Answers

Solving mathematical problems

Swap to order (p.8)

Challenge/What if?

Pupils' statements will vary.

What are the number sequences? (p.9)

Challenge

The number that comes before the first number is in blue, and the number that comes after the last number is in red.

–100, –75, –50, –25, 0, 25, 50, 75, 100

2950, 3950, 4950, 5950, 6950, 7950, 8950, 9950, 10 950

2250, 2350, 2450, 2550, 2650, 2750, 2850, 2950, 3050

22 150, 23 150, 24 150, 25 150, 26 150, 27 150, 28 150, 29 150, 30 150

234 900, 235 000, 235 100, 235 200, 235 300, 235 400, 235 500, 235 600, 235 700

195 150, 205 150, 215 150, 225 150, 235 150, 245 150, 255 150, 265 150, 275 150

What if?

The rules of the six number sequences are:

–100, –75, –50, –25, 0, 25, 50, 75, 100

count forwards in steps of 25

2950, 3950, 4950, 5950, 6950, 7950, 8950, 9950, 10 950

count forwards in steps of 1000

2250, 2350, 2450, 2550, 2650, 2750, 2850, 2950, 3050

count forwards in steps of 100

22 150, 23 150, 24 150, 25 150, 26 150, 27 150, 28 150, 29 150, 30 150

count forwards in steps of 1000

234 900, 235 000, 235 100, 235 200, 235 300, 235 400, 235 500, 235 600, 235 700

count forwards in steps of 100

195 150, 205 150, 215 150, 225 150, 235 150, 245 150, 255 150, 265 150, 275 150

count forwards in steps of 10 000

Pupils' number sequences will vary.

Rounding and sorting (p.10)

Challenge/What if?

Results of the challenge will vary.

999 (p.11)

Challenge

The answers are the same. This method always works. For example:

	Example 1	**Example 2**
Step ①:	256 and 748	365 and 804
Step ②:	999 – 256 = 743	999 – 365 = 634
Step ③:	743 + 748 = 1491	634 + 804 = 1438
Step ④:	1 and 491	1 and 438
Step ⑤:	1 + 491 = 492	1 + 438 = 439
	748 – 256 = 492	804 – 365 = 439

What if?

The method also works for 4-digit numbers when the smaller number is subtracted from 9999 and the answer at **Step ③** is separated into a 1-digit number and a 4-digit number. For example:

	Example 1	**Example 2**
Step ①:	4756 and 9462	1462 and 7201
Step ②:	9999 – 4756 = 5243	9999 – 1462 = 8537
Step ③:	5243 + 9462 = 14 705	8537 + 7201 = 15 738
Step ④:	1 and 4705	1 and 5738
Step ⑤:	4706	5739
	9462 – 4756 = 4706	7201 – 1462 = 5739

5-digit totals and differences (p.12)

Challenge

The 12 five-digit numbers between 59 000 and 63 000 are: 59 236, 59 263, 59 326, 59 362, 59 623, 59 632, 62 359, 62 395, 62 539, 62 593, 62 935 and 62 953.

The pair of numbers are as follows:

- The smallest total: 59 236 + 59 263 = 118 499
- The greatest total: 62 935 + 62 953 = 125 888
- The smallest difference: 59 632 – 59 623 = 9
- The greatest difference: 62 953 – 59 236 = 3717

Pupils' explanations will vary. However, they should include comments such as:

- To find the smallest total, choose the two smallest numbers.
- To find the greatest total, choose the two largest numbers.
- To find the smallest difference, start by looking for pairs of numbers with identical tens of thousands, thousands and hundreds digits; then from these numbers, look for pairs of numbers with identical tens digits or tens digits that differ by 1.
- To find the greatest difference, choose the smallest and the largest numbers.

What if?

The pair of numbers with the smallest total is: 23 569 + 23 596 = 47 165

The pair of numbers with the greatest total is: 96 523 + 96 532 = 193 055

There are 12 pairs of numbers with the same smallest difference. They are:

23 965 – 23 956 = 9	29 365 – 29 356 = 9	32 965 – 32 956 = 9
39 265 – 39 256 = 9	56 932 – 56 923 = 9	59 632 – 59 623 = 9
65 932 – 65 923 = 9	69 532 – 69 523 = 9	92 365 – 92 356 = 9
93 265 – 93 256 = 9	95 632 – 95 623 = 9	96 532 – 96 523 = 9

The pair of numbers with the greatest difference is: 96 532 – 23 569 = 72 963

Palindromic numbers (p.13)

Challenge

Results of the challenge will vary.

What if?

The same applies for four-digit numbers as for three-digit numbers.

Factors (p.14)

Challenge

The following five numbers are the two-digit numbers with the most factors. Each of the following numbers has 12 factors:

60: 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60

72: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72

84: 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84

90: 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90

96: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, 48, 96

Each of these five numbers has four common factors: 1, 2, 3 and 6.

What if?

The following 21 numbers are the two-digit numbers with the fewest factors. Each of the following numbers has two factors: 1 and itself. These numbers are prime numbers.

11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

Square sums and differences (p.15)

Challenge

Each number from 1 to 50 can be made by adding and/or subtracting square numbers.

Other calculations are possible.

$1 = 1^2 + 0^2$	$13 = 7^2 - 6^2$	$25 = 4^2 + 3^2$	$38 = 8^2 - 5^2 - 1^2$
$2 = 1^2 + 1^2$	$14 = 3^2 + 2^2 + 1^2$	$26 = 5^2 + 1^2$	$39 = 8^2 - 5^2$
$3 = 2^2 - 1^2$	$15 = 4^2 - 1^2$	$27 = 6^2 - 3^2$	$40 = 7^2 - 3^2$
$4 = 2^2 + 0^2$	$16 = 5^2 - 3^2$	$28 = 8^2 - 6^2$	$41 = 5^2 + 4^2$
$5 = 3^2 - 2^2$	$17 = 9^2 - 8^2$	$29 = 5^2 + 2^2$	$42 = 5^2 + 4^2 + 1^2$
$6 = 4^2 - 3^2 - 1^2$	$18 = 4^2 + 1^2 + 1^2$	$30 = 5^2 + 2^2 + 1^2$	$43 = 12^2 - 10^2 - 1^2$
$7 = 4^2 - 3^2$	$19 = 10^2 - 9^2$	$31 = 6^2 - 2^2 - 1^2$	$44 = 12^2 - 10^2$
$8 = 3^2 - 1^2$	$20 = 6^2 - 4^2$	$32 = 6^2 - 2^2$	$45 = 6^2 + 3^2$
$9 = 5^2 - 4^2$	$21 = 5^2 - 2^2$	$33 = 7^2 - 4^2$	$46 = 6^2 + 3^2 + 1^2$
$10 = 6^2 - 5^2 - 1^2$	$22 = 12^2 - 11^2 - 1^2$	$34 = 5^2 + 3^2$	$47 = 7^2 - 1^2 - 1^2$
$11 = 6^2 - 5^2$	$23 = 12^2 - 11^2$	$35 = 6^2 - 1^2$	$48 = 7^2 - 1^2$
$12 = 4^2 - 2^2$	$24 = 5^2 - 1^2$	$36 = 10^2 - 8^2$	$49 = 7^2 + 0^2$
		$37 = 6^2 + 1^2$	$50 = 7^2 + 1^2$

What if?

Pupils' numbers will vary.

Decimal multiplications (p.16)

Challenge

Pupils' calculations will vary. However, it is possible to make 24 different O·th × O calculations from the same set of four 1–9 digit cards.

What if?

Pupils' calculations will vary.

Mixed numbers (p.17)

Challenge/What if?

Results of the challenge will vary depending on the mixed numbers written by pupils.

Decimals (p.18)

Challenge

It is possible to make 24 different decimals with one decimal place:

123·4, 124·3, 132·4, 134·2, 142·3, 143·2, 213·4, 214·3, 231·4, 234·1, 241·3, 243·1, 312·4, 314·2, 321·4, 324·1, 341·2, 342·1, 412·3, 413·2, 421·3, 423·1, 431·2, 432·1

It is possible to make 24 different decimals with two decimal places:

12·34, 12·43, 13·24, 13·42, 14·23, 14·32, 21·34, 21·43, 23·14, 23·41, 24·13, 24·31, 31·24, 31·42, 32·14, 32·41, 34·12, 34·21, 41·23, 41·32, 42·13, 42·31, 43·12, 43·21

It is possible to make 24 different decimals with three decimal places:

1·234, 1·243, 1·324, 1·342, 1·423, 1·432, 2·134, 2·143, 2·314, 2·341, 2·413, 2·431, 3·124, 3·142, 3·214, 3·241, 3·412, 3·421, 4·123, 4·132, 4·213, 4·231, 4·312, 4·321

What if?

Decimals less than $\frac{1}{2}$:

0·3, 0·34, 0·348, 0·38, 0·384, 0·4, 0·43, 0·438, 0·48, 0·483

Decimals greater than $\frac{3}{4}$ and less than 1:

0·8, 0·83, 0·834, 0·84, 0·843

1 to 100 percentages (p.19)

Challenge

The percentage of numbers on a 1 to 100 number square are as follows:

- odd numbers: 50%
- 1-digit numbers: 9%
- 2-digit numbers: 90%
- 3-digit numbers: 1%
- contain the digit 7: 19%
- multiples of 5: 20%

- multiples of 2 and 5: 10%
- multiples of 3: 33%
- square numbers: 10%
- factors of 36: 9%
- prime numbers: 25%

What if?

Pupils' answers will vary.

Equivalences (p.20)

The following fractions, decimals and percentages can be made using the given number cards. Teachers will need to use their professional judgement when assessing pupils' ability to recognise equivalent fractions, decimals and percentages. NOTE: Most pupils will not be able to recognise all of the following equivalences. At this stage they are only expected to recognise fraction, decimal and percentage equivalences of $\frac{1}{2}, \frac{1}{4}, \frac{1}{5}, \frac{2}{5}, \frac{4}{5}$ and those fractions with a denominator of a multiple of 10 or 25.

Fraction	Equivalent fraction in its simplest form	Decimal	Percentage
$\frac{1}{2}$		0·5	50%
$\frac{1}{4}$		0·25	25%
$\frac{1}{5}$		0·2	20%
$\frac{1}{10}$		0·1	10%
$\frac{1}{25}$		0·04	4%
$\frac{1}{50}$		0·02	2%
$\frac{2}{4}$	$\frac{1}{2}$	0·5	50%
$\frac{2}{5}$		0·4	40%
$\frac{2}{10}$	$\frac{1}{5}$	0·2	20%
$\frac{2}{25}$		0·08	8%
$\frac{2}{50}$	$\frac{1}{25}$	0·04	4%
$\frac{4}{5}$		0·8	80%
$\frac{4}{10}$	$\frac{2}{5}$	0·4	40%

$\frac{4}{25}$		0·16	16%
$\frac{4}{50}$	$\frac{2}{25}$	0·08	8%
$\frac{5}{10}$	$\frac{1}{2}$	0·5	50%
$\frac{5}{25}$	$\frac{1}{5}$	0·2	20%
$\frac{5}{50}$	$\frac{1}{10}$	0·1	10%
$\frac{10}{25}$	$\frac{2}{5}$	0·4	40%
$\frac{10}{50}$	$\frac{1}{5}$	0·2	20%
$\frac{20}{25}$	$\frac{4}{5}$	0·8	80%
$\frac{20}{50}$	$\frac{2}{5}$	0·4	40%

What if?

Pupils' criteria will vary but may include sorting the fractions into equivalent fractions and/or fraction families, for example:

Equivalent fractions

$\frac{1}{2} = \frac{2}{4} = \frac{5}{10}$

$\frac{1}{4}$

$\frac{1}{5} = \frac{2}{10} = \frac{5}{25} = \frac{10}{50}$

$\frac{1}{10} = \frac{5}{50}$

$\frac{1}{25} = \frac{2}{50}$

$\frac{1}{50}$

$\frac{2}{5} = \frac{4}{10} = \frac{10}{25} = \frac{20}{50}$

$\frac{2}{25} = \frac{4}{50}$

$\frac{4}{5} = \frac{20}{25}$

$\frac{4}{25}$

Fraction families

$\frac{1}{2}$

$\frac{1}{4}$ $\frac{2}{4}$

$\frac{1}{5}$ $\frac{2}{5}$ $\frac{4}{5}$

$\frac{1}{10}$ $\frac{2}{10}$ $\frac{4}{10}$ $\frac{5}{10}$

$\frac{1}{25}$ $\frac{2}{25}$ $\frac{4}{25}$ $\frac{5}{25}$ $\frac{10}{25}$ $\frac{20}{25}$

$\frac{1}{50}$ $\frac{2}{50}$ $\frac{4}{50}$ $\frac{5}{50}$ $\frac{10}{50}$ $\frac{20}{50}$

1 million steps (p.21)

Challenge

Answers will vary depending on the size of individual pupils' steps. However, if a pupil's step is 50 cm, then:

- It would take them 2000 steps to travel 1 km.
- It would take them 20 000 steps to travel 10 km.
- It would take them 24 000 steps to travel 12 km.
- It would take them 50 000 steps to travel 25 km.
- It would take them 200 000 steps to travel 100 km.
- They would travel about 500 kilometres in 1 million steps.

What if?

Pupils' answers will vary.

Expanding quadrilaterals (p.22)

Challenge

The area of shape B is four times that of shape A.

The area of shape C is nine times that of shape A.

The perimeter of shape B is twice that of shape A.

The perimeter of shape C is three times that of shape A.

What if?

The same rules apply for other quadrilaterals.

Sacks (p.23)

Challenge

Wheat: 59 kg, 67 kg and 70 kg

Sugar: 46 kg, 52 kg

Rice: 65 kg

What if?

Wheat: 34 kg, 32 kg and 12 kg

Sugar: 10 kg, 11 kg and 5 kg

Rice: 9 kg and 4 kg

Corn: 13 kg

When will it be? (p.24)

Challenge

Results of the challenge will vary depending on the day's date and the current time. However, dates and times should be based on the following calculations:

- 100 minutes from now: add 1 hour and 40 minutes to the current time
- 1000 minutes from now: add 16 hours and 40 minutes to the current time
- 100 hours from now: add 4 days and 4 hours to the current date/time
- 1000 hours from now: add 41 days and 16 hours to the current date/time
- 100 days from now: add 100 days to the current date
- 1000 days from now: add 2 years and 270 days, or 271 days if one of the years is a leap year, to the current date

What if?

Rounded to the nearest day, week and year:

- 1 million seconds in days is 12 days.

  1 000 000 seconds ÷ 60 seconds = 16 667 minutes

  16 667 minutes ÷ 60 minutes = 278 hours

  278 hours ÷ 24 hours = 12 days

  If pupils cannot carry out the calculations, then their working out may look similar to the following:

  1 000 000 seconds ÷ 60 seconds = □ minutes

  □ minutes ÷ 60 minutes = ◇ hours

  ◇ hours ÷ 24 hours = ○ days

- 1 million minutes in weeks is 99 weeks.

  1 000 000 minutes ÷ 60 minutes = 16 667 hours

  16 667 hours ÷ 24 hours = 694 days

  694 days ÷ 7 days = 99 weeks

  If pupils cannot carry out the calculations, then their working out may look similar to the following:

  1 000 000 minutes ÷ 60 minutes = □ hours

  □ hours ÷ 24 hours = ◇ days

  ◇ days ÷ 7 days = ○ weeks

- 1 million hours in years is 114 years.

  1 000 000 hours ÷ 24 hours = 41 667 days

  41 667 days ÷ 365 days = 114 years

  If pupils cannot carry out the calculations, then their working out may look similar to the following:

  1 000 000 hours ÷ 24 hours = □ days

  □ days ÷ 365 days = ◇ years

NOTE: Numbers are rounded to the nearest whole number.

Coins (p.25)

Challenge

50% of £1 = 50p

20% of £1 = 20p

25% of £2 = 50p

10% of £2 = 20p

10% of £1 = 10p

5% of £2 = 10p

5% of £1 = 5p

2% of £1 = 2p

1% of £2 = 2p

1% of £1 = 1p

What if?

You would receive:

£1.28 pocket money in the 8th week

£20.48 in the 12th week

£163.84 in the 15th week

£5242.88 in the 20th week

£83 886.08 in the 24th week.

Clock angles (p.26)

Challenge

12:10: 60° (acute angle)

2:25: 90° (right angle)

4:45: 120° (obtuse angle, it's almost 5:00)

Pupils' times, and sizes and names of the angles formed by the hands of the clock will vary. However, pupils should realise that the size of the angle formed by the hour and minute hands on the clock can be worked out (calculated), as the distance between consecutive labelled numbers (i.e. 5 minute intervals) on the clock face are equal to 30°, and unlabelled divisions (i.e. 1 minute intervals) are equal to 6°.

What if?

Pupils' times, and sizes and names of the angles formed by the hands of the clock will vary.

Angle polygons (p.27)

Challenge

Pupils' quadrilaterals, and named and measured angles, will vary. However, pupils should realise that:

- the interior angles of a quadrilateral total 360°
- all angles in a square and rectangle are right angles
- opposite angles in a parallelogram and a rhombus are equal.

What if?

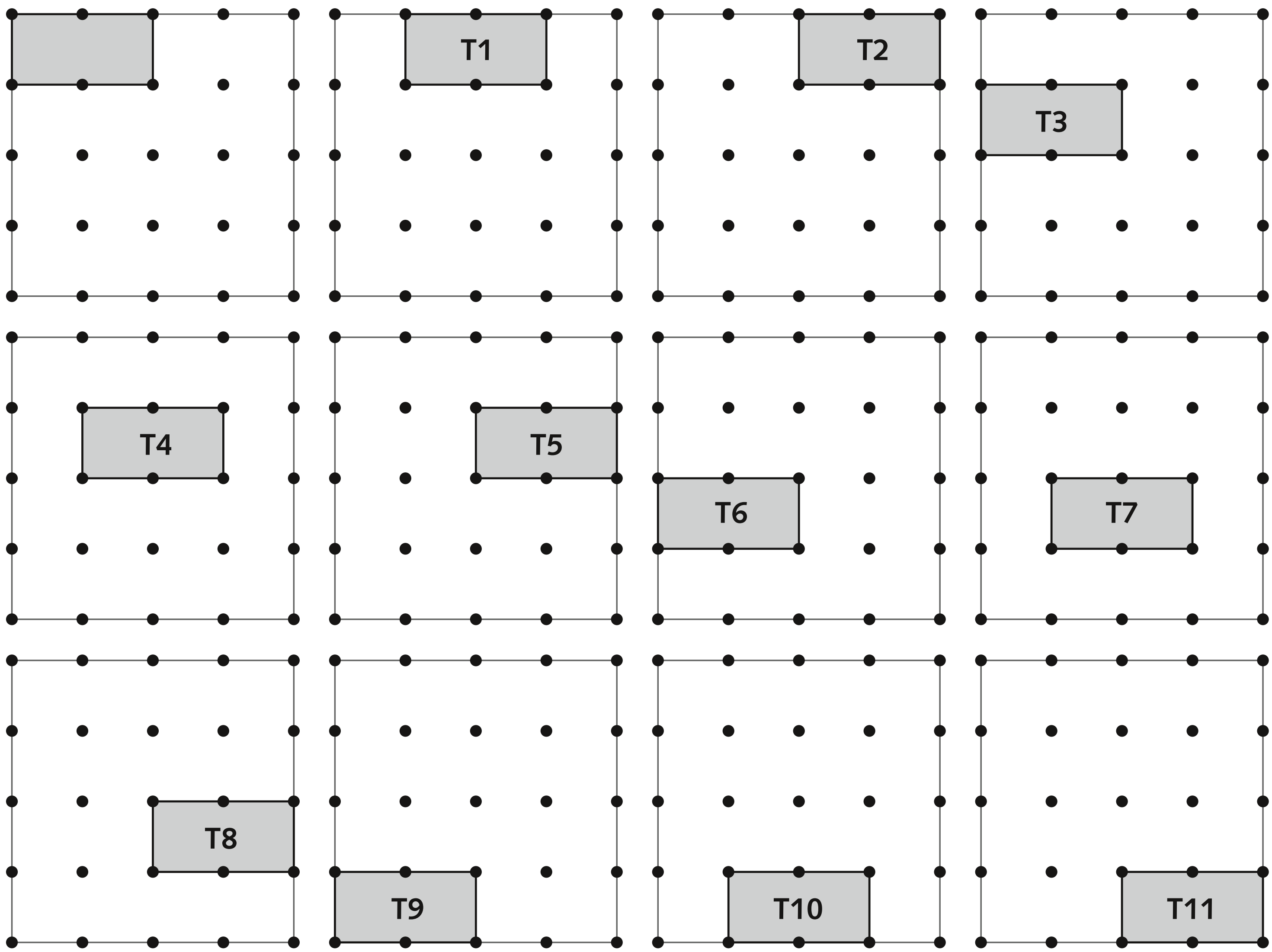
Pupils' triangles and other polygons, and named and measured angles, will vary. However, pupils should realise that:

- the interior angles of a triangle total 180°
- the interior angles of a pentagon total 540°
- the interior angles of a hexagon total 720°
- the interior angles of an octagon total 1080°.

Translating rectilinear shapes (p.28)

Challenge

On a 4 by 4 grid there are 11 possible translations of a 1 by 2 rectangle.



There are 19 possible translations on a 5 by 5 grid.

There are 29 possible translations on a 6 by 6 grid.

Pupils' explanations will vary. However, they may include the following:

Size of grid	Number of translations	Difference between the number of translations
2 by 2 grid	1 translation	
		4
3 by 3 grid	5 translations	
		6
4 by 4 grid	11 translations	
		8
5 by 5 grid	19 translations	
		10
6 by 6 grid	29 translations	

The difference between the number of translations increases by 2 for each successive increase in the size of the square grid.

What if?

Results of the challenge will vary depending on the rectilinear shape chosen by the pupil.

Reflect and translate (p.29)

Challenge/What if?

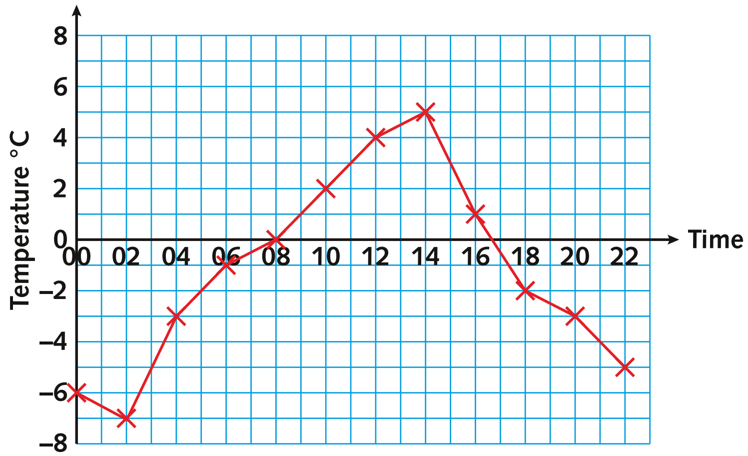
Pupils' designs and explanations will vary.

A winter's day in Thredbo Village (p.30)

Challenge

Pupils' statements will vary.

What if?



Pupils' statements will vary.

Timetables (p.31)

Challenge

The 13:23 train is the fastest train from Birmingham International to Bournemouth.

The 19:22 train is the slowest from Birmingham International to Bournemouth.

The average trip from Birmingham International to Bournemouth takes approximately 2 h 50 min.

The two consecutive train stations that are the furthest apart are Southampton Central and Bournemouth. This is because it takes longer to travel between these two consecutive stations than between any other two consecutive stations.

Pupils' statements will vary.

What if?

Pupils' statements will vary.

Reasoning mathematically

Count down (p.32)

Challenge

Isabel is correct – she will say –3 when she counts back from 12 in steps of 3.

Joshua is incorrect – he will not say –5 when he counts back from 12 in steps of 5.

Georgia is correct – she will say –4 when she counts back from 12 in steps of 4.

Pupils' reasoning and explanations will vary depending on their depth of understanding. However, they should make reference to the following. Use your professional judgment when assessing pupils' reasoning.

Both Isabel and Georgia are correct because their starting number, i.e. 12, is a multiple of the counting step that they are counting in, i.e. 3 and 4, and therefore as they count they will both include zero in their sequence, which means that the next number will be –3 and –4 respectively, i.e. 0 – 3 = –3 and 0 – 4 = –4.

Joshua's starting number, i.e. 12, is not a multiple of his counting steps, i.e. 5, which means he will not include zero in his sequence. The positive number closest to zero that Joshua will say is 2, and when counting back in steps of 5 from 2, the next number will be –3, i.e. 2 – 5 = –3.

What if?

When counting back in steps of 5 from 14, the sixth number that Isabel will say is –16.

When counting back in steps of 4 from 14, the sixth number that Joshua will say is –10.

When counting back in steps of 6 from 14, the sixth number that Georgia will say is –22.

Counting in 10s, 100s and 1000s (p.33)

Challenge

Counting forwards and backwards in steps of 10 from 3642, Isabel will say the following numbers:

– 8

12

642

902

8512

14 742

What if?

Counting forwards and backwards in steps of 100 from 64 781 Isabel will say the following numbers:

–19

381

4581

50 481

69 981

193 881

Counting forwards and backwards in steps of 1000 from 371 497 Isabel will say the following numbers:

–503

497

11 497

353 497

385 497

513 497

Roman numeral calculations (p.34)

Challenge

Pupils' working out and explanations will vary. Use your professional judgment when assessing pupils' reasoning.

A XI + XXIII

XI + XXIII = XXXIV

11 + 23 = 34

B LVI – XXV = XXXI

56 – 25 = 31

C XIII × VIII = CIV

13 × 8 = 104

D LXXII ÷ IX = VIII

72 ÷ 9 = 8

E $\frac{I}{V}$ of XLV = IX

$\frac{1}{5}$ of 45 = 9

What if?

Pupils' working out and explanations will vary. Use your professional judgment when assessing pupils' reasoning.

F DCXLVII + CCCLXXXV = MXXXII

647 + 385 = 1032

G MMMMDCLII – MCMXLVI = MMDCCVI

4652 – 1946 = 2706

H LXIII × LVIII = MMMDCLIV

63 × 58 = 3654

I MCMXLIV ÷ VI = CCCXXIV

1944 ÷ 6 = 324

J $\frac{V}{VIII}$ × XCVI = LX

$\frac{5}{8}$ of 96 = 60

Sort the calculations (p.35)

Challenge

56 + 38 = 94

560 + 380 = 940

3·8 + 5·6 = 9·4

121 – 35 = 86

86 + 35 = 121

12·1 – 8·6 = 3·5

10 – 3·83 = 6·17

6·17 + 3·83 = 10

38·3 + 61·7 = 100

12·67 + 5·82 = 18·49

1849 – 582 = 1267

1·267 + 0·582 = 1·849

Pupils' explanations will vary. Use your professional judgment when assessing pupils' reasoning.

What if?

Pupils' related calculations and explanations will vary.

What are the calculations? (p.36)

Challenge

Pupils' two addition and two subtraction calculations will vary. However, the calculations must contain two 4-digit numbers and the following red digits:

$$\begin{array}{r} 7\,6\,5\,7 \\ +\ 4\,8\,7\,5 \\ \hline 1\,2\,5\,3\,2 \\ \hline \end{array} \qquad \begin{array}{r} 6\,5\,3\,2 \\ -\ 2\,7\,0\,7 \\ \hline 3\,8\,2\,5 \\ \hline \end{array}$$

Pupils' explanations as to how they worked out the possible numbers will also vary. However, they should make mention of the use of inverse relationships. Use your professional judgment when assessing pupils' reasoning.

What if?

Pupils' two addition and two subtraction calculations will vary, as will their explanations. However, the calculations must contain two decimal numbers, each with two decimal places, and the following red digits:

$$\begin{array}{r} 4\,8 \cdot 1\,3 \\ +\ 3\,6 \cdot 5\,4 \\ \hline 8\,4 \cdot 6\,7 \\ \hline \end{array} \qquad \begin{array}{r} 8\,1 \cdot 5\,6 \\ -\ 2\,9 \cdot 6\,3 \\ \hline 5\,1 \cdot 9\,3 \\ \hline \end{array}$$

Pupils' explanations as to how they worked out the possible decimal numbers will also vary. However, they should make mention of the use of inverse relationships. Use your professional judgment when assessing pupils' reasoning.

Estimate the answer (p.37)

Challenge

£11 399 – £8254 = £3145

£4530 – £979 = £3551

£1265 + £1744 = £3009

£945 + £3050 = £3995

£16 750 – £12 999 = £3751

£1476 + £1899 = £3375

Pupils' explanations of how they used rounding will vary. Use your professional judgment when assessing pupils' reasoning.

What if?

The following problems have an answer that is between £41.25 and £41.75:

£62.75 – £21.39

£28.28 + £13.30

£75.11 – £33.39

£21.89 + £19.55

The answers to the six calculations are:

£62.75 – £21.39 = £41.36

£28.28 + £13.30 = £41.58

£75.11 – £33.39 = £41.72

£69.05 – £27.90 = £41.15

£25.36 + £16.44 = £41.80

£21.89 + £19.55 = £41.44

Pupils' explanations of how they used rounding will vary. Use your professional judgment when assessing pupils' reasoning.

Multiplying by 19, 21 and 25 (p.38)

Challenge

Pupils' strategies may vary. However, they may include the following:

If the chosen two-digit number is 32, for example, then:

- to multiply a number by 19: multiply the number by 20 and subtract the number you are multiplying, e.g. $19 \times 32 = 20 \times 32 = 640 - 32 = 608$
- to multiply a number by 21: multiply the number by 20 and add the number you are multiplying, e.g. $21 \times 32 = 20 \times 32 = 640 + 32 = 672$
- to multiply a number by 25: multiply the number by 20, multiply the number by 5 and add the two answers together, e.g. $25 \times 32 = (20 \times 32) + (5 \times 32) = 640 + 160 = 800$
  or multiply the number by 100, and then divide by 4, e.g. $32 \times 100 = 3200 \div 4 = 800$.

What if?

Pupils' adaptation of the strategies above may vary. However, they may include the following:

If the chosen two-digit number is 32, for example, then:

- to multiply a number by 29: multiply the number by 30 and subtract the number you are multiplying, e.g. $29 \times 32 = 30 \times 32 = 960 - 32 = 928$
- to multiply a number by 31: multiply the number by 30 and add the number you are multiplying, e.g. $31 \times 32 = 30 \times 32 = 960 + 32 = 992$
- to multiply a number by 35: multiply the number by 30, multiply the number by 5 and add the two answers together, e.g. $35 \times 32 = (30 \times 32) + (5 \times 32) = 960 + 160 = 1120$.

Divisible by 11? (p.39)

Challenge

Isabel's statement is sometimes true. When you reverse the digits of a 2-digit number to make a new number and find the total of the two numbers, the answer is always divisible by 11, for example:

$87 + 78 = 165$

$165 \div 11 = 15$

$49 + 94 = 143$

$143 \div 11 = 13$

However, this does not apply for 2-digit numbers that are multiples of 10, for example 50 and 80.

This only applies to numbers with an even number of digits, i.e. pairs of 2-digit, 4-digit, 6-digit, … numbers.

What if?

When you reverse the digits of any 2-digit number to make a new number and find the difference between the two numbers, the answer is always divisible by 9.

63 – 36 = 27	91 – 19 = 72
27 ÷ 9 = 3	72 ÷ 9 = 8

Alphabet numerals (p.40)

Challenge

0	1	2	3	4	5	6	7	8	9
J	C	F	A	H	D	G	B	E	I

$$\begin{array}{r} 3\,7\,8\,4 \\ \times \quad\quad 6 \\ \hline 2\,2\,7\,0\,4 \\ \hline \end{array} \qquad \begin{array}{r} 2\,5\,9 \\ \times \quad 6\,7 \\ \hline 1\,5\,5\,4\,0 \\ 1\,8\,1\,3 \\ \hline 1\,7\,3\,5\,3 \\ \hline \end{array} \qquad \begin{array}{r} 4\,6 \\ 8\,\overline{)\,3\,6\,8} \end{array}$$

Pupils' explanations will vary.

What if?

0	1	2	3	4	5
V	Y	U	X	Z	W

$$\begin{array}{r} 3\,0\,1 \\ \times \quad 1\,5 \\ \hline 3\,0\,1\,0 \\ 1\,5\,0\,5 \\ \hline 4\,5\,1\,5 \\ \hline \end{array} \qquad \begin{array}{r} 4\,5 \\ 3\,\overline{)\,1\,3\,5} \end{array} \qquad \begin{array}{l} \quad\;\; 5\,3 \text{ r } 3 \\ 4\,\overline{)\,2\,1\,5\quad\;} \end{array}$$

Pupils' explanations will vary.

Multiplying fractions (p.41)

Challenge

Xavier, Georgia and Joshua are all correct: $\frac{3}{5} \times 9$ is equal to $5\frac{2}{5}$, $\frac{27}{5}$ and $5\frac{4}{10}$, as all three are equivalent fractions. They are just expressed differently, i.e.

$5\frac{2}{5}$: mixed number with the fraction reduced to its simplest form

$\frac{27}{5}$: improper fraction

$5\frac{4}{10}$: mixed number with the fraction not reduced to its simplest form.

Pupils' reasoning and explanations will vary. Use your professional judgment when assessing pupils' reasoning.

What if?

Pupils' calculations and explanations will vary.

Equivalent to 4·532 (p.42)

Challenge

The following expressions are equivalent to 4·532:

$4 + \frac{5}{10} + \frac{3}{100} + \frac{2}{1000}$

4 ones, 5 tenths, 3 hundredths and 2 thousandths

4 + 0·4 + 0·13 + 0·002

4 + 0·5 + 0·03 + 0·002

$4 + \frac{5}{10} + \frac{2}{100} + \frac{12}{1000}$

3 ones, 15 tenths, 3 hundredths and 2 thousandths

Pupils' expressions will vary.

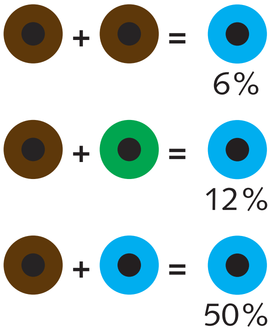
What if?

Pupils' expressions will vary.

Percentage eyes (p.43)

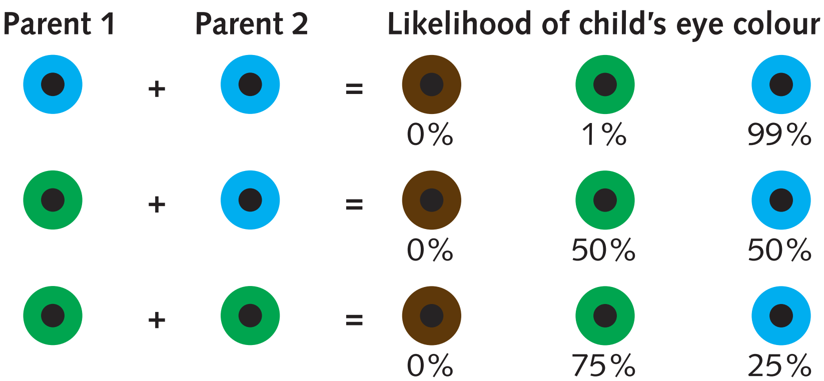
Challenge

Likelihood of child's eye colour being blue:



Pupils' explanations will vary. However, pupils should make mention of the fact that as per cent refers to the number of parts per hundred, in order to work out the likelihood of blue eyes they need to add the percentage given for brown eyes and the percentage given for green eyes then subtract this total from 100 in order to work out the percentage of blue eyes.

What if?



Pupils' explanations will vary.

Making comparisons (p.44)

Challenge

Pupils' statements will vary.

What if?

The order of the nine fractions, smallest first, is:

$\frac{3}{100}, \frac{17}{100}, \frac{3}{10}, \frac{1}{3}, \frac{2}{5}, \frac{11}{20}, \frac{17}{25}, \frac{7}{10}, \frac{99}{100}$

The order of the nine decimals, smallest first, is:

0·09, 0·14, 0·28, 0·35, 0·41, 0·67, 0·73, 0·8, 0·92

The order of the nine percentages, smallest first, is:

7%, 12%, 20%, 39%, 43%, 58%, 76%, 85%, 91%

Pupils' explanations will vary.

The order of the 27 fractions, decimals and percentages, smallest first, is:

$\frac{3}{100}$, 7%, 0·09, 12%, 0·14, $\frac{17}{100}$, 20%, 0·28, $\frac{3}{10}$, $\frac{1}{3}$, 0·35, 39%, $\frac{2}{5}$, 0·41, 43%, $\frac{11}{20}$, 58%, 0·67, $\frac{17}{25}$, $\frac{7}{10}$, 0·73, 76%, 0·8, 85%, 91%, 0·92, $\frac{99}{100}$

Pupils' explanations will vary.

Less than $\frac{1}{4}$: $\frac{3}{100}$, 7%, 0·09, 12%, 0·14, $\frac{17}{100}$, 20%

Between 25% and 0·5: 0·28, $\frac{3}{10}$, $\frac{1}{3}$, 0·35, 39%, $\frac{2}{5}$, 0·41, 43%

Between $\frac{1}{2}$ and 75%: $\frac{11}{20}$, 58%, 0·67, $\frac{17}{25}$, $\frac{7}{10}$, 0·73

Greater than 0·75: 76%, 0·8, 85%, 91%, 0·92, $\frac{99}{100}$

Sort and compare measures (p.45)

Challenge

Length measurements: 1750 m, 18 cm, 20 cm, $3\frac{3}{5}$ cm, 40 cm, 190 mm, 1·7 km, 38 mm, $\frac{2}{5}$ m, 0·2 m

Mass measurements: 0·4 kg, 1·2 kg, 1200 g, 0·25 kg, $\frac{1}{4}$ kg, $\frac{4}{5}$ kg, 600 g

Volume and capacity measurements: $\frac{1}{3}$ litre, 500 ml, 300 ml, 0·75 litre, 0·5 litre, $\frac{3}{4}$ litre, 0·8 litre, $1\frac{1}{4}$ litre

Pupils' statements and explanations will vary.

What if?

The order of each group of measurements, starting with the smallest, is:

Length measurements: $3\frac{3}{5}$ cm, 38 mm, 18 cm, 190 mm, (0·2 m and 20 cm), (40 cm and $\frac{2}{5}$ m), 1·7 km, 1750 m

Mass measurements: ($\frac{1}{4}$ kg and 0·25 kg), 0·4 kg, 600 g, $\frac{4}{5}$ kg, (1200 g and 1·2 kg)

Volume and capacity measurements: 300 ml, $\frac{1}{3}$ litre, (500 ml and 0·5 litre), ($\frac{3}{4}$ litre and 0·75 litre), 0·8 litre, $1\frac{1}{4}$ litre

The same and different (p.46)

Challenge

Pupils' criteria for sorting the measurements into groups that are the same and groups that are different will vary. However, they may include the following:

Metric measurements: kilometre, metre, centimetre, kilogram, gram, litre, millilitre

Imperial measurements: mile, yard, foot, inch, ounce, pound, pint, gallon

Length measurements: kilometre, metre, centimetre, mile, yard, foot, inch

Mass measurements: kilogram, gram, ounce, pound

Capacity/volume measurements: litre, millilitre, pint, gallon

What if?

The following are the cheaper item in each pair:

Carrots: £2 per pound (lb)

Milk: £1.50 per pint

Curtain fabric: £7 per metre

Pupils' explanations will vary, but may include mention of the approximate equivalences between metric and imperial units, for example:

1 pound (lb) = 0·45 kg / 1 kg = 2·2 pounds (lb)

1 pint = 0·57 litre / 1 litre = 1·76 pints

1 yard = 0·91 metres / 1 metre = 1·09 yards

NOTE: Equivalences may differ depending on the degree of accuracy taught.

Squares inside squares (p.47)

Challenge

The length of one side of the **red** square is 3 cm.

The length of one side of the **blue** square is 5 cm.

The area of the square that is shaded **blue** and **red** is 25 cm^2.

The perimeter of the **blue** square is 20 cm.

The length of one side of the **yellow** square is 7 cm.

The area of the square that is shaded **yellow**, **blue** and **red** is 49 cm^2.

The perimeter of the **yellow** square is 28 cm.

The length of one side of the **green** square is 9 cm.

The area of the square that is shaded **green**, **yellow**, **blue** and **red** is 81 cm^2.

The perimeter of the **green** square is 36 cm.

Pupils' explanations will vary.

What if?

The area of the square that is shaded **blue** only is 16 cm^2.

The area of the square that is shaded **yellow** only is 24 cm^2.

The area of the square that is shaded **green** only is 32 cm^2.

Pupils' explanations will vary.

Descendants (p.48)

Challenge

Daphne: (1920), Lorna: (1940), Joan: (1943), Valda: (1948)

Ray: (1965), Wayne: (1967), Johnny: (1968), Jimmy: (1973), Marella: (1978), Peter: (1980)

Alan: (1987), Daniel: (1989), Michael: (1992), Darren: (1994), Justin: (1995), Aaron: (1996), Allison: (1998), Christopher: (1999), Jessie: (2000), Blaze: (2005), Craig: (2007)

Taylah: (2013), Chloe: (2015), Lewis and Ryan: (2016), Harry: (2017)

Pupils' explanations will vary.

What if?

Pupils' answers and explanations will vary.

NOTE:

Great Britain and France both declared war on Germany on 3rd September, 1939.

The first moon landing occurred on 20th July 1969.

What's the new price? (p.49)

Challenge

Joshua has worked out the correct new price of £30.

What if?

Dave's Discounts is selling is selling Roger Robot for the lowest price at £16.

Dave's Discounts: Was £32.

With 50% discount the new price is £16 because 50% of £32 is £16 and £32 – £16 = £16.

Robot World: Was £24.

With 25% discount the new price is £18 because 25% of £24 is £6 and £24 – £6 = £18.

Everything Remote: £20.

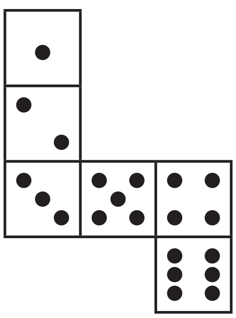
Spot the fake dice (p.50)

Challenge

The following are fake dice:

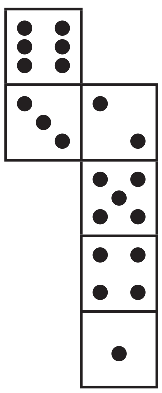
C

(This net does not create a closed cube.)



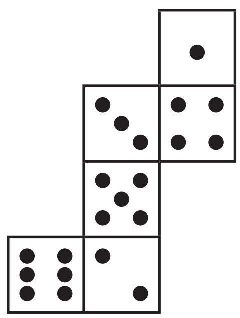
E

(This net does not create a closed cube.)



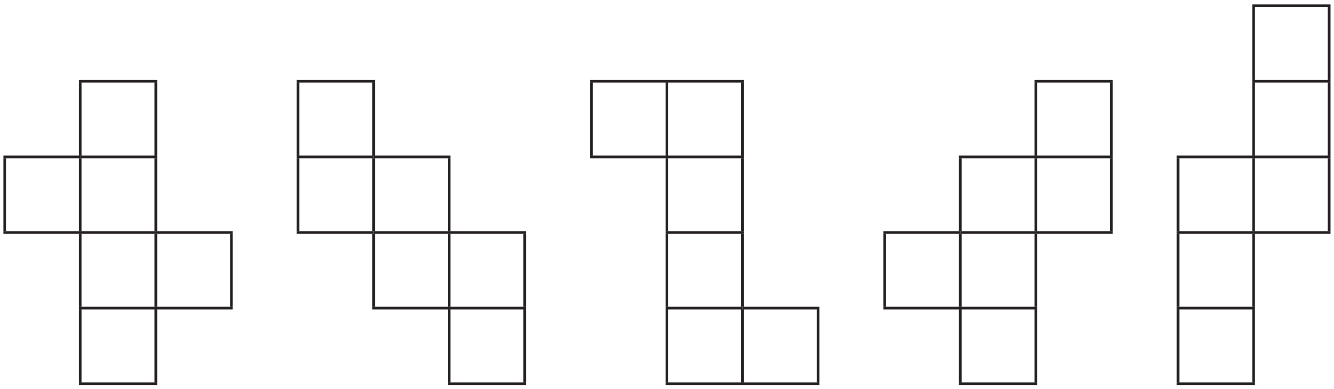
G

(This net does create a closed cube, but the dots do not make a 1–6 dice.)



What if?

Pupils' arrangements of the dots will vary but the net should be one of the following:



Match the missing angle (p.51)

Challenge

Isabel calculated angle c.

Joshua calculated angle b.

Georgia calculated angle a.

Xavier calculated angle d.

Pupils' reasoning will vary, but should make mention of the following:

- Angles in one whole turn total 360°.
- Angles on a straight line total 180°.

Use your professional judgment when assessing pupils' reasoning.

What if?

Isabel calculated an acute angle.

Joshua calculated an obtuse angle.

Georgia calculated a reflex angle.

Xavier calculated obtuse angle.

Compass directions (p.52)

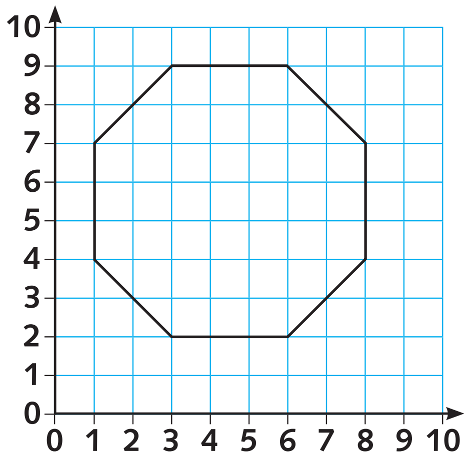
Challenge/What if?

Pupils' statements will vary.

Spot the symmetrical octagon (p.53)

Challenge

Georgia drew the only symmetrical octagon.



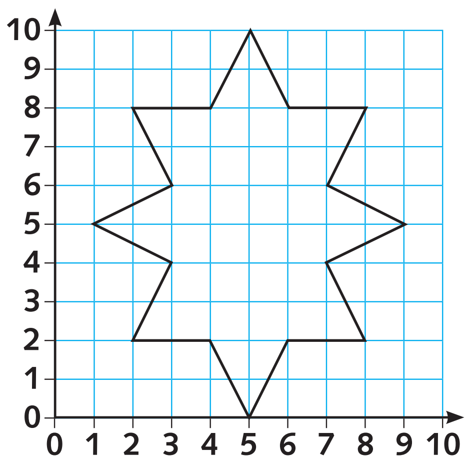
Isabel and Joshua drew octagons but they were irregular and not symmetrical.

Xavier drew a heptagon.

What if?

The coordinates that Joshua used to draw his 8-pointed star with both vertical and horizontal symmetry are:

(5, 10) (6, 8) (8, 8) (7, 6) (9, 5) (7, 4) (8, 2) (6, 2) (5, 0) (4, 2) (2, 2) (3, 4) (1, 5) (3, 6) (2, 8) (4, 8)



A day on each of the planets (p.54)

Challenge

Pupils' statements will vary. However, they should make statements that describe the length of a day on the various planets, for example: 'The length of one day on Jupiter is 10 hours.' And they should make statements comparing the lengths of a day on various planets, for example: 'A day on Mars is 1 hour longer than a day on Earth.'

What if?

Assuming pupils spend 6 hours at school a day, which is $\frac{1}{4}$ of a day, then the equivalent time they would spend at school on each of the other planets is:

6 h 15 m on Mars

2 h 30 m Jupiter

2 h 45 m on Saturn

4 h 15 m on Uranus

4 h on Neptune

350 h on Mercury

1462 h on Venus

Pupils' other approximations will vary depending on the tasks chosen.

The temperature of the planets (p.55)

Pupils' statements will vary. However, they should make statements that describe the average temperature on each of the planets in our solar system, for example: 'The average temperature on Saturn is approximately –150°C.' And they should make statements comparing the temperatures on various planets, for example: 'Uranus and Neptune have similar average temperatures.'

NOTE: Pupils' reading of the graph may differ due to the scale used on the graph.

What if?

Pupils' statements will vary. However, they may include the following:

The difference between the minimum and maximum temperatures on Earth is 160°C.

The difference between the minimum and maximum temperatures on Mars is 173°C.

The difference between the minimum and maximum temperatures on Mercury is 649°C.

Using and applying mathematics in real-world contexts

Your own number system (p.56)

Challenge/What if?

Results of the challenge will vary.

Internet (p.57)

Challenge/What if?

Results of the challenge will vary.

Temperature differences (p.58)

Challenge

Pupils' answers will vary, but may include the following:

The difference between the average minimum and average maximum temperatures in:

- Washington DC is 34°C
- Ottawa is 43°C
- Moscow is 39°C
- Berlin is 27°C
- Stockholm is 27°C.

The difference between the average minimum temperatures in:

- Washington DC and Ottawa is 13°C
- Washington DC and Moscow is 13°C
- Washington DC and Berlin is 0°C
- Washington DC and Stockholm is 2°C
- Ottawa and Moscow is 0°C
- Ottawa and Berlin is 13°C
- Ottawa and Stockholm is 11°C
- Moscow and Berlin is 13°C

- Moscow and Stockholm is 11°C
- Berlin and Stockholm is 2°C.

The difference between the average maximum temperatures in:

- Washington DC and Ottawa is 4°C.
- Washington DC and Moscow is 8°C.
- Washington DC and Berlin is 7°C.
- Washington DC and Stockholm is 9°C.
- Ottawa and Moscow is 4°C
- Ottawa and Berlin is 3°C
- Ottawa and Stockholm is 5°C
- Moscow and Berlin is 1°C
- Moscow and Stockholm is 1°C
- Berlin and Stockholm is 2°C.

What if?

The city with the greatest temperature difference is Ottawa.

You can tell this without having to do any calculations by comparing the height of the mercury (red shading) in the two thermometers. The two thermometers for Ottawa show the greatest difference in height.

Different nationalities (p.59)

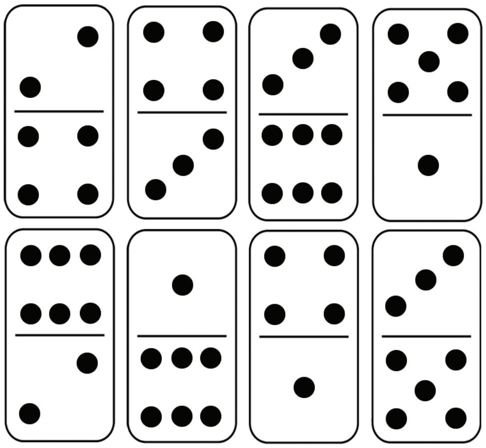
Challenge/What if?

Results of the challenge will vary.

Domino magic square (p.60)

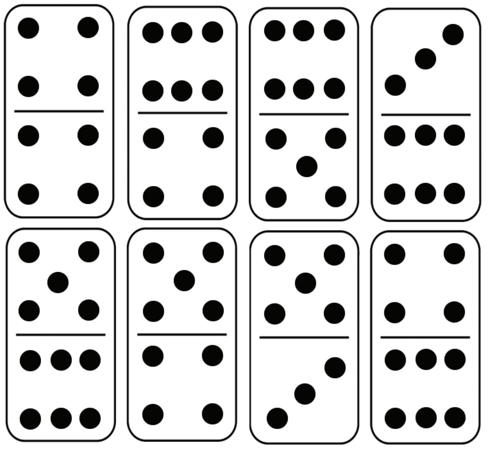
Challenge

Many different magic squares can be made using eight dominoes, for example:



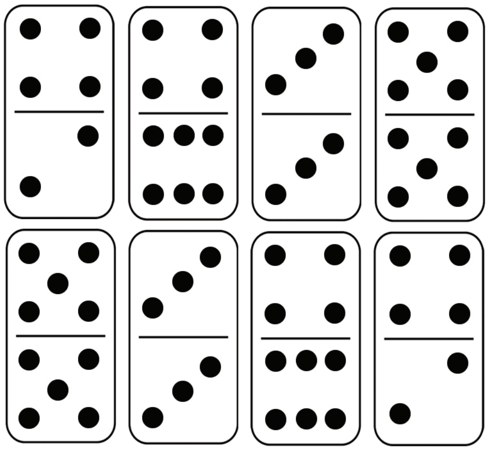
Magic number: 14

(This magic square uses eight different dominoes).



Magic number: 16

(This magic square uses two 6 × 4 dominoes, two 5 × 5 dominoes, two 4 × 2 dominoes and two 3 × 3 dominoes).



Magic number: 19

(This magic square uses two 6 × 4 dominoes and two 6 × 5 dominoes).

What if?

Pupils' magic squares will vary.

Domino calculations (p.61)

Challenge/What if?

Pupils' calculations will vary.

Who am I? (p.62)

Challenge/What if?

Results of the challenge will vary.

Burning calories (p.63)

Challenge/What if?

Results of the challenge will vary.

The triple jump (p.64)

Challenge/What if?

Results of the challenge will vary.

You be the architect (p.65)

Challenge/What if?

Results of the challenge will vary.

News (p.66)

Challenge/What if?

Results of the challenge will vary.

Bath time (p.67)

Challenge

Pupils' answers will vary. However, assuming that pupils are 11 years old, that they have had a bath or shower every day of their life, and that on average the total amount of time to take a bath or shower is half an hour, then most pupils will have spent approximately 84 days in the bath or shower:

11 years old × 365 days = 4015 days (the approximate number of days a pupil has been alive)

4015 days ÷ 48 hours (the number of $\frac{1}{2}$ hours in a day) ≈ 84 days

What if?

Pupils' answers will vary. However, assuming that Mr Menel is 24 years old, that he has had a bath or shower every day of his life, and that on average the total amount of time to take a bath or shower is half an hour, then Mr Menel could have spent 180 days in the bath or shower:

24 years old × 365 days = 8760 days (the approximate number of days Mr Menel has been alive)

8760 days ÷ 48 hours (the number of $\frac{1}{2}$ hours in a day) ≈ 180 days

Shopping (p.68)

Challenge/What if?

Results of the challenge will vary.

Shopping for the family (p.69)

Challenge/What if?

Results of the challenge will vary.

Plants in the playground (p.70)

Challenge/What if?

Results of the challenge will vary.

Mobile phones (p.71)

Challenge/What if?

Results of the challenge will vary.

Local currencies (p.72)

Challenge/What if?

Results of the challenge will vary.

Exchange rates (p.73)

Challenge/What if?

Results of the challenge will vary.

Walking around the school (p.74)

Challenge/What if?

Results of the challenge will vary.

All stand (p.75)

Challenge/What if?

Results of the challenge will vary.

Painting (p.76)

Challenge/What if?

Results of the challenge will vary.

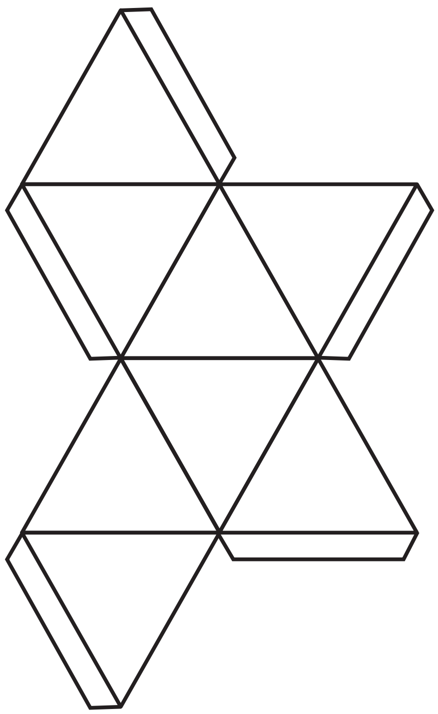
Making an octahedron (p.77)

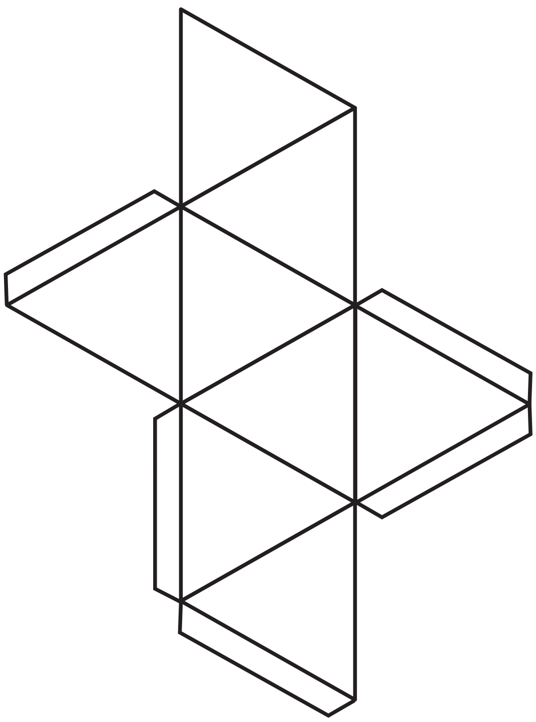
Challenge

Check Pupils' constructed octahedrons.

What if?

Other nets of an octahedron include:





Find the places (p.78)

Challenge/What if?

Results of the challenge will vary.

Your favourite things (p.79)

Challenge/What if?

Results of the challenge will vary.