17 Ratio

Learning objectives

- How to use ratio notation
- How to use ratios to compare quantities
- How to simplify ratios
- How to use ratios to find missing quantities
- The connection between ratios and fractions

Prior knowledge

- How to simplify fractions. In KS2, pupils will have divided numerators and denominators by common factors.
- How to find a fraction of a quantity. Pupils will be familiar with percentage notation.
- How to interpret bar charts and pie charts

Context

- Ratios are a very useful way to compare quantities without the distraction of the actual values. For example, saying that the diameter of Saturn is 10 times the diameter of the Earth (or the ratio is 10 : 1) provides an immediate mental image. This would not be as obvious just by quoting the diameters.
- Many pupils will have experience of riding a bicycle with gears. It may come as a surprise to them to learn that ratios are used to compare one gear with another. If you know the gear ratio, you know if it will be easy or difficult to pedal in that gear.

Discussion points

- Provide pupils with questions to activate prior knowledge and exercise mathematical 'fluency', that is, the ability to manipulate mathematical language and concepts and apply them in different contexts.

Associated Collins ICT resources

- Chapter 17 interactive activities on Collins Connect online platform
- *Zodiac map* Wonder of Maths on Collins Connect online platform
- *Lines of the form x + y = a* Worked solution on Collins Connect online platform
- Ratio and proportion tool on Collins Connect platform

Curriculum references

Reason mathematically
- Extend and formalise their knowledge of ratio and proportion in working with measures and geometry, and in formulating proportional relations algebraically

Solve problems
- Develop their use of formal mathematical knowledge to interpret and solve problems, including in financial mathematics

Fast-track for classes following a 2-year scheme of work

- Pupils will have worked with ratio in KS2, when comparing quantities and in problems involving unequal sharing. Pupils may have been introduced to the *a* : *b* notation. If pupils can show understanding by answering one or more of the later questions in Exercise 17A of the Pupil Book, they can move on to simplifying ratios in Exercise 17B.
- Similarly, if pupils are confident about simple sharing problems, as provided in Exercise 17C, then they can move on to concentrate on the mixed questions in Exercise 17D.

Lesson 17.1 Introduction to ratios

Learning objectives
- To introduce ratio notation
- To use ratios to compare quantities

Resources and homework
- Pupil Book 1.1, pages 319–323
- Intervention Book 2, pages 43–45
- Homework Book 1, section 17.1
- Online homework 17.1, exercises 1–10

Links to other subjects
- **Science** – to compare quantities in experiments or compounds; to compare distances in astronomy
- **Geography** – to compare populations, areas, heights or other physical measures

Key words
- quantity
- ratio

Problem solving and reasoning help
- In the **PS** and **MR** questions in Exercise 17A in the Pupil Book, pupils are required to apply their understanding of ratio to some familiar but increasingly complex real-life situations.
- The investigation requires pupils to decode some quite complex information in less familiar situations. If necessary, go through Example 1 as a group, discussing how you identify the relevant information.

Common misconceptions and remediation
- Pupils sometimes have difficulty remembering how to pronounce 'ratio'. They may pronounce it as rat-i-o, with a 't' sound rather than with an 'sh' sound. Help pupils by saying the word often and asking them to use it. Point out that ratio is related to the word 'ration', and that sometimes spell checkers change 'ratio' to 'ration'.
- The order of the numbers in a ratio is important and relates to the order of the quantities being compared. Emphasise this by asking for the ratio the other way round and what it means when looking at examples.

Probing questions
- The ratio of two distances is 10 : 1. Do you know either of the distances? (No)
- Do you know if they are both a whole number of centimetres? (No)
- What do you know about the distances? (One distance is 10 times greater than the other distance.)

Part 1
- Write or place these numbers on the board, randomly scattered:
  15, 4, 27, 24, 9, 20, 45, 60, 8, 12.
- Ask pupils to arrange the numbers in two sets in any way they choose.
- Ask one or two pupils to give their sets, with reasons.
- Odd and even numbers is one obvious way. Try to find a pupil who has put: 4, 8, 9, 15, 20 in one group and 12, 24, 27, 45, 60 in the other. If no one has done this, do it yourself and ask pupils what is the logic behind your choice. (In these sets, the numbers in one set are three times the numbers in the other set.)
- Write these numbers on the board and ask pupils to divide them into two sets in a similar way: 2, 5, 10, 20, 25, 40, 50, 100, 200, 250.

- Here we are looking for: 2, 5, 20, 40, 50 and 10, 25, 100, 200, 250. Here, the multiple is 5.
- If pupils need extra multiplication practice, give them a number by which to multiply (such as 4 or 20) and then some quick-fire numbers to multiply by that. So if the number is 4, you might ask them to multiply by, for example: 5, 7, 20, 11 ….

Part 2

- Look at the numbers in the first set in Part 1. They can be paired as 4 and 12, 8 and 24 …. Explain that we would say: '*The* ratio *for each pair is 1 to 3 or 1 : 3*'. Explain this notation. (Each time, the second number is three times the first.)
- Ask for some other pairs of numbers in the same ratio.
- **Less able** pupils may find a visual representation helpful. Draw two columns of equal width, one three times the height of the other. Say that the ratio of the heights is 1 : 3 and divide the second column into three parts to demonstrate this.
- Pupils should be able to say that the numbers in the second example in Part 1 give pairs in the ratio of 1 : 5. A visual representation would help **less able** pupils.
- Look at the example of the beads at the start of Lesson 17.1 in Pupil Book 1.1. The ratio of red beads to blue beads is 2 : 1. The ratio tells us is that there are two red beads for every blue bead. Explain that the ratio can be written the other way around, so that the ratio of blue to red beads is 1 : 2.
- **Pupils can now do Exercise 17A in Pupil Book 1.1.**

Part 3

- After completing the investigation at the end of Exercise 17A, ask pupils to share their findings. They should compare the ratios of the number of coins with the ratios of the values.
- If pupils have not done the investigation, ask individuals to draw a diagram to illustrate a particular ratio such as 1 : 6 or 5 : 2.

Answers

Exercise 17A
1   4 : 1
2   Drawing with the number of black beads 3 times the number of white beads
3   **a** 2   **b** 2 : 1
4   3
5   10 : 1
6   **a** 3   **b** 3
7   **a** 4   **b** 4 : 1
8   **a** 8   **b** 8 : 1   **c** 2 : 1
9   **a** 5 : 1   **b** 6 : 1   **c** 3   **d** 3 : 1
10 **a** 4 : 1   **b** 5 : 1
11 **a i** 8   **ii** 16   **b** 2   **c** 1 : 2
    d 2 : 1
12 **a** 3 : 1   **b** 4 : 1

13 **a** Mazda RX7
    b Peugeot to Mazda = 1 : 2 or Mazda to Peugeot = 2 : 1
    c Lexus LFA
    d Fiesta to Lexus = 1 : 3 or Lexus to Fiesta = 3 : 1
    e i 3 : 1 **ii** 4 : 1
14 **a i** 2 : 1 **ii** 3 : 1 **iii** 5 : 1
    b 20 million
Investigation: It's in the bag
A   **a** £20   **b** £10   **c** 2 : 1
B   **a** £10   **b** £1   **c** 10 : 1
C   £1 or £2 : 10p or 5p
D   £1 or £2 : 2p or 1p
E   **a** 100   **b** 50   **c** 2 : 1   **d** 2 : 1
F   **a** own choice   **b** Find the two ratios

Lesson 17.2 Simplifying ratios

Learning objective
- To write a ratio as simply as possible

Resources and homework
- Pupil Book 1.1, pages 323–328
- Homework Book 1, section 17.2
- Online homework 17.2, exercises 1–10

Links to other subjects
- **Science** – to compare the amounts of two elements in a chemical formula
- **Geography** – to compare the area of one country to the area of another, for example, comparing the size of other countries to the size of Wales

Key words
- fraction
- simplify

Problem solving and reasoning help
- The investigation at the end of Exercise 17B requires pupils to combine their understanding with other areas of mathematics, in this case algebra and sequences. This is very useful as ratio is a key concept underpinning work across the strands in mathematics.

Common misconceptions and remediation
- Pupils forget that if quantities being compared must be in the same units. The activity in Part 3, below, addresses this point clearly.
- Pupils can also be careless about which way around they write ratios.

Probing questions
- When you are given a ratio, how do you know if it is in its simplest form?
- Is the ratio 1 : 5 the same as the ratio 5 : 1?

Part 1
- Write some fractions on the board and ask pupils to write them in their simplest forms. Start with fractions that simplify to 1 in the numerator; progress to more complicated examples.
- Emphasise the fact that you want the simplest possible form. For example, the simplest form of $\frac{12}{20}$ is $\frac{3}{5}$, not $\frac{6}{10}$.
- Match the difficulty to the ability of your pupils. Ask individuals to give answers. For **more able** pupils, choose examples where the common factor is more or less obvious (such as 7), or where there are multiple factors.

Part 2
- Draw two lines, one line that is 24 cm long and one that is 40 cm long. Do not reveal the lengths yet.
- Ask pupils to estimate the ratios of the lengths. Do not comment on whether the suggestions are correct or not. If you get several suggestions, ask pupils to say which is best, and why.
- Now write the lengths on the lines and say that we could use these to write the ratio of the shorter length to the longer length as 24 : 40. Imagine each line marked in centimetre sections. There will be 24 on one line and 40 on the other.

- Say that we can simplify ratios in the same way that we simplify fractions – by dividing by a common factor.

- Ask pupils to simplify $\dfrac{24}{40}$ to its simplest form. The answer is $\dfrac{3}{5}$.

- In the same way, the ratio 24 : 40 simplifies to 3 : 5. Did anyone have this as their estimate?

- What does this mean? Divide the 24-cm line into three equal sections (8 cm each) and the 40-cm line into five equal sections (8 cm each). This visually represents the ratio.

- **Pupils can now do Exercise 17B from Pupil Book 1.1.**

Part 3

- At the end of the lesson, ask pupils to share their findings from the investigation at the end of the exercise. Check that pupils can a work out all the ratios required.

Answers

Exercise 17B
1   **a** 3 : 5   **b** 5 : 3
2   **a** 2 : 3   **b** 3 : 2
3   **a** 3 : 2   **b** 2 : 3
4   **a** 18   **b** 9 : 2   **c** 2 : 9
5   **a** 3   **b** 2 : 3   **c** 3 : 2
6   **a** 1 : 7   **b** 5 : 3
7   2 : 5
8   **a** 2 : 3   **b** 3 : 2   **c** 2 : 5   **d** 5 : 2
9   **a** 3 : 2   **b** 1 : 3   **c** 2 : 1
10 **a** 5 : 2   **b** 2 : 5
11 **a** 1 : 4   **b** 2 : 3
12 **a** 3 : 2   **b** 2 : 1
13 **a** 1 : 2   **b** 4 : 1   **c** 1 : 4   **d** 1 : 2
14 **a** 3 : 1   **b** 1 : 3   **c** 2 : 1   **d** 1 : 2
15 5 : 6
16 **a** 1 : 5   **b** 10 and 30   **c** 1 : 3
Investigation: Red and green
A   1 : 2 , 2 : 1 , 4 : 1
B   own drawings
C   13 : 2 and 19 : 2

Lesson 17.3 Ratios and sharing

Learning objective
- To use ratios to find missing quantities

Resources and homework
- Pupil Book 1.1, pages 329–331
- Intervention Book 3, pages 43–44
- Homework Book 1, section 17.3
- Online homework 17.3, exercises 1–10

Links to other subjects
- **Science** – to compare quantities in different types of materials
- **Food technology** – to combine recipe ingredients in the correct quantities

Key words
- No new key words

Problem solving and reasoning help
- In **MR** question 6 of Exercise 17C in the Pupil Book, the crucial step is finding the fraction of the 20 people who are women. This is best achieved by changing the ratio to make the calculation easier.
- In the challenge at the end of the exercise, the link between carat and percentage of gold can be illustrated by a horizontal line with a scale from 0 to 24 carats at the top and 0 to 100% on the bottom. This way, pupils can see easily that 24 carats is 100% gold, 12 carats is 50% gold, 18 carats is 25% gold, and so on.

Common misconceptions and remediation
- Pupils sometimes get confused about the exact relationship between ratios and fractions. Emphasise the link between the ratio numbers and the numerators of the fractions, as outlined in the lesson plan.

Probing questions
- If the ratio of boys to girls in a class is 3 : 1 could there be exactly 30 children in the class?
- Could there be 25 boys? Why?

Part 1
- Pupils can work in pairs on this activity.
- Write £20 on the board. Ask pupils to find what fractions of this will be a whole number of pounds. If they need clarification, show that $\frac{1}{2}$ is £10, which is a whole number of pounds; $\frac{1}{3}$ is £3.33 with 1p left over, which is not a whole number.
- After a few minutes check answers.
- Check that pupils remember how to find a fraction:

  For example, $\frac{1}{5}$ of £20 = 20 ÷ 5 = £4; $\frac{3}{5}$ of £20 = 3 × £4 = £12
- **Less able** pupils may need more practice with finding fractions. If so, repeat the activity with £18 and/or £12.

Part 2

- Draw a 4 × 3 grid of 12 squares. Colour three of the squares. Ask: What fraction is coloured? What fraction is uncoloured? The answers should be one-quarter and three-quarters.
- In the example we have just done, the ratio of shaded to unshaded parts is 1 : 3. Make sure that pupils are happy with this.
- Repeat this activity but shade different numbers of squares each time. For example, 4 or 2, or 1 or 6. Each time, ask for the fraction shaded, the fraction unshaded and the ratio of the two. Pupils should see that the numerators of the fractions give the numbers in the ratio.
- Now work through Example 4 and Example 5 above Exercise 17C in the Pupil Book. In Example 4, £200 is divided into shares in a given ratio. This is done by using a visual image which can reinforce the link between fractions and ratios. In Example 5, one share is given and the total must be found. Again, fractions are used to tackle this problem.
- **Pupils can now do Exercise 17C from Pupil Book 1.1.**

Part 3

- Say that Eve has 20 apples to share with Adam. Can they share them in the ratio 2 : 1 and have an equal number each? What about sharing them in the ratio 2 : 3?
- What is the smallest number that can be divided equally in the ratio 2 : 1 or 2 : 3? (The answer is 15; the lowest whole number is divisible by both 3 and 5.)
- A question for **more able** pupils is to find the smallest number that can be divided equally in the ratio 2 : 1 or 2 : 5.

Answers

Exercise 17C
1   **a** 10   **b** 20
2   **a** 200   **b** 800
3   **a** £6   **b** £18
4   **a** 30   **b** 20
5   **a** 360   **b** 40
6   8
7   24
8   30
9   24
10 75 g
11 400
12 **a** 2 and 5 are the missing numbers.   **b** 35
13 20
Challenge: All that glitters
A   **a** 3 : 1   **b** 18 carat
B   **a** 3 : 5   **b** 9 carat
C   **a** 6 g   **b** 3 g
     c There is twice as much gold in the 18 carat ring. The ratio of 18 carat to 9 carat is 2 : 1.
D   **a** a diagram showing $\frac{3}{4}$ gold   **b** a diagram showing $\frac{3}{8}$ gold

Lesson 17.4 Ratios and fractions

Learning objective
- To understand the connections between fractions and ratios

Resources and homework
- Pupil Book 1.1, pages 332–335
- Homework Book 1, section 17.4
- Online homework 17.4, exercises 1–10

Links to other subjects
- **Science** – to find unknown quantities of substances
- **Geography** – to compare the resources of different countries

Key words
- No new key words for this topic

Problem solving and reasoning help
- In **PS** question 8 of Exercise 17D of the Pupil Book encourages pupils to apply their understanding. The challenge at the end of the exercise is an extension of the investigation at the end of Exercise 17B.

Common misconceptions and remediation
- Pupils can make mistakes by not being clear about whether they know the whole or a part when working out missing values. Encourage pupils always to ask themselves if they know the whole or a part before they try to answer a question.

Probing questions
- Ann and Baz share some money in the ratio 3 : 2
- If you know the total amount, how can you work out what Ann has?
- If you know what Ann has, how can you work out what Baz has?
- If you know what Baz has, how can you work out what Ann has?

Part 1
- Write down the following sentences: '*Tim and Claire bake 60 biscuits in total. Tim bakes 24 of them.*'
- Ask pupils to work in pairs and find different ways to write this information. They can use fractions, percentages or ratios.
- After a few minutes take answers from selected pupils. Possible statements are:
  - Tim bakes $\frac{2}{5}$ and Claire bakes $\frac{3}{5}$.
  - Tim bakes 40% and Claire bakes 60%.
  - Tim and Claire bake biscuits in the ratio 2 : 3.
- Make sure that pupils can remember the equivalences between simple fractions and percentages. If not, ask more questions about this, for example, writing tenths, fifths or quarters as percentages.
- This should remind pupils of the work covered in the last lesson.

Part 2
- This lesson is about using the ideas about ratios that pupils have met so far in questions, in a realistic context.

- Tell pupils that the ratio of teachers to pupils in a school is 1 : 12 (or use your own figure if you know it, but round it to a whole number).
- Ask a volunteer to explain what this means. A possible answer is that there is one teacher for every 12 pupils.
- Ask questions that require pupils to use this information in different ways, for example:
  - If there are 30 teachers, how many pupils are there?
  - If there are 60 teachers, how many pupils are there?
  - If there are 1300 pupils and teachers, how many of each are there?
- Ask pupils to explain how they work out the answers each time and check that their reasoning is correct.
- **Pupils can now do Exercise 17D from Pupil Book 1.1.**

Part 3

- Pupils can work in pairs or small groups of three on this task.
- Ask pupils to produce a concise statement that will help them to remember the key points about ratios. This could be an example, a diagram or a sentence or two.
- Ask one or two groups to share what they have done with the rest of the class.
- As an extension, pupils could use these ideas to produce posters for a wall display.

Answers

Exercise 17D

1 **a** 1 : 3   **b** $\frac{1}{4}$ **c** $\frac{3}{4}$

2 **a** $\frac{1}{6}$ **b** $\frac{5}{6}$

3 **a** 1 : 2   **b** $\frac{2}{3}$

4 **a** The missing numbers are 3 and 2
     b $\frac{3}{5}$ **c** $\frac{2}{5}$

5 $\frac{4}{5}$

6 **a** 3 : 2   **b** $\frac{3}{5}$ **c** $\frac{2}{5}$

7 **a** 8 : 1   **b** $\frac{1}{9}$

8 **a** 1 : 2   **b i** $\frac{1}{3}$ **ii** $\frac{2}{3}$ **c** 1 : 3

     d i $\frac{1}{4}$ **ii** $\frac{3}{4}$

9 **a** 3 : 2   **b** $\frac{3}{5}$ **c** $\frac{2}{5}$

10 **a** $\frac{2}{3}$ **b** 1 : 2

11 **a** $\frac{1}{4}$ **b** 3 : 1

12 4 : 1

13 **a** $\frac{3}{4}$ **b** $\frac{1}{4}$

14 **a** 18 and 12   **b** 3 : 2   **c i** $\frac{3}{5}$ **ii** $\frac{2}{5}$

15 **a** 3 : 1   **b** $\frac{3}{4}$ **c** 2 : 1   **d** $\frac{2}{3}$

Challenge: Coloured patterns

A **a** 1 : 2   **b** $\frac{1}{3}$ **c** $\frac{2}{3}$

B own drawing

Answers to Review questions

1 **a** 5 is the missing number   **b** 5 : 1
2 **a** 2 : 1   **b** 4 : 1
3 **a** 20   **b** 2 : 1   **c** 2 : 1   **d** 3 : 1   **e** 6 : 1
4 **a** 75%   **b** 3 is the missing number   **c** 3 : 1
5 **a** 2 : 1   **b** 3 : 1   **c** 3 : 2
6 **a** red, blue, blue, from the bottom up   **b** 1 : 2   **c** $\frac{1}{3}$ **d** $\frac{2}{3}$
7 **a** 6   **b** 2 : 1
8 **a** 2 : 1   **b** 2 × (8 + 4) = 24 **c** 3 : 1   **d** 6 : 1
9 **a** drawing
    b No, because the lengths are not in the same units. The correct ratio is 2 : 4 = 1 : 2.
10 **a** 4 : 1   **b** 2 : 1

Answers to Problem solving – Smoothie bar

1 **a** 50 g mango, 25 g strawberries, 50 g banana, 125 ml orange juice
    b 300 g mango, 150 g strawberries, 300 g banana, 750 ml orange juice
    c 50 g strawberries, 100 g banana, 50 g yogurt
    d 300 g strawberries, 600 g banana, 300 g yogurt
2 **a** 4 : 3   **b** 3 : 2   **c** size = 2 : 1, cost = 2 : 1   **d** 4 : 3 = 1.333 : 1 and 3 : 2 = 1.5 : 1.
    The medium smoothie is only 1.333 times the size of the small smoothie, but 1.5 times
    the price so is not better value for money
3 **a** 2 : 1   **b** 1 : 1   **c** Yes, all quantities are halved so the ratios will be the same.
    d Yes, all quantities are doubled so the ratios will be the same.
4 **a** 300 g   **b** 3 : 1   **c** $\frac{1}{4}$ **d** $\frac{3}{4}$
5 Fruity Surprise : Tropical Fruit = 2 : 3
    Fruity Surprise : Breakfast Boost = 1 : 3
    Tropical Fruit : Breakfast Boost = 1 : 2
    All these ratios can be reversed.