<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>Method mark</td>
</tr>
<tr>
<td>A</td>
<td>Accuracy mark</td>
</tr>
<tr>
<td>B</td>
<td>Working mark</td>
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<tr>
<td>cao</td>
<td>Correct answer only</td>
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<tr>
<td>oe</td>
<td>Or equivalent</td>
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<tr>
<td>ft</td>
<td>Follow through</td>
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<tr>
<td>Question</td>
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<tr>
<td>1 a</td>
<td>Example</td>
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<td></td>
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</tr>
<tr>
<td>b</td>
<td>$3(c + 5) = 3c + 15$</td>
</tr>
<tr>
<td>c</td>
<td>Example</td>
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<tr>
<td></td>
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<tr>
<td>d</td>
<td>Example</td>
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<td></td>
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</tr>
<tr>
<td>2</td>
<td>A letter, say $f$, stands for an unknown if it is in an equation such as $3f + 2 = 14$. Then $f = 4$ is the only number that satisfies this equation.</td>
</tr>
<tr>
<td></td>
<td>A letter stands for a variable if it is part of an equation that has more than two letters, e.g. $A = \pi r^2$, where both $A$ and $r$ are variables that will be different for different values of $A$ or $r$.</td>
</tr>
<tr>
<td></td>
<td>B1</td>
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<td>B1</td>
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<td>B1</td>
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</tbody>
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2
<p>| | | | | | |</p>
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<thead>
<tr>
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</tr>
</thead>
</table>
| 3 a | $5(c + 4) = 5c + 20$  
Feedback: Don’t forget to multiply out both terms in the brackets. | **M1** | 2 | **M1** for correctly expanding the brackets  
**A1** for suitable feedback | **B** |
| b  | $6(t - 2) = 6t - 12$  
Feedback: Don’t forget $6(......)$ means **multiply** both terms by 6. | **M1** | **A1** | **M1** for correctly expanding the brackets  
**A1** for suitable feedback | **B** |
| c  | $-3(4 - s) = -12 + 3s$  
Feedback: Don’t forget $-3(......)$ means **multiply** both terms by 6 and minus $\times$ minus $= ...$ | **M1** | **A1** | **M1** for correctly expanding the brackets  
**A1** for suitable feedback | **B** |
| d  | $15 - (n - 4) = 15 - n + 4 = 15 + 4 - n = 19 - n$  
Feedback: Don’t forget $-(n - 4)$ means multiply each term inside the brackets by $-1$ and that the $-\text{inside the brackets belongs to the } 4$ to make it $-4$. | **A1** | **A1** for suitable feedback | **B** |
|   |   |   |   |   |   |
| 4  | Start with numbers that work.  
\[
\frac{(6 - 1)}{2} = 2.5
\]  
So $z = \frac{(s - 1)}{t}$ will satisfy conditions.  
Start with a formula. e.g.  
\[
z = \frac{(3s - 4t + x)}{2}
\]  
Substitute $z = 2.5$, $s = 6$, $t = 2$ to find $x$.  
$5 = 18 - 8 + x$  
$x = -5$  
so $z = \frac{(3s - 4t - 5)}{2}$ satisfies conditions. | **M1** | 3 | **M1** for first method, e.g. starting with numbers  
**A1** for an example that works | **B** |
| 5  | $\frac{(2n + 6)}{2} = \frac{2(n + 3)}{2} = n + 3$ | **M1** | **A1** | **M1** for factorising  
**A1** for any correct expression | **B** |
### 6

Let the base length be $b$, then the height will be $3b$.

Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times b \times 3b$

$= \frac{3}{2} b^2$

Where $A = 6$

$\frac{3}{2} b^2 = 6$

$b^2 = \frac{2 \times 6}{3} = 4$

$b = 2$

so height is $3 \times 2$ which is 6 cm.

B1 3 B1 for stating variables
B1 3 B1 for stating triangle formula
B1 1 B1 for correct expression

### 7

Boys get:
- one red egg each from each of 4 girls = 4 red
- one green egg from each other = 2 green

Girls get:
- one blue egg from each of the 2 boys = 8 blue
- one yellow egg from each other = 3 yellow eggs each = 12 yellow altogether.

B1 3 B1 for explanation of 4 red
B1 3 B1 for explanation of 2 green

### 8

<table>
<thead>
<tr>
<th>a</th>
<th>Abi</th>
</tr>
</thead>
<tbody>
<tr>
<td>b</td>
<td>Abi stopped</td>
</tr>
<tr>
<td>c</td>
<td>12 minutes</td>
</tr>
<tr>
<td>d</td>
<td>Bryn</td>
</tr>
<tr>
<td>e</td>
<td>By 1.8 km</td>
</tr>
<tr>
<td>f</td>
<td>4.5 km</td>
</tr>
</tbody>
</table>

Another suitable question

B1 3 B1 for suitable question using a linear function
B1 3 B1 for a suitable graph
| 9 a | Need to find both times when \( h = 0 \).
    Substitute \( u = 16 \) \( \text{m/s} \) into the equation.
    \( 16t - 5t^2 = 0 \)
    \( t(16 - 5t) = 0 \)
    so \( t = 0 \) or \( 16 - 5t = 0 \)
    \( t = 0 \) or \( 5t = 16 \)
    \( t = 3.2 \)   3.2 s | B1 | B1 for clear explanation
    A1 | M1 for setting \( h = 0 \)
    A1 |
| **b** | Maximum height = \( 16 \times 1.6 - 5 \times 1.6^2 \)
    = \( 25.6 - 12.8 \)
    = \( 12.8 \) \( \text{m} \) | A1 | A1 for 0 and 3.2
    A1 | A1 for 3.2 seconds
    B1 | B1 for substituting \( t = 1.6 \)
    B1 | B1 for 12.8 \( \text{m} \)
    B1 |
| **c** | \( h = ut + 5t^2 + 1 \) | B1 | B1 cao |

| 10 a | Use \( s = ut - \frac{1}{2} gt^2 \).
    Assuming \( g = 10 \), given \( u = 8 \) \( \sin \theta \)
    and assuming suitable value for \( \theta \), for example, \( 30^\circ \)
    \( \sin 30^\circ = 0.5 \)
    So \( s = -5t^2 + 4t \)
    Complete the square to give:
    \( (t - \frac{4}{10})^2 - (\frac{4}{10})^2 = 0 \)
    Comparing this to the equation of \( y = x^2 \).
    Then the horse will reach its maximum at \( t = 0.4 \).
    Substitute this into \( s = -5t^2 + 4t \)
    to give \( s = 0.8 \) \( \text{m} \). | Parabola/quadratic equation | B1 | B1 for either of these
    B1 | EC for stating suitable assumptions for the starting point
    M1 | M1 for a suitable method for finding greatest height, could also be sketch graph
    | M1 for suitable comparison
    A1 | A1 for ft from the initial assumption
    A1 | A1 cao |
| **b** | Suitable justification, e.g. Yes it does, the horse is in the air for 0.8 \( \text{s} \) and jumps 0.8 \( \text{m} \) into the air. | | |
This graph shows expected sales for the different prices charged. If he prices his snowboard at more than £257 demand will be 0.

The graph also shows that the cheaper the snowboards, the more he will sell. But he needs to consider his charges to make sense of it all.

Number sold = 450 00 – 175P

Sales = number sold × P
= (45 000 – 175P)P

Costs = set-up fees + manufacturing costs per board

Demand = 45 000 + 95P

From a graph of this quadratic function, his maximum profit would be approximately £2 850 000 if he sold his boards at £135.
### 12 a

ii, v and vi might be difficult as they all involve squaring a term. The typical error made in ii will be to calculate half of \( at \) and then to square that. The same error can be found in vi where \( 2\pi r \) can be calculated first and then squared.

ii and vi are also difficult to rearrange as they involve a quadratic element and it’s not easy to make each variable the subject of the formula.

Typical errors in rearranging the equation \( s = ut + \frac{1}{2}at^2 \) to make \( a \) the subject include:

- Incorrect sign when changing sides, e.g. \( s + ut = \frac{1}{2}at^2 \)
- Incorrect removal of fraction, e.g. \( \frac{1}{2} \) to leave \( s + t = at^2 \).

### B2

2

B1 for identifying some examples with a valid reason
B1 for clear identification and explanation of classic errors

### M

B1 for identifying some examples with a valid reason
B1 for clear identification and explanation of classic errors
c and d can be difficult because they contain minus signs; errors are often made when combining minus signs. In substituting $x = -3$ into $t = -2(3 - x)$, a common error is to assume $3 - -3$ is 0.

In substituting $x = -3$ into $z = \frac{-2(x + 2)}{x}$ a typical error is to assume a negative divided by a negative gives a negative answer.

A suggestion to avoid these errors is to remember that when multiplying or dividing with positive and negative numbers, same signs means positive, different signs means negative.

<table>
<thead>
<tr>
<th>13</th>
<th>B1</th>
<th>2</th>
<th>B1 for identifying some examples with a valid reason</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B2</td>
<td></td>
<td>B1 for clear identification of one typical error with one equation.</td>
<td></td>
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<tr>
<td></td>
<td>B1</td>
<td></td>
<td>B1 for another typical error</td>
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<td></td>
<td>B1</td>
<td></td>
<td>B1 for a satisfactory suggestion</td>
<td></td>
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</tbody>
</table>

The similarities are that both include an equals sign and both require the manipulation of terms.

The difference is that in solving an equation you reach a numerical answer, but in rearranging you still have a formula.

<table>
<thead>
<tr>
<th>14</th>
<th>B1</th>
<th>2</th>
<th>B1 for clear explanation of similarities</th>
<th>M</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>B1</td>
<td></td>
<td>B1 for clear explanation of differences</td>
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</tbody>
</table>

In line 2 Phillip has initially rearranged $x^2 + 2x - 3$ to $x(x + 2) - 3$ when he should have factorised it as $(x + 3)(x - 1)$.

He has incorrectly simplified in line 3. He should have factorised $(x^2 - 9)$ to $(x + 3)(x - 3)$.

Philip has cancelled incorrectly just by looking at the different numbers and not realising that you can only cancel a number on both numerator and denominator if it is a factor of the complete expression.

<table>
<thead>
<tr>
<th>15</th>
<th>B1</th>
<th>2</th>
<th>B1 for identifying the first error</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>B1</td>
<td></td>
<td>B1 for identifying the second error</td>
<td></td>
</tr>
<tr>
<td></td>
<td>B1</td>
<td></td>
<td>B1 for a clear explanation of the errors made</td>
<td></td>
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<tr>
<td><strong>16 a</strong></td>
<td></td>
<td>‘I think of a number and double it’ just means an expression of $2x$, where $x$ is the number I thought of – still unknown at the moment. ‘I think of a number and double it – the answer is 12’ has a solution that I know is 6. One e.g. $10 = p + 3$ Because each solution is $p = 7$.</td>
<td>B1</td>
<td>2</td>
</tr>
<tr>
<td>bi</td>
<td>ii</td>
<td>iii</td>
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</table>

**Example:**

1. $10 = p + 3$
2. Because each solution is $p = 7$.
An expression is any combination of letters and numbers, e.g. $3x + 5y$.
An equation contains an equals sign and at least one variable, e.g. $3x + 5y = 10$.
A formula is like an equation, but it is a rule for working out a particular value, such as the area of a rectangle or the cost of cleaning windows, e.g. $A = lb$, where $A$ is area, $l$ is length and $b$ is breadth.
An identity looks like a formula but it is true for all values, e.g. $(x + 1)^2 = x^2 + 2x + 1$ is true for all values of $x$.

For example:
State whether each item is an expression, equation, formula or identity. Explain why.

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<table>
<thead>
<tr>
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<tbody>
<tr>
<td>a</td>
<td>$x + y$</td>
<td>$2x + y = 6$</td>
</tr>
<tr>
<td>b</td>
<td>$m^2 = m \times m$</td>
<td>$5x^2 - 3$</td>
</tr>
<tr>
<td></td>
<td>$10 = x - 7$</td>
<td>$A = \frac{1}{2} bh$</td>
</tr>
<tr>
<td>c</td>
<td>$v = ut + \frac{1}{2} at^2$</td>
<td>$10 - 5t$</td>
</tr>
<tr>
<td></td>
<td>$x^2 = 16$</td>
<td>$5p = 5 \times p$</td>
</tr>
<tr>
<td>d</td>
<td>$y = x^2 - 1$</td>
<td>$v = \frac{b^2h}{3}$</td>
</tr>
<tr>
<td></td>
<td>$x^2 - 1 = (x + 1)(x - 1)$</td>
<td>$\frac{6}{y}$</td>
</tr>
</tbody>
</table>
| 18 a | The two straight-line graphs will be parallel, with the same gradient of 2. 
\( y = 2x \) crosses the y-axis at the origin, and 
\( y = 2x + 6 \) crosses the y-axis at \( y = 6 \) |
| b | The two straight-line graphs will be parallel, having the same gradient of 1. 
\( y = x + 5 \) crosses the y-axis at \( y = 5 \), and 
\( y = x - 6 \) crosses the y-axis at \( y = -6 \) |
| c | The two straight-line graphs will cross each other at \( \left( \frac{1}{2}, \frac{1}{2} \right) \) and each one is a reflection of the other in a vertical mirror line. |
| d | The two straight-line graphs will both cross the y-axis at the origin, one with gradient 2, the other with gradient \( \frac{1}{2} \). |
| 19 a | The gradients represent how quickly the variable on the y-axis changes as the variable on the x-axis changes. |
| b | The intermediate points will only have any meaning for continuous data, such as mass or height. If the data is discrete then the points will only have values when they coincide with actual data. |
| c | The intercept indicates a value that must be added to a variable value, such as a standing charge of £3.50 for a taxi fare, being included before adding on a rate per km. |
### 20 a

**ii**

The highest power will be 2 with no negative powers, e.g. \( y = x^2 + 3x - 1 \), where 2 is the highest power.

The highest power will be 3 with no negative powers, e.g. \( y = x^2 + 5x^2 - 6 \), the 3 being the highest power.

Find points that cross the axes where possible and then create a table of values including for the turning point and the axis intercepts so that you have sufficient points to plot the curve.

1: \( x^2 \) is always positive for positive or negative values of \( x \), hence \( 2x^2 \) will also be positive as positive multiplied by positive is positive. The +5 moves the graph up 5, so \( y = 2x^2 + 5 \) will also always be positive for all values of \( x \).

2: Draw the graph and illustrate all the points on the graph are above the \( x \) axis.

### 21 a

**b**

A quadratic function always has line symmetry because \( x^2 \) and \((-x)^2\) have the same \( y \) value.

A cubic equation does not have a line of symmetry as \( A^3 \) will have different values depending on whether \( A \) is negative or positive.

Rotational symmetry of order 2 about the point of inflection.

### 22 i

**ii**

D, \( y = 5x^2 \)

B, \( y = \frac{12}{x} \)

**iii**

A, \( y = 0.5x + 2 \)

C, \( d = \sqrt{A} \)

**iv**

B1 2 B1 for clear explanation
B1 2 B1 for also using an example
B1 2 B1 for clear explanation
B1 2 B1 for also using an example
B1 2 B1 for clear explanation
B1 2 B1 for first explanation
B1 7 B1 for second explanation
B1 2 B1 for clear explanation
B1 2 B1 for clear explanation
B1 2 B1 for clear description
B1 3 B1 correct letter with correct example
B1 3 B1 correct letter with correct example
B1 3 B1 correct letter with correct example
B1 3 B1 correct letter with correct example
B1 3 B1 correct letter with correct example
B1 3 B1 correct letter with correct example
B1 3 B1 correct letter with correct example
<table>
<thead>
<tr>
<th>23</th>
<th>As they are balanced ( 12 = xy ), rearranging ( y = \frac{12}{x} ), they are inversely proportional.</th>
<th>B1</th>
<th>3</th>
<th>B1 for a good diagram accompanying the explanation. B1 for the equation B1 for the correct explanation of the relationship</th>
</tr>
</thead>
<tbody>
<tr>
<td>24 a</td>
<td>Any two examples of the form ( y = x^3 + c ), e.g. ( y = x^3 + 7 ), ( y = x^3 - 4 ). They are all quadratics. All of them pass through the origin except ( y = 4x^2 + 3 ). ( y = 4x^2 ) and ( y = 4x^2 + 3 ) have the same shape, but the latter is moved up 3 units.</td>
<td>B1</td>
<td>2</td>
<td>B1 for the first example B1 for the second example B1 for similarities B2 for differences</td>
</tr>
<tr>
<td>b</td>
<td>Sometimes true. It is only true when ( a &gt; 0 ).</td>
<td>B1</td>
<td>2</td>
<td>B1 for sometimes B1 for explanation</td>
</tr>
<tr>
<td>25</td>
<td>When you draw the graphs of ( y = 4x^2 ) and ( y = -4x^2 ) you get the graphs shown here. It can be seen that ( y = 4x^2 ) is a reflection of the graph of ( y = -4x^2 ) in the x-axis.</td>
<td>B1</td>
<td>2</td>
<td>B1 for explanation of drawing a graph of each on the same axes</td>
</tr>
</tbody>
</table>
| 26 | | B1 | 3 | B1 for an accurate diagram of both graphs on the same pair of axes B1 for clear explanation bringing everything together.
Distance travelled = \( \frac{1}{2} (15 \times (u + 3u)) + (10 \times 3u) + \frac{1}{2} (20 \times 3u) \)
\[ = 30u + 30u + 30u = 90u \]
The assumption is that acceleration is at a steady rate when the motorbike speeds up and slows down.

Distance travelled
\[ \frac{1}{2} (15 \times (u + 3u)) + (10 \times 3u) + \frac{1}{2} (20 \times 3u) \]
\[ = 30u + 30u + 30u = 90u \]

The assumption is that acceleration is at a steady rate when the motorbike speeds up and slows down.

\( x = 0 \)

The circle has a radius of 5 and its centre is at (0, 0), halfway between D and E.

Find the distance of F(−3, 4) from the origin. If it is 5 units from the origin, it is on the circumference of the circle.

Using Pythagoras’ theorem:
\[ \text{distance} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5 \]
So F(−3, 4) is on the circumference of the circle.

The tangent at F will be at right angles to the line joining the point to the origin.
Gradient of the line is −\( \frac{4}{3} \).

The product of the gradient of this line and the tangent is −1. Use this to work out the gradient of the tangent.

Substitute the gradient and the coordinates of the point into the general equation of a straight line (y = mx + c) to find the y-intercept c.
Use the method of elimination, in which you combine the equations to eliminate one of the variables leaving an equation in the other variable. Solve this equation then substitute the value into one of the original equations to work out the other value.

Use the substitution method, in which you make one of the variables the subject of one equation and substitute this into the other equation. Solve this equation and then substitute the value into one of the original equations to work out the other value.

Use the graphical method, in which you draw a graph of both equations on the same axes and the solution is the point of intersection.

Use the elimination method when you can eliminate one variable easily by either adding or subtracting the two equations.

Use the substitution method when it is easy to make one of the variables in one of the equations the subject of the equation.

Use the graphical method to solve equations where there is a quadratic.

You might use more than one method: if you used the graphical method and didn't get integer values for the solution, you might then use the elimination method to find the fractional answers.
\[
\begin{align*}
3x - 4y &= 13 \quad \text{(1)} \\
2x + 3y &= 20 \quad \text{(2)}
\end{align*}
\]

Using elimination:

Multiply (1) by 2 and (2) by 3:

\[
\begin{align*}
6x - 8y &= 26 \\
6x + 9y &= 60
\end{align*}
\]

Subtract the first equation from the second equation:

\[
9y - 8y = 60 - 26
\]

\[
y = 2
\]

Substitute \(y = 2\) into (2):

\[
2x + 3 \times 2 = 20
\]

\[
x = 7
\]

Check by substituting both values into (1):

\[
3 \times 7 - 4 \times 2 = 21 - 8 = 13 \quad \text{Correct.}
\]

Using substitution:

Rearrange one of the equations:

\[
2x + 3y = 20
\]

\[
y = \frac{20 - 2x}{3}
\]

Substitute into the other equation and rearrange.

\[
3x - 4\left(\frac{20 - 2x}{3}\right) = 13
\]

\[
3x - \frac{80}{3} + \frac{8x}{3} = 13
\]

\[
3x + \frac{8x}{3} = 13 + \frac{80}{3}
\]

\[
\frac{9x + 8x}{3} = \frac{39 + 80}{3}
\]

\[
\frac{17x}{3} = \frac{119}{3}
\]
The equations look awkward. The method of eliminating one variable by multiplying the first equation by 3 and the second equation by 5 will give new equations with $15y$ and $-15y$ in both of them. These terms can be eliminated by adding the two new equations.

Rearranging one equation to make one variable the subject will give awkward fractions and so is not desirable.

It is not obvious whether drawing a graph of each equation will produce an integer solution.

The first equation already has $y$ as the subject and the substitution method is ideal, substituting for $y$ into the second equation.

The elimination method would mean unnecessary work to eliminate one of the variables.

Drawing a graph would give integer values, but it would take more time than the simple substitution method.

As one of the equations is a quadratic drawing graphs could be the best method; this would only be justified if an integer solution was found.

It seems straightforward to make $x$ the subject of the first equation and then substitute it into the second equation to get a quadratic equation that could then be solved for two solutions. Because one equation is a quadratic it is not suitable to use the elimination method.
| 32 a | If one of the equations is a multiple of
|      | the other then there will be an infinite
|      | number of solutions, e.g.
|      | \( x + y = 5 \)
|      | \( 2x + 2y = 10 \)
|      | Every point on the line \( x + y = 5 \) is a
|      | solution, giving an infinite number of
|      | solutions.
|      | If the equations have graphs that are
|      | parallel to each other then there will be
|      | no intersection and so no solution, e.g.
|      | \( x + y = 5 \)
|      | \( x + y = 6 \)
|     | B1 | B1 for clear explanation
| b i | One solution, as one equation is not a
|     | multiple of the other and they are not
|     | parallel.
| b ii | None – the gradient is the same but the
|      | intercepts are different so they are
|      | parallel.
| b iii | Infinite number of solutions as the first
|      | equation is a multiple of the second, so
|      | drawing graphs gives the same line.
| c i | \( y = 2x = -5 \)
|     | \( y = 0.5x + 1 \)
|     | Substitute for \( y \) in the first equation:
|     | \( 0.5x + 1 - 2x = -5 \)
|     | \(-1.5x = -6\)
|     | \( x = 4 \)
|     | Substitute into second equation:
|     | \( y = 0.5 \times 4 + 1 = 3 \)
|     | Check by substituting both values in
|     | the first equation
|     | \( 3 - 2 \times 4 = 3 - 8 = -5 \) Correct.
| | 2 | B1 for use of a good example accompanying the
| | explanation
| B1 | B1 for use of a good example accompanying the
| explanation
| B1 | B1 for clear explanation
| B1 | B1 for clear explanation
| B1 | B1 for clear explanation
| M2 | M1 for arranging equations in a suitable format
| M1 | M1 equating both equations
| A1 | A1 cao
| d | You can see how many times the
|    | graphs cross each other.
| | 11 | B1 clear explanation
| 33 a | They are the same equation. Multiply the first equation by 3 and it is the same as the second equation, so they have an infinite number of solutions. | B1 | 3 | B1 for clear explanation | M |
|      | Treble the first equation to get $15x - 3y = 27$ They have the same coefficients of $x$ and $y$ but a different constant so they are parallel lines with no intersections and so no solutions. | B1 |   | B1 for clear explanation |   |
| 34 a | $x^2 + 2x - 5 = 6x - 9$  
$x^2 - 4x + 4 = 0$  
$(x - 2)(x - 2) = 0$  
x = 2  
y = 6 × 2 − 9 = 3  
y = 3 | M1 | 2 | M1 for equating both equations  
M1 for arranging to equal 0  
M1 for factorising | M |
|      | $x = 2$  
y = 3 | M1 | 3 | M1 for factorising | |
| b    | There is just one intersection of the two graphs, so it has to be sketch iii, as the straight line touches the curve once. | A1 |   | A1 for $x = 2$ cao | |
|      | B1 |   | B1 for sketch iii | |
|      | B1 |   | B1 for clear explanation | |

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Let the cost of a second class stamp be \( x \).
Let the cost of a first class stamp be \( y \).

\[
10x + 6y = 902 \quad \text{…….(1)}
\]
\[
8x + 10y = 1044 \quad \text{……(2)}
\]
\[
5 \times (1) \quad 50x + 30y = 4510 \quad \text{…….(3)}
\]
\[
3 \times (2) \quad 24x + 30y = 3132 \quad \text{……(4)}
\]

Subtract (4) from (3):

\[
26x = 1378
\]
\[
x = 53
\]

Substitute for \( x \) in (1):

\[
10 \times 53 + 6y = 902
\]
\[
6y = 902 - 530
\]
\[
6y = 372
\]
\[
y = 62
\]

So 3 second-class plus 4 first-class will cost:

\[
3 \times 53 + 4 \times 62 = 407
\]

Cost will be £4.07.

Let the cost of a can of cola be \( c \).
Let the cost of a chocolate bar be \( b \).

Then:

\[
6c + 5b = 437 \quad \text{…….(1)}
\]
\[
3c + 2b = 200 \quad \text{……(2)}
\]
\[
2 \times (2) \quad 6c + 4b = 400 \quad \text{……(3)}
\]

Subtract (3) from (1):

\[
b = 37
\]

Substitute for \( b \) in (2):

\[
3c + 74 = 200
\]
\[
3c = 126
\]
\[
c = 42
\]

So three cans of cola and a chocolate bar will cost:

\[
2 \times 42 + 37 = 121
\]

Cost will be £1.21.

\[
\text{£4.07}
\]

\[
\text{£1.21}
\]
36 a

\[
S = \frac{5B}{4}
\]

\[
B = \frac{2}{5} C
\]

\[
S + C = 75
\]

\[
C = \frac{5}{2} B
\]

\[
\frac{5}{2} B + \frac{5B}{4} = 75
\]

\[
\frac{15B}{4} = 75
\]

\[
B = 75 \times \frac{4}{13}
\]

\[
S = 1.25B
\]

\[
S = 1.25 \times 20 = £25
\]

\[
C = 75 - 25 = £50
\]

The method used was to find equations linking each two persons at a time, and then use those to create one equation that could be solved. Once you had one solution you could find the rest.

Answer checked by going back to beginning statements and ensuring each one works. They do.

b

\[
B = £20
\]

\[
S = £25
\]

\[
C = £50
\]

M1 for creating single equation with one unknown

B1 for first equation

B1 for second equation

B1 for third equation

A1 for £20 cao

A1 for £25 cao

A1 for £50 cao

B1 for clear explanation.

B1 for matching explanation with the work done

B1 for clear explanation
Draw the graph of \( y = x + x^3 \).

Then solve for \( y = 20 \). Let the width be \( x \), then the length is \( x + 2 \). Hence the area is \( x(x + 2) = 67.89 \).

Solve the quadratic equation to find \( x \).

Create a table of values to assist in drawing the graph of \( y = x + x^3 = 20 \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>0</td>
<td>2</td>
<td>10</td>
<td>30</td>
</tr>
</tbody>
</table>

Plot the points and draw the graph.

\[
x^2 + 2x = 67.89
\]

\[
x^2 + 2x - 67.89 = 0
\]

Solve with the formula:

\[
x = \frac{-2 \pm \sqrt{(2^2 - 4 \times 1 \times -67.89)}}{2}
\]

\[
x = \frac{-2 \pm 16.6}{2} \text{ so } x = 7.3 \text{ or } -9.3
\]

Width, \( x \), cannot be negative hence solution is width = 7.3 cm.
### Question 38

**Area of large square** = \( 7^2 = 49 \)

To find the length \( z \) of the side of the small square:

\[
\begin{align*}
z^2 &= \sqrt{3^2 + 4^2} \\
&= \sqrt{25} \\
&= 5
\end{align*}
\]

So the area of the small square is 25.

Shaded area is equal to area large square – area small square.

\[
49 - 25 = 24
\]

So less than half is shaded, as required.

**Mark Scheme**

<table>
<thead>
<tr>
<th>Score</th>
<th>Mark Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>B1 for area of large square</td>
</tr>
<tr>
<td>3</td>
<td>M1 for use of Pythagoras’ theorem to help find the side length of inner square</td>
</tr>
<tr>
<td>1</td>
<td>A1 cao</td>
</tr>
<tr>
<td>1</td>
<td>A1 cao</td>
</tr>
<tr>
<td>1</td>
<td>A1 for area of shaded part</td>
</tr>
<tr>
<td>B2</td>
<td>B1 for clear explanation</td>
</tr>
<tr>
<td></td>
<td>B1 for complete explanation with correct mathematical notation throughout</td>
</tr>
</tbody>
</table>

### Question 39

Set up two simultaneous equations, using the information given.

5 is the first term; from the rule for the sequence, the next term is:

\( 5 \times a - b \), which equals 23

Hence \( 5a - b = 23 \) …… (1)

Doing the same to the next term gives:

\( 23a - b = 113 \) …… (2)

Subtract (1) from (2) to give:

\[
18a = 90
\]

\[
a = \frac{90}{18} = 5
\]

Substitute in (1):

\[
25 - b = 23
\]

\[
b = 2
\]

So \( a = 5, b = 2 \)

Hence, the next 2 terms are 563 and 2813.

**Mark Scheme**

<table>
<thead>
<tr>
<th>Score</th>
<th>Mark Scheme</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>M1 for clear initial explanation</td>
</tr>
<tr>
<td></td>
<td>B1 for first equation</td>
</tr>
<tr>
<td></td>
<td>B1 for second equation</td>
</tr>
<tr>
<td></td>
<td>M1 for clear method of elimination</td>
</tr>
<tr>
<td>1</td>
<td>A1 cao</td>
</tr>
<tr>
<td></td>
<td>M1 for substitution</td>
</tr>
<tr>
<td></td>
<td>A1 cao</td>
</tr>
<tr>
<td></td>
<td>B1 for next two terms correctly found</td>
</tr>
</tbody>
</table>
If the sum of the whole numbers from 1 to 50 is 1275 the sum from 2 to 51 will be 1275 + 51 – 1 = 1325.

Check method, using simple examples.

Using $S_n = \frac{n(n + 1)}{2}$

$S_{52} = \frac{52(52 + 1)}{2} = 1326$

$1326 - 1 + 1325$ as above.

If 1 is the first term then the $n$th term will be $n$, so their sum is $1 + n$.

Because they come in pairs, there will be $\frac{n}{2}$ of these pairs adding to the total.

So the total = $(n + 1) \times \frac{n}{2} = \frac{n(n + 1)}{2}$, the formula given.
### 41 a

Ali receives: £1000 + £2000 + ... + £20 000.
This amount, in pounds, is: 1000 × (sum of the numbers 1 to 20)
The sum of the first $n$ natural or whole numbers is $\frac{1}{2}n(n + 1)$.
So the amount Ali receives after $n$ years is:
$1000 \times \frac{1}{2}n(n + 1)$ or $500n(n + 1)$.

The amounts Ben receives each year are £1, £2, £4. The general term is $£2^{n-1}$. Adding these amounts gives a running total, in pounds, of: 1, 3, 7, 15,...
Looking for a pattern linking the number of years with the amount, we see that after 2 years, it is:
$3 = 4 - 1 = 2^2 - 1$
After 3 years it is:
$7 = 8 - 1 = 2^3 - 1$
After 4 years it is:
$15 = 16 - 1 = 2^4 - 1$
So after $n$ years it is $2^n - 1$.

After 20 years, Ali will have
$500 \times 20 \times 19 = £190 000$
Ben will have $2^{20} - 1 = £1 048 575$
Ben will have more than five times the amount that Ali has.
| 42 a b c d | 2n + 1  
3n + 4  
2001  
3004 | No, because \( \frac{2n}{3n} \) will always equal \( \frac{2}{3} \) no matter what \( n \) is, and the denominator increase of 4 will always give a larger increase than the numerator increase of 1, hence the fraction can never be larger than \( \frac{2}{3} \). | B1  B1  2 | B1 cao  B1 cao  2  B1 cao  B1 for no  B1 for a clear concise explanation | M |

| 43 a i ii iii b c | Neither – it’s the Fibonacci series, where each term is found by adding the previous two.  
Geometric – because each term is multiplied by 2 to find the next term.  
Arithmetic – because to find the next term you add 4 to the previous term. | B1  B1  2 | B1 for neither  B1 for clear explanation  2  B1 for geometric  B1 for clear explanation  B1 for arithmetic  B1 for clear explanation  B1 for arithmetic | M |
### 44 a
- **b**
- **c**
- **d**

*a* is the first term.

*d* is the amount added each time.

For example: use *a* = 2 and *d* = 3 to generate the sequence in part *a* as:

2, 5, 8, 11, 14, ...

The 5th term is 14.

Using the given \( X_n = a + (n - 1)d \):

the 5th term will be \( 2 + 4 \times 3 = 14 \), the same value.

\[ X_n = ar^{n-1} \]

It is a quadratic sequence because it contains a term in \( n^2 \).

### 45 a
- **b**

Evidence of reproducing proof as given in question.

\[ S_n = \frac{n}{2} [2a + (n - 1)d] \]

for the set of integers, \( a = 1 \) and \( d = 1 \).

Hence \( S_n = \frac{n}{2} (2 + (n - 1)) \)

\[ = \frac{n}{2} (n + 1) \]
\[ S_n = a + ar + ar^2 + ar^3 \ldots + ar^n \]
\[ rS_n = ar + ar^2 + ar^3 + ar^4 \ldots + ar^n + ar^{n+1} \]

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Therefore:
\[ S_n - rS_n = a - ar^{n+1} \]
\[ S_n(1 - r) = a(1 - r^{n+1}) \]
\[ S_n = \frac{a(1 - r^{n+1})}{(1 - r)} \]

**46 a**

**B1** for equation showing at least up to \( ar^3 \) and the generalisation

**B1** for equation showing at least up to \( ar^4 \) and the two generalisations

**B1** for top two rows shown correctly

**B1** for the bottom row shown correctly

**B1** cao

**B1** cao

**B1** cao

**46 c d e**

**47**

Considering the area of an \((x + 1)\) by \((x + 1)\) square:

\[
\begin{array}{|c|c|c|}
\hline
x & x^2 & x \\
\hline
1 & x & 1 \\
\hline
\end{array}
\]

The area of each rectangle created above is shown inside that rectangle, so it can be seen that:
\[
(x + 1)^2 = x^2 + x + x + 1 = x^2 + 2x + 1 \text{ as required}
\]

**B1** for explaining the sides of each square are \((x + 1)\)

**B1** for creating the square divided into the rectangles, using the \(x\) and the 1

**B1** for areas of each rectangle indicated in the rectangles

**B1** for clear explanation of required result
For example, \(2x > 10\)
Divide both sides by 2 to get \(x > 5\).

We show the solution with a line and a circle at each end point.
A solid circle means that the solution includes the end point; an open circle means that the solution does not include the end point. For example:

\[
\begin{array}{cccccc}
\cdots & -2 & -1 & 0 & 1 & 2 & 3 \\
\cdots & \bullet & \circ & \circ & \circ & \circ & \circ \\
\end{array}
\]

The top diagram shows \(x \leq 2\). It has a solid circle at the end point \(x = 2\) because that is part of the solution.
The bottom diagram shows \(x > 1\), it has an open circle at the end point \(x = 1\) because \(x = 1\) is not part of the solution.

Starting with an equation \(10 - x > 4\) and solving by adding \(x\) to both sides gives the solution \(6 > x\).
This can also be given as \(x < 6\) \ldots (1)
Consider again \(10 - x > 4\).
This time multiply throughout by \(-1\), keeping the inequality sign the same gives:
\(-10 + x > -4\)
Add 10 to each side to give \(x > 10 - 4\).
This gives the solution as \(x > 6\) \ldots (2)
But comparing this with equation (1) we see that the signs are the other way round, this illustrates that when we multiplied through by a negative number, we should have changed the sign from >
**49 a**

In this example, \( x \) and \( y \) are values satisfying the conditions:

\[
\begin{align*}
  x + y &\leq 5 \\
  x &> 1 \\
  y &> 2
\end{align*}
\]

These are drawn on the diagram.

Any region needs a minimum of three straight lines to enclose it. The region \( R \) above is where the solutions satisfying all three inequalities lie.

Point \((x, y)\) is inside the region if the point satisfies all three inequalities.

- For example, \((1.5, 3)\) is inside the region since \(1.5 + 3 \leq 5\), \(1.5 > 1\) and \(3 > 2\).
- Point \((x, y)\) is outside the region if it does not satisfy at least one of the inequalities.
  - For example, \((2, 4)\) satisfies two of the conditions \((x > 1\) and \(y > 2\)) but does not satisfy \(x + y \leq 5\).
- Point \((x, y)\) is on the boundary of the region if the point satisfies one of the inequalities but only as an equality.
  - For example, \((2, 3)\) is on the boundary of \(x + y \leq 5\) as \(2 + 3 = 5\).

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>B1</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>bi</td>
<td></td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>ii</td>
<td></td>
<td>B1</td>
<td></td>
</tr>
<tr>
<td>iii</td>
<td></td>
<td>B1</td>
<td></td>
</tr>
</tbody>
</table>

B1 for choosing three inequalities that will define a region

B1 for a clear diagram illustrating the chosen inequalities

B1 for clear explanation linking the chosen inequalities with the diagram

B1 for clear correct explanation

B1 for use of an example to illustrate this

B1 for clear correct explanation

B1 for use of an example to illustrate this

B1 for use of an example to illustrate this
| 50 a | The total number of games cannot be greater than 4, hence \( w + d \leq 4 \).
The number of points must be 8 or more, they score 3 for a win, 1 for a draw, hence \( 3w + d \geq 8 \).
The shaded area is the region that satisfies these two inequalities.

In four games, they need to score at least 8 points. The graph shows that to do this they can win at least 3 games or win 2 games and draw two games.

The team would still need to score at least 8 points, but now they have five games in which to do it.
The inequality \( w + d \leq 4 \) would change to \( w + d \leq 5 \). The other inequality is unchanged. The line for \( w + d = 4 \) would move up to go through \((0, 5)\) and \((5, 0)\). The other line would be unchanged. |
| 3 | B3 for explaining \( w + d \leq 4 \)
B1 for explaining \( 3w + d \geq 8 \)
B1 for explaining what the shaded region is |
| B2 | B1 for explaining they need at least 8 points
B1 for showing all the possible ways this could happen |
| B2 | B1 for explanation of how this affects both equations
B1 for complete solution, clearly showing what the new line(s) are |
For example, \( n^2 \) can generate a quadratic sequence:
1, 4, 9, 16, 25 ........ \( n^2 \)

Similarities: you find the \( n \)th term for both types of sequence by looking at the differences between terms.

Differences: in a linear sequence you find the first term by subtracting the difference from the second term.

In a quadratic sequence you also have to look at the second differences. This allows you to extend the differences backwards to find the values of \( a, b \) and \( c \) in the \( n \)th term of \( an^2 + bn + c \).

For a linear sequence, just keep on adding 6 each time to give:
2, 8, 14, 20 ............(6\( n \) – 4)
The \( n \)th term includes 6\( n \) because we add 6 each time, 6\( n \) – 4 because (2 – 6) = –4.

For a quadratic equation, we build up the series by again having the first differences as 6, then choosing a second difference, say 2.
This will give a table such as:

<table>
<thead>
<tr>
<th>( n )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>nth term</td>
<td>2</td>
<td>8</td>
<td>16</td>
<td>26</td>
</tr>
<tr>
<td>1st difference</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>2nd difference</td>
<td>2</td>
<td>2</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Start with the two terms in positions \( n = 1 \) (2) and \( n = 2 \) (8) in the sequence.
Put each second difference as 2.
Then complete the first differences as shown.
Finally the \( n \)th terms can be completed.
as shown.

\[
\begin{array}{c|cccc}
  n & 0 & 1 & 2 & 3 \\
\hline
  c & -2 & 2 & 8 & 16 \\
  a + b & 4 & 6 & 8 & 10 \\
  2a & 2 & 2 & 2 & 2
\end{array}
\]

Extending the table backwards will allow us to find the values of \(a\), \(b\) and \(c\) in the \(n\)th term \(an^2 + bn + c\).

\[
2a = 2 \rightarrow a = 1 \\
a + b = 4 \rightarrow b = 3 \\
c = -2
\]

Hence \(n\)th term is \(n^2 + 3n - 2\).

52

\[
(m^2 - n^2)^2 + (2mn)^2 = \\
\quad m^4 - 2m^2n^2 + n^4 + 4m^2n^2 \\
\quad = m^4 + 2m^2n^2 + n^4 \\
\quad = (m^2 + n^2)^2
\]

B1 for correctly finding the \(n\)th term

M1 choosing two smallest terms, squaring and adding
A1 for cao
A1 for cao
A1 for showing the factorisation leads to the given result

53

Because the differences between consecutive terms are 2, 3, 4, 5, 6, etc. (even and odd alternately); when generating the triangular number sequence starting with the odd 1, add even to odd to generate odd …3; add odd to odd to generate even …6; add even to even to generate even …10. Add odd to even to generate odd …15. We are now back again where we add even to odd to generate odd, and the whole sequence continues in the same way, continually giving two odd, two even, etc.

B1 for explaining the about the differences of the terms being the set of integers

B1 for explaining the pattern is odd, even, odd, even and so on

B1 for explaining the complete sequence of combining odd and even to generate the final sequence

M
### Question 54

The assumption is that $p$ and $q$ are integers.

1. $10p + q = 7n$ where $n$ is also an integer.
2. $7p + 3p + q = 7n$
3. $3p + q = 7n - 7p = 7(n - p)$

As $n$ and $p$ are integers then $n - p$ is also an integer hence $7(n - p)$ is a multiple of 7 and so $3p + q$ must be as well.

### Question 55

1. $2(5(x - 2) + y) = 2(5x - 10 + y) = 10x - 20 + 2y$ ....(1)
2. $10(x - 1) + 2y - 10 = 10x - 10 + 2y - 10 = 10x - 20 + 2y$ ....(2)

Equation (1) = equation (2)

Hence the two expressions are equal.
### 56a

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Take two numbers</strong> (x) and (y) where (x &gt; y).&lt;br&gt;<strong>First step:</strong> (5(x - 2) = 5x - 10)&lt;br&gt;<strong>Second step:</strong> (2(5x - 10 + y))&lt;br&gt;<strong>Third step:</strong> (10x - 20 + 2y = 10x - 11 + y)&lt;br&gt;<strong>Fourth step:</strong> (10x - 11 + y + 11 = 10x + y)</td>
<td>B1 2</td>
<td>B1 for initial explanation</td>
</tr>
<tr>
<td>Hence where the first two numbers might have been 7 and 3, the final outcome would be 70 + 3 = 73.</td>
<td>B1</td>
<td>B1 cao</td>
</tr>
</tbody>
</table>

---

### 56b

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Let the single-digit number be</strong> (x) and the two-digit number be (10a + b).&lt;br&gt;<strong>First step:</strong> (10(10a + b) - 9x = 100a + 10b - 9x)&lt;br&gt;= (100a + 10b - x - 10x)&lt;br&gt;= (100a + 10b - 10x + x)&lt;br&gt;= (100a + 10(b - x) + x)&lt;br&gt;The hundreds unit is (a).&lt;br&gt;The tens unit is ((b - x)).&lt;br&gt;The units term is (x), the same as the single digit we started with.&lt;br&gt;The split then becomes (10a + (b - x)) and (x).&lt;br&gt;Adding these two gives (10a + b - x + x)&lt;br&gt;which is (10a + b), the two-digit term.</td>
<td>A1</td>
<td>A1 for defining each digit</td>
</tr>
</tbody>
</table>

---

### 57

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Marks</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Expand each square and add:</strong>&lt;br&gt;(n^2 - 2n + 1 + n^2 + n^2 + 2n + 1)&lt;br&gt;= (3n^2 + 2n - 2n + 2)&lt;br&gt;= (3n^2 + 2) as given.</td>
<td>M1 2</td>
<td>M1 for expanding brackets, A1 for correct expansion of brackets</td>
</tr>
</tbody>
</table>

---
The difference is 5 so $n$th term is $5n + c$
where $c = \text{first term} - 5$
\[ = 4 - 5 = -1 \]

So $n$th term is $5n - 1$.
Check the $4$th term gives 19.
When $n = 4$, $4n - 1 = 20 - 1 = 19$, correct.

<table>
<thead>
<tr>
<th>$n$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$th term</td>
<td>1</td>
<td>3</td>
<td>6</td>
<td>10</td>
</tr>
<tr>
<td>1st diff'</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2nd diff'</td>
<td>1</td>
<td>1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Extending the table backwards will allow us to find the values of $a$, $b$ and $c$ in the $n$th term $an^2 + bn + c$.

\[ 2a = 1 \rightarrow a = \frac{1}{2} \]
\[ a + b = 1 \rightarrow b = \frac{1}{2} \]
\[ c = 0 \]

Hence $n$th term is $\frac{1}{2}n^2 + \frac{1}{2}n = \frac{1}{2}(n^2 + n)$
\[ = \frac{1}{2}n(n + 1) \]
\[ \begin{align*}
T_n &= \frac{1}{2} n(n + 1) \\
T_2n+1 &= \frac{1}{2} (2n + 1)(2n + 1 + 1) \\
&= \frac{1}{2} (2n + 1)(2n + 2) \\
&= \frac{1}{2} (2n + 1)(n + 1) \\
&= 2n^2 + 3n + 1 \quad \ldots \ldots (1)
\end{align*} \]

\[ \begin{align*}
T_{n+1} &= \frac{1}{2} (n + 1)(n + 1 + 1) \\
&= \frac{1}{2} (n + 1)(n + 2) \\
&= \frac{1}{2} (n^2 + 3n + 2) \quad \ldots \ldots (2)
\end{align*} \]

So \( T_{2n+1} - T_{n+1} \)

\[ \begin{align*}
&= 2n^2 + 3n + 1 - \frac{1}{2} n^2 - \frac{3}{2} n - 1 \\
&= \frac{3}{2} n^2 + \frac{3}{2} n
\end{align*} \]

\[ \sum \begin{array}{l}
\text{B1 for correct } T_n \text{ formula} \\
\text{M1 for substituting } 2n + 1 \\
\text{A1 cao} \\
\text{M1 for substituting } n + 1 \\
\text{A1 cao} \\
\text{B1 for subtracting each equation} \\
\text{B1 for clear full explanation of proving the final connection}
\end{array} \]

\[ \begin{align*}
T_n &= \frac{3}{2} n(n+1) \\
T_n - 1 &= \frac{1}{2} n(n+1) - 1 \\
&= \frac{1}{2} n(n+1) - \frac{3}{2} n(n+1) \\
&= \frac{1}{2} (n^2 + n - 2) \\
&= \frac{n^2 + n - 2}{n(n+1)} \\
&= \frac{(n-1)(n+2)}{n(n+1)}
\end{align*} \]

But \( n^2 + n - 2 \) factorises to \( (n-1)(n+2) \)

So final expression is \( \frac{(n-1)(n+2)}{n(n+1)} \)

\[ \begin{array}{l}
\text{B1 for correct factorisation} \\
\text{B1 for fully clear correct proof with no mathematical notational errors}
\end{array} \]
Let the first number be \( x \), then the next four are \( x + 1, x + 2, x + 3 \) and \( x + 4 \).
The sum of these is \( 5x + 1 + 2 + 3 + 4 \)
which is \( 5x + 10 = 5(x + 2) \),
a multiple of 5.

\[ 10^a = \frac{a}{b} \]
Hence \( b \times 10^a = a \)
Substitute this into \( 10^a = \frac{b}{a} \)
to give \( 10^a = \frac{b}{(b \times 10^a)} \)
Hence \( 10^a = \frac{1}{10^a} \)
So \( 10^a \times 10^a = 1 \)
So \( 10^{a+a} = 1 \)
But \( 10^5 = 1 \)
And so \( x + y = 0 \).
<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Mark</th>
<th>Marking</th>
</tr>
</thead>
</table>
| **64** | Where two terms are \( x \) and \( x + 1 \) the expression required is: \[
(x + x + 1)^2 - (x^2 + (x + 1)^2) = (2x + 1)^2 - (x^2 + x^2 + 2x + 1) = 4x^2 + 4x + 1 - 2x^2 - 2x - 1 = 2x^2 + 2x = 2x(x + 1) \] Where \( T_x = \frac{1}{2}x(x + 1) \) \( 4T_x = 2x(x + 1) \), which is same as the result in equation (1). | M1 2 | M1 for identifying the two terms A1 for correct expression as asked A1 for correct expansion of all brackets A1 for correct simplification A1 for correct factorisation B1 for clear complete proof with correct mathematical notation |
| **65** | Let \( p = x \) Then \( q = x + 1 \) and \( r = x + 2 \) so \( pr = x(x + 2) = x^2 + 2x \) \( q^2 - 1 = (x + 1)^2 - 1 = x^2 + 2x + 1 - 1 = x^2 + 2x = pr \) \[ B1 2 \] B1 for \( q \) expressed algebraically B1 for \( r \) expressed algebraically B1 for product \( pr \) expressed algebraically B1 for \( q^2 - 1 \) expressed algebraically B1 for simplification B1 for complete clear proof | M |
| **66 a** | There are many equivalent expressions. For example, expand the bracketed term: \( \frac{3}{2} - q^2 - q - 4 \) For example: \( \frac{(6x + 4y)}{10} \) For example: \( 4x^2 + 10x \) | B1 2 3 | B1 for a correct example B1 for a correct example B1 for a correct example |
| **67 a** | To make it a product of two linear expressions. The quadratic expression. That the signs of the numbers in the brackets are different. One factor of the constant term is zero. There is only one set of brackets. | B1 2 3 | B1 for clear explanation B1 for clear explanation B1 for clear explanation |

**Explanation:**

- **64**: The expression is derived by expanding and simplifying the given terms. The final expression is simplified to match the result in equation (1).
- **65**: The expression is derived by substituting and simplifying algebraically, ensuring the result matches the given equation.
- **66 a**: Various examples are provided to illustrate equivalent expressions and simplification.
- **67 a**: Conditions and requirements for forming a product of linear expressions are outlined, including the nature of the quadratic expression and the signs of the numbers in the brackets.
| 68 a |  | For example: \(x^2 - 1\). Because each part is a square, \(x^2\) and \(1^2\), one is subtracted from the other. Because: 
\[1000 \times 998 = (999 + 1) \times (999 - 1) = 999^2 - 1\] | B1 | 3 | B1 for clear explanation | H |
|---|---|---|---|---|---|---|
| b |  | Two that can be cancelled, for example: 
\[
\frac{2x}{x} \text{ and } \frac{5(x+1)}{10(x+1)}
\]
I chose two straightforward ones, one that would cancel by a single letter and one that would cancel by an algebraic term.

Two that cannot be cancelled, for example: 
\[
\frac{x}{3} \text{ and } \frac{x^2}{x+1}
\]
I chose two straightforward examples, one being a single term as numerator and denominator, the other one where the denominator was more than a single term. | B1 | 2 | B1 for two examples that cancel |
| 69 |  | B1 | 3 | B1 for a clear explanation | |
|  |  | B1 |  | B1 for two examples that don’t cancel | |
|  |  | B1 |  | B1 for a clear explanation | |
| 70 |  | To get such a term on the top this must be the difference of two squares, hence the two expressions both need multiplying by \((3x - 4)\) to give: 
\[
\frac{(3x + 4)(3x - 4)}{(x+2)(3x - 4)} = \frac{9x^2 - 16}{3x^2 + 2x - 8}
\]
This expands to: | B1 | 2 | B1 for clear explanation | H |
|  |  | B1 |  | B1 for \((3x - 4)\) | |
|  |  | B1 |  | B1 for setting up the expression | |
|  |  | B1 |  | B1 for showing how to find the final expression in suitable format | |
\[(2a + b)(2a + b) = 4a^2 + 4ab + b^2\]
\[(2a + b)(2a - b) = 4a^2 - b^2\]
\[(2a - b)(2a - b) = 4a^2 - 4ab + b^2\]

\[(a + b)(a + b) = a^2 + 2ab + b^2\]
\[(a + b)(a - b) = a^2 - b^2\]
\[(a - b)(a - b) = a^2 - 2ab + b^2\]

The difference in the two is that in the \((2a \pm b)\) product, both the \(a^2\) and \(ab\) terms have a coefficient of 4 (when the \(ab\) term is not zero), but in the \((a \pm b)\) product, the \(a^2\) term has a coefficient of 1 and the \(ab\) term has a coefficient of 2 (when the \(ab\) term is not zero).
Draw a triangle ABC.

Using trigonometric functions:
\[ \sin C = \frac{b}{h} \]

Therefore: \( h = b \sin C \)

Then using the basic formula for area of a triangle:
\[ \text{Area} = \frac{1}{2} a \times h \]
Substituting for \( h \) gives:
\[ \text{Area} = \frac{1}{2} a \times b \sin C \]
\[ \text{Area} = \frac{1}{2} ab \sin C \] as required.

Using the given triangle:
\( C = 45^\circ, a = x + 2 = 6, b = x - 2 = 2 \)
\[ \text{Area} = \frac{1}{2} ab \sin C \]
\[ = \frac{1}{2} \times 6 \times 2 \times \sin 45^\circ \]
\[ = 6 \times \frac{1}{\sqrt{2}} \]
\[ = \frac{6}{\sqrt{2}} \]
Multiply numerator and denominator by \( \sqrt{2} \) .

This gives \[ \frac{6 \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{6 \sqrt{2}}{2 \times 2} \]
\[ = 3 \sqrt{2} \] as required.
f(x) = 3 – 7x
Find f⁻¹(x) from y = 3 – 7x:
7x = 3 – y
x = \frac{3 - y}{7}

So f⁻¹(x) = \frac{3 - x}{7}

Find g⁻¹(x) from y = 7x + 3:
7x = y – 3
x = \frac{y - 3}{7}

So g⁻¹(x) = \frac{x - 3}{7}

So f⁻¹(x) + g⁻¹(x) = \frac{3 - x}{7} + \frac{x - 3}{7} = \frac{3 - x + x - 3}{7} = 0

= 0 as required.

M1 2 M1 for method of finding inverse

A1 A1 cao

M1 M1 for method of finding inverse

A1 A1 cao

M1 M1 for showing how the two functions can be added together

B1 B1 for complete explanation of how they sum to 0

H
| 74 ai | It means that the constant terms in both expressions are positive, or if one is positive and one is negative, their sum is positive. | B1 | 2 | B1 for first condition  
B1 for second condition | H |
| --- | --- | --- | --- | --- | --- |
| ii | It means that the constant terms in both expressions are negative, or if one is positive and one is negative their sum is negative. | B1 | B1 | B1 for first condition  
B1 for second condition |  |
| iii | The expression is the difference of two squares. | B1 |  | B1 for clear explanation |  |
| bi | For example, \((4x + 2)(x - 1)\)  
\[= 4x^2 - 2x - 2\] as required. | B1 |  | B1 for correct example explained |  |
| ii | For example, \((3x + 1)(x + 1)\)  
\[= 3x^2 + 4x + 1\] as required. | B1 |  | B1 for correct example explained |  |
| c | If it is positive then both expressions have the same sign.  
If it is negative then the expressions have different signs. | B1 |  | B1 for complete clear explanation |  |
\[
\left(\frac{1}{2}\right)^2 = \frac{25}{4} \\
2x^2 + 10x - 5 = 0
\]
Divide through by 2:
\[
x^2 + 5x - \frac{5}{2} = 0
\]
Completing the square:
\[
(x + \frac{5}{2})^2 - \frac{5}{2} - \frac{25}{4} = 0
\]
\[
(x + \frac{5}{2})^2 = \frac{(10 + 25)}{2} = \frac{35}{2}
\]
Taking the square root of each side:
\[
x + \frac{5}{2} = \pm \frac{\sqrt{35}}{2} = \pm 2.958
\]
\[
x = 2.958 - 2.5 \text{ or } x = -2.958 - 2.5
\]
\[
x = 0.458 \text{ or } x = -5.458
\]
Consider the 4 lines of his working.
Line 1: he has forgotten the \(-5\) in the equation.
Line 2: he has squared the right-hand side incorrectly.
Line 3: the \(\frac{5}{2}\) should be \(-\frac{5}{2}\) and the \(\frac{5\sqrt{2}}{2}\) should be \(\pm \frac{\sqrt{45}}{2}\).
Line 4: there should be two solutions.
\[
x^2 + 5x - 5 = 0
\]
\[
(x + \frac{5}{2})^2 - \frac{5}{2} = (\frac{5}{2})^2. \text{ Don't forget the } \frac{-5}{2} \text{ in the original equation.}
\]
\[
(x + \frac{5}{2})^2 = \frac{25}{4}. \text{ Square the whole of the bracket on the right-hand side.}
\]
\[
(x + \frac{5}{2})^2 = 5 + \frac{25}{4}. \text{ Remember, when you add a term on one side, you must also add it on the other side.}
\]
\[
(x + \frac{5}{2}) = \pm \frac{\sqrt{45}}{2} = \pm \frac{3\sqrt{5}}{2}
\]
Be careful when taking square roots of fractions. Don't forget that when you find square root there is a positive and a negative root.
| 76 a | If the coefficient of $x^2$ is positive, the turning point is between the two roots, so choose an equation with two positive $x$ roots, say $x = 1$ and $x = 3$. The quadratic that has these roots is $y = (x - 1)(x - 3)$, which is $y = x^2 - 4x + 3$. Complete the square to get $y = (x - 2)^2 - 2^2 + 3$. Hence the turning point of $y = (x - 2)^2 - 1$ will have a positive $x$-value. | B1 | B1 for clear explanation of what equation to look for. B1 for choosing a suitable equation with these characteristics. |
| b | If the equation has no roots, then the turning point will be above the $x$-axis, hence a positive value of $y$. From the general form of the quadratic equation, $y = ax^2 + bx + c$, the value of $b^2$ is less than $4ac$ so, keeping $a$ as 1, we could choose $c$ as 6 and $b$ as 2, giving $y = x^2 + 2x + 6$ Complete the square to get $y = (x + 1)^2 - 1 + 6$. Hence the turning point of $y = (x + 1)^2 + 5$ will have a positive $y$ value. | B1 | B1 for a suitable equation with complete justification B1 for clear explanation of what equation to look for. |
| c | The $y$-intercept will be positive if $y$ is positive when $x = 0$. For example: $y = (x + 2)^2 + 3$, when $x = 0$ $y = 7$, positive so $y = (x + 2)^2 + 3$ has a $y$-intercept that is positive. | B1 | B1 for choosing a suitable equation with these characteristics. B1 for a suitable equation with complete justification B1 for complete clear explanation |
A table of values for the graph will be:

<table>
<thead>
<tr>
<th>x</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>8.25</td>
<td>4.25</td>
<td>2.25</td>
<td>2.25</td>
<td>4.25</td>
<td>8.25</td>
</tr>
</tbody>
</table>

\( f(x + 3) - 2 \) is a translation 3 left and 2 down of \( f(x) \). Therefore, as the turning point moves 2 down, it will now turn on the \( x \)-axis, giving one real root at the point \((-3.5, 0)\).