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Maths

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Rob Ellis

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24 Algebraic fractions and functions

The questions below are differentiated with colours to show progression. Green is the most accessible, moving through blue to the pink questions, which are the most challenging.

24.1 Algebraic fractions

1 Simplify each of these.

a $\frac{2x}{3} + \frac{4x}{5}$

b $\frac{x+1}{3} + \frac{x+3}{2}$

c $\frac{2x-3}{2} + \frac{5x-1}{3}$

2 Simplify each of these.

a $\frac{3x}{4} - \frac{2x}{5}$

b $\frac{x+2}{2} - \frac{x+1}{5}$

c $\frac{4x-1}{2} - \frac{2x-4}{3}$

3 Solve the following equations.

a $\frac{2x}{3} + \frac{4x}{5} = 11$

b $\frac{x+1}{3} + \frac{x+3}{2} = 10$

c $\frac{2x-5}{2} - \frac{x-1}{3} = 1$

4 Simplify each of these.

a $\frac{3x}{2} \times \frac{4x}{5}$

b $\frac{x+1}{4} \times \frac{3}{2x+2}$

c $\frac{2x-1}{2} \times \frac{4}{3x-1}$

5 Simplify each of these.

a $\frac{x}{4} \div \frac{2x}{5}$

b $\frac{x+3}{2} \div \frac{2x+6}{5}$

c $\frac{4x-2}{3} \div \frac{2x-1}{4}$

6 Show that $\frac{3}{x+2} + \frac{5}{2x-1} = 2$ simplifies to $4x^2 - 5x - 11 = 0$.

7 Solve the following equations.

a $\frac{3}{x-1} + \frac{2}{2x+3} = 5$

b $\frac{5}{3x+2} - \frac{3}{2x-3} = 4$

c $\frac{5}{x+3} + \frac{2}{2x+6} = 4$

8 Simplify the expression $\frac{x^2 - 2x - 3}{2x^2 - 10x + 12}$.

24.2 Changing the subject of a formula

1 Make the stated term the subject of each formula.

a $4(x-2y) = 3(2x-y)$ (x)

b $p(a-b) = q(a+b)$ (a)

c $A = 2ab^2 + ac$ (a)

d $s(t+1) = 2r+3$ (r)

e $st - r = 2r - 3t$ (t)

2 Make x the subject of these equations.

a $ax = b - cx$

b $x(a - b) = x + b$

c $a - bx = dx - a$

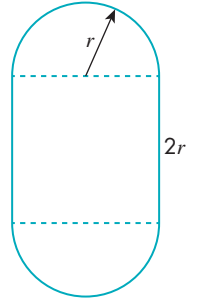
d $x(c - d) = c(d - x)$

3 a The perimeter of the shape on the right is given by the formula $P = 2\pi r + 4r$.

Make r the subject of the formula.

b The area of the same shape is given by $A = \pi r^2 + 4r^2$.

Make r the subject of this formula.



4 a Make x the subject of $y = \frac{x + 2}{x + 3}$.

b Make x the subject of $y = \frac{2 - 3x}{x - 1}$.

5 Make b the subject of $a = \frac{2 + 3a}{b - 2}$.

6 The resistance when two resistors with values a and b are connected in parallel is given by:

$$R = \frac{ab}{a + b}$$

a Make b the subject of the formula.

b Write down the formula when a is the subject.

7 a Make x the subject of the formula $y = \frac{x + 1}{x + 2}$.

b Show that the formula $y = 1 - \frac{1}{x + 2}$ can be rearranged to give $x = -2 - \frac{1}{y - 1}$.

8 a Rearrange the formula $y = \frac{x}{x + 1}$ to make x the subject.

b Rearrange the formula $z = \frac{x + 1}{x - 1}$ to make x the subject.

c Equate the answers to part a and b, cross multiply and expand each side.

Make y the subject of the resultant formula.

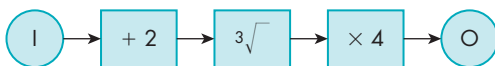
24.3 Functions

1 Find the input for this function machine when the output is:

a 12

b 16

c 8



2 For which input do these function machines also have the same output?



3

$f(x) = 3x^2 - 2$

a Find the value of:

i $f(2)$

ii $f(5)$

iii $f(-1)$

iv $f(-4)$

v $f(\sqrt{3})$

b Given that $f(k) = 25$, find both values of k .**4**

$g(x) = 8 - x^2$

a Find the value of:

i $g(2)$

ii $g(-3)$

iii $g(6)$

iv $g(-4)$

v $g(\sqrt{7})$

vi $g(-0.5)$

b Solve $g(x) = -1$.**5**

$f(x) = 2x^2 - 6x + 4$

a Find the value of:

i $f(-1)$

ii $f(5)$

iii $f(-2)$

b Solve $f(x) = 0$.**6**Find an expression for $f^{-1}(x)$ for:

a $f(x) = 10x - 1$

b $f(x) = \frac{1}{3}x + 4$

c $f(x) = -5x - 10$

d $f(x) = (x - 3)^2$

e $f(x) = \sqrt{x - 4}$

7Given that $f(x) = \frac{x + 3}{2x - 1}$, find an expression for $f^{-1}(x)$.**8**Given that $f(x) = \frac{3x + 5}{2x - 3}$, find an expression for $f^{-1}(x)$.**9**Given that $f(x) = \frac{4x + 3}{x - 4}$, find an expression for $f^{-1}(x)$.**10**

What do you notice about your answers to questions 8 and 9? Make up a question of this type and find the inverse.

24.4 Composite functions

1

Evaluate each composite value

a $f(x) = 3x - 5$

$g(x) = x^2$

Find $fg(3)$.

b $f(x) = -9x - 9$

$g(x) = \sqrt{(x - 9)}$

Find $fg(10)$.

c $f(x) = -4x + 2$

$g(x) = \sqrt{(x - 8)}$

Find $fg(12)$.

d $f(x) = -3x + 4$

$g(x) = x^2$

Find $gf(-2)$.

e $f(x) = -2x + 1$

$g(x) = \sqrt{x^2 - 5}$

Find $gf(2)$.**2**

Find each composite function.

a $f(x) = -9x + 3$

$g(x) = x^4$

Find $fg(x)$.

b $f(x) = 2x - 5$

$g(x) = x + 2$

Find $fg(x)$.

c $f(x) = x^2 + 7$

$g(x) = x - 3$

Find $fg(x)$.

d $f(x) = 4x + 3$

$g(x) = x^2$

Find $gf(x)$.

e $f(x) = x - 1$

$g(x) = x^2 + 2x - 8$

Find $gf(x)$.

- 3 For homework a teacher asks his class to evaluate the composite function $f(x) = x^2 - 3$ $g(x) = 5x$.
Find $fg(-3)$
This is Wayne's answer:
- | | | |
|----------------------|-----------------|-------------------|
| $f(-3) = (-3)^2 - 3$ | $g(x) = 5x$ | $fg(-3) = 6(-15)$ |
| $f(-3) = 9 - 3$ | $g(-3) = 5(-3)$ | $= -90$ |
| $f(-3) = 6$ | $g(-3) = -15$ | |
- What has he done wrong? Solve the problem correctly.

24.5 Iteration

- 1 Find the first five iterations of the following iterative formulae:
- a** $x_{n+1} = \frac{x_n + 2}{6}$ **b** $x_{n+1} = \frac{x_n}{5} + 4$ **c** $x_{n+1} = \frac{2}{x_n - 5}$
- Start each one with $x_1 = 3$.
- 2 Find a root of the quadratic equation $2x^2 + 3x - 4 = 0$ using the iterative formula:
- $$x_{n+1} = \sqrt{\frac{4 - 3x_n}{2}}$$
- Start with $x_1 = 2$ giving your answer to 2 decimal places.
- 3 Show that $x^2 + x - 1 = 0$ can be rearranged into the iterative formula $x_{n+1} = \sqrt{1 - x_n}$
Use the iterative formula together with a starting value of $x_1 = 0.5$ to obtain a root of the equation correct to 2 decimal places.
- 4 Show that $x^2 - 9x + 2 = 0$ can be re-arranged into the iterative formula
- $$x_{n+1} = \sqrt{9x_n - 2}$$
- Use the iterative formula together with a starting value of $x_n = 8$ to obtain a root of the equation correct to 2 decimal places.
- 5 A rectangle has sides of $(x - 3)$ cm and $(x + 4)$ cm and an area of 26 cm^2 .
- a** Show that $x^2 + x - 38 = 0$.
- b** Use the iteration formula $x_{n+1} = \sqrt{38 - x_n}$ and an initial input of $x_1 = 3$ to find the length of each side of the rectangle, correct to 2 decimal places.
- 6 Show that $x = \frac{5}{x} - 3$ can be rearranged into the equation $x^2 + 3x - 5 = 0$.
Use the iterative formula $x_{n+1} = \frac{5}{x_n} - 3$ to find a root of the equation giving your answer to 2 decimal places.
- 7 Solve the equation $x^3 - 2x + 3 = 5$ using an iterative formula.

