of boards between £120 and £140  C1 for a drawn graph of the quadratic equation found | B |
| **10** |
| **12 a**  **b** |  | | | ii, v and vi might be difficult as they all involve squaring a term.  The typical error made in ii will be to calculate half of *at* and then to square that. The same error can be found in vi where 2π*r* can be calculated first and then squared.  ii and vi are also difficult to rearrange as they involve a quadratic element and it’s not easy to make each variable the subject of the formula.  Typical errors in rearranging the equation s = ut + at2 to make a the subject include:   * incorrect sign when changing sides, e.g. s + ut = at2 * incorrect removal of fraction, e.g.  to leave (s + t) = at2. | B2  B2 | 2 | B1 for identifying some examples with a valid reason  B1 for clear identification and explanation of classic errors  B1 for identifying some examples with a valid reason  B1 for clear identification and explanation of classic errors | M |
| **4** |
| **13** |  | | | c and d can be difficult because they contain minus signs; errors are often made when combining minus signs.  In substituting x = –3 into t = –2(3 – x), a common error is to assume 3 – –3 is 0.  In substituting *x* = –3 into z = a typical error is to assume a negative divided by a negative gives a negative answer.  A suggestion to avoid these errors is to remember that when multiplying or dividing with positive and negative numbers, same signs means positive, different signs means negative. | C1  C2  C1 | 2 | C1 for identifying some examples with a valid reason  C1 for clear identification of one typical error with one equation.  C1 for another typical error  C1 for a satisfactory suggestion | M |
| **4** |
| **14** |  | | | The similarities are that both include an equals sign and both require the manipulation of terms.  The difference is that in solving an equation you reach a numerical answer, but in rearranging you still have a formula. | B1  B1 | 2 | B1 for clear explanation of similarities  B1 for clear explanation of differences | M |
| **2** |
| **15** |  | | | In line 2 Phillip has initially rearranged x2 + 2x– 3 to x(x+ 2) – 3 when he should have factorised it as (x + 3)(x – 1).  He has incorrectly simplified in line 3. He should have factorised (x2 – 9) to (x + 3)(x – 3).  Philip has cancelled incorrectly just by looking at the different numbers and not realising that you can only cancel a number on both numerator and denominator if it is a factor of the complete expression. | B1  B1  C1 | 2 | B1 for identifying the first error  B1 for identifying the second error  C1 for a clear explanation of the errors made | M |
| **3** |
| **16 a**  **b i**  **ii**  **iii** |  | | | ‘I think of a number and double it’ just means an expression of 2x, where x is the number I thought of – still unknown at the moment.  ‘I think of a number and double it – the answer is 12’ has a solution that I know is 6.  One  e.g. 10 = p + 3  Because each solution is p = 7. | C1  B1  B1  C1 | 2 | C1 for clear explanation of the difference  B1 cao  B1 a correct example  C1 a clear explanation | M |
| **4** |

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| **17 a**  **b** |  | An expression is any combination of letters and numbers, e.g. 3x + 5y.  An equation contains an equals sign and at least one variable, e.g.  3x + 5y = 10.  A formula is like an equation, but it is a rule for working out a particular value, such as the area of a rectangle or the cost of cleaning windows, e.g. A = lb, where A is area, l is length and b is breadth.  An identity looks like a formula but it is true for **all** values, e.g.  (x + 1)2 = x2 + 2x + 1 is true for all values of x.  For example:  State whether each item is an expression, equation, formula or identity. Explain why.  **a** x + y2x + y = 6A = πr23x = 2x + x  **b** m2 = m × m 5x2 – 3 10 = x – 7 A =  bh  **c** v = ut + at2 10 – 5t x2 = 16 5p = 5 × p  **d** y = x2 – 1 v =   x2 – 1 = (x + 1)(x – 1) | C4  C3 | 2 | C1 for explanation of expression  C1 for explanation of equation  C1 for explanation of formula  C1 for explanation of identity  C1 for an activity that works  C1 for plenty of practice  C1 for quality of activity | M |
| **7** |

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| **18 a**  **b**  **c**  **d** |  | The two straight-line graphs will be parallel, with the same gradient of 2,  y = 2x crosses the y-axis at the origin, and y = 2x + 6 crosses the y axis at  y = 6  The two straight-line graphs will be parallel, having the same gradient of 1,  y = x + 5 crosses the y axis at y = 5, and y = x – 6 crosses the y axis at  y = –6  The two straight-line graphs will cross each other at (, ) and each one is a reflection of the other in a vertical mirror line.  The two straight-line graphs will both cross the y-axis at the origin, one with gradient 2, the other with gradient . | C2  C2  C2  C2 | 2 | C1 for explanation of parallel  C1 for explanation containing points of intersection of axes  C1 for explanation of parallel  C1 for explanation containing points of intersection of axes  C1 for explanation containing point of intersection  C1 for explanation of symmetry  C1 for explanation of passing through origin  C1 for explanation about gradient | M |
| **8** |
| **19 a**  **b**  **c** |  | The gradients represent how quickly the variable on the *y*-axis changes as the variable on the *x*-axis changes.  The intermediate points will only have any meaning for continuous data, such as mass or height. If the data is discrete then the points will only have values when they coincide with actual data.  The intercept indicates a value that must be added to a variable value, such as a standing charge of £3.50 for a taxi fare, being included before adding on a rate per km. | C1  C2  C1 | 3 | C1 for clear explanation  C1 for clarity of continuous data  C1 for clarity of discrete data  C1 for clear explanation, using an example | M |
| **4** |
| **20 a i**  **ii**  **b**  **c** |  | The highest power will be 2 with no negative powers, e.g. y = x2 + 3x – 1, where 2 is the highest power.  The highest power will be 3 with no negative powers, e.g. y = x3 + 5x2 – 6, the 3 being the highest power.  Find points that cross the axes where possible and then create a table of values including for the turning point and the axis intercepts so that you have sufficient points to plot the curve.  1: x2 is always positive for positive or negative values of x, hence 2x2 will also be positive as positive multiplied by positive is positive. The +5 moves the graph up 5, so y = 2x2 + 5 will also always be positive for all values of x.  2: Draw the graph and illustrate all the points on the graph are above the x axis. | C1  C1  C1  C1  C1  C1  C1 | 2 | C1 for clear explanation  C1 for also using an example  C1 for clear explanation  C1 for also using an example  C1 for clear explanation  C1 for first explanation  C1 for second explanation | M |
| **7** |
| **21 a**  **b**  **c** |  | A quadratic function always has line symmetry because x2 and (–x)2 have the same y value.  A cubic equation does not have a line of symmetry as A3 will have different values depending on whether A is negative or positive.  Rotational symmetry of order 2 about the point of inflection. | C1  C1  C1 | 2 | C1 for clear explanation  C1 for clear explanation  C1 for clear description | M |
| **3** |
| **22 i**  **ii**  **iii**  **iv** |  | D, y = 5x2  B, y =  A, y = 0.5x + 2  C, d = √A | B1  B1  B1  B1 | 3 | B1 correct letter with correct example  B1 correct letter with correct example  B1 correct letter with correct example  B1 correct letter with correct example | M |
| **4** |
| **23** |  | As they are balanced 12= xy  Therefore rearranging y = , they are inversely proportional. | C1  B1  C1 | 3 | C1 for a good diagram accompanying the explanation.  B1 for the equation  C1 for the correct explanation of the relationship | M |
| **3** |
| **24 a**  **b** |  | Any two examples of the form y = x3 + c, e.g. y = x3 + 7, y = x3 – 4.  They are all quadratics.  All of them pass through the origin except *y* = 4x2 + 3.  y = 4x2 and y = 4x2 + 3 have the same shape, but the latter is moved up 3 units. | B1  B1  C3 | 2 | B1 for the first example  B1 for the second example  C1 for similarities  C2 for differences | M |
| **5** |
| **25** |  | Sometimes true.  It is only true when a > 0. | B1  C1 | 2 | B1 for sometimes  C1 for explanation | M |
| **2** |
| **26** |  | When you draw the graphs of y = 4x2 and y = –4x2 you get the graphs shown here.  It can be seen that y = 4x² is a reflection of the graph of y = –4x² in the x-axis. | C1  C1  P1 | 2 | C1 for explanation of drawing a graph of each on the same axes  C1 for an accurate diagram of both graphs on the same pair of axes  P1 for clear explanation bringing everything together. | M |
| **3** |
| **27** | Drawing a velocity/time graph allows us to illustrate the journey. The area under the graph is the distance travelled. | Distance travelled = (15 × (u + 3u)) + (10 × 3u) + (20 × 3u)  = 30u + 30u +30u = 90u  The assumption is that acceleration is at a steady rate when the motorbike speeds up and slows down. | C1  M1  M1  A1  A1 | 3 | C1 for a good diagram illustrating the journey  M1 for method of using  diagram  M1 for correct area equation  A1 cao  A1 for clear explanation of assumption. | M |
| **5** |
| **28 a**  **b**  **c** |  | x = 0  The circle has a radius of 5 and its centre is at (0, 0), halfway between D and E.  Find the distance of F(–3, 4) from the origin. If it is 5 units from the origin, it is on the circumference of the circle. Using Pythagoras’ theorem:  distance =  = 5  So F(–3, 4) is on the circumference of the circle.  The tangent at F will be at right angles to the line joining the point to the origin.  Gradient of the line is –.  The product of the gradient of this line and the tangent is –1. Use this to work out the gradient of the tangent.  Substitute the gradient and the coordinates of the point into the general equation of a straight line (y = mx + c) to find the y-intercept c. | B1  C2  C3 | 2 | B1 cao  C1 for explanation referring to a right angle triangle  C1 for explaining how using Pythagoras’ theorem helps in finding the length 5  C1 for explaining the gradient is tangent of the angle  C1 for showing y = mx + c  C1 for complete explanation showing how to work out the equation of the line | M |
| **6** |
| **29 a**  **b** |  | Use the method of elimination, in which you combine the equations to eliminate one of the variables leaving an equation in the other variable. Solve this equation then substitute the value into one of the original equations to work out the other value.  Use the substitution method, in which you make one of the variables the subject of one equation and substitute this into the other equation. Solve this equation and then substitute the value into one of the original equations to work out the other value.  Use the graphical method, in which you draw a graph of both equations on the same axes and the solution is the point of intersection.  Use the elimination method when you can eliminate one variable easily by either adding or subtracting the two equations.  Use the substitution method when it is easy to make one of the variables in one of the equations the subject of the equation.  Use the graphical method to solve equations where there is a quadratic.  You might use more than one method: if you used the graphical method and didn’t get integer values for the solution, you might then use the elimination method to find the fractional answers. | C3  C3  P1 | 2 | C1 for each explanation  C1 for each explanation  P1 for clear explanation of why you might use two methods | M |
| **7** |

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| **30** | |  | 3x – 4y = 13(1)  2x + 3y = 20 (2)  Using elimination:  Multiply (1) by 2 and (2) by 3:  6x – 8y = 26  6x + 9y = 60  Subtract the first equation from the second equation:  9y – –8y = 60 – 26  17y = 34  y = 2  Substitute y = 2 into (2):  2x + 3× 2 = 20  2x = 14  x = 7  Check by substituting both values into (1)  3 × 7 – 4 × 2 = 21 – 8 = 13 Correct.  Using substitution:  Rearrange one of the equations:  2x + 3y = 20  y =  Substitute into the other equation and rearrange.  3x – 4() = 13  3x – +  = 13  3x + = 13 +    =  = 119  x = 7  Substitute this into other equation as before to give y = 2. | M1  A1  M1  A1  M1  A1  M1  A1  M1  A1  M1  A1 | 3 | M1 for method of changing equations in order to be able to eliminate  A1 for correct equations  M1 for subtracting equations  A1 cao  M1 for substitution  A1 cao  M1 for rearrangement to get one variable as a subject  A1 cao  M1 for substitution of the one variable into the other equation  A1 cao  M1 for substitution  A1 cao | M |
| **12** |
| **31 a**  **b**  **c** | |  | The equations look awkward. The method of eliminating one variable by multiplying the first equation by 3 and the second equation by 5 will give new equations with 15y and –15y in both of them. These terms can be eliminated by adding the two new equations.  Rearranging one equation to make one variable the subject will give awkward fractions and so is not desirable.  It is not obvious whether drawing a graph of each equation will produce an integer solution.  The first equation already has y as the subject and the substitution method is ideal, substituting for y into the second equation.  The elimination method would mean unnecessary work to eliminate one of the variables.  Drawing a graph would give integer values, but it would take more time than the simple substitution method.  As one of the equations is a quadratic drawing graphs could the best method; this would only be justified if an integer solution was found.  It seems straightforward to make x the subject of the first equation and then substitute it into the second equation to get a quadratic equation that could then be solved for two solutions. Because one equation is a quadratic it is not suitable to use the elimination method. | C3  C3  C3 | 2 | C1 for explanation of advantage of first  C1 for explanation of first disadvantage  C1 for explanation of second disadvantage  C1 for explanation of advantage of first  C1 for explanation of first disadvantage  C1 for explanation of second disadvantage  C1 for explanation of advantage of first  C1 for explanation of first disadvantage  C1 for explanation of second disadvantage | M |
| **9** |
| **32 a**  **b i**  **ii**  **iii**  **c i**  **ii**  **iii**  **d** | |  | If one of the equations is a multiple of the other then there will be an infinite number of solutions, e.g.  x + y = 5  2x + 2y = 10  Every point on the line x + y = 5 is a solution, giving an infinite number of solutions.  If the equations have graphs that are parallel to each other then there will be no intersection and so no solution, e.g.  x + y = 5  x + y = 6  One solution, as one equation is not a multiple of the other and they are not parallel.  None –the gradient is the same but the intercepts are different so they are parallel.  Infinite number of solutions as the first equation is a multiple of the second, so drawing graphs gives the same line.  y – 2x = –5  y = 0.5x + 1  Substitute for y in the first equation:  0.5x + 1 – 2x = –5  –1.5x = –6  x = 4  Substitute into second equation:  y = 0.5 × 4 + 1 = 3  Check by substituting both values in the first equation  3 – 2 × 4 = 3 – 8 = –5 Correct.  No solution.  Infinite number of solutions.  You can see how many times the graphs cross each other. | C1  C1  C1  C1  C1  C1  C1  M2  A1  C1 | 2 | C1 for clear explanation  C1 for use of a good example accompanying the explanation  C1 for clear explanation  C1 for use of a good example accompanying the explanation  C1 for clear explanation  C1 for clear explanation  C1 for clear explanation  M1 for arranging equations in a suitable format  M1 equating both equations  A1 cao  C1 clear explanation | M |
| **11** |
| **33 a**  **b** | |  | They are the same equation. Multiply the first equation by 3 and it is the same as the second equation, so they have an infinite number of solutions.  Treble the first equation to get 15x– 3y= 27  They have the same coefficients of x and *y* but a different constant so they are parallel lines with no intersections and so no solutions. | C1  C1 | 3 | C1 for clear explanation  C1 for clear explanation | M |
| **2** |
| **34 a**  **b** | | x2 + 2x – 5 = 6x – 9  x2 – 4x + 4 = 0  (x – 2)(x – 2) = 0  x = 2  y = 6 × 2 – 9 = 3  y = 3  There is just one intersection of the two graphs, so it has to be sketch **iii**, as the straight line touches the curve once. | x = 2  y = 3 | M1  M1  M1  A1  A1  C1  C1 | 2  3 | M1 for equating both equations  M1 for arranging to equal 0  M1 for factorising  A1 for x = 2 cao  A1 for y = 3 cao  C1 for sketch iii  C1 for clear explanation | M |
| **7** |
| **35 a**  **b** | | Let the cost of a second class stamp be x.  Let the cost of a first class stamp be y.  10x + 6y = 902 …….(1)  8x + 10y = 1044 ……(2)  5 × (1) 50x + 30y = 4510 ….(3)  3 × (2) 24x + 30y = 3132 ….(4)  Subtract (4) from (3):  26x = 1378  x = 53  Substitute for *x* in (1):  10 × 53 + 6y = 902  6y = 902 – 530  6y = 372  y = 62  So 3 second-class plus 4 first-class will cost:  3 × 53 + 4 × 62 = 407  Cost will be £4.07.  Let the cost of a can of cola be c.  Let the cost of a chocolate bar be b.  Then:  6c + 5b = 437 …….(1)  3c + 2b = 200 ……(2)  2 × (2) .. 6c + 4b = 400 ….(3)  Subtract (3) from (1):  b = 37  Substitute for *b* in (2):  3c + 74 = 200  3c = 126  c = 42  So three cans of cola and a chocolate bar will cost:  2 × 42 + 37 = 121  Cost will be £1.21. | £4.07  £1.21 | P1  M1  M1  M1  M1  A1  M1  A1  A1  P1  M1  M1  M1  M1  A1  M1  A1  A1 | 3 | P1 for clear explanation of variables chosen  M1 for first equation created  M1 for second equation created  M1 for multiplying equation in order to be able to eliminate a variable  M1 for subtracting  A1 cao  M1 for substituting x into an equation  A1 cao  A1 cao  P1 for clear explanation of variables chosen  M1 for first equation created  M1 for second equation created  M1 for multiplying an equation in order to be able to eliminate a variable  M1 for subtracting  A1 cao  M1 for substituting b into an equation  A1 cao  A1 cao | M |
| **18** |
| **36 a**  **b** | S =  B =C  S + C = 75  C = B  B +  = 75  =75  B = 75 ×  = 20  S = 1.25B  S = 1.25 × 20 = £25  C = 75 – 25 = £50  The method used was to find equations linking each two persons at a time, and then use those to create one equation that could be solved. Once you had one solution you could find the rest.  Answer checked by going back to beginning statements and ensuring each one works. They do. | | B = £20  S = £25  C = £50 | C1  C1  C1  M1  A1  A1  A1  C2  C1 | 3 | C1 for first equation  C1 for second equation  C1 for third equation  M1 for creating single equation with one unknown  A1 for £20 cao  A1 for £25 cao  A1 for £50 cao  C1 for clear explanation.  C1 for matching explanation with the work done  C1 for clear explanation | M |
| **10** |

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| **37 a i**  **ii**  **b i**  **ii** | Draw the graph of y = x + x3.  Then solve for y = 20**.**  Let the width be x, then the length is x + 2.  Hence the area is x(x + 2) = 67.89.  Solve the quadratic equation to find x.  Create a table of values to assist in drawing the graph of y = x + x3 = 20.   |  |  |  |  |  | | --- | --- | --- | --- | --- | | x | 0 | 1 | 2 | 3 | | y | 0 | 2 | 10 | 30 |   Plot the points and draw the graph.  Follow the line from y = 20 to the graph and read down to the *x*-axis to find x = 2.6.  x2 + 2x = 67.89  x2 + 2x – 67.89 = 0  Solve with the formula:  x =  x =  so x = 7.3 or –9.3  Width, x , cannot be negative hence solution is width = 7.3 cm. | x = 2.6  Width = 7.3 cm | C2  C2  M1  A1  B1  M1  A1  M1  M1  M1  A1  A1  A1  C1 | 3 | C1 for explanation about drawing a graph  C1 for using it to solve for y = 20  C1 for showing how equation is created  C1 for explaining there will be a quadratic equation that needs solving  M1 for creating a suitable table of values  A1 for at least 4 correct, useful values  B1 for a good, accurate graph drawn  M1 for drawing the line y = 20  A1 cao  M1 for setting up initial equation  M1 for amending it to arrive at equation = 0  M1 for using the formula  A1 for correct intermediate values  A1 for both possible solutions  A1 for picking out 7.3  C1 for explanation of why this solution was selected | M |
| **16** |
| **38** |  | Area of large square = 72 = 49  To find the length (*z*) of the side of the small square:  z2 =  z2 =  z2 = 5  So the area of the small square is 25.  Shaded area is equal to  area large square – area small square.  49 – 25 = 24  So less than half is shaded, as required. | B1  M1  A1  A1  A1  C2 | 2  3 | B1 for area of large square  M1 for use of Pythagoras’ theorem to help find the side length of inner square  A1 cao  A1 cao  A1 for area of shaded part  C1 for clear explanation  C1 for complete explanation with correct mathematical notation throughout | M |
| **7** |
| **39** |  | Set up two simultaneous equations, using the information given.  5 is the first term; from the rule for the sequence, the next term is:  5 × a – b, which equals 23  Hence 5a – b = 23 …… (1)  Doing the same to the next term gives:  23a– b= 113 ……. (2)  Subtract (1) from (2) to give:  18a = 90  a = = 5  Substitute in (1):  25 – b = 23  b = 2  *So* a= 5, b= 2  Hence, the next 2 terms are 563 and 2813. | P1  B1  B1  M1  A1  M1  A1  C1 | 2 | P1 for clear initial explanation  B1 for first equation  B1 for second equation  M1 for clear method of elimination  A1 cao  M1 for substitution  A1 cao  C1 for next two terms correctly found | M |
| **8** |
| **40 a**  **b**  **c**  **d** |  | If the sum of the whole numbers from 1 to 50 is 1275 the sum from 2 to 51 will be 1275 + 51 – 1 = 1325.  Check method, using simple examples.  Using Sn =  S52 == 1326  1326 – 1 + 1325 as above.  If 1 is the first term then the nth term will be n, so their sum is 1 + n.  Because they come in pairs, there will be  of these pairs adding to the total.  So the total = (n + 1) × =, the formula given. | C1  C1  C2  C2 | 2 | C1 for clear explanation.  C1 for clear explanation  C1 for clear use of the formula  C1 for showing same answer as above  C1 for clear explanation.  C1 for use of the generalization and good mathematical language. | M |
| **6** |

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| **41 a**  **b**    **c** |  | Ali receives:  £1000 + £2000 + … + £20 000.  This amount, in pounds, is:  1000 × (sum of the numbers 1 to 20)  The sum of the first *n* natural or whole numbers is n(*n* + 1).  So the amount Ali receives after nyears is:  1000 × n(*n* + 1) or 500n(n + 1).  The amounts Ben receives each year are £1, £2, £4. The general term is  £2*n* -1. Adding these amounts gives a running total, in pounds, of: 1, 3, 7, 15,…  Looking for a pattern linking the number of years with the amount, we see that after 2 years, it is:  3 = 4 – 1 = 22 – 1  After 3 years it is:  7 = 8 – 1 = 23 – 1  After 4 years it is:  15 = 16 – 1 = 24 – 1  So after nyears it is 2*n* – 1.  After 20 years, Ali will have  500 × 20 ×19 = £190 000  Ben will have 220 – 1 = £1 048 575  Ben will have more than five times the amount that Ali has. | C1  C1  C1  C1  C1  C1  C1  C1  C1 | 2 | C1 for clear explanation  C1 for explaining how the formula for sum of integers helps  C1 for explaining the given sum  C1 for explaining how annual totals are found  C1 for explaining how to look for a pattern  C1 for showing the pattern building up  C1 for showing how the generalisation is found  C1 for showing both totals  C1 for explanation linking both totals with formula found | M |
| **9** |

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| **42 a**  **b**  **c**  **d** |  | 2n + 1  3n + 4    No, because  will always equal no matter what *n* is, and the denominator increase of 4 will always give a larger increase than the numerator increase of 1, hence the fraction can never be larger than . | B1  B1  B1  B1  C1 | 2 | B1 cao  B1 cao  B1 cao  B1 for no  C1 for a clear concise explanation | M |
| 5 |
| **43 a i**  **ii**  **iii**  **b**  **c** |  | Neither – it’s the Fibonacci series, where each term is found by adding the previous two.  Geometric – because each term is multiplied by 2 to find the next term.  Arithmetic – because to find the next term you add 4 to the previous term.  Arithmetic.  Arithmetic. | B1  C1  B1  C1  B1  C1  B1  B1 | 2 | B1 for neither  C1 for clear explanation  B1 for geometric  C1 for clear explanation  B1 for arithmetic  C1 for clear explanation  B1 for arithmetic  B1 for arithmetic | M |
| 8 |

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| **44 a**  **b**  **c**  **d** |  | a is the first term.  d is the amount added each time.  For example: use a = 2 and d = 3 to generate the sequence in part a as:  2, 5, 8, 11, 14, …  The 5th term is 14.  Using the given X*n* = a + (n - 1)d:  the 5th term will be 2 + 4 × 3 = 14, the same value.  X*n* = ar*n*-1  It is a quadratic sequence because it contains a term in n2. | B1  B1  C1  C1  P1  B1  C1 | 2 | C1 for explaining a  C1 for explaining d  C1 for creating a suitable example  C1 for generating a value higher than the third term  P1 for showing the X*n* formula gives the same value  B1 cao  C1 cao | M |
| 7 |
| **45 a**  **b** |  | Evidence of reproducing proof as given in question.  *Sn* = (2a + (n – 1)d)  for the set of integers, a = 1 and d = 1.  Hence *Sn* = (2 + (n – 1))  = (n + 1) | C1  C1  C1  P1 | 2 | C1 for correct proof clearly explained  C1 for using a and d equal to 1  C1 for correct substitution into the formula  P1 for showing how this simplifies to the desired formula | M |
| 4 |

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| **46 a**  **B**  **c**  **d**  **e** | |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | *Sn* | *a* | + *ar* | + *ar*2 | … | *+ arn* |  | | *rSn* |  | *ar* | + *ar*2 | … | *+ arn* | *+ arn +* 1 | | *Sn –rSn* | *a* |  |  |  |  | *– arn +* 1 | | S*n* = a + ar + ar2 + ar3 … + ar*n*  rS*n* = ar + ar2 + ar3 + ar4 … + ar*n* + ar*n* + 1  Therefore:  S*n* - rS*n* = a – ar*n* + 1  S*n*(1 – r) = a(1 – r*n* + 1)  S*n* = | | P1  P1  C2  B1  B1  B1 | 2  3 | P1 for equation showing at least up to ar3 and the generalisation  P1 for equation showing at least up to ar4 and the two generalisations  C1 for top two rows shown correctly  C1 for the bottom row shown correctly  B1 cao  B1 cao  B1 cao | M |
| 7 |
| **47** |  | Considering the area of an (x + 1) by  (x + 1) square:  The area of each rectangle created above is shown inside that rectangle, so it can be seen that:  (x + 1)2 = x2 + x + x + 1  = x2 + 2x + 1 as required | | C1  C2  C1 | 2 | C1 for explaining the sides of each square are (x + 1)  C1 for creating the square divided into the rectangles, using the x and the 1  C1 for areas of each rectangle indicated in the rectangles  C1 for clear explanation of required result | M |
| 4 |
| **48 a**  **b**  **c** |  | For example, 2x > 10  Divide both sides by 2 to get x > 5.  We show the solution with a line and a circle at each end point.  A solid circle means that the solution includes the end point; an open circle means that the solution does not include the end point. For example:  The top diagram shows  *x* ≤ 2. It has a solid circle at the end point *x* = 2 because that is part of the solution.  The bottom diagram shows *x* > 1, it has an open circle at the end point *x* = 1 because *x* = 1 is not part of the solution.  Starting with an equation 10 –  *x* > 4 and solving by adding  *x* to both sides gives the solution 6 > x.  This can also be given as x < 6 ….. (1)  Consider again 10 – x > 4.  This time multiply throughout by –1. Keeping the inequality sign the same gives: –10 + x > -4  Add 10 to each side to give x > 10 – 4.  This gives the solution as *x* > 6 ….. (2)  But comparing this with equation (1) we see that the signs are the other way round, this illustrates that when we multiplied through by a negative number, we should have changed the sign from > to <. | | B1  C1  C1  C1  P1  C1  C2 | 2 | B1 for a correct inequality  C1 for clear explanation of how to solve the inequality chosen  C1 for explanation about circles at the end of each line  C1 for clear explanation differentiating between solid and open circles  P1 for use of a clear diagram to support the explanation  C1 for a full, clear explanation showing both aspects of the circles  C1 for clear explanation  C1 for using an example in a way that illustrates the principle | M |
| 8 |
| **49 a**  **b i**  **ii**  **iii** |  | In this example, *x* and *y* are values satisfying the conditions:  x + y ≤ 5 x > 1 y > 2  These are drawn on the diagram.  Any region needs a minimum of three straight lines to enclose it. The region R above is where the solutions satisfying all three inequalities lie.  Point (x, y) is inside the region if the point satisfies all three inequalities.  For example, (1.5 , 3) is inside the region since 1.5 + 3 ≤ 5, 1.5 >1 and 3 > 2.  Point (x, y) is outside the region if it does not satisfy at least one of the inequalities.  For example, (2, 4) satisfies two of the conditions (x > 1 and y > 2) but does not satisfy x + y ≤ 5.  Point (x,y) is on the boundary of the region if the point satisfies one of the inequalities but only as a equality.  For example, (2, 3) is on the boundary of x + y ≤ 5 as 2 + 3 = 5. | | C1  C1  C1  C1  C1  C1  C1  C1  C1 | 3  2 | C1 for choosing three inequalities that will define a region  C1 for a clear diagram illustrating the chosen inequalities  C1 for clear explanation linking the chosen inequalities with the diagram  C1 for clear correct explanation  C1 for use of an example to illustrate this  C1 for clear correct explanation  C1 for use of an example to illustrate this  C1 for clear correct explanation  C1 for use of an example to illustrate this | M |
| 9 |
| **50 a**  **b**  **c** |  | | The total number of games cannot be greater than 4, hence w+ d≤ 4.  The number of points must be 8 or more, they score 3 for a win, 1 for a draw, hence 3*w* + *d* ≥ 8.  The shaded area is the region that satisfies these two inequalities.  In four games, they need to score at least 8 points. The graph shows that to do this they can win at least 3 games or win 2 games and draw two games.  The team would still need to score at least 8 points, but now they have five games in which to do it.  The inequality *w* + *d* ≤ 4 would change to *w* + *d* ≤ 5. The other inequality is unchanged. The line for *w* + *d* = 4 would move up to go through (0, 5) and (5, 0).The other line would be unchanged. | C3  C2  C2 | 3 | C1 for explaining w + d ≤ 4  C1 for explaining 3w + d ≥ 8  C1 for explaining what the shaded region is  C1 for explaining they need at least 8 points C1 for showing all the possible ways this could happen  C1 for explanation of how this affects both equations  C1 for complete solution, clearly showing what the new line(s) are | M |
| 7 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
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| **51 a**  **b**  **c** |  | For example, n2 can generate a quadratic sequence:  1, 4, 9, 16, 25 ……. n2  Similarities: you find the nth term for both types of sequence by looking at the differences between terms.  Differences: in a linear sequence you find the first term by subtracting the difference from the second term.  In a quadratic sequence you also have to look at the second differences. This allows you to extend the differences backwards to find the values of a, b and c in the nth term of an2 + bn + c.  For a linear sequence, just keep on adding 6 each time to give:  2, 8, 14, 20 ………..(6n – 4)  The *n*th term includes 6*n* because we add 6 each time, 6*n* – 4 because (2 – 6) = –4.  For a quadratic equation, we build up the series by again having the first differences as 6, then choosing a second difference, say 2.  This will give a table such as:   |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | n | 1 | | 2 | | 3 | | | | 4 | | | nth term | 2 | | 8 | | 16 | | | | 26 | | | 1st difference | | 6 | | 8 | | | 10 | | | | 2nd difference | | | 2 | | | 2 | |   Start with the two terms in positions *n* = 1 (2) and *n* = 2 (8) in the sequence.  Put each second difference as 2.  Then complete the first differences as shown.  Finally the nth terms can be completed as shown.   |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | n | 0 | | 1 | | | 2 | | 3 | | 4 | | | | c | –2 | | 2 | | | 8 | | 16 | | 26 | | | | a + b | | 4 | | | 6 | | 8 | | 10 | | | 2a | | | | 2 | | 2 | | 2 | |  | |   Extending the table backwards will allow us to find the values of a, b and c in the nth term an2 + bn + c.  2a = 2 → a = 1  a + b = 4 → b = 3  c = –2  Hence nth term is n2 + 3n – 2. | C1  C1  C1  C1  B1  C3  C2 | 2 | C1 for a valid quadratic sequence  C1 for a clear explanation  C1 for a clear explanation  C1 for explaining how you woud find the sequence  B1 for cao  C1 for explaining second differences  C1 for use of a table or equivalent  C1 for correctly finding a quadratic equation with 2 and 8 as starting terms  C1 for explaining how you would find the nth term  C1 for correctly finding the nth term | | M |
| 10 |
| **52** |  | (m2 – n2)2 + (2mn)2 =  *m*4 – 2*m*2*n*2+ *n*4 + 4m2n2  = m4 + 2m2n2 + n4  = (m2 + n2)2 | M1  A1  A1  C1 | 2  3 | M1 choosing two smallest terms, squaring and adding  A1 for cao  A1 for cao  C1 for showing the factorisation leads to the given result | | M |
| 4 |
| **53** |  | Because the differences between consecutive terms are 2, 3, 4, 5, 6, etc. (even and odd alternately); when generating the triangular number sequence starting with the odd 1, add even to odd to generate odd …3; add odd to odd to generate even …6; add even to even to generate even …10. Add odd to even to generate odd …15. We are now back again where we add even to odd to generate odd, and the whole sequence continues in the same way, continually giving two odd, two even, etc. | C3 | 2 | C1 for explaining the about the differences of the terms being the set of integers  C1 for explaining the pattern is odd, even, odd, even and so on  C1 for explaining the complete sequence of combining odd and even to generate the final sequence | | M |
| **3** |
| **54** |  | The assumption is that p and q are integers.  10p + q = 7n where n is also an integer.  7p + 3p + q = 7n  3p + q = 7n – 7p  = 7(n – p)  As n and p are integers then n – p is also an integer hence 7(n – p) is a multiple of 7 and so 3p + q must be as well. | P1  C1  M1  A1  C1 | 2 | P1 for giving the assumption about p and q  C1 for expressing 10p + q as a multiple of 7  M1 for expressing 3p + q in terms of 10p + q  A1 for similar expression  C1 for clear full explanation | M | |
| **5** |
| **55** |  | 2(5(x – 2) + y) = 2(5x – 10 + y)  = 10x – 20 + 2y …….(1)  10(x – 1) + 2y – 10 = 10x – 10 + 2y – 10  = 10x – 20 + 2y ….(2)  Equation (1) = equation (2)  Hence the two expressions are equal. | M1  A1  M1  A1  P1 | 2 | M1 for expanding  A1 cao  M1 for expanding  A1 cao  P1 for explaining the two expressions are the same | M | |
| **5** |

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| **56 a**  **b** |  | | Take two numbers x and y where x > y.  First step: 5(x – 2) = 5x – 10  Second step: 2(5x – 10 + y)  = 10x – 20 + 2y  Third step: 10x – 20 + 2y + 9 – y  = 10x – 11 + y  Fourth step: 10x – 11 + y + 11 = 10x + *y*  Hence where the first two numbers might have been 7 and 3, the final outcome would be 70 + 3 = 73.  Let the single-digit number be x and the two-digit number be 10a + b.  First step: 10(10a + b) – 9x = 100a + 10b – 9x  = 100a + 10b + x – 10x  = 100a + 10b – 10x + x  = 100a + 10(b – x) + x  The hundreds unit is a.  The tens unit is (b – x).  The units term is x, the same as the single digit we started with.  The split then becomes 10a + (b – x) and x.  Adding these two gives 10a + b – x + x  which is 10a + b, the two-digit term. | C1  B1  B1  B1  B1  P1  C1  P1  C1  C1  C1  C1  P1 | 2 | C1 for initial explanation  B1 cao  B1 cao  B1 cao  B1 cao  P1 for explanation of how this shows the final result  C1 for defining each digit  P1 for expressing the first manipulation algebraically  C1 for showing it in terms of hundreds, tens and units  C1 for correct hundreds  C1 for correct tens  C1 for correct unit explanation  P1 for clear explanation as to how the second manipulation gives the two-digit term | M |
| **13** |
| **57** |  | | Expand each square and add:  n2 – 2n + 1 + n2 + n2 + 2n + 1  = 3n2 + 2n – 2n + 2  = 3n2 + 2 as given. | M1  A1  C1  C1 | 2 | M1 for expanding brackets  A1 for correct expansion of brackets  C1 for showing how the n terms cancel  C1 for complete solution with no incorrect notation or terminology | M |
| **4** |
| **58** |  | | The difference is 5 so nth term is 5n + *c*  where c = first term – 5  = 4 – 5 = –1  So nth term is 5n – 1.  Check the 4th term gives 19.  When n = 4, 4n – 1 = 20 – 1 = 19, correct. | C1  M1  A1  A1  C1 | 2 | C1 for obtaining the difference of 5  M1 for finding c  A1 cao  A1 cao  C1 for showing a check works | M |
| **5** |
| **59** | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | n | 0 | | 1 | | | 2 | | | 3 | | | 4 | | | | c | 0 | | 1 | | | 3 | | | 6 | | | 10 | | | | a + b | | 1 | | | 2 | | | 3 | | | 4 | | | | 2a | | | | 1 | | | 1 | | | 1 | | | | Triangular numbers are 1, 3, 6, 10, 15 …..  This will give a table as:   |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | n | 1 | | 2 | | 3 | | | 4 | | | | nth term | 1 | | 3 | | 6 | | | 10 | | | | 1st diff’nce | | 2 | | 3 | | | 4 | | | | 2nd diff’nce | | | 1 | | | 1 | | |   Extending the table backwards will allow us to find the values of a, b and c in the nth term an2 + bn + c.  .  2a = 1 → a =  a + b = 1 → b =  c = 0  Hence nth term is n2 + n = (n2 + n)  = n(n + 1) | | B1  M1  A1  C1  M1  A1  B1  B1  B1  P1 | 2  3 | B1 for showing triangular numbers  M1 for method of finding differences  A1 for correct table  C1 for explanation of extending table backwards  M1 for method of extending table  A1 for correct table  B1 correct a  B1 correct b  B1 correct c  P1 for correct evaluation of generalisation to show given result | M |
| **10** |
| **60** |  | T*n* = n(n + 1)  T2*n* + 1 = (2n + 1)(2n + 1 + 1)  = (2n + 1)(2n + 2)  = (2n + 1)(n + 1)  = 2n2 + 3n + 1 …….. (1)  T*n* + 1 = (n + 1)(n + 1 + 1)  = (n + 1)(n + 2)  = (n2 + 3n + 2) …….(2)  So T2*n* + 1 – T*n* + 1  = 2n2 + 3n + 1 – n2 – n – 1  = n2 + n  = (n2 + n)  = (n + 1) but Tn =  n(n + 1)  = 3T*n* as given | | B1  M1  A1  M1  A1  B1  P1 | 2 | B1 for correct T*n* formula  M1 for substituting 2n + 1  A1 cao  M1 for substituting n + 1  A1 cao  B1 for subtracting each equation  P1 for clear full explanation of proving the final connection | M |
| **7** |
| **61** |  | T*n* = n(n + 1)  =  ==  But n2 + n – 2 factorises to  (n – 1)(n + 2)  So final expression is | | B1  C3  B1  P1 | 2 | B1 for T*n*  formula  C1 for numerator expansion  C1 for denominator expansion  C1 for cancelling  B1 for correct factorisation  P1 for fully clear correct proof with no mathematical notational errors | M |
| **6** |

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| **62** |  | Let the first number be x, then the next four are x + 1, x + 2, x + 3 and x + 4.  The sum of these is 5x + 1 + 2 + 3 + 4  which is 5x +10 = 5(x + 2),  a multiple of 5. | P1  M1  P1 | 2 | P1 for stating each term in algebraic form  M1 for adding all 5 terms  P1 for showing they are multiple of 5 | M |
| **3** |
| **63** |  | 10*x* =  Hence *b* × 10*x* = a  Substitute this into 10*y* =  to give 10*y* =  Hence 10*y* =  So 10*y* × 10*x* = 1  So 10(*x* + *y*) = 1  But 100 = 1  And so x + y = 0. | P1  M1  C1  C1  C1  B1  P1 | 2 | P1 for expressing *a* as subject  M1 for substituting into other expression  C1 for showing the correct substitution  C1 for showing the product of the two terms equal to 1  C1 for showing combination of indices  B1 for 100 = 1  P1 for complete, clear proof with clear mathematical statements | M |
| **7** |

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| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **64** |  | Where two terms are x and x + 1  the expression required is:  (x + x + 1)2 – (x2 + (x + 1)2)  = (2x + 1)2 – (x2 + x2 + 2x + 1)  = 4x2 + 4x + 1 – 2x2 – 2x – 1  = 2x2 + 2x  = 2x(x + 1) …..(1)  Where T*x* = x(x + 1)  4T*x* = 2x(x + 1), which is same as the result in equation (1). | P1  C1  C1  C1  C1  P1 | | 2 | | P1 for identifying the two terms  C1 for correct expression as asked  C1 for correct expansion of all brackets  C1 for correct simplification  C1 for correct factorisation  P1 for clear complete proof with correct mathematical notation | M |
| **6** | |
| **65** |  | Let p = x  Then q = x + 1  and r = x + 2  so pr = x(x + 2)  = x2 + 2x  q2 – 1 = (x + 1)2 – 1  = x2 + 2x + 1 – 1  = x2 + 2x = pr | B1  B1  B1  B1  B1  P1 | | 2 | | B1 for q expressed algebraically  B1 for r expressed algebraically  B1 for product pr expressed algebraically  B1 for q2 – 1 expressed algebraically  B1 for simplification  P1 for complete clear proof | M |
| **6** | |
| **66 a**  **b i**  **ii** |  | There are many equivalent expressions.  For example, expand the bracketed term: – q2 – q – 4  For example:  For example: 4x2 + 10x | B1  B1  B1 | 2  3 | | B1 for a correct example  B1 for a correct example  B1 for a correct example | | H |
| **3** |
| **67 a**  **b**  **c**  **d** |  | To make it a product of two linear expressions.  The quadratic expression.  That the signs of the numbers in the brackets are different.  One factor of the constant term is zero. There is only one set of brackets. | C1  C1  C1  C1 | 2  3 | | C1 for clear explanation  C1 for clear explanation  C1 for clear explanation  C1 for clear explanation | | H |
| **4** |
| **68 a**  **b** |  | For example: x2 – 1.  Because each part is a square, x2 and 12, one is subtracted from the other.  Because:  1000 × 998 = (999 + 1) × (999 – 1)  = 9992 – 1 | C1  C1 | 3 | | C1 for clear explanation  C1 for clear explanation | | H |
| **2** |
| **69** |  | Two that can be cancelled, for example:  and  I chose two straightforward ones, one that would cancel by a single letter and one that would cancel by an algebraic term.  Two that cannot be cancelled, for example:  and  I chose two straightforward examples, one being a single term as numerator and denominator, the other one where the denominator was more than a single term. | B1  C1  B1  C1 | 2  3 | | B1 for two examples that cancel  C1 for a clear explanation  B1 for two examples that don’t cancel  C1 for a clear explanation | |  |
| **4** |
| **70** |  | To get such a term on the top this must be the difference of two squares, hence the two expressions both need multiplying by (3x – 4) to give:  This expands to: | C1  B1  C1  P1 | 2 | | C1 for clear explanation  B1 for (3*x* – 4)  C1 for setting up the expression  P1 for showing how to find the final expression in suitable format | | H |
| **4** |

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| **71** |  | (2a + b)(2a + b) = 4a2 + 4ab + b2  (2a + b)(2a – b) = 4a2 – b2  (2a – b)(2a – b) = 4a2 –4ab + b2  (a + b)(a + b) = a2 + 2ab + b2  (a + b)(a – b) = a2 – b2  (a – b)(a – b) = a2 – 2ab + b2  The difference in the two is that in the (2a ± b) product, both the a2 and ab terms have a coefficient of 4 (when the ab term is not zero), but in the (a ± b) product, the a2 term has a coefficient of 1 and the ab term has a coefficient of 2 (when the ab term is not zero). | C1  C1  C1 | 2 | C1 for showing all the possibilities  C1 for showing all the possibilities  C1 for clear explanation | H |
| **3** |

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| **72 a**  **b** |  | Draw a triangle ABC.  Using trigonometric functions:  sin *C* =  Therefore: *h* = *b* sin C  Then using the basic formula for area of a triangle:  Area = a × h  Substituting for *h* gives:  Area =  a *×* bsin *C*  Area =  absin *C* as required.  Using the given triangle:  *C* = 45°, a = x + 2 = 6, b = x – 2 = 2  Area =  absin *C*  =× 6 × 2 × sin 45°  = 6 ×  =  Multiply numerator and denominator by .  This gives =  = 3as required. | C1  C1  C1  B1  M1  A1  P1  P1  M1  A1  M1  P1 | 2 | C1 for drawing diagram correctly labelled  C1 for trigonometric expression linking C, *h* and *b*  C1 for *h* = *b* sin C  B1 formula for area of triangle  M1 substitution of *h*  A1 cao  P1 for complete correct proof with correct mathematical notation throughout  P1 for expressing *a*, *b* and *C*  M1 for substituting for *a*, *b* and *C*  A1 cao  M1 for dealing with the in denominator  P1 for full explanation showing given result | H |
| **12** |
| **73** |  | f(x) = 3 – 7x  Find f–1(x) from *y* = 3 – 7x:  7*x* = 3 – y  *x* =  So f–1(x) =  Find g–1(x) from y = 7x + 3:  7x = y – 3  x =  So g-1(x) =  So f-1(x) + g-1(x) = +  = =    = 0 as required. | M1  A1  M1  A1  P1  C1 | 2 | M1 for method of finding inverse  A1 cao  M1 for method of finding inverse  A1 cao  P1 for showing how the two functions can be added together  C1 for complete explanation of how they sum to 0 | H |
| **6** |

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| **74 a i**  **ii**  **iii**  **b i**  **ii**  **c** |  | It means that the constant terms in both expressions are positive, or if one is positive and one is negative, their sum is positive.  It means that the constant terms in both expressions are negative, or if one is positive and one is negative their sum is negative.  The expression is the difference of two squares.  For example, (4x + 2)(x – 1)  = 4x2 – 2x – 2 as required.    For example, (3*x* + 1)(*x* + 1)  = 3*x*2 + 4*x* + 1 as required.  If it is positive then both expressions have the same sign.  If it is negative then the expressions have different signs. | P1  P1  P1  P1  C1  C1  C1  C1 | 2 | P1 for first condition  P1 for second condition  P1 for first condition  P1 for second condition  C1 for clear explanation  C1 for correct example explained  C1 for correct example explained  C1 for complete clear explanation | H |
| **8** |

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| **75 a**  **b**  **c**  **d** |  | 2x2 + 10x – 5 = 0  Divide through by 2:  x2 + 5x –  = 0  Completing the square:  (x + )2 –  – = 0  (x + )2 = =  Taking the square root of each side:  x +  =  = ± 2.958  x = 2.958 – 2.5 or x = –2.958 – 2.5  x = 0.458 or x = –5.458  Consider the 4 lines of his working.  Line 1: he has forgotten the –5 in the equation.  Line 2: he has squared the right-hand side incorrectly.  Line 3: the should be and the should be  Line 4: there should be two solutions.  *x*2 + 5*x* – 5 = 0  (*x* + )2 – 5 = ()2 Don’t forget the –5 in the original equation.  (x + )2 – 5 =  Square the whole of the bracket on the right-hand side.  (x + )2 = 5 + Remember, when you add a term on one side, you must also add it on the other side.  (x + ) = ± = ±  Be careful when taking square roots of fractions. Don’t forget that when you find square root there is a positive and a negative root. | M1  A1  M1  M1  A1  A1  C1  C1  B1  B1  B1  B1  C2 | 2 | M1 for squaring half of 5  A1 cao  M1 correctly simplifying equation  M1 completing the square  A1 cao  A1 for 2.958 (3 dp or more)  C1 for showing the two possible solutions  C1 for two correct solutions (3 dp or more)  B1 for line 1 comments  B1 for line 2 comments  B1 for line 3 comments  B1 for line 4 comments  C1 for commenting on each line  C1 for clear positive comments that would be deemed helpful | H |
| **14** |
| **76 a**  **b**  **c** |  | If the coefficient of x2 is positive, the turning point is between the two roots, so choose an equation with two positive x roots, say x = 1 and x = 3.  The quadratic that has these roots is  y = (x – 1)(x – 3), which is  y = x2 – 4x + 3.  Complete the square to get y = (x – 2)2 – 22 + 3.  Hence the turning point of y = (x – 2)2 – 1 will have a positive x-value.  If the equation has no roots, then the turning point will be above the x-axis, hence a positive value of y.  From the general form of the quadratic equation, y = ax2 + bx + c, the value of b2 is less than 4ac so, keeping a as 1, we could choose c as 6 and b as 2, giving y = x2 + 2x + 6  Complete the square to get y = (x + 1)2 – 1 + 6.  Hence the turning point of y = (x + 1)2 + 5 will have a positive y value.  The y-intercept will be positive if *y* is positive when x = 0. for example: y = (x + 2)2 + 3, when x = 0 y = 7, positive so y = (x + 2)2 + 3 has a y-intercept that is positive. | C1  P1  C1  C1  P1  C1  P1 | 2 | C1 for clear explanation of what equation to look for.  P1 for choosing a suitable equation with these characteristics.  C1 for a suitable equation with complete justification  C1 for clear explanation of what equation to look for.  P1 for choosing a suitable equation with these characteristics.  C1 for a suitable equation with complete justification  P1 for complete clear explanation | H |
| **7** |

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| **77 a**  **b** |  | A table of values for the graph will be:   |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | | x | –3 | –2 | –1 | 0 | 1 | 2 | | y | 8.25 | 4.25 | 2.25 | 2.25 | 4.25 | 8.25 |   f(x+ 3) – 2 is a translation 3 left and 2 down of f(x). Therefore, as the turning point moves 2 down, it will now turn on the x*-*axis, giving one real root at the point (–3.5, 0). | C1  C1  C1  C1 | 2  3 | C1 for finding suitable points to assist sketch the graph  C1 for a suitable sketch of the graph  C1 for explaining how the function will change the graph  C1 for explanation about turning point being now on the x-axis | H |
| **4** |