Area and Volume 1

You must be able to:

- Recall and use the formulae for the circumference and area of a circle
- Recall and use the formula for the area of a trapezium
- Recall and use the formulae for the volume and surface area of a prism
- Recall and use the formulae for the volume and surface area of a cylinder.

Circumference and Area of a Circle

LEARN

The circumference of a circle is: $C = 2\pi r$ or $C = \pi d$

The area of a circle is: $A = \pi r^2$

Find the circumference and area of a circle with radius 9cm. Give your answer to 1 decimal place.

Circumference

$C = 2 \times \pi \times 9$

$= 18 \times \pi$

$= 56.5\text{cm}$ (to 1 d.p.)

Area

$A = \pi \times 9^2$

$= \pi \times 81$

$= 254.5\text{cm}^2$ (to 1 d.p.)

Key Point

The symbol π represents the number **pi**.

π can be approximated to 3.14 or $\frac{22}{7}$

Area of a Trapezium

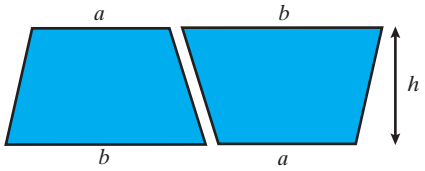
LEARN

The area of a **trapezium** is:

$$A = \tfrac{1}{2}(a + b)h$$

Where the sides labelled a and b are the **parallel** sides and h is the **perpendicular** height

- This formula can be proved:

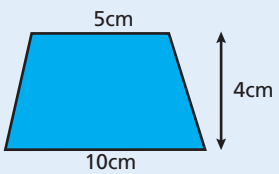


  - Two identical trapeziums fit together to make a parallelogram with base $a + b$ and height h.
  - The area of the parallelogram is $(a + b)h$.
  - Therefore the area of each trapezium is $\frac{1}{2}(a + b)h$.

Work out the area of the trapezium.

The area $= \frac{1}{2} \times (5 + 10) \times 4$

$= 30\text{cm}^2$



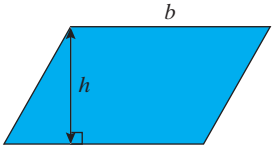
Key Point

Perpendicular means 'at right angles'.

Parallel means 'in the same direction and continuously having the same space between'.

Key Point

The area of a parallelogram is: $A = bh$



Volume of a Prism

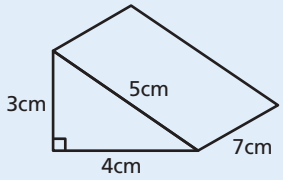
- A right prism is a 3D shape that has the same **cross-section** running all the way through it.

LEARN

Volume of a Prism = Area of Cross-section × Length

- The surface area is the sum of the areas of all the **faces**.

Work out the volume and surface area of the triangular prism.



Volume

Area of the cross-section
$= \frac{1}{2} \times 3 \times 4 = 6\text{cm}^2$
Volume $= 6 \times 7$
$= 42\text{cm}^3$

Surface Area

Five faces:
Two triangular faces: 6 + 6 = 12
Base: 4 × 7 = 28
Side: 3 × 7 = 21
Slanted side: 5 × 7 = 35
Total surface area:
12 + 28 + 21 + 35 = 96cm²

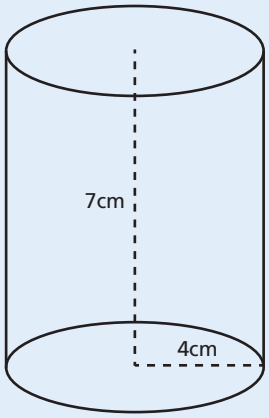
Cylinders

LEARN

Volume of a Cylinder = $\pi r^2 h$

Surface Area of a Cylinder = $2\pi rh + 2\pi r^2$

Work out the volume and surface area of the cylinder. Give your answers in terms of π.



Volume

$V = \pi \times 4^2 \times 7$
$= 112\pi\text{cm}^3$

Surface Area

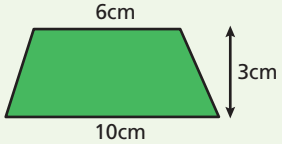
$SA = 2 \times \pi \times 4 \times 7 + 2 \times \pi \times 4^2$
$= 56\pi + 32\pi$
$= 88\pi\text{cm}^2$

Key Point

A cylinder is just like any other right prism. So to find the volume, you multiply the area of the cross-section (circular face) by the height of the cylinder.

Quick Test

1. Calculate the volume and surface area of a cylinder with radius 4cm and height 6cm.
2. Work out the area of the trapezium.



3. Calculate the circumference and area of a circle, diameter 7cm.

Key Words

trapezium
parallel
perpendicular
cross-section
face

Practice Questions

Ratio and Proportion

1. Simplify 5g : 10kg [1]

2. The angles in a triangle are in the ratio of 2 : 3 : 4

   What is the size of the largest angle? [2]

3. Six sticks of celery are needed to make celery soup for four people.

   How many sticks of celery would be needed to make soup for 14 people? [2]

4. It took six people four days to build a wall.

   a) Working at the same rate, how long would it have taken eight people to build the wall? [2]

   b) Working at the same rate, how many people would have been needed if the wall had to be completed in two days? [1]

Total Marks ..... / 8

Variation and Compound Measures

1. A bar of lead has a volume of 400cm^3 and a mass of 4.56kg.

   Work out the density of the bar of lead in g/cm^3. [2]

2. A rabbit runs 200 metres in 22 seconds.

   What is the rabbit's average speed in m/s?
   Give your answer to 2 decimal places. [2]

3. Khalid left his home at 10am and went for a 15km run.
   He arrived back home at 1pm.

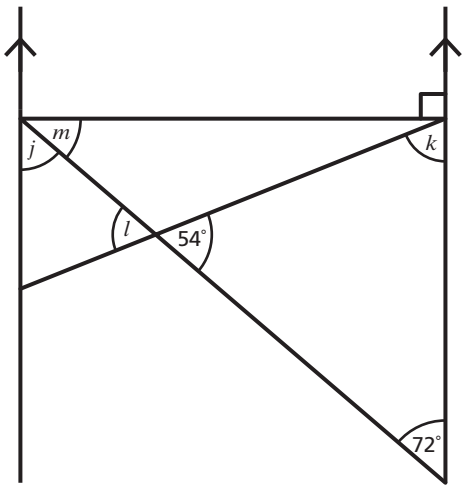
   What was his average speed in km/h? [2]

4. Work out the compound interest on £4000 invested at 4% for four years. [3]

Total Marks ..... / 9

Angles and Shapes 1 & 2

1 Work out the size of angles j, k, l and m, giving a reason for each answer.



[4]

2 $ABCD$ is a parallelogram.
AB is parallel to CD and AD is parallel to BC. Angle BAD = 110°

Work out

a) Angle DCB

b) Angle ABC [2]

3 The angles in a quadrilateral are x, $2.5x$, $3x$ and $2.5x$ degrees.

Calculate the size of the largest angle. [2]

4 Work out the interior angle of a regular decagon. [2]

5 A and B are two points.

If the bearing of B from A is 036°, what is the bearing of A from B? [1]

6 A regular polygon has an exterior angle of 45°.

a) Work out how many sides the polygon has. [1]

b) What is the name of the polygon? [1]

Total Marks ..... / 13

Review Questions

Ratio and Proportion

1 The square of the speed (v) at which a ball is thrown is directly proportional to the height (h) reached.
A ball thrown at a speed of 10 metres per second reaches a height of 5 metres.

Calculate the height reached by a ball thrown at a speed of 30 metres per second. [4]

2 £60 is divided in the ratio of 5 : 7

What is the difference in value between the two shares? [3]

3 Simplify 6.2 hours : 4 minutes [2]

Total Marks ..... / 9

Variation and Compound Measures

1 Martin is investing £400. He can choose between two ways of saving:

Type A – Simple interest at 6% per annum.

Type B – Compound interest at 5% per annum.

Which savings plan will give the better return after four years?
You **must** show your working. [4]

2 **a)** Calculate the distance travelled by a mouse moving at 1.5 metres per second for 1.5 seconds. [2]

b) Misty, a farm cat, can run at 11.25 miles per hour. Misty chases a mouse moving at 1.5 metres per second into a straight, plastic pipe.

If the mouse enters the pipe two seconds before the cat and the pipe is six metres long, would Misty catch the mouse before it escapes out of the other end? You must justify your answer.
Assume 5 miles = 8 kilometres. [3]

3 It takes three dogs 15 days to eat a large sack of dog biscuits.

How long would it take five dogs to eat the same size sack of dog biscuits at the same rate? [2]

Total Marks ..... / 11

Angles and Shapes 1 & 2

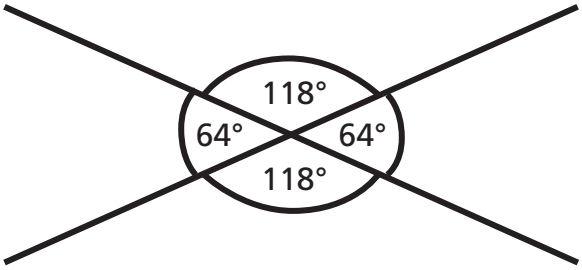
1 The three interior angles of a triangle are $y°$, $2y°$ and $3y°$.

Work out the size of the largest angle. [2]

2 A quadrilateral has one angle of 80°, one angle twice as big and one angle 20° smaller than the original angle.

Work out the size of the fourth angle. [2]

3 This angle diagram is incorrect. Explain why. [1]



4 An aircraft flies from airport A on a bearing of 054°. It arrives at airport B six hours later.

Work out the bearing that the aircraft must fly in order to return to airport A. [1]

5 Work out the exterior angle of a regular 15-sided shape. [2]

6 An irregular hexagon has interior angles of $2h$, $4h$, $4h$, $4h$, $5h$ and $5h$.

Work out the size of the smallest angle. [3]

7 State whether each of the following statements is **True** or **False**.

a) The sum of the interior angles of a heptagon is 900°. [1]

b) A parallelogram has no lines of symmetry and rotational symmetry of order 2. [1]

c) The direction south-east is on a bearing of 145°. [1]

Total Marks ..... / 14

Mixed Exam-Style Questions

28 As part of a health and safety review, a company surveys its employees to find out how many wear glasses or contact lenses.

	Male	Female
Glasses	9	6
Contact Lenses	8	16
Neither	20	15

a) Write down the ratio of the number of females who wear glasses to the number of females who wear contact lenses. Give your answer in its simplest form.

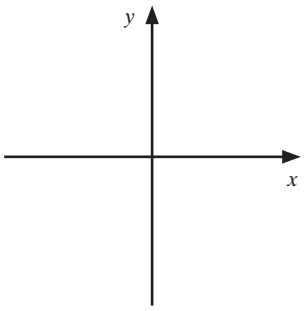
Answer _____ [1]

b) What percentage of all employees are male and do not wear glasses or contact lenses?

Answer _____ [2]

29 **a)** Sketch the graph of

$y = x^2 + 5x + 4$ [1]



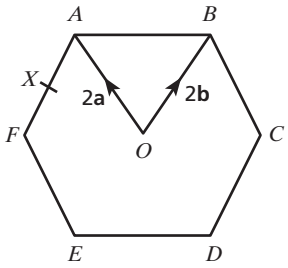
b) Using your graph from part **a)**, solve the inequality

$x^2 + 5x \leqslant -4$

Answer _____ [2]

c) On your graph, shade the region that represents the inequality in part **b)**. [1]

30 The diagram shows a regular hexagon $ABCDEF$ with centre O.



a) $\overrightarrow{OA} = 2\mathbf{a}$ and $\overrightarrow{OB} = 2\mathbf{b}$

Express in terms of **a** and/or **b**

i) $\overrightarrow{AB}$

Answer _____ [1]

ii) $\overrightarrow{EF}$

Answer _____ [1]

Total Marks ..... / 9

b) X is the midpoint of AF.

Express $\overrightarrow{DX}$ in terms of **a** and **b**.

Answer _____ [2]

c) Y is the point on BA extended, such that $BA : AY = 3 : 2$

Prove that D, X and Y lie on the same straight line.

Answer _____ [3]

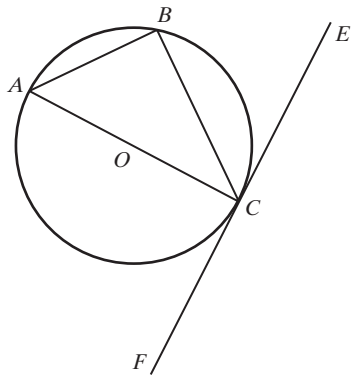
31 Solve the simultaneous equations.

$y = x^2 - 1$

$y = 3x + 3$

Answer _____ [4]

32 In the diagram, A, B and C are points on the circle, centre O.
Angle $BCE = 57°$
FE is a tangent to the circle at point C.



a) Calculate the size of angle ACB.
Give reasons for your answer.

Answer _____ [2]

Total Marks ..... / 11