CHAPTER 1 Algebra 1

This chapter is going to show you

- some simple number patterns that you may have seen before, and how to describe them
- how to create sequences and describe them in words
- how to generate and describe simple whole-number sequences

What you should already know

- Odd and even numbers
- Times tables up to 10 × 10

Sequences and rules

You can make up many different sequences with integers (whole numbers) using simple rules.

Example 1.1

Rule add 3 — Starting at 1 gives the sequence 1, 4, 7, 10, 13, …
Starting at 2 gives the sequence 2, 5, 8, 11, 14, …
Starting at 6 gives the sequence 6, 9, 12, 15, 18, …

Rule double — Starting at 1 gives the sequence 1, 2, 4, 8, 16, …
Starting at 3 gives the sequence 3, 6, 12, 24, 48, …
Starting at 5 gives the sequence 5, 10, 20, 40, 80, …

So you see, with *different* **rules** and *different* **starting points**, there are very many *different* **sequences** you may make.

The numbers in a sequence are called **terms** and the starting point is called the **1st term**. The rule is often referred to as the **term-to-term rule**.

Exercise 1A

1 Use each of the following term-to-term rules with the 1st terms **i** 1 and **ii** 5.

Create each sequence with 5 terms in it.

a	add 3	**b**	multiply by 3	**c**	add 5	**d**	multiply by 10
e	add 9	**f**	multiply by 5	**g**	add 7	**h**	multiply by 2
i	add 11	**j**	multiply by 4	**k**	add 8	**l**	add 105

2 Give the next two terms in each of these sequences. Describe the term-to-term rule you have used.

a	2, 4, 6, …	**b**	3, 6, 9, …	**c**	1, 10, 100, …	**d**	1, 2, 4, …
e	2, 10, 50, …	**f**	0, 7, 14, …	**g**	7, 10, 13, …	**h**	4, 9, 14 , …
i	4, 8, 12, …	**j**	9, 18, 27, …	**k**	12, 24, 36, …	**l**	2, 6, 18 , …

3 Give the next two terms in these sequences. Describe the term-to-term rule you have used.

a	50, 45, 40, 35, 30, …	**b**	35, 32, 29, 26, 23, …
c	64, 32, 16, 8, 4, …	**d**	3125, 625, 125, 25, 5, …
e	20, 19.3, 18.6, 17.9, 17.2, …	**f**	1000, 100, 10, 1, 0.1, …
g	10, 7, 4, 1, –2, …	**h**	27, 9, 3, 1, $\frac{1}{3}$, …

4 For each pair of numbers find at least two different sequences, writing the next two terms. Describe the term-to-term rule you have used.

a	1, 4, …	**b**	3, 7, …	**c**	2, 6, …
d	3, 6, …	**e**	4, 8, …	**f**	5, 15, …

5 Find two terms between each pair of numbers to form a sequence. Describe the term-to-term rule you have used.

a	1, …, …, 8	**b**	3, …, …, 12	**c**	5, …, …, 20
d	4, …, …, 10	**e**	80, …, …, 10	**f**	2, …, …, 54

Extension Work

1 Make up some of your own sequences and describe them.

2 Give your sequences to someone else and see if they can find out what your term-to-term rule is.

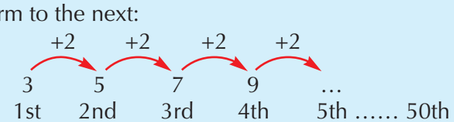
Finding missing terms

In any sequence, you will have a 1st term, 2nd term, 3rd term, 4th term and so on.

Example 1.2

In the sequence 3, 5, 7, 9, …, what is the 5th term, and what is the 50th term?

You first need to know what the term-to-term rule is. You can see that you add 2 from one term to the next:



To get to the 5th term, you add 2 to the 4th term, which gives 11.

To get to the 50th term, you will have to add on 2 a total of 49 times (50 – 1) to the first term, 3. This will give $3 + 2 \times 49 = 3 + 98 = 101$

Exercise 1B

1 In each of the following sequences, find the 5th and the 50th term.

a 4, 6, 8, 10, …
b 1, 6, 11, 16, …
c 3, 10, 17, 24, …
d 5, 8, 11, 14, …
e 1, 5, 9, 13, …
f 2, 10, 18, 26, …
g 20, 30, 40, 50, …
h 10, 19, 28, 37, …
i 3, 9, 15, 21, …

2 In each of the sequences below, find the 1st term, then find the 50th term.

In each case, you have been given the 4th, 5th and 6th terms.

a …, …, …, 13, 15, 17, …
b …, …, …, 18, 23, 28, …
c …, …, …, 19, 23, 27, …
d …, …, …, 32, 41, 50, …

3 In each of the following sequences, find the missing terms and the 50th term.

Term	1st	2nd	3rd	4th	5th	6th	7th	8th	50th
Sequence A	…	…	…	…	17	19	21	23	…
Sequence B	…	9	…	19	…	29	…	39	…
Sequence C	…	…	16	23	…	37	44	…	…
Sequence D	…	…	25	…	45	…	…	75	…
Sequence E	…	5	…	11	…	…	20	…	…
Sequence F	…	…	12	…	…	18	…	22	…

4 Find the 40th term in the sequence with the term-to-term rule ADD 5 and a 1st term of 6.

5 Find the 80th term in the sequence with the term-to-term rule ADD 4 and a 1st term of 9.

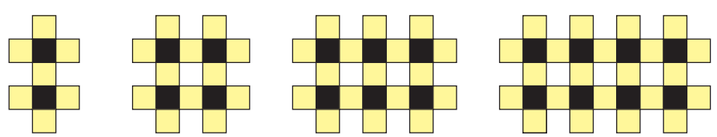
6 Find the100th term in the sequence with the term-to-term rule ADD 7 and 1st term of 1.

7 Find the 30th term in the sequence with the term-to-term rule ADD 11 and 1st term of 5.

Extension Work

1 You have a simple sequence where the 50th term is 349, the 51st is 354 and the 52nd is 359. Find the 1st term and the 100th term.

2 You are building patterns using black and yellow squares.



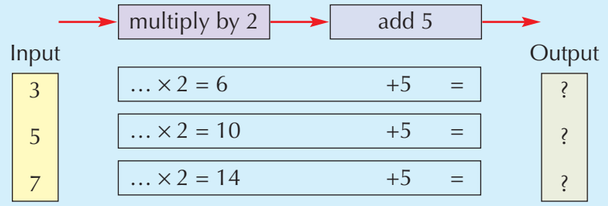
Pattern 1 **Pattern 2** **Pattern 3** **Pattern 4**

You have 50 black squares. How many yellow squares will be in the pattern?

Functions and mappings

Example 1.3

Complete the function machine to show the output.



The output box can be seen to be:

11
15
19

Exercise 1C

1 Complete the input and output for each of the following function machines.

a add 3

input	output
4	?
5	?
8	?
11	?

b subtract 2

input	output
4	?
5	?
8	?
11	?

c multiply by 5

input	output
4	?
5	?
8	?
11	?

d divide by 10

input	output
100	?
80	?
60	?
50	?

e add 4

input	output
3	?
?	9
8	?
?	15

f multiply by 3

input	output
4	?
?	18
8	?
?	36

2 Express each of these functions in words.

a

input	output
2	4
3	5
4	6
5	7

b

input	output
3	9
4	12
5	15
6	18

c

input	output
1	6
2	12
3	18
4	24

d

input	output
24	12
12	6
8	4
6	3

e

input	output
2	9
3	10
4	11
5	12

f

input	output
2	16
4	32
6	48
8	64

3 Fill in the missing values in the following double function machines.

a 4 → [× …] → 8 → [+ …] → 11

b 5 → [× …] → 15 → [− …] → 9

c 4 → [+ …] → 6 → [× …] → 18

d 3 → [+ …] → … → [× 2] → 18

4 Each of the following functions is made up from two operations, as above.

Find the **combined functions** in each case.

a

1	3
2	5
3	7
4	9

b

1	2
2	5
3	8
4	11

c

1	7
2	12
3	17
4	22

Extension Work

1 Work backwards from each output to find the input to each of the following functions.

a ?, ?, ?, ? → [× 3] → [+ 4] → 7, 13, 16, 25

b ?, ?, ?, ? → [+ 5] → [× 2] → 14, 16, 20, 26

c ?, ?, ?, ? → [× 4] → [− 3] → 9, 17, 33, 37

2 From the following single functions, see how many different combined functions you can make.

[× 3] [+ 4] [− 1] [× 2] [+ 5]

Using letter symbols to represent functions

Here is some algebra shorthand that is useful to know:

$2x$ means two multiplied by x
$2g$ means two multiplied by g
$5h$ means five multiplied by h

The idea of algebra is that we use a letter to represent a situation where we don't know a number (value) or where we know the value can vary (be lots of different numbers).

Each of the above is an **expression**. An expression is often a mixture of letters, numbers and signs. We call the letters **variables**, because the values they stand for vary.

For example, $3x$, $x + 5$, $2x + 7$ are expressions, and x is a variable in each case.

When the variable in an expression is a particular number, the expression has a particular value.

For example, in the expression $x + 6$, when $x = 4$, the expression has the value $4 + 6$, which is 10.

Example 1.4

Given that $n = 5$, write down the value of the following expressions.

a $n + 6$ **b** $4n$ **c** $n - 2$

In each case, substitute (replace) the letter n with the number 5.

a $n + 6 = 5 + 6 = 11$

b $4n = 4 \times 5 = 20$

c $n - 2 = 5 - 2 = 3$

Example 1.5

Draw mapping diagrams to illustrate each of the following functions.

a $x \to x + 5$ **b** $x \to 3x$ **c** $x \to 2x + 1$

In each case, substitute (replace) the letter n with the number 5.

a x → [+ 5] → $x + 5$

a x → [× 3] → $3x$

a x → [× 2] → $2x$ → [+ 1] → $2x + 1$

Exercise 1D

1 Write down what the expression [$n + 5$] is equal to when:

i $n = 3$ **ii** $n = 7$ **iii** $n = 10$ **iv** $n = 2$ **v** $n = 21$

2 Write down what the expression [$3n$] is equal to when:

i $n = 4$ **ii** $n = 8$ **iii** $n = 11$ **iv** $n = 5$ **v** $n = 22$

3 Write down what the expression [$x - 1$] is equal to when:

i $x = 8$ **ii** $x = 19$ **iii** $x = 100$ **iv** $x = 3$ **v** $x = 87$

4 Write each of the following rules in symbolic form: for example, $x \to x + 4$

a add 3 **b** multiply by 5 **c** subtract 2 **d** divide by 5

5 Draw mapping diagrams to illustrate each of the following functions.

a $x \to x + 2$ **b** $x \to 4x$ **c** $x \to x + 5$ **d** $x \to x - 3$

6 Express each of the following functions in symbols as in Question 4.

a

2	9
3	10
4	11
5	12

b

2	10
3	15
4	20
5	25

c

2	1
3	2
4	3
5	4

d

2	8
3	12
4	16
5	20

e

12	4
15	5
21	7
30	10

f

2	7
3	8
4	9
5	10

g

2	20
3	30
4	40
5	50

h

12	9
13	10
14	11
15	12

7 Draw mapping diagrams to illustrate each of these functions.

a $x \to 2x + 3$ **b** $x \to 3x - 2$ **c** $x \to 5x + 1$ **d** $x \to 10x - 3$

8 Describe each of the following mappings as functions in the symbolic form, as above.

a

1	1
2	3
3	5
4	7

b

1	7
2	11
3	15
4	19

c

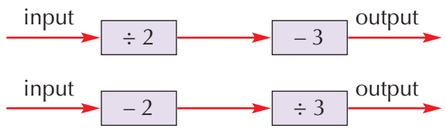
1	1
2	4
3	7
4	10

d

1	11
2	21
3	31
4	41

Extension **Work**

Put the same number through each of these function machines.



Repeat with other numbers.

Can you find an input that gives the same output for both function machines?

The general term (*n*th term)

We can describe sequences by giving a rule for any term. This is called the *n*th term and is an algebraic expression.

Example 1.6

The *n*th term of the sequence 7, 11, 15, 19, 23, … is given by the expression $4n + 3$.

a Show this is true for the first three terms.

b Use the rule to find the 50th term of the sequence.

a Let $n = 1$: $4 \times 1 + 3 = 4 + 3 = 7$
Let $n = 2$: $4 \times 2 + 3 = 8 + 3 = 11$
Let $n = 3$: $4 \times 3 + 3 = 12 + 3 = 15$

b Let $n = 50$: $4 \times 50 + 3 = 200 + 3 = 203$

So, the 50th term is 203.

Example 1.7

The *n*th term of a sequence is given by $3n - 1$.

a Find the first three terms of the sequence.

b Find the 60th term of the sequence.

a Let $n = 1$: $3 \times 1 - 1 = 3 - 1 = 2$
Let $n = 2$: $3 \times 2 - 1 = 6 - 1 = 5$
Let $n = 3$: $3 \times 3 - 1 = 9 - 1 = 8$

So, the first three terms are 2, 5, 8, ….

b Let $n = 60$: $3 \times 60 - 1 = 180 - 1 = 179$

So, the 60th term is 179.

Exercise 1E

1 Find **i** the first three terms and **ii** the 100th term, of sequences whose *n*th term is given by:

a $2n + 1$ **b** $4n - 1$ **c** $5n - 3$

d $3n + 2$ **e** $4n + 5$ **f** $10n + 1$

g $\frac{1}{2}n + 2$ **h** $7n - 1$ **i** $\frac{1}{2}n - \frac{1}{4}$

Extension **Work**

Find **i** the first three terms and **ii** the 100th term, of sequences whose *n*th term is given by:

a n^2 **b** $(n + 2)(n + 1)$ **c** $\frac{1}{2}n(n + 1)$

An *n*th term investigation

Activity **Work**

Here is a list of three sequences and their *n*th term.

4, 9, 14, 19, 24, … $5n - 1$

2, 6, 10, 14, 18, … $4n - 2$

8, 11, 14, 17, 20, … $3n + 5$

Make up at least three more *n*th terms of the form $an \pm b$, for example $2n + 5$, and work out the first five terms.

Copy and complete the table below using the three sequences above and the ones you made up.

Sequence			*n*th term	
Sequence	**Difference between terms**	**First term**	**Coefficient of *n***	**Constant term**
4, 9, 14, 19, 24	5	4	5	–1
2, 6, 10, 14, 18	4	2	4	–2
8, 11, 14, 17, 20				

1 What is the connection between the difference between the terms and the coefficient of *n*?

2 What is the connection between the difference between the terms, the first term and the constant term?

3 Without working out the terms of the sequences, match these sequences to the *n*th term expressions.

	Sequence	***n*th term**
a	3, 9, 15, 21, 27, …	$6n - 1$
b	10, 13, 16, 19, 22, …	$3n - 1$
c	7, 13, 19, 25, 31, …	$3n + 7$
d	2, 5, 8, 11, 14, …	$6n - 3$

4 Can you write down the nth terms of these sequences?

a 4, 11, 18, 25, 32, …

b 5, 7, 9, 11, 13, …

c 9, 13, 17, 21, 25, …

d 5, 13, 21, 29, 37, …

e 11, 21, 31, 41, 51, …

LEVEL BOOSTER

3
I can substitute numbers into simple algebraic expressions.
I can find the output given the input for a single operation function machine.

4
I can write down a sequence given a first term and a term-to-term rule.
I can give the term-to-term rule for how a sequence is building up.
I can write a single operation rule in symbolic form using algebra.
I can find the operation in a single operation function machine given the inputs and outputs.

5
I can find any term in a sequence given the first term and the term-to-term rule.
I can write a double operation rule in symbolic form using algebra.

6
I can find the operations in a double operation function machine given the inputs and outputs.
I can find any term in a sequence given the *n*th term.

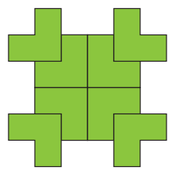
National Test questions

1 *1998 Paper 2*

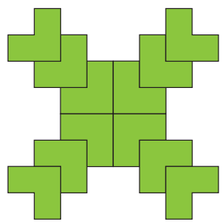
Owen has some tiles like these:

He uses the tiles to make a series of patterns.

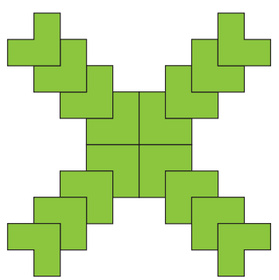
Pattern number 1



Pattern number 2



Pattern number 3



Pattern number 4

a Each new pattern has more tiles than the one before. The number of tiles goes up by the same amount each time.

How many more tiles does Owen add each time he makes a new pattern?

b How many tiles will Owen need altogether to make pattern number 6?

c How many tiles will Owen need altogether to make pattern number 9?

d Owen uses 40 tiles to make a pattern. What is the number of the pattern he makes?

2 *2000 Paper 2*

a Write down the next two numbers in the sequence below.

281, 287, 293, 299, …, …

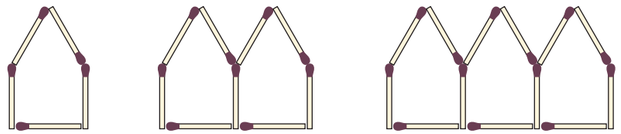
b Write down the next two numbers in the sequence below.

1, 4, 9, 16, 25, …, …

c Describe the pattern in part **b** in your own words.

3 *2000 Paper 2*

You can make 'huts' with matches.



1 hut needs 5 matches | 2 huts need 9 matches | 3 huts need 13 matches

A rule to find how many matches you need is

$$m = 4h + 1$$

m stands for the number of matches

h stands for the number of huts.

a Use the rule to find how many matches you need to make 8 huts. (Show your working.)

b I use 81 matches to make some huts. How many huts do I make? (Show your working.)

CHAPTER 2 Number 1

This chapter is going to show you

- how to work with decimals and whole numbers
- how to use estimation to check your answers
- how to solve problems using decimals and whole numbers, with and without a calculator

What you should already know

- How to write and read whole numbers and decimals
- How to write tenths and hundredths as decimals
- Times tables up to 10×10
- How to use a calculator to do simple calculations

Decimals

Look at this picture. What do the decimal numbers mean? How would you say them?

When you multiply by 100, all the digits are moved two places to the left.

Example 2.1

Work out 3.5×100.

Thousands	Hundreds	Tens	Units	Tenths	Hundredths	Thousandths
			3 .	5		
	3	5	0 .			