Collins

Maths

4th Edition

GCSE

Foundation Teacher Pack

Christine Watson

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Chapter 7 Ratio, speed and proportion

Overview

7.1 Dividing in a given ratio	**7.3** Understand and use direct proportion
7.2 Calculating with speed, distance, time	**7.4** Calculate in a 'best buy' context

Prior learning

Can cancel fractions
Can convert between measurement units (does not include metric to imperial)

Chapter 7

Ensure students understand and can calculate with ratios, speed and proportion.

In the examination, students will be expected to:

A cancel ratios to simplest form, to divide quantities using a ratio of two or three parts including unit ratios. Contexts include recipes and finance.
B calculate speed, distance or time including from information in a travel graph.
C calculate missing values in direct proportion problems including in a Best Buy context.

Extension

Explore real world applications.
Devise a way to compare/order ratios.
Explore the golden ratio in architecture and art.
Explore proportionality in geometry or in graphs.

Curriculum references

Section	*KS3 NC Programmes of Study*	*KS4 NC Programmes of Study*	*GCSE specification*
7.1	R1 R2	R1 R3	R4 R5 R6 R7 N11
7.2	R5	A (EF) 7	N14
7.3	R4	R3	R10
7.4	R4	R3	R11

Route mapping

Exercise	Accessible	Intermediate	Challenging	AO1	AO2 MR CM	AO3 PS EV	Key questions
7A	1–5	6–8	9–12	1–8	9–10	11–12	2, 6, 7, 9
7B	1–4	5–7	8–10	1–5, 7–8	6	9–10	3, 7, 8g
7C	1–4	5–7	8–10	1–6, 9	7	8, 10	5, 7
7D	1–5	6–13	14–16	1–5, 7–8, 11–13	6, 9, 10, 15	14, 16	3, 4, 9, 14
7E	1–5	6–7	8–10	1–6	7–8	8–10	4, 7, 8
7F	1–4	5–6	7–8	1–4	5, 7, 8	6	3, 5, 7

Key questions are those that demonstrate mastery of the concept or which require a step-up in understanding or application. These could be used to identify the questions that students must tackle, to support differentiation, or to identify the questions that should be teacher-marked rather than student-marked.

About this chapter

Making connections: The chapter deals with proportional reasoning, with a focus on using tables to organise information and calculating missing values. The four sections are inter-related, with strong connections between 7.1, 7.3 and 7.4 in methods of representation. It builds on concepts met when working on fractions.

Relevance: Comparison of values is an essential skill, not just for shopping. Students should be encouraged to compare proportions in a range of contexts: financial, test scores, sports records, speed comparisons, betting odds, work rates etc.

Working mathematically: What is the most effective way to present this information? What is the most efficient way to solve the problem? What are the key words when reading a ratio problem, and are the key words the same in a direct proportion problem?

Assessment: Give students a pair of values. Ask them to create a ratio question, a speed question, a direct proportion question and a best buy question each using this pair of values. Ask students to describe what is the same and what is different about their 4 questions and their methods of solution.

See the CD for suggested assessment tracking foci, and the section plans for further suggested Assessment tasks.

Worked exemplars from Student Book– suggestions for use

A Present students with the same question but different numbers. They use the exemplar to mirror the working, in full or just the notes.

B Copy and cut up the exemplar into cards. Students match the working with the notes. (You may need to remove the words ‘first, second’ etc.)

C Copy and cut up the working into cards but split the label/description from the working. Students put the working in order then match with the descriptions.

Answers to Student Book questions at the end of this book (NB: not included in this sample)

Section 7.1 Working with ratios; Dividing in a ratio

Learning objectives

- Understand the concept of ratio including simplest form
- Be able to apply methods for dividing quantities in a ratio

Resources and homework

- Student Book 7.1
- Practice Book 7.1

Making mathematical connections

- Cancelling fractions
- Describing and comparing quantities and size

Making cross-curricular connections

- **Geography** – map scales
- **Technology** – recipes, concrete mixes
- **Relevance** – betting odds; sharing in proportion vs fairly

Prior learning

It is essential that students can convert between time measures, between metric units. This may need a prior homework to refresh the skill, or a full lesson, depending on the students.

Working mathematically

- Encourage students to apply their understanding of fractions (and decimals, percentages) to ratios. Avoid always presenting the simplest case (unit ratios, ratios with two parts) but challenge students to work with more complex ratios and apply their skills to these.
- Offer students the opportunity to discover their own representations for ratios – pictorial, tabular or other – and to make the links between the various representations.

Common misconceptions and remediation

- Students see one part of the ratio as the total, not as a part, or see the ratio 1 : 2 as a half. Ensure that the physical image of a ratio is embedded at the start and make the comparison with fractions explicit.

Probing questions

- Describe what is the same and what is different about a ratio and a fraction.
- Explain how you would set out a ratio calculation – make up a good example to help you.

Literacy focus

- Key vocabulary: cancel ratio common units simplest form
- Be explicit in the use of the vocabulary: fraction – ratio – part – total – sharing – comparing – describing.

Part 1

NB: *This section may take more than 1 hour*

Introduce the concept of ratio as a comparison of proportions and make the link to fractions.

- Use the Top Trumps cards [or a set of photos with characteristics]. Give a set to each group.
- Sort the cards in different ways. For each, show and verbalise the ratio, show the link to ratio in a table (for example F : M : total) and make the link to the associated fractions.
- Ask what the *ratio* tells them about the two groups. Prompt for the idea of *relative size* of the groups and bring in the term *proportion*.
- Ask for other suggestions for different groupings, allowing more than two groups. Students should organise their cards into the new grouping, and write the ratio on their boards. Repeat until it is clear students understand how to decide what the ratio is.

- Ask students to work in pairs to decide what they think a ratio is. Ask for a short definition, a long definition, one with more mathematical words. Ask what the difference is between a ratio and a fraction. Give time for students to write down their 'final' definition.

Part 2

Draw out equivalent ratios visually and using common factors, and make the link to cancelling fractions.

- Return to a two-part ratio that was not in simplest form – ask the students to quickly organise the photos into these two groups, and expect photos to be bunched. Now ask students to create a 'tidy' layout, and look for one that is rectangular. Match this one using the cards on the board [or ask students to suggest a more mathematical way to show the photos].
- Ask students if they can see any other ratios in the grouping (for example from 10 : 5, the new layout should highlight 2 : 1). Make the link between the overall ratio and the simplified ratio – what calculations can you do to simplify or increase the values? Which describes the *proportions* in the two groups? How does this link to other mathematics you have done?
- Ensure the link to equivalent fractions and cancelling is made.
- **Students can now do Exercise 7A from Student Book.**

A 1–5	I 6–8	C 9–12	CM	MR 9–10	PS 11–12	EV	Key 2, 6, 7, 9

Part 3

Explore dividing in a given ratio as the reverse of simplifying – multiplying up as the inverse of dividing.

- Return to a visual display of a ratio, for example 2 : 6. Revisit that it can be cancelled, and show that it can be multiplied up.
- Return to the table layout for the ratio, with 2 : 6 (total) 8. Ask what the values would be when the total number of items is 16 – 80 – 40 – 4 – 12, showing these in the table one by one. Ensure ratio equivalence underpins the understanding and make the link to equivalent fractions.
- **Students can now do Exercise 7B from Student Book.**

A 1–4	I 5–7	C 8–10	CM	MR 6d	PS 9–10	EV	Key 3, 7, 8g

NB: *these involve mixed units – a mini-plenary to ensure students are dealing with the units correctly is appropriate.*

- Students could then apply their understanding to Exercise 7C (optional).

Part 4

Explore dividing in a given ratio when the quantity is not the total amount.

- Pose a problem where the given quantity does not match the total and ask students to decide how they could tackle the problem.
- Discuss their suggestions and ensure all have a correct adapted original method.
- **Students can now do Exercise 7C from Student Book.**

A 1–4	I 5–7	C 8–10	CM	MR 7	PS 8	EV 10	Key 5, 7

- **Plenary: (A)** Students could create an easy and a hard ratio question and challenge their partner to complete one. (**B)** Students could write the steps needed when working with ratios.

Assessment task

- Create a set of Top Trumps cards of your own to meet set criteria – could be heroes/pop stars/sports personalities/tv or film stars… To be peer-assessed in groups next lesson.
- eg female : male is 2 : 3, under 20 : 20 to 30 : over 30 is 2 : 3 : 1, British : American is 4 : 1.

Section 7.2 Speed, distance and time

Learning Objective

- Use the relationship between speed, distance and time to work out unknown values.

Resources and homework

- Student Book 7.2
- Practice Book 7.2

Making mathematical connections

- Substituting into formulae
- Rearranging formulae
- Minutes as a fraction or decimal

Making cross-curricular connections

- **Science/PE** – measuring speed
- **Relevance** – road safety; appreciation of racing

Prior learning

It is essential that students can convert minutes to a decimal of an hour.

Working mathematically

- Encourage students to rearrange the formula or substitute then rearrange their calculation.
- Use real life examples of speeds – speed limits, athletics, animal speeds – to bring the topic to life.

Common misconceptions and remediation

- Students divide the given two quantities without thinking about their meaning. Encourage use of the formula or of the units to avoid blind substitution.
- Ensure students recognise that 30 minutes is not 0·30 of an hour. Use the clock face to link to geometric representations of fractions to consolidate understanding.

Probing questions

- Is there a diagram that helps me understand speed questions?
- How do I use the units in the question to help me decide what to do?
- Which is faster, mph or km/h?

Literacy focus

- Key vocabulary: speed distance time average speed
- Prompt for correct units, and ensure students use the units to help decide what to do, for example km/h means km divided by hours.

Part 1

Explore what students already know about speed.

- Create a mind map: Speed at the centre, ask students to (a) individually add what they know about speed (b) share with a partner (c) share with another pair [1 to 2 to 4].
- Draw out the key points, which may include: speed limits; units of speed; athletics records; land/air/sea records; the formula speed = distance/time; how to graph speed; links to acceleration.

Part 2

Demonstrate methods for calculating with speed, distance, time.

- Ensure students know that 60 mph means 60 miles in 1 hour
  180 miles in 3 hours
  30 miles in half an hour.
- Draw out the formula speed = distance ÷ time and check that students can calculate any value by substituting the other two then calculating.
- **Students can now do Exercise 7D from Student Book.**

A 1–5	I 6–13	C 14–16	CM 6b, 9, 15	MR 10	PS 14	EV 16	Key 3, 4, 9, 14

NB: *Students need to use time as a decimal. Include a mini-plenary to pose the thinking point: 30 miles divided by 30 minutes doesn't give 60 mph – what am I doing wrong?*

Part 3

Plenary

- Display a distance-time graph and challenge students to apply their understanding to find the speed(s).

Assessment task

- Provide a set of examination questions and ask students to (a) rank them in order of difficulty and assign red – amber – green (b) solve an amber question.

Section 7.3 Direct proportion

Learning objective

- Understand and calculate with direct proportion.

Resources and homework

- Student Book 7.3
- Practice Book 7.3

Making mathematical connections

- Link to ratio and fractions
- Link to enlargement

Making cross-curricular connections

- **Science** – correlations and connections, creating formulae
- **Relevance** – personal finance, especially shopping; stretch factors in photography

Prior learning

- This section builds on the ratio work and the links need to be made explicitly.

Working mathematically

- Encourage students to see different representations of direct proportion: in a table, as pairs of values, as a graph plot, as a form of ratio, as a pictorial form of ratio etc and to see the links between each form.
- Explore which parts of the body are in direct proportion for a group of people, for example wrist circumference to waist circumference, leg length to height. Link this to media representations, for example the stretching of female legs in advertising – see example photographs on http://mathspig.wordpress.com/2010/05/04/make-me-feel-real-loose-like-a-long-necked-goose/.

Common misconceptions and remediation

- Students sometimes assume that the unitary method is an essential step rather than using multipliers to move straight to the solution. Include very simple multipliers to draw this out.
- Students may see the original question and the solution as a set of 4 values, isolated from the concept of direct proportion. Include examples in a table where there are many missing values in both variables that can be found in different ways and emphasise this continuity. Use the graph form to show the infinite pairs of values that satisfy the multiplier, noting where the pairs do not connect with the context (for example, if the context is buying quantities the negative pairs do not have a physical meaning).

Probing questions

- What is special about pairs of values that are in direct proportion?
- What mathematical tests can you apply to check that values are in direct proportion?
- When you plot the values as coordinates, what will you get?
- Direct proportion values have a multiplier. How many multipliers can you find?

Literacy focus

- Key vocabulary: proportion direct proportion multiplier unitary method unit cost
- After completing Exercise 7E, discuss the language of the questions. Some make the direct proportion aspect obvious and some are phrased to make it more obscure.
- Select a straightforward wording – ask students to make it more obscure. Select a complex wording – ask students to make it more straightforward. This may need modelling first to build student skills.

Part 1

Recognise the properties of values in direct proportion.

- Create a set of at least six statements for display. For example: ***A*** 2 eggboxes hold 12 eggs ***B*** 2 people plaster a room in 4 hours ***C*** I travel 2 km in 15 minutes ***D*** a square of length 2 m has area 4 m² ***E*** 2 bags of potatoes cost £3 ***F*** a cube of length 2 m has volume 8 m³ ***G*** 2 cold-callers ring a batch of people in 5 hours.
- Ask students to group these into types. Look for a sense of increasing together/increase & decrease, and for a sense of which are direct. If necessary, prompt to change the '2' to 4 (or 10), ask what would happen to the other value.
- Label A, C and E as direct proportion. Ask students to describe in pairs what the criteria might be for direct proportion, and draw this out through questioning. Use the term 'multiplier' to describe the relationships.

Part 2

Calculate missing values in the context of direct proportion.

- Make the link from ratio to direct proportion by setting it out in a table but without a 'total'.
- Use A, C and E as examples of multiplying up to find missing values, and of the unitary method.

	2 boxes	12 eggs		2 km	15 mins		2 bags	£3	
	↓	↓		↓	↓		↓	↓	
(÷2)	1	6	(÷2) (×2·5)	5 km	37.5 mins	(×2·5) (×4)	8 bags	£12	(×4)

- Use the eggbox example to show that there is more than one set of relationships: Downwards, divide by 2; upwards, multiply by 2; to the right, multiply by 6, to the left, divide by 6. Emphasise this property of direct proportion.
- **Students can now do Exercise 7E from Student Book.**

A 1–5	I 6–7	C 8–10	CM 7	MR 8	PS 9	EV 8, 10	Key 4, 7, 8

Part 3

Plenary

- Display a direct proportion problem where a number of values must be found in both variables. Ask students to work in pairs to find at least three different ways to solve it.

Assessment task

- See the Literacy section.

Section 7.4 Best buys

Learning objectives

- Understand the concept of 'value for money'
- Calculate price per unit and units per pound/penny

Resources and homework

- Student Book 7.4
- Practice Book 7.4

Making mathematical connections

- Link to direct proportion and to simplifying ratios

Making cross-curricular connections

- **Relevance** – shopping, value for money

Prior learning

- This section builds on the ratio and direct proportion work and the links need to be made explicitly.

Working mathematically

- This section can be approached as a problem-solving task: Present students (in pairs or groups) with sets of pricing scenarios such as those in the exercise. Ask them to decide which is the best buy in each case and to justify their decisions, presenting solutions as a poster. Ask students to group their responses, showing which ones had been tackled using the same method. Students then present their posters to their peers, with the teacher prompting or rephrasing/clarifying but not leading. Students could then note in general form the most effective methods for deciding a best buy.

Common misconceptions and remediation

- Some students will mix up their division calculations. Encourage them to set out the initial information in a table, making the link to direct proportion and ratio, and to then use their established method to find the missing values.
- It may be appropriate to only look at price per unit as this is an easier concept to understand and leave units per price until revisiting the topic at a later date.

Probing questions

- How do you know which is the best buy? Convince me.
- Which is better, cost divided by quantity, or quantity divided by cost? Why?
- Should you always buy the item that gives you the cheapest costs per unit? Why?

Literacy focus

- Key vocabulary: best buy value for money better value
- Ensure students understand the concepts of cheapest/dearest vs best buy/value for money.
- This may need some specific scenarios where best buy is dependent on product quality, money available, or quantity needed.
- The language can be obscure and may need decoding. Display sets of statements and ask students to match them up. For example:

cost per unit mass	cost of 1 gram (or 1 kg)	cost per unit length	cost of 1 metre (or 1 cm)
mass per unit cost	number of g (kg) for £1 (1p)	length per unit cost	number of m (cm) for £1 (1p)

Part 1

Recognise the properties of values in direct proportion.

- Display price labels from supermarkets.

BASICS MINCE Always Good Value **500g for £1·20** price per kg £2·40	BEST VALUE MINCE MEATY GOODNESS **1 kg for £3·00** price per kg £3·00	BASICS MINCE **2 kg for £4·50** TODAY ONLY! price per kg £2·25

- Ask students: You need a kilogram of mince. Which would you buy? Why?
- Draw out: quality comparisons, size comparisons (you don't need 2 kg, under what circumstances would you buy more than you need), the 'price per…' element of the label.
- Draw out the difference between cheapest and best buy, best value for money.

Part 2

Calculate missing values in the context of direct proportion.

- Introduce price per unit. Ask which is better, a lower price per unit or higher.
- Introduce units per price. Ask which is better, lower or higher.
  NB: *For some classes, ask them to complete some of Exercise 7F using price per unit and check understanding* before *introducing units per price. Note that for some students meeting both in the same session may not be helpful.*
- Ensure students understand the language of the chapter – see Literacy section.
- **Students can now do Exercise 7F from Student Book.**

A 1–4	I 5–6	C 7–8	CM 7, 8	MR 5	PS 6	EV	Key 3, 5, 7

Part 3

Plenary and Assessment task

- Use examples of supermarket pricing including ones where the largest is not cheapest, or where a special offer or BOGOF affects the calculation (There are sets of these via, for example, TES). Ask students to identify which of each pair is best value for money.