# Contents



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Letts

GCSE

# Mathematics

Higher tier

# H

**Paper 1**

Time: 1 hour 30 minutes

**For this paper you must have:**

- mathematical instruments

You must **not** use a calculator.

**Instructions**

- Use black ink or black ball-point pen. Draw diagrams in pencil.
- Read each question carefully before you start to write your answer.
- Diagrams are **not** accurately drawn unless otherwise stated.
- Answer **all** questions.
- You must answer the questions in the space provided.
- In all calculations, show clearly how you work out your answer. Use a separate sheet of paper if needed. Marks may be given for a correct method even if the answer is wrong.

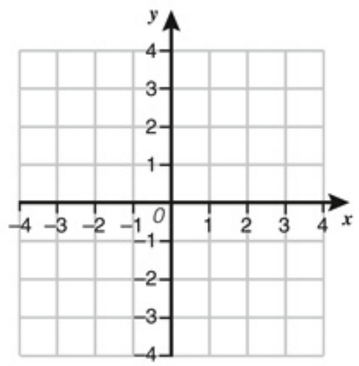
**Information**

- The marks for each question are shown in brackets.
- The maximum mark for this paper is 80.

**Name:** _____

**14.** Shade the region on the graph which satisfies the following inequalities:

$2y + x > 0$ $\quad x < 0$ $\quad y \leqslant 2$ **[3 marks]**

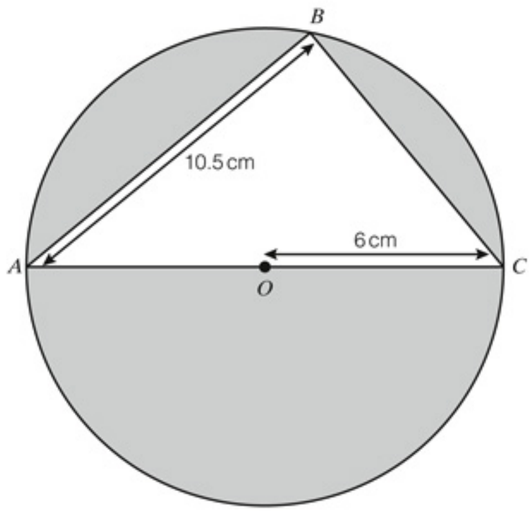



**15.** The diagram shows a triangle $ABC$ with each of the corners touching the circumference of a circle.

$AB$ = 10.5 cm

$OC$ = 6 cm

$AC$ is the diameter of the circle.

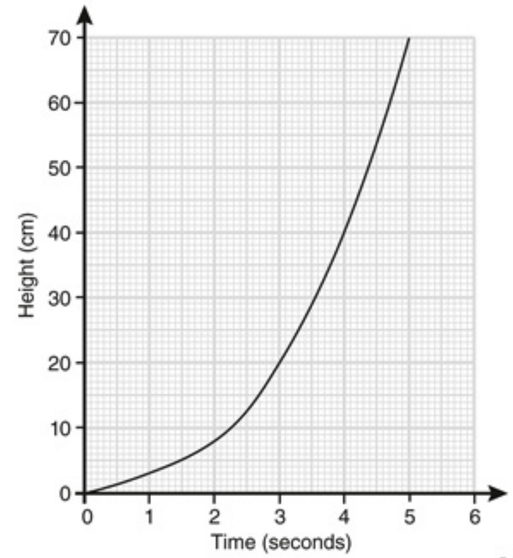



Find the area of the shaded section. **[5 marks]**

Answer ..... $cm^2$



**16.** The graph shows the height of water in a container as it is filled over a period of five seconds. The container is being filled at a constant rate.




**(a)** Draw a sketch of the container. **[1 mark]**

**(b)** Estimate the rate at which the height is increasing at exactly three seconds. **[2 marks]**

Answer ..... cm/s

**17.** Solve the equation $2x^2 + 8x = 3$

Give your answer to 3 decimal places. **[3 marks]**

Answer $x$ = ..... and $x$ = .....

**8.** Volume of ice cubes: $16 \times 2 \times 2 \times 2 = 128\,\text{cm}^3$ **[1]**

Volume of cylinder: $\pi \times 6^2 \times h$ **[1]**

$36\pi h = 128$ **[1]**

$h = \frac{128}{36\pi} = 1.13\,\text{cm}$ **[1]**

**9. (a)** (9, −4) **[1 mark for each correct value]**

Plus 2 in the $x$-direction, subtract 5 in the $y$-direction.

**(b)** For $y = mx + c$, gradient $m = \frac{\text{change in } y}{\text{change in } x} = \frac{-4-1}{9-7} = \frac{-5}{2}$ **[1]**

Substitute (7, 1) into the equation:

$y = \left(\frac{-5}{2}\right)x + c$

$1 = \left(\frac{-5}{2}\right) \times 7 + c$ **[1]**

$c = 1 + \left(\frac{35}{2}\right) = 18.5$

Equation of the line:

$y = \left(\frac{-5}{2}\right)x + 18.5$ or $2y = -5x + 37$ **[1]**

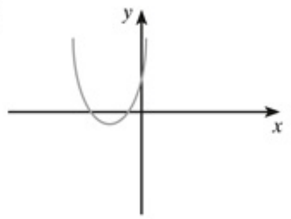
**10. (a)** £632000000 − £12500000 = £619500000 **[1]**

$£6.195 \times 10^8$ **[1]**

**(b)** $(£6.195 \times 10^8) \times (2.1 \times 10^{-3})$ **[1]** = £1300950 **[1]**

Divide $2.1 \times 10^{-1}$% by 100 to find the equivalent decimal $2.1 \times 10^{-3}$

**11. (a)**




**[2 marks if fully correct; 1 mark if graph intercepts positive $y$-axis once; 1 mark if the graph intercepts negative $x$-axis twice]**

When $x = 0$, $y = (0)^2 + 3(0) + 2 = 2$, giving the $y$-intercept as (0, 2). Solve the equation $x^2 + 3x + 2 = 0$ by factorising, $(x + 1)(x + 2) = 0$, giving the $x$-intercepts as (−2, 0) and (−1, 0).

**(b)** When $x = -1.5$: $y = (-1.5)^2 + 3(-1.5) + 2 = -0.25$

Turning point (−1.5, −0.25)

Alternatively, complete the square to get: $y = (x + 1.5)^2 - 0.25$

**12.** $\cos 62° = \frac{\text{adjacent}}{658}$ **[1]**

Adjacent = $658 \times \cos 62°$ **[1] =** 308.91 m **[1]**

The angle of elevation from the boat to the man is 62° since alternate angles are equal.

**13. (a)** $\frac{6^{7-2}}{6^5} = \frac{6^5}{6^5}$ **[1]** $= 6^{-1} = \frac{1}{6}$ **[1]**

**(b)** $(2^3)^x = 2^4$ **[1]**

$3x = 4$

$x = \frac{4}{3}$ **[1]**

Change 8 and 16 so that they have the same base number, 2.

**14.** $x = 0.136363636\ldots$

$10x = 1.363636363\ldots$

$1000x = 136.363636\ldots$ **[1 mark for any one of these]**

$990x = 135$ **[1]**

$x = \frac{135}{990}$ **[1]** $= \frac{3}{22}$

**15. (a)** Elliot or Wiki with a correct reason:

Elliot has the largest interquartile range or Wiki has the largest range.

**(b)** One correct statement **[1]** with correct value **[1]:**

Natasha and Wiki have the same median (5 mins), Niall and Natasha have the same range (6.5 mins) or Niall and Wiki have the same interquartile range (4 mins).

**16.** $x(5x - 4) = 2$ **[1]**

$5x^2 - 4x - 2 = 0$ **[1]**

$\frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 5 \times -2}}{2 \times 5}$ **[1]**

$\frac{4 \pm \sqrt{16 + 40}}{10} = \frac{4 \pm \sqrt{56}}{10}$ **[1]**

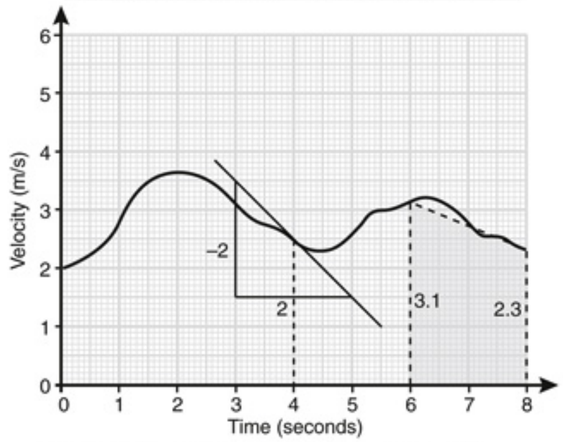
$x = 1.15$ and $x = -0.35$ **[1]**

Use the quadratic formula $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, when $ax^2 + bx + c = 0$

**17. (a)** Tangent drawn at 4 seconds **[1]**

Acceleration = $\frac{-2}{2}$ **[1]** $= -1\,\text{m/s}^2$ **[1] (accept −0.8 to −1.2)**

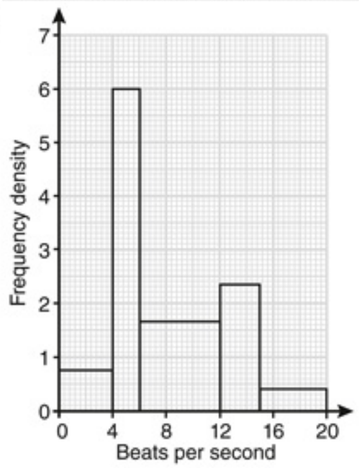
Calculate the gradient of the tangent drawn at 4 s.




**(b)** $\left(\frac{3.1 + 2.3}{2}\right) \times 2$ **[1]** = 5.4 m **[1] (accept 5.2 to 5.6)**

Estimate the area under the graph between 6 and 8 s by using the formula for the area of a trapezium.

**18. (a)**






| Beats per second | $0 < b \leqslant 4$ | $4 < b \leqslant 6$ | $6 < b \leqslant 12$ | $12 < b \leqslant 15$ | $15 < b \leqslant 20$ |
|---|---|---|---|---|---|
| Frequency | **3** | **12** | 10 | 7 | 2 |

**[2 marks for fully correct graph and frequencies; 1 mark for correct attempt to find the height of the last bar]**

Find the frequency density by dividing the frequency by the class width: $2 \div 5 = 0.4$, $7 \div 3 = 2.\dot{3}$, $10 \div 6 = 1.\dot{6}$. Label the frequency density axis and find the height of all of the bars. Find the area of the bars to find the missing frequencies.

**(b)** $\frac{3 + 12 + 10 + 7 + 2}{2} = \frac{34}{2} = 17$ **[1]** qualified

$6 + 1.2 = 7.2$ **[1]** beats per minute

The 17th person is three squares into the third bar since $2 \div 1.\dot{6} = 1.2$

**19.** $\frac{1}{2} \times 5 \times 8 \times \sin x = 10.15$ **[1]**

$x = \sin^{-1}(0.5075)$ **[1]**

$x = 30.5°$ **[1]**

Use the formula: Area of triangle = $\frac{1}{2}ab\sin C$.

**20.** $y = 18 - 2x$

$3x^2 + (18 - 2x)^2 = 139$ **[1]**

$3x^2 + 324 - 36x - 36x + 4x^2 = 139$ **[1]**

$7x^2 - 72x + 185 = 0$

$(7x - 37)(x - 5) = 0$ **[1]**

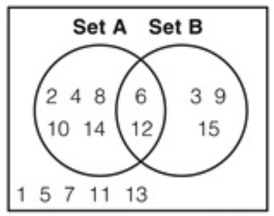
$x = \frac{37}{7}$ or $5\frac{2}{7}$ **[1]**

$y = 18 - 2 \times \frac{37}{7}$

$y = 7\frac{3}{7}$ **[1]**

Substitute the second equation into the first. Expand, simplify, rearrange and then factorise to solve the quadratic equation. Substitute $x$ back into one of the original equations to find $y$.

**21.** $\mathcal{E}$




**[3 marks if fully correct; deduct 1 mark for each error or omission]**

Even numbers in Set A. Multiples of 3 in Set B. Even multiples of 3 in the overlap. Any other numbers in the universal set go outside the circles.

**22.** P(three girls being picked)

$= \frac{13}{25} \times \frac{12}{24} \times \frac{11}{23}$ **[1]** $= \frac{1716}{13800}$ **[1]** $= \frac{143}{1150}$

P(at least one boy being picked) = 1 − P(three girls being picked)

$1 - \frac{143}{1150}$ **[1]** $= \frac{1007}{1150}$ **[1]**

You may find it useful to draw a tree diagram to show all eight possible outcomes.

**Paper 3 (pages 66–78)**

**1. (a)** False: a negative number cubed gives a negative answer, e.g. $-2^3 = -8$

**(b)** True: a negative or a positive number squared gives a positive answer, e.g. $(-2)^2 = 2^2 = 4$

**(c)** False: when $x = 2$, $2^2$ is equal to $2 \times 2$.

**2. (a)** $-3 \leqslant 2x < 6$ **[1]**

$-1.5 \leqslant x < 3$ **[1]**

Add 3 to each part, then divide by 2.

**(b)** −1, 0, 1, 2

**3. (a) Any one of the following reasons:**

There is no time period (in a week/month, etc.)

No option for 'never' visit

Two central tick boxes overlap

**(b)** Add a time period to the original question **[1]**

Have boxes which include '0' or 'never' and which do not overlap **[1]**

**4. (a)** B **(b)** A

**5. (a)** $\frac{38}{203}$ of 48752 $= \frac{40}{200} \times 50000$ **[1]** $= 10000$ **[1]**

Round each number to 1 significant figure.

**(b)** An over-estimate since $\frac{40}{200} > \frac{38}{203}$ and $50000 > 48752$ **[1]**

Assumption: A similar proportion of students from every school in England will not bring a calculator to the exam. **[1]**

**6.** Premium bonds: £15 × 2.5 × 3 **[1]** = £112.50 **[1]**

Local bank: £2500 × 1.01 × 1.015 × 1.02 **[1]** = £2614.13 **[1]**

Local bank is likely to give the best investment **[1]**

**7.** $180 - 8x = 32°$ **[1]**

$x = 18.5°$ **[1]**

Angle $ABC = 180 - 6x$

Angle $ABC = 180 - 6(18.5°)$ **[1]**

Angle $ABC = 69°$ **[1]**

Use the fact that there are two isosceles triangles which each add up to 180°.

**8. (a)** $18x + 8x + 11x + 3x = 1$ **[1]**

$40x = 1$

$x = 0.025$ **[1]**

$0.025 \times 11 = 0.275 = 27.5\%$ **[1]**

**(b)** $3(0.025) \times 80$ **[1]** = 6 people **[1]**

Multiply the probability for vegetarian by 80 people.

**9. (a)** $\frac{10\,\text{km}}{1.6}$ **[1]** 6.25 miles **[1]**

$6.25 \times 133$ **[1]** = 831.25 calories **[1]**

To convert miles into kilometres, you must know that 1 mile = 1.6 km

**(b)** Speed = $\frac{6.25}{1.25}$ **[1]** = 5 mph **[1]**

Answer should be B or D. Since his speed (5 mph) is much slower than his usual average speed, he must be running on difficult terrain. **[1]**

1 hour 15 minutes converts to 1.25 hours. Then use the formula: Speed = $\frac{\text{Distance}}{\text{Time}}$