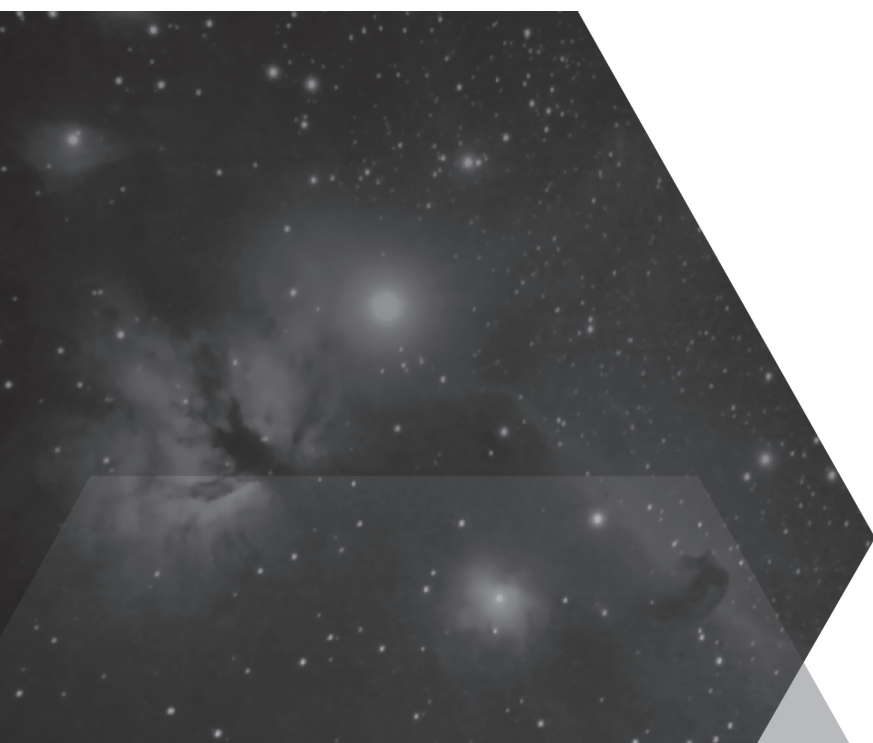




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# AQA GCSE **Maths**

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Answers



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# Answers

## Chapter 1 – Number: Basic number

### Exercise 1A

- 1 a 6000  
b 5 cans cost £1.95, so 6 cans cost £1.95.  
 $32 = 5 \times 6 + 2$ . Cost is £10.53.
- 2 a 288                      b 16
- 3 a 38  
b Coach price for adults = £8, coach price for juniors = £4, money for coaches raised by tickets = £12 400, cost of coaches = £12 160, profit = £240
- 4 (18.81...) Kirsty can buy 18 models.
- 5 £8.40 per year, 70p per copy
- 6 £450
- 7 15
- 8 3 weeks
- 9 £248.75
- 10 Gavin pays  $2926.25 - 1840 = £1086.25$
- 11 a Col is correct  
b Abi has multiplied  $30 \times 50$  as 150 instead of 1500. Baz has lined up the columns wrongly when adding. Instead of lining up the units he has lined up the first digits. Des has forgotten to add a zero on the second line of the multiplication, it should be 1530.

### Exercise 1B

- 1 a 4.6                      b 0.08                      c 45.716  
d 94.85                      e 602.1                      f 671.76  
g 7.1                      h 6.904                      i 13.78  
j 0.1                      k 4.002                      l 60.0
- 2 a 0.028    b 0.09    c 50.96    d 46.512
- 3 a 35, 35.04, 0.04    b 16, 18.24, 2.24  
c 60, 59.67, 0.33    d 140, 140.58, 0.58
- 4 a 18    b 140    c 1.4    d 12    e 6.9
- 5 a 280    b 12    c 240    d 450    e 0.62
- 6 a 572    b i 5.72    ii 1.43    iii 22.88
- 7 a Incorrect as should end in the digit 2  
b Incorrect since  $9 \times 5 = 45$ , so answer must be less than 45
- 8 300
- 9 a 27  
b i 27                      ii 0.027                      iii 0.27
- 10 Mark bought a DVD, some jeans and a pen.
- 11 Headline A does not give the exact figure so does not convey any useful information. Headline B is accurate and records should be given accurately. Headline C may be correct but without the previous record does not convey any useful information.

### Exercise 1C

- 1 a 50 000                      b 90 000                      c 30 000  
d 200                      e 0.5                      f 0.006  
g 0.3                      h 10                      i 0.05  
j 1000
- 2 a 56 000                      b 80 000                      c 31 000  
d 1.7                      e 0.066                      f 0.46  
g 4.1                      h 8.0                      i 1.0  
j 0.80
- 3 a 60 000                      b 5300                      c 89.7  
d 110                      e 9                      f 1.1  
g 0.3                      h 0.7
- 4 a 65, 74                      b 95, 149                      c 950, 1499
- 5 Elsecar 750, 849; Hoyland 1150, 1249; Barnsley 164 500, 165 499
- 6 18 to 23 inclusive
- 7 1, because there could be 450 then 449
- 8 Donte has rounded to 2 significant figures or nearest 10 000
- 9 a Advantage – quick. Disadvantage – assumes 3 penguins a square metre which may not be accurate  
b Advantage. Quite accurate as 5 by 5 is a big enough area to give a reliable estimate. Disadvantage – takes a long time.

### Exercise 1D

- 1 a 60 000                      b 120 000                      c 10 000  
d 15                      e 140                      f 100  
g 200                      h 0.08                      i 0.09  
j 45
- 2 a 5                      b 25                      c 3000  
d 600                      e 2000                      f 5000  
g 400                      h 8000                      i 4 000 000
- 3  $30 \times 90\,000 = 2\,700\,000$   
 $600 \times 8000 = 4\,800\,000$   
 $5000 \times 4000 = 20\,000\,000$   
 $200\,000 \times 700 = 140\,000\,000$
- 4 a 54 400                      b 16 000
- 5 1400 million
- 6 His answer is correct but he had one too many zeros on each value, which cancel each other out. Matt wrote 600,000 rather than 60,000 and 2000 rather than 200. The two mistakes cancelled themselves out due to the zeros involved.
- 7 a Value of the money is about  $66\,000\,000 \times 0.2 = £13\,200\,000$ , so it is enough to buy the yacht.  
b Weight is  $66\,000\,000 \times 5 = 330\,000\,000$  grams = 330 tonnes, so they do not weigh as much as the yacht.
- 8  $1420\,000\,000\,000 \div 64\,000\,000 \approx 22\,200$ , so the National Debt per person is approximately £22 200.

### Exercise 1E

- 1 a 35 000      b 15 000      c 960  
d 5              e 1200      f 500
- 2 a 39 700      b 17 000      c 933  
d 4.44          e 1130      f 550
- 3 a 1.74 m      b 6 minutes      c 240 g  
d 83°C          e 35 000 people      f 15.5 miles  
g 14 m<sup>2</sup>
- 4 a 10      b 1      c 3
- 5 a 8.79      b 1.03      c 3.07
- 6 82°F, 5 km, 110 min, 43 000 people, 6.2 seconds, 67th, 1788, 15 practice walks, 5 seconds  
The answers will depend on the approximations made. Your answers should be to the same order as these.
- 7 a £15 000      b £18 000      c £17 500
- 8 \$1000
- 9 a 40 miles per hour      b 10 gallons      c £70
- 10 a 80 000      b 2000      c 1000      d 30 000  
e 5000      f 2500      g 75      h 100
- 11 a 86 900      b 1760      c 1030      d 29 100  
e 3960      f 2440      g 84.8      h 163
- 12 Approximately 500
- 13 £1 million pounds is 20 million 5p coins. 20 000 000 × 4.2 = 84 000 000 grams = 84 tonnes, so 5 lorries needed.
- 14 22.5° C – 18.2° C = 4.3 Celsius degrees
- 15 a i 27.571 428 57      ii 27.6  
b i 16.896 516 39      ii 16.9  
c i 18 672.586 16      ii 18 700
- 16 a 37.5 × 48.6 ≈ 40 × 50 = 2000      21.7 × 103.6 ≈ 20 × 100 = 2000      985 ÷ 0.54 ≈ 1000 ÷ 0.5 = 2000  
b as both values are rounded down the actual answer must be bigger than 2000. The other two must be less than 2000.  
c Pete is correct it is not possible to tell. 37.5 × 48.6 = 1822.5      985 ÷ 0.54 = 1824.074
- 17 149 000 000 ÷ 300 000 = 496.67 ≈ 500 seconds
- 18 a 58.9 × 4.8 ≈ 60 × 5 = 300  
b Lower as both values are rounded up to get the estimate.
- 19 Macau's population density is approximately 710 000 times the population density of Greenland.
- 20 26.8 ÷ 3.1 ≈ 27 ÷ 3 = 9      36.2 ÷ 3.9 ≈ 36 ÷ 4 = 9. Second calculation must be biggest as first is smaller than 27 ÷ 3 and second is bigger than 36 ÷ 4.

### Exercise 1F

- 1 a 12      b 9      c 6      d 13      e 15      f 14  
g 16      h 10      i 18      j 17      k 8 or 16      l 21

- 2 4 packs of sausages and 5 packs of buns (or multiples of these)
- 3 30 seconds
- 4 12 minutes; Debbie will have run 4 laps; Fred will have run 3 laps.
- 5 1 + 3 + 5 + 7 + 9 = 25, 1 + 3 + 5 + 7 + 9 + 11 = 36, 1 + 3 + 5 + 7 + 9 + 11 + 13 = 49, 1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64
- 6 a -2      b -7      c -12      d -1      e -30
- 7 a 1      b 3      c 4      d 2      e -4
- 8 a 400      b 900      c 2500      d 0.25      e 16
- 9 a Student's own explanation  
b 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105  
c Adding consecutive pairs gives you square numbers.

10

	Square number	Factor of 56
Cube number	64	8
Multiple of 7	49	28

- 11 2, 3 and 12
- 12 a 1, 64, 729, 4096, 15 625  
b 1, 8, 27, 64, 125  
c  $\sqrt{a} = a \times \sqrt{a}$   
d Square numbers
- 13 a 0.2      b 0.5      c 0.6      d 0.9  
e 1.5      f 2.1      g 0.8      h 0.7
- 14 The answers will depend on the approximations made. Your answers should be to the same order as these.  
a 60                      b 1500                      c 150

### Exercise 1G

- 1 a 84 = 2 × 2 × 3 × 7  
b 100 = 2 × 2 × 5 × 5  
c 180 = 2 × 2 × 3 × 3 × 5  
d 220 = 2 × 2 × 5 × 11  
e 280 = 2 × 2 × 2 × 5 × 7  
f 128 = 2 × 2 × 2 × 2 × 2 × 2 × 2  
g 50 = 2 × 5 × 5
- 2 a 84 = 2<sup>2</sup> × 3 × 7      b 100 = 2<sup>2</sup> × 5<sup>2</sup>  
c 180 = 2<sup>2</sup> × 3<sup>2</sup> × 5      d 220 = 2<sup>2</sup> × 5 × 11  
e 280 = 2<sup>3</sup> × 5 × 7      f 128 = 2<sup>7</sup>  
g 50 = 2 × 5<sup>2</sup>
- 3 1, 2, 3, 2<sup>2</sup>, 5, 2 × 3, 7, 2<sup>3</sup>, 3<sup>2</sup>, 2 × 5, 11, 2<sup>2</sup> × 3, 13, 2 × 7, 3 × 5, 2<sup>4</sup>, 17, 2 × 3<sup>2</sup>, 19, 2<sup>2</sup> × 5, 3 × 7, 2 × 11, 23, 2<sup>3</sup> × 3, 5<sup>2</sup>, 2 × 13, 3<sup>3</sup>, 2<sup>2</sup> × 7, 29, 2 × 3 × 5, 31, 2<sup>5</sup>, 3 × 11, 2 × 17, 5 × 7, 2<sup>2</sup> × 3<sup>2</sup>, 37, 2 × 19, 3 × 13, 2<sup>3</sup> × 5, 41, 2 × 3 × 7, 43, 2<sup>2</sup> × 11, 3<sup>2</sup> × 5, 2 × 23, 47, 2<sup>4</sup> × 3, 7<sup>2</sup>, 2 × 5<sup>2</sup>
- 4 a 2 is always the only prime factor  
b 64, 128      c 81, 243, 729  
d 256, 1024, 4096  
e 3, 3<sup>2</sup>, 3<sup>3</sup>, 3<sup>4</sup>, 3<sup>5</sup>, 3<sup>6</sup>; 4, 4<sup>2</sup>, 4<sup>3</sup>, 4<sup>4</sup>, 4<sup>5</sup>, 4<sup>6</sup>

- 5 **a**  $2 \times 2 \times 3 \times 5$       **b**  $2^2 \times 3 \times 5$   
**c**  $120 = 2^3 \times 3 \times 5$ ,  $240 = 2^4 \times 3 \times 5$ ,  
 $480 = 2^5 \times 3 \times 5$
- 6 **a**  $7^2 \times 11^2 \times 13^2$       **b**  $7^3 \times 11^3 \times 13^3$   
**c**  $7^{10} \times 11^{10} \times 13^{10}$
- 7 Because 3 is not a factor of 40 so it does not divide exactly.
- 8  $a = 2$ ,  $b = 7$
- 9 **a**  $2ab a^2 4b$       **b**  $8a^3b 4a^3b^2$

### Exercise 1H

- 1 **a** 20      **b** 56      **c** 6      **d** 28  
**e** 10      **f** 15      **g** 24      **h** 30
- 2 They are the two numbers multiplied together.
- 3 **a** 8      **b** 18      **c** 12      **d** 30
- 4 No. The numbers have a common factor. Multiplying them together would mean using this factor twice, thus increasing the size of the common multiple. It would not be the least common multiple.
- 5 **a** 168      **b** 105      **c** 84      **d** 84  
**e** 96      **f** 54      **g** 75      **h** 144
- 6 3 packs of cheese slices and 4 packs of bread rolls
- 7 **a** 8      **b** 7      **c** 4      **d** 16      **e** 14      **f** 9
- 8 **a** ii and iii      **b** iii
- 9 18 and 24
- 10 **a**  $6x^2y^2$       **b**  $xy$

### Exercise 1I

- 1 **a** 7      **b** -8      **c** -5      **d** -11  
**e** 11      **f** 6      **g** 8      **h** 8  
**i** -2      **j** -1      **k** -9      **l** -5  
**m** 5      **n** -9      **o** 8      **p** 0
- 2 **a** -15      **b** -14      **c** -24      **d** 6  
**e** 14      **f** 2      **g** -2      **h** -8  
**i** -4      **j** 3      **k** -24      **l** -10  
**m** -18      **n** 16      **o** 36
- 3 **a** -9      **b** 16      **c** -3      **d** -32  
**e** 18      **f** 18      **g** 6      **h** -4  
**i** 20      **j** 16      **k** 8      **l** -48  
**m** 13      **n** -13      **o** -8
- 4 **a** -2      **b** 30      **c** 15      **d** -27      **e** -7
- 5 **a** -9      **b** 3      **c** 1
- 6 **a** 16      **b** -2      **c** -12
- 7  $-1 \times 12$ ,  $1 \times -12$ ,  $-2 \times 6$ ,  $2 \times -6$ ,  $-3 \times 4$ ,  $3 \times -4$ ,
- 8 Any appropriate divisions
- 9 **a** -24      **b** 24 degrees      **c**  $3 \times -6$
- 10  $-56 \div -8$ ,  $-72 \div 4$ ,  $-15 \times 4$ ,  $13 \times -6$
- 11 **a**  $32^\circ\text{F}$  and  $212^\circ\text{F}$       **b**  $-40^\circ\text{C} = -40^\circ\text{F}$

12  $-460^\circ\text{F}$

### Exercise 1J

- 1 **a** -4      **b** -6      **c** 4      **d** 45      **e** 6      **f** 6
- 2 **a** 38      **b** 24      **c** -3      **d** -6      **e** -1      **f** 2  
**g** -25      **h** 25      **i** 0      **j** -20      **k** 4      **l** 0
- 3 **a**  $(3 \times -4) + 1 = -11$       **b**  $-6 \div (-2 + 1) = 6$   
**c**  $(-6 \div -2) + 1 = 4$       **d**  $4 + (-4 \div 4) = 3$   
**e**  $(4 + -4) \div 4 = 0$       **f**  $(16 - -4) \div 2 = 10$
- 4 **a** 49      **b** -1      **c** -5      **d** -12
- 5 **a** 40      **b** 1      **c** 78      **d** 4
- 6 Possible answer:  $3 \times -4 \div 2$
- 7 Possible answer:  $(2 - 4) \times (7 - 3)$
- 8  $(-4)^2 = -4 \times -4 = +16$ ,  $-(-4)^2 = -(4 \times 4) = -16$
- 9  $(5 + 6) - (7 \div 8) \times 9$
- 10 -6

### Review questions

- 1 10 weeks
- 2 16
- 3 270
- 4 **a**  $3^2 \times 5 \times 7$       **b** 63
- 5 **a** 11.412 712 21      **b** 11.4
- 6 **a** 412.603252      **b** 400.5
- 7 **a** iii Prime numbers less than 20  
**b** i 252      ii 3780      iii 18
- 8 **a** 10.663 418 78      **b** 11
- 9 1200
- 10 5
- 11 **a** 3.141 592 92      **b** 0.000 009%
- 12 **a** 7:30 pm Monday (7:45 pm on Town Hall Clock)  
**b** 6:00 pm on Tuesday (7:00 pm on Town Hall clock)
- 13 **a**  $2^4 \times 3^3 \times 5 \times 5$       **b**  $2^2 \times 3$
- 14 **a** 90      **b** 240      **c** 6
- 15 27 and 36
- 16 **a** 2000  
**b** Higher as top values rounded down and denominator rounded up.
- 17 **a**  $p$  and  $q$  are 2 and 5.  $r$  is 3      **b** 15
- 18  $m = 5$ ,  $n = 3$

## Chapter 2 – Number: Fractions, ratio and proportion

### Exercise 2A

- a  $\frac{1}{3}$       b  $\frac{1}{5}$       c  $\frac{2}{5}$       d  $\frac{5}{24}$

e  $\frac{2}{5}$       f  $\frac{1}{6}$       g  $\frac{2}{7}$       h  $\frac{1}{3}$
- $\frac{12}{30} = \frac{2}{5}$
- $\frac{1}{5}$
- $\frac{1}{2}$
- Jon saves  $\frac{30}{90} = \frac{1}{3}$   
Matt saves,  $\frac{35}{100} = \frac{7}{20}$  which is greater than  $\frac{1}{3}$ , so Matt saves the greater proportion of his earnings.
- $\frac{13}{20} = \frac{65}{100}$  and  $\frac{16}{25} = \frac{64}{100}$  so 13 out of 20 is the better mark.
- $\frac{3}{8}$
- $\frac{11}{24}$
- $\frac{3}{7}$
- $\frac{9}{22}$

### Exercise 2B

- a  $\frac{8}{15}$       b  $\frac{7}{12}$       c  $\frac{11}{12}$

d  $\frac{1}{10}$       e  $\frac{1}{8}$       f  $\frac{1}{12}$
- Three-quarters of 68
- a  $4\frac{47}{60}$       b  $\frac{41}{72}$       c  $1\frac{109}{120}$       d  $1\frac{23}{30}$
- a  $\frac{1}{6}$       b 30
- No, one eighth is left, which is 12.5 cl, so enough for one cup but not two cups.
- He has added the numerators and added the denominators instead of using a common denominator. Correct answer is  $3\frac{7}{12}$ .
- Possible answer: The denominators are 4 and 5. I first find a common denominator. The lowest common denominator is 20 because 4 and 5 are both factors of 20. So I am changing the fractions to twentieths. One-quarter is the same as five-twentieths (multiplying numerator and denominator by 5). Two-fifths is the same as eight-twentieths (multiplying numerator and denominator by 4).

Five-twentieths plus eight-twentieths = thirteen-twentieths.

- $\frac{11}{20}$  of 900 = 495,  $\frac{2}{11}$  of 495 = 90 left-handed boys.

900 – 495 = 405 girls.  $\frac{2}{9}$  of 405 = 90 left-handed girls. 180 left-handed students altogether so 180 out of 900 =  $\frac{1}{5}$ .
- $\frac{1}{5} + \frac{3}{8} = \frac{23}{40}$ , so  $\frac{17}{40}$  of the counters are yellow.  $\frac{17}{40}$  of 600 = 255
- a because  $\frac{27}{40} + \frac{2}{5} = 1\frac{3}{40}$  which is greater than 1.

b  $\frac{2}{5}$  of 200 = 80.  $\frac{5}{8}$  of 80 = 50 women at least 40.  $\frac{27}{40}$  of 200 = 135 members at least 40. 135 – 50 = 85 men at least 40.  $\frac{3}{5}$  of 200 = 120, so 120 – 85 = 35 men under 40.
- a  $\frac{1}{5}$  is  $\frac{8}{40}$ .  $\frac{3}{4}$  is  $\frac{30}{40}$ . Half-way between 8 and 30 is 19, so the mid-point fraction is  $\frac{19}{40}$ .

b Yes as the mid-point of any two numbers  $a$  and  $b$  is  $(a + b) \div 2$  and adding the same denominator is the same thing as dividing by 2.

### Exercise 2C

- a  $\frac{1}{6}$       b  $\frac{3}{8}$       c  $\frac{7}{20}$       d  $\frac{3}{5}$

e  $\frac{5}{12}$       f  $2\frac{11}{12}$       g  $3\frac{9}{10}$       h  $3\frac{1}{3}$
- a  $\frac{3}{4}$       b  $1\frac{1}{15}$       c 5      d  $\frac{4}{9}$       e  $1\frac{3}{5}$
- a  $\frac{1}{4}$       b 5      c  $\frac{8}{3}$       d  $\frac{4}{5}$
- a  $-\frac{1}{5}$       b 2      c  $-\frac{9}{7}$       d  $\frac{5}{3}$
- $\frac{3}{8}$
- $\frac{1}{8}$
- 40
- $\frac{2}{5}$  of  $6\frac{1}{2}$
- £10.40
- a  $\frac{9}{32}$       b  $\frac{256}{625}$
- After 1 day  $\frac{7}{8}$  of the water is left. On day 2,  $\frac{1}{8} \times \frac{7}{8} = \frac{7}{64}$  is lost so total lost is  $\frac{1}{8} + \frac{7}{64} = \frac{8}{64} + \frac{7}{64} = \frac{15}{64}$ , so  $1 - \frac{15}{64}$  is left =  $\frac{49}{64}$
- $50 \times 1\frac{1}{2} = 75$  kg.  $120 - 75 = 45$ ,  $45 \div 2\frac{1}{2} = 18$ , so 18 of the  $2\frac{1}{2}$  kg bags are packed.

- 13 a 77% is about  $\frac{3}{4}$ . 243 is about 240, so  $\frac{3}{4}$  of 240 = 180.  
b Lower, as both estimates are lower than the original values.

### Exercise 2D

- 1 a  $1\frac{11}{20}$  b  $1\frac{1}{4}$  c  $1\frac{63}{80}$   
d  $\frac{11}{30}$  e  $\frac{61}{80}$  f  $\frac{167}{240}$
- 2 a  $12\frac{1}{4}$  miles b  $3\frac{1}{4}$  miles
- 3 a  $6\frac{11}{20}$  b  $8\frac{8}{15}$  c  $11\frac{63}{80}$   
d  $3\frac{11}{30}$  e  $7\frac{61}{80}$  f  $4\frac{277}{396}$
- 4 a  $-\frac{77}{1591}$  b Answer is negative
- 5  $18\frac{11}{12}$  cm
- 6  $\frac{5}{12}$  (anticlockwise) or  $\frac{7}{12}$  (clockwise)
- 7 a  $\frac{3}{5}$  b  $\frac{27}{128}$  c  $5\frac{2}{5}$   
d  $5\frac{1}{7}$  e  $3\frac{9}{32}$  f  $\frac{11}{18}$
- 8 a  $8\frac{11}{20}$  b  $65\frac{91}{100}$  c  $52\frac{59}{160}$   
d  $2\frac{17}{185}$  e  $2\frac{22}{103}$  f  $7\frac{881}{4512}$
- 9  $18\frac{5}{12}$  m<sup>2</sup>
- 10 3
- 11 a  $6 \times (1\frac{3}{4})^2 = 18\frac{3}{8}$  cm<sup>2</sup>  
b  $34\frac{14}{25} \div 6 = \frac{144}{25}$ ,  $\sqrt{\frac{144}{25}} = \frac{12}{5} = 2\frac{2}{5}$  cm
- 12  $22 \div (2 \times \frac{22}{7}) = \frac{7}{2}$ ,  $\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 38\frac{1}{2}$  cm<sup>2</sup>
- 13 Volume cuboid =  $22\frac{11}{24}$  cm<sup>3</sup>,  $22\frac{11}{24} \div (\frac{22}{7} \times \frac{4}{3}) = \frac{343}{64}$ ,  $\sqrt[3]{\frac{343}{64}} = 1\frac{3}{4}$  cm
- 14 After 1 day  $\frac{7}{8}$  is left, after two days  $\frac{49}{64}$  and after three days  $\frac{343}{512}$  is left
- 15  $120 \times 4\frac{1}{2} = 540$ .  $175 \times 1\frac{1}{2} = 262\frac{1}{2}$ .  $540 - 262\frac{1}{2} = 277\frac{1}{2}$ .  $277\frac{1}{2} \div 2\frac{1}{2} = 111$  bags.

### Exercise 2E

- 1 a 1.1 b 1.03 c 1.2 d 1.07 e 1.12
- 2 a 0.92 b 0.85 c 0.75 d 0.91 e 0.88
- 3 a 391 kg b 824.1 cm c 253.5 g  
d £143.50 e 736 m f £30.24

- 4 a 731 m b 83.52 g c 360 cm  
d 117 min e 81.7 kg f £37.70
- 5 448
- 6 No, as the total is £101. She will save £20.20, which is less than the £25 it would cost to join the club.
- 7 7% pay rise is an increase of £1925 per year which is better than  $£150 \times 12 = £1800$
- 8 a £6.125 (£6.13)  
b  $x \times 0.025$   
c  $y \div 1.175 \times 1.2$
- 9 Offer A gives 360 grams for £1.40, i.e. 0.388 pence per gram.  
Offer B gives 300 grams for £1.12, i.e. 0.373 pence per gram, so Offer B is the better offer.  
Or Offer A is 360 for 1.40 = 2.6 g/p, offer B is 300 for 1.12 = 2.7 g/p, so offer B is better.
- 10 c Both the same as  $1.05 \times 1.03 = 1.03 \times 1.05$
- 11 a Shop A as  $1.04 \times 1.04 = 1.0816$ , so an 8.16% increase.
- 12 £425.25
- 13  $0.9 \times 1.1 = 0.99$  (99%)
- 14 Area of original circle = 200.96  
Enlarged area =  $200.96 \times 1.6 = 321.536$   
Enlarged radius =  $\sqrt{321.536 \div 3.14} = 10.1192885125$   
% increase =  $2.11928/8 \times 100 = 26.49\%$
- 15 a Let  $r = 10$ . Approx formula gives  $V = 4000$ , actual gives  $V = 4188.79$ ,  $188.79 \div 4188.79 = 0.045$  which is 4.5%  
b The value is lower as  $\frac{4}{3} \times \pi$  is greater than 4 as  $\pi$  is 3.14.

### Exercise 2F

- 1 a 25% b 60.6% c 46.3% d 12.5%  
e 41.7% f 60% g 20.8% h 10%  
i 1.9% j 8.3% k 45.5% l 10.5%
- 2 32%
- 3 6.5%
- 4 33.7%
- 5 a 49.2% b 64.5% c 10.6%
- 6 4.9%
- 7 90.5%
- 8 Stacey had the greater percentage increase.  
Stacey:  $(20 - 14) \times 100 \div 14 = 42.9\%$   
Calum:  $(17 - 12) \times 100 \div 12 = 41.7\%$
- 9 Yes, as 38 out of 46 is over 80% (82.6%)
- 10 Let  $z = 100$ .  $y = 75$ ,  $x = 0.6 \times 75 = 45$ , so  $x$  is 45% of  $z$
- 11 Let  $z$  be 100,  $x = 60$ . If  $x$  is 75% of  $y$ ,  $y = 80$ , so  $y$  is 80% of  $z$ .

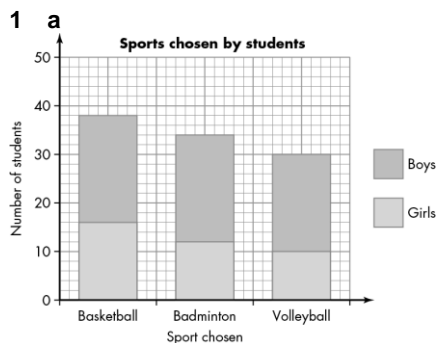
- 12  $30\%$  of  $4800 = 1440$ .  $1.2 \times 4800 = 5760$ .  $70\%$  of  $5760 = 4032$ .  $(4032 - 1440) \div 1440 = 1.8$ , so the increase in numbers owning a mobile phone is  $180\%$ .
- 13  $31 \div 26 = 1.19$  which is a  $19\%$  increase.  $31\%$  is  $5\%$  more of the total votes cast than  $26\%$

### Review questions

- 1 £572
- 2 a 36 seconds  
b i 25.2 seconds ii Eve iii Eve
- 3 £120
- 4 £576
- 5 a £9 b £13.20
- 6 a 0.875 b  $\frac{11}{35}$  c  $5\frac{1}{5}$
- 7 £322
- 8  $\frac{19}{40}$
- 9  $4\frac{1}{12}$
- 10 5
- 11 a  $\frac{54}{17}$ ,  $\frac{312}{99}$ ,  $\frac{22}{7}$ ,  $\frac{221}{71}$  b  $\frac{22}{7}$
- 12  $28\%$
- 13  $77\%$
- 14  $25\%$
- 15 For bag A  $P(\text{red}) = 0.1875$  and for bag B  $P(\text{red}) = 0.186$  so Tomas is wrong.
- 16  $13\%$
- 17 a 150 men, 100 women b  $12\%$

## Chapter 3 – Statistics: Statistical diagrams and averages

### Exercise 3A

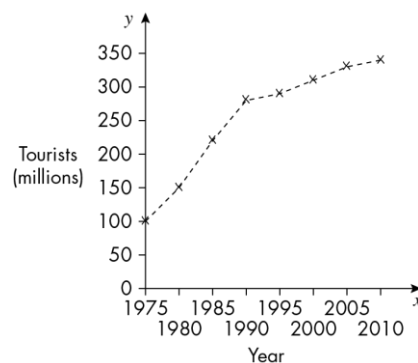


- b 16 c 42

- 2 Pie charts with these angles:  
a  $36^\circ$ ,  $90^\circ$ ,  $126^\circ$ ,  $81^\circ$ ,  $27^\circ$   
b  $90^\circ$ ,  $108^\circ$ ,  $60^\circ$ ,  $78^\circ$ ,  $24^\circ$   
c  $168^\circ$ ,  $52^\circ$ ,  $100^\circ$ ,  $40^\circ$
- 3 a Pictogram with suitable key  
b Bar chart correctly labelled  
c Vertical line chart correctly labelled  
d Pie chart with these angles:  $60^\circ$ ,  $165^\circ$ ,  $45^\circ$ ,  $15^\circ$ ,  $75^\circ$  and correctly labelled  
e Vertical line chart. It shows the frequencies, the easiest one to draw and comparisons can be made.
- 4 a 36  
b Pie charts with these angles:  $50^\circ$ ,  $50^\circ$ ,  $80^\circ$ ,  $60^\circ$ ,  $60^\circ$ ,  $40^\circ$ ,  $20^\circ$   
c Student's bar chart.  
d Bar chart, because easier to make comparisons.
- 5 a Pie charts with these angles:  $124^\circ$ ,  $132^\circ$ ,  $76^\circ$ ,  $28^\circ$   
b Split of total data seen at a glance.
- 6 a  $55^\circ$  b 22 c  $33\frac{1}{3}\%$
- 7 a Pie charts with these angles:  
Strings:  $36^\circ$ ,  $118^\circ$ ,  $126^\circ$ ,  $72^\circ$ ,  $8^\circ$   
Brass:  $82^\circ$ ,  $118^\circ$ ,  $98^\circ$ ,  $39^\circ$ ,  $23^\circ$   
b Overall, the strings candidates did better, as a smaller proportion obtained lower grades. A higher proportion of Brass candidates scored very good grades.
- 8 Work out the angle for 'Don't know' =  $40^\circ$ , so  
 $P(\text{Don't know}) = \frac{40}{360} = \frac{1}{9}$

### Exercise 3B

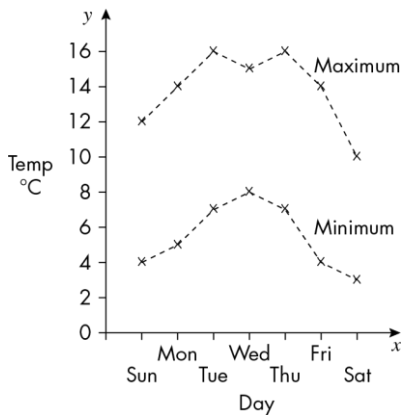
- 1 a



- b About 328 million  
c Between 1980 and 1985  
d Rising steeply at first, but then leveling off. Rise in living standards, cheaper flights, more package holidays

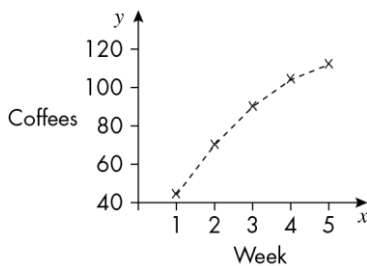


2 a



- b Smallest difference Wednesday and Saturday (7°), greatest difference Friday (10°)

3 a



- b about 120  
 c The same people keep coming back and tell others, but new customers each week become more difficult to find.  
 4 No, you cannot extrapolate the data or the data is likely to change after 5 weeks  
 5 All the temperatures were presumably higher than 20 °C.

### Exercise 3C

- 1 a 47 b 53 c 55 d 65  
 2 Mode  
 3 Three possible answers: 12, 14, 14, 16, 18, 20, 24; or 12, 14, 14, 16, 18, 22, 24; or 12, 14, 14, 16, 20, 22, 24  
 4 a Median (mean could be unduly influenced by results of very able and/or very poor candidates)  
 b Median (mean could be unduly influenced by pocket money of students with very rich or generous parents)  
 c Mode (numerical value of shoe sizes irrelevant, just want most common size)  
 d Median (mean could be distorted by one or two extremely short or tall performers)  
 e Mode (the only way to get an 'average' of non-numerical values)  
 f Median (mean could be unduly influenced by very low weights of premature babies)

- 5 a i £20 000 ii £28 000 iii £34 000  
 b A 6% rise would increase the mean salary to £36 040, a £1500 pay increase would produce a mean of £35 500.

- 6 a Median b Mode c Mean  
 7 Tom – mean, David – median, Mohammed – mode  
 8 11.6  
 9 42.7 kg  
 10 24

### Exercise 3D

- 1 a i 7 ii 6 iii 6.4  
 b i 8 ii 8.5 iii 8.2  
 2 a 1280 b 1.9 c 0 d 328  
 3 a 2.2, 1.7, 1.3 b Better dental care  
 4 a 50 b 2 c 2.8  
 5 a Roger 5, Brian 4 b Roger 3, Brian 8  
 c Roger 5, Brian 4 d Roger 5.4, Brian 4.5  
 e Roger, smaller range f Brian, better mean  
 6 a 40 b 7 c 3 d 2  
 e 2.5 f the mode, 3 g 2.4  
 7 5  
 8 The total frequency could be an even number where the two middle numbers have an odd difference.  
 9 a 34  
 b  $x + 80 + 3y + 104 = 266$ , so  $x + 3y = 82$   
 c  $x = 10$ ,  $y = 24$   
 d 2.5

### Exercise 3E

- 1 a i  $30 < x \leq 40$  ii 29.5  
 b i  $0 < y \leq 100$  ii 158.3  
 c i  $5 < z \leq 10$  ii 9.43  
 d i 7–9 ii 8.41  
 2 a  $100 < m \leq 120$  b 10.86 kg c 108.6 g  
 3 a  $175 < h \leq 200$  b 31% c 193.3 hours  
 d No the mean was under 200 and so was the mode.  
 4 24  
 5 a Yes, average distance is 11.7 miles per day.  
 b Because shorter runs will be run at a faster speed, which will affect the average.  
 c Yes, because the shortest could be 1 mile, the longest 25 miles.  
 6 Soundbuy; average increases are Soundbuy 17.7p, Springfield 18.7p, Setco 18.2p  
 7 a 160 b 52.6 minutes  
 c Modal group d 65%  
 8 The first 5 and the 10 are the wrong way round.

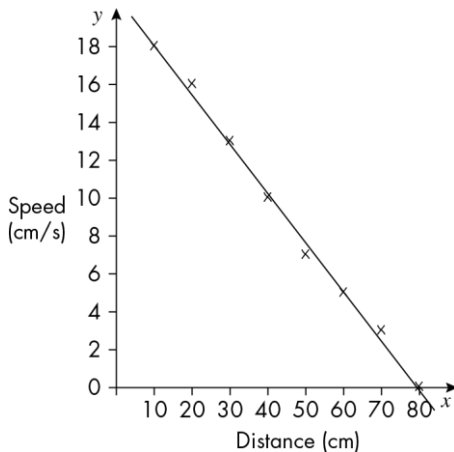
9 Find the midpoint of each group, multiply that by the frequency and add those products. Divide that total by the total frequency.

- 10 a Yes, as total in first two columns is 50, so median is between 39 and 40.  
b He could be correct, as the biggest possible range is  $69 - 20 = 49$ , and the lowest is  $60 - 29 = 31$ .

### Exercise 3F

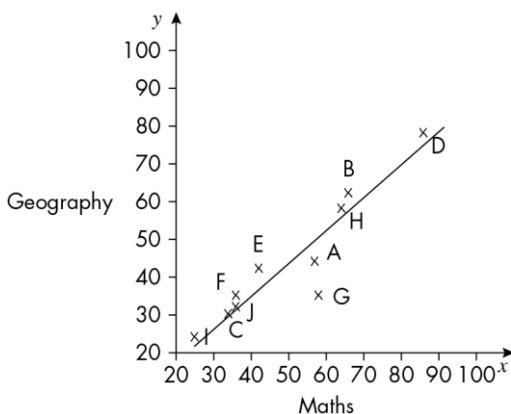
- 1 a good positive correlation, time taken increases with the number of press-ups  
b strong negative correlation, you complete a crossword more quickly as you get older  
c No correlation, speed of cars on M1 is not related to the temperature  
d weak, positive correlation, older people generally have more money saved in the bank

2 a and b



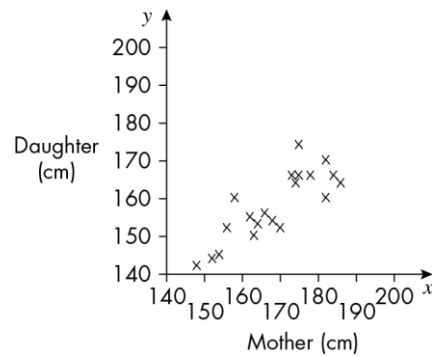
- c about 19 cm/s  
d about 34 cm

3 a and b



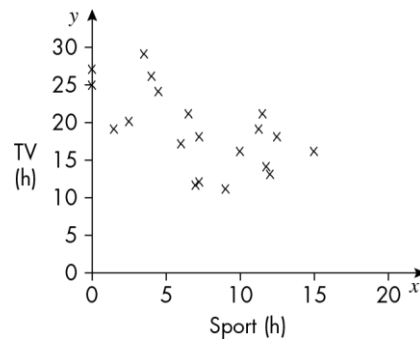
- c Greta  
d about 70  
e about 70

4 a



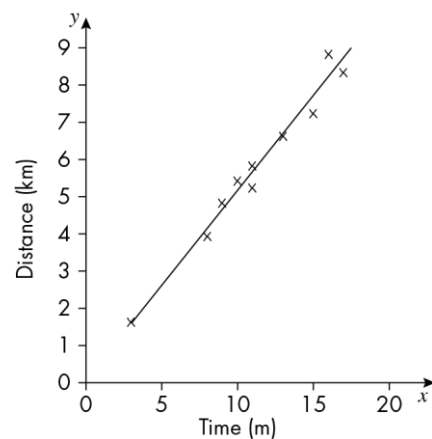
b Yes, as good positive correlation

5 a



b Little correlation, so cannot draw a line of best fit or predict the value

6 a and b



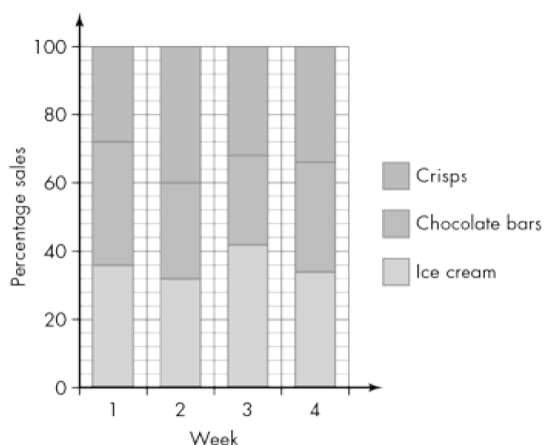
- c about 2.4 km  
d about 8 minutes  
e you cannot extrapolate values from a scatter diagram or the data may change for longer journeys

7 about 23 mph

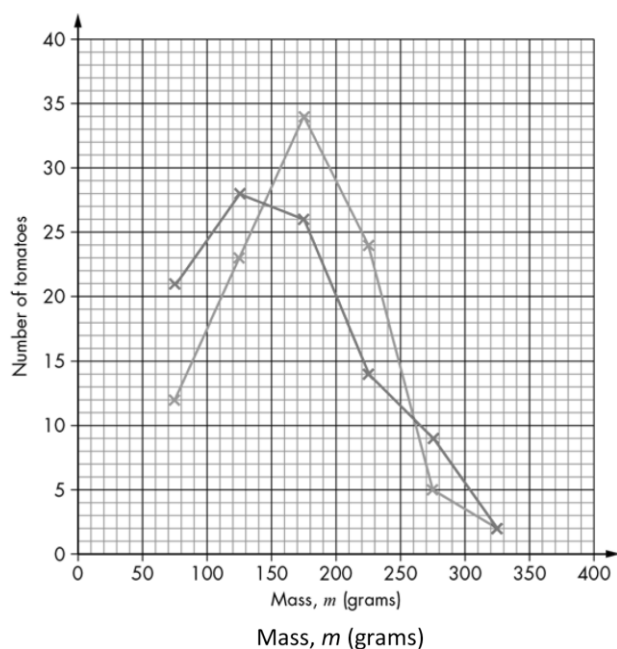
8 Points showing a line of best fit sloping down from top left to bottom right

## Review questions

- 1 a Grade 7  
b  $\frac{100}{360}$  or  $\frac{5}{18}$   
c i 48    iii 216  
d e.g. pie charts show proportions or they are percentages, not actual numbers or do not know how many students, etc.
- 2 43.7 matches
- 3 a  $10 < t \leq 20$   
b  $10 < t \leq 20$   
c 19 minutes
- 4 a because over half the students have more than £10 pocket money, so the mean must be more than £10  
b £11.13
- 5



- 6 a  $100 < m \leq 150$   
b  $150 < m \leq 200$   
c 159 g  
d



e

mass, $m$ (grams)	Margot's tomatoes
$50 < m \leq 100$	12
$100 < m \leq 150$	23
$150 < m \leq 200$	34
$200 < m \leq 250$	24
$250 < m \leq 300$	5
$300 < m \leq 350$	2

f

mass, $m$ (grams)	Margot's tomatoes	mid point $x$	$x \times m$
$50 < m \leq 100$	12	75	900
$100 < m \leq 150$	23	125	2875
$150 < m \leq 200$	34	175	5950
$200 < m \leq 250$	24	225	5400
$250 < m \leq 300$	5	275	1375
$300 < m \leq 350$	2	325	650
totals	100		17150

estimate for the mean = 171.5 g

- g on average Tom's tomatoes had a smaller mass and were therefore probably smaller in size
- 7 a i Diagram C    ii Diagram A    iii Diagram B  
b Diagram A: strong negative correlation, diagram B: no correlation, diagram C: strong positive correlation
- 8 a/b Student's graph as follows: Time on horizontal axis from 0 to 20 and Distance (km) on vertical axis from 0 to 10 with the following points plotted: (3, 1.7) (17, 8.3) (11, 5.1) (13, 6.7) (9, 4.7) (15, 7.3) (8, 3.8) (11, 5.7) (16, 8.7) (10, 5.3) and with line of best fit drawn.  
c/d answers depend on student's line of best fit

## Chapter 4 – Algebra: Number and sequences

### Exercise 4A

- 1 a  $11111 \times 11111 = 123\,454\,321$ ,  
 $111111 \times 111111 = 12\,345\,654\,321$   
b  $99999 \times 99999 = 9\,999\,800\,001$ ,  
 $999999 \times 999999 = 999\,998\,000\,001$
- 2 a  $7 \times 8 = 7^2 + 7$ ,  $8 \times 9 = 8^2 + 8$   
b  $50 \times 51 = 2550$ ,  $60 \times 61 = 3660$
- 3 a  $1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25 = 5^2$ ,  
 $1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1 = 36 = 6^2$   
b  $21 + 23 + 25 + 27 + 29 = 125 = 5^3$ ,  
 $31 + 33 + 35 + 37 + 39 + 41 = 216 = 6^3$
- 4 a  $1 + 6 + 15 + 20 + 15 + 6 + 1 = 64$ ,  
 $1 + 7 + 21 + 35 + 35 + 21 + 7 + 1 = 128$   
b  $12\,345\,679 \times 45 = 555\,555\,555$ ,  
 $12\,345\,679 \times 54 = 666\,666\,666$
- 5 a  $1^3 + 2^3 + 3^3 + 4^3 = (1 + 2 + 3 + 4)^2 = 100$ ,  
 $1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1 + 2 + 3 + 4 + 5)^2 = 225$   
b  $36^2 + 37^2 + 38^2 + 39^2 + 40^2 = 41^2 + 42^2 + 43^2 + 44^2$

$$55^2 + 56^2 + 57^2 + 58^2 + 59^2 + 60^2 = 61^2 + 62^2 + 63^2 + 64^2 + 65^2$$

- 6 a 12 345 678 987 654 321  
b 999 999 998 000 000 001  
c  $12^2 + 12$   
d 8190  
e  $81 = 9^2$   
f  $512 = 8^3$   
g 512  
h 999 999 999  
i  $(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9)^2 = 2025$
- 7  $1 + 500 = 501$ ,  $2 + 499 = 501$ , ...,  $250 + 251 = 501$ ,  
 $250 \times 501 = 125250$

### Exercise 4B

- 1 a 21, 34: add previous 2 terms  
b 49, 64: next square number  
c 47, 76: add previous 2 terms
- 2 15, 21, 28, 36
- 3 61, 91, 127
- 4  $\frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{5}{7}, \frac{3}{4}$
- 5 a 6, 10, 15, 21, 28  
b It is the sums of the natural numbers, or the numbers in Pascal's triangle or the triangular numbers.
- 6 a 2, 6, 24, 720      b 69!
- 7 364: Daily totals are 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78 (these are the triangular numbers). Cumulative totals are: 1, 4, 10, 20, 35, 56, 84, 120, 165, 220, 286, 364.
- 8 X. There are 351 ( $1 + 2 + \dots + 25 + 26$ ) letters from A to Z.  $3 \times 351 = 1053$ .  $1053 - 26 = 1027$ ,  $1027 - 25 = 1002$ , so, as Z and Y are eliminated, the 1000th letter must be X.
- 9 29 and 41
- 10 No, because in the first sequence, the terms are always one less than in the 2nd sequence
- 11  $4n - 2 = 3n + 7$  rearranges as  $4n - 3n = 7 + 2$ , so  $n = 9$

### Exercise 4C

- 1 a 13, 15,  $2n + 1$       b 25, 29,  $4n + 1$   
c 33, 38,  $5n + 3$       d 32, 38,  $6n - 4$   
e 20, 23,  $3n + 2$       f 37, 44,  $7n - 5$   
g 21, 25,  $4n - 3$       h 23, 27,  $4n - 1$   
i 17, 20,  $3n - 1$       j  $-8, -18, 42 - 10n$   
k 4, 0,  $24 - 4n$       l  $-1, -6, 29 - 5n$
- 2 a  $3n + 1$ , 151      b  $2n + 5$ , 105      c  $5n - 2$ , 248  
d  $4n - 3$ , 197      e  $8n - 6$ , 394      f  $n + 4$ , 54  
g  $5n + 1$ , 251      h  $8n - 5$ , 395      i  $3n - 2$ , 148  
j  $3n + 18$ , 168      k  $47 - 7n$ ,  $-303$       l  $41 - 8n$ ,  $-359$
- 3 a 33rd      b 30th      c 100th = 499
- 4 a i  $4n + 1$       ii 401      iii 101, 25th

- b i  $2n + 1$       ii 201  
iii 99 or 101, 49th and 50th
- c i  $3n + 1$       ii 301      iii 100, 33rd
- d i  $2n + 6$       ii 206      iii 100, 47th
- e i  $4n + 5$       ii 405      iii 101, 24th
- f i  $5n + 1$       ii 501      iii 101, 20th
- g i  $3n - 3$       ii 297      iii 99, 34th
- h i  $6n - 4$       ii 596      iii 98, 17th
- i i  $205 - 8n$       ii  $-595$       iii 101, 13th
- j i  $227 - 2n$       ii 27      iii 99 or 101, 64th and 63rd

- 5 a  $\frac{2n+1}{3n+1}$   
b Getting closer to  $\frac{2}{3}$  (0.6)  
c i 0.667 774 (6dp)      ii 0.666 778 (6dp)  
d 0.666 678 (6dp), 0.666 667 (6dp)
- 6 a  $\frac{4n-1}{5n+1}$   
b Getting closer to  $\frac{4}{5}$  (0.8)  
c i 0.796 407 (6dp)      ii 0.799 640 (6dp)  
d 0.799 964 (6dp), 0.799 9996 (7dp)
- 7 a £305      b £600      c 3      d 5
- 8 a  $\frac{3}{4}, \frac{5}{7}, \frac{7}{10}$   
b i 0.666 666 777 8      ii  $\frac{2}{3}$   
c For  $n$ ,  $\frac{2n+1}{3n+1} \approx \frac{2n}{3n} = \frac{2}{3}$
- 9 a  $8n + 2$       b  $8n + 1$       c  $8n$       d £8
- 10 a Sequence goes up in 2s; first term is  $2 + 29$   
b  $n + 108$   
c Because it ends up as  $2n \div n$   
d 79th
- 11 If there was a common term then for some value of  $n$  the expressions would be equal i.e.  $2n = 2n - 1$ , Subtracting  $2n$  from both sides gives  $0 = -1$ , which is impossible.
- 12 Difference is  $19 - 10 = 9$ .  $9 \div 3 = 3$  so  $A = 3$ .  $3 \times 5 + b = 10$ ,  $b = -5$

### Exercise 4D

- 1 a Even,  

+	Odd	Even
Odd	Even	Odd
Even	Odd	Even

b Odd,  

x	Odd	Even
Odd	Odd	Even
Even	Even	Even
- 2 a  $1 + 3 + 5 + 7 = 16 = 4^2$ ,  $1 + 3 + 5 + 7 + 9 = 25 = 5^2$   
b i 100      ii 56
- 3 a 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144  
b because odd + odd = even, odd plus even = odd and even + odd = odd.  
c i  $a + 2b$ ,  $2a + 3b$ ,  $3a + 5b$ ,  $5a + 8b$ ,  $8a + 13b$

ii coefficient of a odd and b even, a even and b odd, both odd

- 4 a Even b Odd c Odd  
d Odd e Odd f Odd  
g Even h Odd i Odd

- 5 a Odd or even b Odd or even  
c Odd or even d Odd  
e Odd or even f Even

- 6 a i Odd ii Even iii Even  
b Any valid answer, e.g.  $x(y+z)$

- 7 a 64, 128, 256, 512, 1024  
b i  $2^n - 1$  ii  $2^n + 1$  iii  $3 \times 2^n$

- 8 a The number of zeros equals the power.  
b 6  
c i  $10^n - 1$  ii  $2 \times 10^n$

- 9 a 125, 216  
b  $1 + 8 = 9$ ,  $1 + 8 + 27 = 36$ ,  $1 + 8 + 27 + 64 = 100$ ... the answers are square numbers

- 10 a 28, 36, 45, 55, 66  
b i 210 ii 5050  
c You get the square numbers.

- 11 a i If  $n$  is odd,  $n + 1$  is even.  
If  $n$  is even,  $n + 1$  is odd.  
Even times odd is always even.  
ii  $2n$  must be even, so  $2n + 1$  must be odd.

- b Odd  
Odd  
Even  
Even  
Odd

- c  $(2n + 1)^2 = 4n^2 + 4n + 1$   
or  $(2n)^2 = 4n^2$   
 $4n^2 + 4n$  is even so adding 1 makes it odd  
 $4n^2$  is  $2 \times 2n^2$  which is even

- 12 11th triangular number is 66, 18th triangular number is 171

- 13 a 36, 49, 64, 81, 100  
b i  $n^2 + 1$  ii  $2n^2$  iii  $n^2 - 1$

- 14 a 6, 24, 96, 384, 1536  
b 21, 147, 1029, 7203, 50 421  
c 2, 10, 50, 250, 1250  
d 6, 60, 600, 6000, 60 000  
e 54, 162, 486, 1458, 4374

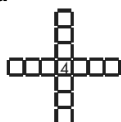
- 15 a  $3 \times 2^{n-1}$  b  $5 \times 4^{n-1}$  c  $20 \times 5^{n-1}$  or  $4 \times 5^n$   
d  $21 \times 3^{n-1}$  or  $7 \times 3^n$  e  $24 \times 8^{n-1}$  or  $3 \times 8^n$

- 16 2 as all other primes are odd, so the sum of two of them will be even, so could not be a prime.

- 17 a There are many answers,  $5 + 31 = 36$ ,  $7 + 29 = 36$ ,  $2 + 47 = 49$  etc.  
b There are many answers,  $49 - 36 = 13$ ,  $81 - 64 = 17$

#### Exercise 4E

- 1 a



- b  $4n - 3$   
c 97  
d 50th diagram



- 2 a  
b  $2n + 1$   
c 121 d 49th set

- 3 a 18 b  $4n + 2$  c 12

- 4 a i 24 ii  $5n - 1$  iii 224  
b 25

- 5 a 5, 8, 11, 14  
b i 20 cm ii  $(3n + 2)$  cm iii 152 cm  
c 332

- 6 a i 20 ii 162  
b 79.8 km

- 7 a i 14 ii  $3n + 2$  iii 41  
b 66

- 8 a i 5 ii  $n$  iii 18  
b 20 tins

- 9 a  $2^n$   
b i  $100 \times 2^{n-1}$  ml ii 1600 ml  
c Next sizes after super giant are 3.2l, 6.4l and 12.8l with weights of 3.2 kg, 6.4 kg and 12.8 kg, so the largest size is 6.4 litres.

- 10 The  $n$ th term is  $\left(\frac{3}{4}\right)^n$ , so as  $n$  gets very large, the unshaded area gets smaller and smaller and eventually it will be zero; so the shaded area will eventually cover the triangle.

- 11 Yes, as the number of matches is 12, 21, 30, 39, ... which is  $9n + 3$ ; so he will need  $9 \times 20 + 3 = 183$  matches for the 20th step and he has  $5 \times 42 = 210$  matches.

- 12 a 20 b 120

- 13 Alex's answer gives  $4(n + 2) = 4n + 8$   
Colin's method gives  $4n + 4$   
Ed's method gives  $4(n + 1) = 4n + 4$   
Gail's method gives  $2 \times n + 2(n + 2) = 2n + 2n + 4 = 4n + 4$   
Linear sequence is 8 12 16 20 .... Which has an  $n$ th term of  $4n + 4$  so they are all valid methods except for Alex who forgot that the corners overlap and should have taken the 4 overlapping corners away to get  $4n + 8 - 4 = 4n + 4$

#### Exercise 4F

- 1 a i 34, 43 ii goes up 3, 4, 5, 6, etc.  
b i 24, 31 ii goes up 1, 2, 3, 4, etc.  
c i 54, 65 ii goes up 5, 6, 7, 8, etc.  
d i 57, 53 ii goes down 10, 9, 8, 7, etc.

- 2 a 4, 7, 12, 19, 28 b 2, 8, 18, 32, 50  
c 2, 6, 12, 20, 30 d 4, 9, 16, 25, 36  
e 2, 8, 16, 26, 38 f 4, 7, 14, 25, 40

- 3 a  $2n + 1$  b  $n$   
c  $n(2n + 1) = 2n^2 + n$   
d  $2n^2 + n + 1$

- 4 a  $n$  b  $n + 1$   
c  $n(n + 1)$  d 9900 square units
- 5 a Yes, constant difference is 1 b No  
c Yes, constant difference is 2 d No  
e Yes, constant difference is 1 f No
- 6 a  $4n + 4$  b  $n^2$   
c  $n^2 + 4n + 4$  d  $n^2 + 4n + 4$   
e The sides of the large squares are of length  $n + 2$  so the total number of squares is  $(n + 2)^2$  which is the same answer as c.
- 7 a Table 10, 15, 21; 6, 10, 15; 16, 25, 36  
b i 45 ii 100
- 8  $n^2 + 2n - 3 = n^2 + n + 3$ , gives  $n = 6$ . Substituting gives 45 for both expressions.
- 9 a Sequences are 4, 7, 14, 25, 40, 59, 82, ... and 4, 11, 20, 31, 44, 59, 76, ... so 59 is the next common term.  
b 59 is the 6th term in each sequence so substitute 6 into each expression. This will give 59 in both cases.
- 10 a There are many answers, for example  $a = -3$  and  $b = 1$ .  
b The only solution is  $c = 2$  and  $d = -3$
- 11 All values of  $n$  from 1 to 39 give a prime number.  $n = 40$  gives 1681 which equals  $41 \times 41$
- 12 a  $(n + 1)(n - 1) = n^2 + n - n - 1 = n^2 - 1$   
b  $n^2 - 1$  as  $50 \times 50 - 1$  is easy to work out but  $51 \times 49$  isn't  
c  $(n + 1)(n - 1)$  as  $100 \times 98$  is easy to work out but  $99^2 - 1$  isn't.

#### Exercise 4G

- 1 a i 36, 49 ii  $n^2$   
b i 35, 48 ii  $n^2 - 1$   
c i 38, 51 ii  $n^2 + 2$   
d i 39, 52 ii  $n^2 + 3$   
e i 34, 47 ii  $n^2 - 2$   
f i 35, 46 ii  $n^2 + 10$
- 2 a i 37, 50 ii  $(n + 1)^2 + 1$   
b i 35, 48 ii  $(n + 1)^2 - 1$   
c i 41, 54 ii  $(n + 1)^2 + 5$   
d i 50, 65 ii  $(n + 2)^2 + 1$   
e i 48, 63 ii  $(n + 2)^2 - 1$
- 3 a i  $n^2 + 4$  ii 2504  
b i  $3n + 2$  ii 152  
c i  $(n + 1)^2 - 1$  ii 2600  
d i  $n(n + 4)$  ii 2700  
e i  $n^2 + 2$  ii 2502  
f i  $5n - 4$  ii 246
- 4 a  $2n^2 - 3n + 2$  b  $3n^2 + 2n - 3$   
c  $\frac{1}{2}n^2 + \frac{5}{2}n + 1$  d  $\frac{1}{2}n^2 + 4\frac{1}{2}n - 2$   
e  $\frac{1}{2}n^2 + 1\frac{1}{2}n + 6$  f  $\frac{1}{2}n^2 + 1\frac{1}{2}n + 2$
- 5  $6n^2$
- 6 a 26 b  $1\frac{1}{2}n^2 + \frac{1}{2}n$  c 8475
- 7 a 45

b  $n$ th term is  $\frac{1}{2}n^2 + \frac{1}{2}n$  so  $\frac{1}{2} \times 15 \times 15 + \frac{1}{2} \times 15 = 120$ , so no.

- 8 Front face is  $n^2$ , sides faces are  $n \times (n + 1) = n^2 + n$  so total surface area is  $2 \times n^2 + 4 \times (n^2 + n) = 6n^2 + 4n$ .
- 9 Sequence is 1, 7, 19, 37.  $n$ th term is  $3n^2 - 3n + 1$  so the 100th hexagonal number is 29 701.
- 10 a Taking the height first. There are  $n + 1$  strips  $m$  feet long. That is  $m(n + 1)$  in total.  
Taking the width. There are  $m + 1$  strips  $n$  feet long. That is  $n(m + 1)$  in total  
 $m(n + 1) + n(m + 1) = mn + m + mn + n = 2mn + m + n$   
b Taking the nails across a width strip. There are  $n + 1$  lots of 2 nails which is  $2(n + 1)$ . There are  $m + 1$  width strips, so the total is  $2(n + 1)(m + 1)$ .

#### Review questions

- 1 No. Sequence is 7, 10, 13, 16, 19, 22, 25, 28, ... so the first 3 odd terms are prime but 25 is not prime.
- 2 a  $4n + 1$   
b Not odd  
c 28th term is 113
- 3  $n$ th term is  $5n + 1$ .  $5 \times 150 + 1 = 751$
- 4 a  $6n + 3$   
b No,  $3n + 2$  generates the sequence 5, 8, 11, 14, 17, 20, 23, ... so the even terms of this sequence are always 1 less than the terms of the original sequence
- 5 a  $2 \times 3^{n-1}$  b Not an even number
- 6 a  $5 \times 6^{n-1}$  b 8th term is 1 399 680
- 7 a The first five terms in the sequence are -27, -21, -11, 3, 21. Of these terms, 3 is a prime number.  
b When  $n^2 = 29$ , the expression can be factorised as  $29(2 \times 29 - 1)$  so is not a prime number
- 8 a 4, 9, 18, 31, 48 b 2, 2, 3, 5, 8
- 9  $n = 1$   $(n - 1) = 0$ ,  $n = 2$   $(n - 2) = 0$ ,  $n = 3$   $2(3 - 1) = 2$   
 $(3 - 2) \div 5 = 2 \times 2 \times 1 \div 5 = 0.8$
- 10  $2n^2 - 2n + 3$
- 11 a  $n$ th term is  $n^2 + 2n$ ,  $12 \times 12 + 2 \times 12 = 168$ , so yes enough squares  
b  $40 \times 40 + 2 \times 40 = 1680$
- 12  $2n^2 - n$ ,  $2 \times 20^2 - 20 = 780$
- 13 The sequence of dots is 5, 15, 30, 50, ...

$n$	0	1	2	3	4
$c$	0	5	15	30	50
$a + b$	5	10	15	20	
$2a$	5	5	5		

$$a = 2\frac{1}{2}, b = 2\frac{1}{2} \text{ and } c = 0,$$

so the  $n$ th term is  $2\frac{1}{2}n^2 + 2\frac{1}{2}n$

$$2\frac{1}{2} \times 50^2 + 2\frac{1}{2} \times 50 = 6375$$

## Chapter 5 – Ratio and proportion and rates of change: Ratio and proportion

### Exercise 5A

- 1 a 1 : 3    b 3 : 4    c 2 : 3    d 2 : 3  
e 2 : 5    f 2 : 5    g 5 : 8    h 25 : 6

- 2 a 1 : 3    b 3 : 2    c 5 : 12  
d 8 : 1    e 17 : 15    f 25 : 7  
g 4 : 1    h 5 : 6    i 1 : 24

3  $\frac{7}{10}$

4  $\frac{2}{5}$

5 a  $\frac{2}{5}$     b  $\frac{3}{5}$

6 7 : 3

7 2 : 1

- 8 a Fruit crush  $\frac{1}{6}$ , lemonade  $\frac{5}{6}$

Fruit Crush	1.25	1	0.2	0.4	0.5
Lemonade	6.25	5	1	2	2.5

- b 0.4 litres    c 2.5 litres

9 a  $\frac{1}{2}$     b  $\frac{7}{20}$     c  $\frac{3}{20}$

10 Sugar  $\frac{5}{22}$ , flour  $\frac{3}{11}$ , margarine  $\frac{2}{11}$ , fruit  $\frac{7}{22}$

11 4

12 1 : 4

13 a 5 : 3 : 2    b 20

14  $13\frac{1}{2}$  litres

15 1 : 1 : 1

### Exercise 5B

- 1 a 160 g, 240 g    b 80 kg, 200 kg  
c 150, 350    d 950 m, 50 m  
e 175 min, 125 min    f £20, £30, £50  
g £36, £60, £144    h 50 g, 250 g, 300 g  
i £1.40, £2, £1.60    j 120 kg, 72 kg, 8 kg

2 a 175    b 30%

3 a 40%    b 300 kg

4 21

- 5 a Mott: no, Wright: yes, Brennan: no, Smith: no, Kaye: yes  
b For example: W26, H30; W31, H38; W33, H37

- 6 a 1 : 400 000    b 1 : 125 000    c 1 : 250 000  
d 1 : 25 000    e 1 : 20 000    f 1 : 40 000  
g 1 : 62 500    h 1 : 10 000    i 1 : 60 000

7 a  $\frac{1}{2}$  km or 500m

- b 78 cm  $\approx$  39 km.  $39 \div 15 \approx 2.6$ . 2.6 hours = 2 h 36 m. Plus 30 mins is 3 h 06 m so he should be back at about 12.06 pm

- 8 a Map A 1 : 250 000, Map B 1 : 1 000 000  
b 2 km  
c 1.2 cm  
d 4.8 cm

- 9 a 1 : 1.6    b 1 : 3.25    c 1 : 1.125  
d 1 : 1.44    e 1 : 5.4    f 1 : 1.5  
g 1 : 4.8    h 1 : 42    i 1 : 1.25

- 10 Diesel : Petrol = 60 : 90.  $\frac{1}{5}$  of 60 = 12.  $\frac{4}{9}$  of 90 = 40. Total red cars = 52 which is more than  $150 \div 3 = 50$  so Yes.

- 11 a 4 : 3    b 90 miles  
c Both arrive at the same time.

12 0.4 metres

13  $13 - 9 = 4$ .  $4 \div 5 = 0.8$ .  $2 \times 0.8 = 1.6$ ,  $9 + 1.6 = 10.6$

- 14 Athos has 3 more parts than Zena.  $24 \div 3 = 8$ , so 1 part is 8. Zena has 8 marbles.

### Exercise 5C

1 a 3 : 2    b 32    c 80

2 a 100    b 160

3 0.4 litres

4 Jamie has 1.75 pints, so he has enough.

5 8100

6 296

7 Kevin £2040, John £2720

8 b C    c F    d T    e T

9 51

10 100

11 40 ml

12 a 160 cans    b 48 cans

- 13 a Lemonade 20 litres, ginger 0.5 litres  
b This one, in part a there are 50 parts in the ratio 40 : 9 : 1, so ginger is  $\frac{1}{50}$  of total amount; in part b there are 13 parts in the ratio 10 : 2 : 1, so ginger is  $\frac{1}{13}$  of total amount.  $\frac{1}{13} > \frac{1}{50}$

- 14 a Will as his multiple of 10 is also a multiple of 9  
b Zeke has rounded off to 1 dp and and Yoko has rounded off to 2 dp. They have not used a recurring decimal notation.

15 54

### Exercise 5D

- 1 60 g
- 2 £5.22
- 3 45
- 4 £6.72
- 5 a £312.50      b 8
- 6 a 56 litres      b 350 miles
- 7 a 300 kg      b 9 weeks
- 8 40 seconds
- 9 a i 100 g, 200 g, 250 g, 150 g  
ii 150 g, 300 g, 375 g, 225 g  
iii 250 g, 500 g, 625 g, 375 g  
b 24
- 10 I can buy four packs (24 sausages) from Peter (£9.20)  
I can only buy two packs (20 sausages) from Paul (£7)  
I should use Peter's shop to get the most sausages for £10.
- 11  $400 \div 10 = 40$  loaves needed.  $1.8 \text{ kg} \div 3 = 0.6 \text{ kg}$  per loaf, so  $40 \times 0.6 = 24 \text{ kg}$  of flour.
- 12 4 buns and 5 cakes
- 13 11 minutes 40 seconds + 12 minutes = 23 minutes 40 seconds
- 14 Possible answer:  
30 g plain flour (rounding to nearest 10 g)  
60 ml whole milk (rounding to nearest 10 ml)  
1 egg (need an egg)  
1 g salt (nearest whole number)  
10 ml beef dripping or lard (rounding to nearest 10 ml)
- 15 30 litres

### Exercise 5E

- 1 a £4.50 for a 10-pack  
b £1.08 for 6  
c £2.45 for 1 litre  
d Same value
- 2 a Large jar as more g per £  
b 75 ml tube as more ml per £  
c Large box as more g per £  
d 400 ml bottle as more ml per £
- 3 a £5.11  
b Large tin (small £5.11/l, medium £4.80/l, large £4.47/l)
- 4 a 95p      b Family size
- 5 Mary
- 6 Kelly
- 7 12-pack  $360 \div 12 = 30\text{p}$  per sachet. 40-pack  $1150 \div 40 = 28.75\text{p}$  per sachet. 4 sachets cost  $4 \times 35 = £1.40$  but you get 5, so  $140 \div 5 = 28\text{p}$  per sachet, so the offer is the best value.

- 8 a Abe uses  $10 \times 0.75 = 7.5$  litres to do 100 km.  
Caryl uses  $100 \div 14 = 7.14$  litres to do 100 km  
and Des uses  $100 \div (55 \times 1.6) \times 4.55 = 5.17$  litres to do 100 km, so Des's car is the most economical.  
b It does not give a 'unit' value, ie miles per gallon or litres per mile.

### Exercise 5F

- 1 a £260      b £307.50      c £323.75      d £289
- 2 a £7.50      b £9.05      c £5.80      d £10.75
- 3 a 38 h      b  $41\frac{1}{2}$  h      c 35 h      d 40 h
- 4 a Fewer hours      b More pay
- 5 a £540      b £702
- 6 £6.90
- 7  $375 - 330 = 45$ ,  $45 \div 6 = £7.50$ .  $(375 - 12 \times 7.50) \div 7.50 = 38$  hours
- 8  $£1\frac{1}{4}x$
- 9 Pay is £442.50 tax is £88.50, NI is  $442.50 - 88.50 - 327.45 = 26.55$ ,  $26.55 \div 442.5 = 0.06$ , so the NI rate is 6%
- 10 407 factorises to  $1 \times 407$  or  $11 \times 37$ , so Jeff works 37 hours a week at £11 per hour.

### Exercise 5G

- 1 18 mph
- 2 280 miles
- 3 52.5 mph
- 4 11:50 am
- 5 500 seconds
- 6 a 75 mph      b 6.5 h  
c 175 miles      d 240 km  
e 64 km/h      f 325 km  
g 4.3 h (4 h 18 min)
- 7 a 2.25 h      b 99 miles
- 8 a 1.25 h      b 1 h 15 min
- 9 a Sheffield to London via Midland mainline 74.38 mph. Sheffield to London via East Coast mainline 78.26 mph, including the wait at Doncaster  
b Doncaster to London 94.12 mph
- 10 a 120 km      b 48 km/h
- 11 6 mph
- 12 a 10 m/s      b 3.3 m/s      c 16.7 m/s
- 13 a 90 km/h      b 43.2 km/h      c 1.8 km/h
- 14 18 m/s is 64.8 km/h. 40 km at 64.8 km/h is  $0.617$  hours  $\approx 37$  minutes so train arrives at 8.07 am



15

Time	10	10.15	10.30	10.45	11
Ajeet	16	20	24	28	32
Bijay	0	6	12	18	24
Time	11.15	11.30	11.45	12	12.15
Ajeet	36	40	44	48	52
Bijay	30	36	42	48	54

Bijay catches Ajeet at 12 noon

- 6 Rebecca: 10 minutes at 50 mph covers 8.333 miles, 10 minutes at 70 mph covers 11.666 miles, so total distance is 20 miles in 20 minutes which is 60 mph, so Rebecca is correct.  
Nick: 10 miles at 40 mph takes 15 minutes, 10 miles at 60 mph takes 10 minutes, so total distance is 20 miles in 25 minutes, which is 48 mph, so Nick is wrong.
- 17 Josh should take 40 minutes. Nell should take  $50 \div 70 \times 60 = 42$  minutes, but Josh is likely to meet traffic through town so is unlikely to travel at anywhere near 30 mph. Nell is likely to be able to travel at 70 mph on the motorway.

### Exercise 5H

- a  $0.75 \text{ g/cm}^3$
- 4 pa
- $8\frac{1}{3} \text{ g/cm}^3$
- $2\frac{1}{2} \text{ N}$
- 32 g
- $5 \text{ m}^2$
- $120 \text{ cm}^3$
- 156.8 g
- $30 \times 20$
- By the handle as smaller area
- So they can walk on sand easier due to less pressure on the surface.
- a 19.3 kg  
b 19.3 kg. Mass is same  
c On largest face 965 Pa, On smallest face 3860 Pa
- First statue is the fake as density is approximately  $26 \text{ g/cm}^3$
- Second piece by  $1 \text{ cm}^3$
- $0.339 \text{ m}^3$
- Areas are  $\frac{1}{2} \text{ m}^2$ .  $0.8 \text{ m}^2$ .  $0.4 \text{ m}^2$ . Sides are 1 m,  $\frac{1}{2} \text{ m}$  and 0.8 m
- a T      b F      c F      d T

### Exercise 5I

- a £400      b £112.50  
c £12.80      d £499.46
- a 8 years      b 12 years
- a i 10.5 g  
ii 11.03 g  
iii 12.16 g  
iv 14.07 g  
b 9 days
- 12 years
- a £14 272.27      b 20 years
- a i 2550  
ii 2168  
iii 1331  
b 7 years
- a £6800      b £5440      c £3481.60
- a i 1.9 million litres  
ii 1.6 million litres  
iii 1.2 million litres  
b 10th August
- a i 51 980  
ii 84 752  
iii 138 186  
b 2021
- a 21 years      b 21 years
- 3 years
- 30 years
- $1.1 \times 1.1 = 1.21$  (21% increase)
- Bradley Bank account is worth £1032, Monastery Building Society account is worth £1031.30, so Bradley Bank by 70p.
- 4 months: fish weighs  $3 \times 1.14 = 4.3923 \text{ kg}$ ; crab weighs  $6 \times 0.94 = 3.9366 \text{ kg}$
- 4 weeks

### Exercise 5J

- a 800 g      b 250 m      c 60 cm  
d £3075      e £200      f £400
- 80
- T-shirt £8.40, Tights £1.20, Shorts £5.20, Sweater £10.75, Trainers £24.80, Boots £32.40
- £833.33
- £300
- 240
- £350
- 4750 blue bottles
- £22
- a £1600  
b With 10% cut each year he earns  $£1440 \times 12 + £1296 \times 12 = £17\,280 + £15\,552 = £32\,832$

With immediate 14% cut he earns  $£1376 \times 24 = £33\ 024$ , so correct decision

11 a 30% b 15%

12 Less by  $\frac{1}{4}\%$

13 £900

14 Calculate the pre-VAT price for certain amounts, and  $\frac{5}{6}$  of that amount. Show the error grows as the amount increases. Up to £280 the error is less than £5.

15 £1250

16 £1250

17  $0.28 \times 5400 = 1512$ .  $1512 \times 2.5 = 3780$ ,  $3780 \div 0.72 = 5250$ , so population has declined by 150 people.

18 Baz has assumed that 291.2 is 100% instead of 112%. He rounded his wrong answer to the correct answer of £260.

19  $35\% = \frac{35}{100}$  which cancels to  $\frac{7}{20}$ , so the smallest number that could have been surveyed is 20.

### Review questions

1 48 mph

2 Definite, as his average speed was 80 miles per hour which is 128 km/h

3 Totals are 40 and 60 giving 2 : 3 and a total of 100. 9 : 11 is a ratio of 45 : 55 so swap 10 and 15

4 a  $1.73 \div 0.04 = 43.25$  so 43 horses,  $2.64 \div 0.065 = 40.61$  so 40 cattle and  $0.95 \div 0.01 = 95$ . Total  $43 + 40 + 95 = 178$  animals.  
b Horses in field A = 43, Sheep in field B =  $2.64 \div 0.01 = 264$ , Cattle in field C,  $0.95 \div 0.065 = 14.61$ , so 14 cattle. Total  $43 + 264 + 14 = 321$  animals

5  $100^\circ$

6 a 22.5 kg b 30 kg c £19.80

7 £8357.35

8 £375

9 a £4945.97

b 5, yes he has £1357.68 in the account so he has rounded to the nearest £10

10 13.04%

11 90

12 Joe pays  $41.4 - 4.4 = £37$ , Lucy pays  $41.4 \div 1.15 = £36$ , so Joe's meal cost more.

13  $680.4 \div 4500 = £0.1512$  per units in 2015.  $0.1512 \div 1.08 = £0.14$  pence per unit in 2014.  $5400 \times £0.14 = £756$ , so she paid more for the units in 2014.

14  $50.50 \div 0.9 \div 0.85 = 66.01$ , so price was £66 and other prices are rounded off.

## Chapter 6 – Geometry and measures: Angles

### Exercise 6A

1 a  $108^\circ$  b  $52^\circ$  c  $59^\circ$

2 a  $57^\circ$  b  $40^\circ$

3 No;  $45^\circ + 125^\circ = 170^\circ$  and for a straight line it should be  $180^\circ$ .

4 a  $x = 100^\circ$  b  $x = 110^\circ$  c  $x = 30^\circ$

5 a  $x = 55^\circ$  b  $x = 45^\circ$  c  $x = 12.5^\circ$

6 a  $x = 34^\circ$ ,  $y = 98^\circ$  b  $x = 70^\circ$ ,  $y = 120^\circ$   
c  $x = 20^\circ$ ,  $y = 80^\circ$

7  $6 \times 60^\circ = 360^\circ$ ; imagine six of the triangles meeting at a point

8  $x = 35^\circ$ ,  $y = 75^\circ$ ;  $2x = 70^\circ$  (opposite angles), so  $x = 35^\circ$  and  $x + y = 110^\circ$  (angles on a line), so  $y = 75^\circ$

### Exercise 6B

1 a-c Students' own drawings d  $180^\circ$

2 a  $60^\circ$  b Equilateral triangle  
c Same length

3 a  $70^\circ$  each b Isosceles triangle  
c Same length

4 a  $109^\circ$  b  $130^\circ$  c  $135^\circ$

5  $65^\circ$

6 Joe is not correct as  $\angle DFE = 30^\circ$ ,  $\angle DEF = 75^\circ$  hence  $\angle D = 180^\circ - 105^\circ = 75^\circ$  but Hannah is correct as  $\angle FED = \angle FDE = 75^\circ$

7  $a = 35^\circ$  (angles in a triangle) because the other angles in the triangle are  $65^\circ$  (angles on a line) and  $80^\circ$  (opposite angles) giving a total of 145, this subtracted from the 180 degrees in a triangle leaves the answer of 35

8 Missing angle =  $y$ ,  $x + y = 180^\circ$  and  $a + b + y = 180^\circ$  so  $x = a + b$

9  $32^\circ$

10  $72^\circ$

### Exercise 6C

1 2, 2,  $360^\circ$

2 3, 3,  $540^\circ$

3 4, 4,  $720^\circ$

4

Shape	Number of sides	Triangles	Angle sum
Triangle	3	1	180
Quadrilateral	4	2	360
Pentagon	5	3	540
Hexagon	6	4	720
Heptagon	7	5	900
Octagon	8	6	1080
Nonagon	9	7	1260
Decagon	10	8	1440

5 18, 18, 3240°

### Exercise 6D

1 a 90° b 150° c 80°

2 a No, total is 350° b Yes, total is 360°  
c No, total is 350° d No, total is 370°  
e Yes, total is 360° f Yes, total is 360°

3 a 90° b Rectangle c Square

4 a 120° b 136° c 149°  
d 126° e 114°

5  $60^\circ + 60^\circ + 120^\circ + 120^\circ + 120^\circ + 240^\circ = 720^\circ$

6  $y = 360^\circ - 4x$ ;  $2x + y + 2x = 360^\circ$ ,  $4x + y = 360^\circ$ , so  $y = 360^\circ - 4x$

7  $x = 40^\circ$ , so the smaller angle is  $60^\circ$

### Exercise 6E

1

Shape	Number of sides	Interior angle sum	Each interior angle
octagon	8	1080	135
nonagon	9	1260	140
decagon	10	1440	144

2

Regular polygon	Number of sides	Interior angle	Exterior angle
square	4	90	90
pentagon	5	108	72
hexagon	6	120	60
octagon	8	135	45
nonagon	9	140	40
decagon	10	144	36

3 a i 45° ii 8 iii 1080°  
b i 20° ii 18 iii 2880°  
c i 15° ii 24 iii 3960°  
d i 36° ii 10 iii 1440°

4 a i 172° ii 45 iii 7740°  
b i 174° ii 60 iii 10 440°  
c i 156° ii 15 iii 2340°  
d i 177° ii 120 iii 21 240°

5 a Exterior angle is  $7^\circ$ , which does not divide exactly into  $360^\circ$   
b Exterior angle is  $19^\circ$ , which does not divide exactly into  $360^\circ$

c Exterior angle is  $11^\circ$ , which does not divide exactly into  $360^\circ$

d Exterior angle is  $70^\circ$ , which does not divide exactly into  $360^\circ$

6 a  $7^\circ$  does not divide exactly into  $360^\circ$   
b  $26^\circ$  does not divide exactly into  $360^\circ$   
c  $44^\circ$  does not divide exactly into  $360^\circ$   
d  $13^\circ$  does not divide exactly into  $360^\circ$

7  $48^\circ$ ;  $\frac{1440^\circ - 5 \times 240^\circ}{5}$

8 10

9  $x = 45^\circ$ , they are the same, true for all regular polygons

10 Three are  $135^\circ$  and two are  $67.5^\circ$

11  $72^\circ$ ,  $72^\circ$ ,  $108^\circ$ ,  $144^\circ$ ,  $144^\circ$

12  $93^\circ$

### Exercise 6F

1 a d b f c d d f e f f e

2 a  $b = c = 70^\circ$  b  $d = 75^\circ$ ,  $e = f = 105^\circ$   
c  $g = 50^\circ$ ,  $h = i = 130^\circ$  d  $n = m = 80^\circ$   
e  $g = i = 65^\circ$ ,  $h = 115^\circ$  f  $j = k = 72^\circ$ ,  $l = 108^\circ$

3 a  $a = 95^\circ$  b  $b = 66^\circ$ ,  $c = 114^\circ$

4 a  $x = 30^\circ$ ,  $y = 120^\circ$  b  $x = 25^\circ$ ,  $y = 105^\circ$   
c  $x = 30^\circ$ ,  $y = 100^\circ$  d  $x = 50^\circ$ ,  $y = 110^\circ$   
e  $x = 25^\circ$ ,  $y = 55^\circ$  f  $x = 20^\circ$ ,  $y = 140^\circ$

5  $290^\circ$ ;  $x$  is double the angle allied to  $35^\circ$ , so is  $2 \times 145^\circ$

6 angle BDC =  $66^\circ$  (angles in a triangle =  $180^\circ$ ) angle BDE =  $114^\circ$  (angles on a line =  $180^\circ$ ) so  $a = 66^\circ$  (corresponding angle or allied angle)

7 Angle PQD =  $64^\circ$  (alternate angles), so angle DQY =  $116^\circ$  (angles on a line =  $180^\circ$ )

8 Use alternate angles to see  $b$ ,  $a$  and  $c$  are all angles on a straight line, and so total  $180^\circ$

9 Third angle in triangle equals  $q$  (alternate angle), angle sum of triangle is  $180^\circ$ .

### Exercise 6G

1 a  $a = 110^\circ$ ,  $b = 55^\circ$  b  $g = i = 63^\circ$ ,  $h = 117^\circ$   
c  $e = f = 94^\circ$

2 a  $a = 58^\circ$ ,  $b = 47^\circ$  b  $c = 141^\circ$ ,  $d = 37^\circ$   
c  $e = g = 65^\circ$ ,  $f = 115^\circ$

3 a  $65^\circ$  b  $60^\circ$  c  $68^\circ$

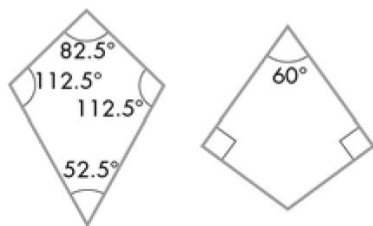
4 both  $129^\circ$

5 Marie is correct, a rectangle is a parallelogram with all angles equal to  $90^\circ$

6 a  $65^\circ$   
b Trapezium, angle A + angle D =  $180^\circ$  and angle B + angle C =  $180^\circ$

7  $135^\circ$

8



- 9 A trapezium; angles add up to  $10x$ , two angles  $x$  and  $4x = 2x + 3x$ , the other pair of angles. Hence each pair adds up to 180 (since  $2 \times 180 = 360$ ). Hence two pairs of allied angles, hence a trapezium. Alternatively you could have found that  $x = 36$  which will give the same result.

### Exercise 6H

- 1 a Student's scale drawing.  
b About 19 m so about 38 plants
- 2 a i 84 km ii 280 km  
iii 144 km iv 48 km  
b i 108 km ii 360 km  
iii 200 km iv 164 km
- 3 a 36 km b 2 000 000
- 4 1 : 63 360
- 5 a  $110^\circ$ , 12.6 km b  $250^\circ$ , 4.5 km  
c  $091^\circ$ , 11.8 km d  $270^\circ$ , 8.4 km  
e  $130^\circ$ , 7.2 km f  $180^\circ$ , 4.2 km
- 6 Students' Sketches
- 7 a Sketch  
b D is due south of B and B is east of A, so A must be west of D. A bearing to the west will be greater than  $180^\circ$
- 8 a  $090^\circ$ ,  $180^\circ$ ,  $270^\circ$  b  $000^\circ$ ,  $270^\circ$ ,  $180^\circ$
- 9 a  $045^\circ$  b  $286^\circ$   
c measure the distance from X to Y and divide 15 by this to find the scale of the map. Then measure the distance from Y to P and multiply by the scale factor
- 10 a  $250^\circ$  b  $325^\circ$  c  $144^\circ$
- 11 a 900 m b  $280^\circ$   
c angle NHS =  $150^\circ$  and HS = 3 cm
- 12  $108^\circ$
- 13  $255^\circ$
- 14 9.92 km

### Review questions

- 1  $16^\circ$
- 2 a i  $115^\circ$   
ii opposite angles are equal and the angle opposite to  $x$  is  $180 - 65$   
b the angles do not add up to  $360^\circ$

3 a  $50^\circ$  b  $32.5^\circ$

4 A five sided shape can be split into 3 triangles  
hence  $3 \times 180$   
 $= 540^\circ$

5  $150^\circ$

6 angle TQP =  $37^\circ$  (alternate angles), PTQ =  $180 - (29 + 37) = 114^\circ$  (angles in a triangle), QTS =  $180 - 114$  (angles on a line) =  $66^\circ$

7  $333^\circ$

8  $360 \div 8 = 45^\circ$ ; exterior angle formula is  $360 \div$  number of sides, in this case 8

9  $180 - (360 \div 6) = 120^\circ$  or  $(180 \times 4) \div 6 = 120^\circ$

10 Selvi might be correct. You will need to draw one example showing this is not a kite, and one example showing that this could be a kite

11 a Student's own sketch b 12.4 km

## Chapter 7 – Geometry and measures: Transformations, constructions and loci

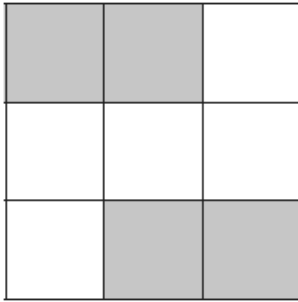
### Exercise 7A

- 1 a SAS b SSS c ASA  
d RHS e SSS f ASA
- 2 a SSS. A to R, B to P, C to Q  
b SAS. A to R, B to Q, C to P
- 3 a  $60^\circ$  b  $80^\circ$  c  $40^\circ$  d 5 cm
- 4 a  $110^\circ$  b  $55^\circ$  c  $85^\circ$  d  $110^\circ$  e 4 cm
- 5 SSS or RHS
- 6 SSS or SAS or RHS
- 7 For example, use  $\angle ADE$  and  $\angle CDG$ . AD = CD (sides of large square), DE = DG (sides of small square),  $\angle ADE = \angle CDG$  (angles sum to  $90^\circ$  with  $\angle ADG$ ), so  $\triangle ADE \equiv \triangle CDG$  (SAS), so AE = CG
- 8 AB and PQ are the corresponding sides to the  $42^\circ$  angle, but they are not equal in length.

### Exercise 7B

- 1 a 4 b 5 c 6 d 4 e 6
- 2 a 2 b 2 c 2 d 2 e 2
- 3 A, B, C, D, E, F, G, J, K, L, M, P, Q, R, T, U, V, W, Y
- 4 a 1  
b the central white star or the large dark green star  
c order 16 – the light green star around the central white star, or order 9 – the light green shape between the outer petals and the inner stars

5 for example:



6

Order of rotational symmetry	Number of lines of symmetry			
	0	1	2	3
	1	D	A	
	2	E	B	

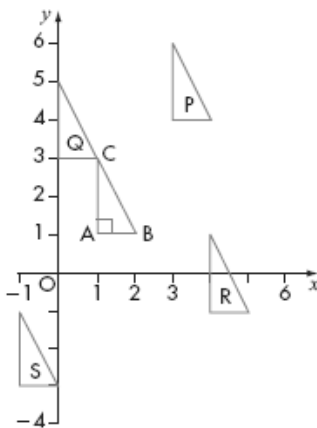
7 She is correct since the angle sum around the centre point is 360 and  $360 \div 3 = 120$

8 Yes she is correct. A triangle can only have 1 or 3 lines of symmetry. If a triangle has 3 lines of symmetry it also has rotational symmetry of order 3, so this triangle must only have 1 line of symmetry. This will mean it has two angles identical and two sides, and hence an isosceles triangle.

### Exercise 7C

- 1 a i  $\begin{pmatrix} 1 \\ 3 \end{pmatrix}$  ii  $\begin{pmatrix} 4 \\ 2 \end{pmatrix}$  iii  $\begin{pmatrix} 2 \\ -1 \end{pmatrix}$  iv  $\begin{pmatrix} 5 \\ 1 \end{pmatrix}$   
 b i  $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$  ii  $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$  iii  $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$  iv  $\begin{pmatrix} 3 \\ 3 \end{pmatrix}$   
 c i  $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$  ii  $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$  iii  $\begin{pmatrix} -5 \\ 4 \end{pmatrix}$  iv  $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$   
 d i  $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$  ii  $\begin{pmatrix} -4 \\ 2 \end{pmatrix}$  iii  $\begin{pmatrix} 5 \\ -4 \end{pmatrix}$  iv  $\begin{pmatrix} -2 \\ -7 \end{pmatrix}$

2



- 3 a  $\begin{pmatrix} -3 \\ -1 \end{pmatrix}$  b  $\begin{pmatrix} 4 \\ -4 \end{pmatrix}$  c  $\begin{pmatrix} -5 \\ -2 \end{pmatrix}$  d  $\begin{pmatrix} 4 \\ 7 \end{pmatrix}$   
 e  $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$  f  $\begin{pmatrix} 1 \\ 6 \end{pmatrix}$  g  $\begin{pmatrix} -4 \\ 4 \end{pmatrix}$  h  $\begin{pmatrix} -4 \\ -7 \end{pmatrix}$

4  $10 \times 10 = 100$  (including  $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ )

- 5 a Check students' designs for a Snakes and ladders board.  
 b i because the ladders always mean moving up the board  
 ii the snakes always mean moving down the board

6  $\begin{pmatrix} -x \\ -y \end{pmatrix}$

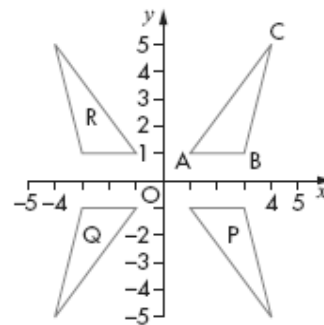
7 a Check student's diagram b  $\begin{pmatrix} -300 \\ -500 \end{pmatrix}$

8  $\begin{pmatrix} -1 \\ 4 \end{pmatrix}$

9 Under a translation every points moves with the same vector, hence all the sides are the same length, so we can use the SSS rule of congruency.

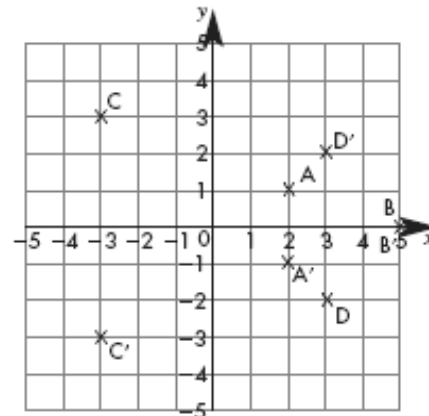
### Exercise 7D

1 a-e



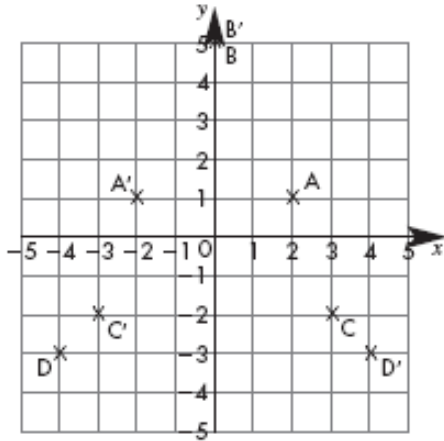
f Reflection in the y-axis

2. a-b



- c y-value changes sign  
 d  $(a, -b)$

3 a-b



c  $x$ -value changes sign

d  $(-a, b)$

e Any three points with  $x$  co-ordinate 0, e.g.  $(0, 1)$ ,  $(0, 2)$ ,  $(0, 3)$

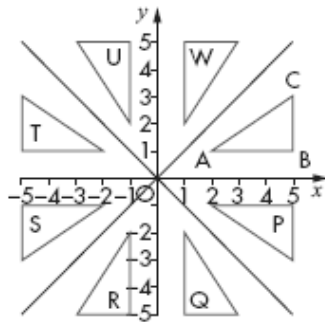
4 Possible answer: Take the centre square as ABCD then reflect this square each time in the line, AB, then BC, then CD and finally AD.

5  $x = -1$

6

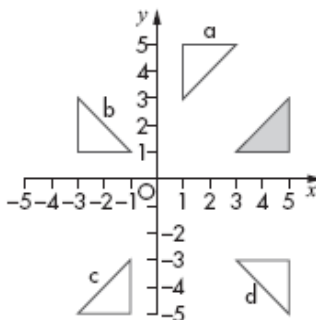


7 a-i

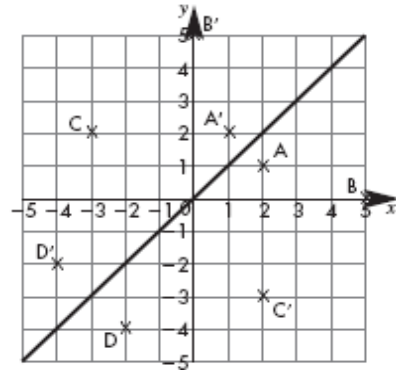


j A reflection in  $y = x$

8



9 a-c

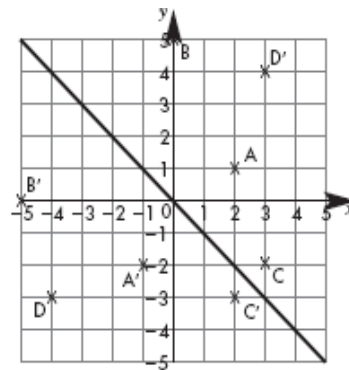


d Coordinates are reversed:  $x$  becomes  $y$  and  $y$  becomes  $x$

e  $(b, a)$

f Any point with  $x$  and  $y$  co-ordinates the same, e.g.  $(1, 1)$ ,  $(2, 2)$

10 a-c



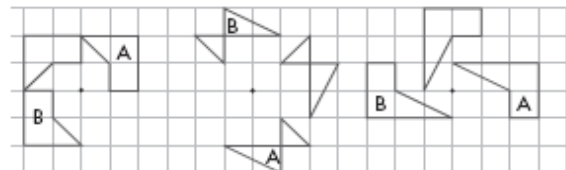
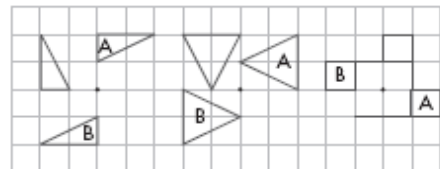
d Coordinates are reversed and change sign,  $x$  becomes  $-y$  and  $y$  becomes  $-x$

e  $(-b, -a)$

11 Because a reflection is exactly the same shape as the original, just in a different orientation, hence we can use the rule SSS to show the two shapes are congruent.

## Exercise 7E

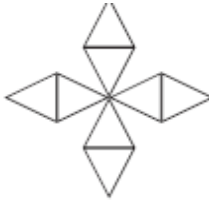
1 a



b i Rotation  $90^\circ$  anticlockwise

ii Rotation  $180^\circ$

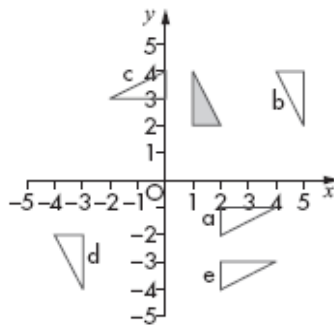
2 a



b rotate shape  $60^\circ$  around point B, then repeat another four times.

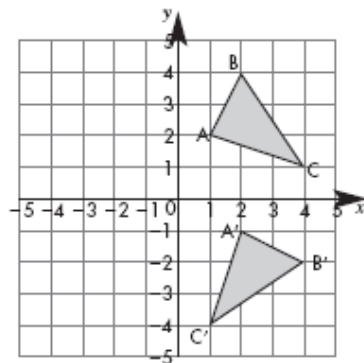
3 Possible answer: If ABCD is the centre square, rotate about A  $90^\circ$  anticlockwise, rotate about new B  $180^\circ$ , now rotate about new C  $180^\circ$ , and finally rotate about new D  $180^\circ$ .

4



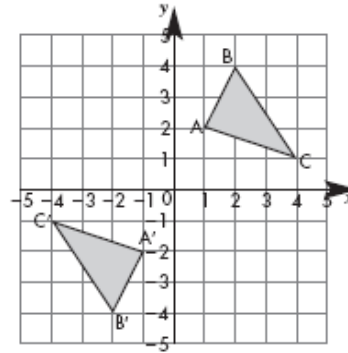
- 5 a  $90^\circ$  anticlockwise  
b  $270^\circ$  anticlockwise  
c  $300^\circ$  clockwise  
d  $260^\circ$  clockwise

6 a b c i



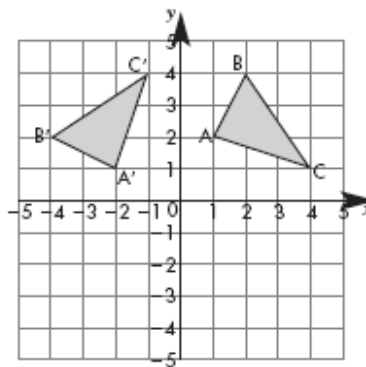
- ii A' (2, -1), B' (4, -2), C' (1, -4)  
iii Original coordinates (x, y) become (y, -x)  
iv Yes

7 i



- ii A' (-1, -2), B' (-2, -4), C' (-4, -1)  
iii Original coordinates (x, y) become (-x, -y)  
iv Yes

8 i



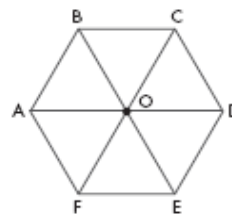
- ii A' (-2, 1), B' (-4, 2), C' (-1, 4)  
iii Original coordinates (x, y) become (-y, x)  
iv Yes

9 The centre of rotation

10 Show by drawing a shape or use the fact that (a, b) becomes (a, -b) after reflection in the x-axis, and (a, -b) becomes (-a, -b) after reflection in the y-axis, which is equivalent to a single rotation of  $180^\circ$ .

11 she is correct

12 a



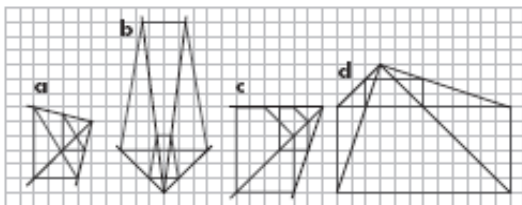
- b i Rotation  $60^\circ$  clockwise about O  
ii Rotation  $120^\circ$  clockwise about O  
iii Rotation  $180^\circ$  about O  
iv Rotation  $240^\circ$  clockwise about O  
c i Rotation  $60^\circ$  clockwise about O  
ii Rotation  $180^\circ$  about O

13 Rotation  $90^\circ$  anticlockwise about  $(3, -2)$

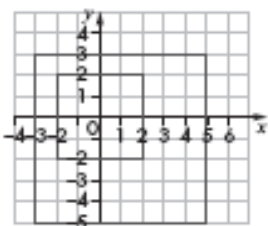
14 Because under a rotation, the lengths of the original shape are preserved, so we can use the rule SSS to show they are congruent.

### Exercise 7F

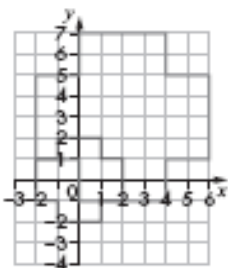
1



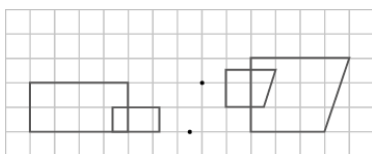
2 a



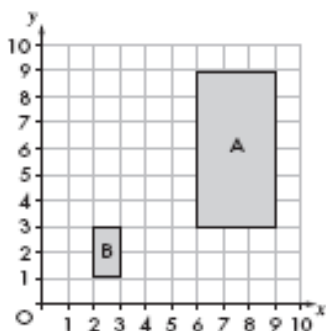
b



3



4 a



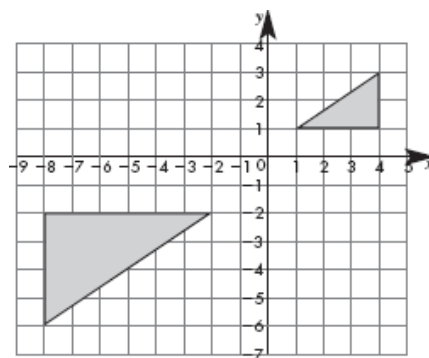
b 3 : 1

c 3 : 1

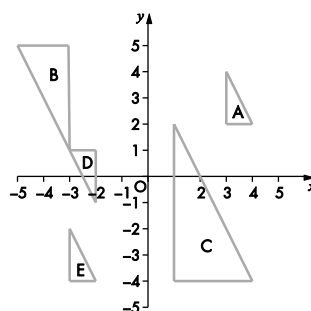
d 9 : 1

5 a  $(1, 1), (3, -3), (-5, -5)$  b  $(1, 1)$

6



7 a-c



d Scale factor  $-\frac{1}{2}$ , centre  $(1, 3)$

e Scale factor  $-2$ , centre  $(1, 3)$

f Scale factor  $-1$ , centre  $(-2.5, -1.5)$

g Scale factor  $-1$ , centre  $(-2.5, -1.5)$

h Same centres, and the scale factors are reciprocals of each other

8 Enlargement, scale factor  $-2$ , about  $(1, 3)$

9 Because the sides of triangle C are all larger than the original triangle B, and so the SSS rule will not apply.

### Exercise 7G

1  $(-4, -3)$

2 a  $(-5, 2)$

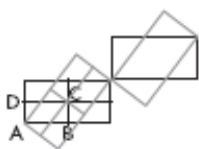
b Reflection in y-axis

3 A: translation  $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$ , B: reflection in y-axis, C:

rotation  $90^\circ$  clockwise about  $(0, 0)$ , D: reflection in  $x = 3$ , E: reflection in  $y = 4$ , F: enlargement by scale factor 2, centre  $(2, -3)$

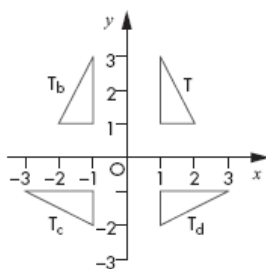


4



- 5 a  $T_1$  to  $T_2$ : rotation  $90^\circ$  clockwise about  $(0, 0)$   
 b  $T_1$  to  $T_6$ : rotation  $90^\circ$  anticlockwise about  $(0, 0)$   
 c  $T_2$  to  $T_3$ : translation  $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$   
 d  $T_6$  to  $T_2$ : rotation  $180^\circ$  about  $(0, 0)$   
 e  $T_6$  to  $T_5$ : reflection in  $y$ -axis  
 f  $T_5$  to  $T_4$ : translation  $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

6 a–d



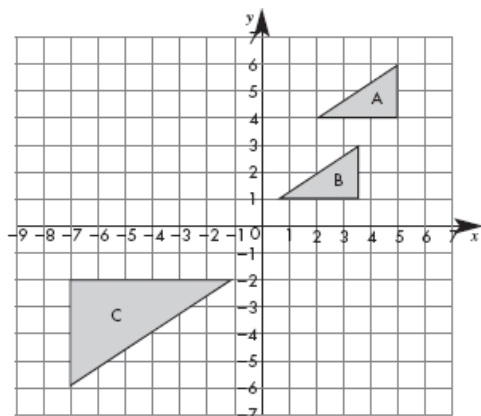
- e  $T_d$  to  $T$ : rotation  $90^\circ$  anticlockwise about  $(0, 0)$

7  $(3, 1)$

- 8 Reflection in  $x$ -axis, translation  $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$ , rotation  $90^\circ$  clockwise about  $(0, 0)$

- 9 Translation  $\begin{pmatrix} 0 \\ -8 \end{pmatrix}$ , reflection in  $x$ -axis, rotation  $90^\circ$  clockwise about  $(0, 0)$

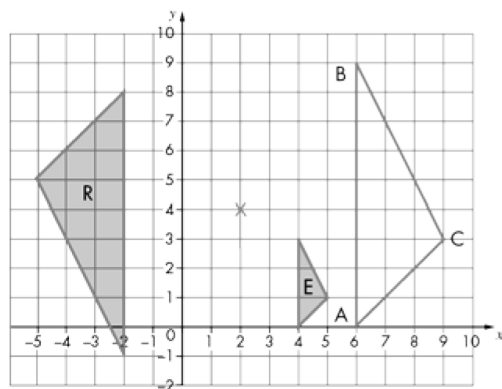
10 a



- b enlargement of scale factor  $-\frac{1}{2}$  about  $(1, 2)$

11 No, this can be shown with an example.

12 a–b



- c An enlargement, scale factor  $-3$ , centre  $(2, 2)$

13  $(10, 10)$

### Exercise 7H

1–9 Practical work; check students' constructions

10 The centre of the circle

11 Start with a base line AB; then construct a perpendicular to the line from point A. At point B, construct an angle of  $60^\circ$ . Ensure that the line for this  $60^\circ$  angle crosses the perpendicular line; where they meet will be the final point C.

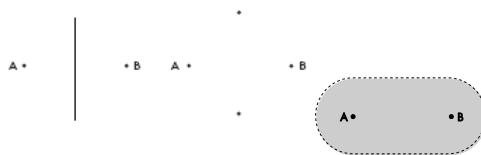
12–14 Practical work; check students' constructions

### Exercise 7I

1 Circle with radius:

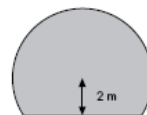
- a 2 cm      b 4 cm      c 5 cm

2 a      b      c



3 a Circle with radius 4 m

b



4 a



b



c



d



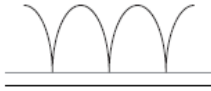
e



f



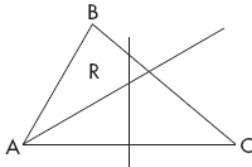
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6



- 7 Construct the bisector of angle BAC and the perpendicular bisector of the line AC.



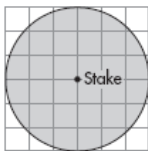
8



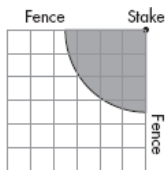
9. Start with a base line, AB, 3 cm long. At point A, draw a few points all 3 cm away from A towards the upper right side. Lightly join these dots with an arc. You can now find the point that is 3 cm away from point B and draw the equilateral triangle.
10. Gary is correct about the triangle inside, but not a triangle outside as there will be three straight lines, parallel to each side of the triangle, then these straight lines will be joined with arcs centred on the vertex between the lines.

### Exercise 7J

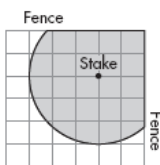
1



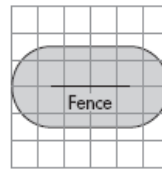
2 a



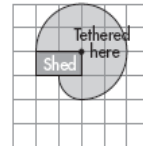
b



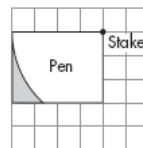
3



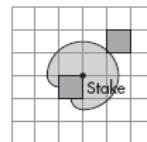
4



5



6



- 7 No, if you accurately draw a diagram showing the path of each boat, you will find the boat leaving from point B meets the path of the other boat in a much shorter time as it's a smaller distance than from A to the cross over point.)

- 8 On a map, draw a straight line from Newcastle to Bristol, construct the line bisector, then the search will be anywhere on the sea along that line.

- 9 a Sketch should show a circle of radius 6 cm around London and one of radius 4 cm around Glasgow.

b No

c Yes

- 10 a Yes

- b Sketch should show a circle of radius 4 cm around Leeds and one of radius 4 cm around Exeter. The area where they overlap should be shaded.

- 11 a This is the perpendicular bisector of the line from York to Birmingham. It should pass just below Manchester and just through the top of Norwich.

- b Sketch should show a circle of radius 7 cm around Glasgow and one of radius 5 cm around London. The area where they overlap should be shaded.

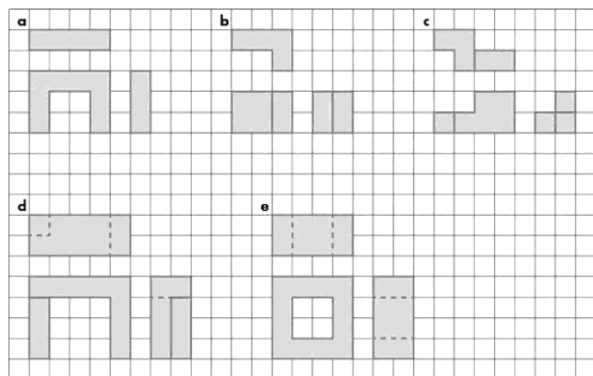
- c The transmitter can be built anywhere on line constructed in part a that is within the area shown in part b.

- 12** Sketch should show two circles around Birmingham, one of radius 3 cm and one of radius 5 cm. The area of good reception is the area between the two circles.
- 13** Sketch should show a circle of radius 6 cm around Glasgow, two circles around York, one of radius 4 cm and one of radius 6 cm and a circle around London of radius 8 cm. The small area in the Irish Sea that is between the two circles around York and inside both the circle around Glasgow and the circle around London is where the boat can be.
- 14** Sketch should show two circles around Newcastle upon Tyne, one of radius 4 cm and one of radius 6 cm, and two circles around Bristol, one of radius 3 cm and one of radius 5 cm. The area that is between both pairs of circles is the area that should be shaded.
- 15** Sketch should show the perpendicular bisector of the line running from Newcastle upon Tyne to Manchester and that of the line running from Sheffield to Norwich. Where the lines cross is where the oil rig is located.
- 16** Sketch should show the perpendicular bisector of the line running from Glasgow to Norwich and that of the line running from Norwich to Exeter. Where the lines cross is where Fred's house is.

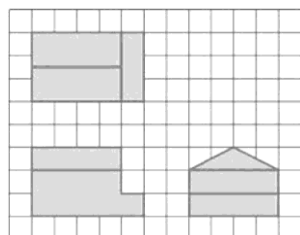
**17** Leeds

### Exercise 7K

**1.**

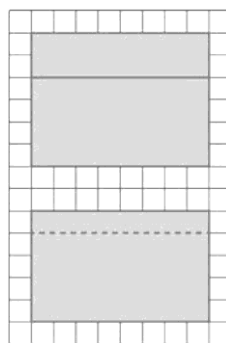


**2**

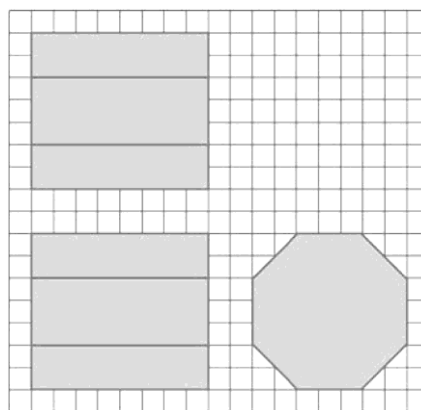


- 3 a** A cylinder  
**b** A hexagonal prism

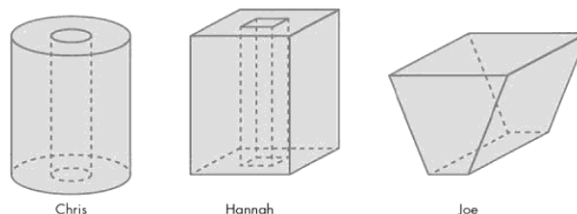
**4**



**5**

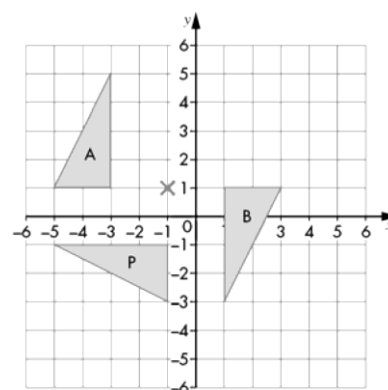


**6**



### Review questions

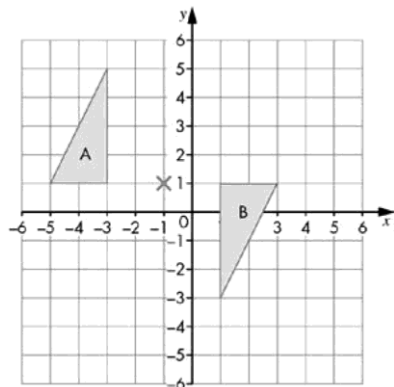
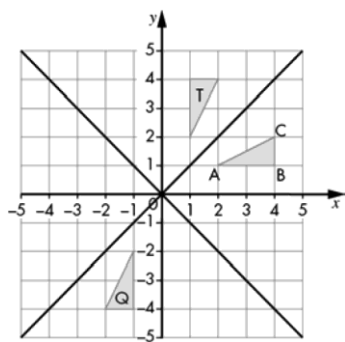
**1 a-b**



- c** rotation of  $180^\circ$  about  $(-1, 1)$   
**d**  $(-1, 1)$

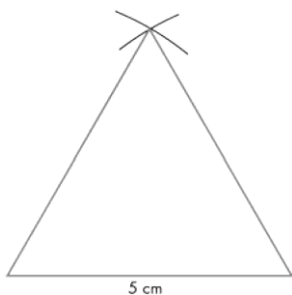
2 a-b

c rotation of  $180^\circ$  about O

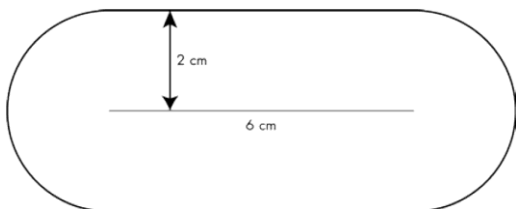


3 You should have measured the error of the angle size and converted that to a percentage error.

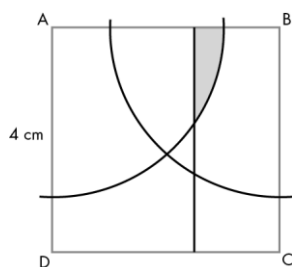
4



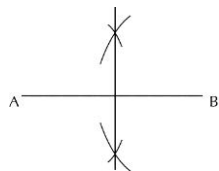
5



6

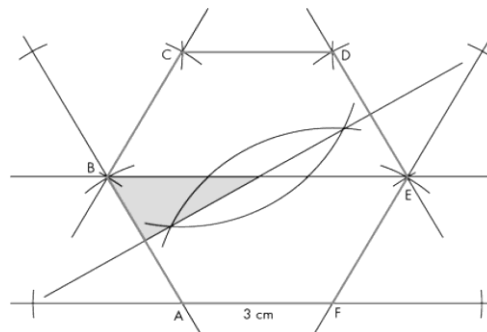


7 a

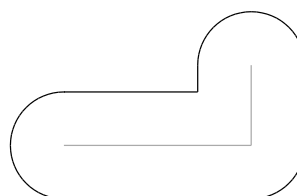


b ... an equal distance from A and B

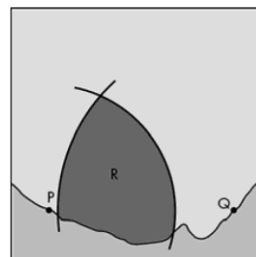
8



9

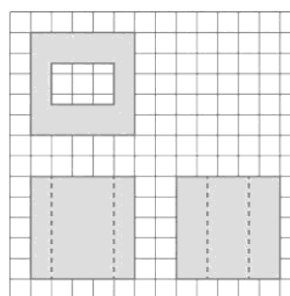


10



11 A (9, 0) B (11, -3) C (2, -1)

12



13 It is not always true

14 In a rhombus all sides are the same length, so  $AB = BC = AD = DC$ , AC is a common length in both triangles, so each triangle has the three sides matching, SSS.

## Chapter 8 – Algebra: Algebraic manipulation

### Exercise 8A

- 1 a 13      b -3      c 5
- 2 a 1.4      b 1.4      c -0.4
- 3 a 13      b 74      c 17
- 4 a 75      b 22.5      c -135
- 5 a 2.5      b -20      c 2.5
- 6 a  $\frac{150}{n}$       b £925
- 7 a  $2 \times 8 + 6 \times 11 - 3 \times 2 = 76$   
b  $5 \times 2 - 2 \times 11 + 3 \times 8 = 12$
- 8 a One odd one even value, different from each other.  
b Any valid combination, e.g.  $x = 1, y = 2$
- 9 a i Odd      ii Odd  
iii Even      iv Odd  
b Any valid expression such as  $xy + z$
- 10 a £20  
b i -£40  
ii Delivery cost will be zero.  
c 40 miles
- 11 A expression, B formula, C identity, D equation
- 12 a First term is cost of petrol, each mile is a tenth of £0.98. Second term is the hire cost divided by the miles.  
b 29.8p per mile

### Exercise 8B

- 1 a  $6 + 2m$       b  $10 + 5l$   
c  $12 - 3y$       d  $20 + 8k$   
e  $6 - 12f$       f  $10 - 6w$   
g  $10k + 15m$       h  $12d - 8n$   
i  $t^2 + 3t$       j  $k^2 - 3k$   
k  $4r^2 - 4t$       l  $8k - 2k^2$   
m  $8g^2 + 20g$       n  $15h^2 - 10h$   
o  $y^3 + 5y$       p  $h^4 + 7h$   
q  $k^3 - 5k$       r  $3t^3 + 12t$   
s  $15d^3 - 3d^4$       t  $6w^3 + 3tw$   
u  $15a^3 - 10ab$       v  $12p^4 - 15mp$   
w  $12h^3 + 8h^2g$       x  $8m^3 + 2m^4$
- 2 a  $5(t - 1)$  and  $5t - 5$   
b Yes, as  $5(t - 1)$  when  $t = 4.50$  is  $5 \times 3.50 = £17.50$
- 3 He has worked out  $3 \times 5$  as 8 instead of 15 and he has not multiplied the second term by 3. Answer should be  $15x - 12$ .
- 4 a  $3(2y + 3)$   
b  $2(6z + 4)$  or  $4(3z + 2)$
- 5 a  $22 + 5t$       b  $21 + 19k$   
c  $22 + 2f$       d  $14 + 3g$
- 6 a  $2 + 2h$       b  $9g + 5$   
c  $17k + 16$       d  $6e + 20$

- 7 a  $4m + 3p + 2mp$   
b  $3k + 4h + 5hk$   
c  $12r + 24p + 13pr$   
d  $19km + 20k - 6m$
- 8 a  $9t^2 + 13t$       b  $13y^2 + 5y$   
c  $10e^2 - 6e$       d  $14k^2 - 3kp$
- 9 a  $17ab + 12ac + 6bc$   
b  $18wy + 6ty - 8tw$   
c  $14mn - 15mp - 6np$   
d  $8r^3 - 6r^2$
- 10 a  $5(f + 2s) + 2(2f + 3s) = 9f + 16s$   
b  $£(270f + 480s)$   
c  $£42\,450 - £30\,000 = £12\,450$
- 11 For  $x$ -coefficients, 3 and 1 or 1 and 4; for  $y$ -coefficients, 5 and 1 or 3 and 4 or 1 and 7
- 12  $5(3x + 2) - 3(2x - 1) = 9x + 13$

### Exercise 8C

- 1 a  $6(m + 2t)$       b  $3(3t + p)$   
c  $4(2m + 3k)$       d  $4(r + 2t)$   
e  $m(n + 3)$       f  $g(5g + 3)$   
g  $2(2w - 3t)$       h  $y(3y + 2)$   
i  $t(4t - 3)$       j  $3m(m - p)$   
k  $3p(2p + 3t)$       l  $2p(4t + 3m)$   
m  $4b(2a - c)$       n  $5bc(b - 2)$   
o  $2b(4ac + 3de)$       p  $2(2a^2 + 3a + 4)$   
q  $3b(2a + 3c + d)$       r  $t(5t + 4 + a)$   
s  $3mt(2t - 1 + 3m)$       t  $2ab(4b + 1 - 2a)$   
u  $5pt(2t + 3 + p)$
- 2 a Mary has taken out a common factor.  
b Because the bracket adds up to £10.  
c £30
- 3 a, d, f and h do not factorise.  
b  $m(5 + 2p)$       c  $t(t - 7)$       e  $2m(2m - 3p)$   
g  $a(4a - 5b)$       i  $b(5a - 3bc)$
- 4 a Bernice  
b Aidan has not taken out the largest possible common factor. Craig has taken  $m$  out of both terms but there isn't an  $m$  in the second term.
- 5 There are no common factors.
- 6 Perimeter =  $2x + 8 + x + 5 + 5x + 4 + 9x - 3 + 10 - x = 16x + 24 = 8(2x + 3)$
- 7  $\frac{4x^2 - 12x}{2x - 6}$

### Exercise 8D

- 1 a  $x^2 + 5x + 6$       b  $t^2 + 7t + 12$   
c  $w^2 + 4w + 3$       d  $m^2 + 6m + 5$
- 2 a  $p^2 + 3p - 70$       b  $u^2 - 12u + 32$   
c  $k^2 + 2k - 15$       d  $z^2 - 12z + 27$
- 3 a should be 35 on the end  
b should be -80  
c should be -10x  
d should be  $12y$   
e should be -9z

### Exercise 8E

- a  $k^2 + 8k + 15$       b  $a^2 + 5a + 4$   
c  $x^2 + 2x - 8$       d  $t^2 + 2t - 15$   
e  $w^2 + 2w - 3$       f  $f^2 - f - 6$
- a  $r^2 - 10r + 16$       b  $s^2 - 17s + 70$   
c  $d^2 - 17d + 16$       d  $m^2 - 9m + 18$   
e  $q^2 - 20q + 99$       f  $y^2 - 13y + 40$
- a 20a      b 3b      c 200  
d -11d      e 12e, 28

### Exercise 8F

- a  $g^2 - 3g - 4$       b  $y^2 + y - 12$   
c  $x^2 + x - 12$       d  $p^2 - p - 2$   
e  $k^2 - 2k - 8$       f  $y^2 + 3y - 10$   
g  $a^2 + 2a - 3$
- a  $x^2 - 9$       b  $t^2 - 25$       c  $m^2 - 16$   
d  $t^2 - 4$       e  $y^2 - 64$       f  $p^2 - 1$   
g  $25 - x^2$       h  $49 - g^2$       i  $x^2 - 36$
- (x + 2) and (x + 3)
- a B:  $1 \times (x - 2)$   
C:  $1 \times 2$   
D:  $2 \times (x - 1)$   
b  $(x - 2) + 2 + 2(x - 1) = 3x - 2$   
c Area A =  $(x - 1)(x - 2)$  = area of square minus areas (B + C + D) =  $x^2 - (3x - 2) = x^2 - 3x + 2$
- a  $x^2 - 9$   
b i 9991      ii 39 991
- a  $y^2 + 14y + 45$   
b i 45.1401      ii 45.4209  
iii 44.7204      iv 11 445

### Exercise 8G

- a  $6x^2 + 11x + 3$       b  $12y^2 + 17y + 6$   
c  $6t^2 + 17t + 5$       d  $8t^2 + 2t - 3$   
e  $10m^2 - 11m - 6$       f  $12k^2 - 11k - 15$   
g  $6p^2 + 11p - 10$       h  $10w^2 + 19w + 6$   
i  $6a^2 - 7a - 3$       j  $8r^2 - 10r + 3$   
k  $15g^2 - 16g + 4$       l  $12d^2 + 5d - 2$   
m  $8p^2 + 26p + 15$       n  $6t^2 + 7t + 2$   
o  $6p^2 + 11p + 4$       p  $-10t^2 - 7t + 6$   
q  $-6n^2 + n + 12$       r  $6f^2 - 5f - 6$   
s  $-10q^2 + 7q + 12$       t  $-6p^2 - 7p + 3$   
u  $-6t^2 + 10t + 4$
- a  $(3x - 2)(2x + 1) = 6x^2 - x - 2$   
 $(2x - 1)(2x - 1) = 4x^2 - 4x + 1$   
 $(6x - 3)(x + 1) = 6x^2 + 3x - 3$   
 $(4x + 1)(x - 1) = 4x^2 - 3x - 1$   
 $(3x + 2)(2x + 1) = 6x^2 + 7x + 2$   
b Multiply the x terms to match the  $x^2$  term and/or multiply the constant terms to get the constant term in the answer.
- a  $4x^2 - 1$       b  $9t^2 - 4$       c  $25y^2 - 9$   
d  $16m^2 - 9$       e  $4k^2 - 9$       f  $16h^2 - 1$   
g  $4 - 9x^2$       h  $25 - 4t^2$       i  $36 - 25y^2$   
j  $a^2 - b^2$       k  $9t^2 - k^2$       l  $4m^2 - 9p^2$   
m  $25k^2 - g^2$       n  $a^2b^2 - c^2d^2$       o  $a^4 - b^4$

- a  $a^2 - b^2$   
b Dimensions:  $a + b$  by  $a - b$ ; Area:  $a^2 - b^2$   
c Areas are the same, so  $a^2 - b^2 = (a + b) \times (a - b)$
- First shaded area is  $(2k)^2 - 1^2 = 4k^2 - 1$   
Second shaded area is  $(2k + 1)(2k - 1) = 4k^2 - 1$
- a  $3w^2 + 22w + 24$   
b i 32 224      ii 23.7803  
iii 24.000 440 0012
- a  $49a^2 - b^2$       b 4896

### Exercise 8H

- a  $x^2 + 10x + 25$       b  $m^2 + 8m + 16$   
c  $t^2 + 12t + 36$       d  $p^2 + 6p + 9$   
e  $m^2 - 6m + 9$       f  $t^2 - 10t + 25$   
g  $m^2 - 8m + 16$       h  $k^2 - 14k + 49$
- a  $9x^2 + 6x + 1$       b  $16t^2 + 24t + 9$   
c  $25y^2 + 20y + 4$       d  $4m^2 + 12m + 9$   
e  $16t^2 - 24t + 9$       f  $9x^2 - 12x + 4$   
g  $25t^2 - 20t + 4$       h  $25r^2 - 60r + 36$   
i  $x^2 + 2xy + y^2$       j  $m^2 - 2mn + n^2$   
k  $4t^2 + 4ty + y^2$       l  $m^2 - 6mn + 9n^2$   
m  $x^2 + 4x$       n  $x^2 - 10x$   
o  $x^2 + 12x$       p  $x^2 - 4x$
- a Bernice has just squared the first term and the second term. She hasn't written down the brackets twice.  
b Pete has written down the brackets twice but has worked out  $(3x)^2$  as  $3x^2$  and not  $9x^2$ .  
c  $9x^2 + 6x + 1$
- Whole square is  $(2x)^2 = 4x^2$ .  
Three areas are  $2x - 1$ ,  $2x - 1$  and 1.  
 $4x^2 - (2x - 1 + 2x - 1 + 1) = 4x^2 - (4x - 1)$   
 $= 4x^2 - 4x + 1$
- a  $9p^6 + 42p^3q^7 + 49q^{14}$
- a  $9k^2 + 24k + 16$   
b i 16.2409      ii 92 416      iii 16.120 225

### Exercise 8I

- a  $x^3 + 6x^2 + 11x + 6$   
b  $x^3 - 49x - 120$   
c  $x^3 + 9x^2 - 4x - 36$
- a  $x^3 + 7x^2 - 17x + 9$   
b  $x^3 + x^2 - x - 10$
- a  $x^3 + 12x^2 + 48x + 64$   
b  $x^3 - 18x^2 + 108x - 216$   
c  $x^3 + 3ax^2 + 3a^2x + a^3$
- abc  $x^3 + 11x^2 + 31x + 21$   
d Can be performed in any order
- a  $x^3 + (a + b + c)x^2 + (ab + ac + bc)x + abc$   
b  $p = 0$ ,  $q = -19$ ,  $r = -30$
- a  $x^3 - 15x^2 - 73x - 57$   
b  $6x^2 - 60x - 146$

- 7 a i  $x^2 + 2x + 1$  ii  $x^3 + 3x^2 + 3x + 1$   
 iii  $x^4 + 4x^3 + 6x^2 + 4x + 1$   
 b  $11^2 = 121$ ,  $11^3 = 1331$ ,  $11^4 = 14\,641$   
 c The digits are the same as the coefficients
- 8 a  $x^3 + 9x^2 + 27x + 27$   
 b 27.027 009 001
- 9 a  $2x^3 + 3x^2 - 29x + 30$   
 b  $3x^3 + 11x^2 + 8x - 4$
- 10 a  $24x^3 + 26x^2 - 173x + 105$   
 b  $50x^3 - 315x^2 + 228x - 44$   
 c  $27x^3 - 108x^2 + 144x - 64$
- 11  $82 - 5x - 32x^2$

### Exercise 8J

- 1 a  $(x+2)(x+3)$  b  $(t+1)(t+4)$   
 c  $(m+2)(m+5)$  d  $(k+4)(k+6)$   
 e  $(p+2)(p+12)$  f  $(r+3)(r+6)$   
 g  $(w+2)(w+9)$  h  $(x+3)(x+4)$   
 i  $(a+2)(a+6)$  j  $(k+3)(k+7)$   
 k  $(f+1)(f+21)$  l  $(b+8)(b+12)$   
 m  $(t-2)(t-3)$  n  $(d-4)(d-1)$   
 o  $(g-2)(g-5)$  p  $(x-3)(x-12)$   
 q  $(c-2)(c-16)$  r  $(t-4)(t-9)$   
 s  $(y-4)(y-12)$  t  $(j-6)(j-8)$
- 2 a  $(p-3)(p-5)$  b  $(y+6)(y-1)$   
 c  $(t+4)(t-2)$  d  $(x+5)(x-2)$   
 e  $(m+2)(m-6)$  f  $(r+1)(r-7)$   
 g  $(n+3)(n-6)$  h  $(m+4)(m-11)$   
 i  $(w+4)(w-6)$  j  $(t+9)(t-10)$   
 k  $(h+8)(h-9)$  l  $(t+7)(t-9)$   
 m  $(d+1)^2$  n  $(y+10)^2$   
 o  $(t-4)^2$  p  $(m-9)^2$   
 q  $(x-12)^2$  r  $(d+3)(d-4)$   
 s  $(t+4)(t-5)$  t  $(q+7)(q-8)$
- 3  $(x+2)(x+3)$ , giving areas of  $2x$  and  $3x$ , or  $(x+1)(x+6)$ , giving areas of  $x$  and  $6x$ .
- 4 a  $x^2 + (a+b)x + ab$   
 b i  $p+q=7$  ii  $pq=12$   
 c 7 can only be  $1 \times 7$  and  $1+7 \neq 12$
- 5 a 440  
 b i  $(x+3)(x+1)$  ii  $22 \times 20 = 440$
- 6 a  $(x^2-3)(x^2-8)$  b  $(y^5-104)(y^5+4)$   
 c  $(z^{1728}-864)(z^{1728}+2)$

### Exercise 8K

- 1 a  $(x+3)(x-3)$  b  $(t+5)(t-5)$   
 c  $(m+4)(m-4)$  d  $(3+x)(3-x)$   
 e  $(7+t)(7-t)$  f  $(k+10)(k-10)$   
 g  $(2+y)(2-y)$  h  $(x+8)(x-8)$   
 i  $(t+9)(t-9)$
- 2 a  $x^2$   
 b i  $(x-2)$  ii  $(x+2)$   
 iii  $x^2$  iv 4  
 c  $A+B-C = x^2-4$ , which is the area of D, which is  $(x+2)(x-2)$ .
- 3 a  $x^2 + 4x + 4 - (x^2 + 2x + 1) = 2x + 3$   
 b  $(a+b)(a-b)$

- c  $(x+2+x+1)(x+2-x-1) = (2x+3)(1) = 2x+3$   
 d The answers are the same.  
 e  $4x$

- 4 a  $(x+y)(x-y)$  b  $(x+2y)(x-2y)$   
 c  $(x+3y)(x-3y)$  d  $(3x+1)(3x-1)$   
 e  $(4x+3)(4x-3)$  f  $(5x+8)(5x-8)$   
 g  $(2x+3y)(2x-3y)$  h  $(3t+2w)(3t-2w)$   
 i  $(4y+5x)(4y-5x)$
- 5 a  $(11x^3-3y^3)(11x^3+3y^3)$   
 b  $(5m^5-9n^9)(5m^5+9n^9)$   
 c  $(24p^{288}-31q^{144})(24p^{288}+31q^{144})$
- 6 a  $(3x-1)(3x+1)$  b 29 and 31
- 7 a  $(2x-7)(2x+7)$  b 3, 23 and 193

### Exercise 8L

- 1 a  $(2x+1)(x+2)$  b  $(7x+1)(x+1)$   
 c  $(4x+7)(x-1)$  d  $(3t+2)(8t+1)$   
 e  $(3t+1)(5t-1)$  f  $(4x-1)^2$   
 g  $3(y+7)(2y-3)$  h  $4(y+6)(y-4)$   
 i  $(2x+3)(4x-1)$  j  $(2t+1)(3t+5)$   
 k  $(x-6)(3x+2)$  l  $(x-5)(7x-2)$
- 2  $4x+1$  and  $3x+2$
- 3 a All the terms in the quadratic have a common factor of 6.  
 b  $6(x+2)(x+3)$ . This has the highest common factor taken out.
- 4  $(3x-1)(x+16)$ ; 1230
- 5 a  $(33x+1)(x-2)$  b  $100 \times 1 = 100$
- 6  $(3x-20)$
- 7  $12x^2 + 14x - 40$ ;  $10x+2$

### Exercise 8M

- 1  $k = \frac{T}{3}$
- 2  $y = X + 1$
- 3  $p = 3Q$
- 4  $r = \frac{A-9}{4}$
- 5 a  $m = p - t$  b  $t = p - m$
- 6  $m = gv$
- 7  $m = \sqrt{t}$
- 8  $l = \frac{P-2w}{2}$
- 9  $p = \sqrt{m-2}$
- 10 a  $-40 - 32 = -72$ ,  $-72 \div 9 = -8$ ,  $5 \times -8 = -40$   
 b  $68 - 32 = 36$ ,  $36 \div 9 = 4$ ,  $4 \times 5 = 20$   
 c  $F = \frac{9}{5}C + 32$

11 Average speeds: outward journey = 72 kph, return journey = 63 kph, taking 2 hours. He was held up for 15 minutes.

12  $r = C/2\pi$ ,  $A = \pi r^2 = \pi C^2/4\pi^2 = C^2/4\pi$

13 a  $y = \frac{5x-75}{9}$  b Pupil's own checks

c  $y = \frac{7x-40}{10}$  d Marlon is incorrect

14 a  $a = \frac{v-u}{t}$  b  $t = \frac{v-u}{a}$

15  $d = \sqrt{\frac{4A}{\pi}}$

16 a  $y = \frac{x+w}{5}$  b  $w = 5y - x$

17  $p = \sqrt{\frac{k}{2}}$

18 a  $t = u^2 - v$  b  $u = \sqrt{v+t}$

19 a  $w = K - 5n^2$  b  $n = \sqrt{\frac{K-w}{5}}$

20 a  $D = \frac{P + Y(K-U)}{3(K-U)}$  or  $\frac{\frac{P}{K-U} + Y}{3}$   
b 16

### Review questions

1 a  $20x + 16$  b  $5x + 4$

2 a  $c = \frac{R-3d-5}{7}$  b 6

3 a i 3.5 ml ii 3.7 ml iii 3.84 ml  
b i 22 ii 38 iii 90

4  $13.5 \text{ m}^2$

5 a  $2\pi r(r+h)$  b  $h = \frac{A-2\pi r^2}{2\pi r}$   
c 5 cm

6 a  $x = \frac{A-y}{0.01y}$  b  $y(1+0.01x)$   
c 38.36 g

7  $x = 5$

8 20 m

9 a  $\frac{19-4x}{5x+18}$  b  $\frac{18-4x}{5x+17}$

10 a  $2(x-8)$  b  $x(x-16)$   
c  $(x-4)(x+4)$  d  $(x-7)(x-9)$

11 a  $3 \times 15 \times 4 = 180$  b  $6x^2 - 51x + 90$

12 a i  $15x^2 - 19x - 56$  ii  $16x - 2$   
b  $162.25 \text{ cm}^2$

13 a  $4x^2 + 4x + 1$  b 441 c 437

14 a  $a^3 + 3a^2b + 3ab^2 + b^3$   
b  $8x^3 + 36x^2 + 54x + 27$  c 27.543 608

15 a i  $12x + 48$  ii  $6x^2 + 48x + 94$   
iii  $x^3 + 12x^2 + 47x + 60$   
b surface area =  $1348 \text{ cm}^2$  and volume =  $3360 \text{ cm}^3$

16 a  $12x^2 - xy - 35y^2$  b  $(3x+7y)(2x-5y)$

17  $\pm 10, \pm 11, \pm 14, \pm 25$

18 a  $(2x+3)(x+2)$   
b i 276 ii 20 706 iii 6.0702

## Chapter 9 – Geometry and measures: Length, area and volume

### Exercise 9A

1 a 8 cm, 25.1 cm,  $50.3 \text{ cm}^2$   
b 5.2 m, 16.3 m,  $21.2 \text{ m}^2$   
c 6 cm, 37.7 cm,  $113 \text{ cm}^2$   
d 1.6 m, 10.1 m,  $8.04 \text{ m}^2$

2 a  $5\pi \text{ cm}$  b  $8\pi \text{ cm}$   
c  $18\pi \text{ m}$  d  $12\pi \text{ cm}$

3 a  $25\pi \text{ cm}^2$  b  $36\pi \text{ cm}^2$   
c  $100\pi \text{ cm}^2$  d  $0.25\pi \text{ m}^2$

4 8.80 m

5 4 complete revolutions

6 1p :  $3.1 \text{ cm}^2$ , 2p :  $5.3 \text{ cm}^2$ , 5p :  $2.3 \text{ cm}^2$ ,  
10p :  $4.5 \text{ cm}^2$

7 0.83 m

8 38.6 cm

9 Claim is correct (ratio of the areas is just over 1.5 : 1)

10 a  $18\pi \text{ cm}^2$  b  $4\pi \text{ cm}^2$

11  $9\pi \text{ cm}^2$

12 Divide 31.3 by  $\pi$  to get about 10 m. This is the diameter of the tree. Is your classroom smaller than  $10 \text{ m} \times 10 \text{ m}$ ? It probably isn't, but you need to check.

13 45 complete revolutions

14 a  $2\pi$   
b i  $8\pi$  ii  $18\pi$  iii  $32\pi$

c  $A = 2\pi \times r^2$

### Exercise 9B

1 a  $96 \text{ cm}^2$  b  $70 \text{ cm}^2$  c  $20 \text{ cm}^2$   
d  $125 \text{ cm}^2$  e  $10 \text{ cm}^2$  f  $112 \text{ m}^2$

2 No, she has used the sloping side instead of the perpendicular height. It should be  $6 \times 4 = 24 \text{ cm}^2$

3 Each parallelogram has an area of  $30 \text{ cm}^2$ . The height of each is 5 cm so the length of each must be 6 cm.  $x = 6 + 4 + 6 = 16 \text{ cm}$  so Freya is incorrect.



- 4 a 500 cm<sup>2</sup> b 3 × 5 = 15

### Exercise 9C

- 1 a 30 cm<sup>2</sup> b 77 cm<sup>2</sup> c 24 cm<sup>2</sup>  
 d 42 cm<sup>2</sup> e 40 m<sup>2</sup> f 6 cm  
 g 3 cm h 10 cm
- 2 Area = 15 cm<sup>2</sup>
- 3 a 27.5 cm, 36.25 cm<sup>2</sup>  
 b 33.4 cm, 61.2 cm<sup>2</sup>  
 c 38.5 m, 90 m<sup>2</sup>
- 4 The area of the parallelogram is  $\frac{(a+b)}{h}$ . This is the same as two trapezia.
- 5 Two of 20 cm<sup>2</sup> and two of 16 cm<sup>2</sup>
- 6 a 57 m<sup>2</sup> b 702.5 cm<sup>2</sup> c 84 m<sup>2</sup>
- 7 trapezium = 56, square area = 9, shaded area = 56 - 9 = 47 cm<sup>2</sup>
- 8 4, because the total area doubled is about 32 m<sup>2</sup>
- 9 80.2%
- 10 1 100 000 km<sup>2</sup>
- 11 160 cm<sup>2</sup>
- 12 a many possible correct answers, eg base 6 cm, top 4cm, height 1.6π. Shaded area is 8π, trapezium must be the same  
 b the dimensions cannot be exact due to the value of π in the area of the circle

### Exercise 9D

- 1 a i 5.59 cm ii 22.3 cm<sup>2</sup>  
 b i 8.29 cm ii 20.7 cm<sup>2</sup>  
 c i 16.3 cm ii 98.0 cm<sup>2</sup>  
 d i 15.9 cm ii 55.6 cm<sup>2</sup>
- 2 2π cm, 6π cm<sup>2</sup>
- 3 a 73.8 cm b 20.3 cm
- 4 area of sector =  $\frac{1}{4} \times \pi \times 8^2 = 16\pi$ ,  
 area of circle =  $\pi \times 4^2 = 16\pi$
- 5 a 107 cm<sup>2</sup>  
 b 173 cm<sup>2</sup>
- 6 43.6 cm
- 7 a  $\frac{180}{\pi}$   
 b If arc length is 10 cm, distance along chord joining the two points of the sector on the circumference will be less than 10 cm, so angle at centre will be less than 60°
- 8 a 66.8° b 10 cm<sup>2</sup>
- 9 Let sector have radius R and arc length C, the angle of the sector is found by  
 $\theta = \frac{360 \times C}{2 \times \pi \times R}$

$$\text{and so the area will be } \frac{360 \times C \times \pi \times R^2}{2 \times \pi \times R \times 360}$$

$$= \frac{CR}{2}$$

- 10  $(36\pi - 72)$  cm<sup>2</sup>
- 11 36.5 cm<sup>2</sup>
- 12 16 cm (15.7)
- 13 Each square has side length of  $r$   
 Shaded part of square  $X = r^2 - \frac{1}{4}\pi r^2$   
 $= r^2(1 - \frac{1}{4}\pi)$   
 In square Y, the four quarter circles will join together to give an area of radius  $\frac{1}{2}r$ , so shaded area  
 in Y =  $r^2 - \pi\left(\frac{r}{2}\right)^2 = r^2 - \frac{1}{4}\pi r^2 = r^2(1 - \frac{1}{4}\pi)$ ,  
 which is the same as square X.

### Exercise 9E

- 1 a i 21 cm<sup>2</sup> ii 63 cm<sup>3</sup>  
 b i 48 cm<sup>2</sup> ii 432 cm<sup>3</sup>  
 c i 36 m<sup>2</sup> ii 324 m<sup>3</sup>
- 2 a 432 m<sup>3</sup> b 225 m<sup>3</sup> c 1332 m<sup>3</sup>
- 3 a A cross-section parallel to the side of the pool always has the same shape.  
 b About  $3\frac{1}{2}$  hours
- 4  $V = \frac{1}{2}(1.5 + 3) \times 1.7 \times 2 = 7.65$  m<sup>3</sup>
- 5  $27 = 3 \times 3 \times 3$ ,  $27 + 37 = 64 = 4^3$ ,  $4 - 3 = 1$ . Hence the side length is 1 small cube longer, hence 2 cm longer
- 6 a i 21 cm<sup>3</sup> ii 210 cm<sup>3</sup>  
 b i 54 cm<sup>2</sup> ii 270 cm<sup>2</sup>
- 7 146 cm<sup>3</sup>
- 8 78 m<sup>3</sup> (78.3 m<sup>3</sup>)
- 9 327 litres
- 10 10.2 tonnes
- 11 She was silly because 188160 is simply all the numbers multiplied together. The volume is 672 cm<sup>2</sup>

### Exercise 9F

- 1 a i 226 cm<sup>3</sup> ii 207 cm<sup>2</sup>  
 b i 14.9 cm<sup>3</sup> ii 61.3 cm<sup>2</sup>  
 c i 346 cm<sup>3</sup> ii 275 cm<sup>2</sup>  
 d i 1060 cm<sup>3</sup> ii 636 cm<sup>2</sup>
- 2 a i 72π cm<sup>3</sup> ii 48π cm<sup>2</sup>  
 b i 112π cm<sup>3</sup> ii 56π cm<sup>2</sup>  
 c i 180π cm<sup>3</sup> ii 60π cm<sup>2</sup>  
 d i 600π m<sup>3</sup> ii 120π m<sup>2</sup>

- 3 Volume =  $\pi \times (0.3)^2 \times 4.2 = 0.378\pi$   
Cost =  $0.378 \times \pi \times £67.50 = £80.16$  which is £80 to 2sf
- 4 1.23 tonnes
- 5 Label should be less than 10.5 cm wide so that it fits the can and does not overlap the rim and more than 23.3 cm long to allow an overlap.
- 6 Volume =  $\pi \times 32.5^2 \times 100 = 331830.7 \text{ cm}^3$   
1 litre =  $1000 \text{ cm}^3$   
volume =  $331830.7 \div 1000 = 331.8307$  litres = 332 litres (3 sf)
- 7 There is no right answer. Students could start with the dimensions of a real can. Often drinks cans are not exactly cylindrical. One possible answer is height of 6.6 cm and diameter of 8 cm.
- 8  $7.78 \text{ g/cm}^3$
- 9 About 127 cm
- 10 A diameter of 10 cm and a length of 5 cm give a volume close to  $400 \text{ cm}^3$  (0.4 litres).

### Exercise 9G

- 1 a  $56 \text{ cm}^3$  b  $168 \text{ cm}^3$  c  $1040 \text{ cm}^3$   
d  $84 \text{ cm}^3$  e  $160 \text{ cm}^3$
- 2  $\frac{1}{3}$  base area  $\times h = \frac{1}{3} \times 9 \times 9 \times 10 = 270 \text{ cm}^3$
- 3 a Put the apexes of the pyramids together. The 6 square bases will then form a cube.  
b If the side of the base is  $a$  then the height will be  $\frac{1}{2}a$ .  
Total volume of the 6 pyramids is  $a^3$ .  
Volume of one pyramid is  $\frac{1}{6}a^3 =$   
 $\frac{1}{3} \times \frac{1}{2} \times a \times a^2 = \frac{1}{3} \times \text{height} \times \text{base area}$
- 4 6.9 m ( $\frac{1}{3}$  height of pyramid)
- 5 a  $73.3 \text{ m}^3$  b  $45 \text{ m}^3$  c  $3250 \text{ cm}^3$
- 6 208 g
- 7 1.5 g
- 8  $\frac{1}{3} \times 6.4 \times 6.4 \times H = 81.3$   
So  $H = \frac{3 \times 81.3}{6.4 \times 6.4} = 5.954 = 6.0$  (2sf)
- 9 14.4 cm
- 10 Volume of pyramid =  $\frac{1}{3} \times 6 \times 9 \times 15 = 270 \text{ cm}^3$   
Volume of part cut off top =  $\frac{1}{3} \times 3 \times 2 \times 5 = 10 \text{ cm}^3$   
So frustum =  $270 - 10 = 260 \text{ cm}^3$   
Hence  $\frac{\text{volume of frustum}}{\text{volume of pyramid}} = \frac{260}{270} = \frac{26}{27}$   
therefore, Hannah is correct.

### Exercise 9H

- 1 a i  $3560 \text{ cm}^3$  ii  $1430 \text{ cm}^2$   
b i  $314 \text{ cm}^3$  ii  $283 \text{ cm}^2$   
c i  $1020 \text{ cm}^3$  ii  $679 \text{ cm}^2$
- 2 935 g
- 3 Total area =  $\pi r l + \pi r^2 = \pi \times 3 \times l + \pi \times 3^2$   
 $= (3l + 9)\pi = 24\pi$   
So  $3l + 9 = 24$ , so  $3l = 24 - 9 = 15$   
 $l = \frac{15}{3} = 5$
- 4 a  $816\pi \text{ cm}^3$  b  $720\pi \text{ mm}^3$
- 5  $24\pi \text{ cm}^2$
- 6 a 4 cm b 6 cm  
c Various answers, e.g.  $60^\circ$  gives 2 cm,  $240^\circ$  gives 8 cm
- 7 If radius of base is  $r$ , slant height is  $2r$ .  
Area of curved surface =  $\pi r \times 2r = 2\pi r^2$ , area of base =  $\pi r^2$
- 8  $2.7 \text{ g/cm}^3$
- 9 2.81 cm
- 10  $252\pi \text{ cm}^2$

### Exercise 9I

- 1 a  $36\pi \text{ cm}^3$  b  $288\pi \text{ cm}^3$  c  $1330\pi \text{ cm}^3$  (3 sf)
- 2 a  $36\pi \text{ cm}^2$  b  $100\pi \text{ cm}^2$  c  $196\pi \text{ cm}^2$
- 3  $65\,400 \text{ cm}^3$ ,  $7850 \text{ cm}^2$
- 4 a  $1960 \text{ cm}^2$  (to 3sf) b  $7444 \text{ cm}^3$  (to nearest unit)
- 5 125 cm
- 6 6232
- 7 7.8 cm
- 8 a The surface area, because this is the amount of material (leather or plastic) needed to make the ball  
b Surface area can vary from about  $1470 \text{ cm}^2$  to  $1560 \text{ cm}^2$ , difference of about  $90 \text{ cm}^2$ . This seems surprisingly large.
- 9 48%
- 10 Radius of sphere = base radius of cylinder =  $r$ ,  
height of cylinder =  $2r$  Curved surface area of cylinder = circumference  $\times$  height =  $2\pi r \times 2r = 4\pi r^2$   
= surface area of sphere

### Review questions

- 1  $29.4 \text{ cm}^2$
- 2  $721 \text{ cm}^2$
- 3  $5740 \text{ cm}^3$  (to 3sf)
- 4 610 g (2sf)
- 5 16.6 cm
- 6 360 g

7 56.5 cm

8 Call length of square  $2x$ , so that radius of arcs is  $x$ .  
Then area of square =  $4x^2$

Area of each semicircle =  $\frac{1}{2} \pi x^2$  so area of 4 semi  
circles is  $2\pi x^2$

Area of shaded part is: area of 4 semicircles – area  
of square =  $(2\pi - 4)x^2$

So percentage shaded =  $\frac{2\pi - 4}{4x^2} x^2 \times 100 =$

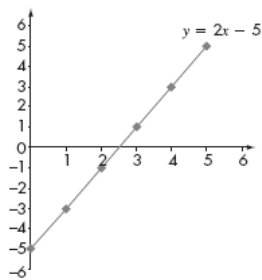
$$\frac{2\pi - 4}{4} \times 100 = 57\%$$

9  $\frac{1}{3}$

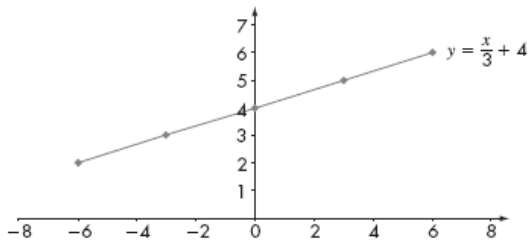
## Chapter 10 – Algebra: Linear graphs

### Exercise 10A

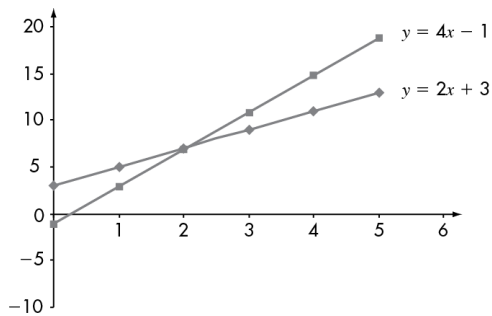
1



2

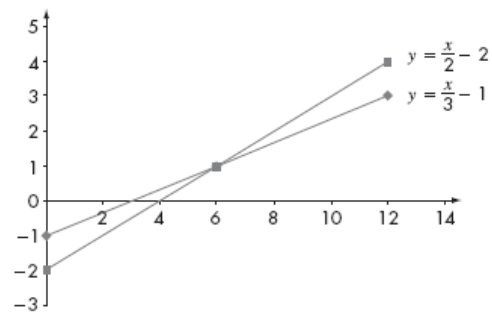


3 a

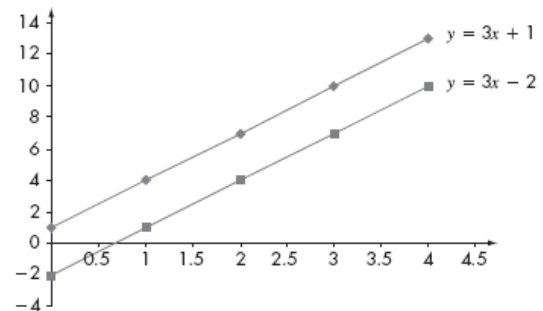


b (2, 7)

4 a



b



4 c lines in part a intersect at (6, 1), lines in part b  
don't intersect because they are parallel

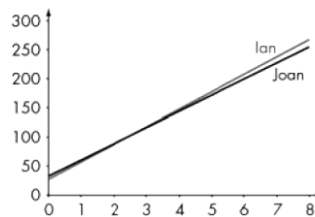
5 a Line isn't straight

b

x	-3	-2	-1	0	1	2	3
y	-5	-3	-1	1	3	5	7

Correct line drawn

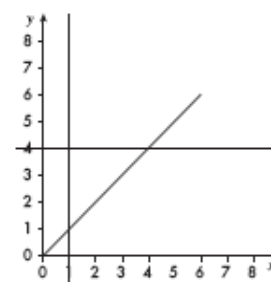
6 a



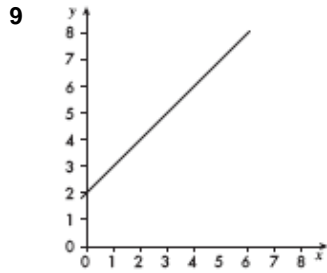
b Ian, Ian only charges £85, whilst Joan charges  
£90 for a 2-hour job.

7 a Jada's method

8 a



b 4.5 units squared

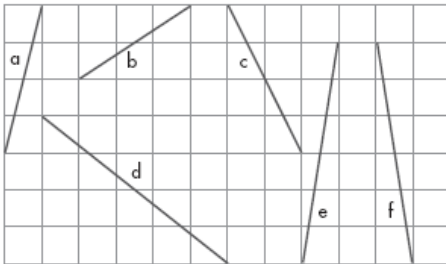


10 18 units squared

### Exercise 10B

- 1 a 2 b  $\frac{1}{3}$  c -3 d 1 e -2  
f  $-\frac{1}{3}$  g 5 h -5 i  $\frac{1}{5}$  j  $-\frac{3}{4}$

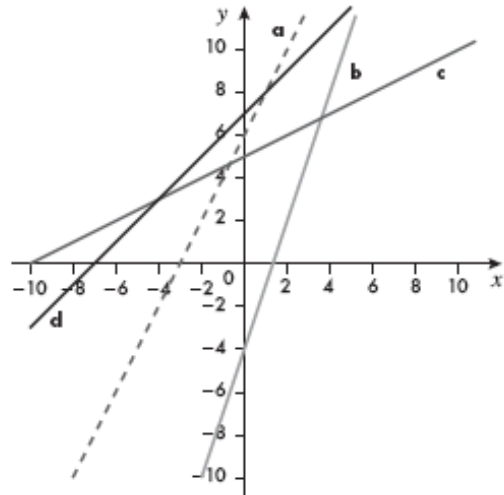
2



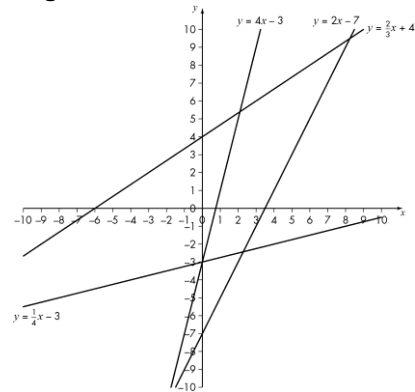
- 3 a Both answers are correct  
b generally the bigger the triangle the more accurate the answer, so Brianna
- 4 a ladder might slip  
b ladder might topple  
c i, ii, v and vi satisfy the safety regulations; iii and iv do not
- 5 a 0.5 b 0.4 c 0.2 d 0.1 e 0
- 6 a  $1\frac{2}{3}$  b 2 c  $3\frac{1}{3}$  d 10 e  $\infty$
- 7 Raisa has misread the scales. The second line has four times the gradient (2.4) of the first (0.6)
- 8 a  $\frac{3}{8}$   
b  $\frac{2}{5}$   
c Although the puzzle appears to be a right-angled triangle, because the gradients of the smaller triangles are different there is actually a bend in the large hypotenuse, so it is actually a quadrilateral. In the first diagram it has a concave angle and in the second diagram the equivalent angle is convex, and the area of the square hole is spread out between them.
- 9 0, 2, -1,  $\frac{1}{2}$ ,  $-\frac{3}{2}$

### Exercise 10C

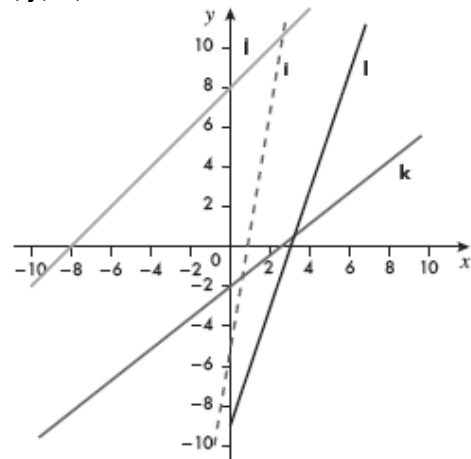
1 a, b, c, d



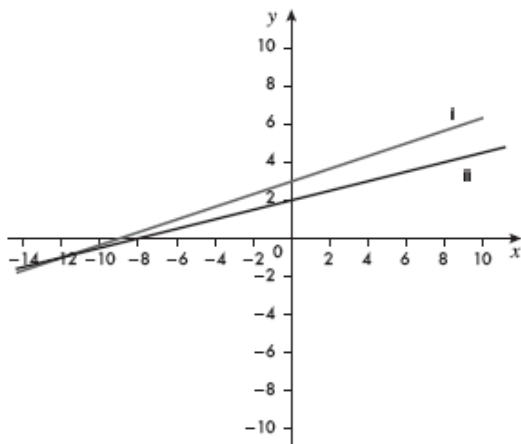
e, f, g, h



i, j, k, l



2 a



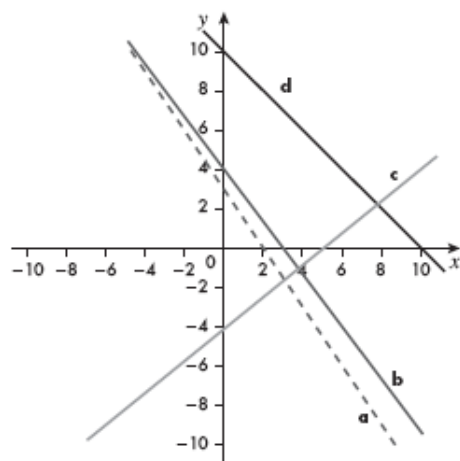
b  $(-12, -1)$

- 3 a They have the same gradient (3).  
 b They intercept the  $y$ -axis at the same point  $(0, -2)$ .  
 c  $(-1, -4)$

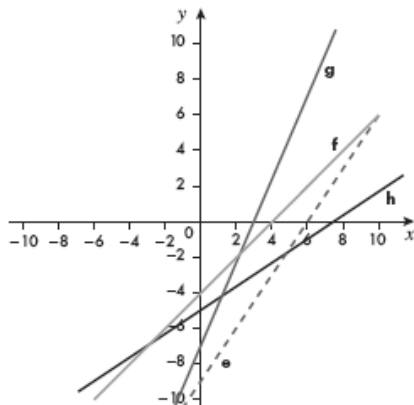
- 4 a  $-2$       b  $\frac{1}{2}$       c  $90^\circ$   
 d Negative reciprocal      e  $-\frac{1}{3}$

### Exercise 10D

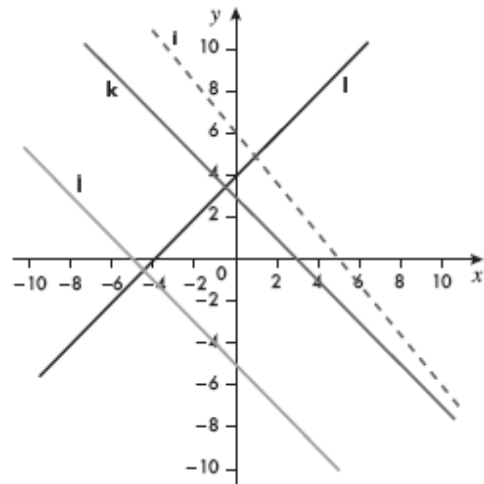
1 a, b, c, d



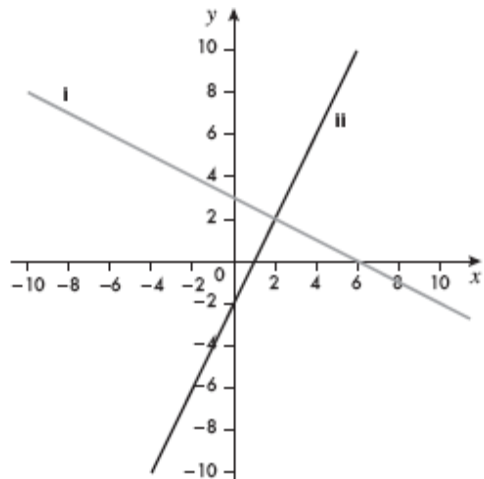
e, f, g, h



i, j, k, l



2 a



b  $(2, 2)$

- 3 a Intersect at  $(6, 0)$   
 b Intersect at  $(0, -3)$   
 c Parallel  
 d  $-2x + 9y = 18$

4 a vi    b iii    c v    d ii    e i    f iv

5  $y = -6x$ ;  $y = 3x + 4$ ;  $2y - 5x = 10$ ;  $2y - x = 7$ ;  $y = 4$

6  $3x + 2y = 18$  and  $y = 9 - x$

- 7 a i  $x = 3$     ii  $x - y = 4$     iii  $y = -3$   
 iv  $x + y = -4$     v  $x = -3$     vi  $y = x + 4$   
 b i  $-3$     ii  $\frac{1}{3}$     iii  $-\frac{1}{3}$

8 Cover-up method for  $2x + y = 10$  and gradient-intercept method for  $y = 11 - 2x$

### Exercise 10E

- 1 a  $y = \frac{7}{5}x - 2$  or  $5y = 7x - 10$   
 b  $y = 2x$     c  $2y = x + 6$

- 2 a i  $y = 2x + 1$ ,  $y = -2x + 1$   
 ii Reflection in  $y$ -axis (and  $y = 1$ )  
 iii Different sign

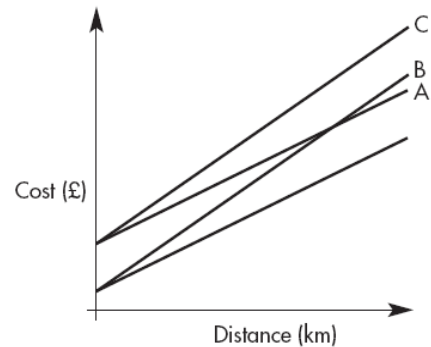
- b i**  $5y = 2x - 5$ ,  $5y = -2x - 5$   
**ii** Reflection in  $y$ -axis (and  $y = -1$ )  
**iii** Different sign
- c i**  $y = x + 1$ ,  $y = -x + 1$   
**ii** Reflection in  $y$ -axis (and  $y = 1$ )  
**iii** Different sign
- 3 a**  $x$ -coordinates go from  $2 \rightarrow 1 \rightarrow 0$  and  
 $y$ -coordinates go from  $5 \rightarrow 3 \rightarrow 1$ .  
**b**  $x$ -step between the points is 1 and  $y$ -step is 2.  
**c**  $y = 3x + 2$
- 4 a**  $y = -x + 1$   
**b**  $5y = -2x - 5$   
**c**  $y = -\frac{3}{2}x - 3$  or  $2y = -3x - 6$
- 5 a i**  $2y = -x + 1$ ,  $y = -2x + 1$   
**ii** Reflection in  $x = y$   
**iii** Reciprocal of each other  
**b i**  $2y = 5x + 5$ ,  $5y = 2x - 5$   
**ii** Reflection in  $x = y$   
**iii** Reciprocal of each other  
**c i**  $y = 2$ ,  $x = 2$   
**ii** Reflection in  $x = y$   
**iii** Reciprocal of each other (reciprocal of zero is infinity)
- 6** All of the lines except  $y = \frac{1}{4}x + 9$
- 7 a**  $y = -3x + 5$       **b**  $y = 2x - 4$   
**c**  $y = 8x - 3$       **d**  $y = 25 - 2x$   
**e**  $y = \frac{2}{3}x - 1$
- 8**  $5x + 6y = 30$
- 9** Chris is correct. The equation of the line is  $y = \frac{1}{2}x + 2$  and  $(12, 8)$  satisfies the equation
- 10 a i**  $x + y = 100$       **ii**  $k = 1$   
**b i**  $x = 46$       **ii**  $k = 46$   
**c i**  $y = 2x + 1$       **ii**  $k = 60$   
**d i**  $y = x + 19$       **ii**  $k = -17$
- 11**  $(4, 11)$

### Exercise 10F

- 1 a** Anya: CabCo £8.50, YellaCabs £8.40, so YellaCabs is best; Bettina: CabCo £11.50, YellaCabs £11.60, so CabCo is best; Calista: CabCo £10, YellaCabs £10, so either  
**b** If they shared a cab, the shortest distance is 16 km, which would cost £14.50 with CabCo and £14.80 with Yellacabs.
- 2 a i**  $8\frac{1}{4}$  kg      **ii**  $2\frac{1}{4}$  kg  
**iii** 9 lb      **iv** 22 lb  
**b** 2.2 lb  
**c** Read off the value for 12 lb (5.4 kg) and multiply this by 4 (21.6 kg)
- 3 a**  $32^\circ \text{ F}$   
**b**  $\frac{9}{5}$  (Take gradient at  $C = 10^\circ$  and  $30^\circ$ )  
**c**  $F = \frac{9}{5}C + 32$
- 4 a** 0.07 (Take gradient at  $U = 0$  and 500)  
**b** £10

**c**  $C = £(10 + 0.07U)$  or Charge = £10 + 7p/unit

- 5 a**  $\$1900 - \$1400 = \$500$   
**b i** \\$7500      **ii** £3783
- 6**



- 7**  $y = 2x + 15$        $0 < x \leq 5$   
 $y = x + 20$        $5 < x \leq 12$   
 $y = \frac{1}{2}x + 26$        $12 < x \leq 22$

### Exercise 10G

- 1**  $(4, 1)$   
**2**  $(2, 3)$   
**3**  $(3, 10)$   
**4**  $(-2, 6)$   
**5**  $(-6, -9)$   
**6**  $(1, -1)$   
**7**  $(2, 6)$   
**8**  $(2, 8)$   
**9**  $(7\frac{1}{2}, 3\frac{1}{2})$   
**10**  $x + 2y = 9.5$ ,  $2x + y = 8.5$   
 Graphs intersect at  $(2.5, 3.5)$ , so a cheesecake costs £2.50 and a gâteau costs £3.50.  
**11 a** P and R      **b** R and S  
**c** P and Q      **d** Q and S  
**12**  $(0, 0)$ ,  $(-3, 3)$ ,  $(-3, -3)$ ,  $(-3, 2)$ ,  $(-2, 2)$ ,  $(2, 2)$   
**13 a** no solutions, lines are parallel  
**b** infinite solutions, lines are same  
**c** one solution, lines intersect once

### Exercise 10H

- 1 a** Line A does not pass through  $(0, 1)$ .  
**b** Line C is perpendicular to the other two.  
**c** (i)
- 2 a**  $-\frac{1}{2}$     **b**  $\frac{1}{3}$     **c** -2    **d**  $\frac{3}{2}$     **e**  $-\frac{2}{3}$     **f**  $-\frac{3}{4}$
- 3**  $y = 3x + 5$ ,  $x + 3y = 10$ ,  $y = 8 - \frac{1}{3}x$ ,  $y = 3(x + 2)$
- 4**  $x = 6$  and  $y = -2$   
 $x + y = 5$  and  $y = x + 4$

$$y = 8x - 9 \text{ and } y = -\frac{1}{8}x + 6$$

$$2y = x + 4 \text{ and } 2x + y = 9$$

$$5y = 2x + 15 \text{ and } 2y + 5x = 2$$

$$y = 0.1x + 2 \text{ and } y = 33 - 10x$$

5 a  $y = \frac{1}{2}x - 2$       b  $y = -x + 3$   
 c  $y = -\frac{1}{3}x - 1$       d  $y = 3x + 5$

6 a  $-4$   
 b  $\frac{1}{4}$   
 c  $(11, 7)$   
 d  $y = \frac{1}{4}x + c$

Substitute in  $(11, 7)$  and solve to get  $c = \frac{17}{4}$ , so  
 $4y - x = 17$

7  $y = -\frac{1}{4}x + 2$

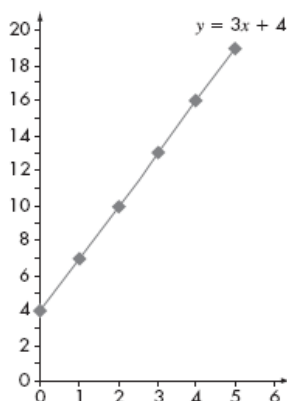
8 i a AB:  $-\frac{1}{5}$ , BC: 1, CD:  $-\frac{1}{5}$ , DA: 1  
 b Parallelogram (two pairs of parallel sides)  
 ii a AB:  $\frac{2}{3}$ , BC:  $-\frac{3}{2}$ , CD:  $\frac{2}{3}$ , DA:  $-\frac{3}{2}$   
 b Rectangle (two pairs of perpendicular sides)  
 iii a AB:  $\frac{2}{5}$ , BC:  $\frac{1}{4}$ , CD:  $\frac{2}{5}$ , DA: 1  
 b Trapezium (one pair of parallel sides)

9  $y = -\frac{1}{2}x + 5$

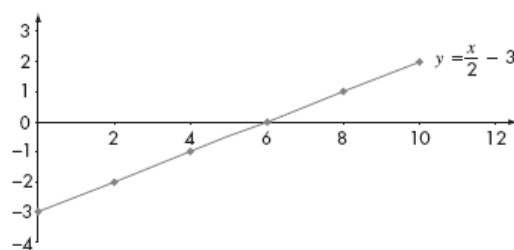
10 a  $y = 3x - 6$   
 b Bisector of AB is  $y = -2x + 9$ , bisector of AC is  
 $y = \frac{1}{2}x + \frac{3}{2}$ , solving these equations shows the  
 lines intersect at  $(3, 3)$ .  
 c  $(3, 3)$  lies on  $y = 3x - 6$  because  $(3 \times 3) - 6 = 3$   
 11  $(3, 10)$

## Review questions

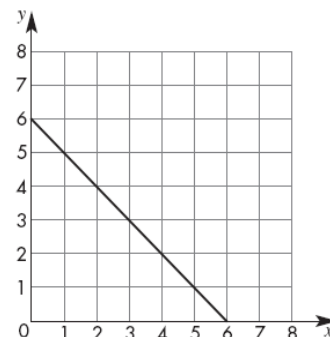
1



2



3



4 a  $\frac{1}{2}$  (Take gradient at  $N = 0$  and  $500$ )  
 b £50  
 c  $C = £(50 + \frac{N}{2})$  or  $£50 + 50p/\text{person}$

5 a  $\frac{1}{10}$   
 b 24.5 cm  
 c 0.1 cm or 1 mm  
 d  $L = 24.5 + \frac{W}{10}$  or Length =  $24.5 + 1 \text{ mm/kg}$

6 a  $(5, 5)$       b  $(1, 5)$       c  $(3, 16)$

7 a 2      b  $y = 2x + 2$       c  $y = -\frac{1}{2}x + 7$

8 a Reflection in  $y$ -axis ( $x = 0$ ); reflection in  $y = 1$   
 (rotations also possible)  
 b Rotation  $90^\circ$  clockwise; rotation  $90^\circ$   
 anticlockwise (reflections also possible)

9 30 square units

10 36 square units

11  $(7, 1)$

## Chapter 11 – Geometry: Right-angled triangles

### Exercise 11A

- Students' own diagrams
- Possible answers include multiples of 3, 4, 5;  
 multiples of 5, 12, 13; multiples of 7, 24, 25;  
 multiples of 8, 15, 17

- 3 a 10.3 cm    b 5.9 cm    c 8.5 cm  
 d 20.6 cm    e 18.6 cm    f 17.5 cm  
 g 13 cm    h 5 cm
- 4 a  $\sqrt{8}$ ,  $\sqrt{12}$ ,  $\sqrt{16}$   
 b Add 4 to 16 to give  $H_4$  as  $\sqrt{20}$
- 5 The square in the first diagram and the sum of the two squares in the second have the same area.

### Exercise 11B

- 1 a 15 cm    b 14.7 cm  
 c 6.3 cm    d 18.3 cm
- 2 a 20.8 m    b 15.5 cm  
 c 15.5 m    d 12.4 cm
- 3 a 5 m    b 6 m  
 c 3 m    d 50 cm
- 4 There are infinite possibilities, e.g. any multiple of 3, 4, 5 such as 6, 8, 10; 9, 12, 15; 12, 16, 20; multiples of 5, 12, 13; multiples of 7, 24, 25 and of 8, 15, 17.
- 5  $498.4 \text{ cm}^2$
- 6 Any of (0, 0), (5, 5), (2, 0), (5, 3), (2, 8), (0, 8), (-3, 3), (-3, 5) are the most likely points
- 7 Use Pythagoras' theorem to find a few possible dimensions of the rectangle, then plot a graph of one side length against the area. You will see that 50 is the highest the area will ever get to.
- 8 The large square is 17 by 17 giving 289 square units.  
 The red and yellow triangles all have shorter lengths of 5 and 12, with an area of 30 square units.  
 The area of the inner square (green and yellow) must be  $289 - 4 \times 30 = 169$ , so the hypotenuse of the yellow triangles must be  $\sqrt{169} = 13$   
 You can see that  $5^2 + 12^2 = 13^2$

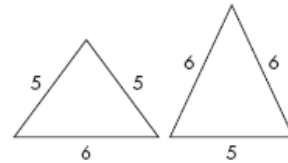
### Exercise 11C

- 1 No. The foot of the ladder is about 6.6 m from the wall.
- 2 2.06 m
- 3 11.3 m
- 4 About 17 minutes, assuming it travels at the same speed.
- 5  $127 \text{ m} - 99.6 \text{ m} = 27.4 \text{ m}$
- 6 4.58 m
- 7 a 3.87 m    b 1.74 m
- 8 3.16 m
- 9 This creates a right-angled triangle with two short sides of 5 and 12. Use Pythagoras' theorem to show length of line =  $\sqrt{5^2 + 12^2} = 13$

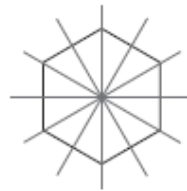
- 10 a 4.85 m  
 b 4.83 m (There is only a small difference.)
- 11 Yes, because  $24^2 + 7^2 = 25^2$
- 12 6 cm
- 13 He is partly correct. The perimeter must be larger than 20 cm or the rectangle has no width, and the area is largest when it's a square, giving a perimeter of 28.3 cm (3sf). So he should have said the perimeter is between 20 and 28.3 cm.

### Exercise 11D

- 1 a  $32.2 \text{ cm}^2$     b  $2.83 \text{ cm}^2$     c  $50.0 \text{ cm}^2$
- 2  $22.2 \text{ cm}^2$
- 3  $15.6 \text{ cm}^2$
- 4 a



- b The areas are  $12 \text{ cm}^2$  and  $13.6 \text{ cm}^2$  respectively, so triangle with 6 cm, 6 cm, 5 cm sides has the greater area.
- 5 a b  $166.3 \text{ cm}^2$



- 6  $259.8 \text{ cm}^2$
- 7 a No, areas vary from  $24.5 \text{ cm}^2$  to  $27.7 \text{ cm}^2$   
 b No, equilateral triangle gives the largest area.  
 c The closer the isosceles triangle gets to an equilateral triangle the larger its area becomes.
- 8 Show the right-angled triangle made with hypotenuse 6.5 m and base  $7.4 \div 2 = 3.7$ , giving the height of the triangle as 5.344 cm. Use area =  $\frac{1}{2} \times 7.4 \times 5.344$  to give 19.7733 which rounds to  $19.8 \text{ m}^2$  (3 sf)
- 9  $48 \text{ cm}^2$
- 10 a 10 cm    b 26 cm    c 9.6 cm
- 11 6 or 8 cm
- 12 Andrew didn't round off any answers until the last calculation, and Olly used a rounded off value to find an intermediate result



### Exercise 11E

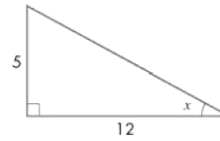
- 1 a i 14.4 cm ii 13 cm iii 9.4 cm  
b 15.3 cm
- 2 No, 6.6 m is longest length
- 3 a 20.6 cm b 15.0 cm
- 4 a 8.49 m b 9 m
- 5  $10^2 + 10^2 + 10^2 = 300$ ,  $\sqrt{300} = 17.3$  cm (3sf)
- 6 20.6 cm
- 7 a 11.3 cm b 7 cm c 8.06 cm
- 8  $AM = \sqrt{(22.5^2 + 15^2 + 40^2)} = 48.283 = 48.3$  cm (3sf)
- 9 21.3 cm

### Exercise 11F

- 1 a 0.682 b 0.829 c 0.922  
d 1 e 0.707 f 0.342  
g 0.375 h 0
- 2 a 0.731 b 0.559 c 0.388  
d 0 e 0.707 f 0.940  
g 0.927 h 1
- 3  $45^\circ$
- 4 a i 0.574 ii 0.574  
b i 0.208 ii 0.208  
c i 0.391 ii 0.391  
d Same  
e i  $\sin 15^\circ$  is the same as  $\cos 75^\circ$   
ii  $\cos 82^\circ$  is the same as  $\sin 8^\circ$   
iii  $\sin x$  is the same as  $\cos (90^\circ - x)$
- 5 a 0.933 b 1.48 c 2.38  
d Infinite (calculator will give a maths error)  
e 1 f 0.364 g 0.404  
h 0
- 6 a 0.956 b 0.899 c 2.16  
d 0.999 e 0.819 f 0.577  
g 0.469 h 0.996
- 7 Has values  $> 1$
- 8 a 4.53 b 4.46 c 6 d 0
- 9 a 10.7 b 5.40  
c 68.58 d 0
- 10 a 3.56 b 8.96 c 28.4 d 8.91
- 11 a 5.61 b 11.3 c 6 d 10
- 12 a 1.46 b 7.77 c 0.087 d 9.33
- 13 a 7.73 b 48.6 c 2.28 d 15.2
- 14 a 29.9 b 44.8 c 20.3 d 2.38
- 15 a  $\frac{4}{5}, \frac{3}{5}, \frac{4}{3}$   
b  $\frac{7}{25}, \frac{24}{25}, \frac{7}{24}$

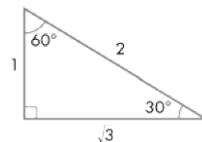
- 16 You should have drawn a right angled triangle as here.  $H = 13$  since this is a 5, 12, 13 Pythagorean triple. See that opposite = 5 and adjacent = 12.

Hence  $\sin x = \frac{O}{H} = \frac{5}{13}$  and  $\cos x = \frac{12}{13}$



### Exercise 11G

- 1 a  $30^\circ$  b  $51.7^\circ$  c  $39.8^\circ$   
d  $61.3^\circ$  e  $87.4^\circ$  f  $45.0^\circ$
- 2 a  $60^\circ$  b  $50.2^\circ$  c  $2.6^\circ$   
d  $45.0^\circ$  e  $78.5^\circ$  f  $45.6^\circ$
- 3 a  $31.0^\circ$  b  $20.8^\circ$  c  $41.8^\circ$   
d  $46.4^\circ$  e  $69.5^\circ$  f  $77.1^\circ$
- 4 a  $53.1^\circ$  b  $41.8^\circ$  c  $44.4^\circ$   
d  $56.4^\circ$  e  $2.4^\circ$  f  $22.6^\circ$
- 5 a  $36.9^\circ$  b  $48.2^\circ$  c  $45.6^\circ$   
d  $33.6^\circ$  e  $87.6^\circ$  f  $67.4^\circ$
- 6 a  $31.0^\circ$  b  $37.9^\circ$  c  $15.9^\circ$   
d  $60.9^\circ$  e  $57.5^\circ$  f  $50.2^\circ$
- 7 a Error message b largest value 1  
c smallest value  $-1$
- 8 a i  $17.5^\circ$  ii  $72.5^\circ$  iii  $90^\circ$   
b Yes
- 9



Adj = 1, Hyp = 2, hence Opp =  $\sqrt{(2^2 - 1^2)} = \sqrt{3}$

- a 30 is the other acute angle in the triangle and so for 30, opp = 1 and adj =  $\sqrt{3}$ , hence  $\tan 30 = \frac{1}{\sqrt{3}}$
- b  $\tan 60 = \sqrt{3}$
- c  $\sin 60 = \frac{\sqrt{3}}{2} = 0.87$
- d  $\cos 30 = \frac{\sqrt{3}}{2} = 0.87$
- e  $\sin 30 = \frac{1}{2}$
- 10 Adj = 1, Opp = 1, hence Hyp =  $\sqrt{(1^2 + 1^2)} = \sqrt{2}$ 
  - a  $\sin 45 = \frac{1}{\sqrt{2}}$
  - b  $\cos 45 = \frac{1}{\sqrt{2}}$

### Exercise 11H

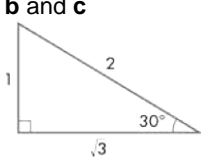
- 1 a  $17.5^\circ$  b  $22.0^\circ$  c  $32.2^\circ$
- 2 a 5.29 cm b 5.75 cm c 13.2 cm
- 3 a 4.57 cm b 6.86 cm c 100 cm

- 4 a 5.12 cm b 9.77 cm  
c 11.7 cm d 15.5 cm

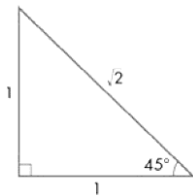
- 5 a  $47.2^\circ$  b 5.42 cm  
c 13.7 cm d  $38.0^\circ$

- 6 a 6 b 15 c 30

- 7 a  $\frac{1}{2}$   
b and c



- 8 a  $\frac{1}{\sqrt{2}}$  b

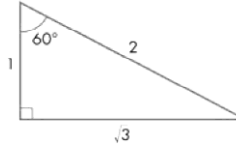


- 4 a 9.02 cm b 7.51 cm  
c 7.14 cm d 8.90 cm

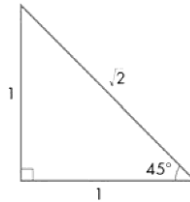
- 5 a 13.7 cm b  $48.4^\circ$   
c 7.03 cm d  $41.2^\circ$

- 6 a 12 b 12 c 2

- 7 a  $\sqrt{3} = 1.73$   
b



- 8 a 1  
b



### Exercise 11I

- 1 a  $51.3^\circ$  b  $75.5^\circ$  c  $51.3^\circ$

- 2 a 6.47 cm b 32.6 cm c 137 cm

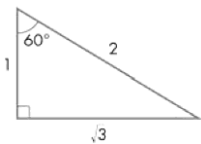
- 3 a 7.32 cm b 39.1 cm c 135 cm

- 4 a 5.35 cm b 14.8 cm  
c 12.0 cm d 8.62 cm

- 5 a 5.59 cm b  $46.6^\circ$   
c 9.91 cm d  $40.1^\circ$

- 6 a 10 b 39 c 2.5

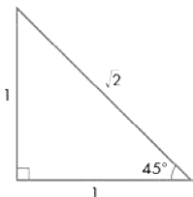
- 7 a  $\frac{1}{2}$



b-c

- 8 a  $\frac{1}{\sqrt{2}}$

b-c



### Exercise 11J

- 1 a  $33.7^\circ$  b  $36.9^\circ$  c  $52.1^\circ$

- 2 a 5.09 cm b 30.4 cm c 1120 cm

- 3 a 8.24 cm b 62.0 cm c 72.8 cm

### Exercise 11K

- 1 a 12.6 b 59.6 c 74.7  
d 16.0 e 67.9 f 20.1

- 2 a  $44.4^\circ$  b  $39.8^\circ$  c  $44.4^\circ$   
d  $49.5^\circ$  e  $58.7^\circ$  f  $38.7^\circ$

- 3 a  $67.4^\circ$  b 11.3 c 134  
d  $28.1^\circ$  e 39.7 f 263  
g  $50.2^\circ$  h  $51.3^\circ$  i 138  
j 22.8

- 4 a Sides of right-hand triangle are sine and cosine  
b Pythagoras' theorem  
c Students should check the formulae

5

	$30^\circ$	$45^\circ$	$60^\circ$
Sine	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
Cosine	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
Tangent	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

### Exercise 11L

- 1  $65^\circ$

- 2 The safe limits are between 1.04 m and 2.05 m.  
The ladder will reach between 5.63 m and 5.90 m up the wall.

- 3  $44^\circ$

- 4 6.82 m

- 5  $31^\circ$

- 6 a  $25^\circ$  b 2.10 m  
c Thickness of wood has been ignored

- 7 a  $20^\circ$  b 4.78 m

- 8 She would calculate  $100 \tan 23^\circ$ . The answer is about 42.4 m
- 9 21.1 m
- 10 One way is stand opposite a feature, such as a tree, on the opposite bank, move a measured distance,  $x$ , along your bank and measure the angle,  $\theta$ , between your bank and the feature. Width of river is  $x \tan \theta$ . This of course requires measuring equipment! An alternative is to walk along the bank until the angle is  $45^\circ$  (if that is possible). This angle is easily found by folding a sheet of paper. This way an angle measurer is not required.

### Exercise 11M

- 1 10.1 km
- 2  $22^\circ$
- 3 429 m
- 4 a 156 m  
b No. the new angle of depression is  $\tan^{-1}(\frac{200}{312}) = 33^\circ$  and half of  $52^\circ$  is  $26^\circ$
- 5 a 222 m                      b  $42^\circ$
- 6 a 21.5 m                      b 17.8 m
- 7 a 13.4 m  
b We don't know if the angle of elevation is from Sunil's feet or head. This would make a difference to the answer as we would need to add Sunil's height if the angle was from his head.
- 8  $\cos \theta = \frac{1}{3}$  so  $\cos^{-1} 0.3333 = 70.5^\circ$  (3sf)
- 9 The angle is  $16^\circ$  so Cara is not quite correct.
- 10 William is 137 m away, Isaac is 107 m away.

### Exercise 11N

- 1 a 73.4 km                      b 15.6 km
- 2 a 14.7 miles                      b 8.5 miles
- 3 Draw a diagram representing the relative places. Your diagram will show the angle of the bearing to fly the direct route as  $90^\circ + \tan^{-1}(70/120) = 120^\circ$ . So the bearing for the direct route is  $120^\circ$ .
- 4 a 59.4 km                      b 8.4 km
- 5 a 15.9 km                      b 24.1 km  
c 31.2 km                      d  $052^\circ$
- 6 2.28 km
- 7  $235^\circ$
- 8 a 66.2 km                      b 11.7 km  
c 13.1 km                      d  $170^\circ$
- 9 48.4 km on a bearing of  $100^\circ$

### Exercise 11P

- 1 a 5.79 cm                      b  $48.2^\circ$   
c 7.42 cm                      d 81.6 cm
- 2 9.86 m
- 3 a  $36.4 \text{ cm}^2$                       b  $115 \text{ cm}^2$   
c  $90.6 \text{ cm}^2$                       d  $160 \text{ cm}^2$
- 4  $473 \text{ cm}^2$
- 5  $39.0 \text{ cm}^2$
- 6 Base radius given by  $8 \tan 31^\circ$ , so volume =  $\frac{8}{3} \times \pi(8 \tan 31^\circ)^2 = 193.57357 = 194$  (3sf)

### Review questions

- 1  $13.6 \text{ cm}^2$
- 2 2 pm
- 3 diagonal =  $\sqrt{3^2 + 4^2 + 12^2} = \sqrt{169} = 13$
- 4  $237^\circ$
- 5  $52.3^\circ$
- 6 a AX and BY are both radii to the tangents at A and B and so perpendicular to AB, hence parallel. So ABYX is a trapezium.  
b Draw in a line, YT, parallel to the base AB, so that T lies on AX. This gives a right-angled triangle with height  $(7 - 2) \text{ cm} = 5 \text{ cm}$ . The hypotenuse is  $(7 + 2) \text{ cm} = 9 \text{ cm}$ , hence the line YT, which is the same length as  $AB = \sqrt{9^2 - 5^2} = \sqrt{56} = 7.4833\dots$   
The area of the trapezium =  $AB \times (7 + 2)/2 = 7.4833 \times 4.5 = 33.6749 = 33.7 \text{ cm}^2$  (3sf)
- 7  $110 \text{ cm}^2$
- 8 Using square roots is dependent on remembering that  $\sin 60^\circ = \frac{\sqrt{3}}{2}$ , then calculating this as 0.866 (3sf).  
Using the equilateral triangle will give  $\sin 60^\circ$  as Height of triangle/10. The height found by Pythagoras as  $\sqrt{(10^2 - 5^2)} = \sqrt{75} = 8.660254$ , so  $\sin 60^\circ = 0.866$  (3sf).  
Both answers are the same.

## Chapter 12 – Geometry and measures: Similarity

### Exercise 12A

- 1 a Yes, 4  
b No, corresponding sides have different ratios.
- 2 a 1 : 3  
b Angle R  
c BA

3 a Angle P b PR

4 a Same angles  
b Angle Q  
c AR

5 a 8 cm  
b 7.5 cm  
c  $x = 6.67$  cm,  $y = 13.5$  cm  
d  $x = 24$  cm,  $y = 13$  cm  
e AB = 10 cm, PQ = 6 cm  
f 4.2 cm

6 a Sides in same ratio b 1 : 3  
c 13 cm d 39 cm

7 5.2 m

8 Corresponding sides are not in the same ratio,  
 $12 : 15 \neq 16 : 19$ .

9 Jay is wrong: DE = 17.5 cm; AC : EC = BA : DE, 5 :  
 $12.5 = 7 : DE$ ,  $DE = 7 \times 12.5 \div 5 = 17.5$  cm

### Exercise 12B

1 a ABC and ADE; 9 cm  
b ABC and ADE; 12 cm

2 a 5 cm  
b 5 cm  
c  $x = 60$  cm,  $y = 75$  cm  
d DC = 10 cm, EB = 8 cm

3 82 m

4  $\frac{\text{pole}}{330} = \frac{400}{600}$ , pole =  $330 \times \frac{4}{6} = 220$

5 15 m

6 3.3 m

7 1.8 m

8  $\frac{BC}{9} = \frac{9}{6}$ , hence BC =  $9 \times \frac{9}{6} = 13.5$  cm

9 c

### Exercise 12C

1 a 5 cm  
b 6 cm  
c 10 cm  
d  $x = 6$  cm,  $y = 7.5$  cm  
e  $x = 15$  cm,  $y = 21$  cm  
f  $x = 3$  cm,  $y = 2.4$  cm

2 a  $\frac{x+12}{12} = \frac{180-1.7}{2-1.7}$ ,  $x = \frac{2136}{0.3} = 7120$  m, just over  
7 km.

b the assumption is that the building, the brick  
wall and Brad are all standing on the same  
level.

### Exercise 12D

1 a i 1 : 9 ii 4 : 25 iii 16 : 49  
b i 1 : 27 ii 8 : 125 iii 64 : 343

2

Linear scale factor	Linear ratio	Linear fraction	Area scale factor	Volume scale factor
2	1 : 2	$\frac{2}{1}$	4	8
3	1 : 3	$\frac{3}{1}$	9	27
$\frac{1}{4}$	4 : 1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$
5	1 : 5	$\frac{5}{1}$	25	125
$\frac{1}{10}$	10 : 1	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$

3  $135 \text{ cm}^2$

4 a  $56 \text{ cm}^2$   
b  $126 \text{ cm}^2$

5 a  $48 \text{ m}^2$   
b  $3 \text{ m}^2$

6 a  $2400 \text{ cm}^3$   
b  $8100 \text{ cm}^3$

7 Length ratio = 1 : 2, so volume ratio = 1 : 8. So  
large tin volume =  $0.5 \times 8 = 4$  litres

8  $1.38 \text{ m}^3$

9 a £6  
b Assume that the cost is only based on the  
volume of paint in the tin.

10 4 cm

11  $8 \times 60\text{p} = £4.80$  so it is better value to buy the large  
tub

12 a 3 : 4  
b 9 : 16  
c 27 : 64

13  $720 \div 8 = 90 \text{ cm}^3$

### Exercise 12E

1 a  $111 \text{ cm}^3$  b  $641 \text{ cm}^3$   
c  $267 \text{ cm}^3$  d  $426 \text{ cm}^3$

2 a Height = 6 cm, Volume =  $25 \text{ cm}^3$   
b Height = 8 cm, Volume =  $51 \text{ cm}^3$   
c Height = 4 cm, Mass = 105 g  
d Height = 3 cm, Volume =  $130 \text{ cm}^3$

3 6.2 cm, 10.1 cm

4 4.26 cm, 6.74 cm

5  $\frac{H^2}{8^2} = \frac{200}{140}$

$$H^2 = 64 \times \frac{200}{140} = 91.428571$$

$$H = \sqrt{91.428571} = 9.56 \text{ (3sf)}$$

6 3.38 m

7 8.39 cm

- 8 26.5 cm  
 9 16.9 cm  
 10 a 4.33 cm, 7.81 cm b 143 g, 839 g  
 11 53.8 kg  
 12 1.73 kg  
 13 8.8 cm  
 14 7.9 cm and 12.6 cm  
 15 b

### Review questions

- 1 Let height of larger triangle =  $h$ .  
 Using similar triangles:  $\frac{h}{35-h} = \frac{40}{30}$   
 Rearrange to  $30h = 1400 - 40h$   
 which gives  $70h = 1400$   
 $h = 20$  cm  
 So small triangle is  $35 - 20 = 15$  cm tall.  
 Thus the difference between the heights is  $20$  cm –  $15$  cm =  $5$  cm
- 2 a For similar shapes, if the ratio of lengths is  $1 : x$ , then the ratio of volumes will be  $1 : x^3$ , so if ratio of lengths is  $1 : 3$ , the ratio of volumes will be  $1 : 3^3 = 1 : 27$   
 b Yes, because if the size (volume) of the plant increases by a factor of 27, the lengths have increased by a factor of 3. Hence the new height should be  $4 \text{ cm} \times 3 = 12 \text{ cm}$ , which it is.
- 3 Andrew is correct, Eve has calculated the length of AD (8 cm) so ED should be 2 cm.
- 4 a 6 cm b  $16 \text{ cm}^3$
- 5 a Area scale factor =  $\frac{324}{100} = 3.24$ , length scale factor =  $\sqrt{3.24} = 1.8$ , length of cylinder B =  $5 \times 1.8 = 9$   
 b  $933 \text{ cm}^3$

## Chapter 13 – Probability: Exploring and applying probability

### Exercise 13A

- 1 a  $\frac{1}{5}, \frac{2}{25}, \frac{1}{10}, \frac{21}{200}, \frac{37}{250}, \frac{163}{1000}, \frac{329}{2000}$   
 b 6 c 1 d  $\frac{1}{6}$  e 1000
- 2 a  $\frac{19}{200}, \frac{27}{200}, \frac{4}{25}, \frac{53}{200}, \frac{69}{200}$   
 b 40  
 c No, it is weighted towards the side with numbers 4 and 5
- 3 a 32 is too high, unlikely that 20 of the 50 throws between 50 and 100 were 5  
 b Yes, all frequencies fairly close to 100
- 4 a B b B c C d A  
 e B f A g B h B

- 5 a 0.2, 0.25, 0.38, 0.42, 0.385, 0.397  
 b 80
- 6 a Caryl, most throws b 0.39, 0.31, 0.17, 0.14  
 c Yes, it is more likely to give a 1 or 2
- 7 Thursday as it had the highest proportion
- 8 The missing top numbers are 4 and 5; the two bottom numbers are likely to be close to 20.
- 9 Although you would expect the probability to be close to  $\frac{1}{2}$ , hence 500 heads, it is more likely that the number of heads is close to 500 rather than actually 500.
- 10 Roxy is correct, as the expected numbers are: 50, 12.5, 25, 12.5. Sam has not taken into account the fact that there are four red sectors.

### Exercise 13B

- 1 a Yes b Yes c No  
 d No e Yes f Yes
- 2 Events a and f
- 3  $\frac{3}{5}$
- 4 a i  $\frac{3}{10}$  ii  $\frac{3}{10}$  iii  $\frac{3}{10}$   
 iv  $\frac{9}{10}$  v  $\frac{4}{5}$   
 b All except iii  
 c Event iv
- 5 a Jane/John, Jane/Jack, Jane/Anne, Jane/Dave, Dave/John, Dave/Jack, Dave/Anne, Anne/John, Anne/Jack, Jack/John  
 b i  $\frac{1}{10}$  ii  $\frac{3}{10}$   
 iii  $\frac{3}{10}$  iv  $\frac{7}{10}$   
 c All except iii  
 d Event ii
- 6 a  $\frac{3}{8}$  b  $\frac{1}{8}$   
 c All except ii  
 d Outcomes overlap
- 7  $\frac{3}{20}$
- 8  $\frac{1}{75}$
- 9 Not mutually exclusive events
- 10 a i 0.25 ii 0.4 iii 0.7  
 b Events not mutually exclusive  
 c Man/woman, American man/American woman  
 d Man/woman
- 11 a i 0.95  
 ii 0.9 (assuming person chooses one or other)  
 iii 0.3  
 b Events not mutually exclusive  
 c Possible answer: pork and vegetarian
- 12 These are not mutually exclusive events.

### Exercise 13C

- 1 25
- 2 1000
- 3 a 260      b 40      c 130      d 10
- 4 5
- 5 a 150      b 100      c 250      d 0
- 6 a 167      b 833
- 7 1050

8 a

Score	1	2	3	4	5	6
Expected occurrences	10	10	10	10	10	10

b  $1 \times 10 + 2 \times 10 + 3 \times 10 + 4 \times 10 + 5 \times 10 + 6 \times 10 = 210 \div 60 = 3.5$

c Find the average of the scores, which is  $\frac{21}{6}$   
 $\frac{21}{6} = 3.5$

- 9 a 0.111      b 40
- 10 281 days
- 11 Multiply the number of tomato plants by 0.997
- 12 400

### Exercise 13D

- 1 a 23      b 20%      c  $\frac{4}{25}$       d 480
- 2 a 10      b 7      c 14%      d 15%
- 3 a

1	2	3	4
5	6	7	8
6	7	8	9
7	8	9	10
8	9	10	11
9	10	11	12

b 4

c i  $\frac{1}{4}$       ii  $\frac{3}{16}$       iii  $\frac{1}{4}$

- 4 a 16      b 16      c 73      d  $\frac{51}{73}$

5 a

1	2	3	4	5	6
1	2	3	4	5	6
2	4	6	8	10	12

b 3      c  $\frac{1}{4}$

- 6 a  $\frac{2}{45}$       b 40%      c 45%

d No, as you don't know how much the people who get over £350 actually earn

7  $\frac{22}{36} = \frac{11}{18}$

8 a

Score of second spinner	10	10	11	13	15	17	19
8	8	8	9	11	13	15	17
6	6	6	7	9	11	13	15
4	4	4	5	7	9	11	13
2	2	2	3	5	7	9	11
0	0	0	1	3	5	7	9
Score of first spinner	0	1	3	5	7	9	

b 9 or 11

c 0

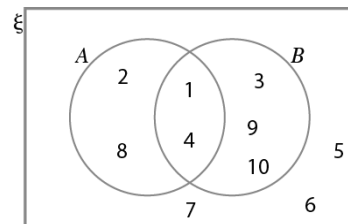
d  $\frac{15}{36} = \frac{5}{12}$

e  $\frac{30}{36} = \frac{5}{6}$

### Exercise 13E

- 1 a 0.9      b 0.7
- 2 a 0.75      b 0.45

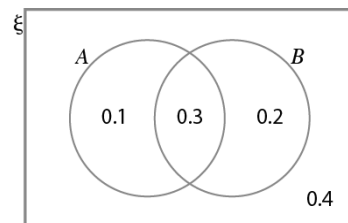
3 a



- b i  $\frac{2}{5}$       ii  $\frac{3}{5}$       iii  $\frac{1}{2}$   
 iv  $\frac{1}{2}$       v  $\frac{7}{10}$       vi  $\frac{1}{5}$

- 4 a i 0.52      ii 0.48      iii 0.65  
 iv 0.35      v 0.82      vi 0.35  
 b 0.3

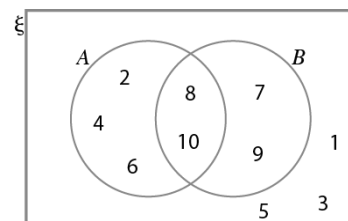
5 a



- b i 0.5      ii 0.6      iii 0.3

- 6 a 65      b 70      c 90

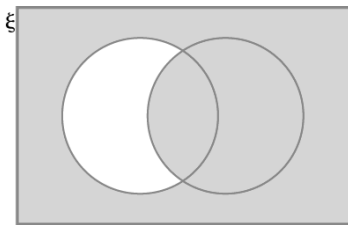
7



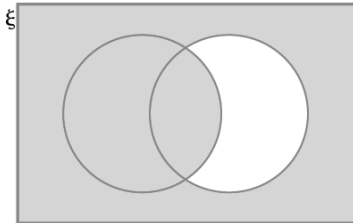
- a  $\frac{1}{2}$       b  $\frac{2}{5}$   
 c  $\frac{7}{10}$       d  $\frac{1}{5}$

- 8 a 130  
 b i  $\frac{8}{13}$   
 ii The probability that a student chosen at random walks to and from school  
 c  $\frac{5}{26}$
- 9 0.4
- 10 0.5
- 11 a  $(A \cup B)'$  b  $(A \cap B)'$
- 12  $\frac{37}{80}$

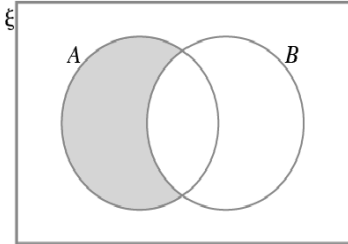
13 a



b



c



### Review questions

- 1 a  $\frac{1}{2}$  b  $\frac{7}{10}$
- 2 a 0.28  
 b the frequencies should all be close to 25
- 3 a i  $\frac{3}{10}$  ii 10 red, 6 green, 4 blue  
 b she may not have taken one in the 10 trials
- 4
- |                       | Boys | Girls | Total |
|-----------------------|------|-------|-------|
| Walk to school        | 9    | 21    | 30    |
| Do not walk to school | 18   | 2     | 20    |
| <b>Total</b>          | 27   | 23    | 50    |
- 5 a 110 b  $\frac{19}{55}$  c  $\frac{3}{22}$  d  $\frac{36}{55}$
- 6 a 0.18 b 0.49  
 c No as the probabilities are close to 0.2

- 7 a i 0.8 ii 12  
 b No as  $P(\text{six}) = \frac{1}{6}$ , so Tom is likely to throw 10 sixes
- 8 Draw a two-way table to show the outcomes
- | + | 1 | 2 | 3 | 4 | 5  |
|---|---|---|---|---|----|
| 1 | 2 | 3 | 4 | 5 | 6  |
| 2 | 3 | 4 | 5 | 6 | 7  |
| 3 | 4 | 5 | 6 | 7 | 8  |
| 4 | 5 | 6 | 7 | 8 | 9  |
| 5 | 6 | 7 | 8 | 9 | 10 |
- $P(\text{score greater than 6}) = \frac{10}{25}$ , as  $\frac{10}{25} < \frac{1}{2}$ , she is likely to lose the game
- 9 a 0.3 b 0.5 c 0.6 d 0.3

## Chapter 14 – Number: Powers and standard form

### Exercise 14A

- 1 a  $2^4$  b  $3^5$  c  $7^2$  d  $5^3$   
 e  $10^7$  f  $6^4$  g  $4^1$  h  $1^7$   
 i  $0.5^4$  j  $100^3$
- 2 a  $3 \times 3 \times 3 \times 3$   
 b  $9 \times 9 \times 9$   
 c  $6 \times 6$   
 d  $10 \times 10 \times 10 \times 10 \times 10$   
 e  $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$   
 f 8  
 g  $0.1 \times 0.1 \times 0.1$   
 h  $2.5 \times 2.5$   
 i  $0.7 \times 0.7 \times 0.7$   
 j  $1000 \times 1000$
- 3 a 16 b 243 c 49  
 d 125 e 10 000 000 f 1296  
 g 4 h 1 i 0.0625  
 j 1 000 000
- 4 a 81 b 729 c 36  
 d 100 000 e 1024 f 8  
 g 0.001 h 6.25 i 0.343  
 j 1 000 000
- 5  $125 \text{ m}^3$
- 6 b  $10^2$  c  $2^3$  d  $5^2$
- 7 3: 3, 9, 27, 81, 243, 729 – pattern is 3, 9, 7, 1  
 4: 4, 16, 64, 256, 1024, 4096 – pattern is 4, 6  
 5: 5, 125, 625, 3125, 15 625, 78 125 – all 5
- 8 a 1 b 4 c 1  
 d 1 e 1
- 9 Any power of 1 is equal to 1.
- 10  $10^6$
- 11  $10^6$

- 12 a 1 b -1 c 1 d 1 e -1  
 13 a 1 b -1 c -1 d 1 e 1  
 14  $2^{24}, 4^{12}, 8^8, 16^6$   
 15  $x = 6$

### Exercise 14B

- 1 a  $5^4$  b  $5^3$  c  $5^2$  d  $5^3$  e  $5^{-5}$   
 2 a  $6^3$  b  $6^0$  c  $6^6$  d  $6^{-7}$  e  $6^2$   
 3 a  $a^3$  b  $a^5$  c  $a^7$   
 d  $a^4$  e  $a^2$  f  $a^1$   
 4 a Any two values such that  $x + y = 10$   
 b Any two values such that  $x - y = 10$   
 5 a  $4^6$  b  $4^{15}$  c  $4^6$   
 d  $4^{-6}$  e  $4^6$  f  $4^0$   
 6 a  $6a^5$  b  $9a^2$  c  $8a^6$   
 d  $-6a^4$  e  $8a^8$  f  $-10a^{-3}$   
 7 a  $3a$  b  $4a^3$  c  $3a^4$   
 d  $6a^{-1}$  e  $4a^7$  f  $5a^{-4}$   
 8 a  $8a^5b^4$  b  $10a^3b$  c  $30a^{-2}b^{-2}$   
 d  $2ab^3$  e  $8a^{-5}b^7$  f  $4a^5b^{-5}$   
 9 a  $3a^3b^2$  b  $3a^2c^4$  c  $8a^2b^2c^3$   
 10 a Possible answer:  $6x^2 \times 2y^5$  and  $3xy \times 4xy^4$   
 b Possible answer:  $24x^2y^7 \div 2y^2$  and  $12x^6y^8 \div x^4y^3$   
 11 12 ( $a = 2, b = 1, c = 3$ )  
 12  $1 = a^x \div a^x = a^{x-x} = a^0$

### Exercise 14C

- 1 a 60 000 b 120 000 c 150  
 d 42 000 e 1400 f 300  
 g 400 h 8000 i 160 000  
 j 4500 k 8000 l 250 000  
 2 a 5 b 50 c 25 d 30 e 7  
 f 300 g 6 h 30 i 1 j 15  
 k 40 l 5 m 40 n 320  
 3 a 54 400 b 16 000  
 4  $30 \times 90\,000 = 2\,700\,000$   
 $600 \times 8000 = 4\,800\,000$   
 $5000 \times 4000 = 20\,000\,000$   
 $200\,000 \times 700 = 140\,000\,000$   
 5 1400 million  
 6 a 31 b 310 c 3100 d 31 000  
 7 a 65 b 650 c 6500 d 65 000  
 8 a 0.31 b 0.031 c 0.0031 d 0.000 31  
 9 a 0.65 b 0.065 c 0.0065 d 0.000 65  
 10 a 250 b 34.5 c 4670  
 d 346 e 207.89 f 56 780  
 g 246 h 0.76 i 999 000  
 j 23 456 k 98 765.4 l 43 230 000  
 m 345.78 n 6000 o 56.7  
 p 560 045  
 11 a 0.025 b 0.345

- c 0.004 67 d 3.46  
 e 0.207 89 f 0.056 78  
 g 0.0246 h 0.0076  
 i 0.000 000 999 j 2.3456  
 k 0.098 7654 l 0.000 043 23  
 m 0.000000034578 n 0.000 000 000 06  
 o 0.000 0005 67 p 0.005 600 45

- 12 a 230 b 578 900  
 c 4790 d 57 000 000  
 e 216 f 10 500  
 g 0.000 32 h 9870

13 a, b and c

14 Power 24 means more digits in the answer, so Venus is heavier.

15 6

### Exercise 14D

- 1 a 0.31 b 0.031 c 0.0031 d 0.000 31  
 2 a 0.65 b 0.065 c 0.0065 d 0.000 65  
 3 a  $9999999999 \times 10^{99}$   
 b  $0.000000001 \times 10^{-99}$  (depending on number of digits displayed)  
 4 a 31 b 310 c 3100 d 31 000  
 5 a 65 b 650 c 6500 d 65 000  
 6 a 250 b 34.5 c 0.004 67  
 d 34.6 e 897 000 f 0.00865  
 g 60 000 000 h 0.000 567  
 7 a  $2.5 \times 10^2$  b  $3.45 \times 10^{-1}$   
 c  $4.67 \times 10^4$  d  $3.4 \times 10^9$   
 e  $2.078 \times 10^{10}$  f  $5.678 \times 10^{-4}$   
 g  $6 \times 10^{-4}$  h  $5.67 \times 10^{-3}$   
 i  $5.60045 \times 10^1$   
 8  $2.81581 \times 10^5, 3 \times 10^1, 1.382101 \times 10^6$   
 9  $1.298 \times 10^7, 2.997 \times 10^9, 9.3 \times 10^4$   
 10 100

### Exercise 14E

- 1 a  $5.67 \times 10^3$  b  $6 \times 10^2$   
 c  $3.46 \times 10^{-1}$  d  $7 \times 10^3$   
 e 1.6 f  $2.3 \times 10^7$   
 g  $3 \times 10^{-6}$  h  $2.56 \times 10^6$   
 i  $4.8 \times 10^2$  j  $1.12 \times 10^2$   
 k  $6 \times 10^{-1}$  l  $2.8 \times 10^6$   
 2 a  $4.81 \times 10^8$  b  $9.15 \times 10^{12}$   
 c  $5.67 \times 10^9$  d  $1.46 \times 10^{14}$   
 e  $1.63 \times 10^{22}$  f  $1.2 \times 10^9$   
 g 1.08 h  $6.4 \times 10^2$   
 i  $1.2 \times 10^1$  j 2.88  
 k  $2.5 \times 10^7$  l  $8 \times 10^{-6}$   
 3 a  $2.64 \times 10^{14}$  b  $1.22 \times 10^8$   
 c  $1.6 \times 10^9$  d  $3.9 \times 10^{-2}$   
 e  $9.6 \times 10^8$  f  $4.6 \times 10^{-7}$   
 g  $2.1 \times 10^3$  h  $3.6 \times 10^7$   
 i  $1.5 \times 10^2$  j  $3.5 \times 10^9$   
 k  $1.6 \times 10^4$  l  $3.81 \times 10^8$   
 m  $7.18 \times 10^{12}$   
 4 a  $2.7 \times 10$  b  $1.6 \times 10^{-2}$



$$\begin{array}{ll} \text{c} & 2 \times 10^{-1} \\ \text{e} & 2 \times 10^5 \\ \text{g} & 2 \times 10^{-5} \\ \text{i} & 2 \times 10 \end{array} \quad \begin{array}{ll} \text{d} & 4 \times 10^{-8} \\ \text{f} & 6 \times 10^{-2} \\ \text{h} & 5 \times 10^2 \end{array}$$

$$\begin{array}{ll} 5 \text{ a} & 5.4 \times 10 \\ \text{c} & 1.1 \\ \text{e} & 2.8 \times 10^2 \\ \text{g} & 4.9 \times 10^2 \end{array} \quad \begin{array}{ll} \text{b} & 2.9 \times 10^{-3} \\ \text{d} & 6.3 \times 10^{-10} \\ \text{f} & 5.5 \times 10^{-2} \\ \text{h} & 8.6 \times 10^6 \end{array}$$

$$6 \quad 2 \times 10^{13}, 1 \times 10^{-10}, \text{mass} = 2 \times 10^3 \text{ g (2 kg)}$$

$$\begin{array}{ll} 7 \text{ a} & (2^{63}) = 9.2 \times 10^{18} \text{ grains} \\ \text{b} & 2^{64} - 1 = 1.8 \times 10^{19} \end{array}$$

$$8 \text{ a} \quad 6 \times 10^7 \text{ sq miles} \quad \text{b} \quad 30\%$$

$$\begin{array}{ll} 9 & 5 \times 10^4 \\ 10 & 2.3 \times 10^5 \\ 11 & 4.55 \times 10^8 \text{ kg} \\ 12 \text{ a} & 2.048 \times 10^6 \quad \text{b} \quad 4.816 \times 10^6 \\ 13 & 2.5 \times 10^2 \\ 14 & 9.41 \times 10^4 \end{array}$$

#### Review questions

$$\begin{array}{ll} 1 \text{ a i} & 2^4 \\ \text{b i} & 10^3 \end{array} \quad \begin{array}{ll} \text{ii} & 2^8 \\ \text{iii} & 10^9 \end{array}$$

$$2 \text{ a} \quad 1\,500\,000 \quad \text{b} \quad 6\,000\,000\,000$$

$$3 \text{ a} \quad 196 \\ \text{b} \quad \text{units digits is } 5 \times 5 = 25 \text{ so it should end in } 5$$

$$4 \text{ a} \quad 7^9 \quad \text{b} \quad x^4 \\ \text{c} \quad \text{Adds numbers and multiplies powers, but should be the other way round. } 15x^9$$

$$5 \text{ a} \quad t^8 \quad \text{b} \quad m^5 \quad \text{c} \quad 9x^6 \quad \text{d} \quad 10a^7h^5$$

$$6 \text{ a} \quad x^{11} \quad \text{b} \quad m^{-5} \quad \text{c} \quad 8k^5m^3$$

$$7 \text{ a} \quad 7.5 \times 10^4 \quad \text{b} \quad 0.009$$

$$8 \text{ a} \quad 2y \quad \text{b} \quad 8m^9p^{12}$$

$$9 \quad 2.48 \times 10^{-7}$$

$$10 \quad 1000 \text{ litres in a cubic metre, } 5.3 \times 10^{24} \div 2000 = 2.65 \times 10^{21}$$

$$11 \text{ Yes: } (6.5 \times 10^4)^2 = 4.225 \times 10^9 \\ (6 \times 10^4)^2 = 3.6 \times 10^9 \\ (2.5 \times 10^4)^2 = 6.25 \times 10^8 = 0.625 \times 10^9 \\ 3.6 \times 10^9 + 0.625 \times 10^9 = 4.225 \times 10^9$$

$$12 \quad 2.6 \times 10^7 \div 2 = 1.3 \times 10^7, \\ \text{short side} = 1.3 \times 10^7 - 8 \times 10^6 = 13 \times 10^6 - 8 \times 10^6 = 5 \times 10^6 \\ \text{Area} = 5 \times 10^6 \times 8 \times 10^6 = 40 \times 10^{12} = 4 \times 10^{13}$$

$$13 \quad 1.5 \times 10^7 \text{ sq miles}$$

$$14 \quad 13\,875\,000$$

$$15 \quad \text{Any value from } 1 \times 10^8 \text{ to } 1 \times 10^9 \text{ (excluding } 1 \times 10^8 \text{ and } 1 \times 10^9), \text{ i.e. any value of the form } a \times 10^8 \text{ where } 1 < a < 10$$

$$16 \quad 38.625$$

$$17 \text{ a} \quad 4.16 \times 10^7 \text{ cm} \quad \text{b} \quad 1.056 \times 10^{14} \text{ cm}^2$$

## Chapter 15 – Algebra: Equations and inequalities

### Exercise 15A

$$\begin{array}{llll} 1 \text{ a} & 30 & \text{b} & 21 \\ \text{e} & 6 & \text{f} & 10\frac{1}{2} \\ \text{i} & 11 & \text{j} & 2 \\ \text{m} & 1 & \text{n} & 11\frac{1}{2} \end{array} \quad \begin{array}{llll} \text{c} & 72 \\ \text{g} & -10 \\ \text{k} & 7 \\ \text{o} & \frac{1}{5} \end{array} \quad \begin{array}{l} \text{d} \quad 12 \\ \text{h} \quad 7 \\ \text{l} \quad 2\frac{4}{5} \end{array}$$

$$2 \quad \text{Any valid equations}$$

$$3 \text{ a} \quad \text{Amanda} \\ \text{b} \quad \text{First line: Betsy adds 4 instead of multiplying by 5.} \\ \text{Second line: Betsy adds 5 instead of multiplying by 5.} \\ \text{Fourth line: Betsy subtracts 2 instead of dividing by 2.}$$

$$4 \text{ a} \quad \frac{x+10}{5} = 9.50 \\ \text{b} \quad \text{£}37.50$$

$$5 \text{ a} \quad \frac{8}{3} \quad \text{b} \quad \text{student's own checks}$$

### Exercise 15B

$$\begin{array}{llll} 1 \text{ a} & \frac{1}{2} & \text{b} & 1\frac{1}{5} \\ \text{e} & -1 & \text{f} & -2 \end{array} \quad \begin{array}{llll} \text{c} & 2 \\ \text{g} & -2 \end{array} \quad \begin{array}{l} \text{d} \quad -2 \\ \text{h} \quad -1 \end{array}$$

$$2 \quad \text{Any values that work, e.g. } a = 2, b = 3 \text{ and } c = 30.$$

$$3 \quad 55$$

$$4 \quad 3 \text{ cm}$$

$$5 \quad 5$$

$$6 \quad \text{Multiplying out the brackets and simplifying gives } 4x - 24 = 0 \text{ which has the solution } x = 6$$

$$7 \quad 168^\circ$$

### Exercise 15C

$$\begin{array}{llll} 1 \text{ a} & x = 2 & \text{b} & y = 1 \\ \text{e} & p = 2 & \text{f} & k = -1 \end{array} \quad \begin{array}{llll} \text{c} & a = 7 \\ \text{g} & m = 3 \end{array} \quad \begin{array}{l} \text{d} \quad t = 4 \\ \text{h} \quad s = -2 \end{array}$$

$$2 \quad 3x - 2 = 2x + 5, x = 7$$

$$3 \text{ a} \quad d = 6 \quad \text{b} \quad x = 11 \quad \text{c} \quad y = 1 \quad \text{d} \quad h = 4 \\ \text{e} \quad b = 9 \quad \text{f} \quad c = 6$$

$$4 \text{ a} \quad 6x + 3 = 6x + 10; 6x - 6x = 10 - 3; 0 = 7, \text{ which is obviously false. Both sides have } 6x, \text{ which cancels out.} \\ \text{b} \quad \text{Multiplying out the brackets gives } 12x + 18 = 12x + 18, \text{ which is true for all values of } x$$

$$5 \quad 8x + 7 + x + 4 = 11x + 5 - x - 4, x = 10$$

$$6 \text{ a} \quad \text{They are both equal to the length of the rectangle} \\ \text{b} \quad 70 \text{ cm}^2$$

$$7 \text{ a} \quad 15 \\ \text{b} \quad -1 \\ \text{c} \quad 2(n+3), 2(n+3) - 5$$

- d  $2(n+3) - 5 = n$ ,  $2n+6-5 = n$ ,  $2n+1 = n$ ,  $n = -1$
- 8  $4x + 18 = 3x + 1 + 50$ ,  $x = 33$ . Large bottle 1.5 litres, small bottle 1 litre
- 9 8

### Exercise 15D

- 1 a  $x = 4\frac{1}{2}$ ,  $y = 1\frac{1}{2}$       b  $x = -2$ ,  $y = 4$   
 c  $x = 2\frac{1}{2}$ ,  $y = -1\frac{1}{2}$
- 2 a  $a = 7$ ,  $b = 10$       b  $c = 4$ ,  $d = 11$   
 c  $e = 5$ ,  $f = 3$
- 3  $x = 12$ ,  $y = 2$

### Exercise 15E

- 1 a  $x = 9$ ,  $y = -2$       b  $x = \frac{1}{2}$ ,  $y = 5$       c  $x = -3$ ,  $y = -10$
- 2 a  $x = 2\frac{1}{4}$ ,  $y = 6\frac{1}{2}$   
 b  $x = 4$ ,  $y = 3$       c  $x = 5$ ,  $y = 3$
- 3 a  $x = 1$ ,  $y = 3$       b  $x = 5$ ,  $y = 9$

### Exercise 15F

- 1 a  $x = 2$ ,  $y = 5$       b  $x = 4$ ,  $y = -3$   
 c  $x = 1$ ,  $y = 7$       d  $x = \frac{1}{2}$ ,  $y = -\frac{3}{4}$   
 e  $x = -1$ ,  $y = 5$       f  $x = 1\frac{1}{2}$ ,  $y = \frac{3}{4}$
- 2 a  $x = 5$ ,  $y = 1$       b  $x = 3$ ,  $y = 8$   
 c  $x = 9$ ,  $y = 1$       d  $x = 7$ ,  $y = 3$   
 e  $x = 4$ ,  $y = 2$       f  $x = 6$ ,  $y = 5$

### Exercise 15G

- 1 a 3 is the first term. The next term is  $3 \times a + b$ , which equals 14.  
 b  $14a + b = 47$       c  $a = 3$ ,  $b = 5$   
 d 146, 443
- 2 Amul £7.20, Kim £3.50
- 3 a  $3t + 5b = 810$ ,  $3t + 3b = 630$   
 b £10.20
- 4 a They are the same equation. Divide the first by 2 and it is the second, so they have an infinite number of solutions.  
 b Double the second equation to get  $6x + 2y = 14$  and subtract to get  $9 = 14$ . The left-hand sides are the same if the second is doubled so they cannot have different values.
- 5 a  $10x + 5y = 840$ ,  $8x + 10y = 1044$   
 b £4.07
- 6 a My age minus 6 equals 2 × (my son's age minus 6)  
 b  $x = 46$  and  $y = 26$
- 7 (1, -2) is the solution to equations A and C; (-1, 3) is the solution to equations A and D; (2, 1) is

the solution to B and C; (3, -3) is the solution to B and D.

- 8 84p
- 9 10.3 kg
- 10 £4.40
- 11  $p = 36$ ,  $c = 22$ . Total weight for Baz is 428 pounds so he can carry the load safely on his trailer.
- 12  $b = £3.50$ ,  $p = £1.75$ . Camilla needs £35 so she will not have enough money.
- 13 a Intersection points are (0, 6), (1, 3) and (2, 4). Area is  $2 \text{ cm}^2$ .  
 b Intersection points are (0, 3), (3, 3) and (4, -1). Area is  $6 \text{ cm}^2$ .
- 14 When Carmen worked out (2) - (3), she should have got  $y = 6$   
 When Jeff rearranged  $2x + 8 - x = 10$ , he should have got  $x = 2$   
 They also misunderstood 'two, six' as this means  $x = 2$  and  $y = 6$ , not the other way round.

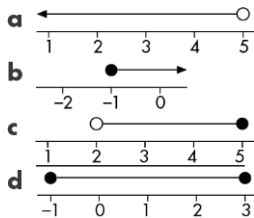
### Exercise 15H

- 1 a  $y \leq 3$       b  $x < 6$       c  $t \geq 18$   
 d  $x < 7$       e  $x \leq 3$       f  $t \geq 5$
- 2 a 16      b 3      c 7
- 3  $2x + 3 < 20$ ,  $x < 8.50$ , so the most each could cost is £8.49
- 4 a Because  $3 + 4 = 7$ , which is less than the third side of length 8  
 b  $x + x + 2 > 10$ ,  $2x + 2 > 10$ ,  $2x > 8$ ,  $x > 4$ , so smallest value of  $x$  is 5
- 5 a  $x = 6$  and  $x < 3$  scores -1 (nothing in common),  $x < 3$  and  $x > 0$  scores +1 (1 in common for example),  $x > 0$  and  $x = 2$  scores +1 (2 in common),  $x = 2$  and  $x \geq 4$  scores -1 (nothing in common), so we get  $-1 + 1 + 1 - 1 = 0$   
 b  $x > 0$  and  $x = 6$  scores +1 (6 in common),  $x = 6$  and  $x \geq 4$  scores +1 (6 in common),  $x \geq 4$  and  $x = 2$  scores -1 (nothing in common),  $x = 2$  and  $x < 3$  scores +1 (2 in common).  $+1 + 1 - 1 + 1 = 2$   
 c Any acceptable combination, e.g.  $x = 2$ ,  $x < 3$ ,  $x > 0$ ,  $x \geq 4$ ,  $x = 6$
- 6 a  $y \leq 4$       b  $x \geq -2$       c  $x \leq \frac{14}{5}$   
 d  $x > 38$       e  $x < 6\frac{1}{2}$       f  $y \leq \frac{7}{5}$
- 7 a  $3 < x < 6$       b  $2 < x < 5$       c  $-1 < x \leq 3$   
 d  $1 \leq x < 4$       e  $2 \leq x < 4$       f  $0 \leq x \leq 5$
- 8 a {4, 5}      b {3, 4}      c {0, 1, 2, 3}  
 d {1, 2, 3}      e {2, 3}      f {0, 1, 2, 3, 4, 5}
- 9 6

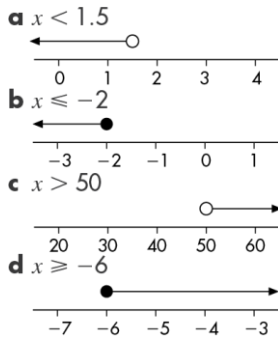
### Exercise 15I

- 1 a  $x < 2$       b  $x \geq -1$       c  $3 < x < 6$

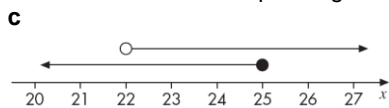
2



3



- 4 a Because 3 apples plus the chocolate bar cost more than £1.20:  $x > 22$   
b Because 2 apples plus the chocolate bar left Max with at least 16p change:  $x \leq 25$



d Apples could cost 23p, 24p or 25p.

- 5 Any two inequalities that overlap only on the integers -1, 0, 1 and 2 – for example,  $x > -1$  and  $x < 3$

6 1 and 4

7  $4(35 - 7x) = 84$

$35 - 7x = 21$

$7x = 14$

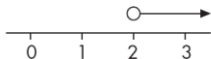
$x = 2$

Also  $35 - 7x > 0$

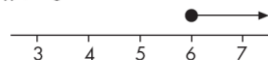
$x < 5$

So the diagram represents this.

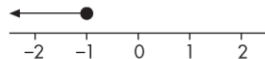
8 a  $x > 2$



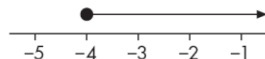
b  $x \geq 6$



c  $x \leq -1$

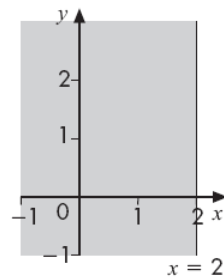


d  $x \geq -4$

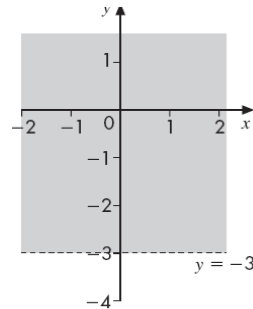


## Exercise 15J

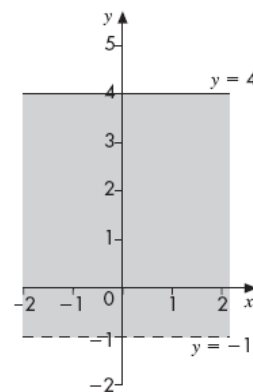
1



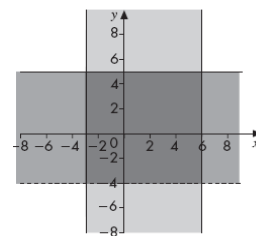
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3



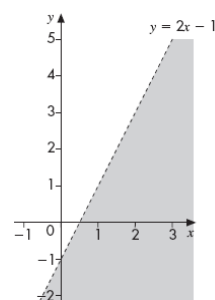
4 a



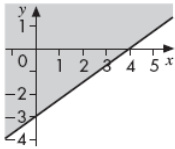
b i Yes ii Yes

iii No

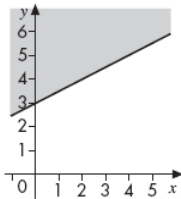
5



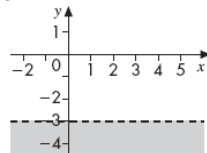
6 a



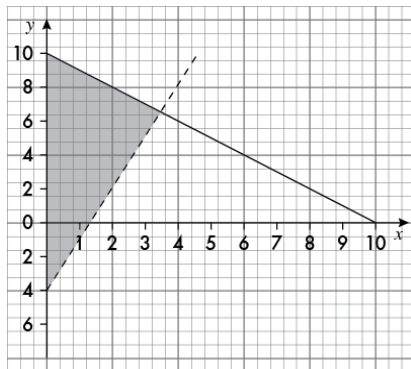
b



c

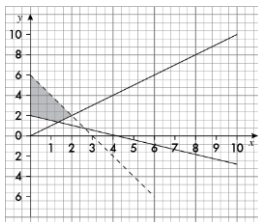


7 a-d



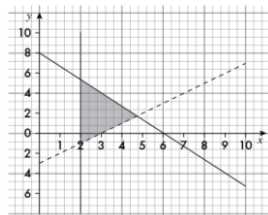
e i No ii Yes iii Yes

8 a-f



g i No ii No iii Yes

9 a



b i No ii Yes  
iii Yes iv No

10 For example,  $x \geq 1$ ,  $y \leq 3$  and  $y \geq x + 1$ . There are many other valid answers.

11 May be true: a, c, d, g  
Must be false: b, e, h  
Must be true: f

12 Test a point such as the origin (0, 0), so  $0 < 0 + 2$ , which is true. So the side that includes the origin is the required side.

13 a (3, 0) b (4, 5)

14 £59.50

### Exercise 15K

1 a 4 and 5 b 4 and 5 c 2 and 3

2  $x = 3.5$

3 a  $x = 3.7$   
b i  $x = 2.4$  ii  $x = 2.8$  iii  $x = 3.2$   
c  $x = 5.8$

4 Student's own working

5  $x = 1.5$

6

Guess	$3x^3 + 2x$	Comment
6	660	Too low
7	1043	Too high
6.5	836.875	Too low
6.8	956.896	Too high
6.7	915.689	Too high
6.6	875.688	Too low
6.65	895.538875	Too low

7 a Area =  $x(x + 5) = 100$   
b Width = 7.8 cm, length = 12.8 cm

8 Volume =  $x \times 2x(x + 8) = 500$ ,  $x^3 + 8x^2 = 250$ ,  $4 \Rightarrow 192$ ,  $5 \Rightarrow 325$ ,  $4.4 \Rightarrow 240.064$ ,  $4.5 \Rightarrow 253.125$ ,  $4.45 \Rightarrow 246.541125$ , so dimensions are 4.5 cm, 9 cm and 12.5 cm

9 Steph is correct because if 7.05 is too low then the answer will round up to 7.1

10 a Cube is  $x^3$ , hole is  $\frac{x}{2} \times \frac{x}{2} \times 8 = 2x^2$ .

Cube minus hole is 1500

b  $12 \Rightarrow 1440$ ,  $13 \Rightarrow 1859$ ,  $12.1 \Rightarrow 1478.741$ ,  $12.2 \Rightarrow 1518.168$ ,  $12.15 \Rightarrow 1498.368$  375 so the value of  $x = 12.2$  (to 1 dp)

11 2.76 and 7.24

### Review questions

1 8

2 3 years

3 Length is 5.5 m, width is 2.5 m and area is  $13.75 \text{ m}^2$ . Carpet costs £123.75

4 a B:  $\frac{3}{8}x$ , C:  $\frac{3}{8}x$ , D:  $\frac{1}{4}x$

**b**  $\frac{3}{8}x = 300$ , 800 cars

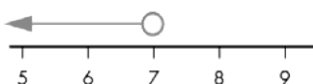
**c**  $\frac{1}{4}x = 500$ , 750 cars

- 5** No, as  $x + x + 2 + x + 4 + x + 6 = 360$  gives  $x = 87^\circ$  so the consecutive numbers (87, 89, 91, 93) are not even but odd

**6** 2 hr 10 min

**7** -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8

**8** **a**  $x = 7$ , **b**  $x < 7$



**9** **a** 6.3 **b** Solve as a linear equation

- 10** **i**  $-3 < x < 1$ , number line **b**;  
**ii**  $-2 < x < 4$ , number line below;  
**iii**  $-1 < x < 2$ , number line **a**



**11** 2.78

**12** £62

**13** £195

**14** **a**  $x = 4$ ,  $y = 3$

**b** **i**  $1000x + 1000y = 7000 \rightarrow x + y = 7$

**ii**  $984x - 984y = 984 \rightarrow x - y = 1$

**c**  $a = 9$ ,  $b = 5$

**15** Let straight part of track =  $D$ , inner radius of end =  $r$ , outer radius =  $r + x$

$x$  being the width of the track

Length of inner track =  $2D + 2\pi r = 300$  (**i**)

Length of outer track =  $2D + 2\pi(r + x) = 320$  (**ii**)

Subtract equation **i** from **ii** to give

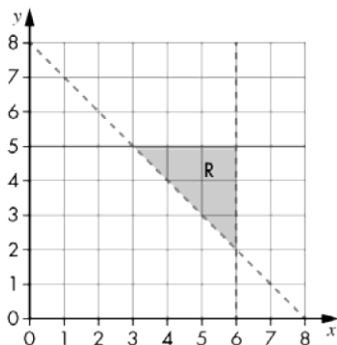
$2\pi(r + x) - 2\pi r = 20$

$2\pi r + 2\pi x - 2\pi r = 20$

$2\pi x = 20$

$x = 3.2$  2sf

**16** **a**



**b**  $4\frac{1}{2}$  square units

**c** It's infinite

**17** -4, -3, -2, -1, 0, 1, 2, 3, 4

**18** **a**  $x + y \geq 7$ ,  $y \leq 2x - 1$ ,  $y \geq \frac{1}{2}x$

**b**  $y \leq x - 3$ ,  $x > 2$ ,  $x + y < 8$

## Chapter 16 – Number: Counting, accuracy, powers and surds

### Exercise 16A

- 1** **a** 0.5 **b**  $0.\dot{3}$  **c** 0.25  
**d** 0.2 **e**  $0.1\dot{6}$  **f**  $0.\dot{1}4285\dot{7}$   
**g** 0.125 **h** 0.1 **i** 0.1  
**j**  $0.\dot{0}7692\dot{3}$

**2** **a**  $\frac{4}{7} = 0.5714285\dots$

$\frac{5}{7} = 0.7142857\dots$

$\frac{6}{7} = 0.8571428\dots$

**b** They all contain the same pattern of digits, starting at a different point in the pattern.

**3**  $0.\dot{1}$ ,  $0.\dot{2}$ ,  $0.\dot{3}$ , etc. Digit in decimal fraction same as numerator.

**4**  $0.\dot{0}\dot{9}$ ,  $0.\dot{1}\dot{8}$ ,  $0.\dot{2}\dot{7}$ , etc. Sum of digits in recurring pattern = 9. First digit is one less than numerator.

**5** 0.444 ..., 0.454 ..., 0.428 ..., 0.409 ..., 0.432 ..., 0.461 ...;

$\frac{9}{22}$ ,  $\frac{3}{7}$ ,  $\frac{16}{37}$ ,  $\frac{4}{9}$ ,  $\frac{5}{11}$ ,  $\frac{6}{13}$

**6** **a**  $\frac{1}{8}$  **b**  $\frac{17}{50}$  **c**  $\frac{29}{40}$  **d**  $\frac{5}{16}$

**e**  $\frac{89}{100}$  **f**  $\frac{1}{20}$  **g**  $2\frac{7}{20}$  **h**  $\frac{7}{32}$

**7** **a**  $0.08\dot{3}$  **b** 0.0625 **c** 0.05  
**d** 0.04 **e** 0.02

**8** **a**  $\frac{4}{3}$  **b**  $\frac{6}{5}$  **c**  $\frac{5}{2}$   
**d**  $\frac{10}{7}$  **e**  $\frac{20}{11}$  **f**  $\frac{15}{4}$

**9** **a** 0.75,  $1.\dot{3}$ ;  $0.8\dot{3}$ , 1.2; 0.4, 2.5; 0.7,  $1.\dot{4}2857\dot{1}$ ; 0.55,  $1.\dot{8}\dot{1}$ ;  $0.2\dot{6}$ , 3.75

**b** Not always true, e.g. reciprocal of  $0.4$  ( $\frac{2}{5}$ ) is  $\frac{5}{2} = 2.5$

**10**  $1 \div 0$  is infinite, so there is no finite answer.

**11** **a** 10

**b** 2

**c** The reciprocal of a reciprocal is always the original number.

**12** The reciprocal of  $x$  is greater than the reciprocal of  $y$ . For example, 2, 10, reciprocal of 2 is 0.5, reciprocal of 10 is 0.1, and  $0.5 > 0.1$

**13** Possible answer:  $-\frac{1}{2}x - 2 = 1$ ,  $-\frac{1}{3}x - 3 = 1$

**14** **a** 24.24242 ... **b** 24

**c**  $\frac{24}{99} = \frac{8}{33}$

**15** **a**  $\frac{8}{9}$  **b**  $\frac{34}{99}$  **c**  $\frac{5}{11}$  **d**  $\frac{21}{37}$

$$\begin{array}{llll} \text{e } \frac{4}{9} & \text{f } \frac{2}{45} & \text{g } \frac{13}{90} & \text{h } \frac{1}{22} \\ \text{i } 2\frac{7}{9} & \text{j } 7\frac{7}{11} & \text{k } 3\frac{1}{3} & \text{l } 2\frac{2}{33} \end{array}$$

16 a true      b true      c recurring

17 a  $\frac{9}{9}$       b  $\frac{45}{90} = \frac{1}{2} = 0.5$

### Exercise 16B

- 1 a 14      b 100      c 5      d 13  
 2 8, 25 and 27  
 3 13 and 14  
 4 5 and 6  
 5 Answers can be about the same as these  
 a i  $\sqrt{(66 \times 100)} \approx 8.1 \times 10 = 81$   
 ii  $\sqrt{49} = 7$ , so  $\sqrt{45} \approx 6.7$   
 iii  $\sqrt[3]{64} = 4$ ,  $\sqrt[3]{27} = 3$ , so  $\sqrt[3]{40} \approx 3.4$   
 iv  $5.8^4 \approx 6^4 = 36 \times 36 \approx 30 \times 40 = 1200$   
 v  $\sqrt[3]{45\,000} = \sqrt[3]{45 \times 10^3} \approx 35$   
 b i 81.24      ii 6.708      iii 3.42  
 iv 1132      v 35.57

### Exercise 16C

- 1 a  $\frac{1}{5^3}$       b  $\frac{1}{6}$       c  $\frac{1}{10^5}$       d  $\frac{1}{3^2}$   
 e  $\frac{1}{8^2}$       f  $\frac{1}{9}$       g  $\frac{1}{w^2}$       h  $\frac{1}{t}$   
 i  $\frac{1}{x^m}$       j  $\frac{4}{m^3}$   
 2 a  $3^{-2}$       b  $5^{-1}$       c  $10^{-3}$       d  $m^{-1}$       e  $t^{-n}$   
 3 a i  $2^4$       ii  $2^{-1}$       iii  $2^{-4}$       iv  $-2^3$   
 b i  $10^3$       ii  $10^{-1}$       iii  $10^{-2}$       iv  $10^6$   
 c i  $5^3$       ii  $5^{-1}$       iii  $5^{-2}$       iv  $5^{-4}$   
 d i  $3^2$       ii  $3^{-3}$       iii  $3^{-4}$       iv  $-3^5$   
 4 a  $\frac{5}{x^3}$       b  $\frac{6}{t}$       c  $\frac{7}{m^2}$       d  $\frac{4}{q^4}$   
 e  $\frac{10}{y^5}$       f  $\frac{1}{2x^3}$       g  $\frac{1}{2m}$       h  $\frac{3}{4t^4}$   
 i  $\frac{4}{5y^3}$       j  $\frac{7}{8x^5}$   
 5 a  $7x^{-3}$       b  $10p^{-1}$       c  $5t^{-2}$   
 d  $8m^{-5}$       e  $3y^{-1}$   
 6 a i 25      ii  $\frac{1}{125}$       iii  $\frac{4}{5}$   
 b i 64      ii  $\frac{1}{16}$       iii  $\frac{5}{256}$   
 c i 8  
 ii  $\frac{1}{32}$   
 iii  $\frac{9}{2}$  or  $4\frac{1}{2}$   
 d i 1 000 000  
 ii  $\frac{1}{1000}$   
 iii  $\frac{1}{4}$   
 7 24 (32 - 8)

8  $x = 8$  and  $y = 4$  (or  $x = y = 1$ )

9  $\frac{1}{2097152}$

10 a  $x^{-5}, x^0, x^5$       b  $x^5, x^0, x^{-5}$       c  $x^5, x^{-5}, x^0$

### Exercise 16D

- 1 a 5      b 25      c 3      d 5  
 e 20      f 5      g 3      h 10  
 i 3      j  $1\frac{1}{7}$       k  $\frac{1}{4}$       l  $\frac{1}{2}$   
 m  $\frac{1}{3}$       n  $\frac{1}{5}$       o  $\frac{1}{10}$   
 2 a  $\frac{5}{6}$       b  $1\frac{2}{3}$       c  $\frac{8}{9}$       d  $1\frac{4}{5}$   
 e  $\frac{5}{8}$       f  $\frac{3}{5}$       g  $\frac{1}{4}$       h  $2\frac{1}{2}$   
 i  $\frac{4}{5}$       j  $1\frac{1}{7}$   
 3  $\left(x^{\frac{1}{n}}\right)^n = x^{\frac{1}{n} \times n} = x^1 = x$ , but  
 $\left(\sqrt[n]{x}\right)^n = \sqrt[n]{x} \times \sqrt[n]{x} \dots n \text{ times} = x$ , so  
 $x^{\frac{1}{n}} = \sqrt[n]{x}$

4  $64^{-\frac{1}{2}} = \frac{1}{8}$ , others are both  $\frac{1}{2}$

5 Possible answer: The negative power gives the reciprocal, so  $27^{-\frac{1}{3}} = \frac{1}{27^{\frac{1}{3}}}$

The power one-third means cube root, so you need the cube root of 27 which is 3, so  $27^{\frac{1}{3}} = 3$  and  $\frac{1}{27^{\frac{1}{3}}} = \frac{1}{3}$

6 Possible answers include  $x = 16$  and  $y = 64$ ,  
 $x = 25$  and  $y = 125$

### Exercise 16E

- 1 a 16      b 25      c 216      d 81  
 2 a  $t^{\frac{2}{3}}$       b  $m^{\frac{3}{4}}$       c  $k^{\frac{2}{5}}$       d  $x^{\frac{3}{2}}$   
 3 a 4      b 9      c 64      d 3125  
 4 a  $\frac{1}{5}$       b  $\frac{1}{6}$       c  $\frac{1}{2}$       d  $\frac{1}{3}$   
 e  $\frac{1}{4}$       f  $\frac{1}{2}$       g  $\frac{1}{2}$       h  $\frac{1}{3}$   
 5 a  $\frac{1}{125}$       b  $\frac{1}{216}$       c  $\frac{1}{8}$       d  $\frac{1}{27}$   
 e  $\frac{1}{256}$       f  $\frac{1}{4}$       g  $\frac{1}{4}$       h  $\frac{1}{9}$   
 6 a  $\frac{1}{100000}$       b  $\frac{1}{12}$       c  $\frac{1}{25}$       d  $\frac{1}{27}$   
 e  $\frac{1}{32}$       f  $\frac{1}{32}$       g  $\frac{1}{81}$       h  $\frac{1}{13}$   
 7  $8^{-\frac{2}{3}} = \frac{1}{4}$ , others are both  $\frac{1}{8}$

- 8 Possible answer: The negative power gives the

reciprocal, so  $27^{-\frac{2}{3}} = \frac{1}{27^{\frac{2}{3}}}$

The power one-third means cube root, so we need the cube root of 27 which is 3 and the power 2 means

square, so  $3^2 = 9$ , so  $27^{\frac{2}{3}} = 9$  and  $\frac{1}{27^{\frac{2}{3}}} = \frac{1}{9}$

9  $3 = x^{-\frac{2}{3}} \div x^{-1}$ ,  $3 = x^{\frac{1}{3}}$ ,  $x = 27$

### Exercise 16F

- 1 a  $\sqrt{6}$  b  $\sqrt{15}$  c 2 d 4  
e  $2\sqrt{10}$  f 3 g  $2\sqrt{3}$  h  $\sqrt{21}$   
i  $\sqrt{14}$  j 6 k 6 l  $\sqrt{30}$
- 2 a 2 b  $\sqrt{5}$  c  $\sqrt{6}$  d  $\sqrt{3}$   
e  $\sqrt{5}$  f 1 g  $\sqrt{3}$  h  $\sqrt{7}$   
i 2 j  $\sqrt{6}$  k 1 l 3
- 3 a  $2\sqrt{3}$  b 15 c  $4\sqrt{2}$  d  $4\sqrt{3}$   
e  $8\sqrt{5}$  f  $3\sqrt{3}$  g 24 h  $3\sqrt{7}$   
i  $2\sqrt{7}$  j  $6\sqrt{5}$  k  $6\sqrt{3}$  l 30
- 4 a  $\sqrt{3}$  b 1 c  $2\sqrt{2}$  d  $\sqrt{2}$   
e  $\sqrt{5}$  f  $\sqrt{3}$  g  $\sqrt{2}$  h  $\sqrt{7}$   
i  $\sqrt{7}$  j  $2\sqrt{3}$  k  $2\sqrt{3}$  l 1
- 5 a a b 1 c  $\sqrt{a}$
- 6 a  $3\sqrt{2}$  b  $2\sqrt{6}$  c  $2\sqrt{3}$  d  $5\sqrt{2}$   
e  $2\sqrt{2}$  f  $3\sqrt{3}$  g  $4\sqrt{3}$  h  $5\sqrt{3}$   
i  $3\sqrt{5}$  j  $3\sqrt{7}$  k  $4\sqrt{2}$  l  $10\sqrt{2}$   
m  $10\sqrt{10}$  n  $5\sqrt{10}$  o  $7\sqrt{2}$  p  $9\sqrt{3}$
- 7 a 36 b  $16\sqrt{30}$  c 54 d 32  
e  $48\sqrt{6}$  f  $48\sqrt{6}$  g  $18\sqrt{15}$  h 84  
i 64 j 100 k 50 l 56
- 8 a  $20\sqrt{6}$  b  $6\sqrt{15}$  c 24 d 16  
e  $12\sqrt{10}$  f 18 g  $20\sqrt{3}$  h  $10\sqrt{21}$   
i  $6\sqrt{21}$  j 36 k 24 l  $12\sqrt{30}$
- 9 a 6 b  $3\sqrt{5}$  c  $6\sqrt{6}$  d  $2\sqrt{3}$   
e  $4\sqrt{5}$  f 5 g  $7\sqrt{3}$  h  $2\sqrt{7}$   
i 6 j  $2\sqrt{7}$  k 5  
l Does not simplify
- 10 a  $2\sqrt{3}$  b 4 c  $6\sqrt{2}$  d  $4\sqrt{2}$   
e  $6\sqrt{5}$  f  $24\sqrt{3}$  g  $3\sqrt{2}$  h  $\sqrt{7}$   
i  $10\sqrt{7}$  j  $8\sqrt{3}$  k  $10\sqrt{3}$  l 6
- 11 a abc b  $\frac{a}{c}$  c  $c\sqrt{b}$
- 12 a 20 b 24 c 10  
d 24 e 3 f 6
- 13 a  $\frac{3}{4}$  b  $8\frac{1}{3}$  c  $\frac{5}{16}$  d 12 e 2
- 14 a False b False

15 Possible answer:  $\sqrt{3} \times 2\sqrt{3} (=6)$

16  $(\sqrt{a} + \sqrt{b})^2 = (\sqrt{a+b})^2$ ,  $a + 2\sqrt{ab} + b = a + b$ ,  
Cancel a and b,  $2\sqrt{ab} = 0$ , so  $a = 0$  and/or  $b = 0$ .

### Exercise 16G

- 1 Expand the brackets each time.
- 2 a  $2\sqrt{3} - 3$  b  $3\sqrt{2} - 8$  c  $10 + 4\sqrt{5}$   
d  $12\sqrt{7} - 42$  e  $15\sqrt{2} - 24$  f  $9 - \sqrt{3}$
- 3 a  $2\sqrt{3}$  b  $1 + \sqrt{5}$  c  $-1 - \sqrt{2}$   
d  $\sqrt{7} - 30$  e -41 f  $7 + 3\sqrt{6}$   
g  $9 + 4\sqrt{5}$  h  $3 - 2\sqrt{2}$  i  $11 + 6\sqrt{2}$
- 4 a  $3\sqrt{2}$  cm b  $2\sqrt{3}$  cm c  $2\sqrt{10}$  cm
- 5 a  $\sqrt{3} - 1$  cm<sup>2</sup> b  $2\sqrt{5} + 5\sqrt{2}$  cm<sup>2</sup>  
c  $2\sqrt{3} + 18$  cm<sup>2</sup>
- 6 a  $\frac{\sqrt{3}}{3}$  b  $\frac{\sqrt{2}}{2}$  c  $\frac{\sqrt{5}}{5}$  d  $\frac{\sqrt{3}}{6}$   
e  $\sqrt{3}$  f  $\frac{5\sqrt{2}}{2}$  g  $\frac{3}{2}$  h  $\frac{5\sqrt{2}}{2}$   
i  $\frac{\sqrt{21}}{3}$  j  $\frac{\sqrt{2}+2}{2}$  k  $\frac{2\sqrt{3}-3}{3}$  l  $\frac{5\sqrt{3}+6}{3}$
- 7 a i 1 ii -4 iii 2  
iv 17 v -44  
b They become whole numbers. Difference of two squares makes the 'middle terms' (and surds) disappear.
- 8 a Possible answer:  $\sqrt{2}$  and  $\sqrt{2}$  or  $\sqrt{2}$  and  $\sqrt{8}$   
b Possible answer:  $\sqrt{2}$  and  $\sqrt{3}$
- 9 a Possible answer:  $\sqrt{2}$  and  $\sqrt{2}$  or  $\sqrt{8}$  and  $\sqrt{2}$   
b Possible answer:  $\sqrt{3}$  and  $\sqrt{2}$
- 10 Possible answer:  $80^2 = 6400$ , so  $80 = \sqrt{6400}$  and  $10\sqrt{70} = \sqrt{7000}$   
Since  $6400 < 7000$ , there is not enough cable.
- 11  $9 + 6\sqrt{2} + 2 - (1 - 2\sqrt{8} + 8) = 11 - 9 + 6\sqrt{2} + 4\sqrt{2} = 2 + 10\sqrt{2}$
- 12  $x^2 - y^2 = (1 + \sqrt{2})^2 - (1 - \sqrt{8})^2 = 1 + 2\sqrt{2} + 2 - (1 - 2\sqrt{8} + 8) = 3 - 9 + 2\sqrt{2} + 4\sqrt{2} = -6 + 6\sqrt{2}$   
 $(x+y)(x-y) = (2 - \sqrt{2})(3\sqrt{2}) = 6\sqrt{2} - 6$
- 13  $4\sqrt{2} - (\sqrt{2} - 1) = 3\sqrt{2} + 1$ .  $(\sqrt{2} - 1)(3\sqrt{2} + 1) = 5 - 2\sqrt{2}$
- 14 a i  $3 + n\sqrt{2}$  ii  $\frac{n+1}{\sqrt{3}}$   
b i  $(\sqrt{5})^n$  ii  $(5 + \sqrt{2}) \times (\sqrt{2})^{n-1}$
- ### Exercise 16H
- 1 a  $6.5 \text{ cm} \leq 7 \text{ cm} < 7.5 \text{ cm}$   
b  $115 \text{ g} \leq 120 \text{ g} < 125 \text{ g}$   
c  $3350 \text{ km} \leq 3400 \text{ km} < 3450 \text{ km}$

- d  $49.5 \text{ mph} \leq 50 \text{ mph} < 50.5 \text{ mph}$   
 e  $\pounds 5.50 \leq \pounds 6 < \pounds 6.50$   
 f  $16.75 \text{ cm} \leq 16.8 \text{ cm} < 16.85 \text{ cm}$   
 g  $15.5 \text{ kg} \leq 16 \text{ kg} < 16.5 \text{ kg}$   
 h  $14\,450 \text{ people} \leq 14\,500 \text{ people} < 14\,550 \text{ people}$   
 i  $54.5 \text{ miles} \leq 55 \text{ miles} < 55.5 \text{ miles}$   
 j  $52.5 \text{ miles} \leq 55 \text{ miles} < 57.5 \text{ miles}$
- 2 a  $5.5 \text{ cm} \leq 6 \text{ cm} < 6.5 \text{ cm}$   
 b  $16.5 \text{ kg} \leq 17 \text{ kg} < 17.5 \text{ kg}$   
 c  $31.5 \text{ min} \leq 32 \text{ min} < 32.5 \text{ min}$   
 d  $237.5 \text{ km} \leq 238 \text{ km} < 238.5 \text{ km}$   
 e  $7.25 \text{ m} \leq 7.3 \text{ m} < 7.35 \text{ m}$   
 f  $25.75 \text{ kg} \leq 25.8 \text{ kg} < 25.85 \text{ kg}$   
 g  $3.35 \text{ h} \leq 3.4 \text{ h} < 3.45 \text{ h}$   
 h  $86.5 \text{ g} \leq 87 \text{ g} < 87.5 \text{ g}$   
 i  $4.225 \text{ mm} \leq 4.23 \text{ mm} < 4.235 \text{ mm}$   
 j  $2.185 \text{ kg} \leq 2.19 \text{ kg} < 2.195 \text{ kg}$   
 k  $12.665 \text{ min} \leq 12.67 \text{ min} < 12.675 \text{ min}$   
 l  $24.5 \text{ m} \leq 25 \text{ m} < 25.5 \text{ m}$   
 m  $35 \text{ cm} \leq 40 \text{ cm} < 45 \text{ cm}$   
 n  $595 \text{ g} \leq 600 \text{ g} < 605 \text{ g}$   
 o  $25 \text{ min} \leq 30 \text{ min} < 35 \text{ min}$   
 p  $995 \text{ m} \leq 1000 \text{ m} < 1050 \text{ m}$   
 q  $3.95 \text{ m} \leq 4.0 \text{ m} < 4.05 \text{ m}$   
 r  $7.035 \text{ kg} \leq 7.04 \text{ kg} < 7.045 \text{ kg}$   
 s  $11.95 \text{ s} \leq 12.0 \text{ s} < 12.05 \text{ s}$   
 t  $6.995 \text{ m} \leq 7.00 \text{ m} < 7.005 \text{ m}$
- 3 a 7.5 m, 8.5 m                      b 25.5 kg, 26.5 kg  
 c 24.5 min, 25.5 min              d 84.5 g, 85.5 g  
 e 2.395 m, 2.405 m                  f 0.15 kg, 0.25 kg  
 g 0.055 s, 0.065 s                  h 250 g, 350 g  
 i 0.65 m, 0.75 m                    j 365.5 g, 366.5 g  
 k 165 weeks, 175 weeks            l 205 g, 215 g
- 4 There are 16 empty seats and the number getting on the bus is from 15 to 24 so it is possible if 15 or 16 get on.
- 5 C: The chain and distance are both any value between 29.5 and 30.5 metres, so there is no way of knowing if the chain is longer or shorter than the distance.
- 6 2 kg 450 grams
- 7 a  $< 65.5 \text{ g}$                       b  $64.5 \text{ g}$   
 c  $< 2620 \text{ g}$                       d  $2580 \text{ g}$
- 8 345, 346, 347, 348, 349
- 9 Any number in range  $4 < a < 5$ , eg 4.5

### Exercise 16I

- 1 Minimum 65 kg, maximum 75 kg
- 2 Minimum is 19, maximum is 20
- 3 a 12.5 kg                      b 20
- 4 3 years 364 days (Jack is on his fifth birthday; Jill is 9 years old tomorrow)
- 5 a  $38.25 \text{ cm}^2 \leq \text{area} < 52.25 \text{ cm}^2$   
 b  $37.1575 \text{ cm}^2 \leq \text{area} < 38.4475 \text{ cm}^2$   
 c  $135.625 \text{ cm}^2 \leq \text{area} < 145.225 \text{ cm}^2$
- 6 a  $5.5 \text{ m} \leq \text{length} < 6.5 \text{ m}$ ,  $3.5 \text{ m} \leq \text{width} < 4.5 \text{ m}$   
 b  $29.25 \text{ m}^2$   
 c 18 m
- 7  $79.75 \text{ m}^2 \leq \text{area} < 100.75 \text{ m}^2$

- 8  $216.125 \text{ cm}^3 \leq \text{volume} < 354.375 \text{ cm}^3$
- 9 12.5 metres
- 10 Yes, because they could be walking at 4.5 mph and 2.5 mph meaning that they would cover 4.5 miles + 2.5 miles = 7 miles in 1 hour
- 11  $20.9 \text{ m} \leq \text{length} < 22.9 \text{ m}$  (3 sf)
- 12  $16.4 \text{ cm}^2 \leq \text{area} < 21.7 \text{ cm}^2$  (3 sf)
- 13 a i  $64.1 \text{ cm}^3 \leq \text{volume} < 69.6 \text{ cm}^3$  (3 sf)  
 ii  $\pounds 22\,578 \leq \text{price} < \pounds 24\,515$  (nearest £)  
 b  $23\,643 \leq \text{price} < \pounds 23\,661$  (nearest £)  
 c Errors in length compounded by being used 3 times in a, but errors in weight only used once in b
- 14 a  $14.65 \text{ s} \leq \text{time} < 14.75 \text{ s}$   
 b  $99.5 \text{ m} \leq \text{length} < 100.5 \text{ m}$   
 c  $6.86 \text{ m/s}$  (3 sf)
- 15 a 1.25% (3 sf)  
 b 1.89% (3 sf)
- 16  $3.41 \text{ cm} \leq \text{length} < 3.43 \text{ cm}$  (3 sf)
- 17  $5.80 \text{ cm} \leq \text{length} < 5.90 \text{ cm}$  (3 sf)
- 18  $14 \text{ s} \leq \text{time} < 30 \text{ s}$
- 19 Cannot be certain as limits of accuracy for all three springs overlap:  
 Red: 12.5 newtons to 13.1 newtons  
 Green: 11.8 newtons to 13.2 newtons  
 Blue: 9.5 newtons to 12.9 newtons  
 For example, all tensions could be 12 newtons

### Exercise 16J

- 1 Number of possible permutations is  $7! \div 2!5! = 21$ . Of these any pair of the first 5 coins will be less than a £1, which is  $5! \div 3!2! = 10$ . Hence 11 pairs will have a value greater than £1.
- 2 6, 16, etc. up to 196 which is 19 plus 60 up to 69, which is 9 (66 already counted) plus 160 up to 169 which is 9 (166 already counted) giving a total of 37
- 3 a i 5040                      ii  $2.43 \times 10^{18}$  (3 sf)  
 b This depends on your calculator but  $69! = 1.71 \times 10^{98}$ , which is about the number of atoms in QUINTILLION (look it up) universes.
- 4 a  $10^4 = 10\,000$                       b  $13^4 = 28\,561$
- 5  $3 \times 13^3 = 6591$
- 6 a  $10^3 = 1000$                       b 6
- 7  $8 \times 7 \times 6 = 336$
- 8 a 16 ways of choosing an Ace followed by a King out of  $52 \times 52$  ways of picking 2 cards with replacement, so  $\frac{16}{2704} = \frac{1}{169}$   
 b Still 16 ways of taking an ace followed by a King but out of  $52 \times 51$  so  $\frac{16}{2652} = \frac{4}{663}$
- 9 a 1 6 15 20 15 6 1, 1 7 21 35 35 21 7 1, 1 8 28 56 70 56 28 8 1, 1 9 36 84 126 126 84 36 9 1, 1 10 45 120 210 252 210 120 45 10 1  
 b i 10                      ii 15  
 c  ${}^7C_3$  is the 3<sup>rd</sup> value in the 7<sup>th</sup> row, which is 35  
 d 126



- 10 a 31      b  $8(2^8 = 256)$
- 11 No The number of people he can expect to throw six 6s is  $\frac{1}{46656} \times 200 = 0.0043$  Explanations may vary.
- 12 There are 4 ways to choose the first letter, 3 ways for the second, and so on. But there are two Is, so  $4! \div 2 = 12$ . Explanations may vary.
- 13 60
- 14 6
- 15 a 435    b 48    c 12    d 32  
e 455    f 1296    g 7776    h 67 6000  
i 3 628 800
- 16 a i 79 380 ii 35 280 iii 52 920  
b Answers will vary. For example, none of the players is repeated in another group.
- 17 a 40 320 b 109 600
- 18 They are both correct.  $8^3 - 1 = 2^9 - 1 = 511$

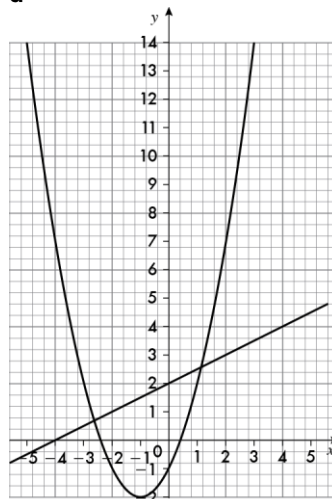
### Review questions

- 1 a 13    b 10    c 13
- 2 a 1845    b 1854
- 3 8, 16 and 36
- 4 19
- 5  $12! \div 10!$  is greater
- 6 a  $12^4 = 20736$     b  $3 \times 12^3 = 5184$
- 7 a  $5 \times 4 \times 3 \times 2 \times 1 = 5! = 120$   
b There will be 24 starting with each letter and CODES will be the first CO word so 13<sup>th</sup> in the list
- 8 a  $3^4 = 81$     b 8    c -3
- 9 a  $\frac{1}{25}$     b  $6^4 = 1296$
- 10  $\frac{7}{15}$
- 11  $a = 7, b = -1$
- 12  $6\sqrt{2}$
- 13 a  $x = 0.5454\ldots, 100x = 54.5454\ldots, 99x = 54$   
 $x = \frac{54}{99}$ , cancel by 9  
b  $0.35454\ldots = 0.3 + 0.05454\ldots = \frac{3}{10} + \frac{6}{110} = \frac{33}{110} + \frac{6}{110} = \frac{39}{110}$
- 14  $\frac{11}{45}$
- 15 a 9    b  $5\sqrt{2}$
- 16 a  $\frac{\sqrt{5}}{5}$     b 2
- 17 a i  $\sqrt{32} = \sqrt{16 \times 2} = \sqrt{16} \times \sqrt{2} = 4\sqrt{2}$   
ii  $14 + 4\sqrt{6}$   
b  $2^2 + (2 + \sqrt{6})^2 = 4 + 4 + 6 + 4\sqrt{6} = 14 + 4\sqrt{6} = (\sqrt{2} + \sqrt{12})^2$  so the sides obey Pythagoras' theorem
- 18 a  $\sqrt{27} = 5.20\text{m}$   
b Cube side 2.95 m has diagonal 5.07 m. Max length pole is 5.005 m so it will fit round the corner.

## Chapter 17 – Algebra: Quadratic Equations

### Exercise 17A

- 1 a Values of  $y$ : 27, 16, 7, 0, -5, -8, -9, -8, -5, 0, 7  
b -8.8    c 3.4 or -1.4
- 2 a Values of  $y$ : 2, -1, -2, -1, 2, 7, 14  
b 0.25    c 0.7 or -2.7  
d



- e (1.1, 2.6) and (-2.6, 0.7)
- 3 a Values of  $y$ : 15, 9, 4, 0, -3, -5, -6, -6, -5, -3, 0, 4, 9  
b -0.5 and 3
- 4 a Same answer  
b

x	-3	-2	-1	0	1	2	3	4	5	6	7
y	28	19	12	7	4	3	4	7	12	19	28

Since the quadratic graph has a vertical line of symmetry and the  $y$ -values for  $x = 1$  and  $x = 3$  are the same, this means that the  $y$ -values will be symmetric about  $x = 2$ . Hence the  $y$ -values will be the same for  $x = 0$  and  $x = 4$ , and so on.

- 5 Points plotted and joined should give a parabola.
- 6 Line A has a constant in front, so is 'thinner' than the rest.  
Line B has a negative in front, so is 'upside down'.  
Line C does not pass through the origin.

### Exercise 17B

- 1 a -2, -5    b 4, 9    c -6, 3
- 2 a -4, -1    b 2, 4    c -2, 5  
d -3, 5    e -6, 3    f -1, 2  
g -5    h 7
- 3  $x(x + 40) = 48\,000, x^2 + 40x - 48\,000 = 0,$   
 $(x + 240)(x - 200) = 0$   
Fence is  $2 \times 200 + 2 \times 240 = 880\text{ m}$
- 4 a -10, 3    b -4, 11    c -8, 9  
d 8, 9    e 1    f -6, 7  
g -2, 3

- 5 Mario was correct.  
Sylvan did not make it into a standard quadratic and only factorised the  $x$  terms. She also incorrectly solved the equation  $x - 3 = 4$ .

6 40 cm

7 48 km/h

- 8 a 4, 9  
b i 2, -2, 3, -3 ii 16, 81  
iii 5, 6, 10, 11

### Exercise 17C

- 1 a  $\frac{1}{3}, -3$  b  $1\frac{1}{3}, -\frac{1}{2}$  c  $-\frac{1}{5}, 2$   
d  $-2\frac{1}{2}, 3\frac{1}{2}$  e  $-\frac{1}{6}, -\frac{1}{3}$  f  $\frac{2}{3}, 4$   
g  $\frac{1}{2}, -3$  h  $\frac{5}{2}, -\frac{7}{6}$  i  $-1\frac{2}{3}, 1\frac{2}{5}$   
j  $1\frac{3}{4}, 1\frac{2}{7}$  k  $\frac{2}{3}, \frac{1}{8}$  l  $\pm\frac{1}{4}$   
m  $-2\frac{1}{4}, 0$  n  $\pm 1\frac{2}{5}$  o  $-\frac{1}{3}, 3$
- 2 a 7, -6 b  $-2\frac{1}{2}, 1\frac{1}{2}$  c  $-1, \frac{11}{13}$   
d  $-\frac{2}{5}, \frac{1}{2}$  e  $-\frac{1}{3}, -\frac{1}{2}$  f  $\frac{1}{5}, -2$   
g 4 h  $-2, \frac{1}{8}$  i  $-\frac{1}{3}, 0$   
j  $\pm 5$  k  $-1\frac{2}{3}$  l  $\pm 3\frac{1}{2}$   
m  $-2\frac{1}{2}, 3$
- 3 a Both only have one solution:  $x = 1$ .  
b B is a linear equation, but A and C are quadratic equations.
- 4 a  $(5x - 1)^2 = (2x + 3)^2 + (x + 1)^2$ , when expanded and collected into the general quadratic, gives the required equation.  
b  $(10x + 3)(2x - 3)$ ,  $x = 1.5$ ; area =  $7.5 \text{ cm}^2$ .
- 5 a Show by substituting into the equation  
b  $-\frac{24}{5}$
- 6 5, 0.5
- 7 Area =  $22.75 \text{ m}^2$ , width 3.5 m

### Exercise 17D

- 1 a 1.77, -2.27 b 3.70, -2.70  
c -0.19, -1.53 d -1.39, -2.27  
e 1.37, -4.37 f 0.44, -1.69  
g 1.64, 0.61 h 0.36, -0.79  
i 1.89, 0.11
- 2 13
- 3  $x^2 - 3x - 7 = 0$
- 4 Terry gets  $x = \frac{4+\sqrt{0}}{8}$  and June gets  $(2x - 1)^2 = 0$  which only give one solution  $x = \frac{1}{2}$
- 5 6.54, 0.46
- 6 1.25, 0.8
- 7 a i -0.382, -2.618 ii 6.414, 3.586  
iii 7.531, -0.531 iv 1.123, -7.123

b Since  $a = 1$ , answers are  $\frac{-b + \sqrt{b^2 - 4c}}{2}$  and  $\frac{-b - \sqrt{b^2 - 4c}}{2}$ . When added,  
$$\frac{-b + \sqrt{b^2 - 4c} - b - \sqrt{b^2 - 4c}}{2} = \frac{-b - b}{2} = \frac{-2b}{2} = -b$$

### Exercise 17E

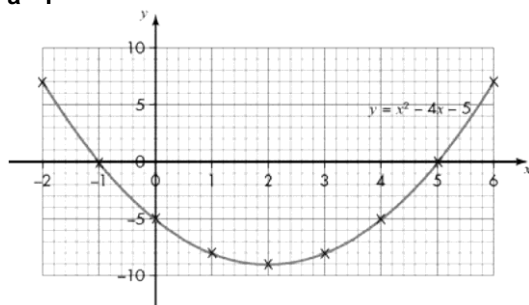
- 1 a 52 (TWO) b 0 (ONE)  
c -23 (NONE) d -7 (NONE)  
e 68 (TWO) f -35 (NONE)  
g -4 (NONE) h 0 (ONE)  
l 409 (TWO)
- 2 300
- 3  $x^2 + 3x - 1 = 0$ ;  $x^2 - 3x - 1 = 0$ ;  $x^2 + x - 3 = 0$ ;  
 $x^2 - x - 3 = 0$
- 4 2 or -10
- 5 Can be factorised:  $b^2 - 4ac = 1849, 1, 49, 1024, 900$   
Cannot be factorised:  $b^2 - 4ac = 41, 265, 3529, 216, 76$   
For those that can be factorised,  $b^2 - 4ac$  is a square number

### Exercise 17F

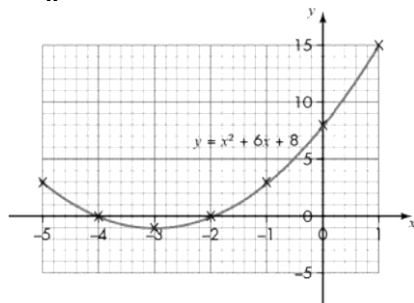
- 1 a  $(x - 2)^2 - 4$  b  $(x + 7)^2 - 49$   
c  $(x - 3)^2 - 9$  d  $(x + 3)^2 - 9$   
e  $(x - 5)^2 - 25$  f  $(x + 10)^2 - 100$   
g  $(x - 2)^2 - 5$  h  $(x + 3)^2 - 6$   
i  $(x + 4)^2 - 22$  j  $(x + 1)^2 - 2$   
k  $(x - 1)^2 - 8$  l  $(x + 9)^2 - 11$
- 2 a 4th, 1st, 2nd and 3rd – in that order  
b Write  $x^2 - 4x - 3 = 0$  as  $(x - 2)^2 - 7 = 0$ , Add 7 to both sides, square root both sides, Add 2 to both sides  
c i  $x = -3 \pm \sqrt{2}$  ii  $x = 2 \pm \sqrt{7}$
- 3 a  $-2 \pm \sqrt{5}$  b  $-7 \pm 3\sqrt{6}$   
c  $3 \pm \sqrt{6}$  d  $5 \pm \sqrt{30}$   
e  $-10 \pm \sqrt{101}$  f  $-4 \pm \sqrt{22}$
- 4 a 1.45, -3.45 b 5.32, -1.32  
c -4.16, 2.16
- 5 Check for correct proof.
- 6  $p = -14, q = -3$
- 7 a  $x^2 - 12x + 40 = (x - 6)^2 + 4 \geq 4$  for all  $x$   
b Doesn't intersect the  $x$ -axis
- 8 The answers are 42, -58. The equation can be factorised as  $(x - 42)(x + 58) = 0$  but it would be hard to find the factors of 2436. Completing the square works well because  $x^2 + 16x - 2436 = (x + 8)^2 - 2500$  and you can find the square root of 2500 without a calculator. Completing the square is therefore the better of the two non-calculator methods. The formula could also be used without a calculator because  $b^2 - 4ac = 10\,000$  so the square root can be taken, but you would have to work out  $16^2 + 4 \times 2436$  in order to get there.
- 9 H, C, B, E, D, J, A, F, G, I

## Exercise 17G

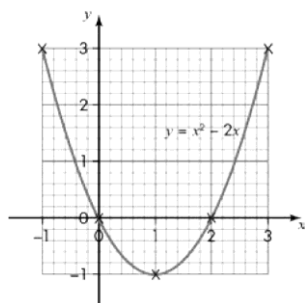
a i



ii



iii



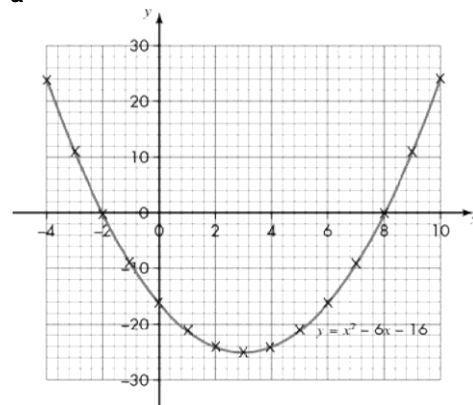
- b Each equation is written in the form  $x^2 + ax + b$ . You should find that the y-intercept is the value of  $b$ . Graph (i) has its y-intercept at  $(0, -5)$ , graph (ii) has its y-intercept at  $(0, 8)$  and graph (iii) has its y-intercept at  $(0, 0)$ . Note that the graph (iii)'s equation has no value for  $b$ , so  $b = 0$ .
- c i  $x = 5$  or  $-1$     ii  $x = -2$  or  $-4$   
iii  $x = 0$  or  $2$
- d The two x-intercepts have a product of  $b$  and add up to  $-a$ . This works because the x-intercepts are the answers of the quadratic equations when  $y = 0$ .
- e The value of  $x$  for the turning point is exactly halfway between the values of  $x$  for the x-intercepts. By completing the square, you should also be able to see that the x co-ordinate is the value that makes the brackets zero and the y co-ordinate is the value at the end.

## Exercise 17H

- 1 a i  $(0, -3)$     ii  $(-1, 0)$  and  $(3, 0)$   
iii  $(1, -4)$   
b i  $(0, 5)$     ii  $(-5, 0)$  and  $(1, 0)$   
iii  $(-2, 9)$

2

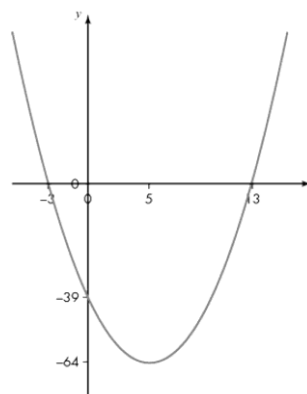
a



- b i  $(0, -16)$     ii  $(-2, 0)$  and  $(8, 0)$   
iii  $(3, -25)$

- 3 a roots:  $(-2, 0)$  and  $(2, 0)$ ; y-intercept  $(0, -4)$   
b roots:  $(0, 0)$  and  $(6, 0)$ ; y-intercept  $(0, 0)$   
c roots:  $(-1, 0)$  and  $(3, 0)$ ; y-intercept  $(0, -3)$   
d roots:  $(-11, 0)$  and  $(-3, 0)$ ; y-intercept  $(0, 33)$
- 4  $(3, -6)$
- 5  $-14$
- 6  $-5$
- 7 a  $(2, 0)$     b 2 is the only root
- 8 roots:  $(-0.5, 0)$  and  $(5, 0)$ ; y-intercept  $(0, -5)$ ; turning point:  $(2.25, -15.125)$
- 9 roots:  $(4.65, 0)$  and  $(7.85, 0)$ ; turning point:  $(6.25, -5.13)$

10



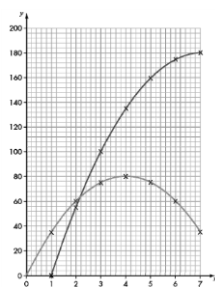
- 11  $y = (x - 3)^2 - 7$ ,  $y = x^2 - 6x + 9 - 7$ ,  $y = x^2 - 6x + 2$
- 12 a  $(-2, -7)$   
b i  $(a, 2b - a^2)$     ii  $(2a, b - 4a^2)$
- 13  $y = 2x^2 + 16x + 14$
- 14 a 60 m    b 80 m, 2 s    c 6 s

## Exercise 17I

- 1 a  $(0.7, 0.7)$ ,  $(-2.7, -2.7)$   
b  $(6, 12)$ ,  $(-1, -2)$   
c  $(4, -3)$ ,  $(-3, 4)$   
d  $(0.8, 1.8)$ ,  $(-1.8, -0.8)$   
e  $(4.6, 8.2)$ ,  $(0.4, -0.2)$   
f  $(3, 6)$ ,  $(-2, 1)$

- g (4.8, 6.6), (0.2, -2.6)  
h (2.6, 1.6), (-1.6, -2.6)
- 2 a (1, 0)  
b Only one intersection point  
c  $x^2 + x(3 - 5) + (-4 + 5) = 0$   
d  $(x - 1)^2 = 0 \Rightarrow x = 1$   
e Only one solution as line is a tangent to curve.
- 3 a There is no solution.  
b The graphs do not intersect.  
c  $x^2 + x + 4 = 0$   
d  $b^2 - 4ac = -15$   
e No solution as the discriminant is negative and there is no square root of a negative number.
- 4 a  $x = 4, y = 31$   
b There is only one solution because the graphs have the same shape and are at a constant distance apart.

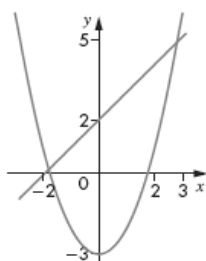
- 5 a Proof  
b



c 2.17 seconds

### Exercise 17J

- 1 a i -1.4, 4.4 ii -2, 5 iii -0.6, 3.6  
b 2.6, 0.4
- 2 a i -1.6, 2.6 ii 1.4, -1.4  
b i 2.3, -2.3 ii 2, -2
- 3



- a 2.2, -2.2 b -1.8, 2.8
- 4 -3.8, 1.8
- 5 a C and D b A and D  
c  $x^2 + 4x - 1 = 0$  d (-1.5, -10.25)
- 6 a i  $y = 5$  ii  $y = x + 3$   
iii  $y = -10$  iv  $y = x$   
v  $y = 3x - 9$  vi  $y = 2 - x$   
vii  $y = -3x$   
b  $y = \frac{1}{2}x + 3$
- 7 a i  $5 - 5x - x^2 = 0$  ii  $11 - 6x - x^2 = 0$   
iii  $9 - 4x - x^2 = 0$  iv  $30 - 16x - 3x^2 = 0$

b Equation would be  $-5 - 4x - x^2 = 0$ .  $b^2 - 4ac = -4$ .  
Negative  $b^2 - 4ac$  has no solutions.

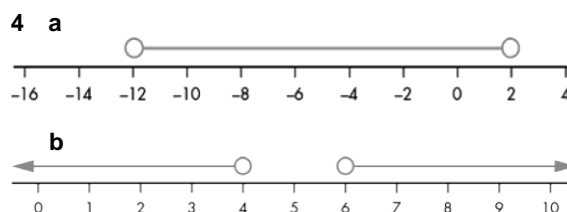
- 8 a  $(x + 2)(x - 1) = 0$  b  $5 - -2 = +7$ , not  $-7$   
c  $y = 2x + 7$

### Exercise 17K

- 1 a (5, -1) b (4, 1) c (8, -1)
- 2 a (2, 5) and (-2, -3)  
b (-1, -2) and (4, 3)  
c (3, 3) and (1, -1)
- 3 a (1, 2) and (-2, -1)  
b (-4, 1) and (-2, 2)
- 4 a (3, 4) and (4, 3)  
b (0, 3) and (-3, 0)
- 5 a (3, 2) and (-2, 3) b  $\sqrt{26}$
- 6 a Proof  
b  $x = -\frac{1}{5}, y = -\frac{43}{5}$  or  $x = 5, y = 7$
- 7 a Proof  
b  $x = 4, y = -13$  or  $x = 8, y = 11$
- 8 a  $x = 6, y = 7$  or  $x = -2, y = -9$   
b  $x = -1, y = 2$  or  $x = -2, y = -1$   
c  $x = 3, y = -5$  or  $x = 5, y = 3$   
d  $x = 1, y = -8$  or  $x = 4, y = 7$
- 9 a (1, 0)  
b iii as the straight line just touches the curve
- 10 a (-2, 1)  
b i (2, 1) ii (-2, -1) iii (2, -1)
- 11 a (2, 4) b (1, 0)  
c The line is a tangent to the curve.
- 12 16 m by 14 m
- 13 30 km/h
- 14 10p

### Exercise 17L

- 1 a  $x < -4, x > 4$  b  $-10 \leq x \leq 10$   
c  $0 < x < 1$  d  $x \leq -5, x \geq 0$   
e  $-23 < x < 23$  f  $x \leq -\frac{3}{2}, x \geq \frac{3}{2}$   
g  $x < 0, x > \frac{8}{3}$  h  $-\frac{19}{2} \leq x \leq 0$
- 2 a  $\{-3, -2, -1, 0, 1, 2, 3\}$   
b  $\{3, 4, 5, 6\}$
- 3 a  $x < -2, x > 5$  b  $-7 < x < -5$   
c  $x \leq 1, x \geq 5$  d  $-8 \leq x \leq 9$   
e  $\frac{1}{3} \leq x \leq 3$  f  $x < -\frac{11}{2}, x > -1$   
g  $x \leq \frac{3}{5}, x \geq 2$  h  $-\frac{3}{2} < x < \frac{2}{3}$

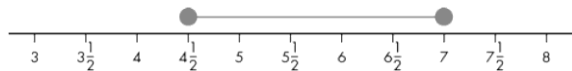


5 a  $3 < x \leq 6\frac{1}{2}$  b  $4 < x \leq 5$

6  $x < 2, x > 10$

7  $x < -\frac{3}{2}, x > 5$

8



9 a  $-692 < x < 708$

b  $x < -4 - \sqrt{5}, x > -4 + \sqrt{5}$

c  $-0.84 \leq x \leq 1.44$

10 £288, £364

11  $x < -4, -1 < x < 1, x > 4$

12 a  $\frac{30}{-13} = -2.31 > -6$  and  $\frac{30}{2} = 15 > 9$

b  $x < -7, 3 < x < 6$

### Review questions

1 a 9 b 5

2 a Two b One c None

3 b  $-5.27, 1.67$

4 b 3.18

5 15 m, 20 m

6 b i  $-0.3, 3.3$  ii  $0.6, 3.4$

7 a  $(0, 36)$  b  $(2, 0), (18, 0)$   
c  $(10, -64)$

8  $(1, 7), (7, 1)$

9 a  $-6$  b 3

10 a  $x^2 - 3x - 550 = 0$  b 25

11 a  $x < -35, x > 45$  b  $-298 < x < 302$   
c  $x \leq -589, x \geq 611$

12 2.54 m, 3.54 m

13  $210 \text{ cm}^2$

14  $(6, 8), (0, -10)$

15 a  $(p + q)(p - q)$   
b  $30^2 - 1^2 = (30 + 1)(30 - 1)$   
c 3600  
d  $-31, 29$

16 0.75 m

17 a  $48 - (x - 6)^2$  b 48

18 complete the square  $-113, 87$

19 a, b  $(1, 4) (5, 20)$  c  $x \leq 1, x \geq 5$

20  $x^2 - 8x + 19 = (x - 4)^2 + 3$   
Because  $(x - 4)^2$  is a squared term, the smallest possible value it can have is zero.  
Hence 3 is the smallest possible value of  $(x - 4)^2 + 3$ , so  $x^2 - 8x + 19$  is always positive.

## Chapter 18 – Statistics: sampling and more complex diagrams

### Exercise 18A

- 1 a secondary data  
b primary data  
c primary or secondary data  
d primary or secondary data  
e primary data  
f primary or secondary data

- 2 *Plan the data collection.* Choose a random sample of 30 boys and 30 girls from Year 11.

*Collect the data.* Ask each student to spell the same 10 words. This will avoid bias. Pick words that are often misspelt, eg accommodation, necessary

*Choose the best way to process and represent the data.* Calculate the mean number and range for the number of correct spellings for the boys and for the girls. Draw a dual bar chart to illustrate the data.

*Interpret the data and make conclusions.* Compare the mean and range to arrive at a conclusion. Is there a clear conclusion or do you need to change any of the 10 words or take a larger sample?

- 3 *Plan the data collection.* Choose a random sample of 20 boys and 20 girls from Year 11.

*Collect the data.* Ask each student, on average, how many hours of sport they play and how many hours of TV they watch each week.

*Choose the best way to process and represent the data.* Calculate the mean number of hours for the number of hours playing sport and the number of hours watching TV. Draw a scatter diagram to illustrate the data.

*Interpret the data and make conclusions.* Compare the means and write down the type and strength of correlation for the scatter diagram to arrive at a conclusion. Is there a clear conclusion or do you need to take a larger sample?

4 6

5 a 3 : 1

b The number of males to females in the survey is in the same ratio as the numbers at the match.

6 28.5%

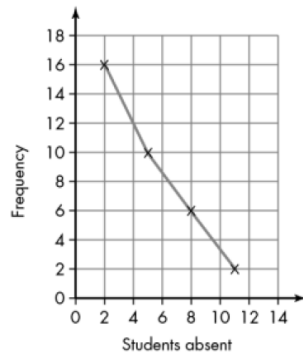
7a

	Male	Female	Total
Full time	13	7	20
Part time	4	6	10
			30

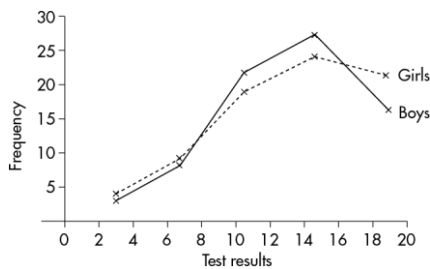
- b The numbers in the sample have been rounded.

## Exercise 18B

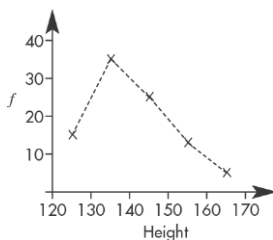
1 a b 4



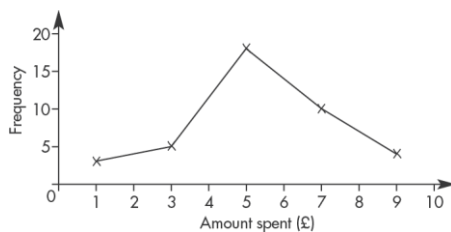
2 a b boys 12.9, girls 13.1  
c the girls did slightly better than the boys



3 a b 140.4 cm



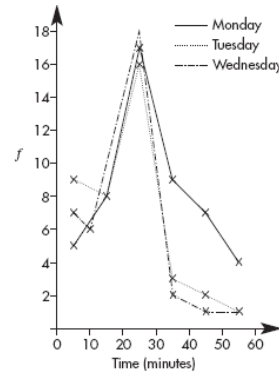
4 a i 17, 13, 6, 3, 1 ii £1.45  
b i



ii £5.35

c Much higher mean. Early morning, people just want a paper or a few sweets. Later people are buying food for the day.

5 a



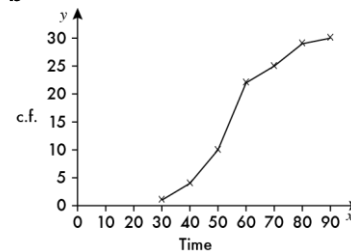
b Monday 28.4 min, Tuesday 20.9 min, Wednesday 21.3 min  
c There are more patients on a Monday, and so longer waiting times, as the surgery is closed during the weekend.

6 2.19 hours

7 That is the middle value of the time group 0 to 1 minute. It would be very unusual for most of them to be exactly in the middle at 30 seconds.

## Exercise 18C

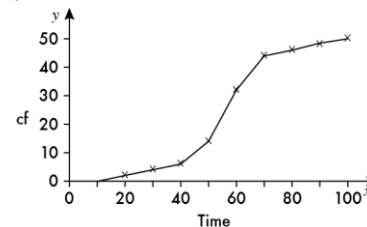
1 a cumulative frequencies 1, 4, 10, 22, 25, 28, 30  
b



c  $m = 54$  s and IQR = 16 s

2 a cumulative frequencies 1, 3, 5, 14, 31, 44, 47, 49, 50

b

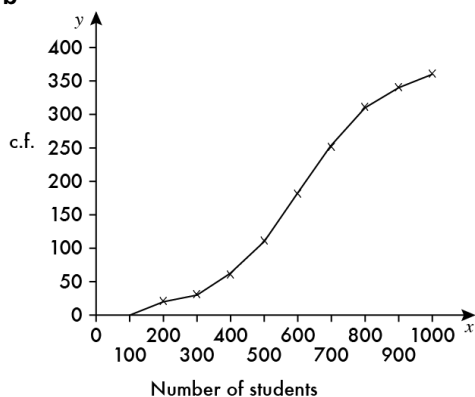


c  $m = 56$  s and IQR = 17 s

d pensioners as the median is closer to 1 minute and the IQRs are almost the same

3 a cumulative frequencies 12, 30, 63, 113, 176, 250, 314, 349, 360

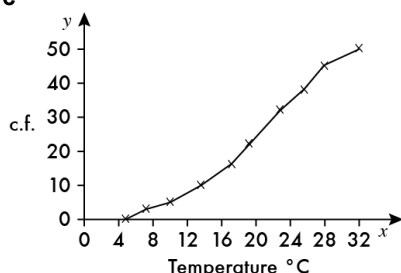
b

c  $m = 606$  studentsd  $Q_1 = 455$ ,  $Q_3 = 732$  and  $IQR = 277$ 

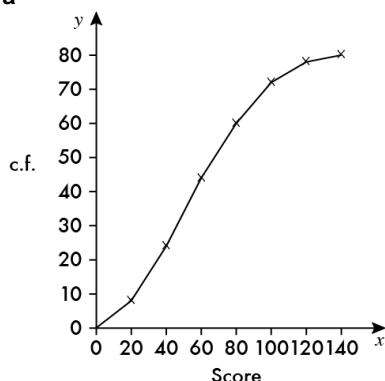
e approximately 13%

- 4 a cumulative frequency 2, 5, 10, 16, 22, 31, 39, 45, 50
- b because the temperature was recorded to the nearest degree, so for example the highest temperature in the first group could have been  $7.5^\circ$

c

d  $m = 20.5^\circ\text{C}$  and  $IQR = 10^\circ\text{C}$ 

5 a

b  $m = 56$ ,  $Q_1 = 37$  and  $Q_3 = 100$ 

c approximately 18%

- 6 a Paper A  $m = 66$ , Paper B  $m = 56$
- b Paper A  $IQR = 25$ , Paper B  $IQR = 18$
- c Paper B is the harder paper, it has a lower median and a lower upper quartile.
- d i Paper A 43, Paper B 45
- ii Paper A 78, Paper B 67

7 create a grouped frequency table:

Age, $a$ , (years)	Cumulative frequency	Frequency, $f$	Mid-point, $x$	$x \times f$
$0 < a \leq 20$	30	30	10	300
$20 < a \leq 40$	95	65	30	1950
$40 < a \leq 60$	150	55	50	2750
$60 < a \leq 80$	185	35	70	2450
$80 < a \leq 100$	200	15	90	1350
<b>Total</b>		200		8800

$$\text{mean} = \frac{8800}{200} = 44 \text{ years}$$

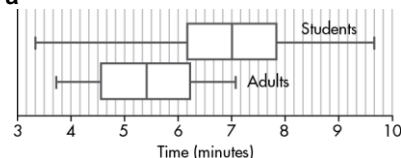
8 create a grouped frequency table:

Time, $t$ , (minutes)	Cumulative frequency	Frequency, $f$	Mid-point, $x$	$x \times f$
$0 < t \leq 5$	6	6	2.5	15
$5 < t \leq 10$	34	28	7.5	210
$10 < t \leq 15$	56	22	12.5	275
$15 < t \leq 20$	60	4	17.5	70
<b>Total</b>		60		570

$$\text{mean} = \frac{570}{60} = 9.5 \text{ minutes}$$

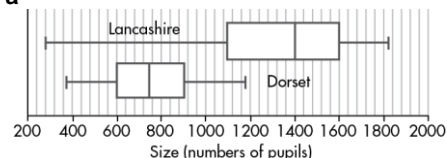
## Exercise 18D

1 a



- b The adults are much quicker than the students. Both distributions have the same interquartile range, but the range is smaller for the adults showing that they are more consistent. The students' median and upper quartiles are 1 minute, 35 seconds higher. The fastest person to complete the calculations was a student, but so was the slowest.

2 a



- b Schools are much larger in Lancashire than in Dorset since it has a greater median. The interquartile range in Dorset is smaller, showing that they have a more consistent size.
- 3 a The resorts have similar median temperatures, but Resort A has a smaller interquartile range, showing that the temperatures are more consistent. Resort B has a much wider temperature range, where the greatest extremes of temperature are recorded.

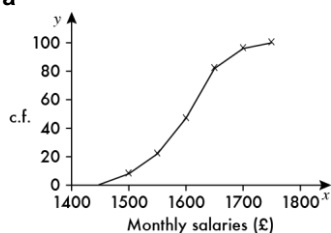
- b Resort A is probably a better choice as the weather seems more consistent.

4 a



- b Both distributions have a similar interquartile range, and there is little difference between the upper quartile values. Men have a wider range of salaries and the men have a higher median. This indicates that the men are better paid than the women.

5 a



- b  $m = £1605$

- c  $Q_1 = £1550$  and  $Q_3 = £1640$

d



- 6 a i 24 min ii 12 min iii 42 min

- b i 6 min ii 17 min iii 9 min

- c Either doctor with a plausible reason, e.g. Dr Khan because on average, her waiting times are always shorter or Dr Collins because he takes more time with each patient as the interquartile range is more spread out.

- 7 Many possible answers but not including any numerical values: eg Bude had a higher median amount of sunshine. Bude had a smaller interquartile range, showing more consistent sunshine in Bude. So overall this indicates that Bude had more sunshine on any one day.

- 8 create a grouped frequency table using the quartiles:  
for the boys

mark, $m$	Cumulative frequency	Frequency, $f$	Mid-point, $x$	$x \times f$
$39 < m \leq 65$	25	25	52	1300
$65 < m \leq 78$	50	25	71.5	1787.5

$78 < m \leq 87$	75	25	82.5	2062.5
$87 < m \leq 112$	100	25	99.5	2487.5
Total	100			7637.5

$$\text{mean} = \frac{7637.5}{100} = 76.4 \text{ marks (1 dp)}$$

for the girls

mark, $m$	Cumulative frequency	Frequency, $f$	Mid-point, $x$	$x \times f$
$49 < m \leq 69$	25	25	59	1475
$69 < m \leq 78$	50	25	73.5	1837.5
$78 < m \leq 91$	75	25	84.5	2112.5
$91 < m \leq 106$	100	25	98.5	2462.5
Total	100			7887.5

$$\text{mean} = \frac{7887.5}{100} = 78.9 \text{ marks (1 dp)}$$

The mean is 2.5 marks higher for the girls

### Exercise 18E

- 1 The respective frequency densities on which each histogram should be based are:

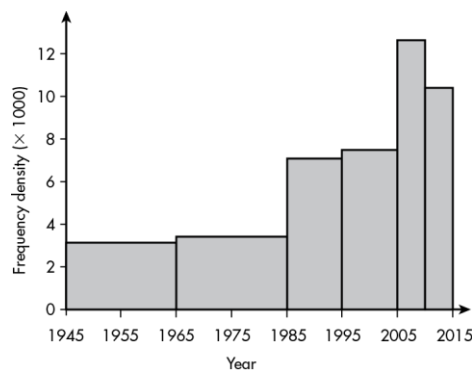
a 2.5, 6.5, 9, 2, 1.5 b 3, 6, 10, 4.5

- 2 The respective frequency densities on which each histogram should be based are:

a 7, 12, 10, 5 b 0.4, 1.2, 2.8, 1

c 9, 12, 13.5, 9

3



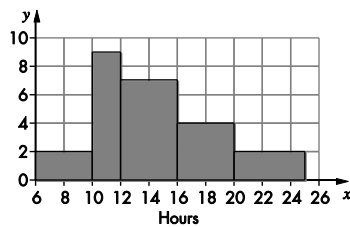
- 4 a i work out the class width  $\times$  frequency density for each bar and add these together ie  $5 \times 25 + 5 \times 30 + 10 \times 20 + 10 \times 10 + 20 \times 5 + 10 \times 10$

ii 775

b 400

- 5 ab 14 kg c 14.6 kg d 33 plants





6 a

<b>Speed, <math>v</math> (mph)</b>	$0 < v \leq 40$	$40 < v \leq 50$	$50 < v \leq 60$	$60 < v \leq 70$	$70 < v \leq 80$	$80 < v \leq 100$
<b>Frequency</b>	80	10	40	110	60	60

b 360 c 64.5 mph d 59.2 mph

7 a 100 b 32.5 c 101.5  
d 10% of 300 = 30, so the pass mark will be in the 70-80 interval. There are 60 students in this interval and 30 is half of 60. So the pass mark is half way between 70 and 80 = 75

8 a

<b>Temperature, <math>t</math> (°C)</b>	$10 < t \leq 11$	$11 < t \leq 12$	$12 < t \leq 14$	$14 < t \leq 16$	$16 < t \leq 19$
<b>Frequency</b>	11	12	14	16	19

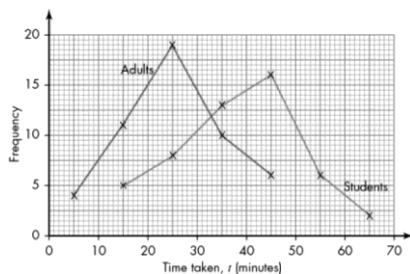
b 12–14°C c 14.5°C  
d 12.6°C, 17°C, 4.4°C e 14.8°C

9 0.45

### Review questions

1 Choose a suitable sample size and decide whether to use a random sample or a stratified sample. Make sure that the sample is reliable and unbiased. Remember that the greater the accuracy required, the larger the sample size needs to be. But the larger the sample size, the higher the cost will be and the time taken. Therefore, the benefit of achieving high accuracy in a sample will always have to be set against the cost of achieving it.

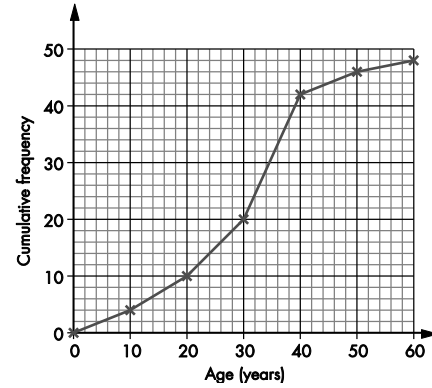
2 a



b The adults completed the puzzle quicker as their average time was better. Also their range was smaller which makes them more consistent.

3 a cumulative frequencies: 4, 10, 20, 42, 46, 48

b

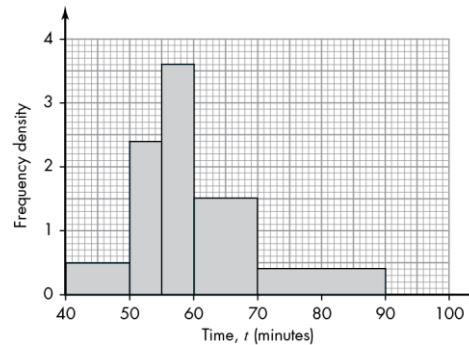


c 32 d  $Q_1 = 22$ ,  $Q_3 = 37$  and IQR = 15

4

a i £7200 ii £6400  
b i £6000 ii £4700  
c On average the men's wages are higher as their median is greater. The women's wages are more consistent as their interquartile range is smaller.

5



6 a

<b>Age, <math>t</math> (years)</b>	$9 < t \leq 10$	$10 < t \leq 12$	$12 < t \leq 14$	$14 < t \leq 17$	$17 < t \leq 19$
<b>Frequency</b>	4	12	8	9	5

b 10–12 c 13  
d 11, 16, 5 e 13.4

7 create a grouped frequency table using the quartiles:

Amount, $m$ (£)	Cumulative frequency	Frequency, $f$	Mid-point, $x$	$x \times f$
$0.50 < m \leq 2.00$	20	20	1.25	25
$2.00 < m \leq 3.00$	40	20	2.50	50
$3.00 < m \leq 4.00$	60	20	3.50	70
$4.00 < m \leq 6.00$	80	20	5.00	100
<b>Total</b>		80		245

$$\text{mean} = \frac{245}{80} = £3.06$$

## Chapter 19 – Probability: Combined events

### Exercise 19A

- 1 a  $\frac{1}{4}$  b  $\frac{1}{4}$  c  $\frac{1}{2}$
- 2 a  $\frac{2}{11}$  b  $\frac{4}{11}$  c  $\frac{6}{11}$
- 3 a  $\frac{1}{3}$  b  $\frac{2}{5}$  c  $\frac{11}{15}$  d  $\frac{11}{15}$  e  $\frac{1}{3}$
- 4 a 60 b  $\frac{4}{5}$
- 5 a 0.8 b 0.2
- 6 a i 0.75 ii 0.6 iii 0.5 iv 0.6  
b i cannot add P(red) and P(1) as events are not mutually exclusive  
ii  $0.75 (= 1 - P(\text{blue}))$
- 7 0.46
- 8 Probabilities cannot be summed in this way as events are not mutually exclusive.
- 9 a i 0.4 ii 0.5 iii 0.9  
b 0.45  
c 2 hours 12 minutes
- 10  $\frac{5}{52}$  or 0.096 to 3 decimal places

### Exercise 19B

- 1 a 7  
b 2, 12  
c
- | Score       | 2              | 3              | 4              | 5             | 6              | 7             | 8              | 9             | 10             | 11             | 12             |
|-------------|----------------|----------------|----------------|---------------|----------------|---------------|----------------|---------------|----------------|----------------|----------------|
| Probability | $\frac{1}{36}$ | $\frac{1}{18}$ | $\frac{1}{12}$ | $\frac{1}{9}$ | $\frac{5}{36}$ | $\frac{1}{6}$ | $\frac{5}{36}$ | $\frac{1}{9}$ | $\frac{1}{12}$ | $\frac{1}{18}$ | $\frac{1}{36}$ |
- d i  $\frac{1}{12}$  ii  $\frac{5}{9}$  iii  $\frac{1}{2}$   
iv  $\frac{7}{36}$  v  $\frac{5}{12}$  vi  $\frac{5}{18}$
- 2 a  $\frac{1}{12}$  b  $\frac{11}{36}$  c  $\frac{1}{6}$  d  $\frac{5}{9}$
- 3 a  $\frac{1}{36}$  b  $\frac{11}{36}$  c  $\frac{5}{18}$
- 4 a
- | Score on second dice | 6                   | 5 | 4 | 3 | 2 | 1 | 0 |
|----------------------|---------------------|---|---|---|---|---|---|
| 5                    | 4                   | 3 | 2 | 1 | 0 | 1 |   |
| 4                    | 3                   | 2 | 1 | 0 | 1 | 2 |   |
| 3                    | 2                   | 1 | 0 | 1 | 2 | 3 |   |
| 2                    | 1                   | 0 | 1 | 2 | 3 | 4 |   |
| 1                    | 0                   | 1 | 2 | 3 | 4 | 5 |   |
|                      | 1                   | 2 | 3 | 4 | 5 | 6 |   |
|                      | Score on first dice |   |   |   |   |   |   |
- b i  $\frac{5}{18}$  ii  $\frac{1}{6}$  iii  $\frac{1}{9}$   
iv 0 v  $\frac{1}{2}$

- 5 a i  $\frac{1}{4}$  ii  $\frac{1}{2}$  iii  $\frac{3}{4}$  iv  $\frac{1}{4}$   
b All possibilities are included

- 6 a  $\frac{1}{12}$  b  $\frac{1}{4}$  c  $\frac{1}{6}$

- 7 a

Score on second spinner	5	6	7	8	9	10
	4	5	6	7	8	9
	3	4	5	6	7	8
	2	3	4	5	6	7
	1	2	3	4	5	6
	Score on first spinner					
	1	2	3	4	5	

- b 6
- c i  $\frac{4}{25}$  ii  $\frac{13}{25}$  iii  $\frac{1}{5}$  iv  $\frac{3}{5}$
- 8 a HHH, HHT, HTH, THH, HTT, THT, TTH, TTT  
b i  $\frac{1}{8}$  ii  $\frac{3}{8}$  iii  $\frac{1}{8}$  iv  $\frac{7}{8}$
- 9 a 16 b 32 c 1024 d  $2^n$
- 10 a

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

- b  $\frac{1}{18}$  c 18 d twice
- 11  $\frac{1}{2}$
- 12 You would need a 3D diagram or there would be too many different events to list.

### Exercise 19C

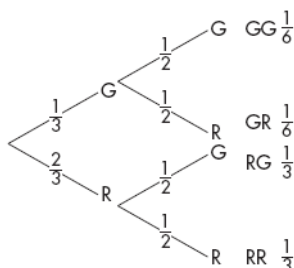
- 1 a
- | First event   | Second event | Outcome | Probability                                    |
|---------------|--------------|---------|--|
| $\frac{1}{2}$ | H            | (H, H)  | $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ |
|               | T            | (H, T)  | $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ |
| $\frac{1}{2}$ | H            | (T, H)  | $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ |
|               | T            | (T, T)  | $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ |
- b i  $\frac{1}{4}$  ii  $\frac{1}{2}$  iii  $\frac{3}{4}$

2 a  $\frac{2}{13}$  b  $\frac{11}{13}$

c i  $\frac{1}{169}$  ii  $\frac{25}{169}$

3 a  $\frac{2}{3}$  b  $\frac{1}{2}$

c



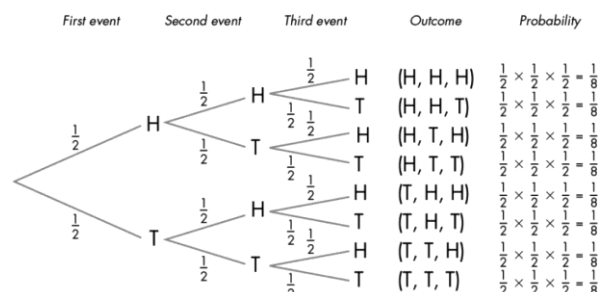
d i  $\frac{1}{6}$  ii  $\frac{1}{2}$  iii  $\frac{5}{6}$

e 15 days

4 a  $\frac{2}{5}$

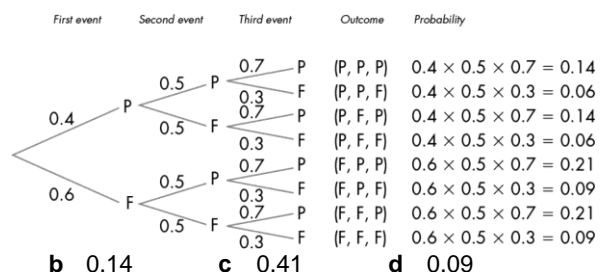
b i  $\frac{4}{25}$  ii  $\frac{12}{25}$

5 a



b i  $\frac{1}{8}$  ii  $\frac{3}{8}$  iii  $\frac{7}{8}$

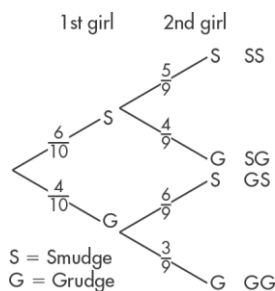
6 a



b 0.14 c 0.41 d 0.09

7 a  $\frac{3}{5}$

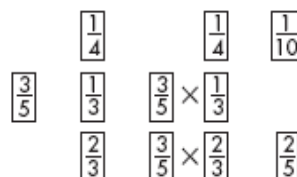
b



c i  $\frac{1}{3}$  ii  $\frac{7}{15}$  iii  $\frac{8}{15}$

8 a 1 b 1

c



9 0.036

10 It will help to show all the 27 different possible events and which ones give the three different coloured sweets, then the branches will help you to work out the chance of each.

11 a  $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$  b  $\left(\frac{1}{2}\right)^n$

### Exercise 19D

1 a  $\frac{4}{9}$  b  $\frac{4}{9}$

2 a  $\frac{1}{169}$  b  $\frac{2}{169}$

3 a 0.08 b 0.32 c 0.48

4  $\left(\frac{1}{6}\right)^5 \times 6 = 0.00077$

5 a  $\frac{4}{25}$  b  $\frac{9}{25}$  c  $\frac{16}{25}$

6 a  $\frac{3}{8}$  b  $\frac{1}{120}$  c  $\frac{119}{120}$

7 a i  $\frac{1}{216} = 0.005$  ii  $\frac{125}{216} = 0.579$

iii  $\frac{91}{216} = 0.421$

b i  $\left(\frac{1}{6}\right)^n$  ii  $\left(\frac{5}{6}\right)^n$  iii  $1 - \left(\frac{5}{6}\right)^n$

8 a 0.54 b 0.216

9 a 0.343 b independent events

c P(exactly two of the three cars are foreign) =  
P(FFB) + P(FBF) + P(BFF) =  $3 \times 0.7 \times 0.7 \times 0.3$   
= 0.441

10  $10 \times 0.6^9 \times 0.4 + 0.6^{10} = 0.046$

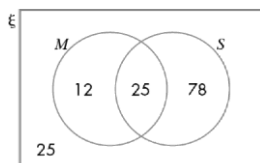
11 0.8

- 12 the events are not independent as he may already have a 10 or Jack or Queen or King in his hand, in which case the probability fraction will have a different numerator

### Exercise 19E

- 1 a  $\frac{7}{10}$  b  $\frac{2}{3}$  c  $\frac{3}{8}$
- 2 a i  $\frac{3}{8}$  ii  $\frac{5}{8}$   
b i  $\frac{5}{12}$  ii  $\frac{7}{12}$   
c i  $\frac{3}{20}$  ii  $\frac{7}{20}$  iii  $\frac{1}{2}$
- 3 a i  $\frac{5}{9}$  ii  $\frac{4}{9}$   
b i  $\frac{2}{3}$  ii  $\frac{1}{3}$   
c i  $\frac{1}{3}$  ii  $\frac{2}{15}$  iii  $\frac{8}{15}$
- 4 a  $\frac{1}{6}$  b 0  
c i  $\frac{2}{3}$  ii  $\frac{1}{3}$  iii 0
- 5 a i  $\frac{1}{120}$  ii  $\frac{7}{40}$  iii  $\frac{21}{40}$  iv  $\frac{7}{24}$   
b they are mutually exclusive and exhaustive events
- 6 Both events are independent, the probability of seeing a British made car is always  $\frac{1}{4}$
- 7 a 0.54 b 0.38 c 0.08  
d they should add up to 1
- 8 First work out P(first blue) and P(second blue) remembering that the numerator and the denominator will each be one less than for P(first blue). Now work out P(first blue)  $\times$  P(second blue). Then work out P(first white) and P(second white) remembering that the numerator and the denominator will each be one less than for P(first white). Now work out P(first white)  $\times$  P(second white). Finally add together the two probabilities.
- 9  $\frac{1}{270725}$

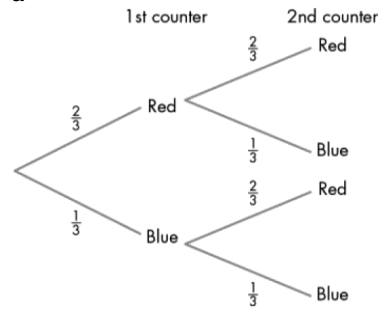
10 a



- b i  $\frac{12}{140} = 0.086$  ii  $\frac{25}{103} = 0.243$

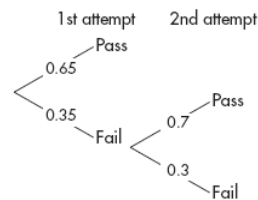
### Review questions

1 a



- b  $\frac{5}{9}$  c  $\frac{4}{9}$

2 a



- b i 0.895 ii 0.105  
c calculate  $0.895^2$

3  $\frac{4}{15}$

4  $\frac{11}{30}$

5 work out  $\frac{5}{16} \div \frac{3}{8} (= \frac{5}{6})$

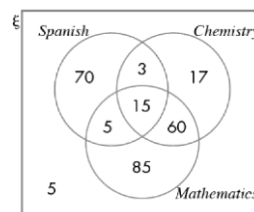
6 0.045

7 0.375

8 a  $\frac{13}{20}$  as it cannot be square rooted

b  $\frac{1}{9}$  as this gives a ratio of red to blue of 1 : 2

9 a



- b i  $\frac{70}{260} = 0.269$  ii  $\frac{60}{260} = 0.231$   
iii  $\frac{5}{260} = 0.0192$  iv  $\frac{20}{165} = 0.121$

## Chapter 20 – Geometry and Measures: Properties of circles

### Exercise 20A

Students' own work.

### Exercise 20B

1 a  $56^\circ$  b  $62^\circ$  c  $105^\circ$  d  $55^\circ$  e  $45^\circ$   
f  $30^\circ$  g  $60^\circ$  h  $145^\circ$

2 a  $55^\circ$  b  $52^\circ$  c  $50^\circ$  d  $24^\circ$  e  $39^\circ$   
f  $80^\circ$  g  $34^\circ$  h  $30^\circ$

3 a  $41^\circ$  b  $49^\circ$  c  $41^\circ$

4 a  $72^\circ$  b  $37^\circ$  c  $72^\circ$

5  $\angle AZY = 35^\circ$  (angles in a triangle),  $a = 55^\circ$  (angle in a semicircle  $= 90^\circ$ )

6 a  $x = y = 40^\circ$  b  $x = 131^\circ, y = 111^\circ$   
c  $x = 134^\circ, y = 23^\circ$  d  $x = 32^\circ, y = 19^\circ$   
e  $x = 59^\circ, y = 121^\circ$  f  $x = 155^\circ, y = 12.5^\circ$

7  $\angle BED = 15^\circ$  (angles at circumference from chord BD are equal);  $\angle EBC = 180^\circ - (15^\circ + 38^\circ) = 127^\circ$  (angles in a triangle);  $\angle ADC = 180^\circ - (15^\circ + 38^\circ) = 127^\circ$  (angles in a triangle);  $x$  = its vertically opposite angle which equals  $360^\circ - (127^\circ + 127^\circ + 38^\circ) = 68^\circ$  (angles in a quadrilateral). So Lana is correct, but Lex probably misread his calculator answer.

8  $\angle ABC = 180^\circ - x$  (angles on a line),  $\angle AOC = 360^\circ - 2x$  (angle at centre is twice angle at circumference), reflex  $\angle AOC = 360^\circ - (360^\circ - 2x) = 2x$  (angles at a point)

9 a  $x$  b  $2x$

c Circle theorem 1 states from an arc AB, any point subtended from the arc on the circumference is half the angle subtended at the centre. So where this arc AB is the diameter, the angle subtended at the centre is 1 straight line and so  $180^\circ$ , the angle at the circumference then is half of 180 which is  $90^\circ$ , a right angle.

10  $20^\circ$

11 It follows from theorem 1 that wherever point C is on the circumference, the angle subtended from arc AB at the circumference is always half the angle subtended at the centre. So every possible angle subtended at the circumference from arc AB will have the same angle at the centre and hence the same angle at the circumference. This proves circle theorem 3.

### Exercise 20C

1 a  $a = 50^\circ, b = 95^\circ$   
b  $c = 92^\circ, x = 90^\circ$   
c  $d = 110^\circ, e = 110^\circ, f = 70^\circ$   
d  $g = 105^\circ, h = 99^\circ$   
e  $j = 89^\circ, k = 89^\circ, l = 91^\circ$   
f  $m = 120^\circ, n = 40^\circ$

g  $p = 44^\circ, q = 68^\circ$

h  $x = 40^\circ, y = 34^\circ$

2 a  $x = 26^\circ, y = 128^\circ$  b  $x = 48^\circ, y = 78^\circ$

c  $x = 133^\circ, y = 47^\circ$  d  $x = 36^\circ, y = 72^\circ$

e  $x = 55^\circ, y = 125^\circ$  f  $x = 35^\circ$

g  $x = 48^\circ, y = 45^\circ$  h  $x = 66^\circ, y = 52^\circ$

3 a Each angle is  $90^\circ$  and so opposite angles add up to  $180^\circ$  and hence a cyclic quadrilateral.

b One pair of opposite angles are obtuse, i.e. more than  $90^\circ$ , hence their sum will be more than  $180^\circ$ .

4 a  $x = 49^\circ, y = 49^\circ$  b  $x = 70^\circ, y = 20^\circ$

c  $x = 80^\circ, y = 100^\circ$  d  $x = 100^\circ, y = 75^\circ$

5 a  $x = 50^\circ, y = 62^\circ$  b  $x = 92^\circ, y = 88^\circ$

c  $x = 93^\circ, y = 42^\circ$  d  $x = 55^\circ, y = 75^\circ$

6 a  $x = 95^\circ, y = 138^\circ$  b  $x = 14^\circ, y = 62^\circ$

c  $x = 32^\circ, y = 48^\circ$  d  $x = 52^\circ$

7 a  $54.5^\circ$  b  $125.5^\circ$  c  $54.5^\circ$

8 a  $x + 2x - 30^\circ = 180^\circ$  (opposite angles in a cyclic quadrilateral), so  $3x - 30^\circ = 180^\circ$

b  $x = 70^\circ$ , so  $2x - 30^\circ = 110^\circ$   $\angle DOB = 140^\circ$  (angle at centre equals twice angle at circumference),  $y = 80^\circ$  (angles in a quadrilateral)

9 a  $x$

b  $360^\circ - 2x$

c, d  $\angle ADC = \frac{1}{2}$  reflex  $\angle AOC = 180^\circ - x$ , so  $\angle ADC + \angle ABC = 180^\circ$

10 Let  $\angle AED = x$ , then  $\angle ABC = x$  (opposite angles are equal in a parallelogram),  $\angle ADC = 180^\circ - x$  (opposite angles in a cyclic quadrilateral), so  $\angle ADE = x$  (angles on a line)

11  $18^\circ$

### Exercise 20D

1 a  $38^\circ$  b  $110^\circ$  c  $15^\circ$  d  $45^\circ$

2 a 6 cm b 10.8 cm c 3.21 cm d 8 cm

3 a  $x = 12^\circ, y = 156^\circ$  b  $x = 100^\circ, y = 50^\circ$

c  $x = 62^\circ, y = 28^\circ$  d  $x = 30^\circ, y = 60^\circ$

4 a  $62^\circ$  b  $66^\circ$  c  $19^\circ$  d  $20^\circ$

5 19.5 cm

6  $\angle OCD = 58^\circ$  (triangle OCD is isosceles),  $\angle OCB = 90^\circ$  (tangent/radius theorem), so  $\angle DCB = 32^\circ$ , hence triangle BCD is isosceles (2 equal angles)

7 a  $\angle AOB = \cos^{-1} \frac{OA}{OB} = \cos^{-1} \frac{OC}{OB} = \angle COB$

b As  $\angle AOB = \angle COB$ , so  $\angle ABO = \angle CBO$ , so OB bisects  $\angle ABC$

8 If the tangent XY touches the circle at C, then  $CY = 10$  cm.  $\angle OYC = 30^\circ$  (theorem 7). Hence where  $r$  is the radius of the circle, then  $r/10 = \tan 30^\circ$ , hence  $r = 10 \times \tan 30^\circ = 5.7735$ , So Ling is correct to one decimal place.

9  $38^\circ$

### Exercise 20E

Students' own work.

### Exercise 20F

- 1 **a**  $a = 65^\circ, b = 75^\circ, c = 40^\circ$   
**b**  $d = 79^\circ, e = 58^\circ, f = 43^\circ$   
**c**  $g = 41^\circ, h = 76^\circ, i = 76^\circ$   
**d**  $k = 80^\circ, m = 52^\circ, n = 80^\circ$
- 2 **a**  $a = 75^\circ, b = 75^\circ, c = 75^\circ, d = 30^\circ$   
**b**  $a = 47^\circ, b = 86^\circ, c = 86^\circ, d = 47^\circ$   
**c**  $a = 53^\circ, b = 53^\circ$   
**d**  $a = 55^\circ$
- 3 **a**  $36^\circ$     **b**  $70^\circ$
- 4 **a**  $x = 25^\circ$   
**b**  $x = 46^\circ, y = 69^\circ, z = 65^\circ$   
**c**  $x = 38^\circ, y = 70^\circ, z = 20^\circ$   
**d**  $x = 48^\circ, y = 42^\circ$
- 5  $\angle ACB = 64^\circ$  (angle in alternate segment),  $\angle ACX = 116^\circ$  (angles on a line),  $\angle CAX = 32^\circ$  (angles in a triangle), so triangle ACX is isosceles (two equal angles)
- 6  $\angle AXY = 69^\circ$  (tangents equal and so triangle AXY is isosceles),  $\angle XZY = 69^\circ$  (alternate segment),  $\angle XYZ = 55^\circ$  (angles in a triangle)
- 7 **a**  $2x$     **b**  $90^\circ - x$   
**c**  $\text{OPT} = 90^\circ$ , so  $\text{APT} = x$

### Review questions

- 1 **a**  $44^\circ$ , both angles subtended from the same chord  
**b**  $52^\circ$ , each vertex touches the circumference  
**c**  $140^\circ$ , the three points not the centre are touching the circumference
- 2 **a**  $55^\circ$     **b**  $75^\circ$
- 3 **a** DOB is double DAB  
**b** DAB and DCB add up to  $180^\circ$  since ABCD is a cyclic quadrilateral
- 4 To be a rhombus, DOB must equal DCB, you know that  $\text{DOB} = 2x$  (double DAB), you know that  $\text{DCB} = 180^\circ - x$  (as ABCD is a cyclic quadrilateral), so  $2x = 180^\circ - x$ , hence  $3x = 180^\circ \rightarrow x = 60^\circ$
- 5  $\text{TPR} = 42^\circ$ , alternate segment;  $\text{PRQ} = 42^\circ$ , alternate angles in parallel lines;  $\text{RPQ} = 42^\circ$ , isosceles triangle;  $\text{PQR} = 180^\circ - 2 \times 42^\circ = 96^\circ$ , angles in a triangle;  $\text{PTR} = 180^\circ - 96^\circ = 84^\circ$ , opposite angles in a cyclic quadrilateral.
- 6  $\text{CAO} = 90^\circ - 66^\circ = 24^\circ$ ;  $\text{ACO} = 24^\circ$ , isosceles triangle;  $\text{AOB} = 360^\circ - (2 \times 90^\circ + 50^\circ) = 130^\circ$ , angles in quadrilateral;  $\text{ACB} = 130^\circ \div 2 = 65^\circ$ , angles at centre double angle at circumference;  $\text{OCB} = 65^\circ - 24^\circ = 41^\circ$ ;  $x = \text{OCB}$ , isosceles triangle hence  $x = 41^\circ$ .
- 7  $\text{OCA} = 90^\circ$  and  $\text{OBA} = 90^\circ$  as AB and AC are both tangents to the circle, centre O. This is a pair of opposite angles having the sum of  $180^\circ$ . Since the sum of the angles is  $360^\circ$ , the other pair of angles BC and BOC will add up to  $360^\circ - 180^\circ = 180^\circ$ , so

both pairs of opposite angles sum to  $180^\circ$ , hence it is a cyclic quadrilateral.

- 8  $\text{OBA} = 90^\circ - x$ ;  $\text{OAB} = 90^\circ - x$ , angles in an isosceles triangle.  $\text{BOA} = 180^\circ - (90^\circ - x + 90^\circ - x) = 180^\circ - (180^\circ - 2x) = 180^\circ - 180^\circ + 2x = 2x$
- 9 Using Pythagoras' theorem,  $\text{OC} = \sqrt{8^2 + 12^2} = 14.4$  (3sf);  $\text{PC} = 14.4 - 8 = 6.4$  (2 sf); the answer is given to 2 sf with the assumption that the 12 is 2 sf.

## Chapter 21 – Ratio, proportion and rates of change: Variation

### Exercise 21A

- 1 **a** 15    **b** 2
- 2 **a** 75    **b** 6
- 3 **a** 150    **b** 6
- 4 **a** 22.5    **b** 12
- 5 **a** 175 miles    **b** 8 hours
- 6 **a** £66.50    **b** 175 kg
- 7 **a** 44    **b**  $84 \text{ m}^2$
- 8 **a** 50    **b** Spaces =  $\frac{1}{14}$  area
- 9 17 minutes 30 seconds
- 10 22.5 cm

### Exercise 21B

- 1 **a** 100    **b** 10
- 2 **a** 27    **b** 5
- 3 **a** 56    **b** 1.69
- 4 **a** 192    **b** 2.25
- 5 **a** 25.6    **b** 5
- 6 **a** 80    **b** 8
- 7 **a** £50    **b** 225
- 8 **a**  $3.2^\circ \text{C}$     **b** 10 atm
- 9 **a** 388.8 g    **b** 3 mm
- 10 **a** 2 J    **b** 40 m/s
- 11 **a** £78    **b** 400 miles
- 12  $4000 \text{ cm}^3$
- 13 £250
- 14 **a** B    **b** A    **c** C
- 15 **a** B    **b** A
- 16  $S = kM^{\frac{2}{3}}$

### Exercise 21C

- 1  $Tm = 12$     **a** 3    **b** 2.5
- 2  $Wx = 60$     **a** 20    **b** 6
- 3  $Q(5 - t) = 16$     **a** -3.2    **b** 4

- 4  $Mt^2 = 36$       a 4      b 5
- 5  $W\sqrt{T} = 24$       a 4.8      b 100
- 6  $x^3y = 32$       a 32      b 4
- 7  $gp = 1800$       a £15      b 36
- 8  $tD = 24$       a 3 °C      b 12 km
- 9  $ds^2 = 432$       a 1.92 km      b 8 m/s
- 10  $p\sqrt{h} = 7.2$       a 2.4 atm      b 100 m
- 11  $W\sqrt{F} = 0.5$       a 5 t/h      b 0.58 t/h
- 12 B – This is inverse proportion, as x increases y decreases.
- 13
- |   |   |               |               |
|---|---|---------------|---------------|
| x | 8 | 27            | 64            |
| y | 1 | $\frac{2}{3}$ | $\frac{1}{2}$ |
- 14 4.3 miles
- 15  $F = 2.02 \times 10^{19}$  N

### Review questions

- 1
- |   |    |     |     |
|---|----|-----|-----|
| x | 25 | 100 | 400 |
| y | 10 | 20  | 40  |
- 2 a  $E = 4000v$       b 3.6 m/s
- 3 a  $y = 4x^{-\frac{1}{3}}$  or  $y = \frac{4}{\sqrt[3]{x}}$   
b i 20      ii 8
- 4 19.4 cm
- 5 128
- 6 a  $D = 5M^2$       b 245      c 3
- 7 80
- 8 a 2.5      b 0.25      c 250      d 50, -50
- 9 a 10      b 3.375
- 10 a  $48\pi$       b 9
- 11 a  $A = \frac{100}{B^2}$  or  $AB^2 = 100$       b 4
- 12 125
- 13 a 27 hertz      b Cannot divide by 0
- 14  $a = 9$ ,  $b = 144$
- 15 40

## Chapter 22 – Geometry and measures: Triangles

### Exercise 22A

- 1 13.1 cm

- 2 73.7°
- 3 9.81 cm
- 4 33.5 m
- 5 a 10.0 cm      b 11.5°      c 4.69 cm
- 6  $PS = 4\tan 25 = 1.8652306$ , angle QRP =  $\tan^{-1} \frac{7.8652306}{4} = 63.0^\circ$ , angle QRP =  $63.0 - 25 = 38.0^\circ$
- 7 774 m
- 8 a  $\sqrt{2}$  cm  
b i  $\frac{\sqrt{2}}{2}$  (an answer of  $\frac{1}{\sqrt{2}}$  would also be accepted)  
ii  $\frac{\sqrt{2}}{2}$       iii 1
- 9 The calculated answer is 14.057869, so Eve is correct to give 14° as her answer. She could also have been correct to round off to 14.1°

### Exercise 22B

- 1 25.1°
- 2 a 24.0°      b 48.0°  
c 13.5 cm      d 16.6°
- 3 a 58.6°      b 20.5 cm  
c 2049 cm<sup>3</sup>      d 64.0°
- 4 a 73.2°      b £7541.53
- 5 a 3.46 m      b 70.5°
- 6 For example, the length of the diagonal of the base is  $\sqrt{b^2 + c^2}$  and taking this as the base of the triangle with the height of the edge, then the hypotenuse is  
$$\sqrt{(a^2 + (\sqrt{b^2 + c^2})^2)} = \sqrt{a^2 + b^2 + c^2}$$
- 7 It is 44.6°; use triangle XDM where M is the midpoint of BD; triangle DXB is isosceles, as X is over the point where the diagonals of the base cross; the length of DB is  $\sqrt{656}$ , the cosine of the required angle is  $0.5 \sqrt{656} \div 18$ .

### Exercise 22C

1a

x	sin x	x	sin x	x	sin x	x	sin x
0°	0	180°	0	180°	0	360°	0
15°	0.259	165°	0.259	195°	-0.259	345°	-0.259
30°	0.5	150°	0.5	210°	-0.5	330°	-0.5
45°	0.707	135°	0.707	225°	-0.707	315°	-0.707
60°	0.866	120°	0.866	240°	-0.866	300°	-0.866
75°	0.966	105°	0.966	255°	-0.966	285°	-0.966
90°	1	90°	1	270°	-1	270°	-1

- b They are the same for values between 90° and 180°. They have the opposite sign for values between 180° and 360°

- 2 a Sine graph  
b Line symmetry about  $x = 90$ , 270 and rotational symmetry about (180, 0)

### Exercise 22D

1a

x	cos x	x	cos x	x	cos x	x	cos x
0°	1	180°	-1	180°	-1	360°	1
15°	0.966	165°	-0.966	195°	-0.966	345°	0.966
30°	0.866	150°	-0.866	210°	-0.866	330°	0.866
45°	0.707	135°	-0.707	225°	-0.707	315°	0.707
60°	0.5	120°	-0.5	240°	-0.5	300°	0.5
75°	0.259	105°	-0.259	255°	-0.259	285°	0.259
90°	0	90°	0	270°	0	270°	0

- b Negative cosines are between 90 and 270, the rest are positive,  
2 a Cosine graph  
b Line symmetry about  $x = 180^\circ$ , rotational symmetry about (90°, 0), (270°, 0)

### Exercise 22E

- 1 a 36.9°, 143.1°      b 53.1°, 126.9°  
c 48.6°, 131.4°      d 224.4°, 315.6°  
e 194.5°, 345.5°      f 198.7°, 341.3°  
g 190.1°, 349.9°      h 234.5°, 305.5°  
i 28.1°, 151.9°      j 185.6°, 354.4°  
k 33.6°, 146.4°      l 210°, 330°
- 2 Sin 234°, as the others all have the same numerical value.
- 3 a 438° or 78° + 360n°  
b -282° or 78° - 360n°  
c Line symmetry about  $\pm 90n^\circ$  where  $n$  is an odd integer.  
Rotational symmetry about  $\pm 180n^\circ$  where  $n$  is an integer.
- 4 30°, 150°, 199.5°, 340.5°
- 5 a 53.1°, 306.9°      b 54.5°, 305.5°  
c 62.7°, 297.3°      d 54.9°, 305.1°  
e 79.3°, 280.7°      f 143.1°, 216.9°  
g 104.5°, 255.5°      h 100.1°, 259.9°  
i 111.2°, 248.8°      j 166.9°, 193.1°  
k 78.7°, 281.3°      l 44.4°, 315.6°
- 6 Cos 58°, as the others are negative.
- 7 a 492° or 132° + 360n°  
b -228° or 132° - 360n°  
c Line symmetry about  $\pm 180n^\circ$  where  $n$  is an integer.  
Rotational symmetry about  $\pm 90n^\circ$  where  $n$  is an odd integer.
- 8 a i High tides 0940, 2200, low tides 0300, 1520  
ii 12hrs 20min  
b i same periodic shape  
ii The period of the cycle is in time not degrees, no negative values on the y axis

### Exercise 22F

- 1 a 0.707      b -1 (-0.9998)  
c -0.819      d 0.731  
2 a -0.629      b -0.875

c -0.087      d 0.999

- 3 a 21.2°, 158.8°      b 209.1°, 330.9°  
c 50.1°, 309.9°      d 150.0°, 210.0°  
e 60.9°, 119.1°      f 29.1°, 330.9°
- 4 30°, 150°
- 5 -0.755
- 6 a 1.41      b -1.37      c -0.0367      d -0.138  
e 1.41      f -0.492
- 7 True
- 8 a Cos 65°      b Cos 40°
- 9 a 10°, 130°      b 12.7°, 59.3°
- 10 38.2°, 141.8°
- 11  $\sin^{-1} 0.9659 = 74.99428$ , which is 75 to 2 sf.  
435 = 75 + 360. From the sine curve extended,  $\sin 75 = \sin 435$ .  
Rose is therefore correct as she has rounded her solution.  
Keiren could also be correct as the answer could also be given more accurately as 434.9942838

### Exercise 22G

- 1 a Maths error      b 89.999999
- 2 5 729 577 951
- 3 ab Graph of  $\tan x$       c All 0
- d Students' own explanations
- 4 a Tan is positive for angles between 0–90° and 180–270°  
b Yes, 180°

### Exercise 22H

- 1 a 41.2°, 221.2°      b 123.7° and 303.7°
- 2 a 14.5°, 194.5°      b 61.9°, 241.9°  
c 68.6°, 248.6°      d 160.3°, 340.3°  
e 147.6°, 327.6°      f 105.2°, 285.2°  
g 54.4°, 234.4°      h 130.9°, 310.9°  
i 174.4°, 354.4°      j 44.9°, 224.9°
- 2 Tan 235°, as the others have a numerical value of 1
- 3 a 425° or 65° + 180n°,  $n > 2$   
b -115° or 65° - 180n°  
c No Line symmetry  
Rotational symmetry about  $\pm 180n^\circ$  where  $n$  is an integer.
- 5  $\tan^{-1} 0.4040 = 21.9987$  which is 22 to 2 sf, so  $\tan^{-1} (-0.4040)$  is same as  $\tan 180 - 22 = 158$ .  
If you calculate  $\tan^{-1} (-0.4040)$  on your calculator it will give  $-21.9987 = -22$  (2 sf).  
Mel is therefore correct as he has rounded his solution. Jose is also correct.

### Exercise 22I

- 1 a 3.64 m      b 8.05 cm      c 19.4 cm  
2 a 46.6°      b 112.0°      c 36.2°  
3 50.3°, 129.7°



- 4 This statement can be shown to be true by using  $\frac{a}{\sin A} = \frac{b}{\sin B}$ . As  $a = b \times \frac{\sin A}{\sin B}$ ,

if  $a > b$  then  $\sin A > \sin B$  and so  $\frac{\sin A}{\sin B} > 1$ ,

hence  $b \times \frac{\sin A}{\sin B} > b$ .

- 5 2.88 cm, 20.9 cm  
 6 a i  $30^\circ$  ii  $40^\circ$   
 b 19.4 m  
 7 36.5 m  
 8 22.2 m  
 9 3.47 m  
 10 The correct height is 767 m. Paul has mixed the digits up and placed them in the wrong order.  
 11 26.8 km/h  
 12 64.6 km  
 13 Check students' answers.  
 14  $134^\circ$   
 15 Check that proof is valid.

### Exercise 22J

- 1 a 7.71 m b 29.1 cm c 27.4 cm  
 2 a  $76.2^\circ$  b  $125.1^\circ$  c  $90^\circ$   
 d Right-angled triangle  
 3 5.16 cm  
 4 65.5 cm  
 5 a 10.7 cm b  $41.7^\circ$  c  $38.3^\circ$   
 d 6.69 cm e  $54.4 \text{ cm}^2$   
 6  $72.3^\circ$   
 7 25.4 cm, 38.6 cm  
 8 58.4 km at  $092.5^\circ$   
 9  $21.8^\circ$   
 10 a  $82.8^\circ$  b 8.89 cm  
 11 42.5 km  
 12 Check students' answers.  
 13  $111^\circ$ ; the largest angle is opposite the longest side

### Exercise 22K

- 1 a 8.60 m b  $90^\circ$  c 27.2 cm  
 d  $26.9^\circ$  e  $41.0^\circ$  f 62.4 cm  
 2 7 cm  
 3 11.1 km  
 4 19.9 knots  
 5 a 27.8 miles  
 b  $262^\circ$   
 6 a  $A = 90^\circ$ ; this is Pythagoras' theorem  
 b A is acute  
 c A is obtuse

- 7 The answer is correct to 3sf but the answer could be slightly less accurate (as 140m to 2sf) since the question data is given to 2sf

### Exercise 22L

- 1 a  $24.0 \text{ cm}^2$  b  $26.7 \text{ cm}^2$  c  $243 \text{ cm}^2$   
 d  $21\,097 \text{ cm}^2$  e  $1224 \text{ cm}^2$   
 2 4.26 cm  
 3 a  $42.3^\circ$  b  $49.6^\circ$   
 4  $103 \text{ cm}^2$   
 5  $2033 \text{ cm}^2$   
 6  $21.0 \text{ cm}^2$   
 7 a  $33.2^\circ$  b  $25.3 \text{ cm}^2$   
 8 Check that proof is valid.  
 9  $21 \text{ cm}^2$   
 10  $726 \text{ cm}^2$   
 11  $\frac{a^2\sqrt{3}}{4}$   
 12 c

### Review questions

- 1  $\cos A = \frac{12^2 + 10^2 - 15^2}{2 \times 12 \times 10} = 0.079166$ ,  $\cos^{-1} 0.079166 = 85.459371 = 85.5^\circ$  (3 sf), so Oliver is incorrect, he has truncated the final answer to 3 figures instead of rounding off.  
 2  $\text{area} = \frac{1}{2} \times 7 \times 13 \times \sin 116 = 40.895129 = 40.9$  (3 sf)  
 3  $AB^2 = 10^2 + 11^2 - 2 \times 10 \times 11 \times \cos 70 = 145.75557$ ,  $AB = 12.1$  (3 sf). The longest side of a triangle is opposite the largest angle, so as AB is the longest side, then angle C must be the largest angle  
 4 a i Let  $QP = 1$ , then  $QT = \frac{1}{2}$ ,  
 $\text{angle } QPT = 30^\circ$ ,  $\sin 30 = \frac{\frac{1}{2}}{1} = \frac{1}{2}$   
 ii  $PT^2 = 1^2 - (\frac{1}{2})^2 = \frac{3}{4}$ , so  $PT = \frac{\sqrt{3}}{2}$ , hence  
 $\cos 30 = \frac{\frac{\sqrt{3}}{2}}{1} = \frac{\sqrt{3}}{2}$   
 b  $(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2 = \frac{3}{4} + \frac{1}{4} = 1$   
 c Assume QPT is any right angled triangle with angle PQT as  $\theta$  and  $PQ = 1$ . Then  $QT = \cos \theta$ ,  $PT = \sin \theta$ , so using Pythagoras, where  $QT^2 + PT^2 = PQ^2$ ,  $(\sin x)^2 + (\cos x)^2 = 1$   
 5 22.2 m  
 6  $60^\circ$ ,  $109.5^\circ$ ,  $250.5^\circ$ ,  $300^\circ$   
 7  $58.8^\circ$   
 8  $FB = \sqrt{5^2 + 6^2} = 7.81 \text{ cm}$

$$AF = \sqrt{9^2 + 7.81^2} = 11.92 \text{ cm}$$

$$\text{So } \angle AFD = \sin^{-1}\left(\frac{6}{11.92}\right) = 30.2^\circ$$

- 9 Jamil is correct to 1dp

## Chapter 23 – Algebra: Graphs

### Exercise 23A

- i 2 h      ii 3 h      iii 5 h
  - i 40 km/h   ii 120 km/h   iii 40 km/h
- 10 mph
  - Faster. The graph is steeper.
  - $13\frac{1}{3}$  mph
- 30 km
  - 40 km
  - 100 km/h
- i 263 m/min (3 sf)   ii 15.8 km/h (3 sf)
  - i 500 m/min
  - Paul by 1 minute
- Patrick ran quickly at first, then had a slow middle section but he won the race with a final sprint. Araf ran steadily all the way and came second. Sean set off the slowest, speeded up towards the end but still came third.
  - i 1.67 m/s      ii 6 km/h
- There are three methods for doing this question. This table shows the first, which is writing down the distances covered each hour.

Time	9am	9:30	10:00	10:30	11:00	11:30	12:00	12:30
Walker	0	3	6	9	12	15	18	21
Cyclist	0	0	0	0	7.5	15	22.5	30

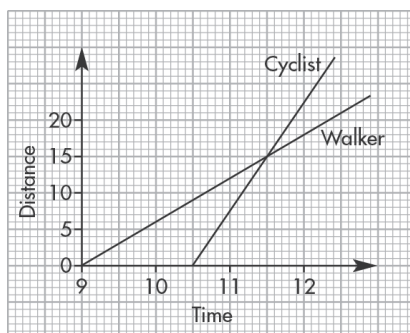
The second method is algebra:

Walker takes  $T$  hours until overtaken, so  $T = \frac{D}{6}$ ;

Cyclist takes  $T - 1.5$  to overtake, so  $T - 1.5 = \frac{D}{15}$ .

Rearranging gives  $15T - 22.5 = 6T$ ,  $9T = 22.5$ ,  $T = 2.5$ .

The third method is a graph:



All methods give the same answer of 11:30 when the cyclist overtakes the walker.

- Vehicle 2 overtook Vehicle 1
  - Vehicle 1 overtook Vehicle 2
  - Vehicles travelling in different directions
  - Vehicle 2 overtook Vehicle 1
  - 17:15
  - 32.0 mph if you only count travelling time, or 11.3 mph if you count total time.

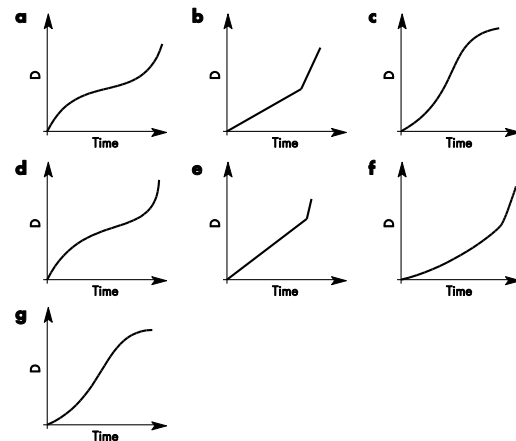
### Exercise 23B

- Two taps on
  - One tap on
  - Shejuti gets in the bath
  - Shejuti has a bath
  - Shejuti takes the plug out, water leaves the bath
  - Shejuti gets out of the bath
  - Water continues to leave the bath until the bath is empty

- Graph C

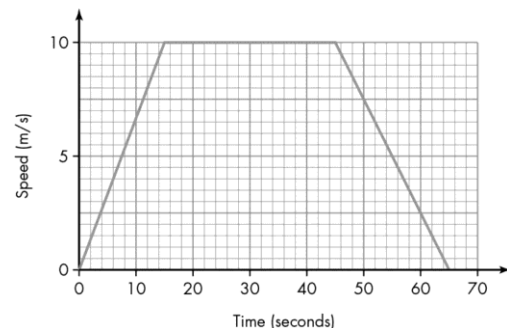


3



### Exercise 23C

- 20 m/s
  - 100 metres
  - 150 metres
  - 750 metres
- 15 km
  - 5 km/h
- AB, greatest area
  - 45 miles
  - 135 miles
- 15 m/s
- 



- 475 metres
- Could be true or false
  - Must be true
  - Must be true
  - Could be true or false
  - Could be true or false

### Exercise 23D

- $6 \text{ m/s}^2$
  - $3 \text{ m/s}^2$
  - 20 s

- d 1200 m    e 2100 m  
 2 40 km/h<sup>2</sup>, 30 km/h<sup>2</sup>, 100 km/h<sup>2</sup>    b 52.5 km  
 3 a  $\frac{3v}{10}$  m/s<sup>2</sup>    b 337.5 m

### Exercise 23E

- 1 a 80 miles, underestimate  
 b 250 metres, overestimate  
 c 75 metres, underestimate  
 d 180 metres, underestimate  
 2 a 8 miles, underestimate  
 b 40 miles, overestimate  
 3 a i 10 m/s    ii 40 m/s  
 b i 900m  
 ii Around 1070 m, depending on student's division of the shape  
 c is more accurate because the shapes are closer to the curve  
 4 a Car starts from rest and speeds up to 10 m/s after 20 seconds. It then travels at a constant speed of 10 m/s for 30 seconds, and then speeds up again to reach 20m/s in the next 10 seconds.  
 b 120 metres  
 5 a The lorry increases speed at a steady rate whereas the car speeds up quickly at first but then levels off to a constant speed and then speeds up at an increasing rate to reach 20 m/s.  
 b Lorry travels further (600 metres as against car, approximately 550 metres) as area under graph is greater.

### Exercise 23F

- 1 a tangent drawn    b 10 m/s    c 0 m/s  
 2 a i 10–12 km/h    ii 20–22 km/h  
 b 2 hours  
 c i 10 km/h    ii 19 km/h  
 3 a About 1.8 m/s<sup>2</sup>  
 b About 0.9 m/s<sup>2</sup>  
 c About 1.8 m/s<sup>2</sup>  
 d 20 s, gradient is zero because this is a maximum point  
 4 Any two points where the gradient of one is the negative of the other, e.g. at 1 s and 3 s.

### Exercise 23G

- 1 a 6    b  $2\sqrt{3}$     c  $5\sqrt{3}$     d 24  
 2 a  $6\sqrt{13}$     b 176    c 114    d  $\frac{3}{2}$   
 3 a inside    b outside    c on circumference  
 d on circumference  
 e outside    f inside  
 4 a any three points such that  $x^2 + y^2 = 25$   
 b 12  
 5 a  $\frac{1}{2}$     b -2

c  $y = -2x + 10$

6 Check proof is valid

7 a  $y = \frac{3}{5}x - \frac{34}{5}$     b  $y = -\frac{1}{3}x - \frac{20}{3}$

c  $y = -\frac{p}{q}x + \frac{p^2 + q^2}{q}$  (or  $\frac{a^2}{q}$ )

8  $y = 2x - 15$ ,  $y = 2x + 15$

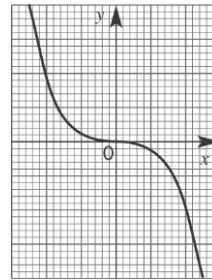
9 b  $x + y = 10$ ,  $x + y = -10$

10 a 10

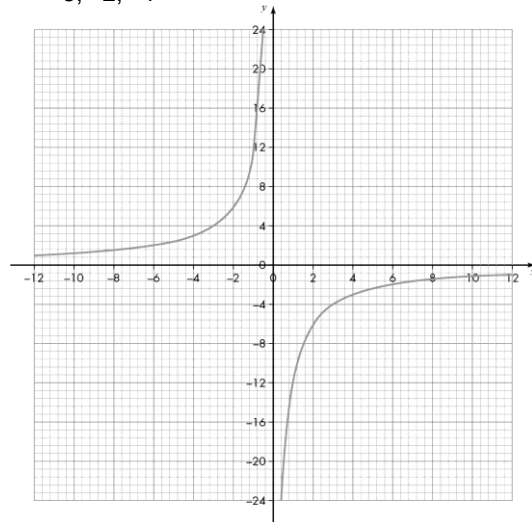
b  $x^2 + y^2 = 90$

### Exercise 23H

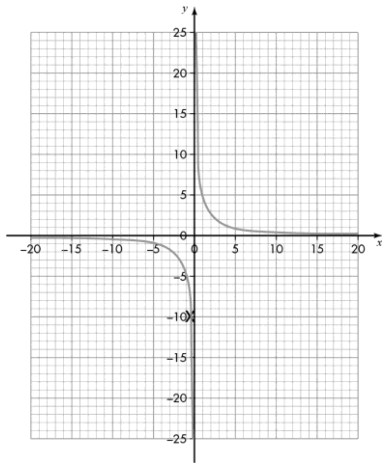
1



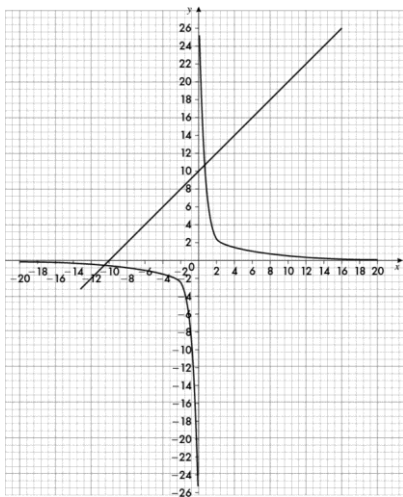
- 2 a Values of y: -54, -31.25, -16, -6.75, -2, -0.25, 0, 0.25, 2, 6.75, 16, 31.25, 54  
 b 39.4  
 3 a Values of y: -24, -12.63, -5, -0.38, 2, 2.9, 3, 3.13, 4, 6.38, 11, 18.63, 30  
 b 4.7    c -1.4 to -1.5  
 4 a Values of y: -16, -5.63, 1, 4.63, 6, 5.88, 5, 4.13, 4, 5.38, 9, 15.63, 26  
 b i -2.1    ii (-0.8, 6)  
 iii (0.7, 3.9)    iv (0, 5)  
 5 Values of y: 1, 2, 3, 4, 6, 12, 24, -24, -12, -6, -4, -3, -2, -1



- 6 a Values of y: -0.25, -0.33, -0.5, -1, -2.5, -5, -10, -12.5, -25, 25, 12.5, 10, 5, 2.5, 1, 0.5, 0.33, 0.25

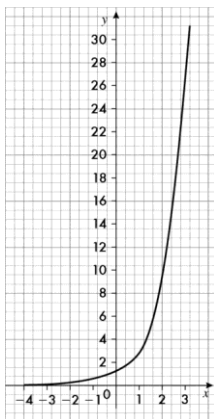


- b** Can't divide by 0  
**c**



- d** 0.48 and -10.48

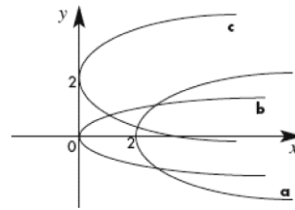
- 7 a** Values of  $y$ : 0.01, 0.04, 0.11, 0.33, 1, 3, 9, 27  
**b** 15.6      **c** -0.63



- 8 a** Quadratic    **b** Linear      **c** Exponential  
**d** Reciprocal    **e** None      **f** Cubic  
**g** Linear      **h** None      **i** Quadratic
- 9 a** The numbers go 1, 2, 4, ... which is equivalent to  $2^0, 2^1, 2^2, \dots$  so the formula is  $2^{(n-1)}$   
**b**  $2^{63} = 9.22 \times 10^{18}$       **c**  $\text{£}4.61 \times 10^{14}$
- 10 a** Number of pieces =  $2^n$  so  $2^{50} = 1.1 \times 10^{15}$  pieces  
**b**  $1.13 \times 10^8$  km

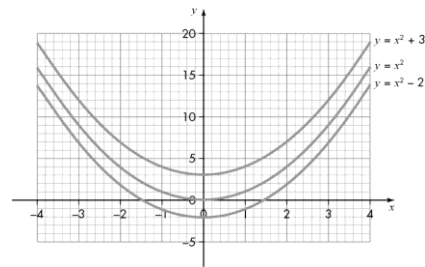
- 11**  $a = 5, b = 3.87$

**12**



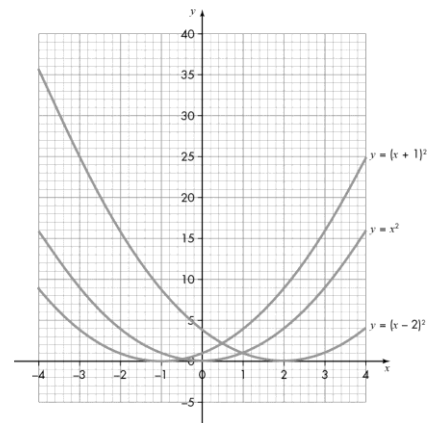
### Exercise 231

- 1 a, b**



- c**  $y = x^2 + 3$  is 3 units higher than  $y = x^2$   
**d**  $y = x^2 - 2$  is 2 units lower than  $y = x^2$   
**e i**  $y = x^2 + 6$  is 6 units higher than  $y = x^2$   
**ii**  $y = x^2 - 6$  is 6 units lower than  $y = x^2$

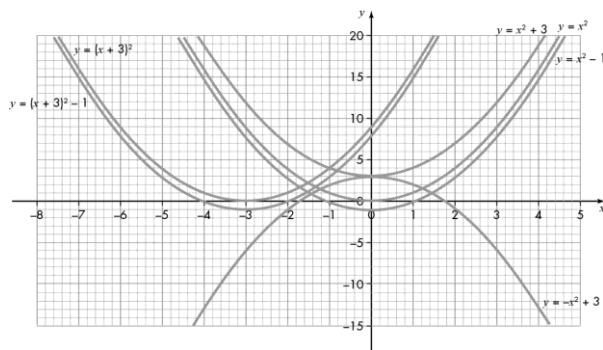
- 2 a, b**



- c**  $y = (x - 2)^2$  is 2 units to the right of  $y = x^2$   
**d**  $y = (x + 1)^2$  is 1 unit to the left of  $y = x^2$   
**e i**  $y = (x - 3)^2$  is 3 units to the right of  $y = x^2$   
**ii**  $y = (x + 4)^2$  is 4 units to the left of  $y = x^2$

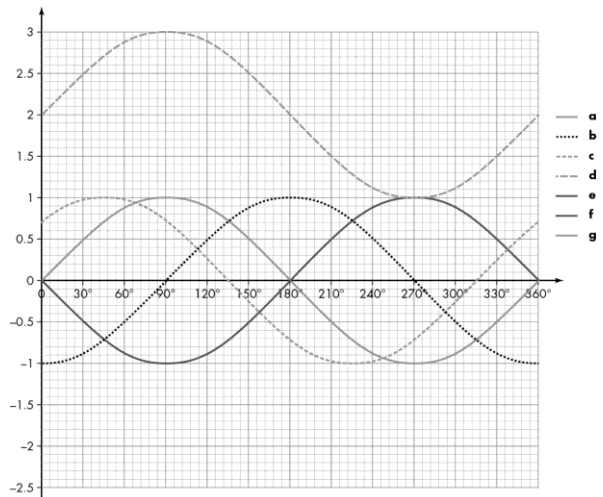
## Exercise 23J

1 a



- b Up 3      c Down 1      d 3 to the left  
e 3 to the left and down 1  
f Reflect in the x-axis and move up 3

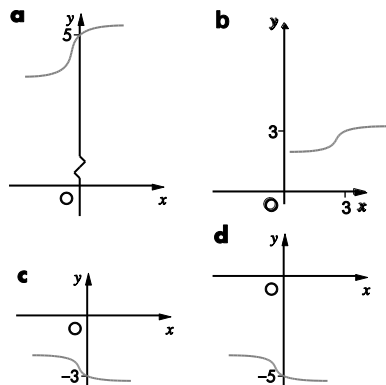
2 a



- b 90° to the left      c 45° to the right  
d up 2      e reflect in the x-axis  
f Reflect in the y-axis  
g Reflect in both axes

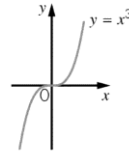
3 All of them.

4

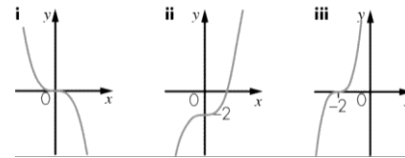


- 5 a  $y = \cos x + 3$       b  $y = \cos(x + 30^\circ)$   
c  $y = \cos(x - 45^\circ) - 2$

6 a



b



- c i  $y = -x^3$       ii  $y = x^3 - 2$   
iii  $y = (x + 2)^3$

7 No, as  $f(-x) = (-x)^2 = x^2$ , and  $-f(x) = -(x)^2 = -x^2$

- 8 a  $y = x^2 + 2$       b  $y = (x - 2)^2$   
c  $y = -x^2 + 4$

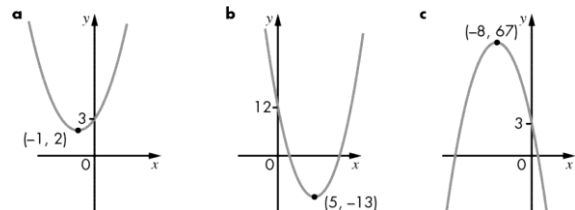
9 a Translation

- b i Equivalent      ii Equivalent

## Exercise 23K

- 1 a  $y = f(x - 3) + 2$ ; 3 right and 2 up  
b  $y = f(x + 7) - 14$ ; 7 left and 14 down  
c  $y = f(x - 11) - 21$ ; 11 right and 21 down

2



- 3 a  $y = x^2 - 8x + 7$       b  $y = -x^2 + 6x + 5$   
c  $y = x^2 - 14x + 59$

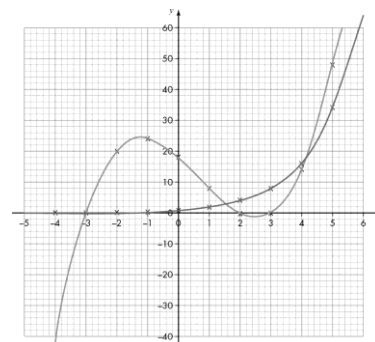
## Review questions



1

- 2 a 19      b  $0.7 \text{ m/s}^2$

3 a



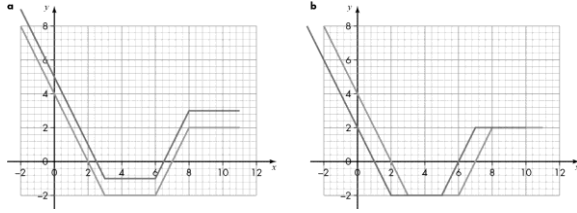
- b -3.0, 1.6, 4.2

4 50

5 1F, 2C, 3D, 4A, 5B, 6E

6 a 5 b 4 c 320

7



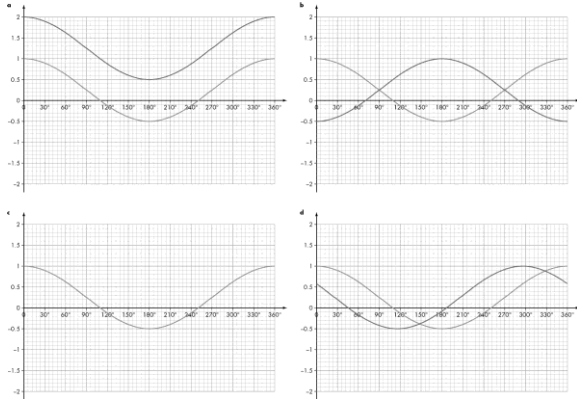
8 a (5, 5) and (7, 1)

b  $(\frac{20}{3}, \frac{10}{3})$

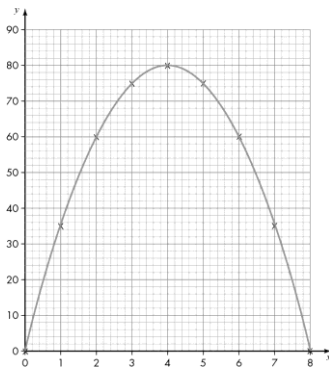
9 a  $-(0.8-0.9) \text{ m/h}^2$

b 8.6 miles

10



11 a



b 10 m/s

## Chapter 24 – Algebra: Algebraic fractions and functions

### Exercise 24A

1 a  $\frac{5x}{6}$  b  $\frac{23x}{20}$  c  $\frac{x^2y+8}{4x}$   
d  $\frac{5x+7}{6}$  e  $\frac{13x+5}{15}$  f  $\frac{5x-10}{4}$

2 a  $\frac{11x}{20}$  b  $\frac{3x-2y}{6}$  c  $\frac{xy^2-8}{4y}$   
d  $\frac{x+1}{4}$  e  $\frac{x-1}{4}$  f  $\frac{2-3x}{4}$

3 a  $x=3$  b  $x=2$  c  $x=0.75$  d  $x=3$

4 a  $\frac{x^2}{6}$  b  $\frac{8}{3}$  c  $\frac{x^2-2x}{10}$   
d  $\frac{2x^2+x}{15}$  e  $\frac{1}{2x}$

5 a  $\frac{x}{y}$  b  $\frac{2xy}{3}$  c  $\frac{2x^2-12x+18}{75}$

d 1 e  $\frac{1}{4x+2}$  f  $\frac{x^2-5x+6}{48}$

g  $\frac{1}{2x}$

6 a  $x$  b  $\frac{x}{2}$  c  $\frac{3x^2}{16}$  d 3

e  $\frac{17x+1}{10}$  f  $\frac{13x+9}{10}$  g  $\frac{3x^2-5x-2}{10}$

h  $\frac{x+3}{2}$  i  $\frac{2}{3}$  j  $\frac{2x^2-6y^2}{9}$

7 All parts: students' own working

8 a  $\frac{x^2-8}{x^2-2}$  b 7

9  $\frac{x^2+14x+37}{x^3+12x^2+47x+60}$

10  $\frac{x+3}{x-6}$

11 First, he did not factorise and just cancelled the  $x^2$ s. Then he cancelled 2 and 6 with the wrong signs. Then he said two minuses make a plus when adding, which is not true.

12  $\frac{2x^2+x-3}{4x^2-9}$

13 a 3, -1.5 b 4, -1.25 c 3, -2.5 d 0, 1

14 a  $\frac{x-1}{2x+1}$  b  $\frac{2x+1}{x+3}$  c  $\frac{2x-1}{3x-2}$

d  $\frac{x+1}{x-1}$  e  $\frac{2x+5}{4x-1}$

15 a Proof b 2 or  $\frac{-10}{3}$

16 a Proof b 2 m/s

17 a  $x^3+3\sqrt{2}x^2+6x+2\sqrt{2}$   
b Proof c  $99+70\sqrt{2}$

18 a  $\frac{72-4x^2}{(x+3)^2(x-3)^2}$   
b  $\frac{4x^3+40x^2+122x+110}{(x+1)(x+2)(x+3)(x+4)}$

$$19 \frac{x - \sqrt{3}}{x - 3\sqrt{3}}$$

### Exercise 24B

$$1 \text{ a } c = \frac{p}{5} + 3 \text{ or } \frac{p+15}{5} \quad \text{b } c = \frac{15}{5-p}$$

$$2 \text{ a } G = \frac{R}{F} - 3 \text{ or } \frac{R-3F}{F} \\ \text{b } G = \frac{R-3F}{F-1} \text{ or } \frac{3F-R}{1-F}$$

$$3 \text{ a } a = \frac{b(q+p)}{q-p} \quad \text{b } b = \frac{a(q-p)}{q+p} \\ \text{c } a = \frac{b+5c}{4} \quad \text{d } r = \frac{A}{\pi(2h+k)} \\ \text{e } v = \frac{u}{\sqrt{1-a}} \quad \text{f } x = \frac{2R-3}{R-1}$$

$$4 \text{ a } r = \frac{P}{\pi + 2k} \quad \text{b } r = \sqrt{\frac{2A}{\pi + \sqrt{k^2 - 1}}}$$

$$5 \text{ P} = \frac{100A}{100 + RY}$$

$$6 \text{ a } b = \frac{Ra}{a-R} \quad \text{b } a = \frac{Rb}{b-R}$$

$$7 \text{ a } x = \frac{2+2y}{y-1} \\ \text{b } y-1 = \frac{4}{x-2}, (x-2)(y-1) = 4, x-2 = \frac{4}{y-1}, \\ x = 2 + \frac{4}{y-1}$$

$$\text{c } y = 1 + \frac{4}{x-2} = \frac{x-2+4}{x-2} = \frac{x+2}{x-2} \text{ and } \\ x = \frac{2+2y}{y-1}$$

d Same formulae as in a

8 a Cannot take  $r$  as a common factor

$$\text{b } \pi = \frac{3V}{r^2(2r+3h)}$$

$$\text{c } \text{Yes, } r = \sqrt[3]{\frac{3V}{5\pi}}$$

$$9 \text{ x} = \frac{2W - 2zy}{z + y}$$

$$10 \text{ x} = \frac{1-3y}{2y-5}$$

The first number at the top of the answer is the constant term on the top of the original.  
The coefficient of  $y$  at the top of the answer is the negative constant term on the bottom of the original.  
The coefficient of  $y$  at the bottom of the answer is the coefficient of  $x$  on the bottom of the original.  
The constant term on the bottom is negative the coefficient of  $x$  on the top of the original.

- 11 a Both are correct  
b Alice's answer is easier to substitute into

### Exercise 24C

$$1 \text{ a i } 8 \quad \text{ii } 14 \quad \text{iii } 2 \quad \text{iv } 4 \\ \text{b i } 36 \quad \text{ii } -9 \quad \text{iii } 1241 \quad \text{iv } -1.5$$

$$2 \text{ a } 6 \quad \text{b } \frac{x+7}{4} - 3 \quad \text{c } 45$$

$$3 \text{ a } 29 \quad \text{b } 218 \quad \text{c } 7.832$$

$$4 \text{ 7}$$

$$5 \text{ a } 25 \quad \text{b } 249 \quad \text{c } 15 \quad \text{d } 1807 \\ \text{e } 1807 \quad \text{f } 13 \quad \text{g } \pm 5$$

$$6 \text{ a } 9 \quad \text{b } -39 \quad \text{c } -56 \quad \text{d } -56 \\ \text{e } 12 \quad \text{f } 24.84 \quad \text{g } \pm 5$$

$$7 \text{ a i } 54 \quad \text{ii } 44 \quad \text{b } 6 \text{ and } -1$$

### Exercise 24D

$$1 \text{ a } f^{-1}(x) = \frac{x+5}{4} \quad \text{b } f^{-1}(x) = \sqrt[3]{x-2}$$

$$\text{c } f^{-1}(x) = \frac{10}{x} - 1 \quad \text{d } f^{-1}(x) = \frac{10-x}{2}$$

$$\text{e } f^{-1}(x) = 6x+7 \quad \text{f } f^{-1}(x) = \frac{3}{x-5}$$

$$2 \text{ a } f^{-1}(x) = \frac{5x+2}{3x-1} \quad \text{b } -\frac{3}{2} \quad \text{c } f^{-1}(-\frac{3}{2}) = 1$$

- 3 a Both inverse functions are the same as the original function  
b The inverse function is the same as the original function  
c Proof

### Exercise 24E

$$1 \text{ a } 8\frac{1}{2} \quad \text{b } 6\frac{1}{2} \quad \text{c } 43 \quad \text{d } -2.25 \quad \text{e } 5.8$$

$$2 \text{ a } 48 \quad \text{b } 229 \quad \text{c } 18 \\ \text{d } 29 \quad \text{e } -8 \quad \text{f } -141$$

$$3 \text{ a i } 4x^3 - 32 \quad \text{ii } 11 - 4x \\ \text{iii } 21 - 27x + 9x^2 - x^3 \\ \text{iv } 16x - 40 \\ \text{v } x^3 - 18x^2 + 108x^3 - 222 \\ \text{b } gh(x) = 4 - 4x, hg(x) = 11 - 4x, 4 - 4x \neq 11 - 4x$$

$$4 \frac{1}{2}(b+1)$$

### Exercise 24F

$$1 \text{ x}_2 = 4 \quad \text{x}_3 = -10 \quad \text{x}_4 = 88 \quad \text{x}_5 = -598$$

$$2 \text{ a } 1878 \quad \text{b } -4372 \quad \text{c } -54.048 \quad \text{d } 3$$

$$3 \text{ 5.0701}$$

$$4 \text{ x}_2 = 3.1414 \quad \text{x}_3 = 3.1745 \quad \text{x}_4 = 3.1821 \\ \text{x}_5 = 3.1839 \quad \text{x}_6 = 3.1843$$

$$5 \text{ a } 2.115 = 2.12 \text{ (2 dp)} \\ \text{b } f(2.12) = 0.03 \text{ (2 dp)}$$

6 Proof

$$7 \text{ a } 3 \text{ and } 7 \\ \text{b } 7$$

- c i converges on 7  
 ii diverges, towards square root of a negative,  
 iii converges on 7 iv stays on 3  
 d  $x < 3$ : diverges, towards square root of a negative  
 $x = 3$ : stays on 3  
 $x > 7$ : converges on 7

8 a Proof

b  $x_2 = \frac{7}{2}$ ,  $x_3 = 3$ ,  $x_4 = \frac{7}{2}$

c  $x_2 = \frac{29}{9}$ ,  $x_3 = \frac{13}{4}$ ,  $x_4 = \frac{29}{9}$

d  $x_2 = \frac{73}{23}$ ,  $x_3 = \frac{33}{10}$ ,  $x_4 = \frac{73}{23}$

e  $x_2 = 1 + \sqrt{5}$ ,  $x_3 = 1 + \sqrt{5}$ ,  $x_4 = 1 + \sqrt{5}$

f  $x = 1 + \sqrt{5}$

9 a Proof b  $67 \text{ cm}^2$

c this will depend upon how accurate the final value of  $x_{n+1}$  is

10 a 1 b 3

11 a oscillates between 8.046, 0.148 and  $-2.262$   
 b diverges  
 c converges on 2.707

### Review questions

1 8

2 a  $x = \frac{a+K}{C+6}$  b 3.5

3 a  $\frac{x+3}{x}$  b  $f^{-1}(x) = \frac{3}{x-1}$

4  $\frac{7x}{3x+1}$

5 a  $x_1 = 2.54$ ,  $x_2 = 2.57$ ,  $x_3 = 2.58$ ,  $x_4 = 2.59$   
 b 2.59 – it's the same

6 a  $fg(x) = 3x^3 + 14$  b  $3 \times 3^3 + 14 = 95$

7 a  $f^{-1}(x) = \frac{x+q}{p}$  b  $f^{-1}(x) = \sqrt[3]{a-x}$

c  $f^{-1}(x) = \frac{a}{x} - c$

8 a i 2 ii 8 iii 18  
 iv 32 v 50  
 b  $2n^2$

9 Proof

10 a  $\frac{2x+7}{x-3}$  b  $\frac{5}{6}$  or 16

11  $2 - 3xy = 4 - x$   
 mistake here expanding the brackets  
 $x = \frac{1-3y}{2}$  should be 2 divided by  $(1-3y)$

corrected:  $y = \frac{4-x}{2-3x}$

$y(2-3x) = 4-x$

$2y - 3xy = 4 - x$

$x - 3xy = 4 - 2y$

$x(1-3y) = 4-2y$

$$x = \frac{4-2y}{1-3y}$$

$$\text{Hence } f^{-1}(x) = \frac{4-2x}{1-3x}$$

12 i  $\frac{3}{4}$  ii  $-\frac{2}{3}$  iii 5 iv  $\frac{8}{5}$

13  $-\frac{4}{x+1}$

14 a 9.51,  $-10.5$

b  $x \leq 10$ ,  $x \geq 28$

c  $-10 < x < 10$

15 a Proof b 5.31

16 a 21 b  $a = -3$ ,  $b = 2$

17 a Proof b 12

## Chapter 25 – Geometry and measures: Vector geometry

### Exercise 25A

1 a Any three of:  $\overline{AC}$ ,  $\overline{CF}$ ,  $\overline{BD}$ ,  $\overline{DG}$ ,  $\overline{GI}$ ,  $\overline{EH}$ ,  $\overline{HJ}$ ,  $\overline{JK}$

b Any three of:  $\overline{BE}$ ,  $\overline{AD}$ ,  $\overline{DH}$ ,  $\overline{CG}$ ,  $\overline{GJ}$ ,  $\overline{FI}$ ,  $\overline{IK}$

c Any three of:  $\overline{AO}$ ,  $\overline{CA}$ ,  $\overline{FC}$ ,  $\overline{IG}$ ,  $\overline{GD}$ ,  $\overline{DB}$ ,  $\overline{KJ}$ ,  $\overline{JH}$ ,  $\overline{HE}$

d Any three of:  $\overline{BO}$ ,  $\overline{EB}$ ,  $\overline{HD}$ ,  $\overline{DA}$ ,  $\overline{JG}$ ,  $\overline{GC}$ ,  $\overline{KI}$ ,  $\overline{IF}$

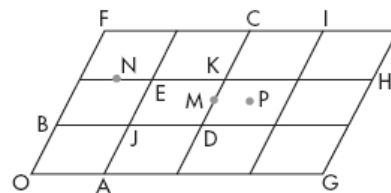
2 a 2a b 2b c a + b  
 d 2a + b e 2a + 2b f a + 2b  
 g a + b h 2a + 2b i 3a + b  
 j 2a k b l 2a + b

3 a Equal b  $\overline{AI}$ ,  $\overline{BJ}$ ,  $\overline{DK}$

4 a  $\overline{OJ} = 2\overline{OD}$  and parallel

b  $\overline{AK}$  c  $\overline{OF}$ ,  $\overline{BI}$ ,  $\overline{EK}$

5



6 a Lie on same straight line  
 b All multiples of  $a + b$  and start at O  
 c H

7 a  $-b$  b  $3a - b$  c  $2a - b$   
 d  $a - b$  e  $a + b$  f  $-a - b$   
 g  $2a - b$  h  $-a - 2b$  i  $a + 2b$   
 j  $-a + b$  k  $2a - 2b$  l  $a - 2b$

8 a Equal but in opposite directions

b Any three of:  $\overline{DA}$ ,  $\overline{EF}$ ,  $\overline{GJ}$ ,  $\overline{FI}$ ,  $\overline{AH}$



9 a Opposite direction and  $\overrightarrow{AB} = -\frac{1}{2}\overrightarrow{CK}$

b  $\overrightarrow{BJ}$ ,  $\overrightarrow{CK}$

c  $\overrightarrow{EB}$ ,  $\overrightarrow{GO}$ ,  $\overrightarrow{KH}$

10 506 mph on a bearing of 009°

11 12 km/h on a bearing of 107°

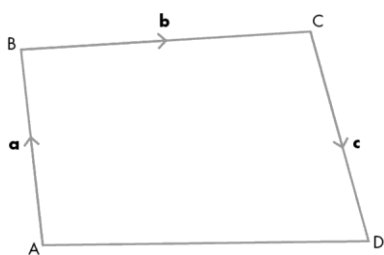
12 a i  $\mathbf{a} + \mathbf{b}$  ii  $3\mathbf{a} + \mathbf{b}$   
iii  $2\mathbf{a} - \mathbf{b}$  iv  $2\mathbf{b} - 2\mathbf{a}$

b  $\overrightarrow{DG}$  and  $\overrightarrow{BC}$

13 Parts **b** and **d** could be, parts **a** and **c** could not be

14 a Any multiple (positive or negative) of  $3\mathbf{a} - \mathbf{b}$   
b Will be a multiple of  $3\mathbf{a} - \mathbf{b}$

15 For example, let ABCD be a quadrilateral as shown.



Then  $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BD} = \mathbf{a} + (\mathbf{b} + \mathbf{c})$ .

But  $\overrightarrow{AD} = \overrightarrow{AC} + \overrightarrow{CD} = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$ .

Hence  $\mathbf{a} + (\mathbf{b} + \mathbf{c}) = (\mathbf{a} + \mathbf{b}) + \mathbf{c}$ .

16 a i  $2\mathbf{b} - 2\mathbf{a}$  ii  $\mathbf{a} - \mathbf{c}$   
iii  $2\mathbf{c} - 2\mathbf{b}$  iv  $\mathbf{b} + \mathbf{c} - \mathbf{a}$

b  $\overrightarrow{RQ} = \mathbf{a} - \mathbf{c} = \overrightarrow{SP}$ , similarly  $\overrightarrow{PQ} = \mathbf{b} = \overrightarrow{SR}$ , so opposite sides are equal and parallel, hence PQRS is a parallelogram

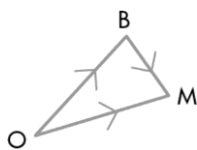
### Exercise 25B

1 a i  $-\mathbf{a} + \mathbf{b}$  ii  $\frac{1}{2}(-\mathbf{a} + \mathbf{b})$   
iii



iv  $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

b i  $\mathbf{a} - \mathbf{b}$  ii  $\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$   
iii



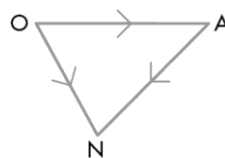
iv  $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

c



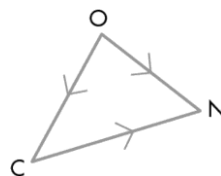
d M is midpoint of parallelogram of which OA and OB are two sides.

2 a i  $-\mathbf{a} - \mathbf{b}$  ii  $-\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$   
iii



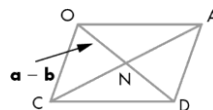
iv  $\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$

b i  $\mathbf{b} + \mathbf{a}$  ii  $\frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{a}$   
iii



iv  $\frac{1}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$

c



d N is midpoint of parallelogram of which OA and OC are two sides

3 a i  $-\mathbf{a} + \mathbf{b}$  ii  $\frac{1}{3}(-\mathbf{a} + \mathbf{b})$  iii  $\frac{2}{3}\mathbf{a} + \frac{1}{3}\mathbf{b}$   
b  $\frac{3}{4}\mathbf{a} + \frac{1}{4}\mathbf{b}$

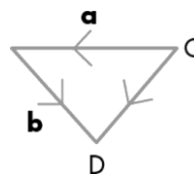
4 a i  $\frac{2}{3}\mathbf{b}$  ii  $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$  iii  $-\frac{2}{3}\mathbf{b}$

b  $\frac{1}{2}\mathbf{a} - \frac{1}{6}\mathbf{b}$

c  $\overrightarrow{DE} = \overrightarrow{DO} + \overrightarrow{OE}$   
 $= \frac{3}{2}\mathbf{a} - \frac{1}{2}\mathbf{b}$

d  $\overrightarrow{DE}$  parallel to  $\overrightarrow{CD}$  (multiple of  $\overrightarrow{CD}$ ) and D is a common point

5 a

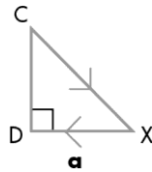


$\overrightarrow{CD} = -\mathbf{a} + \mathbf{b} = \mathbf{b} - \mathbf{a}$

b i  $-\mathbf{a}$  ii  $-\mathbf{b}$  iii  $\mathbf{a} - \mathbf{b}$

- c 0, vectors return to starting point  
 d i  $2\mathbf{b}$  ii  $2\mathbf{b} - 2\mathbf{a}$  iii  $-2\mathbf{a}$   
 iv  $2\mathbf{b} - \mathbf{a}$  v  $-\mathbf{a} - \mathbf{b}$

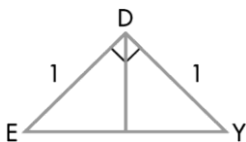
6 a



$$\overrightarrow{CX} = \sqrt{1^2 + 1^2} \mathbf{b} = \sqrt{2} \mathbf{b}$$

$$\overrightarrow{CD} = \overrightarrow{CX} + \overrightarrow{XD} = \sqrt{2} \mathbf{b} - \mathbf{a}$$

b



$$\overrightarrow{YE} = \sqrt{1^2 + 1^2} \mathbf{a} = \sqrt{2} \mathbf{a}$$

$$\overrightarrow{DE} = \overrightarrow{DY} + \overrightarrow{YE} = \mathbf{b} - \sqrt{2} \mathbf{a}$$

- c i  $-\mathbf{a}$  ii  $-\mathbf{b}$   
 iii  $\mathbf{a} - \sqrt{2} \mathbf{b}$  iv  $\sqrt{2} \mathbf{a} - \mathbf{b}$   
 v  $\sqrt{2} \mathbf{a} + \mathbf{a}$  vi  $\sqrt{2} \mathbf{b} + \mathbf{b}$   
 vii  $2\mathbf{b} + \sqrt{2} \mathbf{b} - \mathbf{a} - \sqrt{2} \mathbf{a}$   
 viii  $2\mathbf{b} + \sqrt{2} \mathbf{b} - 2\mathbf{a} - \sqrt{2} \mathbf{a}$

7 a i  $-\mathbf{a} + \mathbf{b}$  ii  $\frac{1}{2}(-\mathbf{a} + \mathbf{b}) = -\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

iii  $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

b i  $\frac{1}{2}\mathbf{b} + \frac{1}{2}\mathbf{c}$  ii  $-\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}$

c i  $-\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}$  ii Equal  
 iii Parallelogram

d  $\overrightarrow{AC} = -\mathbf{a} + \mathbf{c} = 2(-\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}) = 2 \overrightarrow{QM}$

8 a i  $\frac{1}{2}\mathbf{a}$  ii  $\mathbf{c} - \mathbf{a}$

iii  $\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{c}$  iv  $\frac{1}{2}\mathbf{c}$

b i  $-\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$  ii  $-\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

c Opposite sides are equal and parallel  
 d NMRQ and PNLQ

9 a  $-\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

b i Rhombus

ii They lie on a straight line,  $\overrightarrow{OM} = \frac{1}{2}\overrightarrow{OC}$

10  $k = 8$

11 a  $\overrightarrow{YW} = \overrightarrow{YZ} + \overrightarrow{ZW} = 2\mathbf{a} + \mathbf{b} + \mathbf{a} + 2\mathbf{b}$   
 $= 3\mathbf{a} + 3\mathbf{b} = 3(\mathbf{a} + \mathbf{b}) = 3\overrightarrow{XY}$

b 3 : 1

c They lie on a straight line.

d Points are A(6, 2), B(1, 1) and C(2, 24). Using Pythagoras' theorem,  $AB^2 = 26$ ,  $BC^2 = 26$  and  $AC^2 = 52$  so  $AB^2 + BC^2 = AC^2$  hence  $\angle ABC$  must be a right angle.

12 In parallelogram ABCD,  $\overrightarrow{AB} = \overrightarrow{DC} = \mathbf{a}$ ,  $\overrightarrow{BC} = \overrightarrow{AD} = \mathbf{b}$ .

Let X be the midpoint of diagonal AC. Then  $\overrightarrow{DX}$

$$= -\mathbf{b} + \frac{1}{2}(\mathbf{a} + \mathbf{b}) = \frac{1}{2}(\mathbf{a} - \mathbf{b}) = \frac{1}{2}\overrightarrow{DB} \text{ which is } \mathbf{a} - \mathbf{b},$$

hence the midpoint of one diagonal is the same as the midpoint of the other diagonal, hence they bisect each other.

### Review questions

1 a  $2\mathbf{b} - \mathbf{a}$  b  $-3\mathbf{b}$  c  $\mathbf{a} + \mathbf{b}$

2 a i  $2\mathbf{y} - 2\mathbf{b}$  ii  $2\mathbf{b} - 2\mathbf{x}$

b  $\overrightarrow{WZ} = \overrightarrow{WB} + \overrightarrow{BZ} = \frac{1}{2}(2\mathbf{b} - 2\mathbf{x}) + \frac{1}{2}(2\mathbf{y} - 2\mathbf{b}) = \mathbf{b} - \mathbf{x} + \mathbf{y} - \mathbf{b} = \mathbf{y} - \mathbf{x}$

c  $\overrightarrow{XY} = \mathbf{y} - \mathbf{x}$ , so parallel and equal in length to WZ, so Tim must be correct.

3 Let  $\overrightarrow{OF} = \mathbf{x} = \overrightarrow{DE}$  and  $\overrightarrow{OD} = \mathbf{y} = \overrightarrow{FE}$ , then

$$\overrightarrow{DF} = \mathbf{x} - \mathbf{y} \text{ and } \overrightarrow{AB} = \frac{1}{2}\mathbf{x} - \frac{1}{2}\mathbf{y} = \frac{1}{2}(\mathbf{x} - \mathbf{y})$$

4 a i  $\mathbf{p} + \mathbf{r}$  ii  $\mathbf{r} - \mathbf{p}$

b  $\frac{1}{2}(\mathbf{r} - \mathbf{p})$

c  $\overrightarrow{OX} = \mathbf{p} + \frac{1}{2}(\mathbf{r} - \mathbf{p}) = \frac{1}{2}(\mathbf{p} + \mathbf{r}) = \frac{1}{2}\overrightarrow{OQ}$

5 a i  $\mathbf{y} - \mathbf{x}$  ii  $\frac{1}{2}(\mathbf{y} - \mathbf{x})$  iii  $\frac{1}{2}(\mathbf{x} + \mathbf{y})$

iv  $\frac{1}{3}(\mathbf{x} + \mathbf{y})$  v  $-\frac{1}{6}(\mathbf{x} + \mathbf{y})$  vi  $\frac{1}{3}(\mathbf{y} - 2\mathbf{x})$

vii  $\frac{1}{2}\mathbf{y} - \mathbf{x}$

b  $\overrightarrow{BG} = \frac{1}{3}(\mathbf{y} - 2\mathbf{x})$  and  $\overrightarrow{BE} = \frac{1}{2}\mathbf{y} - \mathbf{x}$

$$= \frac{1}{2}(\mathbf{y} - 2\mathbf{x}), \text{ both are multiples of } (\mathbf{y} - 2\mathbf{x}) \text{ so}$$

are parallel, and with a common point, they must all be collinear.

c  $\frac{2}{3}\overrightarrow{BE} = \frac{2}{3} \times \frac{1}{2}(\mathbf{y} - 2\mathbf{x}) = \frac{1}{3}(\mathbf{y} - 2\mathbf{x}) = \overrightarrow{BG}$

6 a i  $\mathbf{x} + \mathbf{y}$  ii  $\frac{1}{3}(\mathbf{x} + \mathbf{y})$  iii  $\frac{2}{3}(\mathbf{x} + \mathbf{y})$

iv  $\frac{1}{3}(\mathbf{x} - 2\mathbf{y})$  v  $\frac{1}{3}(\mathbf{x} - 2\mathbf{y})$

b  $\overrightarrow{SA} = \frac{1}{3}(\mathbf{x} - 2\mathbf{y}) = \overrightarrow{BQ}$ ,  $\overrightarrow{SB} = \overrightarrow{SA} + \overrightarrow{AB}$ ,  $\overrightarrow{AQ} =$

$\overrightarrow{AB} + \overrightarrow{BQ} = \overrightarrow{AB} + \overrightarrow{SA}$ , hence  $\overrightarrow{SB} = \overrightarrow{AQ}$ , hence SAQB has opposite sides equal and parallel, so a parallelogram.

7 a i  $6\mathbf{a} - 2\mathbf{b}$  ii  $3\mathbf{a} - \mathbf{b}$

b  $\overrightarrow{BP} = 2(3\mathbf{a} - \mathbf{b})$  hence it is parallel to BQ with a common point Q, so the points B, Q and P are collinear.

8 In triangle ABC the midpoint of AB is M and the midpoint of AC is N

Let  $\overrightarrow{AM} = \mathbf{x}$  and  $\overrightarrow{AN} = \mathbf{y}$ , then  $\overrightarrow{MN} = \mathbf{y} - \mathbf{x}$ ,  $AB = 2\mathbf{x}$  and  $AC = 2\mathbf{y}$ , so  $\overrightarrow{BC} = 2\mathbf{y} - 2\mathbf{x} = 2(\mathbf{y} - \mathbf{x})$ .  $\overrightarrow{BC}$  is a

multiple of  $\overrightarrow{MN}$  and so parallel,  $\overrightarrow{MN} = \frac{1}{2} \overrightarrow{BC}$  and so half its length.

**9 a**  $\mathbf{m} - \mathbf{r}$

**b**  $\overrightarrow{RT} = 2\overrightarrow{RM} = 2(\mathbf{m} - \mathbf{r})$ , so  $\overrightarrow{NT} = -2\mathbf{m} + 2\mathbf{m} - 2\mathbf{r} = -2\mathbf{r}$ , parallel to  $\mathbf{r}$ , hence NT is parallel to OR.