

**Collins**

# Maths

4th Edition

**GCSE**

Higher Teacher Pack

Christine Watson

William Collins' dream of knowledge for all began with the publication of his first book in 1819. A self-educated mill worker, he not only enriched millions of lives, but also founded a flourishing publishing house. Today, staying true to this spirit, Collins books are packed with inspiration, innovation and practical expertise. They place you at the centre of a world of possibility and give you exactly what you need to explore it.

Collins. Freedom to teach.

Published by Collins  
An imprint of HarperCollins*Publishers*  
1 London Bridge Street  
London  
SE1 9GF

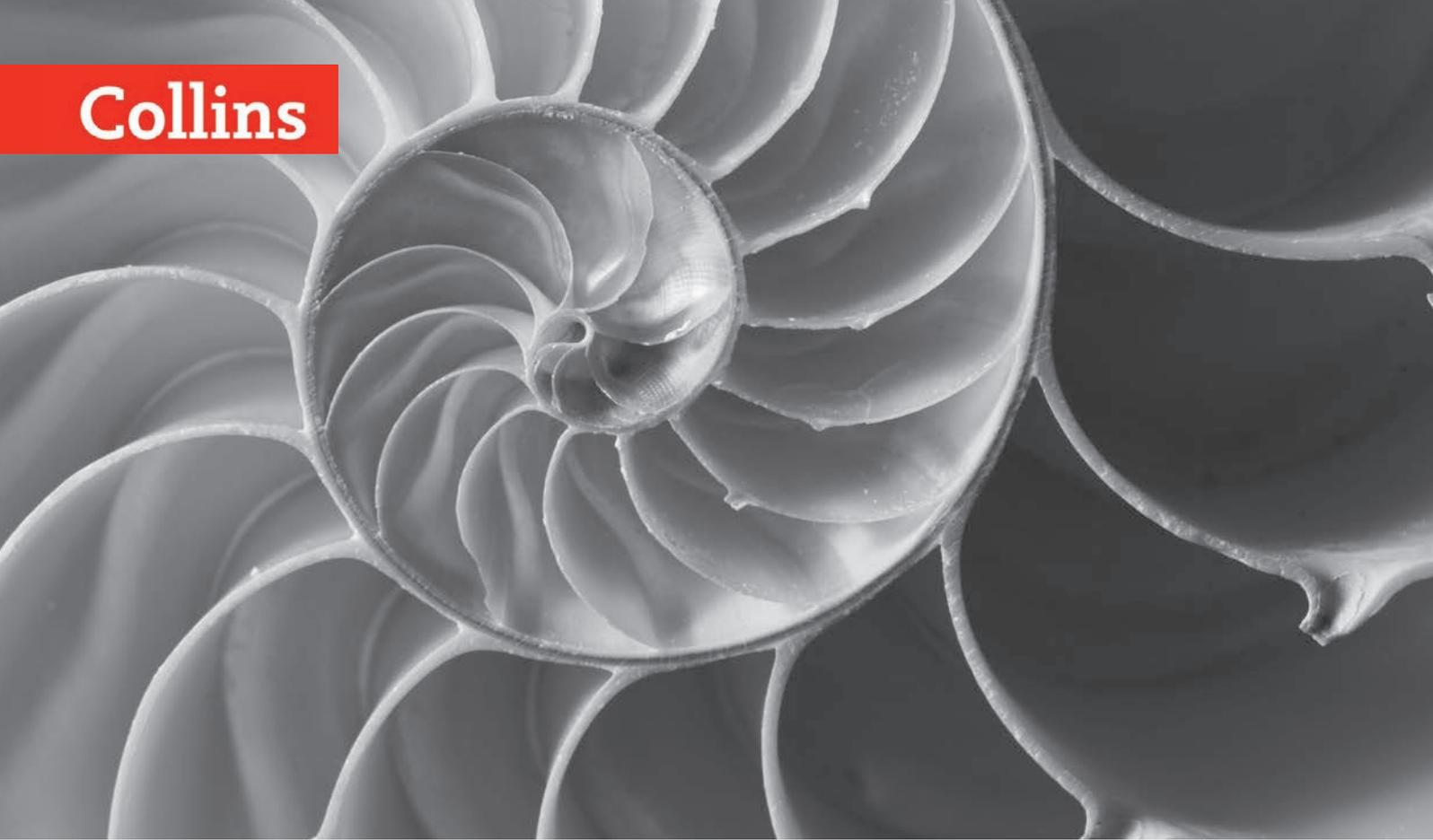
Browse the complete Collins catalogue at  
[www.collins.co.uk](http://www.collins.co.uk)

© HarperCollins*Publishers* Limited 2015

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system or transmitted in any form or by any means – electronic, mechanical, photocopying, recording or otherwise – without the prior written consent of the Publisher or a licence permitting restricted copying in the United Kingdom issued by the Copyright Licensing Agency Ltd, 90 Tottenham Court Rd, London W1T 4LP.

Cover design by We are Laura  
Cover images © Procy/Shutterstock, joingate/  
Shutterstock  
Printed by Fuller Davies [www.fullerdavies.com](http://www.fullerdavies.com)

Every effort has been made to contact the copyright holders but if any have been inadvertently overlooked, the publishers will be pleased to make the necessary arrangements at the first opportunity.



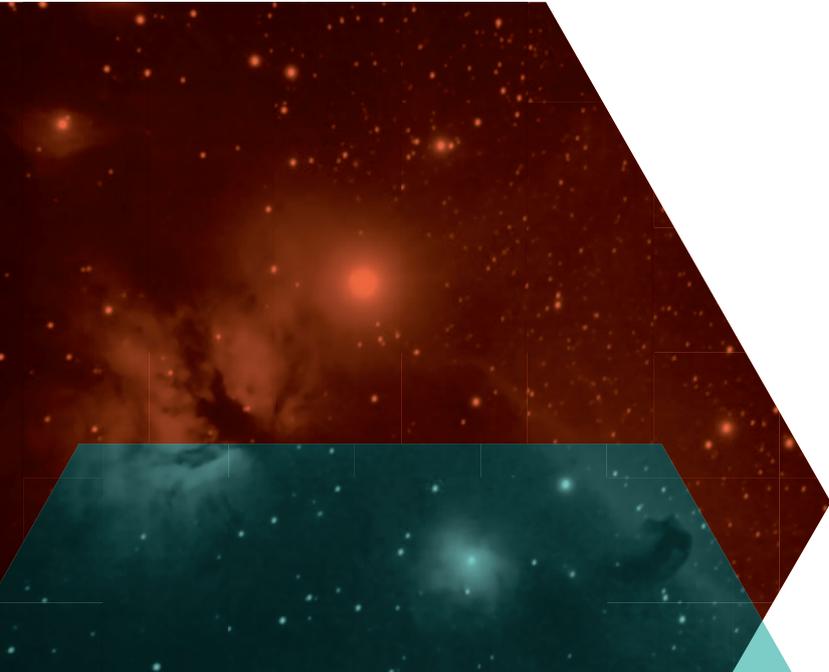
**Collins**

# Maths

4th Edition

**GCSE**

Higher Teacher Pack



Christine Watson

# Chapter 24 Algebraic fractions and functions

## Overview

|   |                                 |
|---|---------------------------------|
| <b>24.1</b> Algebraic fractions               | <b>24.4</b> Composite functions |
| <b>24.2</b> Changing the subject of a formula | <b>24.5</b> Iteration           |
| <b>24.3</b> Functions                         |                                 |

## Prior learning

Manipulate algebraic expressions and solve equations.  
Use rules to generate sequences.  
Use trial and improvement to solve equations.

## Chapter 24

**Ensure students can solve equations involving algebraic fractions by manipulation, find inverse and composite functions, and use iteration to find solutions.**

In the examination, students will be expected to

- A manipulate, simplify and rearrange algebraic expressions involving fractions
- B substitute into functions, find inverse and composite functions
- C rearrange polynomials to the iterative form; solve iterations to a given number of iterations or number of decimal places.

## Extension

Explore more complex algebraic fractions, for example partial fractions. Use a wider range of mathematics in functions including trigonometric functions.  
Explore iteration in coding, in particular the generation of fractals.

## Curriculum references

| <i>Section</i> | <i>KS4 NC Programmes of Study</i> | <i>GCSE specification</i> |
|----------------|-----------------------------------|---------------------------|
| 24.1           | A (ER) 10                         | A4, A6                    |
| 24.2           | A (ER) 3                          | A5, A6                    |
| 24.3           | A (ER) 6                          | A6, A7                    |
| 24.4           | A (EF) 7                          | A6, A7                    |
| 24.5           | A (EF) 2                          | A6, A20                   |

## Route mapping

| Exercise | Accessible | Intermediate | Challenging | AO1                     | AO2<br>MR CM     | AO3<br>PS EV   | Key<br>questions    |
|----------|------------|--------------|-------------|-------------------------|------------------|----------------|---------------------|
| 24A      |            | 1–7          | 8–19        | 1–6, 8–10,<br>13–14, 18 | 7, 12, 15,<br>17 | 11, 16,<br>19  | 6, 8, 10,<br>12, 16 |
| 24B      |            | 1–6          | 7–11        | 1–6, 9                  |                  | 7–8, 10–<br>11 | 4, 7, 9             |
| 24C      | 1–4        | 5–7          |             | 1–3, 5–7                | 4, 7             |                | 2, 7                |
| 24D      |            | 1–2          | 3           | 1–2                     | 3                |                | 1e, 1f, 2           |
| 24E      |            | 1–2          | 3–4         | 1–2                     | 3                | 4              | 2a, 2f, 3i,<br>3v   |
| 24F      |            | 1–7          | 8–11        | 1–5                     | 5–6, 8           | 7, 9–11        | 6, 9                |

*Key questions are those that demonstrate mastery of the concept or which require a step-up in understanding or application. These could be used to identify the questions that students must tackle, to support differentiation, or to identify the questions that should be teacher-marked rather than student-marked.*

### About this chapter

**Making connections:** The chapter brings together algebraic manipulation, solving equations, rearranging formulae. The work on iteration builds on understanding of sequences and graphical representation of sequences is a clear way of demonstrating convergence/divergence. For much of the chapter, the next step is A level content.

**Relevance:** There is an emphasis on the use of logic and thinking in steps. There are applications in engineering, architecture, manufacturing, project management and many more areas, with STEM careers being a strong focus.

**Working mathematically:** Which is more complex, having a linear algebraic numerator or denominator? Why? What is the connection between  $fg(x)$  and  $gf(x)$ ? When will they be equal? When will the inverse of a function be equal to the function? Find a chain of steps from an equation involving algebraic fractions to the iterative form. How efficient can you make this chain?

**Assessment:** Give students a set of fractions with linear denominators and simple numerators. Ask them to select two to create an equation of the form  $y = A + B$ , where  $A$  and  $B$  are their chosen fractions. They should then represent this in as many forms as they can: rearrange, simplify, graph, iteration etc.

See the CD for suggested assessment tracking foci, and the section plans for further suggested Assessment tasks.

### Worked exemplars from Student Book – suggestions for use

- A Present students with the same question but different numbers. They use the exemplar to mirror the working, in full or just the notes.
- B Copy and cut up the exemplar into cards. Students match the working with the notes. (You may need to remove the words ‘first, second’ etc.)
- C Copy and cut up the working into cards but split the label/description from the working. Students put the working in order then match with the descriptions.

### Answers to Student Book questions at the end of this book (NB: not included in this sample)

## Section 24.1 Algebraic fractions

### Learning objectives

- Apply methods for calculating with numerical fractions to algebraic fractions

### Resources and homework

- Student Book 24.1
- Practice Book 24.1

### Making mathematical connections

- Equivalent fractions
- Simplifying linear and quadratic expressions

### Making cross-curricular connections

- **Science** – use of formulae
- **Relevance** – developing logical thinking

### Prior learning

It is essential that students can confidently calculate with numerical fractions; this may need refreshing before the lesson via a homework task. Students should be able to expand, factorise, simplify and solve linear and quadratic expressions.

### Working mathematically

- Encourage students to articulate their methods for numerical fractions (equivalence including cancelling; addition and subtraction; multiplication and division) and then to apply these to algebraic fractions.
- Structure tasks so students can work out the methods for themselves, either by increasing the difficulty incrementally or through one straightforward and one complex example.

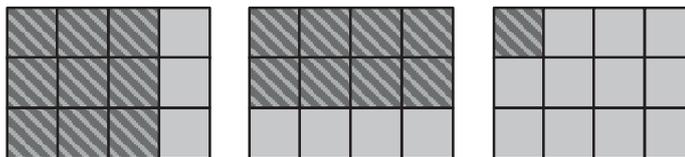
### Common misconceptions and remediation

- Students can be confused about cancelling terms across two fractions. If this arises – or to force the issue into the open – try this example:

$\frac{3}{4} - \frac{2}{3} \rightarrow (\text{cancel } 3 \text{ and } 3) \frac{1}{4} - \frac{2}{1} \rightarrow (\text{cancel } 2 \text{ and } 4) \frac{1}{2} - \frac{1}{1} = -\frac{1}{2}$  Is this a correct final answer? If not, what has gone wrong?

Link to the pictorial representation.

Ensure the distinction is made between cancelling within a single fraction  $\frac{3 \times 2}{4 \times 3}$  or its equivalent  $\frac{3}{4} \times \frac{2}{3}$ , and cancelling separate fractions.



Link to an algebraic version such as  $\frac{6x}{x(x+1)} - \frac{2(x+1)}{6x-4}$ , asking what can be cancelled and what cannot.

### Probing questions

- What is the same and what is different about simplifying these two expressions?  $\frac{4x}{x+3} + \frac{x+3}{x}$      $\frac{x}{x+3} \times \frac{x+3}{4x}$
- Which types of algebraic fractions lead to quadratic numerators and which to linear numerators?
- How does simplifying algebraic fractions link to the work on irrational fractions?

### Literacy focus

- Key vocabulary: cancel   algebraic fraction   LCM (lowest common multiple)
- Be explicit about the language of equivalence and avoid 'cross-multiplication'.

## Part 1

**NB:** This section may take more than 1 hour

### Review calculating with numerical fractions.

- Display fraction calculations on the board. Ask students to complete these then to describe their method(s) as efficiently as possible.

For example:  $\frac{7}{10} - \frac{2}{6}$     $\frac{3}{16} + \frac{11}{12}$     $\frac{4}{15} - \frac{5}{8}$     $\frac{1}{6} + \frac{1}{12} + \frac{1}{3}$     $\frac{4}{15} \times \frac{5}{18}$     $\frac{3}{16} \div \frac{11}{12}$

- Draw out the principles: Cancel first for efficiency. For add/subtract, find lowest common multiple (LCM) of denominators & use to find the equivalent fractions then add/subtract. For division, multiply first fraction by the reciprocal of the second fraction.
- It may be useful to show  $\frac{7}{10} - \frac{2}{6} = \frac{7}{10} - \frac{1}{3} = \frac{21}{30} - \frac{10}{30} = \frac{21-10}{30}$  and pause to check the equivalence of the last two parts.

## Part 2

### Apply the principles of numerical fraction calculations to algebraic fractions, ensuring that cancelling of algebraic terms is correct.

- Present the students with simple algebraic fractions such as  $\frac{6x}{4} + \frac{7x}{5}$  and ask them to apply the same principle – can they work out what to do?
- Extend the challenge to algebraic denominators such as  $\frac{4}{6x} + \frac{5}{7x}$  – can the students extend their method?
- Ask students what difference it makes if this becomes an equation:  $\frac{6x}{4} - \frac{5x}{6} = 1$ .

Draw out the equivalent method of multiplying every term by the lowest common multiple (LCM) then ask which is more efficient: finding the single algebraic fraction then solving, or multiplying all terms by the LCM.

**NB:** there is no right answer; it depends on the starting point!

- **Students can now begin Exercise 24A from Student Book.**

|   |       |   |      |    |    |    |       |
|---|-------|---|------|----|----|----|-------|
| A | I 1–7 | C | CM 7 | MR | PS | EV | Key 6 |
|---|-------|---|------|----|----|----|-------|

## Part 3

- **Apply the principles to algebraic fractions with more complex terms.**

**NB:** If this extends to a second lesson, clarifying cancelling of algebraic fractions is an appropriate starter for the second lesson.

- Return to  $\frac{4}{6x} + \frac{5}{7x}$  and ask students to suggest more complex terms for the denominators – but not too complex! Look for linear expressions as a starting point and use these to challenge students to extend their method again.
- Ensure that students are allowed to try methods in pairs, then use the example to build the general method: (A) simplify by cancelling if possible (B) find the LCM of the denominators (C) find the equivalent fractions, using brackets if needed (D) add/subtract (E) simplify.
- Emphasise the importance of the brackets to avoid errors.
- If appropriate, pause during this phase to ensure understanding through a more complex example such as  $\frac{3x}{x^2+8x} - \frac{2x+2}{x^2+x}$ .

- **Students can now continue Exercise 24A from Student Book.**

|   |   |        |           |       |       |           |                   |
|---|---|--------|-----------|-------|-------|-----------|-------------------|
| A | I | C 8–19 | CM 15, 17 | MR 12 | PS 19 | EV 11, 16 | Key 8, 10, 12, 16 |
|---|---|--------|-----------|-------|-------|-----------|-------------------|

## Assessment task

- Use some of these terms to create two algebraic fractions – you can use each more than once. It is a good idea to think ahead!
- $2x$     $(x+2)$     $x$     $(2-x)$     $x^2$     $2$     $(x-2)$     $2x^2$     $4$
- Add, subtract, multiply and then divide your two fractions – good luck!

## Section 24.2 Changing the subject of a formula

### Learning objectives

- Rearrange formulae where the subject appears more than once

### Resources and homework

- Student Book 24.2
- Practice Book 24.2

### Making mathematical connections

- Rearranging simple formulae
- Solving equations

### Making cross-curricular connections

- **Science** – use of formulae
- **Computing** – application of logic
- **Relevance** – programming languages; business use of flow diagrams

### Prior learning

Students should be able to solve equations and rearrange simple formulae using the concept of 'balance'.

An understanding of inverse operations is an advantage.

### Working mathematically

- Encourage students to be able to move forwards and backwards within a rearrangement and to be able to articulate how they are 'balancing' at each step.
- This activity lends itself to sorting the steps into order (see the Unjumble Assessment task). Ordering the steps and then writing the descriptions of the operations, then doing the same for the reverse order, provides a foundation for the work on inverse functions in section 24.3.
- Extend students by asking them to create formulae that include more complicated elements – powers, roots, trigonometry – and work out how to rearrange these.

### Common misconceptions and remediation

- Students do not link solving equations with rearranging formulae. Make the parallels explicit by asking students to work in pairs, one solving the equation and the other rearranging the formula, then to swap solutions and discuss.

### Probing questions

- How many steps will I need to rearrange this 4-term formula? Will it always be the same number of steps? Can you find an exception?
- How do you 'undo' (find the inverse of) an add 4? Identify as many types of operations as you can, then list how to 'undo' each one – be creative!

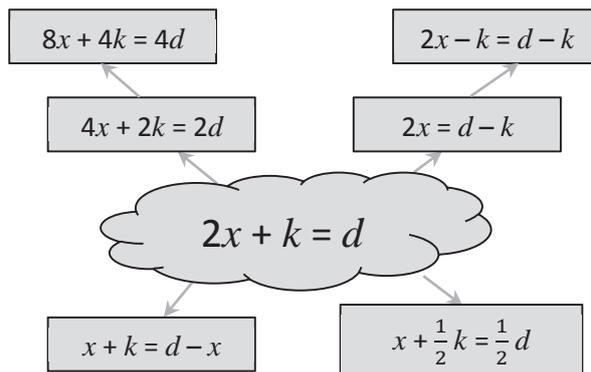
### Literacy focus

- Key vocabulary: formula rearrange balance equivalent inverse
- Ask students to describe the steps in their rearrangement, but insist on the language of '... to both sides'. For example: *I subtracted  $3x$  from both sides, then added 4 to both sides, then divided by 5 on both sides.* This can be turned into a game similar to the children's game 'May I...', where children paid a forfeit if they did not say 'May I'. Students could have a set of formulae to rearrange then describe their steps in groups, getting points each time they do not say 'to both sides'; the student with the least points wins.

## Part 1

### Review equivalent expressions/equations.

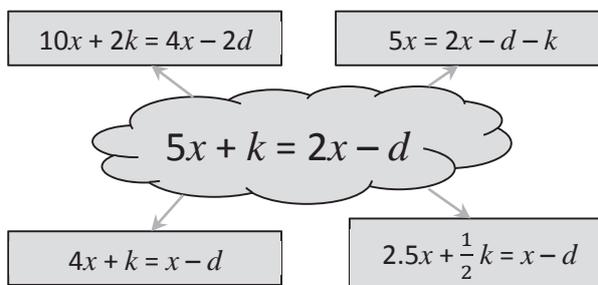
- Use the *Clouding the Picture* frame.
- Ask students what is happening along each of the four branches: make explicit the operation and how it is applied.
- Identify the operation that is a step towards  $x$  being the subject, and that which is a step towards  $k$  being the subject.
- Ask students to identify the most efficient step for each rearrangement. If appropriate, provide additional examples for the discussion.
- Ensure students recognise (A) operations happen to both sides equally (B) multiplication and division happen to every term on each side (C) some operations lead to a simpler form and some lead towards the desired rearrangement.



## Part 2

### Apply the principles of rearranging simple formulae to those where the subject occurs more than once.

- Use the same *Clouding the Picture* frame but change the central formula to one with a repeated subject ( $x$ ).
- Repeat the key question from part one: Which operation(s) are the most efficient ways to move towards a formula with only one subject ( $x$ )?
- Ask students to find a single step that will lead to each variable only occurring once.
- If appropriate, repeat for other formulae.
- Challenge students to extend the principles to equations involving algebraic fractions such as  $\frac{5x+k}{2x-d} = t$  – what would be the most efficient first move? And the next move?
- **Students can now do Exercise 24B from Student Book.**



|   |       |        |    |    |    |               |             |
|---|-------|--------|----|----|----|---------------|-------------|
| A | I 1–6 | C 7–11 | CM | MR | PS | EV 7–8, 10–11 | Key 4, 7, 9 |
|---|-------|--------|----|----|----|---------------|-------------|

## Part 3

- **Assessing understanding at the end of the session.**
- Provide students with the *Clouding the Picture* frame, with the end four boxes completed. They can be challenged to step back towards the centre to find a formula that can be rearranged into all four outer variations.

### Assessment task

- **Unjumble:** Create a set of cards that show the steps in rearranging a formula. Students should be able to place these in the correct order. For a more challenging task, include incorrect steps.
- Challenge students to create their own formula and matching set of Unjumble cards, including distractors.

## Section 24.3 Functions

### Learning objectives

- Know what a function is and use simple function notation
- Use functions including finding the inverse function

### Resources and homework

- Student Book 24.3
- Practice Book 24.3

### Making mathematical connections

- 'Doing and undoing'
- Equations, expressions identities and functions
- Mappings, coordinates and graphing

### Making cross-curricular connections

- **Science** – use of formulae
- **Computing** – logic gates; formulae in spreadsheets; flow diagrams
- **Relevance** – developing logical thinking

### Prior learning

Students should be familiar with the concept of 'flow' via flowcharts or flow diagrams, and of inverse operations. Previous experience of function machines and mapping diagrams is an advantage. Students should be confident with manipulating expressions.

### Working mathematically

- Research: find the definition of a function. Write an algebraic statement that is not a function then compare with your class. Can you classify these statements?
- Display a function and ask students to identify the steps in order (see Literacy focus – this could be verbal). Ask "What happens if the steps are not in the same order? Try the steps in different orders. Can you write the function?" Differentiate by providing steps on card to order, or by using matching cards: one set of cards with the steps in various orders, and one set with the functions on. This can include partially complete cards and distractors to extend other students.

### Common misconceptions and remediation

- Students do not use brackets when converting function machines into algebraic statements – remind them the order of operations applies to algebra as well as number.
- Students forget to reverse the 'x' and 'f(x)' (or y) when finding inverse functions. When marking/going through questions, award marks for method, for answer, and for remembering to reverse to highlight the issue.
- Students read  $f(x)$  as f multiplying an-x-in-brackets. Be consistent in the use of the correct language and insist on 'f of x' from students rather than 'f x'.

### Probing questions

- What are the pitfalls when working with function notation? How can you avoid them?

### Literacy focus

- Key vocabulary: function machine function of  $x$  function inverse
- Ask students to verbalise functions as if they are the variable. If appropriate model this first, for example for the function  $f(x) = \frac{6(x-5)^2}{11}$  Say: "I am  $x$ . Subtract 5, then square, then multiply by 6 and divide by 11." Repeat for other functions with the emphasis on verbal descriptions and the use of 'then', 'and' to discriminate between steps that must be in sequence and those that can be reversed.

## Part 1

### Introduce function machine cards to represent mathematical operations.

- Display the start (3) and the finish (10) and a set of function machines. Ask students to find different ways to get from 3 to 10 using these cards only:  $\times 2$   $\times 3$   $\div 2$  square  $+1$   $+2$   $+3$   $+4$
- Ask students for their routes from 3 to 10. List some on the board. Discuss the different ways found: shortest v longest, most efficient, easiest, complex etc. Refer to order of operations and the need for brackets. Is there an operation that students wish had been in the list?

Some routes:  $+4 +3$   $\times 2 +4$   $\times 3 +1$   $+1 \times 2 +2$   $+4 +2 +1$   $+2 \times 2$   $+1 \div 2 +3 \times 2$  square  $+1$   $+1$  square  $\div 2 +2$

## Part 2

### Introducing function notation and the concept of a function.

- Using the ways found in part 1, model conversion to algebraic function notation – replace the 3 with  $x$ , then the 10 with  $y$ . Then replace the  $y$  with  $f(x)$  and introduce the language ‘function of  $x$ ’. For example:



Talk through the steps:  $x \rightarrow 2x \rightarrow 2x + 4 \rightarrow 2x + 4 = y \rightarrow 2x + 4 = f(x) \rightarrow f(x) = 2x + 4$



$x \rightarrow x + 1 \rightarrow (x + 1) \div 2 \rightarrow \frac{1}{2}(x + 1) \rightarrow \frac{1}{2}(x + 1) + 3 \rightarrow [\frac{1}{2}(x + 1) + 3] \times 2 \rightarrow (x + 1) + 6 = f(x) \rightarrow f(x) = x + 7$

- This may be an appropriate point to do the activity in the Literacy focus.
- **Students can now do Exercise 24C from Student Book.**

|       |       |   |    |         |    |    |          |
|-------|-------|---|----|---------|----|----|----------|
| A 1-4 | I 5-7 | C | CM | MR 4, 7 | SP | EM | Key 2, 7 |
|-------|-------|---|----|---------|----|----|----------|

## Part 3

### Extend to finding inverse functions.

- Use the function machine model to demonstrate the inverse, for example:



- Talk through how to complete the reverse function machine one cell at a time from the ‘end’:  $f(x)$  becomes the starting point  $x \rightarrow$  fill in the inverses  $+2$ ,  $\div 3$ , square root  $\rightarrow x$  becomes the end point  $f(x)$ .



- Ask students to write this algebraically:  $x = \sqrt{\frac{x+2}{3}}$ .
- Students work in pairs. Create a function and its matching function machine. Student A: from the function, write it as a function machine then find the inverse function machine, then convert to algebraic form. Student B: from the function machine, convert to algebraic form, rearrange to make  $x$  the subject (replace  $f(x)$  with  $y$  for convenience). Students then compare their work.
- Draw out that each method gives the inverse – method A replaces the  $f(x)$  with  $x$  at the start, method B the  $x$  and  $y$  also need reversing at the start (or end) of the process.
- **Students can now do Exercise 24D from Student Book.**

|   |       |     |    |      |    |    |               |
|---|-------|-----|----|------|----|----|---------------|
| A | I 1-2 | C 3 | CM | MR 3 | PS | EV | Key 1e, 1f, 2 |
|---|-------|-----|----|------|----|----|---------------|

## Assessment task

- Ask students what is different about the function in Exercise 24D Question 2, compared with the functions in Question 1. Challenge them to create a function machine diagram for this function.

## Section 24.4 Composite functions

### Learning objectives

- Use function notation
- Find composite functions

### Resources and homework

- Student Book 24.4
- Practice Book 24.4

### Making mathematical connections

- Equations, expressions identities and functions
- Mappings , coordinates and graphing

### Making cross-curricular connections

- **Science** – use of formulae
- **Computing** – logic gates; formulae in spreadsheets; flow diagrams
- **Relevance** – developing logical thinking

### Prior learning

Students should be able to use function notation and substitute numerical values into a function.

### Working mathematically

- Give students a function  $f(x)$  and a function  $h(x)$  where  $h(x) = gf(x)$ , and ask students to deduce  $g(x)$ . This can easily be differentiated: Support – provide a set of  $g(x)$  functions to choose from. Challenge – only give  $h(x)$  and a set of functions, so students have to identify both  $f(x)$  and  $g(x)$ .

### Common misconceptions and remediation

- It is easy to forget that  $fg(x)$  means 'do  $g$  of  $x$ , then substitute the result into  $f$  of  $x$ '. Be consistent in the use of language.

### Probing questions

- Which method is best for finding composite functions in simple cases? Does it change if one function has more operations? If the second (last) function has more than one  $x$ ?

### Literacy focus

- Key vocabulary: function machine   function of  $x$    function   composite

### Part 1

#### Introduce composite function notation.

- Display  $f(x) = 3x + 1$ ,  $g(x) = 2(x - 1)$ ,  $fg(x)$ ,  $gf(x)$ .
- Ask students to consider what  $fg(x)$  and  $gf(x)$  might mean, and what  $fg(5)$  and  $gf(5)$  might give. This could be done as a Think, Pair, Share. Take initial thoughts, without indicating whether right or wrong.
- Ask students what difference it would make if the notation was  $f(g(x))$  – would this change their ideas?

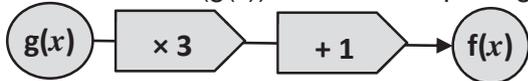
## Part 2

**Introducing conventions for composite function notation and applying this to substitution and algebraic notation.**

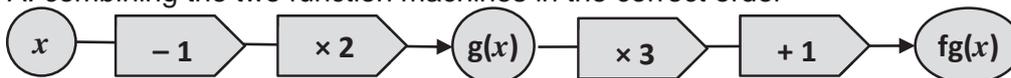
- Display on the board the function machine version:



- Draw out that  $f(g(x))$  would mean putting  $g(x)$  where the  $x$  is in  $f(x)$ :



- Model the two methods:
- A: combining the two function machines in the correct order



$$(x - 1) \times 2 \rightarrow [(x - 1) \times 2] \times 3 \rightarrow \{[(x - 1) \times 2] \times 3\} + 1 \rightarrow [2x - 2] \times 3 + 1 \rightarrow 6x - 5$$

- B: substituting  $g(x)$  into  $f(x)$  and simplifying
- $f(g(x)) = 3[2(x - 1)] + 1 = 3[2x - 2] + 1 = 6x - 5$
- Ask students to try both methods for  $gf(x)$  and  $ff(x)$ , then to choose the method that they find most effective.

Answers:  $gf(x) = 2[(3x + 1) - 1] = 6x$      $ff(x) = 3(3x + 1) + 1 = (9x + 3) + 1 = 9x + 4$

- Students can now do Exercise 24E from Student Book.**

|   |       |       |      |    |      |    |                    |
|---|-------|-------|------|----|------|----|--------------------|
| A | I 1-2 | C 3-4 | CM 3 | MR | PS 4 | EV | Key 2a, 2f, 3i, 3v |
|---|-------|-------|------|----|------|----|--------------------|

## Part 3 – Assessment task

- Design functions  $f(x)$  and  $g(x)$  such that  $f(1) = 1$ ,  $g(1) = 1$  and  $fg(1) \neq 1$ ,  $gf(1) \neq 1$ .

## Section 24.5 Iteration

### Learning objectives

- Use term-to-term sequence notation to generate sequences
- Use iteration to find solutions to polynomials

### Resources and homework

- Student Book 24.5
- Practice Book 24.5

### Making mathematical connections

- Trial and improvement
- Sequences
- Solving equations

### Making cross-curricular connections

- **Computing** – efficient coding using ‘repeat’ or loops
- **Art** - fractals
- **Relevance** – applications to manufacturing and project management

### Prior learning

Students should be able to solve polynomials – although not know that name – using trial and improvement. They should be able to generate sequences using simple term-to-term rules, and algebraic position-to-term rules.

### Working mathematically

- Students use graphing software to identify where the roots of an equation lie: to the nearest whole number, to one decimal place, etc. They then describe how this helps with both trial and improvement and iteration.
- Students use graphing software to generate (for example)  $y = x^4 - 10x - 14$  and  $y = \sqrt[4]{10x + 14}$  to ensure understanding of equivalence.
- Students work in groups of 4, one pair using iteration and one pair using trial and improvement to solve an equation. They then identify the most efficient method for doing this on their calculator, then compare methods: which is easiest? Quickest? Which is more efficient when the equation is longer?

### Common misconceptions and remediation

- Students may confuse the work on rearranging formulae – where the subject must appear once – with the rearranging here, where the ‘subject’ must appear on both sides.

### Probing questions

- What is the same and what is different about ‘trial and improvement’ and ‘iteration’?
- Give me an example of a convergent sequence ... and another one ... and a more general one... [prompt towards an algebraic form].

### Literacy focus

- Key vocabulary: iteration convergence divergence oscillation

## Part 1

### Introduce the concept of convergence, divergence and oscillation in sequences.

- Display these sequences on the board and ask students to find the first 6 terms of each.

$$2^n \quad \frac{1}{2^n} \quad (-1)^n \quad (-2)^n \quad \left[\frac{-1}{2}\right]^n$$

- Challenge the students to sketch the graph for each sequence, or to use graph-plotting software to construct the graphs.
- Draw out the 5 sketches. Ask students to classify the 5 sequences and use this to bring in the language convergence, divergence, oscillation.

## Part 2

### Introduce conventions for sequence notation.

- Use the sequence  $2^n$  to model the notation  $u_{n+1} = u_n \times 2$ , or  $u_{n+1} = 2u_n$ .
- Ask students to find the next 3 terms of the sequence if  $u_1 = 5$ .
- Ask: Does the starting term change the behaviour of the sequence, or does it still diverge?
- **Students can now begin Exercise 24F from Student Book – questions 1 & 2.**

|   |       |   |    |    |    |    |        |
|---|-------|---|----|----|----|----|--------|
| A | I 1–2 | C | CM | RM | SP | EM | Key 2b |
|---|-------|---|----|----|----|----|--------|

## Part 3

### Make the link from sequence notation to iteration.

- Use the equation  $y = x^4 - 10x - 14$ . Ask students to find  $y$  when  $x = 2$  and when  $x = 3$ .  $x = 2$  gives a negative ( $-18$ ),  $x = 3$  gives a positive ( $37$ ) therefore there must be a value between 2 and 3 that gives 0.  
**NB:** If students are not convinced that this is the case, construct the graph using graph plotting software.
- Explain that this mathematics helps to solve equations such as  $x^4 - 10x - 14 = 0$ .
- Show the rearrangement to  $x = \sqrt[4]{10x + 14}$  then the conversion to sequence notation  $u_n = \sqrt[4]{10u_n + 14}$ .
- Ask students to find the next 5 terms of the sequence beginning with  $u_1 = 2$ . What value does their sequence converge towards? (To 1 dp this should be 2.5.)
- If appropriate, model a second example or ask students to try Exercise 24F question 3, then go through it as a class.
- **Students can now continue Exercise 24F from Student Book.**

|   |       |        |            |    |       |             |          |
|---|-------|--------|------------|----|-------|-------------|----------|
| A | I 3–7 | C 8–11 | CM 5, 6, 8 | MR | PS 10 | EV 7, 9, 11 | Key 6, 9 |
|---|-------|--------|------------|----|-------|-------------|----------|

## Assessment task

- Use the Worked Exemplars – example 1. Cut up the exemplar into separate cards, add some distractors, and ask students to put the cards in order. Students could then choose Exercise 24F Question 11a or 11b or 11c and generate their own complete working with notes and distractors to challenge one another.

