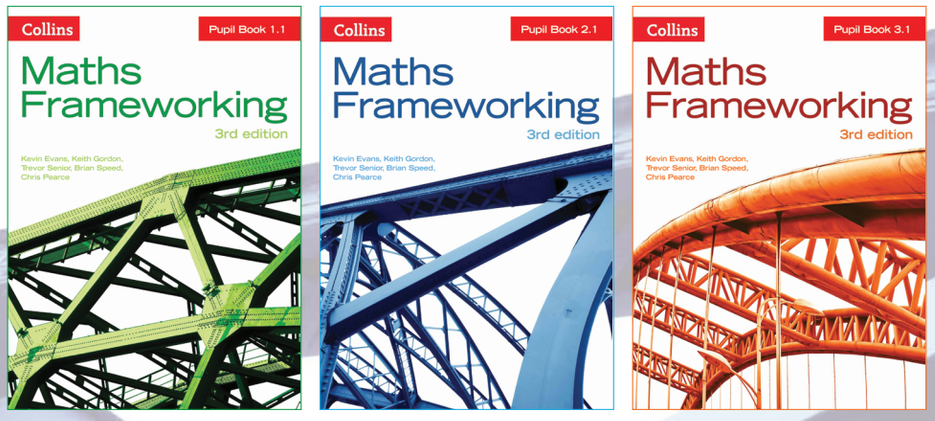
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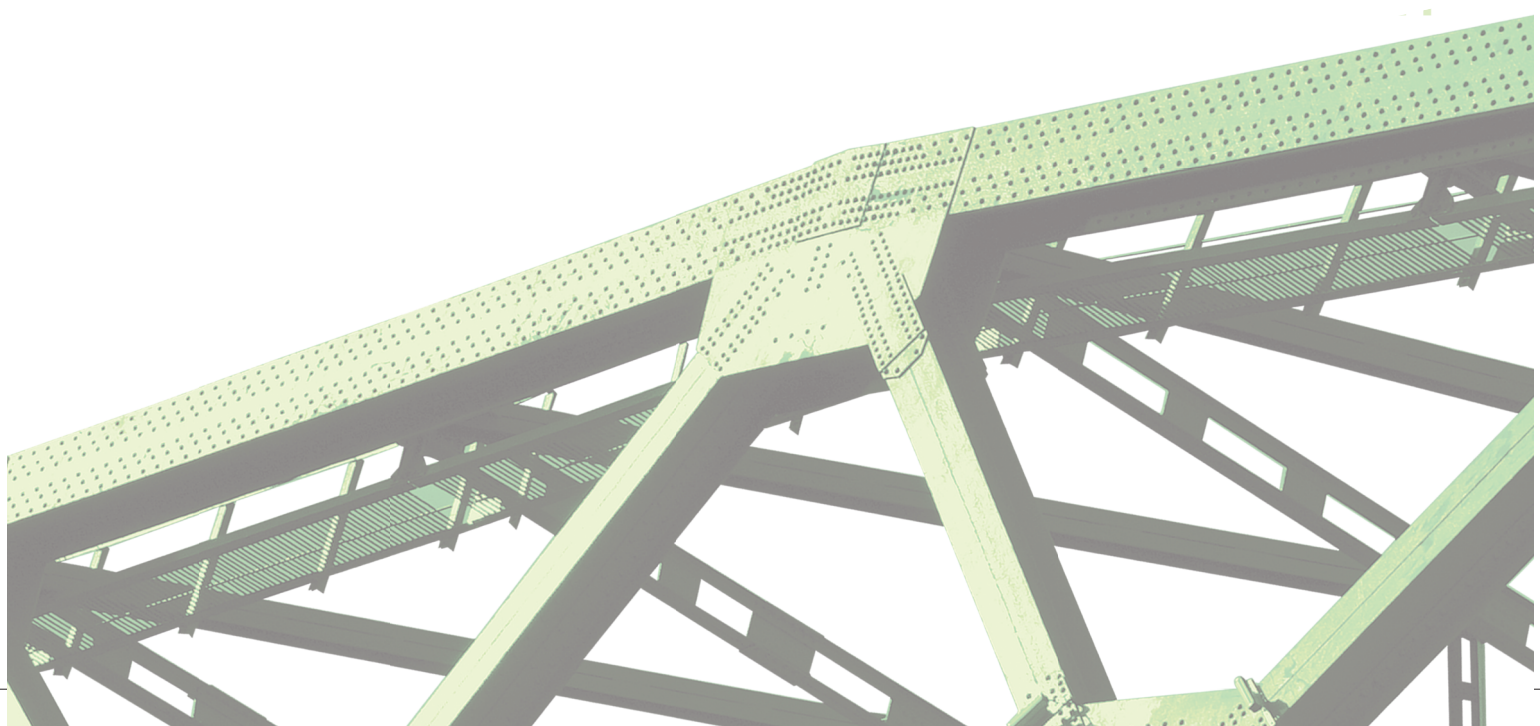
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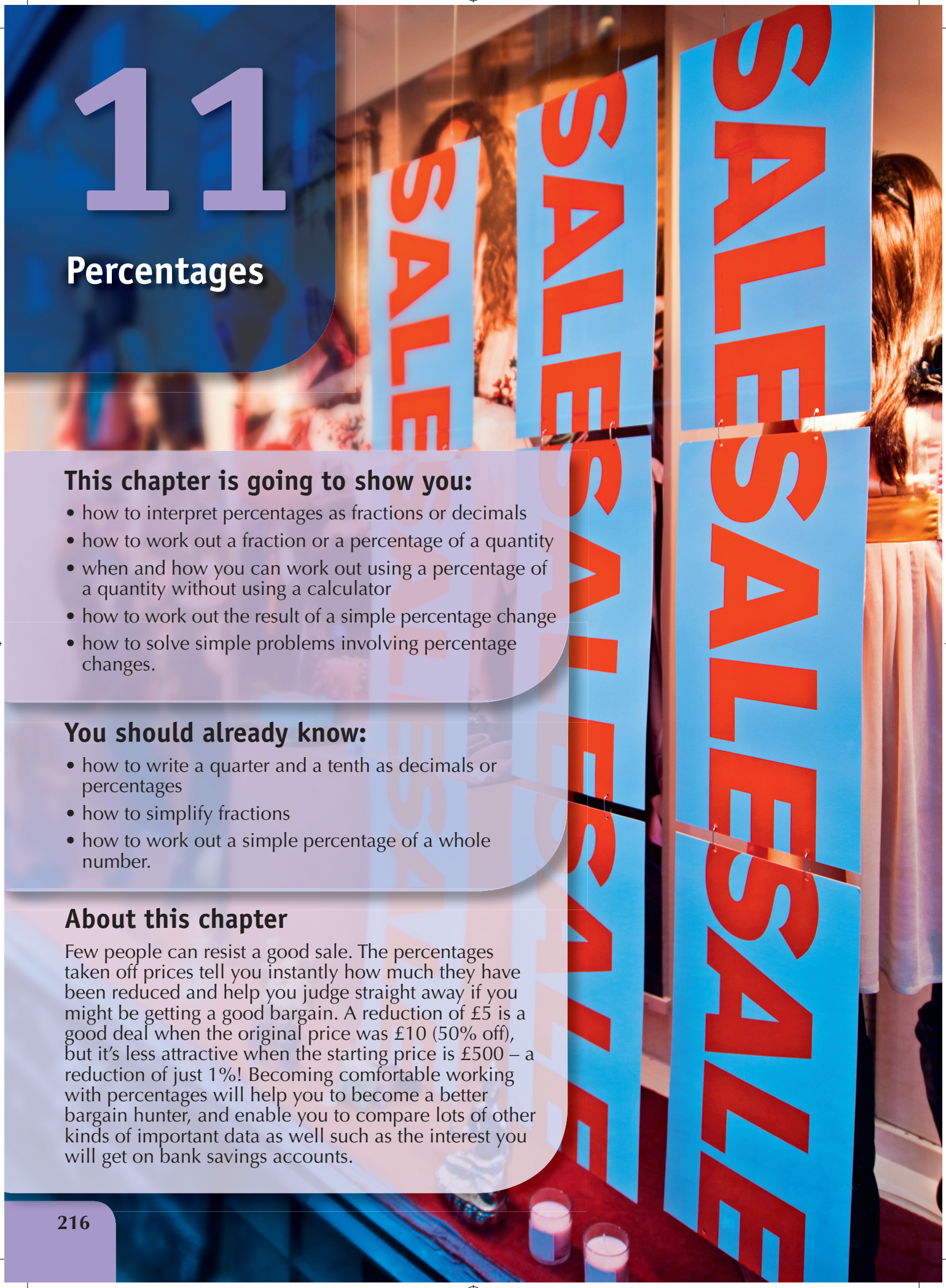
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Kevin Evans, Keith Gordon,
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Chris Pearce



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11 Percentages



This chapter is going to show you:

- how to interpret percentages as fractions or decimals
- how to work out a fraction or a percentage of a quantity
- when and how you can work out using a percentage of a quantity without using a calculator
- how to work out the result of a simple percentage change
- how to solve simple problems involving percentage changes.

You should already know:

- how to write a quarter and a tenth as decimals or percentages
- how to simplify fractions
- how to work out a simple percentage of a whole number.

About this chapter

Few people can resist a good sale. The percentages taken off prices tell you instantly how much they have been reduced and help you judge straight away if you might be getting a good bargain. A reduction of £5 is a good deal when the original price was £10 (50% off), but it's less attractive when the starting price is £500 – a reduction of just 1%! Becoming comfortable working with percentages will help you to become a better bargain hunter, and enable you to compare lots of other kinds of important data as well such as the interest you will get on bank savings accounts.

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11.1 Fractions, decimals and percentages

Learning objective

- To understand the equivalence between a fraction, a decimal and a percentage

Keywords	
decimal	fraction
per cent (%)	percentage

Fractions, **decimals** and **percentages** can all be used to compare quantities or measurements.

This pie chart shows the favourite colours of a group of children.

The red sector is a quarter of the whole circle.

You can write that as $\frac{1}{4}$ or 0.25 or 25%.

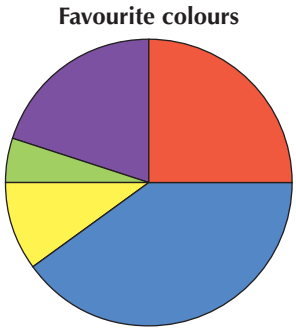
These all mean the same thing.

Remember that **per cent** means 'for every 100'.

So 25% is 25 parts out of 100 in the whole, or $\frac{25}{100}$.

As a fraction, this can be simplified to $\frac{1}{4}$.

As a decimal, this can be expressed as 0.25.



Example 1

The blue sector is 40% of the pie chart.

What fraction is that?

$40\% = \frac{40}{100} = \frac{2}{5}$

Example 2

The yellow sector is 0.1 of the pie chart.

What percentage is that?

$0.1 = \frac{1}{10} = 10\%$

Hint Remember that:

$\frac{1}{10} = 0.1$

$\frac{1}{100} = 0.01$

You should know simple equivalences such as these.

Example 3

The green sector is 5% of the pie chart.

What percentage is the purple sector?

The total of the four sectors that you already know is 25% + 40% + 10% + 5% = 80%.

The whole pie chart is 100% so the purple sector must be 100% – 80% = 20%.

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Exercise 11A

1 Change these fractions to percentages.

a $\frac{1}{2}$ **b** $\frac{1}{4}$ **c** $\frac{3}{10}$ **d** $\frac{9}{10}$ **e** $\frac{3}{4}$

2 Write these decimals as percentages.

a 0.5 **b** 0.8 **c** 0.05 **d** 0.08 **e** 0.77

3 Write these percentages as fractions.

Give your answers as simply as possible.

a 20% **b** 30% **c** 5% **d** 65% **e** 95%

4 Copy and complete this table.

The first row has been done for you.

Percentage	Fraction	Decimal
20%	$\frac{1}{5}$	0.2
80%		
		0.35
4%		
	$\frac{9}{25}$	

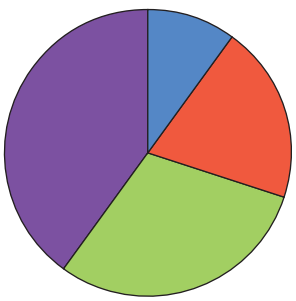
5 $\frac{1}{3}$ is $33\frac{1}{3}$%.

Write $\frac{2}{3}$ as a percentage.

MR **6** This pie chart shows the most popular colours chosen by a group of children to paint a pattern.

Three of the sectors of this pie chart represent 10%, 20% and 30%.

Colours to paint a pattern



a What percentage does the fourth sector represent?

b What fractions of the whole are the four sectors?

c What percentage of the pie chart is not red?

d What colour is 0.3 of the pie chart?

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7 Match each fraction with a percentage.
One has been done for you.

$\frac{3}{5}$	70%
$\frac{4}{5}$	60%
$\frac{7}{10}$	74%
$\frac{17}{20}$	80%
$\frac{37}{50}$	85%

8 $\frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

a Write the fractions as percentages to make a new addition.

b Write the fractions as decimals to make a new addition.

9 20% + 75% = 95%

a Rewrite this as an addition of decimals.

b Rewrite this as an addition of fractions.

10 $\frac{1}{4}$ = 25% $\frac{1}{8}$ is half of $\frac{1}{4}$

a Write $\frac{1}{8}$ as a percentage.

b Write $\frac{3}{8}$, $\frac{5}{8}$ and $\frac{7}{8}$ as percentages.

MR **11** Percentages can be larger than 100%.

1 = 100% 1.5 = 150% 2 = 200%

Write these decimal numbers as percentages.

a 2.5 **b** 1.2 **c** 1.9 **d** 1.75 **e** 1.88

12 Write these percentages as mixed numbers.
Remember that a mixed number is a whole number plus a fraction.

a 150% **b** 125% **c** 130% **d** 160% **e** 275%

Investigation: Looking into percentages

A Here is a sequence of percentages.

10%, 20%, 30%, 40%, …, …, …, …, 90%

a Copy the sequence and fill in the missing numbers.

b Write each percentage as a fraction, as simply as possible.

c Which percentages give the fractions that have the smallest denominator?

B Repeat part **A** with this sequence of percentages.

5%, 15%, 25%, …, …, …, …, …, …, 95%

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11.2 Fractions of a quantity

Learning objective

- To find a fraction of a quantity

Keyword

quantity

You can work out a fraction of a number or a **quantity** by using multiplication and division.

Example 4

There are 360 animals in a farm.

$\frac{1}{5}$ are cows and $\frac{3}{8}$ are sheep.

a How many cows are there?

b How many sheep are there?

a $360 \div 5 = 72$ Divide by 5 to work out $\frac{1}{5}$.

There are 72 cows.

b $360 \div 8 = 45$ To work out $\frac{3}{8}$ first work out $\frac{1}{8}$ by dividing by 8.

$45 \times 3 = 135$ Then $\frac{3}{8}$ of 360 = $\frac{1}{8}$ of 360×3.

There are 135 sheep.

Exercise 11B

1 Work out these quantities.

a $\frac{1}{3}$ of £96 **b** $\frac{2}{3}$ of £96

2 Work out these quantities.

a $\frac{1}{10}$ of £600 **b** $\frac{3}{10}$ of £600 **c** $\frac{7}{10}$ wof £600 **d** $\frac{9}{10}$ of £600

3 Work out these quantities.

a $\frac{2}{3}$ of 120 cm **b** $\frac{3}{5}$ of 120 cm **c** $\frac{5}{8}$ of 120 cm **d** $\frac{11}{20}$ of 120 cm

PS **4** A marathon is approximately 26 miles.

a How far is half a marathon?

b Janice says: 'When I have run 20 miles I shall have completed more than three-quarters of the marathon.'

Is this correct?

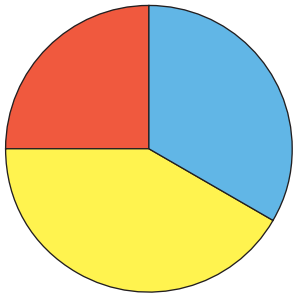
Give a reason for your answer.

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5 Copy the questions and answers below.
Draw an arrow from each part of the question to the correct answer.

$\frac{3}{4}$ of 72	49
$\frac{2}{5}$ of 150	54
$\frac{7}{8}$ of 64	56
$\frac{7}{20}$ of 140	60

6 The complete angle at the centre of a circle is 360°.



a One-third of this circle is coloured blue.
What is the angle at the centre of the blue sector?

b $\frac{5}{12}$ of this circle is coloured yellow.
What is the angle at the centre of the yellow sector?

7 Work out each quantity.

a $\frac{1}{12}$ of 2 hours.
Give your answer in minutes.

b $\frac{5}{6}$ of 2 days.
Give your answer in hours.

c $\frac{2}{3}$ of 2 minutes.
Give your answer in seconds.

MR **8** Tom has £600.
He spends $\frac{3}{5}$ of it on a bed.
He spends $\frac{3}{4}$ of the remainder on curtains.
Show that he has £60 left.

PS **9** Sam has a large bag of apples.
She gives away two-thirds of them.
She has 6 left.
How many did she have at the start?

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Challenge: Fractions larger than 1

A Work out each quantity.

a $\frac{2}{5}$ of 30 **b** $\frac{4}{5}$ of 30 **c** $\frac{6}{5}$ of 30 **d** $\frac{8}{5}$ of 30

B Work out each quantity.

a $\frac{2}{3}$ of 90 **b** $\frac{4}{3}$ of 90 **c** $\frac{3}{8}$ of 40 **d** $\frac{11}{8}$ of 40

C a Copy and complete each of these.

i $\frac{3}{5}$ of 60 is … $\frac{8}{5}$ of 60 is …

ii $\frac{3}{4}$ of 60 is … $\frac{7}{4}$ of 60 is …

iii $\frac{3}{10}$ of 60 is … $\frac{13}{10}$ of 60 is …

b What do you notice about the answers in part **a**?

c What are the answers to part **a** if you change 60 to 100?

d What are the answers to part **a** if you change 60 to 200?

11.3 Percentages of a quantity

Learning objective

- To find a percentage of a quantity

You have learnt how to find a fraction of a quantity. You can find a percentage of a quantity in a similar way.

Example 5

There are 450 people in a village. 60% are over 18 years old.

How many people is that?

$60\% = \frac{60}{100} = \frac{3}{5}$

$\frac{3}{5}$ of $450 = (450 \div 5) \times 3 = 90 \times 3 = 270$

270 people are over 18 years old.

Example 6

A woman earns £3600. She has to pay 23% of this in income tax.

How much tax must she pay?

Here are two different methods.

Method 1

$23\% = \frac{23}{100}$ and this fraction cannot be simplified.

$\frac{23}{100}$ of 3600 is $(3600 \div 100) \times 23 = 36 \times 23 = 828$

She must pay £828.

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Method 2

23% = 20% + 1% + 1% + 1%

20% of 3600 = $\frac{1}{5}$ of 3600 = 720

1% of 3600 = $\frac{1}{100}$ of 3600 = 36

So 23% of 3600 = 720 + (3 × 36) = 720 + 108 = 828

She must pay £828.

You can use either method. If you do not have a calculator the second one could be easier.

Exercise 11C

1 Work out 1% of each amount.

a £700 **b** £7000 **c** £760 **d** £723 **e** £7 **f** £74

2 Find these amounts.

a 50% of 320 kg **b** 25% of 48 m **c** 20% of 45 litres **d** 10% of 60 cm
e 5% of 500 g **f** 2% of £1000 **g** 75% of 640 people **h** 30% of 410 km

3 **a** Work out 10% of 32 kg.

b Use your answer to part **a** to find these amounts.

i 20% of 32 kg **ii** 30% of 32 kg **iii** 5% of 32 kg **iv** 90% of 32 kg

4 **a** Find: **i** 25% of £28 **ii** 10% of £28.

b Use your answers to part **a** to work these out.

i 35% of £28 **ii** 15% of £28 **iii** 30% of £2 **iv** 5% of £28
v 12.5% of £28

5 Work out 63% of each quantity.

a £1 **b** £2 **c** £10 **d** £20

6 Work out these percentages of 640 kg.

a 50% **b** 25% **c** 35% **d** 90% **e** 99%

PS **7** 38% of £49.00 is £18.62.

Use this fact to work out 62% of £49.00.

8 Work out these amounts.

a 9% of £300 **b** 19% of £400 **c** 32% of £2000 **d** 91% of £3000

FS **9** The cost of a holiday for a couple is £1320.

They must make a first payment of 30%.

a How much is the first payment?

b How much is left to pay?

PS **10** At a conference, 25% of the people attending are men.

There are 30 men.

How many people are there at the conference altogether?

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Financial skills: Buying furniture

Here is a notice in a furniture store.

Easy payments on furniture
Initial payment of just 20%
Pay the remainder in
10 monthly installments

A Adele buys some furniture for £2700.

a She makes an initial payment of 20%.
How much did she pay?

b She pays the remainder in 10 monthly payments.
How much is each monthly payment?

B a Complete this table to show the initial and the monthly payments for each of these items.

Item	Table	Chair	Sideboard	Cabinet
Cost	£800	£120	£1400	£500
Initial payment				
Monthly payments				

b David buys a table and four chairs. Work out his total monthly payment.

11.4 Percentages with a calculator

Learning objectives

- To use a calculator to find a percentage of a quantity
- To know when it is appropriate to use a calculator

Keyword
deposit

Sometimes you can work out a percentage easily, without using a calculator.
If the calculation is complicated it is more efficient to use a calculator.
It is easy to work out 25% of £48 without using a calculator.
If you need to work out 27% of £49, it is easier to use a calculator.
Can you explain why?

Example 6

In an election, 850 people vote. 28% vote for Ms White.
How many people vote for Ms White?

Write 28% as a decimal and multiply by 850. Use a calculator to do this.
28% of 850 = 0.28 × 850 = 238
238 people vote for Ms White.

How would you do Example 6 without using a calculator? Look back at section 11.3 if you need to.

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Exercise 11D

1 **a** Work out these percentages without using a calculator.

i 20% of 70 kg **ii** 75% of 240 m **iii** 15% of 600 people

b Now use a calculator to work out the percentages in part **a**.

Check that you get the same answers.

2 Work out:

a 13% of £29.00 **b** 39% of 8200 km **c** 72% of 9500 people

d 93% of 6200 years **e** 42% of 65.5 cm **f** 88% of £88.

3 **a** Work out:

i 17% of 4300 people **ii** 54% of 4300 people **iii** 29% of 4300 people.

b Show that the three answers in part **a** add up to 4300. Explain why.

4 **a** Work out:

i 64% of 380 kg **ii** 32% of 380 kg **iii** 16% of 380 kg **iv** 8% of 380 kg.

b The questions and answers in part **a** form a sequence.

What is the next term in the sequence?

5 In an election, 17 600 people voted.

The Red party gained 23% of the votes. The Blue party gained 36%.

The Yellow party gained 19%.

How many votes did each party get?

6 A politician is talking to a meeting of 350 people.

He says: 'I know that 95% of the people in this room agree with me.'

How many people is that?

7 **a** Write 1.5% as a decimal.

FS

b A concert hall charges an extra 1.5% if you pay for tickets with a credit card. Jason buys a ticket for £68.

How much will he be charged for paying with a credit card?

8 This information was found on a website.

> 15.3% of the population of England live in Greater London.
>
> 4.9% of the population of England live in the North East.
>
> The population of England is 53.0 million.

Work out the population of Greater London and of the North East.

Give your answers in millions, correct to one decimal place (1 dp).

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PS **9** In the general election in the UK, in 2010, 29.6 million people voted.

This table shows the percentages of voters that voted for the three main parties.

Conservative	36%
Labour	29%
Liberal Democrat	23%

a Work out how many people voted for each party. Give your answers correct to one decimal place (1 dp).

b How many of the people who voted did not vote for one of the main parties? Give your answer in millions, correct to one decimal place.

10 The members of a club are voting about a change in the rules.

65% must be in favour for the change to take place.

The club has 327 members.

How many must vote in favour for the change to take place?

MR **11** **a** Use a calculator to work out 32% of 76 g.

b Find a simple way of working these out, without using your calculator.

i 16% of 76 g **ii** 32% of 38 m **iii** 16% of £38

Financial skills: Don't lose your deposit

When you book a holiday, or order something you want to buy, you often have to pay a **deposit**.

The deposit will not be returned if you change your mind about taking the holiday or buying the item you have ordered.

A Mr Brown books a holiday for his family. The holiday costs £3250.

He pays a deposit of 15%.

Work out the deposit.

B Ms Green is getting new glasses. The cost is £286.

She is asked to pay a deposit of 40%.

How much is that?

C Sam orders a new car.

The car costs £12 456.

She adds metallic paint for £475 and alloy wheels for £395.

She is asked to pay a deposit of 30%. How much is that?

D Three friends are going on holiday together. The cost for each person is £789.

They must pay a deposit of 35%.

a Work out the total deposit for all three.

b They must pay the rest a month before the holiday starts.

How much is that?

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11.5 Percentage increases and decreases

Learning objective

- To work out the result of a simple percentage change

Keywords

decrease
increase
reduction

In a sale, prices are often reduced by a certain percentage.

When the value of something **increases** or **decreases**, the change is often described in terms of a percentage. This is the value of the increase or decrease as a percentage of the original price or value.

You add it on for an increase. You subtract it for a decrease. A decrease is often called a **reduction**.

Example 7

Here is a sign in a shop window.

The original price of a dress is £135.

In the sale the price is reduced by 30%.

What is the sale price?

The reduction is 30% of £135.

30% of 135 = 0.3 × 135 = 40.5

The reduction is £40.50.

The sale price is £135 – £40.50 = £94.50.



If, instead of a reduction, the price had been increased by 30%, then the new price would be £135 + £40.50 = £175.50.

For an increase, you calculate the percentage in the same way and then add it to the original value.

Exercise 11E

1 A tree is 5.60 m high. In a year the height increases by 10%.

a Work out the increase in height.

b Work out the new height.

2 A woman's mass is only 32 kg and she is trying to increase it.

She successfully increases her mass by 12%.

a Work out the increase in her mass.

b Work out her new mass.

FS **3** The price of a TV is £720.

In a sale the price is reduced by 15%.

a Work out the reduction.

b Work out the new price.

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FS **4** The price of a car is decreased by 3%.
The original price was £14 800.

a Work out the decrease in price.

b Work out the new price.



5 The population of a village has increased by 45% over the last ten years.
Ten years ago the population of the village was 360 people.
Work out the population now.

FS **6** The price of a washing machine is £440.
Work out the new price after an increase of:

a 5% **b** 10% **c** 15%.

FS **7** In a sale, prices are reduced. Work out the sale price of each of these items.

a	A jacket	Original price £85.00	Reduction 40%
b	A pair of trainers	Original price £63.50	Reduction 20%
c	A bag	Original price £44.00	Reduction 70%

8 Census figures show that in 2001 the population of England and Wales was 52.4 million.
In the next ten years the population increased by 7%.
What was the population of England and Wales in 2011?

PS **9** This is an extract from a newspaper article.

GAS AND ELECTRICITY PRICES INCREASE

A fuel company has announced that domestic gas bills will rise by 16% and electricity bills will rise by 18%.

The Watsons' gas bill last year was £625.

Their electricity bill was £441.

a Work out the expected bills for the Watsons for gas and electricity after the increases.

b What is the total increase in their bills for gas and electricity?

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FS **10** A woman earns £36 600 per year.
She is given a pay rise of 4%.

a Work out her new salary.

b She is paid $\frac{1}{12}$ of her salary each month.
How much extra will she earn each month?

Financial skills: Add on the tax

VAT is a tax that is added to goods and services.
If you buy something, or have work done by someone, VAT is probably included in your bill.
The rate of VAT can vary.
In January 2011 the rate of VAT in the UK increased from 17.5% to 20%.

A The cost of a computer game before adding VAT is £38.
20% VAT must be added to this.

a How much is the VAT?

b What is the price including VAT?

B **a** Write 17.5% as a decimal.

b The cost of fitting a new kitchen, before adding VAT, is £8200.
If the rate of VAT is 17.5% work out the tax.

c Work out the cost including VAT.

C When the rate of VAT went up from 17.5% to 20%, prices increased.
The cost of a camera before VAT is added is £354.

a If the rate of VAT is 17.5%, what is the price of the camera including VAT?

b If the rate of VAT is 20%, what is the price of the camera including VAT?

c What is the increase in the price of the camera because of the increase in the VAT?



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Ready to progress?

I can interpret percentages as fractions or decimals.
I can work out a fraction or a percentage of a quantity.
I can work out some percentages of quantities without using a calculator.

I can work out the result of a simple percentage change.
I can solve simple problems involving percentage changes.

Review questions

1 Write these fractions as percentages.

a $\frac{3}{4}$ **b** $\frac{3}{10}$ **c** $\frac{3}{20}$ **d** $\frac{3}{50}$

2 Write each percentage as a fraction, as simply as possible.

a 70% **b** 60% **c** 4% **d** $12\frac{1}{2}\%$ **e** 35%

3 Write each percentage as a decimal.

a 90% **b** 9% **c** 0.9%

4 Here are five numbers.

0.65 69% 0.8 $\frac{2}{3}$ $\frac{3}{4}$

a Write down the largest number. **b** Write down the smallest number.

5 Work out these quantities.

a $\frac{2}{3}$ of 60 cm **b** $\frac{3}{4}$ of 60 cm **c** $\frac{4}{5}$ of 60 cm **d** $\frac{5}{6}$ of 60 cm

6 There are 600 people in a concert hall. $\frac{2}{5}$ of them are teenagers. $\frac{3}{8}$ of the teenagers are girls. How many teenage girls are there in the concert hall?

7 Work out these quantities.

a 60% of 750 kg **b** 15% of 40 cm **c** 30% of 1.5 litres **d** 99% of £17.00

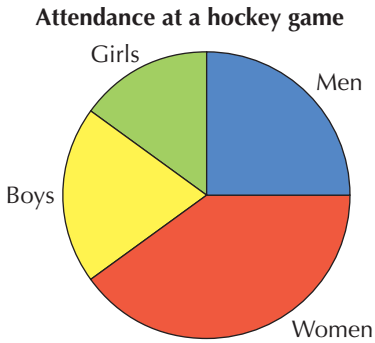
8 Work out these percentages of 7.5 kg.

a 12% **b** 72% **c** 92% **d** 2%

9 This pie chart shows the proportions of people in the crowd at a hockey game. The percentage of the crowd who are men is 25%.

a Estimate, to the nearest ten per cent, the percentage of the crowd that are women.

b There are 2300 people in the crowd.
Estimate the number of women.



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PS **10** This information was found on a website.

> Modern bronze is typically 88% copper and 12% tin



A small bronze statue has a mass of 160 g.

Work out the masses of copper and tin in the statue.

FS **11** Arthur is given £417.

He decides to donate 20% to charity.

a How much does he donate? **b** How much does he have left?

FS **12** Max pays a bill of £726 with a credit card.

There is a charge of 2% for doing this.

a What is the charge, in pounds?

b What is the total amount he has to pay?

c If Max pays a bill of £N, which of these expressions gives the credit card charge?

$2N$ $0.2N$ $0.02N$ $0.2 + N$ $0.02 + N$

13 Jan draws a rectangle. The sides are 16 cm and 24 cm long.

a Work out the area of the rectangle.

b Lou draws another rectangle. Each side is 25% longer than Jan's.

Work out:

i the lengths of the sides of Lou's rectangle **ii** the area of Lou's rectangle.

c Show that the area of Lou's rectangle is more than 50% greater than the area of Jan's rectangle.

FS **14** The price of a second-hand car is £4900. In a sale, the price is reduced by 5%.
Work out the sale price of the car.

FS **15** Josie's salary is £22 400 a year. She gets a pay rise of 7%.

Work out her salary after the increase.

16 On Friday a website gets 150 hits.

On Saturday the number of hits increases by 40%.

a Work out the number of hits on Saturday.

b On Sunday the number of hits is 40% fewer than on Friday.

Work out the number of hits on Sunday.

c Draw a bar chart to show the number of hits each day.

FS **17** Here are three sale notices.

Which one gives the greatest reduction?

Explain your answer.



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Financial skills

Income tax

Most people who earn money have to pay income tax.
You can earn a certain amount before you have to pay tax.
This is called your tax allowance.
You pay a percentage of anything over your tax allowance as income tax.
This percentage is called the tax rate.

Peter earns £20 000 in one year. His tax allowance is £10 000. The tax rate is 20%.

1 Copy and complete these sentences.

a Peter pays tax on £20 000 – £10 000 = £…

b Peter's tax bill is 20% of … = £…

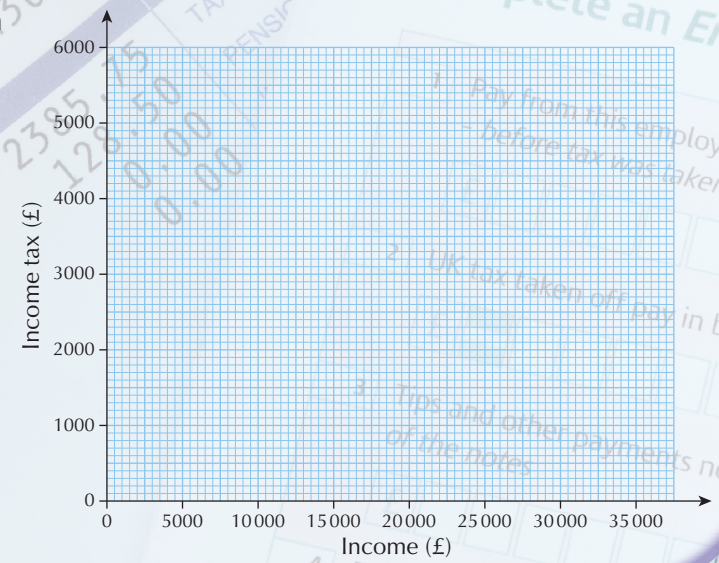
2 Work out the tax Peter will pay if he earns £30 000 in one year.

3 **a** Copy and complete this table for Peter. Fill in the answers from questions 1 and 2.

Income	£15 000	£20 000	£25 000	£30 000	£35 000
Tax					

b Use the table to draw a graph that shows how much tax Peter pays. Draw axes like this.

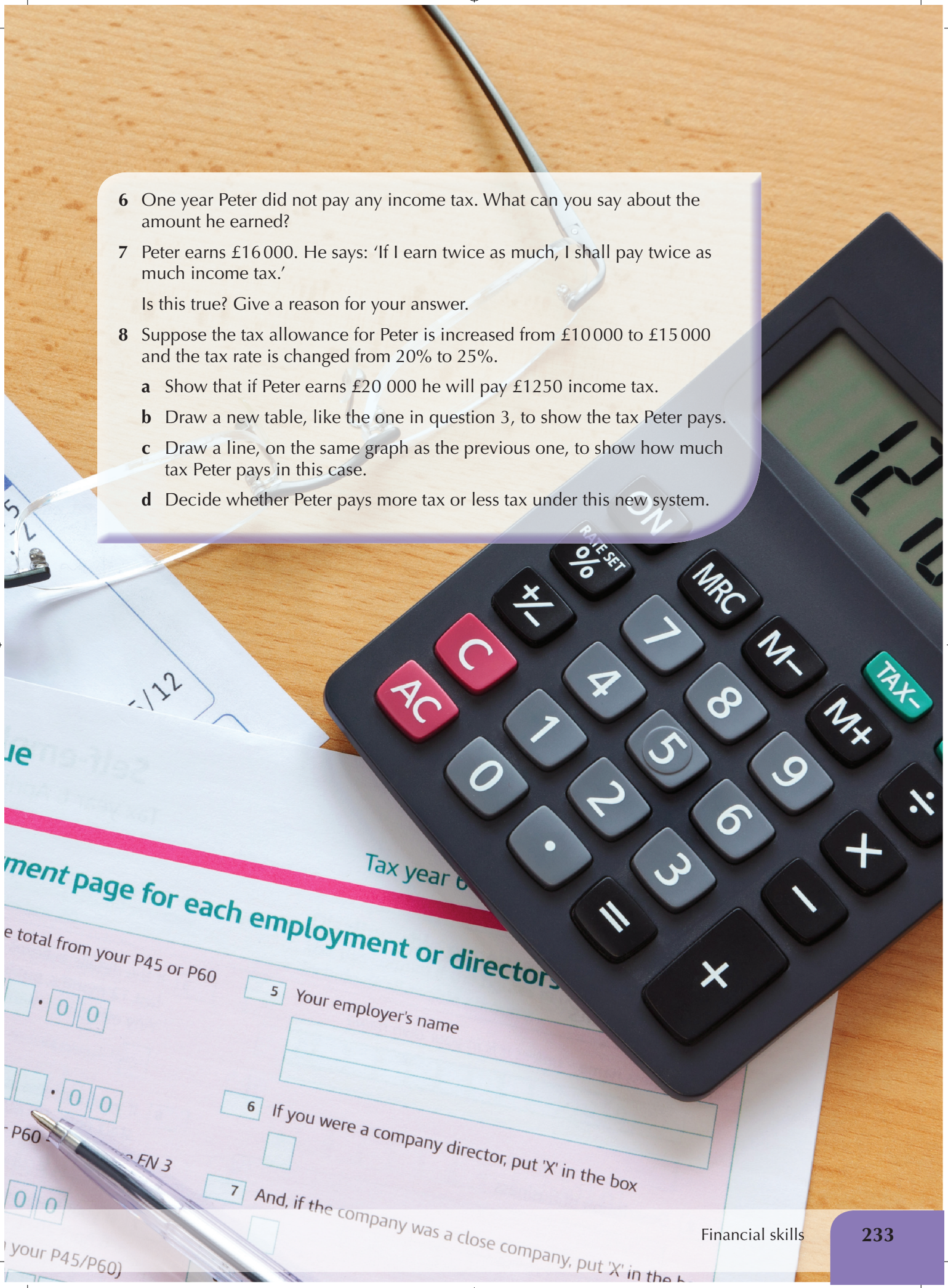
Plot five points and join them with a straight line that starts at 10 000 on the 'income' axis.



4 Use your graph to find out how much tax Peter will pay if he earns £27 500 in one year.

5 One year Peter pays £2400 income tax. Use the graph to find out how much he earned.

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6 One year Peter did not pay any income tax. What can you say about the amount he earned?

7 Peter earns £16 000. He says: 'If I earn twice as much, I shall pay twice as much income tax.'

Is this true? Give a reason for your answer.

8 Suppose the tax allowance for Peter is increased from £10 000 to £15 000 and the tax rate is changed from 20% to 25%.

a Show that if Peter earns £20 000 he will pay £1250 income tax.

b Draw a new table, like the one in question 3, to show the tax Peter pays.

c Draw a line, on the same graph as the previous one, to show how much tax Peter pays in this case.

d Decide whether Peter pays more tax or less tax under this new system.

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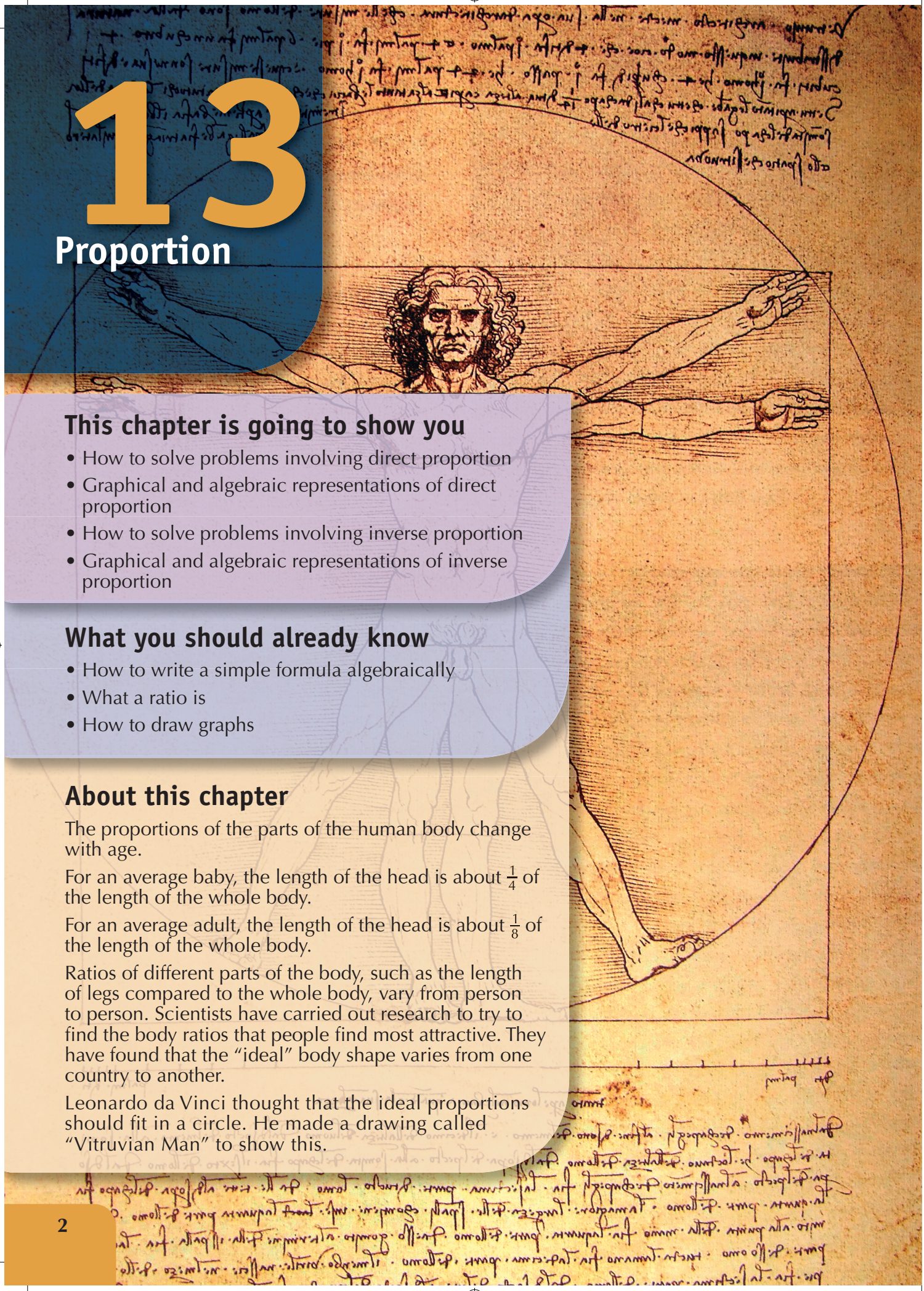
Maths Frameworking

3rd edition

Sample chapter



13 Proportion



This chapter is going to show you

- How to solve problems involving direct proportion
- Graphical and algebraic representations of direct proportion
- How to solve problems involving inverse proportion
- Graphical and algebraic representations of inverse proportion

What you should already know

- How to write a simple formula algebraically
- What a ratio is
- How to draw graphs

About this chapter

The proportions of the parts of the human body change with age.

For an average baby, the length of the head is about $\frac{1}{4}$ of the length of the whole body.

For an average adult, the length of the head is about $\frac{1}{8}$ of the length of the whole body.

Ratios of different parts of the body, such as the length of legs compared to the whole body, vary from person to person. Scientists have carried out research to try to find the body ratios that people find most attractive. They have found that the "ideal" body shape varies from one country to another.

Leonardo da Vinci thought that the ideal proportions should fit in a circle. He made a drawing called "Vitruvian Man" to show this.

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13.1 Direct proportion

Learning objectives:

- To understand the meaning of direct proportion
- To find missing values in problems involving proportion

Keywords	
direct proportion	proportional
scale factor	variable

When you buy petrol you pay a fixed price for each litre. If you buy twice or three times as much, you pay twice or three times as much. This is an example of **direct proportion**.

Two **variables** (such as the number of litres and the price) are in direct proportion if, when you multiply one by a number (such as 2 or 3 or 0.5) you multiply the other by the same number.

Here are some other pairs of variables that are in direct proportion.

- The distance travelled by a car moving at 100 km/hour and the time taken
- The volume of water flowing out of a tap and the time in seconds
- The volume of a fizzy drink and the amount of sweetener in it
- The mass of some loose carrots bought in a shop and the cost in pounds
- The time a light has been on and the cost of the electricity used
- The length of a journey in miles and the length of the same journey in kilometres

Example 1

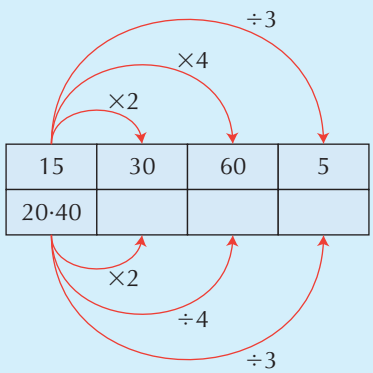
At a petrol station 15 litres of petrol cost £20.40.

Work out the cost of **a** 30 litres **b** 60 litres **c** 5 litres

Answer

It helps to put the numbers in a table.

Petrol (litres)	15	30	60	5
Cost (£)	20.40			



a $30 = 15 \times 2$ The number of litres is multiplied by 2, so the cost is also multiplied by 2.

The cost of 30 litres = £20.40 × 2 = £40.80.

b $60 = 15 \times 4$ The number of litres is multiplied by 4.

The cost of 60 litres = £20.40 × 4 = £81.60.

c $5 = 15 \div 3$ The number of litres is divided by 3. Do the same to the cost.

The cost of 5 litres is £20.40 ÷ 3 = £6.80.

If you multiply or divide the number of litres by any number, you multiply or divide the cost by the same number.

We say that the number of litres and the cost in pounds are directly **proportional**.

Sometimes we leave out the word "directly" and just say they are proportional.

You will learn about another sort of proportion later in this chapter.

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Exercise 13A

1 A train is travelling at a constant speed. The distance travelled is proportional to the time taken. In five minutes it travels 13 km. Copy and complete this table.

Time taken (minutes)	5	10	20	30	45
Distance (km)	13				

FS **2** Jacob buys 300 g of carrots and they cost 84 pence.

Work out the cost of

a 600 g of carrots **b** 900 g of carrots **c** 150 g of carrots **d** 100 g of carrots

3 250 ml of cola contains 27 g of sugar.

Work out the amount of sugar in

a 500 ml of cola **b** 1 litre of cola **c** 2 litres of cola **d** 125 ml of cola

4 1.5 kg of flour is enough to make 4 small loaves.

a How much flour is needed to make 16 small loaves?

b How many small loaves can be made from 9 kg of flour?

5 Five miles is approximately the same as eight kilometres. Copy and complete this table to show equivalent lengths.

miles	5	40	100			
kilometres	8			24	40	200

6 The perimeter of a circle is called the circumference. The circumference of a circle is proportional to the diameter of the circle. A circle with a diameter of 3.5 m has a circumference of 11 m.

Work out the circumference of a circle with a diameter of

a 7 m **b** 10.5 m **c** 35 m **d** 1.75 m

7 Water is dripping from a tap at a steady rate. In 15 minutes there are 80 drips.

a Work out the number of drips in one hour.

b Work out the time taken by 800 drips.

8 In a shop, 100 g of sweets cost 84 pence.

a Work out the cost of

i 300 g of sweets **ii** 500 g of sweets **iii** 25 g of sweets

b What mass of sweets can you buy for:

i £1.68 **ii** £8.40 **iii** 42p?

FS **9** The exchange rate between pounds and US dollars is £41 = $63. Copy and complete this table.

Pounds	£41	£205		£328	
US dollars	$63		$126		$630

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10 The pressure of a car tyre can be measured in two different units, bar or psi. 2.1 bar is the same as 30 psi. Copy and complete this table to show conversions between the two units.

bar	2.1			8.4	12.6
psi	30	10	20		

11 The mass of a steel cable is proportional to its length.

Five metres of a particular cable has a mass of 8.2 kg.

a Work out the mass of 20 metres of the cable.

b Another cable of the same type has a mass of 49.2 kg. How long is it?

12 This table shows the exchange rate between pounds and New Zealand dollars.

Pounds	£50	£150
NZ dollars	$96	

a Work out the missing value.

b Work out the ratio of the two amounts of pounds.

c Work out the ratio of the two amounts of dollars.

MR **13** Energy content on food labels is given in two different units, kilocalories (kcal) and kilojoules (kJ). Here is part of a conversion table:

kilocalories	38 kcal	
kilojoules	160 kJ	800 kJ

a Work out the missing value.

b Show that the ratio of the two amounts of kilojoules is the same as the ratio of the two amounts of kilocalories.

MR **14** Temperature can be measured in degrees C or degrees F. Here is a table of values.

Degrees C	20	30	50	100
Degrees F	68	86	122	212

Is temperature in degrees C proportional to temperature in degrees F? Give a reason for your answer.

Investigation

This table is from a US website.

It shows the average height and the average mass of a boy at different ages.

Because it is a US website, the heights are in inches and the masses are in pounds.

Age (years)	2	4	6	8
Average height (inches)	31	37	42	45
Average mass (pounds)	28.4	36.0	46.2	57.2

a Is the average height proportional to age? Use numbers from the table to justify your answer.

b Is the average mass proportional to age? Use numbers from the table to justify your answer.

c Is the average mass proportional to the average height? Use numbers from the table to justify your answer.

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13.2 Graphs and direct proportion

Learning objective:

- To represent direct proportion graphically and algebraically

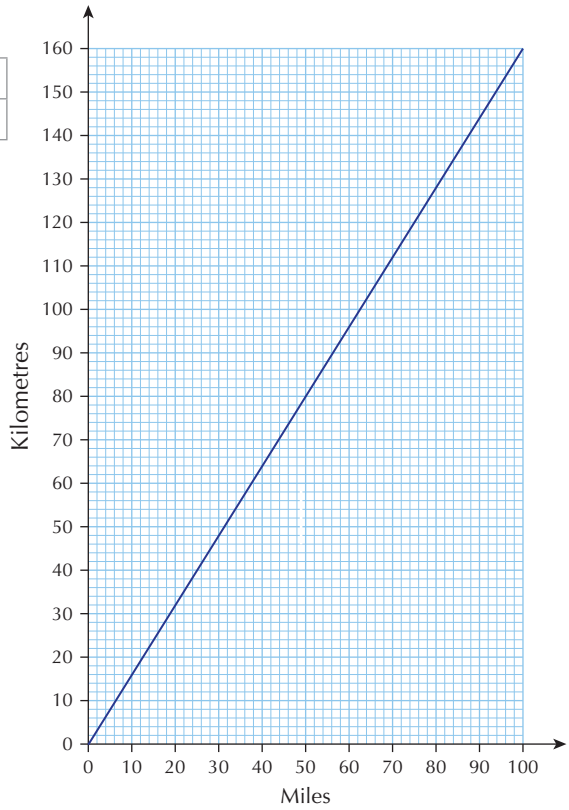
Keywords	
graph	formula

Here is a table to show the relationship between distances measured in miles and in kilometres.

They are in direct proportion.

Distance in miles	20	30	50	60	70	100
Distance in kilometres	32	48	80	96	112	160

We can plot these values on a graph and join them with a line.



You can see two things:

- The points are in a straight line
- The line passes through the origin

A graph of values of two variables in direct proportion always has these properties.

Look at the pairs of values in the table.

Check that $20 \times \mathbf{1.6} = 32$ $30 \times \mathbf{1.6} = 48$
$50 \times \mathbf{1.6} = 80$ and so on.

If x miles is the same as y kilometres we can write this formula:
$y = 1.6x$

If two variables, x and y, are in direct proportion we can always write a formula $y = mx$ where m is a number.

Example 2

Ribbon is sold by the metre. Lucy buys 6 metres and the cost is £5.04.

a Find a formula for the cost, y pence, of x metres of ribbon.

b Draw a graph to show the cost of different lengths of ribbon.

Answer

a The formula is $y = mx$ and you need to find the number m.

You know that if x is 6 then y is 504. *Notice that y is the cost in pence*

$504 = m \times 6$

$m = 504 \div 6 = 84$

The formula is $y = 84x$.

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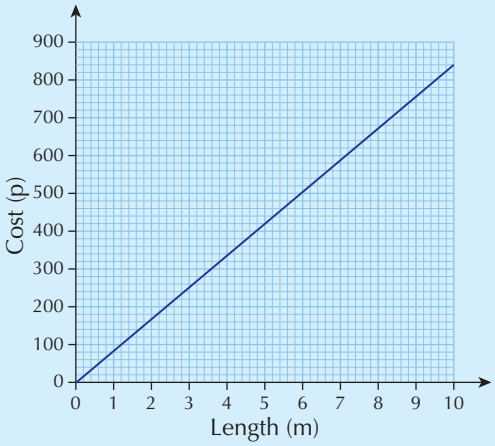
b Use the formula to find the costs of different lengths. Multiply the length by 84 to find the cost.

Choose some values for the length.

Length (x metres)	1	2	3	5	7	10
Cost (y pence)	84	168	252	420	588	840

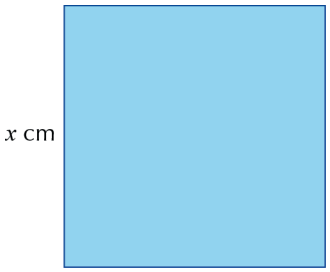
Plot the points on graph paper and join them up.

The points should be in a straight line. The line should go to the origin.



Exercise 13B

1 The perimeter (y cm) of a square of side x cm is given by the formula $y = 4x$.



a Complete this table to show values of x and y.

Side (x cm)	2	5	7	8	10
Perimeter (y cm)					

b Draw a graph to show the values in the table. Label the axes.

2 The cost, £y, of x litres of petrol is given by the formula $y = 1.3x$.

a Show that 10 litres cost £13.

b Copy and complete this table.

Petrol (x litres)	10	20	25	30	40
Cost (£y)					

c Draw a graph to show the cost of petrol.

3 A 200 ml glass of a fizzy drink contains 20 g of sugar.

a Copy and complete this table.

Drink (x ml)	200	100	500	1000
Sugar (y g)	20			

b If x ml of fizzy drink contain y g of sugar, show that $y = 0.1x$.

c Draw a graph to show the values in the table.

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4 This table shows the price of different masses of potatoes.

Mass (x kg)	0.5	1	1.5	2	3
Price (y pence)		48		96	

a Work out the missing values of y.

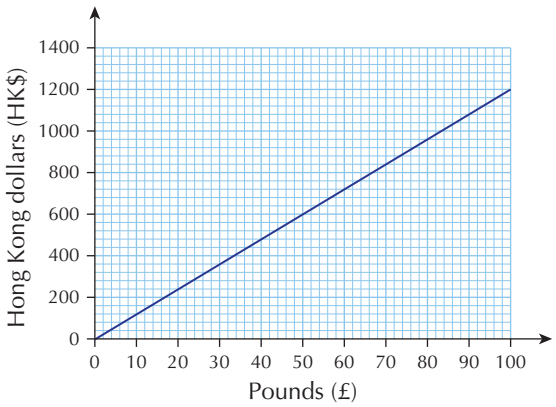
b What do you multiply the mass by to find the price?

c Work out a formula for y in terms of x.

d Use the formula to find the cost of 7.5 kg of potatoes.

e Draw a graph to show the cost of potatoes.

5 This graph shows the exchange rate between pounds and Hong Kong dollars.



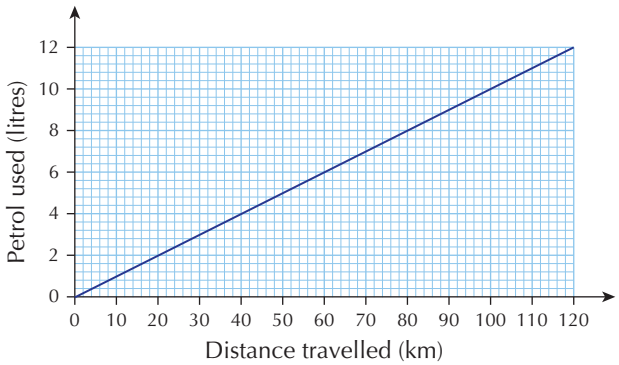
a Use the graph to complete this table.

Pounds (£x)	25	50	75	100
Hong Kong dollars (HK$$y$)				

b Work out a formula for y in terms of x.

Use the formula to change £1270 into Hong Kong dollars.

6 A car is travelling at 80 km/h. The amount of petrol used for different distances travelled is shown in this graph.



a Use the graph to complete this table.

Distance (x km)	30	60	70	85	110
Petrol (y litres)					

b Write a formula for y in terms of x.

c Use your formula to work out the amount of petrol used to travel 430 km.

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7 The speed of a car can be measured in metres per second (m/s) or in kilometres per hour (km/h). 5 m/s is the same as 18 km/h.

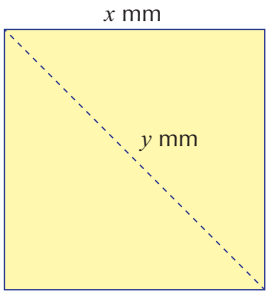
a Write 10 m/s in km/h.

b What do you multiply a speed in m/s by to find the speed in km/h?

c y km/h is the same as x m/s

d Write down a formula for y in terms of x.

8 The lengths of the side and the diagonal of a square are proportional.



a This table shows possible values of x and y. Fill in the missing values.

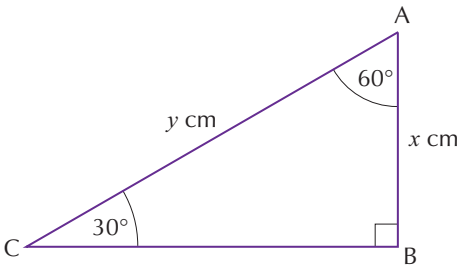
Side (x mm)	5	10	15	20	25
Diagonal (y mm)	7				

b Work out a formula for y in terms of x.

c Use the formula to find the diagonal of a square if the length of the side is

i 12 mm **ii** 19 mm **iii** 31 mm

9 The angles of this triangle are 30°, 60° and 90°.



The lengths of AB and AC are proportional.

Here is an incomplete table showing possible values of x and y.

AB (x cm)	4.1	5.2	12.9		
AC (y cm)	8.2			18.8	6.4

a Write down a formula for y in terms of x.

b Work out the missing values in the table.

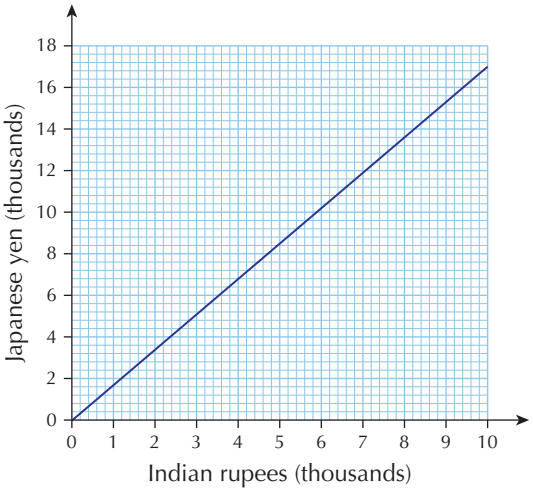
c Draw your own triangle with angles of 30°, 60° and 90°, like the one in the diagram. Measure x and y and check that they agree with your formula.

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Challenge

FS

This graph shows the exchange rate between two currencies, the Indian rupee and the Japanese yen.



a Work out a formula connecting the values of the two currencies. Show your method.

b Explain how to use your formula to convert Indian rupees into Japanese yen.

c Could you use your formula to convert Japanese yen into Indian rupees? If so, explain how to do this.

13.3 Inverse proportion

Learning objectives:

- To understand what is meant by inverse proportion
- To use graphical and algebraic representations of inverse proportion

Keywords	
inverse proportion	formula

A car journey is 120 km.

The time it takes depends on the speed of the car.

If the car travels at 60 km/h the journey will take 120 ÷ 60 = 2 hours.

If the car travels at 40 km/h the journey will take 120 ÷ 40 = 3 hours.

Here is a table to show the times taken at different speeds.

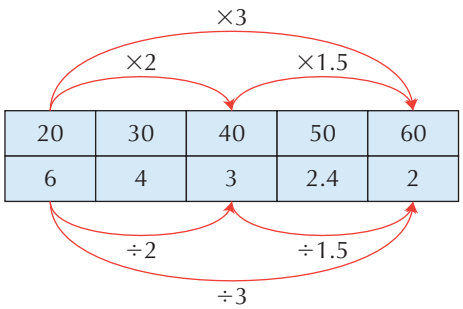
Speed (x km/h)	20	30	40	50	60
Time (y hours)	6	4	3	2.4	2

It you multiply the speed by any number, you divide the time by the same number.

For example, $20 \times 2 = 40$ and $6 \div 2 = 3$

or $40 \times 1.5 = 60$ and $3 \div 1.5 = 2$.

x and y are in **inverse proportion**.



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When two variables are in inverse proportion, if you multiply one by a number you divide the other by the same number.

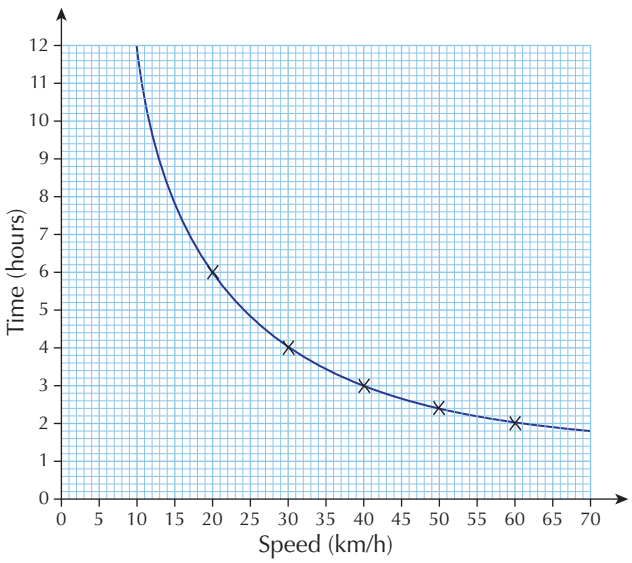
If you multiply the speed by the time then the answer is always 120 because speed × time = distance.

For example, $20 \times 6 = 120$ or $50 \times 2.4 = 120$.

If the speed is x km/h and the time is y hours, you can write this formula: $xy = 120$

When x and y are in inverse proportion you can always write a formula $xy = k$ where k is a number.

Here is a graph to show the numbers in the table:



The points are not in a straight line. A smooth curve is drawn through them.

Notice that the curve passes through (10, 12). What does that tell you about a speed and a time?

Graphs of inverse proportion are always this shape.

The variables do not have to be x and y. You can use any letters.

Example 3

The area of a rectangular field is 1200 m^2.

The sides of the field are a metres and b metres long.

Here is a table showing possible sides of the field.

a metres	40	50	
b metres			60

a Find the missing values.

b Work out a formula connecting a and b.

Answer

a The area of the field is the lengths of the two sides multiplied together.

If $a = 40$, $40 \times b = 1200 \rightarrow b = 1200 \div 40 = 30$.

If $a = 50$, $50 \times b = 1200 \rightarrow b = 1200 \div 50 = 24$.

If $b = 60$, $b \times 60 = 1200 \rightarrow b = 1200 \div 60 = 20$.

b The formula is $ab = 1200$.

In this case the variables, a and b, are inversely proportional.

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Exercise 13C

1 A train is travelling a distance of 600 km.

a How long does the train take if it travels at 100 km/h?

b If the train travels at 150 km/h, how long will it take?

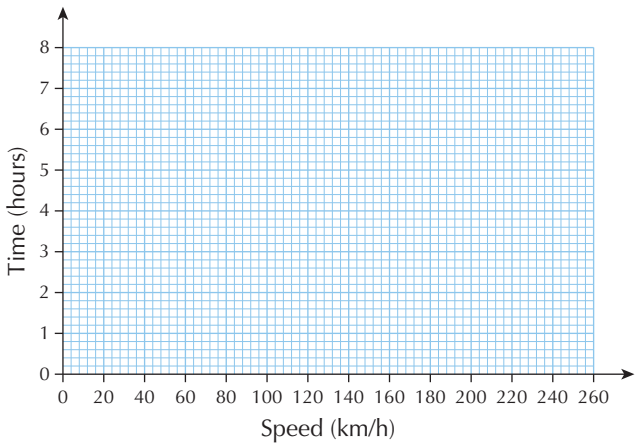
c Copy and complete this table.

Speed (x km/h)	100	120	150	200	300
Time (y hours)					

d Show that x and y are inversely proportional.

e Write down a formula connecting x and y.

f Draw a pair of axes like this.



g Plot the points from the table in part **c** and join them with a smooth curve.

2 A teacher has £1000 to spend on books.

a If books cost £10 each, how many can the teacher buy?

b If books cost £5 each, how many can the teacher buy?

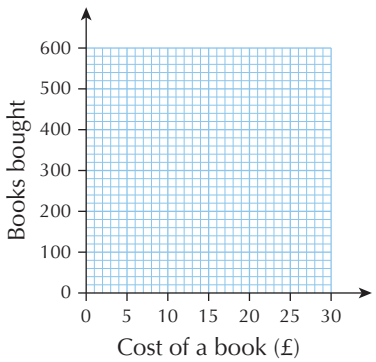
c Copy and complete this table.

Cost of a book (£x)	2	2.50	5	10	20	25
Number bought (y)						

d Show that x and y are inversely proportional.

e Write down a formula connecting x and y.

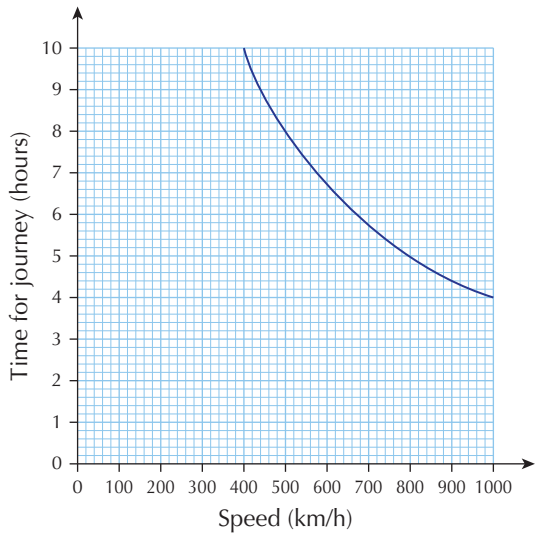
f Draw a pair of axes like this.



g Draw a graph to show the information in the table.

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3 The graph shows the time taken by a plane to travel between two airports at different speeds.



a Use the graph to find the time taken if the plane flies at 800 km/h.

b Use the graph the find the speed if the plane takes 4 hours.

The time (y hours) is inversely proportional to the speed (x km/h).

c Use your answers to **a** and **b** to find a formula connecting x and y.

4 Children are measuring the length of a pace and how many paces they take to walk 12 metres.

a If the length of a pace is 0.5 metres, how many paces are needed to walk 12 metres?

b Find the missing values in this table.

Length of pace (p metres)	0.5	0.6	1	1.2
Number of paces (n)				

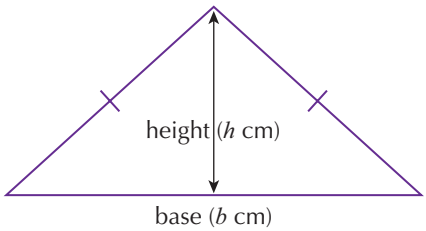
c Show that p and n are inversely proportional.

d Write down a formula connecting p and n.

MR **5** An isosceles triangle has an area of 100 cm^2.

The length of the base (b cm) is inversely proportional to the height (h cm).

If the base is 20 cm then the height is 10 cm.



a Write down a formula connecting b and h.

b Copy and complete this table to show possible values of b and h.

Base (b cm)	20	16	12	10	8
Height (h cm)	10				

c Draw a graph to show how the height varies with the base.

d Use your graph to find the height if the base is 15 cm.

e Use your formula to check the answer to part **d**.

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FS **6** Some families want to buy play equipment for their local park. The total cost is £30 000. The families agree to share the cost between them equally.

a If there are 20 families, work out the cost for each family.

b If there are 30 families, work out the cost for each family.

c Copy and complete this table of values.

Number of families (n)	10	20	30	40	50	60
Cost for each family (£c)						

d Is the cost for each family inversely proportional to the number of families? Justify your answer.

e Draw a graph to show how the cost varies with the number of families.

f They decide that each family should not pay more than £800. Use your graph to work out the smallest number of families that are needed.

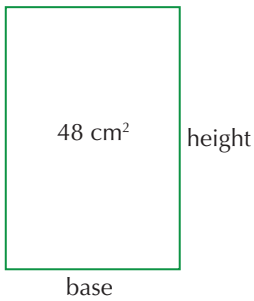
g Write down a formula connecting n and c and use it to check your answer to part **f**.

Activity

1 Draw two different rectangles with an area of 48 cm^2

2 Copy and complete this table showing possible lengths for the height and base of a rectangle with an area of 48 cm^2.

Base (x cm)	4	5	6	8	10	12
Height (y cm)						



3 Draw a graph to show your values.

4 Write down a formula to show the connection between x and y.

5 Use your graph to find the length of the side of a square with an area of 48 cm^2.

13.4 Comparing direct proportion and inverse proportion

Learning Objective:

- To recognise direct and inverse proportion and work out missing values

You have learnt about direct proportion and inverse proportion. Here is a summary.

Direct proportion

x	3	4.5	6	15	20
y	24	36	48	120	160

x and y are directly proportional

If you multiply (or divide) a value of x by a number, you multiply (or divide) y by the same number.

You can always write $y = mx$ where m is a number.

In this example $y = 8x$.

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Inverse proportion

x	3	4.5	6	15	20
y	24	16	12	4.8	3.6

x and y are inversely proportional

If you multiply a value of x by a number, you divide y by the same number.

You can always write $xy = k$ where k is a number.

In this example $xy = 72$.

Check that each pair of numbers does multiply to make 72.

Example 4

Here are the values of two variables, p and q.

p	20	50
q	90	

a Find the missing value if q is directly proportional to p.

b Find the missing value if q is inversely proportional to p.

Answer

a $q \div p = 90 \div 20 = 4.5$ So $q = 4.5p$

If $p = 50$ then $q = 4.5 \times 50 = 225$

b $pq = 20 \times 90 = 1800$

If $pq = 1800$ and $p = 50$ then $50 \times q = 1800 \rightarrow q = 1800 \div 50 = 36$

Exercise 13D

1 George is walking at a constant speed. In 5 minutes he walks 400 metres.

a Is the distance travelled (d metres) proportional to the time taken (t minutes)?

b Write down a formula for d in terms of t.

c Use the formula to work out how far George walks in 8.5 minutes.

2 Anne is doing a sponsored walk of 20 km.

a If she walks at 5 km/ h how long will it take?

b If she walks at 8 km/ h how long will it take?

c Explain why the time taken (t hours) is inversely proportional to her walking speed (w km/h).

d Write down a formula connecting t and w.

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MR **3** Here are some tables of values. In each case, say whether they show direct proportion, inverse proportion or neither of these. Give a reason in each case. If the variables are directly or inversely proportional, work out the formula.

a

x	12	17	5	14
y	36	51	20	42

b

c	18	30	12	1.5
d	5	3	7.5	60

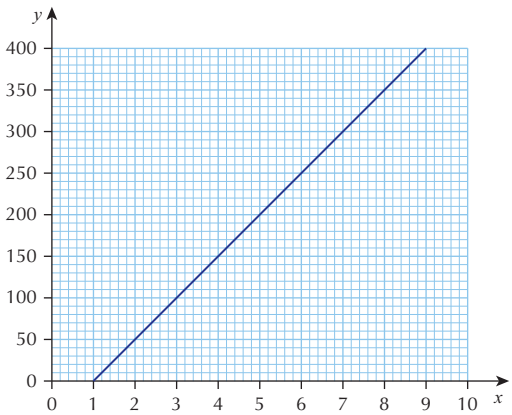
c

f	5.6	9.4	63.8	3.6
r	75.6	126.9	861.3	48.6

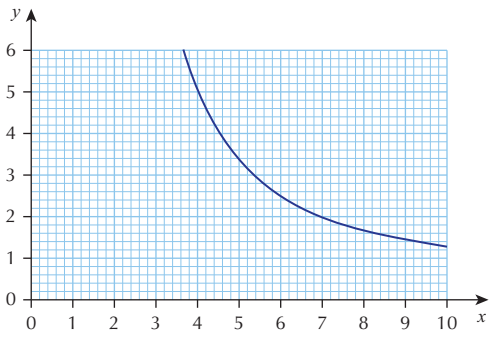
d

u	12	16	14.8	46.25
w	12	9	10	3.2

MR **4** How can you tell that this graph does not show direct proportion?



5



a Two points on this line are (4, ….) and (7, ….).
Find the missing y coordinates.

b Show that x and y are <u>not</u> in inverse proportion.

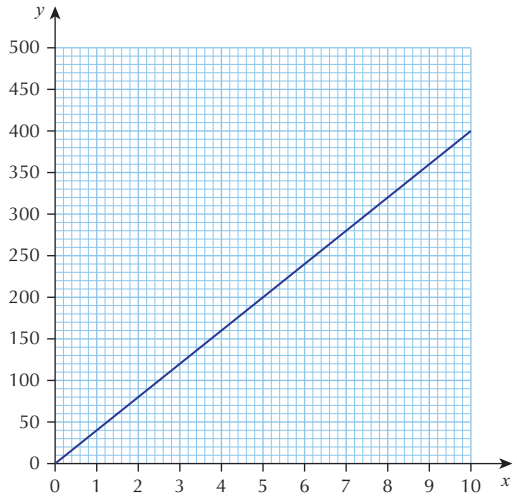
6 x and y are two variables.

a They are in direct proportion and when $x = 40$, $y = 10$.
Work out a formula for y in terms of x.

b They are in inverse proportion and when $x = 40$, $y = 10$.
Work out a formula connecting x and y.

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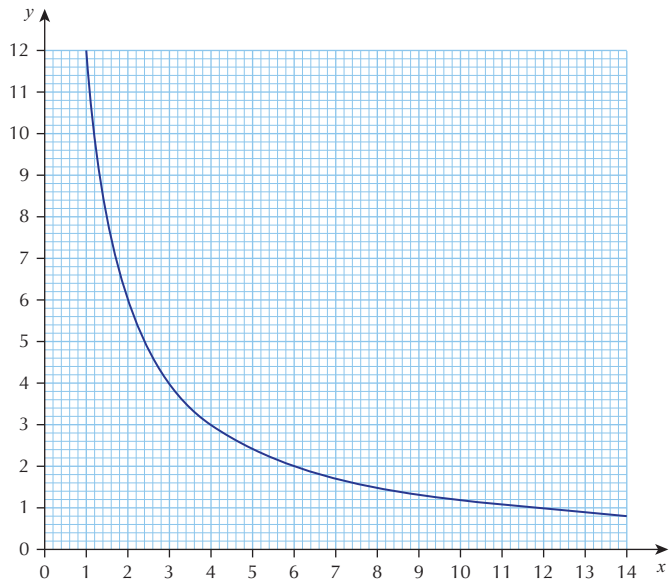
7 This graph shows two variables in direct proportion.



a Write down the coordinates of two points on the line.

b Work out a formula for y in terms of x.

8 This graph shows two variables in inverse proportion.



a Write down the coordinates of three points on the line.

b Work out a formula for y in terms of x.

Reasoning

The perimeter of a rectangle is 20 cm.

a If one side of the rectangle is 3 cm, show that the other side is 7 cm.

b Work out three other pairs of values for the two sides of the rectangle.

c Draw a graph to show the pairs of values and draw a line through them.

d Are the lengths of the two sides in direct proportion? Justify your answer.

e Are the lengths of the two sides in inverse proportion? Justify your answer.

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Ready to progress?

I can decide whether two variables are in direct proportion or inverse proportion

I can find a formula for two variables in direct proportion
I can recognise and draw a graph for two variables in direct proportion
I can find a formula connecting two variables in inverse proportion
I can recognise and draw a graph for two variables in inverse proportion

Review questions

1 The number of pages of a particular book and the time to read them are proportional.
Work out the missing numbers in this table.

Number of pages	20	40	100
Time to read them (minutes)	17 minutes		

2 A rectangle where the length is 1.6 times the width is called a "Golden Rectangle".
Copy and complete this table to show some possible sizes for a "Golden Rectangle".

Length	5 cm	20 cm	50 cm	1.5 m	4 m
Width					

3 The mass of five maths text books is 3.48 kg.
Work out the mass of

a 10 books **b** 20 books **c** 200 books

4 The cost of oranges is proportional to the number bought. Eight oranges cost £2.24.
Work out the cost of

a 16 oranges **b** 24 oranges **c** 4 oranges

5 The average cost of heating the water for a shower is £0.20.

a Work out the cost, in pounds of

i 7 showers (one each day for a week) **ii** 30 showers (one each day for a month)
iii 365 showers (one each day for a year)

b Write down a formula for the cost (£c) of n showers.

6 The cost of a taxi journey is proportional to the distance travelled.

a Work out the missing values in this table.

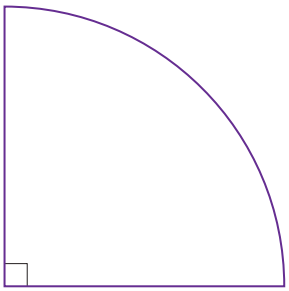
Distance (x km)	4	8	20	2
Cost (£y)	14			

b Work out a formula for the cost (£y) of travelling x kilometres.

c Draw a graph to show the values in the table.

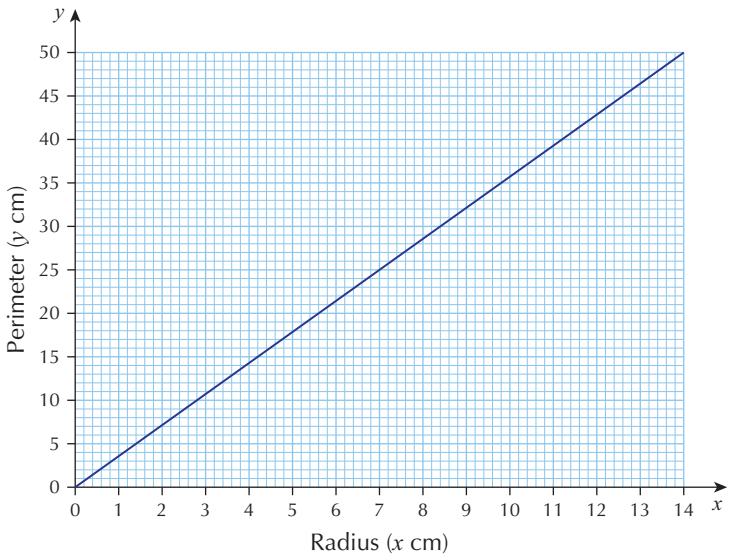
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7 This shape is called a quadrant. It is a quarter of a circle.



radius x cm

This graph shows the perimeter (y cm) of a quadrant of radius x cm.



a Explain why the graph shows that the perimeter of the quadrant is proportional to the radius.

b Find the perimeter of a quadrant of radius 10 cm.

c Work out a formula for the perimeter (y cm) of the quadrant in terms of the radius (x cm).

d Use your formula to find the perimeter of a quadrant of radius 43 cm.

8 A survey of one hundred 50-year olds shows that 27 of them wear glasses.
If the sample in the survey is a fair representation of all 50-year olds, how many out of 2000 do you expect will wear glasses?

9 The time taken to complete a rail journey is inversely proportional to the average speed of the train. At a speed of 100 km per hour the journey takes 4.5 hours.
Find the missing values in this table.

Speed (km/h)	100	50	150
Time (hours)	4.5		

10 The time to walk to the end of a playing field is inversely proportional to walking speed. Amelia walks at 1.6 m/s and takes 200 seconds.

a Lily walks at 0.8 m/s. How long does she take?

b Emily runs at 4.8 m/s. How long does she take?

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11 Here are the lengths and widths of four rectangles with the same area.

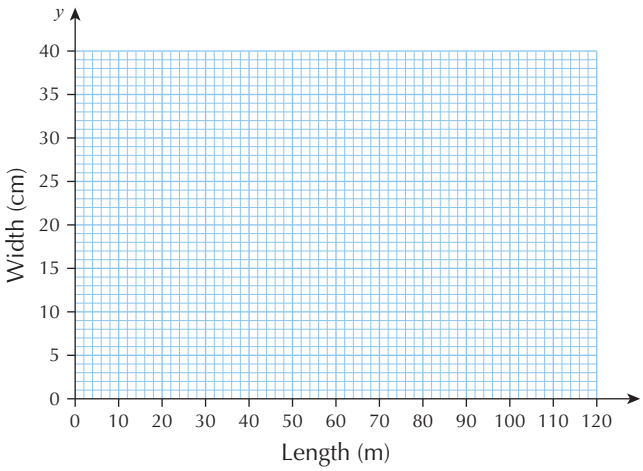
Length (x cm)	120	100	80	60
Width (y cm)	10	12	15	20

a Show that the length is inversely proportional to the width.

b The length of a fifth rectangle is 40 cm. Work out the width.

c Work out a formula connecting x and y.

d Use the dimensions of the five rectangles to draw a graph to show values of x and y. Use axes like this:



12 A fence is being put along one side of a school playground.



The fence is made from separate panels. The number of panels needed is inversely proportional to the length of each panel.

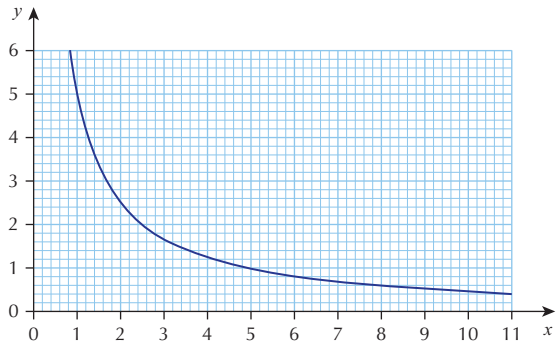
a Work out the missing numbers in the table.

Length of panel (l m)	1.2	1.5	1.8	2.4	3.0
Number of panels (n)	150	120			

b Work out a formula connecting l and n.

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13 This graph shows the values of two variables in inverse proportion.



a Find the missing coordinate for these points on the line:
i (5, ….) **ii** (10, ….) **iii** (2, ….) **iv** (….. ,1)

b Work out a formula connecting x and y.

c Use your formula to find the value of y if $x = 25$.

14 Here is a table of values of r and s.

r	12	48
s	8	?

a If s is directly proportional to r, work out the missing value of s.

b If s is inversely proportional to r, work out the missing value of s.

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Problem Solving, Reasoning, Financial Skills

A Grand Day Out

A youth leader is planning a coach trip for some youth club members.

1. Cost of the coach

The total cost of the coach is £800.

This must be shared equally by the people on the coach.

a Copy and complete this table.

Number of passengers (x)	50	40	32	25	20
Cost for each person (£y)					

b Show that the number of passengers and the cost per person are in inverse proportion.

c Draw a graph to show how the cost per person varies with the number of people.

2. Time and distance

On the motorway the coach travels at a constant speed of 90 km/h.

a How far does the coach travel in 30 minutes on the motorway?

b How far does the coach travel in 10 minutes on the motorway?

c Copy and complete this table.

Time (minutes)	10	20	30	40	50	60
Distance (km)						

d Is the distance proportional to the time? Justify your answer.

e Draw a graph to show the data in the table.

3. Speed and time

The time to complete the whole journey is inversely proportional to the average speed.

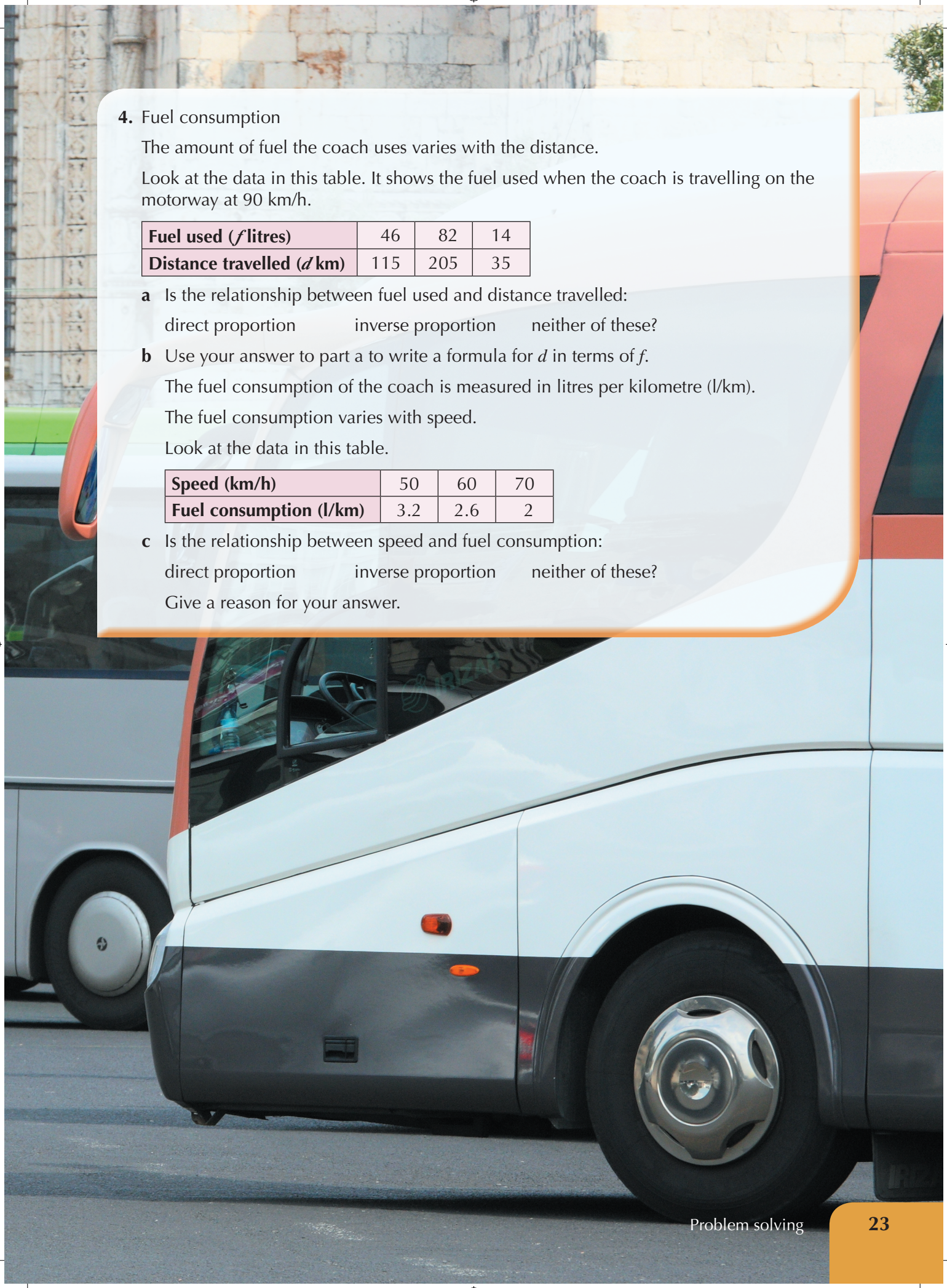
a Complete this table

Average speed (x km/h)	50	60	75	80	100
Time (y hours)	4.8	4			

b Write down a formula connecting x and y.

c Draw a graph to show how the time varies with the average speed.

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4. Fuel consumption

The amount of fuel the coach uses varies with the distance.

Look at the data in this table. It shows the fuel used when the coach is travelling on the motorway at 90 km/h.

Fuel used (f litres)	46	82	14
Distance travelled (d km)	115	205	35

a Is the relationship between fuel used and distance travelled:

direct proportion inverse proportion neither of these?

b Use your answer to part a to write a formula for d in terms of f.

The fuel consumption of the coach is measured in litres per kilometre (l/km).

The fuel consumption varies with speed.

Look at the data in this table.

Speed (km/h)	50	60	70
Fuel consumption (l/km)	3.2	2.6	2

c Is the relationship between speed and fuel consumption:

direct proportion inverse proportion neither of these?

Give a reason for your answer.

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Collins

Pupil Book 3.2

Maths Frameworking

3rd edition

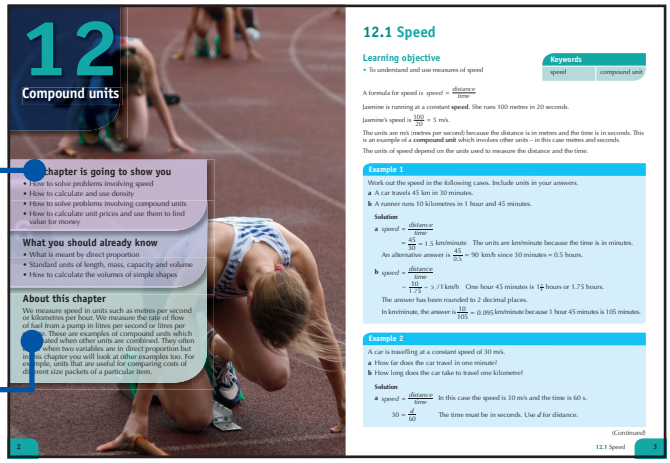
Sample chapter



How to use this book

Learning objectives

See what you are going to cover and what you should already know at the start of each chapter.



About this chapter

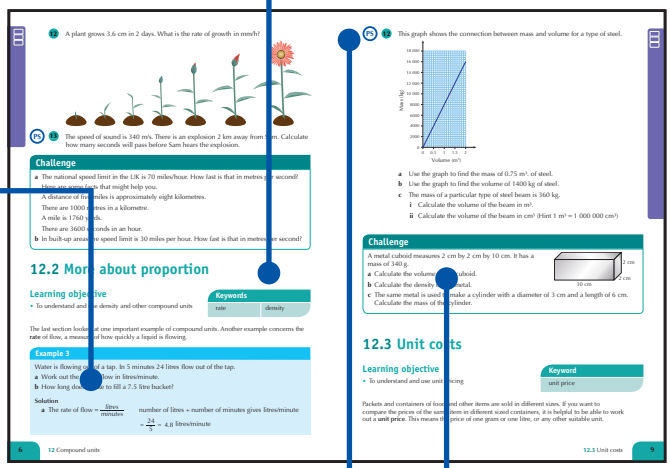
Find out the history of the maths you are going to learn and how it is used in real-life contexts.

Key words

The main terms used are listed at the start of each topic and highlighted in the text the first time they come up, helping you to master the terminology you need to express yourself fluently about maths. Definitions are provided in the glossary at the back of the book.

Worked examples

Understand the topic before you start the exercises, by reading the examples in blue boxes. These take you through how to answer a question step by step.



Skills focus

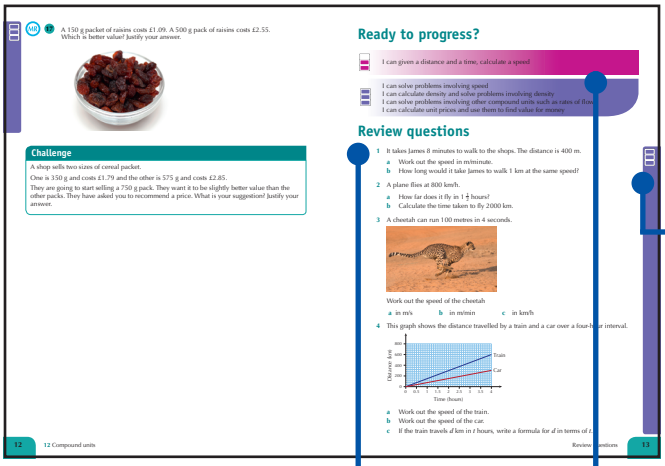
Practise your problem-solving, mathematical reasoning and financial skills.

PS MR FS

Take it further

Stretch your thinking by working through the **Investigation**, **Problem solving**, **Challenge** and **Activity** sections. By tackling these you are working at a higher level.

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Progress indicators

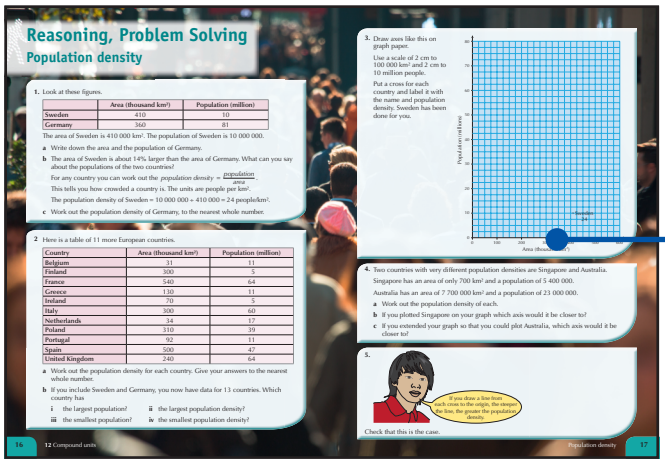
Track your progress with indicators that show the difficulty level of each question.

Ready to progress?

Check whether you have achieved the expected level of progress in each chapter. The statements show you what you need to know and how you can improve.

Review questions

The review questions bring together what you've learnt in this and earlier chapters, helping you to develop your mathematical fluency.



Activity pages

Put maths into context with these colourful pages showing real-world situations involving maths. You are practising your problem-solving, reasoning and financial skills.

Interactive book, digital resources and videos

A digital version of this Pupil Book is available with interactive classroom and homework activities, assessments, worked examples and tools that have been specially developed to help you improve your maths skills. Also included are engaging video clips that explain essential concepts, and exciting real-life videos and images that bring to life the awe and wonder of maths.

Find out more at www.collins.co.uk/connect

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12 Compound units



This chapter is going to show you

- How to solve problems involving speed
- How to calculate and use density
- How to solve problems involving compound units
- How to calculate unit prices and use them to find value for money

What you should already know

- What is meant by direct proportion
- Standard units of length, mass, capacity and volume
- How to calculate the volumes of simple shapes

About this chapter

We measure speed in units such as metres per second or kilometres per hour. We measure the rate of flow of fuel from a pump in litres per second or litres per minute. These are examples of compound units which are created when other units are combined. They often occur when two variables are in direct proportion but in this chapter you will look at other examples too. For example, units that are useful for comparing costs of different size packets of a particular item.

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12.1 Speed

Learning objective

- To understand and use measures of speed

Keywords	
speed	compound unit

A formula for speed is $speed = \frac{distance}{time}$

Jasmine is running at a constant **speed**. She runs 100 metres in 20 seconds.

Jasmine's speed is $\frac{100}{20} = 5$ m/s.

The units are m/s (metres per second) because the distance is in metres and the time is in seconds. This is an example of a **compound unit** which involves other units – in this case metres and seconds.

The units of speed depend on the units used to measure the distance and the time.

Example 1

Work out the speed in the following cases. Include units in your answers.

a A car travels 45 km in 30 minutes.

b A runner runs 10 kilometres in 1 hour and 45 minutes.

Solution

a $speed = \frac{distance}{time}$

$= \frac{45}{30} = 1.5$ km/minute The units are km/minute because the time is in minutes.

An alternative answer is $\frac{45}{0.5} = 90$ km/h since 30 minutes = 0.5 hours.

b $speed = \frac{distance}{time}$

$= \frac{10}{1.75} = 5.71$ km/h One hour 45 minutes is $1\frac{3}{4}$ hours or 1.75 hours.

The answer has been rounded to 2 decimal places.

In km/minute, the answer is $\frac{10}{105} = 0.095$ km/minute because 1 hour 45 minutes is 105 minutes.

Example 2

A car is travelling at a constant speed of 30 m/s.

a How far does the car travel in one minute?

b How long does the car take to travel one kilometre?

Solution

a $speed = \frac{distance}{time}$ In this case the speed is 30 m/s and the time is 60 s.

$30 = \frac{d}{60}$ The time must be in seconds. Use d for distance.

(*Continued*)

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$30 \times 60 = d$ Multiply by 60 to solve the equation.

$d = 1800$

The distance is 1800 m or 1.8 km. You must include the units.

b $speed = \frac{distance}{time}$ In this case the distance is 1000 m.

$30 = \frac{1000}{t}$ The distance must be in metres. Use t for time.

$30t = 1000$ First multiply by t.

$t = \frac{1000}{30} = 33.33 \ldots$ Now divide by 30.

It takes 33 seconds. It is sensible to round the answer to the nearest second.

Exercise 12.1

1 A marathon runner runs 40 km in $2\frac{1}{2}$ hours. Work out his speed in km/h.

2 A train is travelling at a constant speed. It takes 30 minutes to travel 45 km.



a Work out the speed in km/minute.

b Work out the speed in km/h.

3 Calculate the speed in each case. Put units in each answer.

a Peter runs 320 metres in 50 seconds.

b A car travels 15 km in 10 minutes.

c A plane flies 400 km in half an hour.

d A cyclist travels 1500 m in 4 minutes.

4 Matthew is cycling at 18 km/h.
Calculate how far he travels in

a 1 hour **b** 4 hours **c** 1.5 hours **d** 2 hours and 30 minutes.

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5 Natalie can swim at a constant speed of 3 km/h.



Calculate how far she can swim in

a 1.5 hours **b** $\frac{1}{2}$ hour **c** $\frac{1}{4}$ hour **d** 1 minute.

6 Calculate the distance travelled in each case.

a Sharon walks at 3 m/s for two minutes.

b Nathan drives at 80 km/h for 15 minutes.

c A plane flies at 700 km/h for 4.5 hours.

d A snail moves at 0.2 m/minute for 150 seconds.

7 The top speed of a sprinter is 8 m/s.

Calculate the time it takes at that speed to sprint

a 40 m **b** 80 m **c** 50 m **d** 200 m.

8 Calculate the times taken for these journeys.

a 40 km at 120 km/h

b 22 km at 4 km/h

c 200 m at 5 m/s

d 5 km at 4 m/s

9 Anita is walking in the countryside at 6 km/h.

a Copy and complete this table.

Time (t hours)	0.5	1	1.5	2	2.5
Distance (d km)					

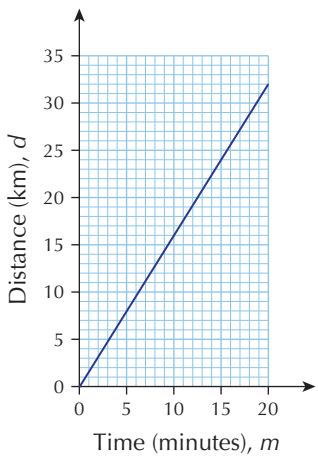
b Work out how long it takes Anita to travel 8 km.

10 A plane is flying at 850 km/h.

a Calculate how far the plane flies in $3\frac{1}{2}$ hours.

b Calculate the time the plane takes to fly 5000 km.

11 This graph shows the journey of a car.



a How far does the car travel in 20 minutes?

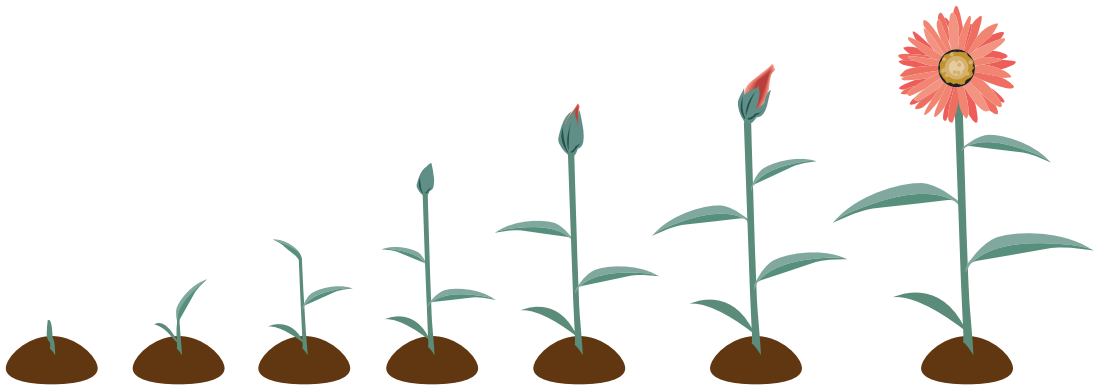
b Explain how you know the car is travelling at a constant speed.

c Work out the speed of the car in km/minute.

d How long does it take the car to travel 50 km?

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12 A plant grows 3.6 cm in 2 days. What is the rate of growth in mm/h?



PS **13** The speed of sound is 340 m/s. There is an explosion 2 km away from Sam. Calculate how many seconds will pass before Sam hears the explosion.

Challenge

a The national speed limit in the UK is 70 miles/hour. How fast is that in metres per second?

Here are some facts that might help you.

A distance of five miles is approximately eight kilometres.

There are 1000 metres in a kilometre.

A mile is 1760 yards.

There are 3600 seconds in an hour.

b In built-up areas the speed limit is 30 miles per hour. How fast is that in metres per second?

12.2 More about proportion

Learning objective

- To understand and use density and other compound units

Keywords

rate	density

The last section looked at one important example of compound units. Another example concerns the **rate** of flow, a measure of how quickly a liquid is flowing.

Example 3

Water is flowing out of a tap. In 5 minutes 24 litres flow out of the tap.

a Work out the rate of flow in litres/minute.

b How long does it take to fill a 7.5 litre bucket?

Solution

a The rate of flow $= \frac{\textit{litres}}{\textit{minutes}}$ number of litres ÷ number of minutes gives litres/minute

$= \frac{24}{5} = 4.8$ litres/minute

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b $4.8 = \frac{7.5}{m}$ — Call the number of minutes m.

$4.8m = 7.5$ — Multiply by m.

$m = \frac{7.5}{4.8} = 1.5625$ — Divide by 4.8.

It takes 1.56 minutes. — Round off the answer. It is just over 1½ minutes.

Another example of a compound unit is **density**.

For a quantity of a substance, $density = \frac{mass}{volume}$.

Examples of possible units are g/cm³ or g/litre.

If you compare equal volumes of different substances, the denser one will be heavier.

Example 4

A piece of iron has a volume of 20 cm³ and a mass of 158 g.

a Calculate the density of iron.

b Calculate the mass of 36 cm³ of iron.

Solution

a $density = \frac{mass}{volume} = \frac{158}{20}$

$= 7.9$ g/cm³ — The units must involve grams and cm³.

b $density = \frac{mass}{volume}$

$7.9 = \frac{m}{36}$ — Use the answer from part a. Use m for the mass.

$m = 7.9 \times 36 = 284.4$ — Multiply by 36.

The mass is 284 g.

Exercise 12.2

1 A driver takes 20 seconds to put 56 litres of petrol in a fuel tank. What is the rate of flow of the petrol? Give units in your answer.

2 A tap is dripping. In 20 minutes 0.3 litres drip from the tap. What is the rate in litres/h?



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3 The water flows down a stream at a rate of 6 litres/s.

a Calculate how much water flows in one minute.

b Calculate how much water flows in half an hour.

c Work out how long it takes for 1000 litres to flow past a particular point.

4 A shower has a rate of flow of 12 litres/minute.

a Work out how much water is used if Sam has a shower for four minutes.

b Work out how long it takes to use 1 litre of water.

5 Work out the density of the following.

a A piece of metal with a mass of 24 g and a volume of 108 cm^3.

b A piece of wood which has a mass of 400 g and a volume of 320 cm^3.

c A piece of plastic with a mass of 75 g and a volume of 165 cm^3.

6 Balsa wood is used to make model aeroplanes because it has a low density of 0.16 g/cm^3.

a A piece of balsa wood has a volume of 25 cm^3. Calculate its mass.

b Calculate the volume of 1 g of balsa wood.

7 Oxygen has a density of 1.43 g/litre. Work out the mass of 200 litres of oxygen.

8 Oak has a density of about 0.75 g/cm^3.



a Work out the mass of 1000 cm^3 of oak.

b Work out the volume of a piece of oak with a mass of 150 g.

c An oak plank measures 3 cm by 15 cm by 120 cm. Work out its mass.

9 Graphite is used to make pencils. It has a density of 2.3 g/cm^3.

a Work out the mass of 50 g of graphite.

b A pencil contains 1.2 cm^3 of graphite. Work out the mass of graphite in 10 pencils.

c Work out the mass of 100 cm^3 of graphite.

10 A piece of copper has a mass of 45 g and a volume of 5 cm^3.

a Work out the density of copper.

b Work out the volume of 120 g of copper.

c Work out the mass of a cube of copper with a side of 5 cm.

PS **11** The density of gold is 19.3 g/cm^3.

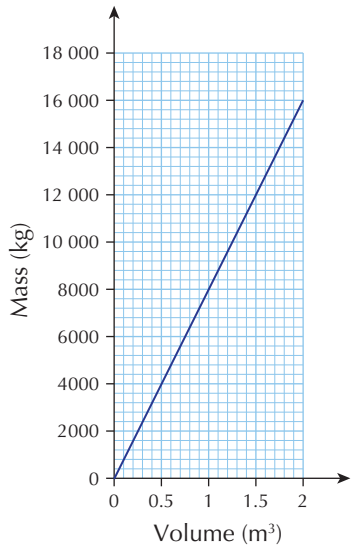
a Work out the mass of 3.5 cm^3 of gold.

b Work out the volume of 3.5 g of gold.

c The largest gold bar in the world has a mass of 250 kg. Work out its volume.

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PS 12 This graph shows the connection between mass and volume for a type of steel.



a Use the graph to find the mass of 0.75 m³. of steel.

b Use the graph to find the volume of 1400 kg of steel.

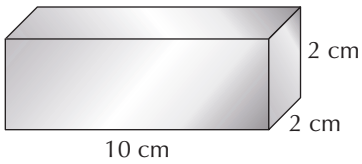
c The mass of a particular type of steel beam is 360 kg.

i Calculate the volume of the beam in m³.

ii Calculate the volume of the beam in cm³ (Hint 1 m³ = 1 000 000 cm³)

Challenge

A metal cuboid measures 2 cm by 2 cm by 10 cm. It has a mass of 340 g.



a Calculate the volume of the cuboid.

b Calculate the density of the metal.

c The same metal is used to make a cylinder with a diameter of 3 cm and a length of 6 cm. Calculate the mass of the cylinder.

12.3 Unit costs

Learning objective

- To understand and use unit pricing

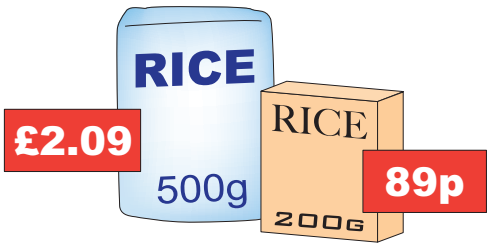
Keyword

unit price

Packets and containers of food and other items are sold in different sizes. If you want to compare the prices of the same item in different sized containers, it is helpful to be able to work out a **unit price**. This means the price of one gram or one litre, or any other suitable unit.

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Here are two packets of rice. Which is better value?



- You could find the cost/gram of rice from each packet.

The answers then are £0.89 ÷ 200 = £0.00445 and £2.09 ÷ 500 = £0.00418.

The larger packet is better value as it has the lower cost per gram.

- You could find how much you can buy for 1p.

For the smaller packet the number of grams/p is 200 ÷ 89 = 2.247… g/p.

For the larger packet it is 500 ÷ 209 = 2.392…. g/p.

The larger packet is better value because you can buy more for 1p.

This exercise will give you the chance to practice different methods.

Note: Items in shops often indicate the weight of an item in grams. It is more correct to use the word mass instead of weight. The word mass will be used in this exercise.

Exercise 12.3

1 A bag of pasta has a mass of 250 g and costs 87p.

a Calculate the cost per 100 g.

b Calculate the cost per gram.

c Calculate the number of grams for 1p.

d Calculate the number of grams for £1.

2 A bag of sugar has a mass of 1 kg and costs £2.35.

a Calculate the cost per 100 g.

b Calculate the cost per gram.

c Calculate the number of grams for 1p.

d Calculate the number of grams for £1.

3 Gina buys a 600 g bag of satsumas for £2.40. Work out the cost per kilogram.

4 Raspberries cost £12.50 per kilogram.

Calculate the mass you can buy for £1.00.



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FS **5** A 160 g can of tuna costs £1.85. A pack of four cans costs £6.20.

a Work out the cost per 100 g for one can.

b Work out the cost per 100 g if you buy the pack of four cans.

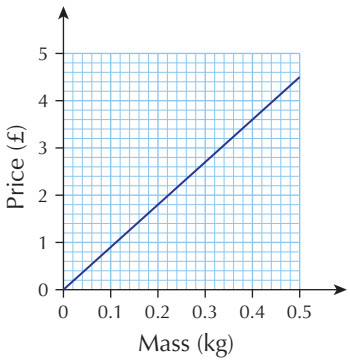
c Which is better value? Give a reason for your answer.

FS **6** You can buy tomato puree in tubes or jars. A shop sells 140 g tubes for 69p and 200 g jars for £1.09.

a Work out the cost per 100 g in a tube of tomato puree.

b Work out which is better value for money. Justify your answer.

7 This graph shows the price of asparagus.



a Use the graph to find the cost of 400 g of asparagus.

b Use the graph to find the mass you can buy for £2.00.

c Explain how the graph shows that cost is proportional to mass.

d Work out the cost of asparagus in £/kg.

FS **8** Aziz buys 700 g of apples for £1.68. Calculate the cost per kilogram.

FS **9** A supermarket sells two cheese selection packs. One has a mass of 490 g and costs £6.00. The other has a mass of 265 g and costs £3.49.
Calculate the cost per kg of each pack.

MR **10** A shop sells cans of pineapple in two sizes. The prices are in this table.

Mass	225 g	435 g
Price	83p	£1.45

Which is better value? Justify your answer.



MR **11** Milk is sold in litres or pints. One pint is 568 ml.

A supermarket sells 1 litre of milk for 95p or 1 pint for 49p.
Which is better value? Justify your answer.

MR **12** A 600 g box of muesli costs £2.65. An 850 g box costs £3.95. Which is better value? Justify your answer.

MR **13** Toothpaste comes in different size tubes. A 125 ml tube costs £2.99 and a 75 ml tube of the same brand costs £1.89.
Which is better value for money? Justify your answer.

PS **14** A 400 g packet of beef costs £4.60. What price would you expect to pay for a 750 g pack? Justify your answer.

MR **15** A pack of four 120 g pots of yogurt costs £2.00. A 450 g pot of the same yogurt costs £1.79.
Show that the large pot is better value.

MR **16** A 350 g block of Cheddar cheese costs £2.90. A 250 g packet of grated Cheddar cheese costs £1.95.
Show that the block of cheese is better value.

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MR **17** A 150 g packet of raisins costs £1.09. A 500 g pack of raisins costs £2.55. Which is better value? Justify your answer.



Challenge

A shop sells two sizes of cereal packet.

One is 350 g and costs £1.79 and the other is 575 g and costs £2.85.

They are going to start selling a 750 g pack. They want it to be slightly better value than the other packs. They have asked you to recommend a price. What is your suggestion? Justify your answer.

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Ready to progress?

I can given a distance and a time, calculate a speed

I can solve problems involving speed
I can calculate density and solve problems involving density
I can solve problems involving other compound units such as rates of flow
I can calculate unit prices and use them to find value for money

Review questions

1 It takes James 8 minutes to walk to the shops. The distance is 400 m.

a Work out the speed in m/minute.

b How long would it take James to walk 1 km at the same speed?

2 A plane flies at 800 km/h.

a How far does it fly in $1\frac{1}{2}$ hours?

b Calculate the time taken to fly 2000 km.

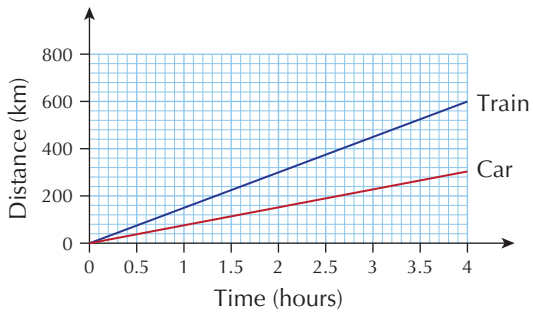
3 A cheetah can run 100 metres in 4 seconds.



Work out the speed of the cheetah

a in m/s **b** in m/min **c** in km/h

4 This graph shows the distance travelled by a train and a car over a four-hour interval.



a Work out the speed of the train.

b Work out the speed of the car.

c If the train travels d km in t hours, write a formula for d in terms of t.

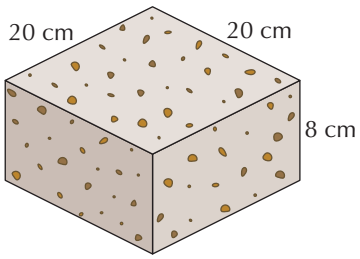
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5 20 litres of water flows out of a tap every minute. Work out the rate of flow in

a litres/second **b** litres/hour

6 A paddling pool has a capacity of 200 litres. It is filled at a rate of 25 litres/minute. Work out how long it takes to fill the pool.

7 Here is a block of concrete in the shape of a cuboid.



a Calculate the volume of the block.

b The density of concrete is 2.4 g/cm³. Calculate the mass of the block of concrete.

8 A piece of metal has a volume of 150 cm³ and a mass of 1.23 kg. Calculate the density of the metal in g/cm³.

9 Densities can be given in g/cm³ or in kg/m³.
1 g/cm³ = 1000 kg/m³

a The density of the wood in a tree trunk is 0.85 g/cm³. Write this in kg/m³.

b The volume of the tree trunk is 3.8 m³. Work out the mass of the tree trunk.

10 A 250 g piece of cheese costs £2.10.



a Work out the cost per 100 g of the cheese.

b Work out the cost per kg of the cheese.

c What mass of cheese costs £5.00?

PS **11** The cost of petrol is £1.35 per litre. Petrol flows into the tank of a car at a rate of 1.2 litre/second.

Jess puts petrol into her car for half a minute.

a Work out the quantity of petrol Jess puts in her car.

b Work out the cost of the petrol.

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PS **12** A 350 g packet of biscuits costs £1.49. A 450 g packet of the same biscuits costs £1.99. Which is better value? Justify your answer.

PS **13** A box of cereal has a mass of 300 g and costs £2.85.

a Work out the cost per 100 g.

b The manufacture decides to give 25% extra for the same price. Calculate the new cost per 100 g.



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Reasoning, Problem Solving

Population density

1. Look at these figures.

	Area (thousand km²)	Population (million)
Sweden	410	10
Germany	360	81

The area of Sweden is 410 000 km². The population of Sweden is 10 000 000.

a Write down the area and the population of Germany.

b The area of Sweden is about 14% larger than the area of Germany. What can you say about the populations of the two countries?

For any country you can work out the *population density* $= \frac{population}{area}$.

This tells you how crowded a country is. The units are people per km².

The population density of Sweden = 10 000 000 ÷ 410 000 = 24 people/km².

c Work out the population density of Germany, to the nearest whole number.

2 Here is a table of 11 more European countries.

Country	Area (thousand km²)	Population (million)
Belgium	31	11
Finland	300	5
France	540	64
Greece	130	11
Ireland	70	5
Italy	300	60
Netherlands	34	17
Poland	310	39
Portugal	92	11
Spain	500	47
United Kingdom	240	64

a Work out the population density for each country. Give your answers to the nearest whole number.

b If you include Sweden and Germany, you now have data for 13 countries. Which country has

i the largest population?

ii the largest population density?

iii the smallest population?

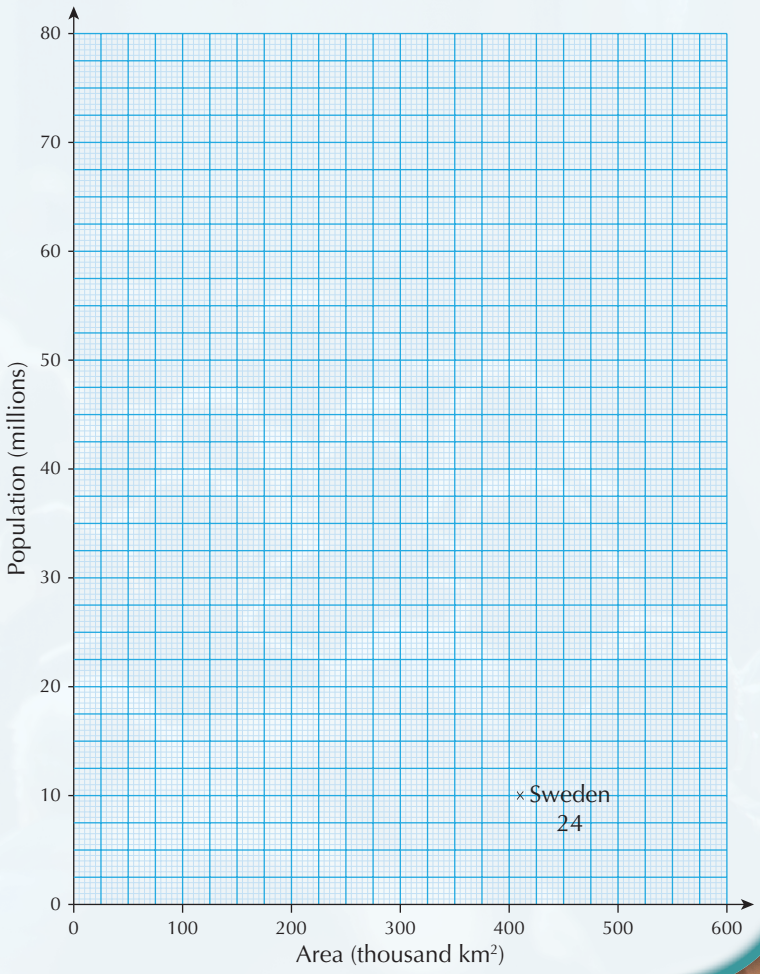
iv the smallest population density?

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3. Draw axes like this on graph paper.

Use a scale of 2 cm to 100 000 km² and 2 cm to 10 million people.

Put a cross for each country and label it with the name and population density. Sweden has been done for you.



4. Two countries with very different population densities are Singapore and Australia.

Singapore has an area of only 700 km² and a population of 5 400 000.

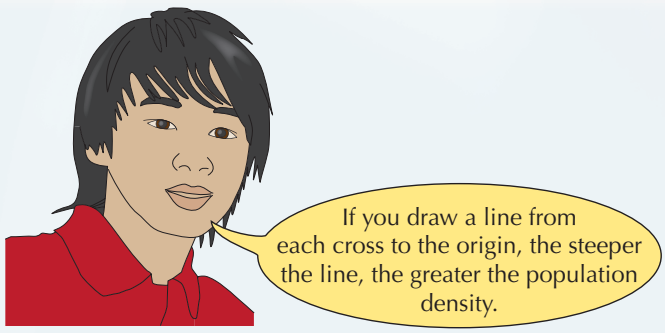
Australia has an area of 7 700 000 km² and a population of 23 000 000.

a Work out the population density of each.

b If you plotted Singapore on your graph which axis would it be closer to?

c If you extended your graph so that you could plot Australia, which axis would it be closer to?

5.



Check that this is the case.

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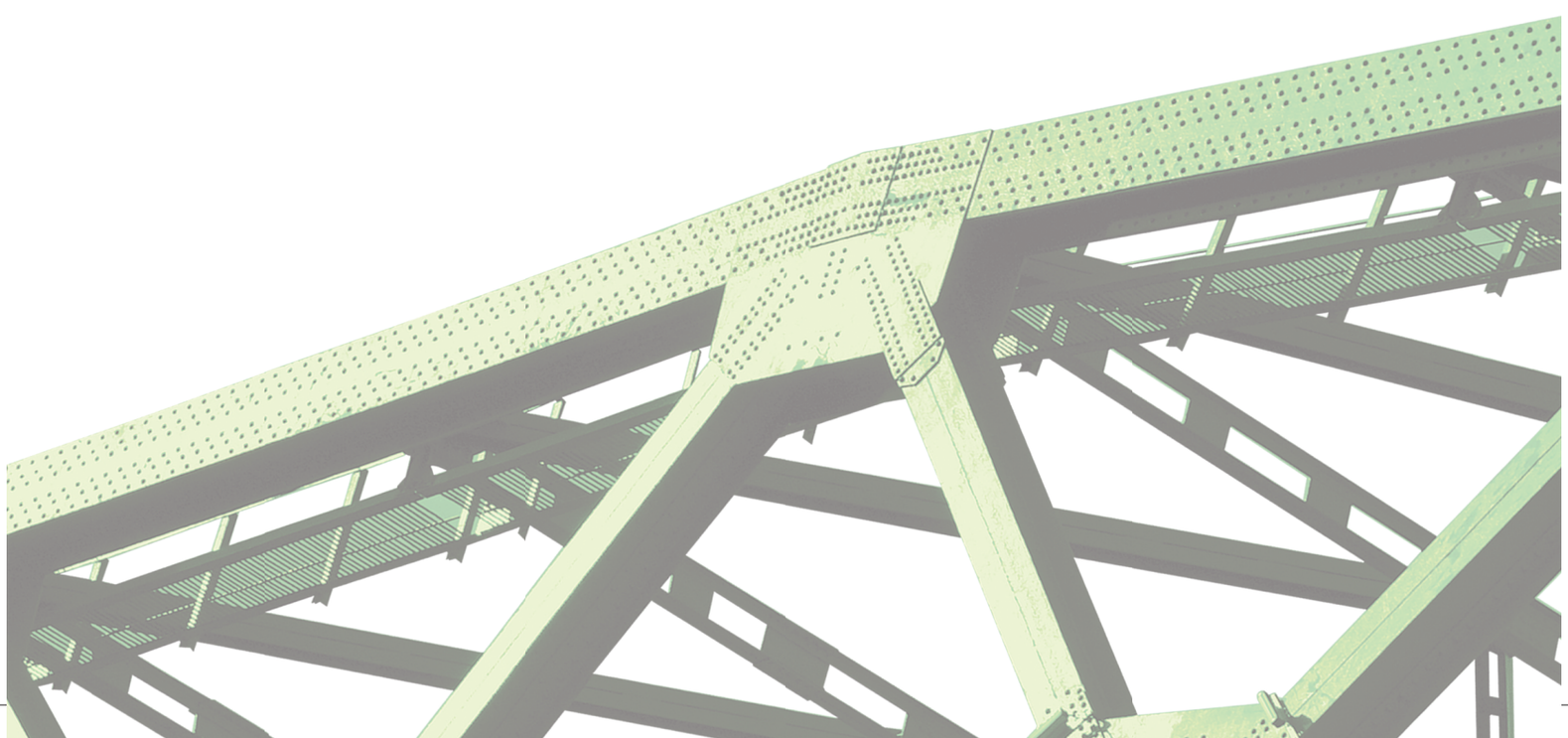
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Teacher Pack 1.2

Maths Frameworking

3rd edition

Kevin Evans, Keith Gordon,
Chris Pearce, Trevor Senior,
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17 Ratio

Learning objectives

- How to use ratio notations
- How to compare quantities with ratios
- How to simplify ratios
- How to use ratios to find totals or missing quantities
- The connection between ratios and fractions
- How to use ratios to solve problems in everyday life

Prior knowledge

- How to simplify fractions. In KS2, pupils will have divided numerators and denominators by common factors.
- How to find a fraction of a quantity. Pupils will be familiar with percentage notation.
- The equivalence between simple fractions and percentages. Pupils should be familiar with simple fractions such as three-quarters or one-tenth, and the corresponding percentages.
- How to interpret bar charts and pie charts

Context

- Ratios are a very useful way to compare quantities without the distraction of the actual values. For example, saying that the diameter of Saturn is 10 times the diameter of the Earth (or the ratio is 10 : 1) provides an immediate mental image. This would not be as obvious just by quoting the diameters.
- Many pupils will have experience of riding a bicycle with gears. It may come as a surprise to them to learn that ratios are used to compare one gear with another. If you know the gear ratio, you know if it will be easy or difficult to pedal in that gear.

Discussion points

- Questions to activate prior knowledge and exercise mathematical ‘fluency’, that is, the ability to manipulate mathematical language and concepts and apply them in different contexts.

Associated Collins ICT resources

- Chapter 17 interactive activities on Collins Connect online platform
- *Zodiac map* Wonder of Maths on Collins Connect online platform
- *Lines of the form x + y = a* Worked solution on Collins Connect online platform
- Ratio and proportion tool on Collins Connect platform

Curriculum references

Develop fluency

- Move freely between different numerical, algebraic, graphical and diagrammatic representations (for example, equivalent fractions, fractions and decimals, and equations and graphs)

Ratio, proportion and rates of change

- Use ratio notation, including reduction to simplest form
- Divide a given quantity into two parts in a given part to part ratio
- Express the division of a quantity into two parts as a ratio
- Relate the language of ratios and the associated calculations to the arithmetic of fractions

Fast track for classes following a 2 year scheme of work

- Pupils should have worked with ratio in KS2, when comparing quantities and in problems involving unequal sharing. Pupils may have been introduced to the a : b notation. If pupils can show understanding by answering one or more of the later questions in Exercise 17A of the Pupil Book, they can move on to simplifying ratios in Exercise 17B. Similarly, if pupils are confident about simple sharing problems, as provided in Exercise 17C, then they can move on to concentrate on the mixed questions in Exercise 17D.

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Lesson 17.1 Introduction to ratios

Learning objectives

- To use ratio notation
- To use ratio to compare quantities

Resources and homework

- Pupil Book 1.2, pages 315–318
- Intervention Workbook 2, pages 43–45
- Intervention Workbook 3, pages 40–42
- Homework Book 1, pages 111–112
- Online homework Chapter 17.1, exercises 1–10

Links to other subjects

- **Science** – to compare quantities in experiments or compounds; comparing distances in astronomy
- **Geography** – to compare populations, areas, heights or other physical measures

Keywords

- ratio
- quantity

Problem solving and reasoning help

- **MR** question 4 of Exercise 17A has an application of algebra in creating a simple formula. The number in the ratio becomes the coefficient of *N*.
- **PS** questions 7 and 8 require pupils to work from a ratio to a value. They will look in more detail in a future section at ways to do this. In these simple cases the earlier part of the question should help them to find the missing value.
- The investigation at the end of the exercise encourages pupils to choose their own ratios to evaluate.

Common misconceptions and remediation

- Pupils sometimes have difficulty remembering how to pronounce ‘ratio’. They may pronounce it as rat-i-o, with a ‘t’ sound rather than with an ‘sh’ sound. Help pupils by saying the word often and asking them to use it. Point out that ratio is related to the word ‘ration’, and that sometimes spell checkers change ‘ratio’ to ‘ration’.
- The order of the numbers in a ratio is important and relates to the order of the quantities being compared. Emphasise this by asking for the ratio the other way round and what it means when looking at examples.

Probing questions

- The ratio of two distances is 10 : 1. Do you know either of the distances? Do you know if they are both a whole number of centimetres? What do you know about the distances? (One distance is 10 times greater than the other distance.)

Part 1

- Write or place these numbers on the board, randomly scattered:
  15, 4, 27, 24, 9, 20, 45, 60, 8, 12
- Ask pupils to arrange the numbers in two sets in any way they choose.
- Ask one or two pupils to give their sets, with reasons.
  Try to find a pupil who has put 4, 8, 9, 15, 20 in one group and 12, 24, 27, 45, 60 in the other. If no one has done this, do it yourself and ask pupils what is the logic behind your choice. (In these sets, the numbers in one set are three times the numbers in the other set.)

- Write these numbers on the board and ask pupils to divide them into two sets in a similar way: 2, 5, 10, 20, 25, 40, 50, 100, 200, 250.
- We are looking for: 2, 5, 20, 40, 50 and 10, 25, 100, 200, 250. In this case the multiple is 5.
- If pupils need extra multiplication practice, give them a number to multiply by (such as 4 or 20) and then some quick-fire numbers to multiply by that. So if the number is 4, you might ask them to multiply by, for example: 5, 7, 20, 11…

Part 2

- Look at the numbers in the first set in Part 1. They can be paired as 4 and 12, 8 and 24… Explain that we would say: 'The ratio for each pair is 1 to 3 or 1 : 3'. Explain this notation.
- Ask for some other pairs of numbers in the same ratio.
- **Less able** pupils may find a visual representation helpful. Draw two columns of equal width, one three times the height of the other. Say that the ratio of the heights is 1 : 3 and divide the second column into three parts to demonstrate this.
- Pupils should be able to say that numbers in the second example give pairs in the ratio 1 : 5.
- Look at the example of the giraffe and the antelope at the start of section 17.1. The ratio of the height of the giraffe to the height of the antelope is 4 : 1. Point out that we do not know the height of either. The ratio tells us is that one animal is four times the height of the other.
- Explain that the ratio can be written the other way around, so that the ratio of the antelope's height to that of the giraffe is 1 : 4.
- Have two bottles. The capacity of one bottle should be one-and-a-half times the capacity of the other bottle. If you do not have bottles, use Example 2 on page 315 of the Pupil Book.
- Explain how we can write the ratio as 3 : 2 by dividing each bottle into sections of equal size.
- **Pupils can now do Exercise 17A from Pupil Book 1.2.**

Part 3

- Once pupils have finished the investigation at the end of Exercise 17A, ask them to share their findings and compare the ratios of the number of coins with the ratios of the values.
- If **less able** pupils are unable to do the investigation, ask individuals to draw a diagram to illustrate a particular ratio such as 1 : 6 or 5 : 2.

Answers

Exercise 17A

1 **a** 4 **b** 1 : 4 **c** 4 : 1

2 **a** 2 : 1 **b** 1 : 2 **c** 4 : 1 **d** 1 : 2

3 **a** 2 : 1 **b** 4 : 1 **c** 1 : 8

4 **a** **i** 3 : 1 **ii** 6 : 1 **iii** 5 : 1 **iv** 1 : 2

b $1\frac{1}{2}$ **c** 3*N* buckets of gravel, $2\frac{1}{2}N$ buckets of sand, $\frac{1}{2}N$ buckets of water

5 The ratio of maths pages to science pages is 1 : 4

6 **a** The ratio of Cara's age to Dani's age is 1 : 2

b The ratio of Elfine's age to Frank's age is 2 : 3

7 **a** 5 times **b** **i** 1 : 5 **ii** 2 : 1 **iii** 3 : 2 **c** 4.2 litres

8 **a** **i** 3 : 1 **ii** 1 : 3 **iii** 2 : 1 **iv** 8 : 1 **b** 1200 million **c** 9*P*, 8*P*, 4*P*, 3*P*, 1.5*P*

9 **a** 4 : 1 **b** 1 : 3 **c** 3 : 2

10 3 : 1

Investigation: Bagging coins

A 50 **B** 100

C **a** 1 : 2 **b** No. The values are in the ratio 2 : 1

D Pupils' own answers

Lesson 17.2 Simplifying ratios

Learning objective

- To write a ratio as simply as possible

Resources and homework

- Pupil Book 1.2, pages 318–321
- Homework Book 1, pages 112–114
- Online homework Chapter 17.2, exercises 1–10

Links to other subjects

- **Science** – to compare the amounts of two elements in a chemical formula
- **Geography** – to compare the area of a country to the area of another, for example, comparing the size of other countries to the size of Wales

Keywords

- simplify
- fraction

Problem solving and reasoning help

- In **PS** question 7 of Exercise 17B in the Pupil Book, percentages are given rather than quantities. This is normal practice on clothing labels. The ratio can be written using the percentages in the same way as if quantities had been given. You could ask pupils why percentages rather than quantities are used here. (Food labelling provides quantities.)
- In **PS** question 9b, pupils should realise that they are looking for numbers where one number is double the other number.
- In the investigation at the end, pupils should realise that the ratios of any corresponding lengths (width, length or perimeter) will be the same. They will not be the same as the area ratios. Pupils may need a hint to recognise square numbers occurring in the area ratios.

Common misconceptions and remediation

- Pupils do not have units. In this section of the Pupil Book, pupils are comparing lengths, masses and so on, but the units of measurement do not appear in the ratio. Emphasise this point when calculating or working with ratios.
- The quantities being compared must be in the same units. Part 3 addresses this point.

Probing questions

- When you are given a ratio, how do you know if it is in its simplest form?

Part 1

- Write some fractions on the board and ask pupils to simplify them. Start with fractions that simplify to 1 in the numerator; then progress to more complicated fractions.
- Emphasise the fact that you want the simplest possible form. For example, the simplest form of $\frac{12}{20}$ is $\frac{3}{5}$, not $\frac{6}{10}$.
- Match the difficulty to the ability of your pupils. For **more able** pupils, choose examples where the common factor is more or less obvious, or where there are multiple factors.

Part 2

- Draw two lines, one 24 cm long and one 40 cm long. Do not reveal the lengths yet.
- Ask pupils to estimate the ratios of the lengths. Do not comment on whether they are correct or not. If you get several suggestions, ask pupils to say which they think is best, and why.

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- Now write the lengths on the lines and say that we could use these to write the ratio of the shorter length to the longer length as 24 : 40. Imagine each line marked in centimetre sections. There will be 24 on one line and 40 in the other.
- Say that we can simplify ratios in the same way that we simplify fractions – by dividing by a common factor.
- Ask pupils to simplify $\frac{24}{40}$ to its simplest form. The answer is $\frac{3}{5}$.
- In the same way, the ratio 24 : 40 simplifies to 3 : 5. Did anyone have this as their estimate?
- What does this mean? Divide the 24-cm line into three equal sections and the 40 cm line into five equal sections. This provides a visual representation of the ratio.
- For **more able** pupils, you could explain that we could also write the ratio as $1:1\frac{2}{3}$.
- This is because $40 \div 24 = 1\frac{2}{3}$ and the length of the longer line is $1\frac{2}{3}$ times the length of the smaller line. We tend to use the whole number representation of a ratio when the numbers are reasonably small.
- Give an example of two masses. Two books of different sizes would be suitable or any two objects you have in the classroom. Ask how we could find the ratio of these masses.
- Tell pupils the masses (make them up), as long as the masses seem reasonable and simplify to small numbers. Ask pupils to find the ratio of the masses in its simplest form. Ask for it both ways: lighter to heavier and heavier to lighter.
- Point out that ratios can be used in this way to compare any measurable quantity. You have looked at length and mass. Ask for other examples. Area or volume are possible answers.
- **Pupils can now do Exercise 17B from Pupil Book 1.2.**

Part 3

- Write on the board: The lengths of two lines are 25 cm and 100 mm.
- Say that the ratio of the lengths is *not* 1 : 4 and ask a pupil to explain why this is the case.
- Ask for the correct ratio in its simplest form. The answer is 5 : 2. Ensure that all pupils understand this. Emphasise that units must be the same when comparing two quantities.
- Give more examples using mixed units and ask for the ratio. Possible examples are 50 cm and 2 m; 200 g and 1 kg. **Less able** pupils will benefit if you use very simple examples.
- At the end of the lesson, ask pupils to share their findings from the investigation at the end of the exercise. Check that pupils can a work out all the ratios required.

Answers

Exercise 17B

1 **a** $\frac{2}{3}$ **b** $\frac{1}{5}$ **c** $\frac{3}{2}$ **d** $\frac{3}{5}$ **e** $\frac{1}{30}$

2 **a** 1 : 4 **b** 2 : 3 **c** 2 : 1 **d** 8 : 1 **e** 2 : 3

3 **a** 2 : 5 **b** 5 : 2

4 4 : 3

5 2 : 7

6 **a** 5 : 2 **b** 2 : 3 **c** 3 : 5

7 **a** 6 : 1 **b** Some possible answers are: viscose to polyamide = 4 : 3; polyamide to angora = 9 : 2; angora to cashmere = 1 : 1

8 **a** 3 : 2 **b** 13 : 7

9 **a** **i** 40 : 1 **ii** 5 : 2 **iii** 5 : 4 **b** sulphur and oxygen

10 **a** 2 : 1 **b** 1 : 2 **c** 2 : 3 **d** 5 : 6

11 **a** 2 : 1 **b** 4 : 3 **c** 3 : 5

Investigation: Ratios in patterns

A **a** 1 : 3 **b** 1 : 3 **c** 1 : 3 **d** 1 : 9 **B** **a** 3 : 2 **b** 3 : 2 **c** 3 : 2 **d** 9 : 4

C For **a**, **b** and **c** it is 2 : 1; for **d** it is 4 : 1

D **a**, **b** and **c** are the same and in **d** the numbers are the numbers in **a** squared.

Lesson 17.3 Ratios and sharing

Learning objective

- To use ratios to find totals or missing quantities

Resources and homework

- Pupil Book 1.2, pages 321–323
- Intervention Workbook 3, pages 43–44
- Homework Book 1, pages 114–115
- Online homework Chapter 17.3, exercises 1–10

Links to other subjects

- **Science** – to compare quantities in different types of materials

Keywords

- No new keywords for this topic

Problem solving and reasoning help

- In **MR** question 5 of Exercise 17C in the Pupil Book, pupils need to realise that the information given implies that the fractions are $\frac{1}{3}$ and $\frac{2}{3}$.
- In **MR** question 6, the crucial step is finding the fraction of the crowd who are women.
- In the challenge at the end of the exercise, the link between carat and percentage of gold can be illustrated by a horizontal line with a scale from 0 to 24 carats at the top and 0 to 100% on the bottom. This way, pupils can see easily that 24 carats is 100% gold, 12 carats is 50% gold, 18 carats is 25% gold, and so on.

Common misconceptions and remediation

- Pupils sometimes get confused about the exact relationship between ratios and fractions. Emphasise the link between the ratio numbers and the numerators of the fractions, as outlined in the lesson plan.

Probing questions

- A liquid is poured into two glasses. Three-fifths is in one glass. What is the ratio of the contents of the two glasses? Justify your answer.

Part 1

- Pupils can work in pairs on this activity.
- Write £20 on the board. Ask pupils to find what fractions of this will be a whole number of pounds. If they need clarification, show that $\frac{1}{2}$ is £10, which is a whole number of pounds; $\frac{1}{3}$ is £3.33 with 1p left over, which is not a whole number.
- After a few minutes check answers.
- Check that pupils remember how to find a fraction:
  For example, $\frac{1}{5}$ of £20 = 20 ÷ 5 = £4; $\frac{3}{5}$ of £20 = 3 × £4 = £12
- Less able pupils may need more practice with finding fractions. If so, repeat the activity with £18 and/or £12.

Part 2

- Draw a 4 by 3 grid of 12 squares. Colour three of the squares. Ask what fraction is coloured, and then what fraction is uncoloured. The answers should be one-quarter and three-quarters.
- Say that so far they have used ratios to compare two quantities. They can also use ratios to compare the shares when dividing one quantity.
- In the example we have just done, the ratio of shaded to unshaded parts is 1 : 3. Make sure that pupils are happy with this.
- Repeat this activity but shade different numbers of squares each time. For example, 4 or 2, or 1 or 6. Each time, ask for the fraction shaded, the fraction unshaded and the ratio of the two. Pupils should be able to see that the numerators of the fractions give the numbers in the ratio.
- Now work through Example 4 and Example 5 on pages 321 and 322 of the Pupil Book. In Example 4, £20 000 is divided into shares in a given ratio. This is done by working out the fraction required, and reinforcing the link between fractions and ratios. In Example 5 one share is given and the whole is found. Again, fractions are used to tackle this problem.
- **Pupils can now do Exercise 17C from Pupil Book 1.2.**

Part 3

- Say that Eve has 20 apples to share with Adam. Can they share them in the ratio 2 : 1 and have an equal number each? What about sharing them in the ratio 2 : 3?
- What is the smallest number that can be divided equally in the ratio 2 : 1 or 2 : 3? (The answer is 15; the lowest whole number is divisible by both 3 and 5.)
- A question for more able pupils is to find smallest number that can be divided equally in the ratio 2 : 1 or 2 : 5.

Answers

Exercise 17C

1 **a** $\frac{5}{6}$ **b** 25

2 **a** $\frac{4}{5}$ **b** 500

3 **a** $\frac{2}{5}$ **b** £18

4 **a** $\frac{1}{9}$ **b** 1080

5 44

6 60

7 20

8 48

9 600

Challenge: Mixing gold

A **a** 3.0 g **b** 3.5 g **c** 1.5 g

B **a** 3 : 1 **b** 7 : 1 **c** 3 : 5

C The ratio is 1 : 2

D The price of gold varies from day to day. At the time of writing it was about £29 per gram. You can find the price on the internet.

Lesson 17.4 Solving problems

Learning objectives

- To understand the connections between fractions and ratios
- To understand how ratios can be useful in everyday life

Links to other subjects

- **Science** – to find unknown quantities of substances
- **Geography** – to compare the resources of different countries

Resources and homework

- Pupil Book 1.2, pages 324–325
- Homework Book 1, pages 115–118

Keywords

- No new keywords for this topic

Problem solving and reasoning help

- Note that all questions in this lesson are Level 6 of the previous curriculum.
- In PS question 4 of Exercise 17D in the Pupil Book, pupils need to be clear about the ratio of the number of coins and the ratio of their values.
- In PS question 5, pupils first need to work out the number of green and brown bottles and then remember to note the change in the total number of bottles. A possible extension for more able pupils is to ask for the ratio if another green bottle falls (1 : 12).
- In the challenge, the fraction received by each child changes each time, but the total remains the same (£60). It is interesting to note that Jack's share increases each year, nearing half (£30), and Jill's share decreases.

Common misconceptions and remediation

- Pupils can make mistakes by not being clear about whether they know the whole or only a part when working out missing values. Encourage pupils always to ask themselves if they know the whole or a part before they try to answer a question.

Probing questions

Ann and Baz share some money in the ratio 3 : 2.

- If you know the total amount, how can you work out what Ann has?
- If you know what Ann has, how can you work out what Baz has?
- If you know what Baz has, how can you work out what Ann has?

Part 1

- Write down the following sentences: 'Tim and Claire bake 60 biscuits in total. Tim bakes 24 of them.'
- Ask pupils to work in pairs and find different ways to write this information. They can use fractions, percentages or ratios.
- After a few minutes take answers from selected pupils. Possible statements are:
- Tim bakes $\frac{2}{5}$ and Claire bakes $\frac{3}{5}$.
- Tim bakes 40% and Claire bakes 60%.
- Tim and Claire bake biscuits in the ratio 2 : 3.
- Make sure that pupils can remember the equivalences between simple fractions and percentages. If not, ask more questions about this, for example, writing tenths, fifths or quarters as percentages.
- This should remind pupils of the work covered in the last lesson.

Part 2

- This lesson is about using the ideas about ratios that pupils have met so far in questions, in a realistic context.
- Tell pupils that the ratio of teachers to pupils in a school is 1 : 12 (or use your own figure if you know it, but round it to a whole number).
- Ask a volunteer to explain what this means. A possible answer is that there is one teacher for every 12 pupils.
- Ask questions that require pupils to use this information in different ways, for example:
- If there are 30 teachers, how many pupils are there?
- If there are 60 teachers, how many pupils are there?
- If there are 1300 pupils and teachers, how many of each are there?
- Ask pupils to explain how they work out the answers each time and check that their reasoning is correct.
- Set the following problem for pupils to tackle in pairs: 100 pupils are going on a trip and the ratio of teachers to pupils must be at least 1 : 9. What is the smallest possible number of teachers?
- The answer is 12. Discuss with pupils different ways to work out the answer. A suitable method for less able pupils is to say that one teacher can take nine pupils, two teachers can take 18 pupils, three teachers can take 27 pupils, and so on.
- More able pupils may prefer a method based on fractions. $\frac{9}{10}$ of the total number is 100, so $\frac{1}{10}$ of the total number is 9, remainder 1. Nine teachers is not quite enough.
- Example 6 in section 17.4 has a similar question for comparison.
- **Pupils can now do Exercise 17D in Pupil Book 1.2.**

Part 3

- Pupils can work in twos or threes on this task.
- Ask pupils to produce a concise statement that will help them to remember the key points about ratios. This could be an example, a diagram or a sentence or two.
- Ask one or two groups to share what they have done with the rest of the class.
- As an extension, pupils could use these ideas to produce posters for a wall display.

Answers

Exercise 17D

1 210 boys and 140 girls

2 3 : 1

3 2 : 1

4 £5.75

5 1 : 6

6 **a** 2 : 3 **b** 36 years

7 **a** 10 : 1 **b** 3 : 2 **c** $\frac{5}{6}$

8 **a** 2 : 3 **b** $\frac{2}{5}$ **c** 1 : 2 **d** $\frac{1}{3}$

Challenge: Presents from Uncle Fred

A Jack £15 and Jill £45

B Jack £20 and Jill £40

C Jack £106.50 and Jill £193.50

Review Questions (Pupil Book pages 326–327)

- The review questions will help to determine pupils' abilities with regard to the material within Chapter 17.
- These questions also draw on the maths covered in earlier chapters of the book to encourage pupils to make links between different topics.

Problem solving – Smoothie bar (Pupil Book pages 328–329)

- This activity is designed to reinforce the use of ratios by putting them in a realistic context.
- Pupils will find the information they need in various places on the double page spread of the Pupil Book. They will need to use the appropriate information in each question.
- Pupils should be familiar with grams and millilitres as measures of mass and capacity respectively.

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Answers to Review Questions

1 **a** 2 : 1 **b** $\frac{1}{6}$ **c** $33\frac{1}{3}\%$ **d** 600 : 300 is 2 : 1

2 **a** $\frac{2}{5}$ **b** 120 ml **c** 3.75 litres

3 **a** 2 : 1 **b** 2 : 3

4 **a** Number of dry days = 365 – 146 = 219 and 146 : 219 = 2 : 3
b 60%

5 280 m

6 36 years old

7 **a** 5 : 2 **b** 24000

8 **a** 5 : 2 **b** $2\frac{1}{2}$ **c** **i** 3 : 1 **ii** 3 : 2

9 **a** 30 **b** 5
c For example, Jupiter and Saturn, or Saturn and Uranus, or Venus and Mars
d For example, Venus and Earth, or Uranus and Neptune

Answers to Problem Solving – Smoothie bar

1 75 g mango, 37.5 g strawberries, 56.25 g bananas, 187.5 ml orange juice

2 100 g bananas, 50 g chocolate spread, 400 ml cranberry juice

3 5.625 litres

4 Six can be made. An extra 2 litres of milk would be required to make 20.

5 7.5 kg

6 52.6%

7 6 Tropical Fruit, 9 Breakfast Boost

8 £1.25

9 75p

10 £10.25

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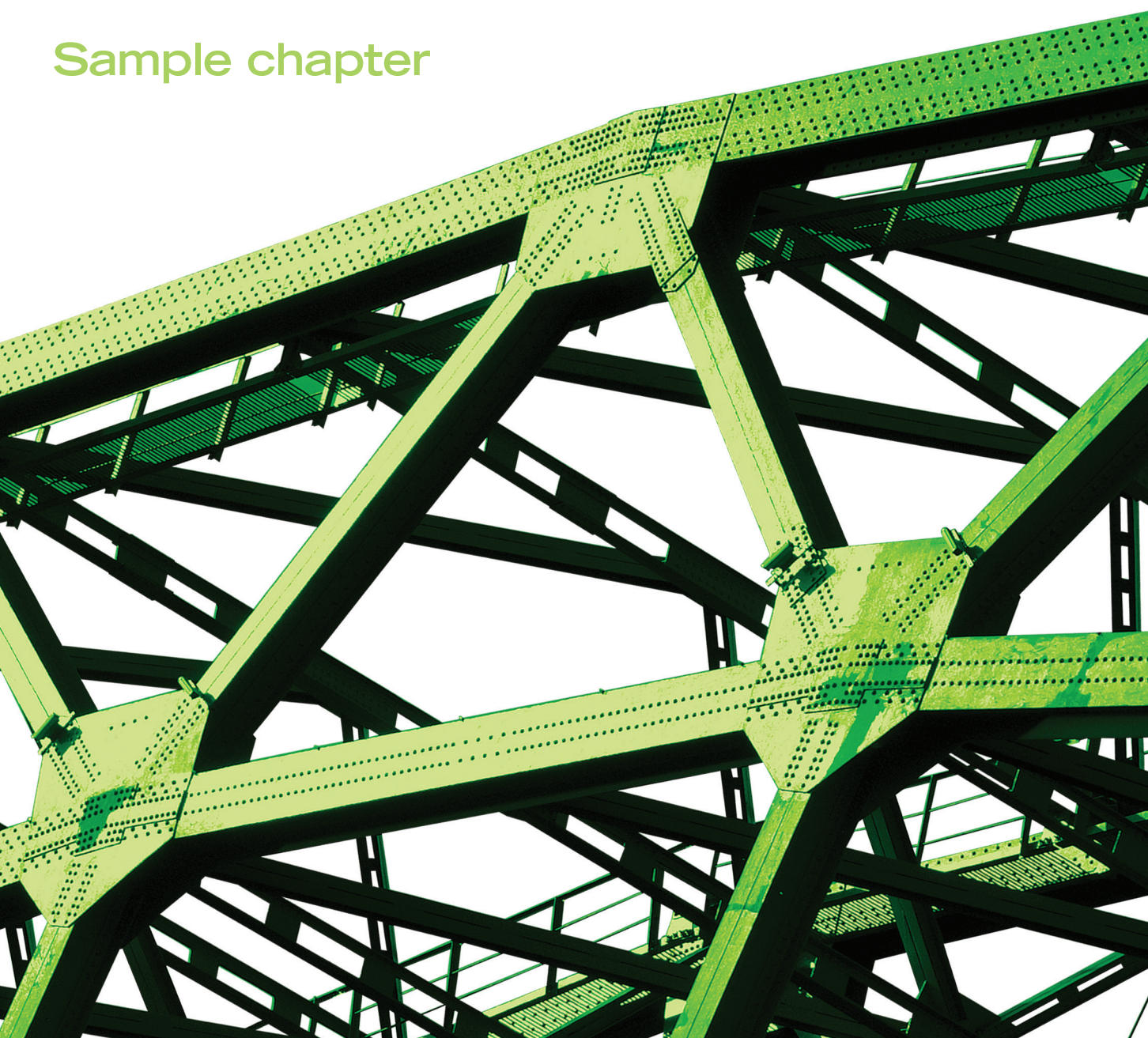
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Homework Book 1

Maths Frameworking

3rd edition

Sample chapter

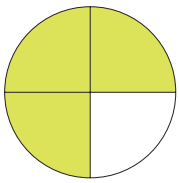


17 Ratio

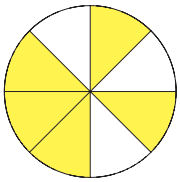
17.1 Introduction to ratio

1 **a** For each of these shapes, work out the proportion that is shaded.

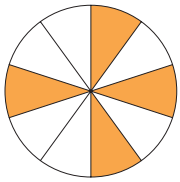
A



B



C



b For each shape, complete these statements.
i There are … parts shaded out of … parts altogether.
ii … in every … parts are shaded.

c What is the ratio of shaded to non-shaded parts?

2 **a** Sarah is one third of Danielle's age. What is the ratio of their ages?

b The ratio of Sarah's age to Fiona's is 3 to 5. What fraction of Fiona's age is Sarah?

3 On a draughts board, there are two black pieces for every one white piece. There are 18 pieces on the board.

a How many black pieces are there?

b What proportion of the pieces is white?

4 Look at the squares below. One in every six squares is black.



Copy and complete the table.

Number of black squares	Number of white squares
1	5
2	10
3	
5	
8	
	50
	100

5 The number of pages in a Michael Morpurgo book is 75% of the number of pages in an Anthony Horowitz book. Work out the ratio of the number of pages in these two books.

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6 **a** The film Spiderman 2 cost $200 000 000 to make, but Spiderman 3 had a budget of $300 000 000. Express this increase as a ratio in its simplest terms.

b Pirates of the Caribbean cost $225 000 000 to make. Express this as a ratio against Spiderman 3's budget.

7 A juice drink is supposed to be made by mixing one part juice to four parts water.

a Write this as a ratio.

The company produce a double concentrate version, which claims you can get the same flavour by mixing nine parts of water to one part juice.

b Is the new bottle really double the strength of the original?

c The original claims to offer 20 servings. According to their ratios, how many similar servings would you expect from the double version?

8 It is estimated that there are around 63 million pet dogs in the USA, 31 million in Brazil and 12 million in Japan.

a Round these amounts to the nearest 10 million, and write them using figures not words.

b How many times more pet dogs are there in the USA compared to Brazil?

c How many times more pet dogs have the USA than Japan? Is it more sensible to round off the Japan figure first or leave it as it is?

17.2 Simplifying ratios

1 Simplify these fractions as much as possible

a $\frac{12}{15}$ **b** $\frac{10}{80}$ **c** $\frac{30}{18}$ **d** $\frac{135}{90}$

2 Reduce each of the following ratios to its simplest form.

a 9 : 6 **b** 8 : 20 **c** 24 : 30 **d** 200 : 900

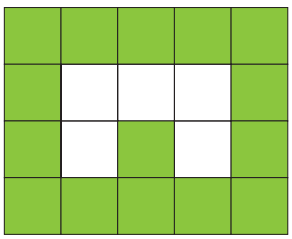
e 75 : 45 **f** 21 : 56

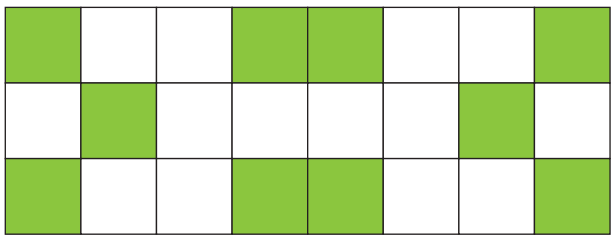
3 The Earth is reckoned to be about 70% water.

a What fraction of the Earth is land?

b Work out the ratio of water to land.

4 Write down the ratio of coloured to white squares for these grids.





5 The atmosphere of Uranus is made up of hydrogen and other gases in the ratio 6 : 1. A space probe filled two flasks with samples of the atmosphere.

a One flask contained 18 litres of hydrogen. How many litres of other gases did it contain?

b One flask contained $\frac{1}{2}$ litre of other gases. How much hydrogen did it contain?

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6 Four football teams scored 250 goals altogether. Beetwood Rangers scored 50 goals, Trenton FC scored 100, Middleton Wanderers scored 25 and Pinkhams scored the rest.

a Write down the percentage of the total goals that each team scored.
b Write down each of the following goal ratios in its simplest form.
i Beetwood Rangers : Trenton FC
ii Middleton Wanderers : Pinkhams
iii Pinkhams : Beetwood Rangers

7 The table shows the values for some common elements in the Universe.

Element	Relative atomic mass
Hydrogen	1
Helium	4
Carbon	12
Nitrogen	14
Oxygen	16
Silicon	28
Sulfur	32
Potassium	39
Copper	63

a Work out the ratio of the masses of the following elements:
i carbon to oxygen **ii** potassium to carbon **iii** silicon to copper.
PS **b** Work out two elements where one mass is four times the other.
c Find two elements in the ratio 3 : 1.

8 The number of Number 1 singles by several artists in the UK is shown below:

Elvis Presley	21
Cliff Richard	14
Madonna	12
The Spice Girls	9

a Simplify the following ratios:
i Elvis to Cliff **ii** Madonna to Elvis **iii** male to female.
b What fraction of the total did Cliff Richard achieve?
c In numerical terms, how many more times successful was Elvis than the Spice Girls?

9 One weekend there were rugby league matches at Leeds, Wigan and Huddersfield.

The attendances to the nearest thousand were 15 000 at Leeds and 12 000 at Wigan.

a Write the ratio of the Leeds attendance to the Wigan attendance as simply as possible.
b The ratio of the Wigan attendance to the Huddersfield attendance was 6 : 5. How many were in the crowd at Huddersfield to the nearest thousand?
c What are the lowest and highest figures possible at Huddersfield?

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MR **10** A recipe for chocolate brownies is given as shown:

Plain chocolate	2 oz	50 g
Margarine	4 oz	100 g
Self-raising flour	4 oz	100 g
Brown sugar	6 oz	180 g
Eggs	2	2

a Ignoring the eggs, write the other amounts as ratios in their simplest form for both types of unit.

b A mistake has been made for one item when converting from imperial units to metric. What is the error, and what should the correct amount be?

Brainteaser

On a bicycle, the gear ratio is the ratio of the number of teeth on the chainring on the front to the number of teeth on the sprocket at the back.

The higher the ratio from front to back, the more power you get downhill.

To make it easier going uphill you need the smallest possible ratio, preferably less than 1.

Mike's front chainring goes from 42 down to 24, and his rear sprocket set goes from 10 up to 28.

Bob's bike goes from 48 to 25 at the front and 12 to 30 at the back.

Use ratios and show your working to decide who has more power going downhill, and who has it slightly easier going uphill.



17.3 Ratios and sharing

1 Divide £120 in each of these ratios.

a 1 : 5 **b** 4 : 1 **c** 19 : 1

2 Andrea divides up her free time in the ratio:

games console : reading = 4 : 1

She has 200 minutes of free time each day. How much time does she spend on each activity?

3 160 children go to a cinema one day. The ratio of those watching Planes to those watching The Lone Ranger is 3 : 5. How many children see each film?

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4 Dominic has 20 coloured sweets left in a packet. These are in the ratio:
red : green = 1 : 4

a How many of each colour are there?
b If he eats one red sweet and four green sweets, what is the new ratio red : green? Comment on your answer.
c If he only ate one of each colour, what would the ratio change to?

5 A bakery normally makes white and brown bread in the ratio 5 : 2. One morning they count 350 white and 154 brown loaves, and realise they've baked too many brown loaves. How many brown loaves should they have baked?

6 Share 400 kg in each of these ratios.

a 1 : 2 : 5 **b** 6 : 11 : 3

7 During the holidays, Andrew does the following activities:
playing cricket : reading : doing puzzle books = 9 : 6 : 4
He plays cricket for 6 hours. How many minutes does he spend on each activity?

8 Diane has 10 pairs of socks left in her drawer. These are in the ratio:
white : blue : black = 1: 2 : 3

a How many of each colour are there? How many are there altogether?
b If she wears one black pair and two blue pairs, what is the new ratio black : blue?
c She then wears three pairs of white socks in a row. What is the new ratio for all three colours? Comment on your answer.

17.4 Solving problems

1 The table shows the seeds contained in four packets of Summer Border.

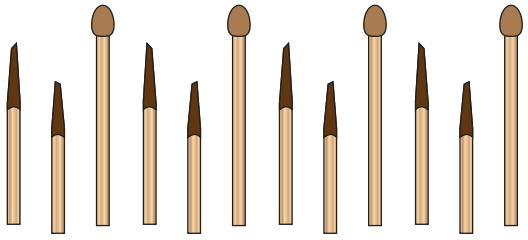
Flower	Number of seeds
Cloth of Gold	20
Delphinium	12
Penstemon	8
Mrs Perry	6

Calculate the number of each seed contained in the following:

a 12 packets **b** 6 packets.

2 In an orchestra, the ratio of cellists to violinists is 3 : 5. There are 20 violinists in the orchestra. How many cellists are there?

3 In the pattern below, one match in every three is unused. The proportion of unused matches is $\frac{1}{3}$.



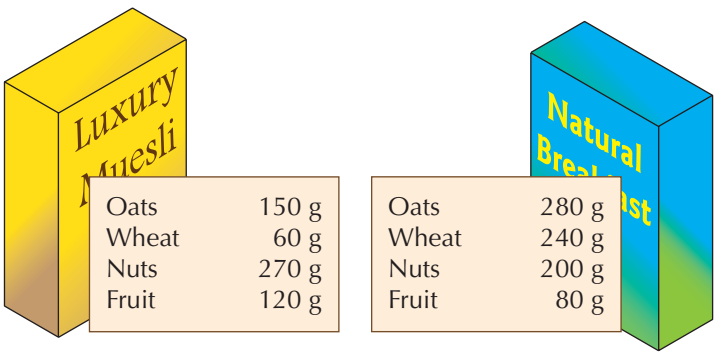
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a What is the proportion of used matches?
b Copy and complete the table to show the sequence of used and unused matches.

Unused matches	Used matches	Total matches
1	2	3
2	4	
3		
6		
	20	

c If you know the number of unused matches, how would you work out the number of used matches?
d If you know the number of used matches, how would you work out the number of unused matches?
e If you know the total number of matches, how would you work out the number of unused matches?
f If you know the total number of matches, how would you work out the number of used matches?

4 The contents of two boxes of muesli are shown below.



a What is the total weight of each box?
b Copy and complete the table below, which shows the proportion of each ingredient.

	Oats	Wheat	Nuts	Fruit
Luxury Muesli	25%			
Natural				10%
Breakfast				

5 Five kilograms of potatoes costs £2.80. Find the cost of the following:

a 2.5 kg **b** 8 kg

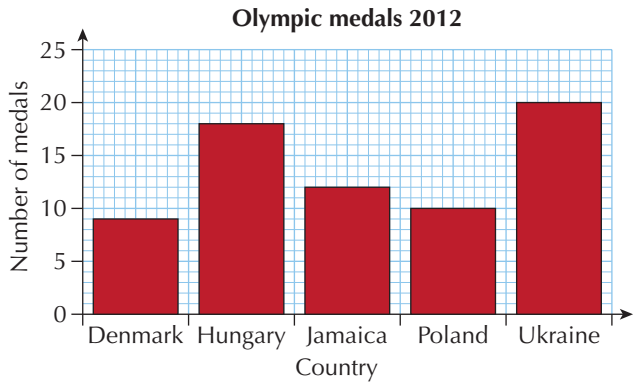
6



Look at the designs above.
a What is the ratio of red to white in design A?
b In designs A and B, why is the ratio of red to yellow ***not*** 5 : 4?
c Why is the ratio of yellow to red in designs B and C ***not*** 5 : 6?

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7 This bar chart shows the number of medals won by different countries in the 2012 Olympics.



a Work out the ratio of medals won between the following countries:
  i Denmark and Jamaica
  ii Hungary and Poland.

b Ukraine won twice as many medals as Poland. Which country won twice as many medals as Denmark?

c Which two countries have the ratio of medals won of 3 : 5?

d Which country only won 45% of the amount of medals won by Ukraine?

8 In a pack of cards, there are four suits: Spades and Clubs are black, Hearts and Diamonds are red. Each suit has 13 cards: the numbers from one to 10 plus three picture cards, Jack, Queen and King. The one (Ace) is usually counted as the highest in the suit.

a How many cards are there in a pack?

b What is the ratio of red cards to black cards?

c **i** Find the ratio of Spades to all other suits. What fraction of a whole pack are Spades?
  ii Find the ratio of Aces to all other cards. What fraction of a pack are Aces?
  iii Find the ratio of picture cards (Jack, Queen and King) to other cards. What fraction of a pack are picture cards?
  iv What do you notice about the figures in corresponding fractions and ratios?

Brainteaser

In the card game Estimation, players have to predict how many tricks they think can win each round, getting points for each trick won, and 10 bonus points for a correct prediction.

Toby calls first. He gets the ten of Spades and has to decide what chance this card has of winning. He knows only the Jack, Queen, King and Ace of Spades can beat it, plus any one of thirteen trumps (Hearts). He decides to bid to win the round and plays his card. The other players are a little surprised. Rosa says she would have bid zero with that card, but Toby reckons his chance of winning is twice as high as his chance of losing. Use ratios to see if he was right or not.

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Collins

Step 1



Maths Frameworking

Intervention Workbook

Chris Pearce

Developed in partnership with

The Colne Community School & College

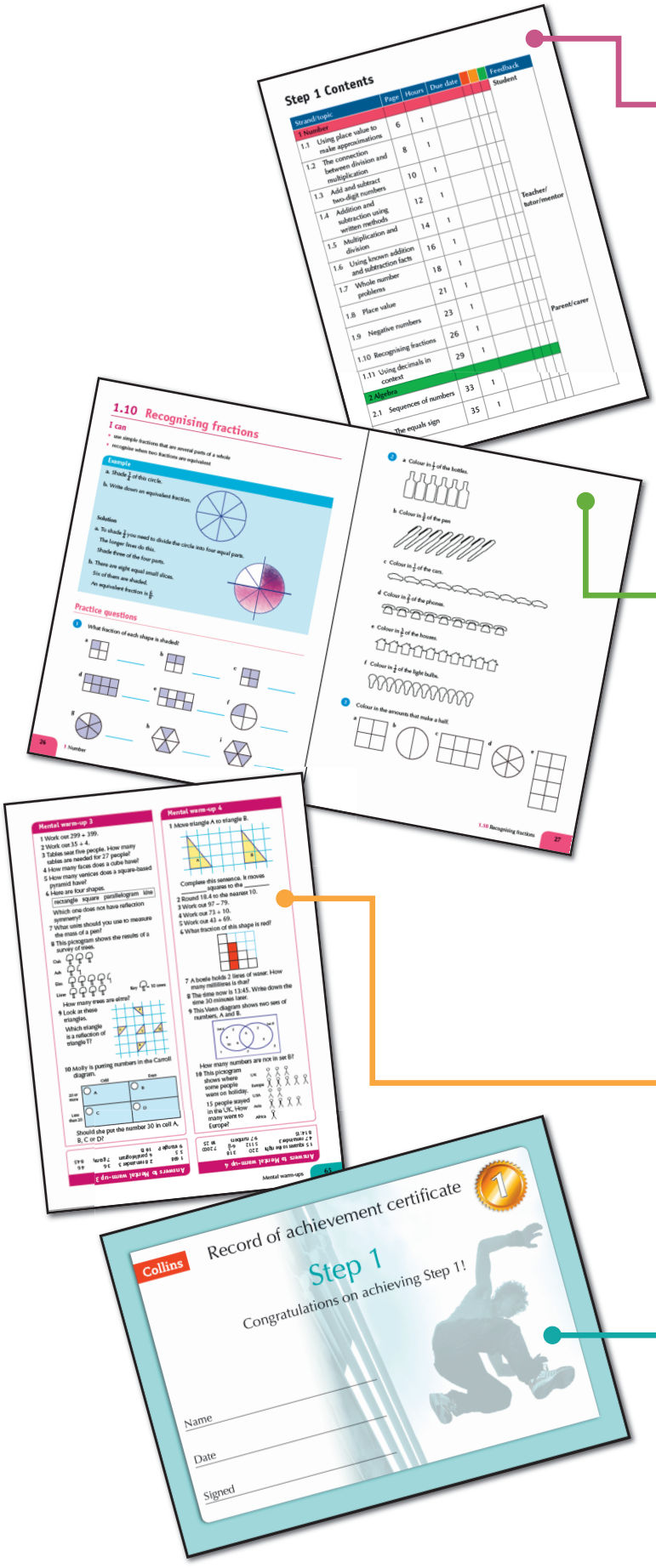
Ofsted Outstanding School 2012|2013

Name _____

Class _____

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How to use this book



Organise your learning

The Contents table at the start of the Workbook shows the topics you are going to cover.

- Your teacher or tutor can set a date for you to complete each topic by.
- You can give a traffic light colour for each topic to show how you feel it went.
- You, your teacher and your parent or carer can write comments.

Work through each topic step by step

For each topic, there are:

- Clear learning objectives
- Worked examples to show you how to answer the questions
- Practice questions to help you consolidate what you have learnt. A glossary and answers are available on the Collins website.

At the end of each chapter, there's a comments box for your teacher or tutor to fill in on how you did.

Practise your mental maths

Try the mental maths questions at the end of the Workbook to see what you have learned.

Celebrate your progress

When you finish the Workbook, your teacher or tutor can fill in the Record of achievement certificate for you to keep.

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Step 1 Contents

Strand/topic	Page	Hours	Due date				Feedback
1 Number							**Student**
1.1 Using place value to make approximations	6	1					
1.2 The connection between division and multiplication	8	1					
1.3 Add and subtract two-digit numbers	10	1					
1.4 Addition and subtraction using written methods	12	1					
1.5 Multiplication and division	14	1					**Teacher/ tutor/mentor**
1.6 Using known addition and subtraction facts	16	1					
1.7 Whole number problems	18	1					
1.8 Place value	21	1					
1.9 Negative numbers	23	1					
1.10 Recognising fractions	26	1					**Parent/carer**
1.11 Using decimals in context	29	1					
2 Algebra							
2.1 Sequences of numbers	33	1					
2.2 The equals sign	35	1					

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Strand/topic	Page	Hours	Due date				Feedback
3 Geometry and measures							**Student**
3.1 Classify 2D and 3D shapes	37	1					
3.2 Nets of 3D shapes	40	1					
3.3 Working with 2D shapes	42	1					
3.4 Position and movement	44	1					**Teacher/ tutor/mentor**
3.5 Measuring	46	1					
3.6 Time	49	1					
4 Statistics							
4.1 Gathering information	52	1					**Parent/carer**
4.2 Statistical diagrams	54	1					
4.3 Sorting and classifying information	57	1					
4.4 Interpreting information	60	1					
Mental maths warm-ups							
Certificate of achievement							

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1 Number

1.1 Using place value to make approximations

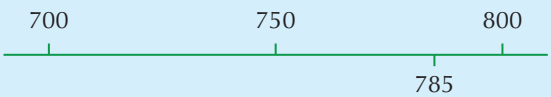
I can

- round any positive integer less than 1000 to the nearest 10 or 100
- round some four-digit numbers to the nearest 1000

Example

The price of a second-hand car is £785. Round this number to the nearest 100.

Solution



785 is between 700 and 800.

750 is halfway between 700 and 800.

785 is larger than 750.

7 is the hundreds digit in 785.

It is closer to 800 than it is to 700.

785 is 800 to the nearest 100.

Practice questions

1 Round each of these numbers to the nearest 10.

a 64 _____ **b** 72 _____ **c** 89 _____

d 23 _____ **e** 65 _____ **f** 14 _____

g 26 _____ **h** 31 _____ **i** 25 _____

j 77 _____ **k** 66 _____ **l** 34 _____

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2 Round each of these numbers to the nearest 100.

a 734 _____ **b** 576 _____ **c** 212 _____

d 788 _____ **e** 899 _____ **f** 150 _____

g 129 _____ **h** 233 _____ **i** 643 _____

j 349 _____ **k** 743 _____ **l** 379 _____

3 Round each of these numbers to the nearest 1000.

a 3400 _____ **b** 1200 _____ **c** 7800 _____

d 8900 _____ **e** 6100 _____ **f** 6500 _____

g 9200 _____ **h** 5500 _____ **i** 6800 _____

j 3200 _____ **k** 4500 _____ **l** 1300 _____

4 Round the prices in this table to the nearest £10 and £100. The first one has been done for you.

Price	To the nearest £10	To the nearest £100
£339	**£340**	**£300**
£182		
£715		
£528		
£891		
£219		
£665		

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1.2 The connection between division and multiplication

I can

- use multiplication facts to write down associated division facts

Example

Here is a multiplication fact: **12 × 9 = 108**

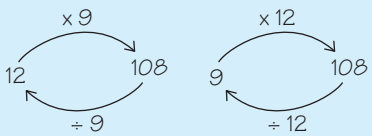
Use this to write down two division facts.

Solution

One fact is 108 ÷ 9 = 12 *because twelve 9s make 108.*

Another fact is 108 ÷ 12 = 9 *because nine 12s make 108.*

If you are given a multiplication fact you can always write down two division facts.



Practice questions

1 Here is a multiplication fact: 7 × 8 = 56

Write down these answers. 56 ÷ 7 = _____ 56 ÷ 8 = _____

2 Here is a multiplication fact: 6 × 12 = 72

Write down these answers. 72 ÷ 6 = _____ 72 ÷ 12 = _____

3 Look at this multiplication: 15 × 5 = 75

Use this to write down two divisions. _____ and _____.

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4 Here are some multiplication facts:

$9 \times 7 = 63$ $12 \times 7 = 84$ $13 \times 4 = 52$ $16 \times 5 = 80$

Use them to complete these divisions.

$84 \div 12 =$ _____ $52 \div 4 =$ _____ $80 \div 5 =$ _____ $63 \div 9 =$ _____

5 Here are some multiplication facts:

$2 \times 50 = 100$ $4 \times 25 = 100$ $5 \times 20 = 100$ $10 \times 10 = 100$

Use them to complete these divisions.

$100 \div 2 =$ _____ $100 \div 5 =$ _____ $100 \div 10 =$ _____

$100 \div 4 =$ _____ $100 \div 50 =$ _____ $100 \div 20 =$ _____

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1.3 Add and subtract two-digit numbers

I can

- add and subtract two-digit numbers in my head

Example

a Add together this pair of numbers. 52 + 29

b Subtract this pair of numbers. 52 – 29

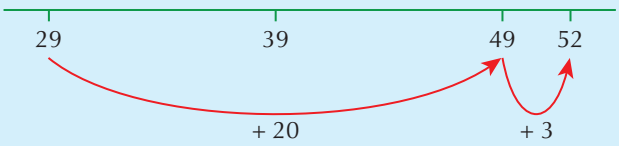
Solution

a 50 + 20 = 70 Add the tens of each number.

2 + 9 = 11 Add the units of each number.

70 + 11 = 81 Then add the two separate answers.

b You can draw or imagine a line if you wish.



Count on from 29 to 52.

From 29 to 49 is 20.

From 49 to 52 is 3.

52 – 29 = 20 + 3 = 23

You may know different methods. Use those if you prefer them.

Practice questions

1 Add together each pair of numbers.

a 47 + 21 = _____ **b** 57 + 35 = _____ **c** 62 + 32 = _____

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d 76 + 54 = _____ **e** 47 + 34 = _____ **f** 56 + 32 = _____

g 72 + 41 = _____ **h** 88 + 33 = _____ **i** 96 + 21 = _____

2 Work out these subtractions.

a 47 – 21 = _____ **b** 57 – 35 = _____ **c** 62 – 32 = _____

d 76 – 54 = _____ **e** 47 – 34 = _____ **f** 56 – 32 = _____

g 72 – 41 = _____ **h** 88 – 33 = _____ **i** 96 – 21 = _____

3 Use any method to work out these additions and subtractions.

a 70 + 34 = _____ **b** 70 – 34 = _____ **c** 66 + 23 = _____

d 66 – 23 = _____ **e** 46 + 39 = _____ **f** 46 – 39 = _____

g 67 + 21 = _____ **h** 67 – 21 = _____

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1.4 Addition and subtraction using written methods

I can

- add and subtract three-digit numbers using written methods

Example

a Work out this addition. 654 + 183

b Work out this subtraction. 654 – 183

Solution

Write the numbers in columns.

a

$$\begin{array}{r} 654 \\ +\,183 \\ \hline 837 \\ \hline {\scriptstyle 1} \end{array}$$

First add the units: 4 + 3 = 7.

Then add the tens: 5 + 8 = 13; write down 3, carry 1.

Now add the hundreds: 6 + 1 + 1 carried = 8.

The answer is 837.

b

$$\begin{array}{r} {\scriptstyle 5}\not{6}\,{\scriptstyle 1}5\,4 \\ -\,183 \\ \hline 471 \end{array}$$

First subtract the units: 4 – 3 = 1.

Then subtract the tens: 5 is smaller than 8, so borrow from the hundreds and do 15 – 8 = 7.

Now subtract the hundreds: 5 – 1 = 4

The answer is 471.

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Practice questions

1 Use a written method to work out these additions. Show your working.

a 174 + 75 = _____ **b** 345 + 128 = _____ **c** 173 + 566 = _____

d 286 + 45 = _____ **e** 365 + 209 = _____ **f** 376 + 545 = _____

g 75 + 333 = _____ **h** 208 + 334 = _____ **i** 265 + 716 = _____

2 Use a written method to work out these subtractions. Show your working.

a 286 – 142 = _____ **b** 765 – 234 = _____ **c** 718 – 34 = _____

d 174 – 57 = _____ **e** 362 – 149 = _____ **f** 634 – 509 = _____

g 977 – 326 = _____ **h** 423 – 66 = _____ **i** 265 – 119 = _____

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1.5 Multiplication and division

I can

- multiply and divide two-digit numbers by 2, 3, 4 or 5
- divide two-digit numbers by 2, 3 4 or 5, including remainders

Example

a Work out this multiplication. 78 x 3

b Work out this division. 78 ÷ 3

Solution

Write out the calculations neatly.

a

$$\begin{array}{r} 78 \\ \times\ \ 3 \\ \hline 234 \\ {\scriptstyle 2} \end{array}$$

First 8 × 3 = 24; write down 4, carry 2.

Then 7 × 3 = 21; 21 + 2 carried = 23.

The answer is 234.

b

$$3\overline{)\,7\,{}^{1}8}\ \ 26$$

First 7 ÷ 3 = 2 remainder 1.

Then 18 ÷ 3 = 6.

The answer is 26.

You should know the multiplication tables for 2, 3, 4, and 5.

Practice questions

1 Work out these multiplications. Show your working.

a 37 x 2 = _____ **b** 19 x 5 = _____ **c** 23 x 3 = _____

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d 62 x 3 = _____ **e** 74 x 4 = _____ **f** 55 x 5 = _____

g 58 x 4 = _____ **h** 32 x 8 = _____ **i** 54 x 8 = _____

2 Work out these divisions. Show your working.

a 48 ÷ 2 = _____ **b** 46 ÷ 3 = _____ **c** 60 ÷ 4 = _____

d 63 ÷ 5 = _____ **e** 54 ÷ 3 = _____ **f** 62 ÷ 5 = _____

g 95 ÷ 4 = _____ **h** 75 ÷ 5 = _____ **i** 87 ÷ 2 = _____

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1.6 Using known addition and subtraction facts

I can

- recall addition and subtraction facts up to 20
- use known facts to solve problems involving larger numbers

Example

Work out the following additions and subtraction.

a 65 + 7 **b** 88 – 30 **c** 63 + 26

Solution

Look for easy methods, like these:

a 65 is 60 + 5 60 + 5 + 7 = 60 + 12 = 72

b 8 – 3 = 5 so 80 – 30 = 50 and 88 – 30 = 50 + 8 = 58

c 63 + 26 = 60 + 3 + 20 + 6 = 60 + 20 + 3 + 6 = 80 + 9 = 89

You can probably do these in your head. There is no need to write them out.

Practice questions

1 Work out these additions.

a 7 + 12 = _____ **b** 13 + 14 = _____ **c** 13 + 6 = _____

d 44 + 30 = _____ **e** 50 + 35 = _____ **f** 65 + 4 = _____

g 216 + 13 = _____ **h** 350 + 14 = _____ **i** 257 + 13 = _____

2 Work out these subtractions.

a 19 – 7 = _____ **b** 18 – 6 = _____ **c** 17 – 13 = _____

d 90 – 30 = _____ **e** 73 – 30 = _____ **f** 82 – 50 = _____

g 92 – 40 = _____ **h** 106 – 30 = _____ **i** 114 – 40 = _____

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3 Add together these numbers.

a 15 and 16 _____ **b** 25 and 16 _____ **c** 7 and 13 _____

d 17 and 13 _____ **e** 7, 8 and 9 _____ **f** 7, 18 and 19 _____

4 Fill in the missing number in these calculations.

a _____ + 7 = 18 **b** 24 – _____ = 15 **c** 13 + _____ = 37

d _____ – 18 = 9 **e** 23 – _____ = 9 **f** 14 + _____ = 59

5 Look at the numbers in the box, then answer the questions.

40	50	60	70	90	120	140	160

a Write down two numbers that add to make 100. _____ and _____

b Write down two numbers with a sum of 200. _____ and _____

c Write down two numbers with a difference of 40. _____ and _____

d Write down two numbers with a difference of 90. _____ and _____

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1.7 Whole number problems

I can

- decide which operation to use to solve a problem
- solve division problems with remainders

Example

There are 32 people waiting to travel in a taxi.

Each taxi can take five passengers.

How many taxis do they need?

Solution

This is a division question because you have to divide the group of people up between the taxis.

$32 \div 5 = 6$ remainder 2

They can fill six taxis but there are still two people left over.

They need seven taxis.

Practice questions

1 Work out these divisions. They all have remainders.

a $29 \div 4 =$ _____ remainder _____ **b** $31 \div 5 =$ _____ remainder _____

c $15 \div 2 =$ _____ remainder _____ **d** $20 \div 3 =$ _____ remainder _____

2 There are 23 biscuits in a packet. Four people share them equally.



How many does each person have? _____ biscuits

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3 There are five cereal bars in a box. Jasmine buys eight boxes.



How many cereal bars does Jasmine buy? _____ bars

4 There are 45 guests at a meal. They are sitting at tables. Each table can seat six people.

How many tables are needed? _____ tables

5 Ali, Rob and Carol are looking at how much money they have. Ali has £32, Rob has £14 and Carol has £57.

a How much do they have all together? _____

b How much more than Rob does Carol have? _____

c Carol has six two-pound coins and the rest is in five-pound notes.

Work out how many five-pound notes she has. _____

d Carol and Rob put their money together and share it out equally between the two of them.

How much does each one get? _____

6 Mintees are sweets that come in packets of 36



a Work out the number of Mintees in five packets. _____ Mintees

b If five people share one packet equally, how many Mintees will each person get?

_____ Mintees

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c Merry has a packet of Mintees. He eats 14 and gives five away. How many are left?

_____ Mintees

7 A bus has 52 seats and 23 passengers. At a bus stop four people get off and 12 get on.

How many empty seats are there on the bus? _____ empty seats

8 There are eight seats in a large taxi.

How many taxis are needed to carry 50 people? _____ taxis

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1.8 Place value

I can

- understand place value in numbers up to thousands

Example

Here is a number: 1062

a Write the number in words.

b Write down the tens digit.

Solution

a The number is one thousand and sixty-two.

b The tens digit is 6. *The 1 is the thousands digit, the 0 is the hundreds digit and the 2 is the units digit.*

Practice questions

1 Write these numbers in figures.



a twenty-seven _____

b thirty-four _____

c one hundred and fifty-two _____

d three hundred and eight _____

e seven hundred and forty _____

f one thousand two hundred and three _____

2 Write these numbers in words.

a 35 _____

b 179 _____

c 204 _____

d 990 _____

e 1678 _____

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3 Write down

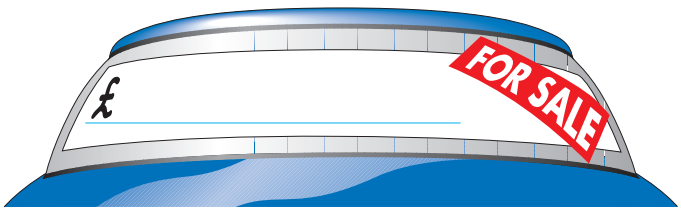
a the tens digit in 3412. _____

b the hundreds digit in 3052. _____

c the units digit in three hundred and sixty-seven. _____

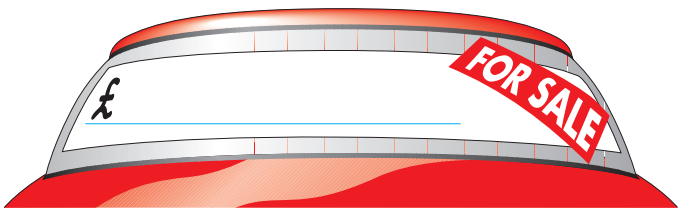
4 Write the price of each car on its windscreen in numbers.

a



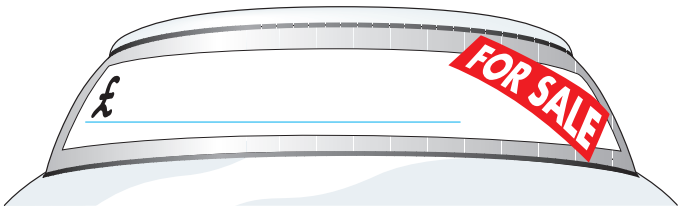
two thousand five hundred pounds

b



one thousand one hundred and ninety-nine pounds

c



two thousand and ninety-nine pounds

5 Which is the cheapest car in Question 4?

_____

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1.9 Negative numbers

I can

- order a set of positive and negative numbers
- calculate a temperature rise and fall across 0 °C

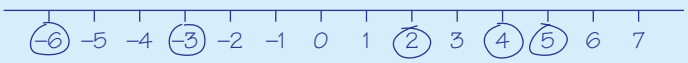
Example

Here are some temperatures in degrees C: **2, –3, 4, 5, –6**

Put them in order, from lowest to highest.

Solution

You can draw a number line to help you,and mark the numbers on it.



The lowest are on the left, the highest are on the right.

In order they are –6, –3, 2, 4, 5.

Practice questions

1 Here are some sets of temperatures in degrees C.

Put each set of temperatures in order, from lowest to highest.

a 2, 7, –2, 3, –3, 0 _____

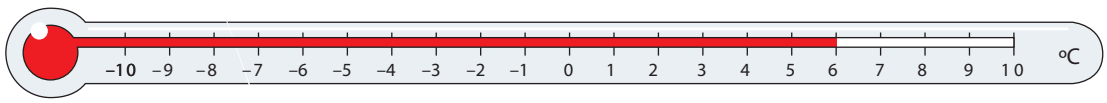
b –7, 4, –5, 6, 2 _____

c 9, 6, –5, 4, –3, 0 _____

d 6, 4, –2, –3, 9, –1 _____

2 Look at the thermometers and answer the questions.

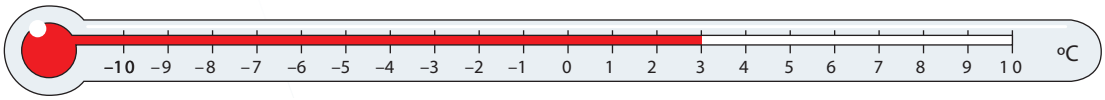
a



If the temperature increases by 3 °C what will the temperature be? _____°C

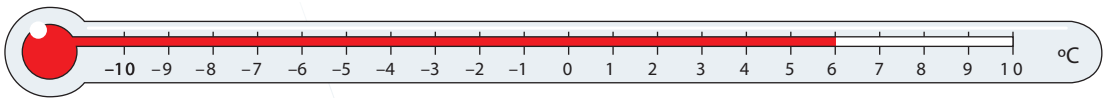
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b



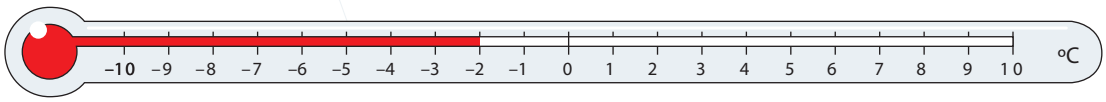
If the temperature falls by 5 °C what will the temperature be? _____ °C

c



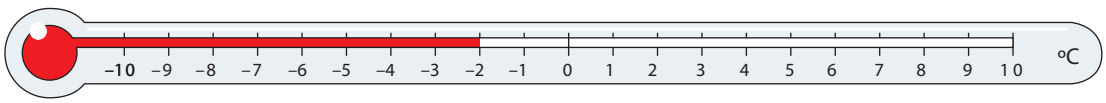
If the temperature falls by 4 °C what will the temperature be? _____ °C

d



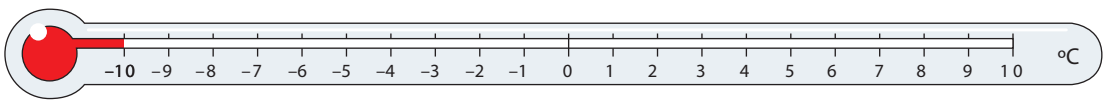
If the temperature increases by 6 °C what will the temperature be? _____ °C

e



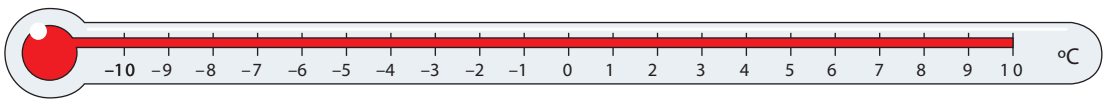
If the temperature increases by 2 °C what will the temperature be? _____ °C

f



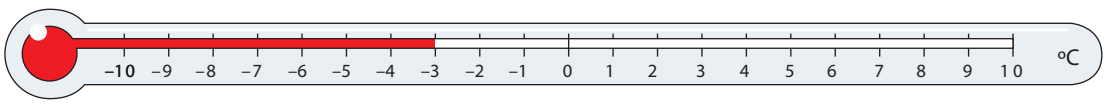
If the temperature increases by 15 °C what will the temperature be? _____ °C

g



If the temperature decreases by 20 °C what will the temperature be? _____ °C

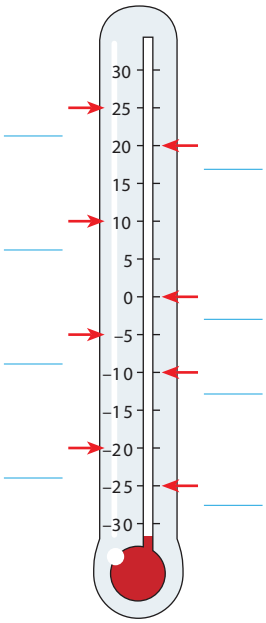
h



If the temperature increases by 12 °C what will the temperature be? _____ °C

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3 Write down the temperatures shown on the thermometer.



4 Fill in the missing numbers on the number lines.

a



b



c



d



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1.10 Recognising fractions

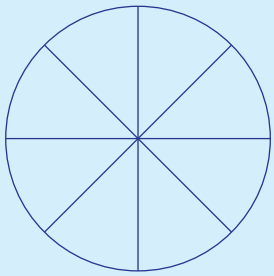
I can

- use simple fractions that are several parts of a whole
- recognise when two fractions are equivalent

Example

a Shade $\frac{3}{4}$ of this circle.

b Write down an equivalent fraction.

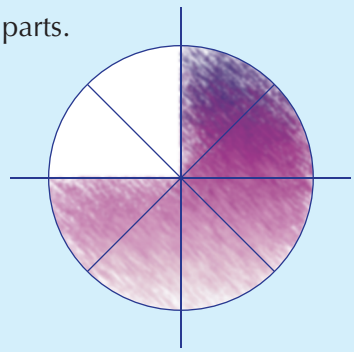


Solution

a To shade $\frac{3}{4}$ you need to divide the circle into four equal parts.

The longer lines do this.

Shade three of the four parts.



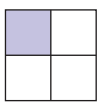
b There are eight equal small slices.

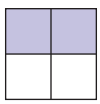
Six of them are shaded.

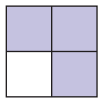
An equivalent fraction is $\frac{6}{8}$.

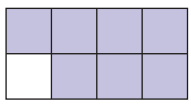
Practice questions

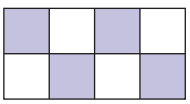
1 What fraction of each shape is shaded?

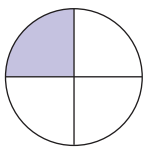
a  _____

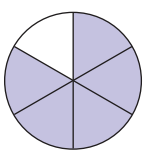
b  _____

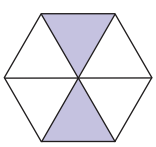
c  _____

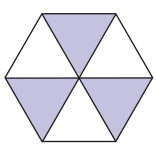
d  _____

e  _____

f  _____

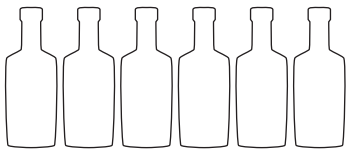
g  _____

h  _____

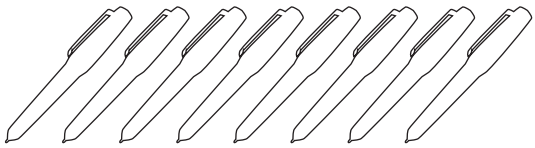
i  _____

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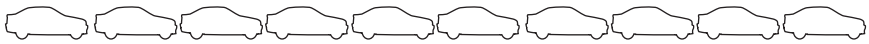
2 **a** Colour in $\frac{1}{3}$ of the bottles.



b Colour in $\frac{3}{4}$ of the pen



c Colour in $\frac{1}{5}$ of the cars.



d Colour in $\frac{2}{3}$ of the phones.



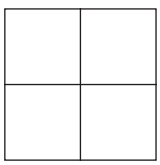
e Colour in $\frac{3}{5}$ of the houses.

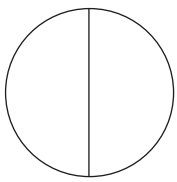


f Colour in $\frac{1}{4}$ of the light bulbs.

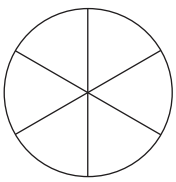


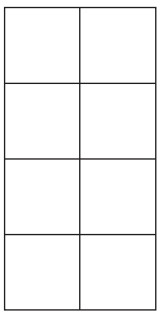
3 Colour in the amounts that make a half.

a 

b 

c 

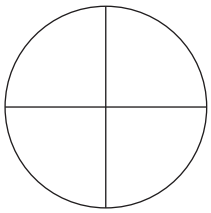
d 

e 

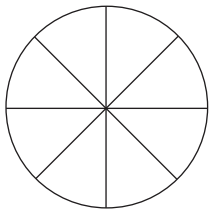
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4 Colour in the amounts that make a quarter.

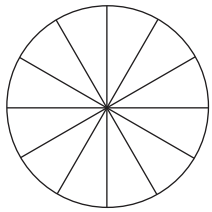
a



b

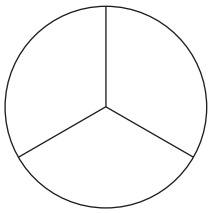


c

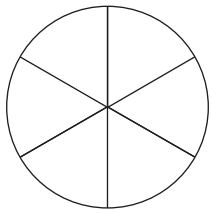


5 Colour in the equivalent fractions for a third.

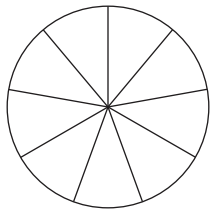
a



b

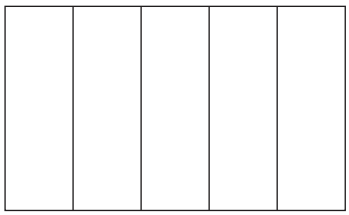


c

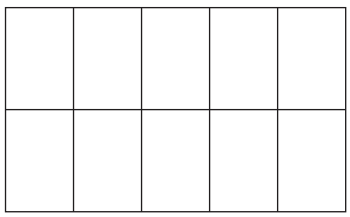


6 Colour in the equivalent fractions for a fifth.

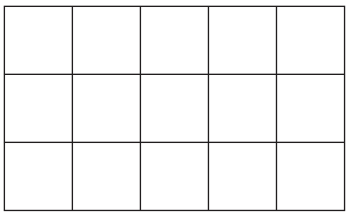
a



b



c



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1.11 Using decimals in context

I can

- use decimals and understand place value in decimals in a context, such as money

Example

a Write 1620 pence in pounds.

b Write 124 millimetres in centimetres.

Solution

a There are 100 pence in one pound.

1620 pence = 16 pounds and 20 pence = £16.20.

b There are 10 millimetres in one centimetre.

124 millimetres = 12 centimetres and 4 millimetres = 12.4 centimetres.

Practice questions

1 **a** Write down the amount of money in each of the piles below. Give your answers in pounds.






_____ _____ _____ _____

b Put a circle around the largest amount. Put a square around the smallest amount.

2 Write these amounts of money in pounds.

a 250 pence = £ _____

b 408 pence = £ _____

c 1287 pence = £ _____

d 4000 pence = £ _____

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3 Write these amounts in pence.

a £7 = _____ pence

b £12.50 = _____ pence

c £0.80 = _____ pence

d £0.05 = _____ pence

4 45 millimetres is equal to 4.5 centimetres.

Write these lengths in centimetres.

a 23 mm = _____ cm

b 51 mm = _____ cm

c 60 mm = _____ cm

d 8 mm = _____ cm

5 5.6 centimetres is equal to 56 millimetres.

Write these lengths in millimetres.

a 6.2 cm = _____ mm

b 12.5 cm = _____ mm

c 25 cm = _____ mm

d 0.2 cm = _____ mm

6 Write the tenths from 0 to 1 as decimals.



7 Fill in the missing numbers on the number lines. The first one has been done for you.

a



b



c



d



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8 Write the hundredths from 0 to 0.1 as decimals.



9 Fill in the missing numbers on the number lines. The first one has been done for you.

a



b



c



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Comments, next steps, misconceptions



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2 Algebra

2.1 Sequences of numbers

I can

- recognise patterns in sequences of numbers

Example

Here is a sequence of numbers: **9** **13** **17** **21** **25**

Work out the next two numbers.

Solution

The numbers increase by 4 each time.

The next number is 25 + 4 = 29.

The number after 29 is 29 + 4 = 33.

Practice questions

1 Write the next two numbers in each of these number sequences using the rule shown.

a The rule is add on 2 each time.

22 24 26 28 _____ _____

b The rule is add on 5 each time.

7 12 17 22 _____ _____

c The rule is subtract 3 each time.

30 27 24 21 _____ _____

d The rule is subtract 10 each time.

95 85 75 65 _____ _____

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e The rule is double each time.

1 2 4 8 _____ _____

2 Write the next two numbers in each of these number sequences.

a 3 5 7 9 _____ _____

b 42 44 46 48 _____ _____

c 30 35 40 45 _____ _____

d 1 7 13 19 _____ _____

e 40 37 34 31 _____ _____

f 50 44 38 32 _____ _____

g 10 21 32 43 _____ _____

h 120 116 112 108 _____ _____

3 Write in the missing numbers in each of these number sequences.

a 12 16 _____ 24 28 32 _____ 40

b 17 22 27 _____ _____ 42 47 52 _____ 62

c 99 96 93 _____ 87 _____ 81 78 _____ 72 69

d 46 42 _____ _____ _____ 26 22 18 _____

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2.2 The equals sign

I can

- use the equals sign (=) correctly

Example

Here are some numbers with gaps between them: **9 _____ 3 _____ 4 _____ 2**

Put the signs +, – and = in the gaps to make a correct statement.

Solution

Try putting the signs in different places.

The correct way is: $9 - 3 = 4 + 2$.

$9 - 3$ and $4 + 2$ are both equal to 6.

Practice questions

1 Put the signs + and = in each of these to make a correct statement.

a 9 _____ 3 _____ 6 **b** 12 _____ 2 _____ 14

c 6 _____ 1 _____ 7 **d** 11 _____ 3 _____ 8

2 Put the signs – and = in each of these to make a correct statement.

a 9 _____ 3 _____ 6 **b** 12 _____ 14 _____ 2

c 6 _____ 7 _____ 1 **d** 11 _____ 3 _____ 8

3 Put the signs +, – and = in each of these to make a correct statement.

a 12 ____ 3 ____ 18 ____ 3 **b** 3 ____ 8 ____ 5 ____ 6 **c** 10 ____ 6 ____ 8 ____ 4

4 Put the signs +, × and = in each of these to make a correct statement.

a 3 ____ 5 ____ 2 ____ 4 **b** 3 ____ 5 ____ 8 ____ 7 **c** 6 ____ 3 ____ 3 ____ 3

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5 Put signs in each of these to make correct statements.

a 24 = 4 _____ 6

b 24 _____ 4 = 6

c 19 = 12 _____ 7

d 19 _____ 12 = 7

6 Put signs in each of these to make correct statements.

a 12 = 3 _____ 4 _____ 5

b 4 _____ 3 = 2 _____ 6

c 5 _____ 4 = 8 _____ 7

d 2 _____ 5 = 6 _____ 4

e 5 _____ 1 = 12 _____ 2

f 50 _____ 8 = 25 _____ 17

Comments, next steps, misconceptions



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