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Edexcel GCSE
Maths

Foundation Student Book
Answers

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Chapter 1 – Number: Basic number

Exercise 1A

1 **a** 25, 62, 86, 151, 219, 310, 400, 501
b 97, 357, 368, 740, 888, 2053, 4366
c –9,–6, –2, 0, 2, 3, 4, 8

2 **a** 700, 401, 174, 117, 80, 65, 18, 2
b 6227, 3928, 2034, 762, 480, 395, 89, 59

3 **a** Number 4 (£128 250)
b Number 1 (£129 100)
c £850

4 **a** 368, 386, 638, 683, 836, 863
b 368
c 863

5 408, 480, 804, 840

6 33, 35, 38, 53, 55, 58, 83, 85, 88

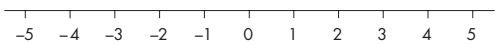
7 7045 or 7405

8 **a** is smaller than **b** is smaller than
c is smaller than **d** is bigger than
e is smaller than **f** is smaller than

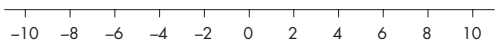
9 **a** 973 + 85 = 1058 or
975 + 83 = 1058 or
985 + 73 = 1058 or
983 + 75 = 1058
b 357 – 98 = 259

10 **a** is smaller than **b** is bigger than
c is smaller than **d** is smaller than
e is bigger than **f** is smaller than
g is smaller than **h** is bigger than
i is bigger than **j** is smaller than
k is smaller than **l** is bigger than

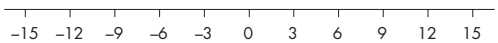
11 **a**



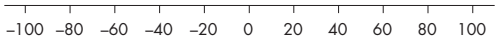
b



c

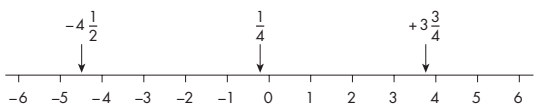


d



12 6 °C –2 °C –4 °C 2 °C

13



14 **a** 7, 8, 9, 10, 11
b 6, 7, 8, 9, 10
c 2, 3, 4, 5, 6, 7, 8

Exercise 1B

1 **a** 11 **b** 6 **c** 10 **d** 12
e 11 **f** 13 **g** 11 **h** 12
i 12 **j** 4 **k** 13 **l** 3

2 **a** 16 **b** 2 **c** 10 **d** 10
e 6 **f** 18 **g** 6 **h** 15
i 9 **j** 12 **k** 3 **l** 8

3 **b** $3 + (2 \times 4) = 11$ **c** $(9 \div 3) - 2 = 1$
d $9 - (4 \div 2) = 7$ **e** $(5 \times 2) + 3 = 13$
f $5 + (2 \times 3) = 11$ **g** $(10 \div 5) - 2 = 0$
h $10 - (4 \div 2) = 8$ **i** $(4 \times 6) - 7 = 17$
j $7 + (4 \times 6) = 31$ **k** $(6 \div 3) + 7 = 9$
l $7 + (6 \div 2) = 10$

4 **a** 38 **b** 48 **c** 3 **d** 2
e 5 **f** 14 **g** 20 **h** 19
i 54 **j** 7 **k** 2 **l** 7
m 7 **n** 10 **o** 2

5 **a** (4 + 1)
b No brackets needed
c (2 + 1)
d No brackets needed
e (4 + 4)
f (16 – 4)
g No brackets needed
h (5 + 5)
i (4 + 2)
j No brackets needed
k No brackets needed
l (8 – 2)

6 **a** 8 **b** 6 **c** 13 **d** 9
e 12 **f** 8 **g** 15
h 16 **i** 1

7 No, correct answer is 5 + 42 = 47

8 **a** $2 + 3 \times 4 = 14$
b $8 - 4 \div 4 = 7$ (correct)
c $6 + 3 \times 2 = 12$ (correct)
d $7 - 1 \times 5 = 2$
e $2 \times 7 + 2 = 16$ (correct)
f $9 - 3 \times 3 = 0$

9 **a** $2 \times 3 + 5 = 11$ **b** $2 \times (3 + 5) = 16$
c $2 + 3 \times 5 = 17$ **d** $5 - (3 - 2) = 4$
e $5 \times 3 - 2 = 13$ **f** $5 \times 3 \times 2 = 30$

10 $4 + 5 \times 3 = 19$
$(4 + 5) \times 3 = 27$. So $4 + 5 \times 3$ is smaller

11 $(5 - 2) \times 6 = 18$

12 $8 \div (5 - 3) = 4$

13 $10 - 3 \times 1.5$ and $10 - 1.5 - 1.5 - 1.5$

14 **a** T **b** F **c** F
d T **e** F **f** T
g F **h** F **i** T
j T

15 **a** **i** $(4 + 4) \div (4 + 4) = 1$
ii $(4 \div 4) + (4 \div 4) = 2$
iii $(4 + 4 + 4) \div 4 = 3$
iv $(4 - 4) \times 4 + 4 = 4$
v $(4 \times 4 + 4) \div 4 = 5$
b There are other ways to do these:
$4 + (4 + 4) \div 4 = 6$, $4 + 4 - (4 \div 4) = 7$
$(4 + 4) \times (4 \div 4) = 8$, $(4 + 4) + (4 \div 4) = 9$
c There are other ways to do this
$4 + \sqrt{4} + \sqrt{4} + \sqrt{4} = 10$

16

8	÷	4	+	6	=	8
×		+		−		
2	+	9	−	1	=	10
−		÷		×		
7	+	3	−	5	=	5
=		=		=		
9		7		1		

Exercise 1C

1 **a** 713 **b** 151 **c** 6381
d 968 **e** 622

2 **a** 646 **b** 826 **c** 3818
d 75.5 **e** 92.92 **f** 85.33

3 **a** 450 **b** 563 **c** 482
d 414 **e** 285 **f** 2892
g 4417 **h** 3767 **i** 4087
j 1828

4 **a** 128 **b** 29 **c** 334
d 17.8 **e** 61.63 **f** 6.882

5 **a** 558 miles **b** 254 miles

6 252

7 Units digit should be 6 (from 14 – 8)

8 For example: 181 – 27 = 154

9 **a** 115 **b** 204 **c** 294
d 212 **e** 425 **f** 150
g 800 **h** 960 **i** 1360
j 1518

10 **a** 294 **b** 370 **c** 288
d 83.2 **e** 21.63 **f** 25.2
g 16.44 **h** 321.5 **i** 30

11 **a** 219 **b** 317 **c** 315
d 10.6 **e** 0.99 **f** 12.1
g 25.2 **h** 1.41 **i** 1.44

12 **a** 705 miles **b** £3525

13 **a** 47 miles
b Three numbers with a total of 125. First number must be less than 50, second number less than first number, third number less than second number, for example 48, 42 and 35.

14 **a** 119 **b** 96 **c** 144
d 210 **e** 210

15 **a** 13 **b** 37 weeks **c** 4.3 m
d 36 **e** 45

16 **a** $152 + 190 = 324$
b $(10 \times 190) + 76 = 1976$
c $(100 \times 38) + 190 = 3990$

Exercise 1D

1 **a** –1 **b** –1 **c** 2 **d** –3
e –4 **f** –6 **g** –6 **h** –1
i –5 **j** –4 **k** 4 **l** –1

2 **a** –4 **b** –4 **c** –10 **d** 2
e –32 **f** –30 **g** –8 **h** 10
i –36 **j** –14 **k** 12 **l** –101

3 **a** 6 **b** –5 **c** 6 **d** –1
e –2 **f** –6 **g** –9 **h** –5
i –80 **j** –7 **k** –1 **l** –47

4 **a** $-5 + 7 = -2$
b $+5 - 6 = -1$
c $-3 - 8 = -11$

5 **a** 7 degrees **b** –6 °C

6 **a** 2 – 8
b 2 + 5 – 8 or 2 + 4 – 7 or 8 – 4 – 5 or 8 – 2 – 7 or 5 – 4 – 2
c 2 – 5 – 7 – 8 **d** 2 + 5 – 4 – 7 – 8

7 500 feet

Exercise 1E

1 **a** 6 **b** 7 **c** 8 **d** 6
e 8 **f** 10 **g** 2 **h** –3
i 2 **j** –3 **k** 1 **l** –5
m 3 **n** –4 **o** –3 **p** –8

2 **a** 7 **b** –8 **c** –5 **d** –11
e 11 **f** 6 **g** 8 **h** 8
i –2 **j** –1 **k** –9 **l** –5
m 5 **n** –9 **o** 8 **p** 0

3 **a** 3 °C **b** 0 °C **c** –3 °C
d –5 °C **e** –11 °C

4 **a** 10 degrees Celsius
b 7 degrees Celsius
c 9 degrees Celsius

5 –9, –6, –5, –1, 1, 2, 3, 8

6 **a** 2 **b** –3 **c** –5
d –7 **e** –10 **f** –20

7 **a** 2 **b** 4 **c** –1
d –5 **e** –11 **f** 8

8 **a** 13 **b** 2 **c** 5
d 4 **e** 11 **f** –2

9 **a** –10 **b** –5 **c** –2
d 4 **e** 7 **f** –4

10 **a** –5 **b** 6 **c** 0
d –6 **e** –2 **f** 212
g –67 **h** 7 **i** 25

11 Yes. The worst case is at when the temperature is just below 0 °C and the thermometer error causes a measurement of 0 + 2 = 2 °C which is below 3 °C so the alert will come on.

12 **a** –1, 0, 1, 2, 3
b –9, –8, –7, –6, –5
c –2, –1, 0, 1, 2, 3; –4, –3, –2, –1, 0, 1
d –12, –11, –10, –9, –8, –7;
–14, –13, –12, –11, –10, –9
e –8, –7, –6, –5, –4, –3, –2;
–5, –4, –3, –2, –1, 0, 1
f –10, –9, –8, –7, –6, –5, –4; –1, 0, 1, 2, 3, 4, 5

13 **a** –4 **b** +3 **c** +4 **d** –6
e +7 **f** +2 **g** 0 **h** 0
i –6 **j** –7 **k** –9 **l** +4
m –3 **n** –9 **o** 0

14 **a** +6 + +5 = 11 **b** +6 + –9 = –3
c +6 – –9 = 15 **d** +6 – +5 = 1

15 **a** +5 + +7 – –9 = +21
b +5 + –9 – +7 = –11
c +7 + –7 or +4 + –4

16 –3 and 8

17 $-7\frac{1}{2}$ and $-4\frac{1}{2}$

Exercise 1F

1 **a** –15 **b** –14 **c** –24 **d** 6
e 14 **f** 2 **g** –2 **h** –8
i –4 **j** 3 **k** –24 **l** –10
m –18 **n** 16 **o** 36 **p** –4
q –12 **r** –4 **s** 7 **t** 25
u 18 **v** –8 **w** –45
x 3 **y** –40

2 **a** –9 **b** 16 **c** –3 **d** –32
e 18 **f** 18 **g** 6 **h** –4
i 20 **j** 16 **k** 8 **l** –48
m 13 **n** –13 **o** –8 **p** 0
q 16 **r** –42 **s** 6 **t** 1
u –14 **v** 6 **w** –4
x 7 **y** 0

3 **a** –2 **b** 30 **c** 15
d –27 **e** –7

4 **a** 4 **b** –9 **c** –3
d 6 **e** –4

5 **a** –9 **b** 3 **c** 1

6 **a** 16 **b** –2 **c** –12

7 **a** 24 **b** 6 **c** –4 **d** –2

8 For example: 1 × (–12), –1 × 12, 2 × (–6), 6 × (–2), 3 × (–4), 4 × (–3)

9 For example: 4 ÷ (–1), 8 ÷ (–2), 12 ÷ (–3), 16 ÷ (–4), 20 ÷ (–5), 24 ÷ (–6)

10 **a** 21 **b** –4 **c** 2 **d** –16
e 2 **f** –5 **g** –35 **h** –17
i –12 **j** 6 **k** 45 **l** –2
m 0 **n** –1 **o** –7 **p** –36
q 9 **r** 32 **s** 0 **t** –65

11 **a** –12 **b** 12 degrees **c** 3 × –6

12 –5 × 4, 3 × –6, –20 ÷ 2, –16 ÷ –4

13 **a** 16 **b** 41 **c** 21 **d** 64

Exercise 1G

1 **a** 12 138 **b** 45 612 **c** 29 988
d 20 654 **e** 51 732 **f** 25 012
g 19 359 **h** 12 673 **i** 140 224
j 92 851 **k** 520 585 **l** 78 660

2 3500

3 No, 62 pupils cannot get in

4 Yes, he walks 57.6 km

5 Yes, 1 204 000 letters

6 5819 litres

7 No, she raised £302.40

Exercise 1H

1

a 25	**b** 15	**c** 37
d 43	**e** 27	**f** 48
g 53	**h** 52	**i** 32
j 57	**k** 37 rem 15	**l** 25 rem 5
m 34 rem 11	**n** 54 rem 9	**o** 36 rem 11
p 17 rem 4	**q** 23	**r** 61 rem 14
s 42	**t** 27 rem 2	

2 68

3 38

4 4 months

5 34 hours

6 45p

7 **a** £46.75

b 13 weeks

8 An odd number divided by an even number cannot give a whole number

9 **a** 64 **b** 22 **c** 22

Exercise 1I

1

a 89.28	**b** 298.39	**c** 66.04
d 167.98	**e** 2352.0	**f** 322.4
g 1117.8	**h** 4471.5	**i** 464.94
j 25.55	**k** 1047.2	**l** 1890.5

2 **a** £224.10 **b** £223.75 **c** £29.90

3 $5 \times 7 = 35$

4 £54.20

5 **a** £120.75 **b** £17 – £3.45 = £13.55

Review questions

1 **a** £1505

b 19 people

2 Murray did the addition first, Harry did the multiplication first.

3 **a** Adam has calculated 5×2 instead of 5^2, Bekki has added 3 to 5 first, instead of doing the power first.
b 26

4 **a** –2 °C, –1 °C, 0 °C, 1 °C
b 3 degrees

5 **i** Any number between –196 °C and –210 °C, e.g. –200 °C
ii Any number over –196 °C, e.g. –10 °C or any positive number
iii Any number below –210 °C, e.g. –250 °C

6 answer in the region of £7.50 to £8

7 DVD, jeans and pen

8 Pack of 8 at £0.625 each

9 **a** £49.90
b Small, so that all four children can have an ice cream (cost £4.80)

10 Yes. She only needed 8 paving stones.

11 2.5×8.4, 5×4.2, $10 \times 2.1 = 21$

12 £133.05

13 **a** £141.75 **b** £119.25

14 **a** 7542
b 2457
c $27 \times 2 = 54$

Chapter 2 – Geometry and measures: Measures and scale drawings

Exercise 2A

1 **a** 26 cm, 1.5 kg. Check individual answers.
b-j Check individual answers

2 The 5 metre ladder, since his height is about 175 cm and the lamp post will be about 525 cm.

3 Metres and centimetres are too small as units; needs to be a large unit as this is a large distance.

4

a 3400 mm	**b** 135 mm	**c** 67 cm
d 7030 m	**e** 2400 ml	**f** 590 cl
g 84 ml	**h** 5200 l	**i** 3750 kg
j 0.000 94 l	**k** 15 200 g	**l** 0.19 ml

5 **a** 1.25 m **b** 8.2 cm **c** 0.55 m
d 2.1 km **e** 4.2 kg **f** 5.75 t
g 2.58 l **h** 3.4 l **i** 0.6 t
j 63 cl **k** 35 cm^3 **l** 1.035 m^3

6 He should choose the 2000 mm × 15 mm × 20 mm.

7 1 000 000

8 **a** 1 000 000
b 1 000 000 000 000

Exercise 2B

1 **a** 24 in **b** 12 ft **c** 3520 yd
d 80 oz **e** 56 lb **f** 6720 lb
g 40 pt **h** 48 in **i** 36 in
j 30 ft **k** 64 oz **l** 5 ft
m 70 lb **n** 12 yd **o** 224 oz

2 **a** 5 miles **b** 120 pt **c** 5280 ft
d 8 ft **e** 7 st **f** 7 gal
g 2 lb **h** 5 yd **i** 5 tons
j 63 360 in **k** 8 lb **l** 9 gal
m 10 st **n** 3 miles **o** 35 840 oz

3 The 32-ounce bag

4 4 014 489 600 in^2

5 **a** miles, feet, ounces, gallons
b 108 inches
c 3 gallons

Exercise 2C

1 **a** 20 cm **b** 13.2 lb **c** 48 km
d 67.5 l **e** 2850 ml **f** 10 gal
g 12 in **h** 50 miles **i** 5 kg
j 3 pints **k** 160 km **l** 123.2 lb
m 180 l **n** 90.9 kg **o** 1100 yd
p 30 cm **q** 6.4 kg **r** 90 cm

2 ton

3 metre

4 **a** **i** 1000 g **ii** 1 kg
b **i** 4500 g **ii** 4.5 kg

5 **a** 135 miles **b** 50 mph **c** 2 h 42 min

6 4 hours 10 minutes

7 288 in^3

Exercise 2D

1 **a** Onions: 40 m × 10 m; Soft fruits: 50 m × 10 m; Apple trees: 20 m × 20 m; Lawn: 30 m × 20 m; Potatoes: 50 m × 20 m
b Onions: 400 m^2; Soft fruit: 500 m^2; Apple trees: 400 m^2; Lawn: 600 m^2; Potatoes: 1000 m^2

2 **a** 33 cm **b** 9 cm

3 **a** 6 m × 6 m **b** 8 m × 2 m **c** 4 m × 3 m
d 4 m × 6 m **e** 6 m × 4 m **f** 3 m × 2 m

4 **a** Student's scale drawing
b 39 plants (the edge is about 19 m)

5 **a** 15 km **b** 9 km **c** 12 km
d 13.5 km **e** 7.8 km **f** 5.4 km

6 **a** On the map, Carlisle to Scarborough is 5.8cm, so 1cm represents 232 × 100000 / 5.8 = 4000000
b **i** 176 km **ii** 66 km

7 **a** 80 km **b** 70 km **c** 122 km

8 **d** 1:600

9 **a** **i** 160 miles **ii** 310 miles

Exercise 2E

1 Bicycle about 2 m, bus about 10 m, train about 17 m

2 Height about 4 m, length about 18 or 19 m

3 **a** About 4 m
b About 5 m
c About 5.5 m

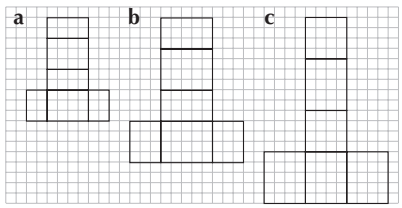
4 About 9 m; the ratio of Joel's height in the photograph to his real height must be the same as the ratio of the height on the statue in the photograph to its actual height

5 Assuming the man's height is about 1.8 metres, the T. Rex height is about 4 times the height of the man and 4 × 1.8 is 7.2m. So yes, Ben's estimate of the height is correct. The length of T. Rex looks about 5 times the height of the man. 5 × 1.8 = 9m, so Ben has overestimated the length of the T. Rex, but not by much.

Exercise 2F

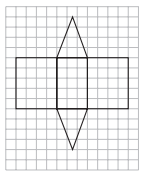
1 A, C, D

2



3 Yes. For example, Jenny could make it from a card 10 cm × 15 cm

4



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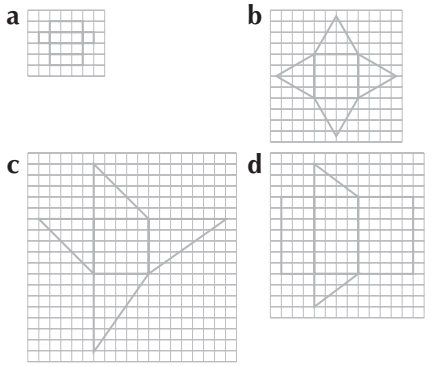

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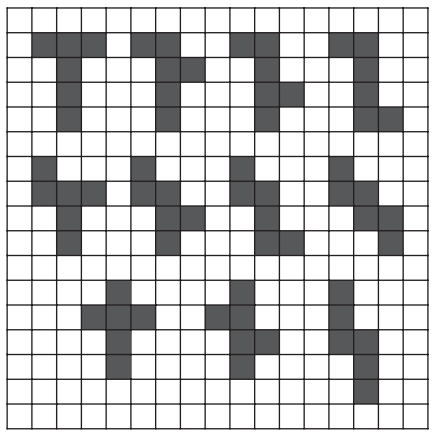
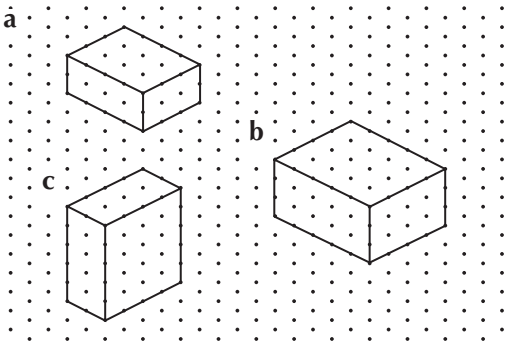


7 a and b

8 The length and height of the net needs to be at least 5.45 cm + 5 cm + 5.45 cm = 15.9 cm, so a card 15 cm by 15 cm is too small.

Exercise 2G

1



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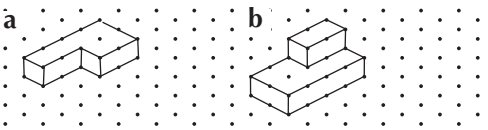


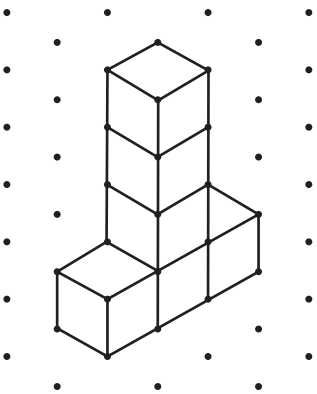
3 **a** **i** **ii** **iii**

b **i** **ii** **iii**

c **i** **ii** **iii**

4 Students' own diagram

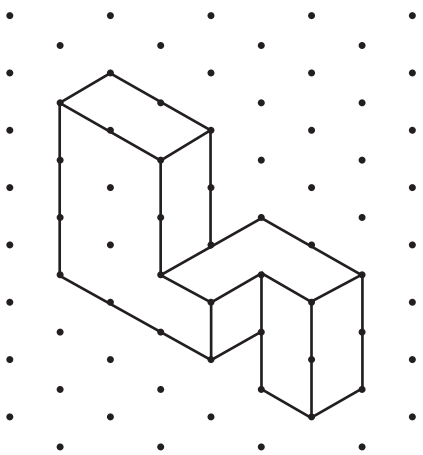
5



6

b She could be correct as 4 bricks could be put together to form a 2 cm by 12 cm block, 4 such blocks creates the bottom layer of bricks almost filling the base of 12 cm × 9 cm, making 4 × 4 =16 bricks in a layer. There will be 5 such layers, making 5 × 16 = 80 bricks, leaving a 1 cm × 12 cm × 5 cm gap at the end of the box. You can put two of the blocks sideways into this space, giving 8 extra bricks and a total of 88 bricks.

7



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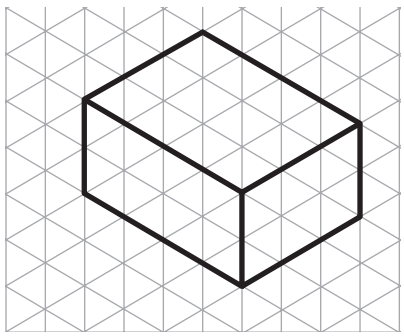


Review questions

1

The length of a garden	metres
The mass of a puppy	kilograms
The length of a brick	centimetres

2 **a** 40 000 metres **b** 25 miles

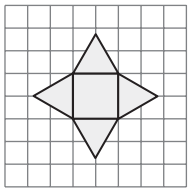
3 **a** **i** Kilometres **ii** Litres
b **i** 90 mm **ii** 3 kg

4 Andrew is correct as $130 \times \frac{5}{8} = 81.25$

5 **a** 1 m **b** 2.82 m

6 **a** 160 mm **b** 150 m

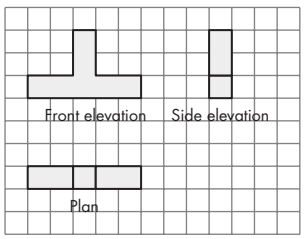
7



8 Centimetres is too small and this distance is an approximation of a large distance and so needs to be a large unit such as kilometres.

9 **a** Seven and a half miles
b 2.3 kg potatoes, 23 grams of butter, 2.3 litres of milk

10



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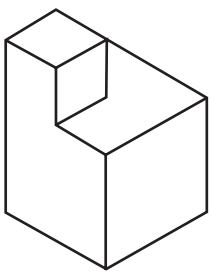


Chapter 3 – Statistics: Charts, tables and averages

Exercise 3A

1 a

Goals	0	1	2	3
Frequency	6	8	4	2

b 1 goal **c** 22

2 a/b

Temperature (°C)	Frequency
14–16	5
17–19	10
20–22	8
23–25	5
26–28	2

c 17–19 °C

d Getting warmer in the first half of the month and then getting cooler towards the end.

3 a

Score	1	2	3	4	5	6
Frequency	5	6	6	6	3	4

b 30

c Yes, frequencies are similar.

4 a/b

Height(cm)	Frequency
151–155	2
156–160	5
161–165	5
166–170	7
171–175	5
176–180	4
181–185	3
186–190	1

c 166–170 cm

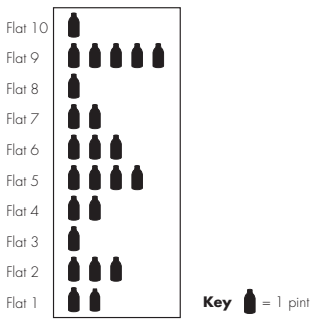
5 a Observation **b** Sampling
c Observation **d** Sampling
e Experiment **f** Observation
g Experiment

6 Draw a tally chart with class intervals 1–10, 11–20, 21–30, 31–40, 41–50 and 51–60. Then complete a grouped frequency table from the tally chart.

7 The class intervals overlap as the ages 20, 30, 40 and 50 are in two different class intervals

Exercise 3B

1



2 a May 9 h, Jun 11 h, Jul 12 h, Aug 11 h, Sep 10 h

b July **c** Visual impact, easy to understand.

3 a Simon **b** £330

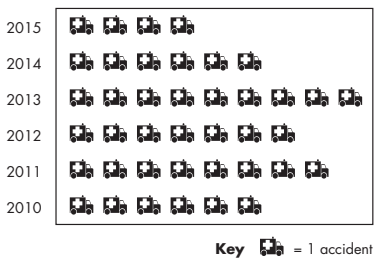
c Difficult to show fractions of a £ symbol.

4 a Swimming **b** 74

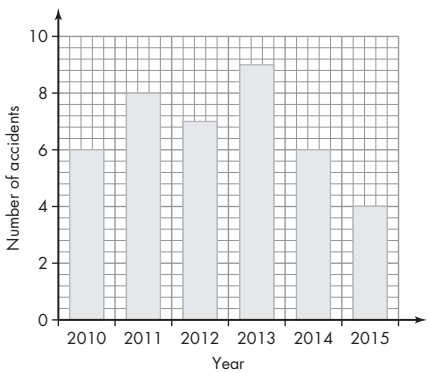
c a better visual impact and easier to read the frequencies

d No. It may not include people who are not fit or not interested in sport.

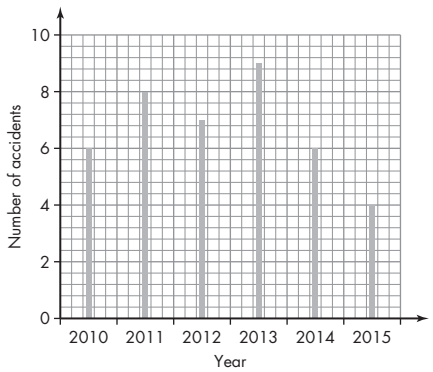
5 a



b



c

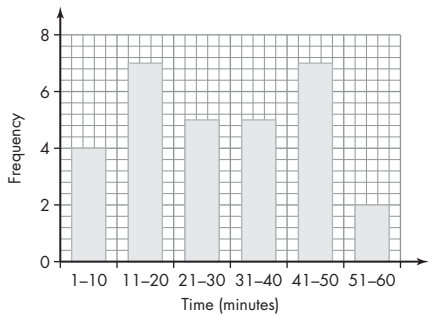


d Use the pictogram because an appropriate symbol makes more impact.

6 a

Time (min)	Frequency
1–10	4
11–20	7
21–30	5
31–40	5
41–50	7
51–60	2

b

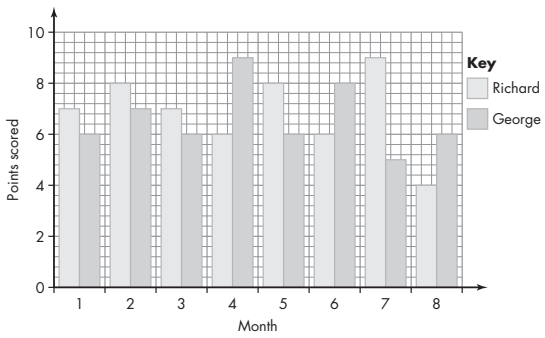


c Some students live close to the school and some live a good distance away and probably travel to school by bus.

7 Yes, in most cities. If you double the minimum temperature each time, it is very close to the maximum temperature.

8 The bar chart is misleading because the temperature axis does not start at zero.

9



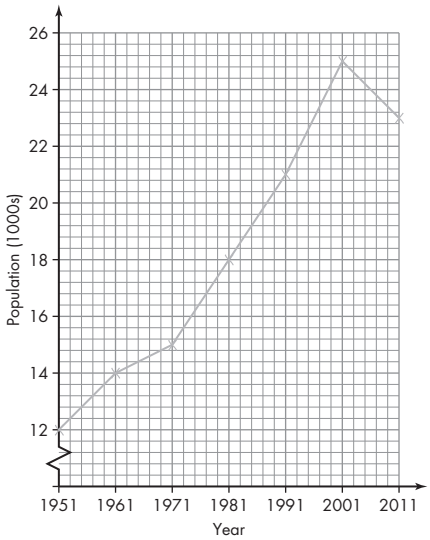
10 a 102 **b** 16 **c** 42

11 £88.80

Exercise 3C

1 a Tuesday, 52p **b** 2p
c Friday **d** £90

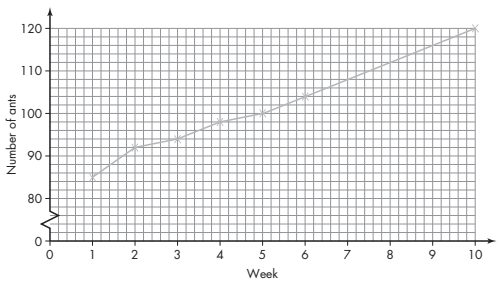
2 a



b about 16 500
c 1991 and 2001
d No; do not know the reason why the population started to decrease after 2001

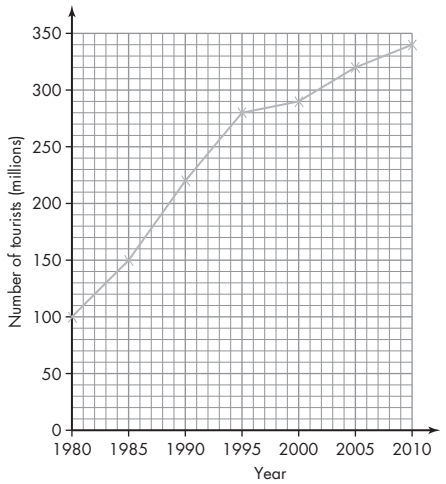
3 All the temperatures were higher than 20 degrees.

4 a



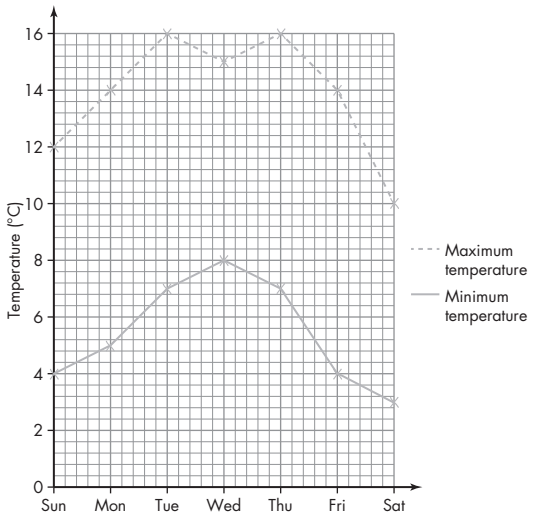
b 112

5 a



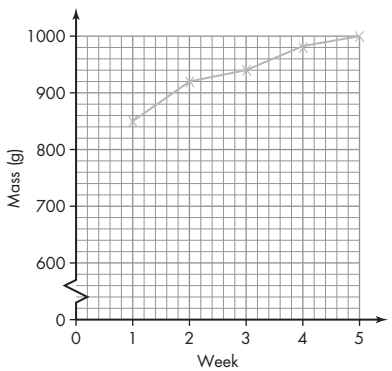
b About 60 million
c 1985 and 1990
d e.g. the trend in tourism is rising over the 30 year period. This is probably because holidays have become cheaper and the rise in popularity of package holidays

6 a



b 7 °C and 10 °C

7 a



b it is not possible to predict the trend as the puppy's mass is likely to level out.

Exercise 3D

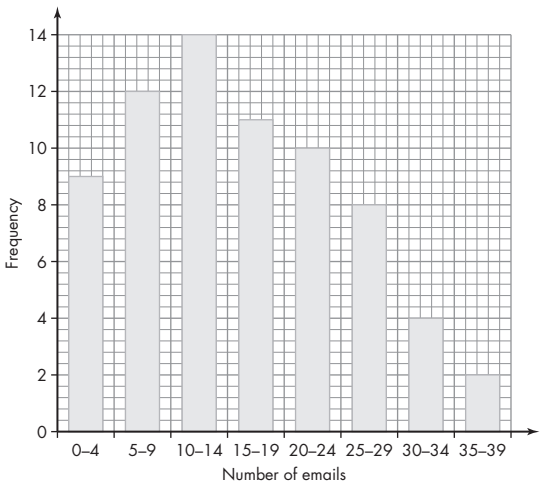
1 a 4 **b** 48 **c** –1
d $\frac{1}{4}$ **e** no mode **f** 3.21

2 a red **b** sun **c** β **d** M

3 a 32 **b** 6 **c** no
d no; boys generally take larger shoe sizes

4 a 5
b no; more than half the class got a higher mark

5 a



b 70
c 24
d cannot tell; know only that 9 households had between 0 and 4 emails
e 10–14

6 a 28
b **i** brown **ii** blue **iii** brown
c Both students had blue eyes.

7 a May lose count.
b Put in a table, or arrange in order
c 4

8 The mode will be the most popular item or brand sold in a shop.

Exercise 3E

1 a 5 **b** 33 **c** $7\frac{1}{2}$ **d** 24
e $8\frac{1}{2}$ **f** 0 **g** 5.25

2 a £2.20 **b** £2.25
c median, because it is the central value

3 **a** 5
b **i** 5 **ii** 215 **iii** 10 **iv** 10

4 **a** 13 years, Bella
b 162 cm, Dolly
c 46 kg, Amina
d Bella, because she is closest to the 3 medians

5 A median of £15 does not take into account the large payment of the £200 so is in no way representative.

6 **a** Possible answer: 11, 15, 21, 21 (one below or equal to 12 and three above or equal)
b Any four numbers higher than or equal to 12, and any two lower or equal to 12
c Eight, all 4 or under

7 12, 14, 14, 16, 20, 22, 24, or 12, 14, 14, 16, 18, 20, 24 or 12, 14, 14, 16, 18, 22, 24

8 **a** 21 **b** 16
c

Mark	Frequency
12	1
13	3
14	4
15	3
16	6
17	3
18	1

d There are 21 students altogether, so the median is the 11th mark when the marks are put in order.
There are 11 values up to a mark of 15, so the median is 15.

Exercise 3F

1 **a** 6 **b** 24 **c** 45
d 1.57 **e** 2

2 **a** 55.1 **b** 324.7 **c** 58.5
d 44.9 **e** 2.3

3 **a** 61 **b** 60 **c** 59
d Blake **e** 2

4 42 min

5 **a** £200 **b** £260 **c** £278
d Median, because the extreme value of £480 is taken into account when calculating the mean and the mode would be too small to represent the data.

6 **a** 35 **b** 36

7 **a** 6
b 16; all the numbers and the mean are 10 more than those in part **a**
c **i** 56 **ii** 106 **iii** 7

8 Possible answers: Speed – Kath, James, John, Joseph; Roberts – Frank, Jim, Helen, Evie.

9 36

10 24

Exercise 3G

1 **a** **i** Mode 3, median 4, mean 5
ii 6, 7, $7\frac{1}{2}$ **iii** 4, 6, 8
b **i** Mean: balanced data
ii Mode: 6 appears five times
iii Median: 28 is an extreme value

2 **a** **i** 29 **ii** 28 **iii** 27.1
b The mean as it is balanced data

3 **a** Mode 73, median 76, mean 80
b The mean, because it is the highest average

4 **a** Mean – takes all marks into account
b Median – avoids any extreme amounts
c Mode – non numerical data
d Median – avoids any extreme heights
e Mode – non numerical data
f Mean - takes all masses into account

5 Yes, mode is 31, median is 31, and mean is $31\frac{1}{2}$; all are close to the stated average.

6 **a** **i** £20 000 **ii** £28 000 **iii** £34 000
b **i** The 6% rise, because it gives a greater increase in salary for the higher paid employees.
ii 6% increase: £21 200, £29 680, £36 040; £1500 pay rise: £21 500, £29 500, £35 500

7 **a** Median **b** Mode **c** Mean

8 Tom-mean, takes all scores into account; David - median, avoids the extreme scores; Mohammed - mode, the two highest scores

9 The boss chose the mean while the worker chose the mode.

Exercise 3H

1 **a** 7 **b** 26 **c** 5
d 2.4 **e** 7

2 **a** 5°, 3°, 2°, 7°, 3°
b Variable or inconsistent weather over England

3 **a** £31, £28, £33 **b** £8, £14, £4
c Not particularly consistent

4 **a** 82 and 83 **b** 20 and 12
c Shona, because the ranges show that her scores are more consistent

5 **a** **i** 5 min and 4 min
ii 9 min and 13 min
b Number 50, because the times are more consistent

6 **a** Isaac, Oliver, Andrew, Niamh, Zoe, Jake and Zac
b For example: 70 cm to 90 cm

7 **a** The teachers because they have a high mean and students could not have a range of 20.
b Year 11 students as the mean is between 15 and16 and the range is 1.

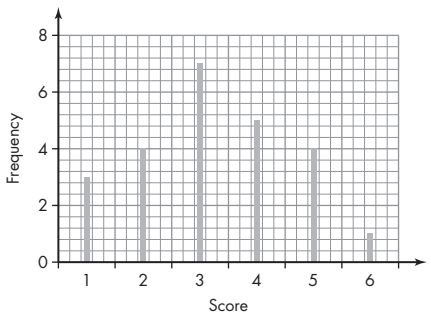
8 For example: **a** 1, 6, 6, 6, 6 **b** 2, 5, 5, 6, 7

Review questions

1 **a**

Number		Frequency
1	|||	3
2	||||	4
3	𝍸 |	7
4	𝍸	5
5	||||	4
6	|	1

b

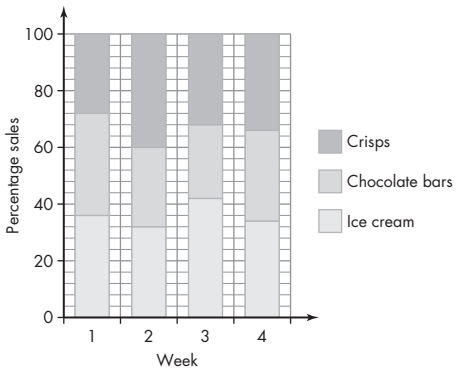


2 **a** 84 cm **b** 73 cm **c** 70 cm **d** the median, avoids the extreme height of 43 cm

3 The modal age of 16, as most of the team are this age

4 **a** **i** 75 minutes **ii** 80 minutes
b **i** 110 minutes **ii** 140 minutes
c on average Josh spent more time on his computer, but Hannah's times were more consistent.

5



6 **a** 20 **b** 25 **c** 46 **d** 43
e On average the boys scored higher and were more consistent.

7 The vertical scale does not start at zero, the horizontal scale is not linear, no £ sign for sales, no information to explain what it is about

8 **a** 12 **b** 11 **c** 11

9 11.6

Chapter 4 – Geometry and measures: Angles

Exercise 4A

1 **a** 48° **b** 307° **c** 108°
d 52° **e** 59° **f** 81°

2 **a** 82° **b** 105° **c** 75°

3 No, as 45° + 125° = 170° and for a straight line it should be 180°

4 **a** $x = 100°$ **b** $x = 110°$ **c** $x = 30°$

5 **a** $x = 55°$ **b** $x = 45°$ **c** $x = 12.5°$

6 **a** $x = 34°$, $y = 98°$ **b** $x = 70°$, $y = 120°$
c $x = 20°$, $y = 80°$

7 $6 \times 60° = 360°$; imagine six of the triangles meeting at a point

8 $x = 35°$, $y = 75°$; $2x = 70°$ (opposite angles), so $x = 35°$ and $x + y = 110°$ (angles on a line), so $y = 75°$

Exercise 4B

1 **a** 70° **b** 50° **c** 80° **d** 60°
e 75° **f** 109° **g** 38° **h** 63°

2 **a** No, total is 190° **b** Yes, total is 180°
c No, total is 170° **d** Yes, total is 180°
e Yes, total is 180° **f** No, total is 170°

3 **a** 80° **b** 67° **c** 20°
d 43° **e** 10° **f** 1°

4 **a–c** Students' own drawings **d** 180°

5 **a** 60° **b** Equilateral triangle
c Same length

6 **a** 70° each **b** Isosceles triangle
c Same length

7 **a** 109° **b** 130° **c** 135°

8 65°

9 Joe is not correct as DFE = 30°, DEF = 75° hence angle D = 180° – 105° = 75° but Hannah is correct as FED = FDE = 75°

10 $a = 35°$ (angles in a triangle) because the other angles in the triangle are 65° (angles on a line) and 80° (opposite angles) giving a total of 145, this subtracted from the 180 degrees in a triangle leaves the answer of 35. As a is less than 90°, it is acute.

11 Missing angle = y, $x + y = 180°$ and $a + b + y = 180°$ so $x = a + b$

12 32°

13 72°

Exercise 4C

1c 2, $2 \times 180° = 360°$

2c 3, $3 \times 180° = 540°$

3c 4, $4 \times 180° = 720°$

4

Shape	Number of sides	Triangles	Angle sum
triangle	3	1	180°
quadrilateral	4	2	360°
pentagon	5	3	540°
hexagon	6	4	720°
heptagon	7	5	900°
octagon	8	6	1080°
nonagon	9	7	1260°
decagon	10	8	1440°

5 18, $18 \times 180° = 3240°$

Exercise 4D

1 **a** 90° **b** 150° **c** 80°
d 80° **e** 77° **f** 131°

2 **a** No, total is 350° **b** Yes, total is 360°
c No, total is 350° **d** No, total is 370°
e Yes, total is 360° **f** Yes, total is 360°

3 **a** 100° **b** 67° **c** 120°
d 40° **e** 40° **f** 1°

4 **a** 90° **b** Rectangle **c** Yes, square

5 **a** 120° **b** 170° **c** 125°
d 136° **e** 149° **f** 126°

6 $60° + 60° + 120° + 120° + 120° + 240° = 720°$

7 $y = 360° - 4x$; $2x + y + 2x = 360°$, $4x + y = 360°$, so $y = 360° - 4x$

8 $x = 40°$, smallest angle = 60°

Exercise 4E

1

Regular polygon	Number of sides	Interior angle sum	Each interior angle
octagon	8	1080°	135°
nonagon	9	1260°	140°
decagon	10	1440°	144°

2

Regular polygon	Number of sides	Interior angle	Exterior angle
square	4	90°	90°
pentagon	5	108°	72°
hexagon	6	120°	60°
octagon	8	135°	45°
nonagon	9	140°	40°
decagon	10	144°	36°

3 **a** **i** 45° **ii** 8 **iii** 1080°
b **i** 20° **ii** 18 **iii** 2880°
c **i** 15° **ii** 24 **iii** 3960°
d **i** 36° **ii** 10 **iii** 1440°

4 **a** **i** 172° **ii** 45 **iii** 7740°
b **i** 174° **ii** 60 **iii** 10 440°
c **i** 156° **ii** 15 **iii** 2340°
d **i** 177° **ii** 120 **iii** 21 240°

5 **a** Exterior angle is 7°, which does not divide exactly into 360°
b Exterior angle is 19°, which does not divide exactly into 360°

c Exterior angle is 11°, which does divide exactly into 360°
d Exterior angle is 70°, which does not divide exactly into 360°

6 **a** 7° does not divide exactly into 360°
b 26° does not divide exactly into 360°
c 44° does not divide exactly into 360°
d 13° does not divide exactly into 360°

7 $x = 45°$, they are the same, true for all regular polygons

8 Three are 135°and two are 67.5°

9 48°; $\frac{1440° - 5 \times 240°}{5}$

10 10

11 72°, 72°, 108°, 144°, 144°

12 93°

Exercise 4F

1 **a** d **b** f **c** d
d f **e** f **f** e

2 **a** 40°
b $b = c = 70°$
c $d = 75°$, $e = f = 105°$
d $g = 50°$, $h = i = 130°$
e $j = k = l = 70°$
f $n = m = 80°$
g $a = 50°$, $b = 130°$
h $c = d = 65°$, $e = f = 115°$
i $g = i = 65°$, $h = 115°$
j $j = k = 72°$, $l = 108°$
k $m = n = o = p = 105°$
l $q = r = s = 125°$

3 **a** $a = 95°$ **b** $b = 66°$, $c = 114°$

4 **a** $x = 30°$, $y = 120°$ **b** $x = 25°$, $y = 105°$
c $x = 30°$, $y = 100°$ **d** $x = 50°$, $y = 110°$
e $x = 25°$, $y = 55°$ **f** $x = 20°$, $y = 140°$

5 290°; x is double the angle allied to 35°, so is $2 \times 145°$

6 $a = 66°$; angle BDC = 66° (angles in a triangle = 180°) and a is the corresponding angle

7 Angle PQD = 64° (alternate angles), so angle DQY = 116° (angles on a line = 180°)

8 Use alternate angles to see b, a and c are all angles on a straight line, and so total 180°

9 Third angle in triangle equals q (alternate angle), angle sum of triangle is 180°

Exercise 4G

1 **a** $a = 110°$, $b = 55°$ **b** $c = 75°$, $d = 115°$
c $e = 87°$, $f = 48°$

2 **a** $a = c = 105°$, $b = 75°$ **b** $d = f = 70°$, $e = 110°$
c $g = i = 63°$, $h = 117°$

3 **a** $a = c = 105°$, $b = 75°$ **b** $d = f = 93°$, $e = 87°$
c $g = i = 49°$, $h = 131°$

4 **a** $a = 135°$, $b = 25°$ **b** $c = d = 145°$
c $e =$ f $= 94°$

5 **a** $a = 58°$, $b = 47°$ **b** $c = 141°$, $d = 37°$
c $e =$ g $= 65°$, $f = 115°$

6 **a** 65° **b** 60° **c** 68°

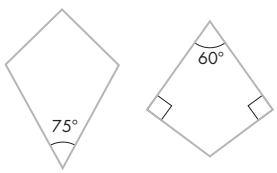
7 both 129°

8 Marie is correct as a rectangle is a special form of parallelogram with all angles = 90°

9 **a** 65°
b Trapezium, angle A + angle D = 180° and angle B + angle C = 180°

10 135°

11



12 A trapezium; angles add up to $10x$, two angles x and $4x = 2x + 3x$, the other pair of angles. Hence each pair adds up to 180 (since $2 \times 180 = 360$). Hence two pairs of allied angles, hence a trapezium. Alternatively you could have found that $x = 36$ which will give the same result.

Exercise 4H

1 **a** 110°, 12.6 km **b** 250°, 4.5 km
c 091°, 11.8 km **d** 270°, 8.4 km
e 130°, 7.2 km **f** 180°, 4.2 km

2 Students' sketches

3 **a** Sketch
b D is due south of B and B is east of A, so A must be west of D. A bearing to the west will be greater than 180°

4 **a** 090°, 180°, 270° **b** 000°, 270°, 180°

5 **a** 250° **b** 325° **c** 144°

6 **a** 045° **b** 286°
c Measure the distance from X to Y and divide 15 by this to find the scale of the map. Then measure the distance from Y to P and multiply by the scale factor.

7 **a** 900 m **b** 280°
c angle NHS = 150° and HS = 3 cm

8 108°

9 255°

10 9.2 km

Review questions

1 16°

2 **a** **i** 115°
ii opposite angles are equal and the angle opposite to x is 180 - 65
b the angles do not add up to 360°

3 **a** 50° **b** 32.5°

4 $3 \times 180 = 540°$ A five-sided shape can be split into three triangles hence 3×180

5 150°

6 angle TQP = 37°, alternate angles, PTQ = 180 – (29 + 37) = 114 angles in a triangle, QTS = 180 – 114, angles on a line, = 66.

7 333°

8 $360 \div 8 = 45°$, exterior angle formula is $\frac{360}{\text{number of slide}}$, in this case

9 180 – (360 ÷ 6) = 120°

10 Selvi might be correct. You will need to draw one example showing this is not a kite, and one example showing that this could be a kite

11 **b** 12.4 km

Chapter 5 – Number: Number properties

Exercise 5A

1 **a** 3, 6, 9, 12, 15 **b** 7, 14, 21, 28, 35
c 9, 18, 27, 36, 45 **d** 11, 22, 33, 44, 55
e 16, 32, 48, 64, 80

2 **a** 254, 108, 68, 162, 98, 812, 102, 270
b 111, 255, 108, 162, 711, 615, 102, 75, 270
c 255, 615, 75, 270 **d** 108, 162, 711, 270

3 **a** 72, 132, 216, 312, 168, 144
b 161, 91, 168, 294
c 72, 102, 132, 78, 216, 312, 168, 144, 294

4 **a** 98 **b** 99 **c** 96
d 95 **e** 98 **f** 96

5 **a** 1002 **b** 1008 **c** 1008

6 No, 50 is not a multiple of 6.

7 4 or 5 (as 2, 10 and 20 are not realistic answers)

8 **a** 18 **b** 28 **c** 15

9 66

10 5 numbers: 18, 36, 54, 72, 90

11 **a** 1, 2, 5, 10
b 1, 2, 3, 5, 6, 10, 15, 50
c 1, 2, 4, 5, 10, 20, 25, 50, 100
d For example, 60, 72, 84, 96

Exercise 5B

1 **a** 1, 2, 5, 10 **b** 1, 2, 4, 7, 14, 28
c 1, 2, 3, 6, 9, 18 **d** 1, 17
e 1, 5, 25 **f** 1, 2, 4, 5, 8, 10, 20, 40
g 1, 2, 3, 5, 6, 10, 15, 30
h 1, 3, 5, 9, 15, 45 **i** 1, 2, 3, 4, 6, 8, 12, 24
j 1, 2, 4, 8, 16

2 8 ways (1, 2, 3, 4, 6, 8, 12, 24 per box)

3 **a** 1, 2, 3, 4, 5, 6, 8, 10, 12, 15, 20, 24, 30, 40, 60, 120
b 1, 2, 3, 5, 6, 10, 15, 25, 30, 50, 75, 150
c 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, 48, 72, 144
d 1, 2, 3, 4, 5, 6, 9, 10, 12, 15, 18, 20, 30, 36, 45, 60, 90, 180
e 1, 13, 169
f 1, 2, 3, 4, 6, 9, 12, 18, 27, 36, 54, 108
g 1, 2, 4, 7, 14, 28, 49, 98, 196
h 1, 3, 9, 17, 51, 153
i 1, 2, 3, 6, 9, 11, 18, 22, 33, 66, 99, 198
j 1, 199

4 **a** 55 **b** 67 **c** 29 **d** 39
e 65 **f** 80 **g** 80 **h** 70
i 81 **j** 50

5 **a** 2 **b** 2 **c** 3 **d** 5
e 3 **f** 3 **g** 7 **h** 5
i 10 **j** 11

6 4 in each layer, which is a 2×2 square

7 It does not have a factor of 3.

8 5

Exercise 5C

1 23 and 29

2 97

3 All these numbers are not prime.

4 3, 5, 7

5 only if all 31 bars are in a single row, as 31 is a prime number and its only factors are 1 and 31

6 **a** E **b** C **c** C

7 **a** as this would leave 27 and taking off any other prime, which must be odd, would leave an even number

b 3, 7 and 19 or 5, 7 and 17 or 5, 11 and 13

8 All prime numbers greater than 2 are odd and Odd – Odd = Even.

9 97 – 61 = 36, 89–73 = 16, 89 – 53 = 36, 83 – 67 = 16, 83 – 47 = 36 , 83 – 19 = 64, 79 – 43 = 36, 73 – 37 = 36, 67 – 31 = 36, 59 – 43 = 16, 59 – 23 = 36, 53 – 37 = 16, 53 –17 = 36, 47 – 31 = 16, 47–11=36, 29–13=16

Exercise 5D

1 **a** $84 = 2 \times 2 \times 3 \times 7$
b $100 = 2 \times 2 \times 5 \times 5$
c $180 = 2 \times 2 \times 3 \times 3 \times 5$
d $220 = 2 \times 2 \times 5 \times 11$
e $280 = 2 \times 2 \times 2 \times 5 \times 7$
f $128 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
g $50 = 2 \times 5 \times 5$
h $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$
i $576 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$
j $650 = 2 \times 5 \times 5 \times 13$

2 **a** $2^2 \times 3 \times 7$ **b** $2^2 \times 5^2$
c $2^2 \times 3^2 \times 5$ **d** $2^2 \times 5 \times 11$
e $2^3 \times 5 \times 7$ **f** 2^7
g 2×5^2 **h** $2^3 \times 5^3$
i $2^6 \times 3^2$ **j** $2 \times 5^2 \times 13$

3 1, 2, 3, 2^2, 5, 2×3, 7, 2^3, 3^2, 2×5, 11, $2^2 \times 3$, 13, 2×7, 3×5, 2^4, 17, 2×3^2, 19, $2^2 \times 5$, 3×7, 2×11, 23, $2^3 \times 3$, 5^2, 2×13, 3^3, $2^2 \times 7$, 29, $2 \times 3 \times 5$, 31, 2^5, 3×11, 2×17, 5×7, $2^2 \times 3^2$, 37, 2×19, 3×13, $2^3 \times 5$, 41, $2 \times 3 \times 7$, 43, $2^2 \times 11$, $3^2 \times 5$, 2×23, 47, $2^4 \times 3$, 7^2, 2×5^2

4 **a** Each is double the previous number
b 64, 128
c 81, 243, 729
d 256, 1024, 4096
e 3, 3^2, 3^3, 3^4, 3^5, 3^6, …; 4, 4^2, 4^3, 4^4, 4^5, …

5 **a** $2 \times 2 \times 3 \times 5$
b $2^2 \times 3 \times 5$
c $120 = 2^3 \times 3 \times 5$, $240 = 2^4 \times 3 \times 5$, $480 = 2^5 \times 3 \times 5$

6 **a** $7^2 \times 11^2 \times 13^2$
b $7^3 \times 11^3 \times 13^3$
c $7^{10} \times 11^{10} \times 13^{10}$

7 because 3 is not a factor of 40 so it does not divide exactly

8 **a** $5 \times 7 \times 11$ **b** 3×7^2, $x = 3$, $y = 7$

Exercise 5E

1 **a** 20 **b** 56 **c** 6 **d** 28
e 10 **f** 15 **g** 24 **h** 30

2 It is their product.

3 **a** 8 **b** 18 **c** 12 **d** 30

4 No. Because the numbers in each part have common factors

5 **a** 168 **b** 105 **c** 84 **d** 84
e 96 **f** 54 **g** 75 **h** 144

6 3 packs of cheese slices and 4 packs of bread rolls

7 **a** 8 **b** 7 **c** 4 **d** 14
e 4 **f** 9 **g** 5 **h** 4
i 3 **j** 16 **k** 5 **l** 18

8 **a** **i** no **ii** yes **iii** yes **iv** no
b **i** no **ii** no **iii** yes **iv** no

9 18 and 24, or 6 and 72

Exercise 5F

1 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400

2 4, 9, 16
$1 + 3 + 5 + 7 + 9 = 25$
$1 + 3 + 5 + 7 + 9 + 11 = 36$
$1 + 3 + 5 + 7 + 9 + 11 + 13 = 49$

3 **a** 3 **b** 5 **c** 7
d odd numbers

4 **a** 529 **b** 3249 **c** 5929
d 15 129 **e** 23 104 **f** 10.24
g 90.25 **h** 566.44 **i** 16
j 144

5 **a** 169 **b** 196
c Answer between 169 and 196 (exact answer is 174.24)

6 **a** 50, 65, 82 **b** 98, 128, 162
c 51, 66, 83 **d** 48, 63, 80
e 149, 164, 181

7 **a** 25, 169, 625, 1681, 3721
b Answers in each row are the same

8 £144

9 29

10 36 and 49

Exercise 5G

1 **a** 6, 12, 18, 24, 30 **b** 13, 26, 39, 52, 65
c 8, 16, 24, 32, 40 **d** 20, 40, 60, 80, 100
e 18, 36, 54, 72, 90

2 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

3 **a** 1, 2, 3, 4, 6, 12 **b** 1, 2, 4, 5, 10, 20
c 1, 3, 9 **d** 1, 2, 4, 8, 16, 32
e 1, 2, 3, 4, 6, 8, 12, 24 **f** 1, 2, 19, 38
g 1, 13 **h** 1, 2, 3, 6, 7, 14, 21, 42
i 1, 3, 5, 9, 15, 45
j 1, 2, 3, 4, 6, 9, 12, 18, 36

4 **a** 12, 24, 36 **b** 20, 40, 60
c 15, 30, 45 **d** 18, 36, 54
e 35, 70, 105

5 square numbers

6 13 is a prime number

7 2, 3, 5, 7, 11, 13, 17, 19

8 $1 + 3 + 5 + 7 + 9 = 25$
$1 + 3 + 5 + 7 + 9 + 11 = 36$
$1 + 3 + 5 + 7 + 9 + 11 + 13 = 49$
$1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 = 64$

9

	Square number	Factor of 70
Even number	16	10
Multiple of 7	49	35

10 4761 (69^2)

11 4 packs of sausages, 5 packs of buns

12 24 seconds

13 30 seconds

14 12 minutes; Debbie: 4 and Fred: 3

15 **a** 12 **b** 9 **c** 6 **d** 13
e 15 **f** 14 **g** 16 **h** 10
i 18 **j** 17 **k** 8 **l** 21

16 **b** 21, 28, 36, 45, 55

Exercise 5H

1 **a** 2 **b** 5 **c** 7 **d** 1
e 9 **f** 10 **g** 8 **h** 3
i 6 **j** 4 **k** 11 **l** 12
m 20 **n** 30 **o** 13

2 **a** +5, –5 **b** +6, –6 **c** +10, –10
d +7, –7 **e** +8, –8 **f** +4, –4
g +3, –3 **h** +9, –9 **i** +1, –1
j +12, –12

3 **a** 81 **b** 40 **c** 100 **d** 14
e 36 **f** 15 **g** 49 **h** 12
i 25 **j** 21 **k** 121 **l** 16
m 64 **n** 17 **o** 441

4 **a** 24 **b** 31 **c** 45 **d** 40
e 67 **f** 101 **g** 3.6 **h** 6.5
i 13.9 **j** 22.2

5 $\sqrt{50}$, 3^2, $\sqrt{90}$, 4^2

6 4 and 5

7 $\sqrt{324} = 18$

8 15 tiles

9 **a** 0.36 **b** 0.6 **c** $\frac{3}{5}$
d **i** $\frac{2}{3}$ **ii** $\frac{5}{7}$ **iii** $\frac{8}{9}$

10 Because $8^2 = 64$ and $9^2 = 81$ or $\sqrt{64} = 8$ and $\sqrt{81} = 9$. 8.7 is between 8 and 9

11 **a** False **b** True
c False **d** True

12 **a** $2^2 \times 3^4$ **b** $2 \times 3^2 = 18$ **c** 22

13 **a** 16 900 **b** 3600

Exercise 5I

1 **a** 45 **b** 52 **c** 66

2 **a** 200 **b** 115 **c** 236

3 **a** 56 **b** 157 **c** 76 **d** 193

4 **a** 144 **b** 108

5 **a** 12.54 **b** 27.45

6 **a** 26.7 **b** 24.5 **c** 145.3 **d** 1.5

7 Sovereign is 102.48p per litre, Bridge is 102.73p per litre so Sovereign is cheaper.

8 Abby 1.247, Bobby 2.942, Col 5.333, Donna 6.538
Col is correct.

9 $31 \times 3600 \div 1610 = 69.31677 \approx 70$

10 **a** 167.552 **b** 196.48

11 **a** 2.77 **b** 6.00

12 **a** 497.952 **b** 110.978 625

13 **a** True **b** False

Review questions

1 **a** 90 **b** 105

2 **a** Last digits: 4, 6, 4. $4 \times 4 \times 4 \times 4 \times 4 = 1024$
b 4

3 **a** 6, 12, 18, 24, 30 **b** 1, 2, 3, 4, 6, 12
c 25 **d** 23, 29

4 13.69

5 53, 59

6 **a** 8 **b** 8 **c** 125

7 9th February

8 5 packets of pies and 2 packets of breadsticks

9 **a** $2 \times 3 \times 3 \times 5$ **b** 5

10 630

11 **a** 15 **b** 3150

12 **a** 196 **b** 35×35 must end in 5

13 **a** Clock A 3 am, clock B 3.15 am
b Clock A 12 midday, clock B 1.00 pm

14 **a** 30 seconds **b** 50 seconds
c 75 seconds **d** 150 seconds

15 **a** Newton's method = 7.746103896, $\sqrt{60}$ from calculator = 7.745966692 so the root is correct to 2 dp or 3dp if rounded.
b Newton's method = 6.245 1612 9, $\sqrt{39}$ from calculator = 6.244 997 998 so the root is correct to 2 dp or 3 dp if rounded.

16 $x = y$

Chapter 6 – Number: Approximations

Exercise 6A

1 **a** 20 **b** 60 **c** 80 **d** 50
e 100 **f** 20 **g** 90 **h** 70
i 10 **j** 30 **k** 30 **l** 50
m 80 **n** 50 **o** 90

2 **a** 200 **b** 600 **c** 800 **d** 500
e 1000 **f** 100 **g** 600 **h** 400
i 1000 **j** 1100 **k** 400 **l** 700
m 800 **n** 1000 **o** 1100

3 **a** 1 **b** 2 **c** 1 **d** 1
e 3 **f** 2 **g** 3 **h** 2
i 1 **j** 1 **k** 3 **l** 2
m 74 **n** 126 **o** 184

4 **a** 2000 **b** 6000 **c** 8000 **d** 5000
e 10 000 **f** 1000 **g** 6000 **h** 3000
i 9000 **j** 2000 **k** 3000 **l** 5000
m 8000 **n** 5000 **o** 9000 **p** 4000
q 7000 **r** 8000 **s** 1000 **t** 2000

5 **a** 230 **b** 570 **c** 720 **d** 520
e 910 **f** 230 **g** 880 **h** 630
i 110 **j** 300 **k** 380 **l** 630
m 350 **n** 1010 **o** 1070

6 **a** True **b** False **c** True
d True **e** True **f** False

7 Welcome to Swinton. Population 1400 (to the nearest 100)

8 **a** Man Utd v Chelsea
b QPR V Aston Villa
c 20 000, 45 000, 75 000, 18 000, 30 000, 44 000, 20 000, 36 000, 25 000, 35 000
d 19 900, 44 600, 75 300, 18 000, 30 000, 44 400, 20 300, 35 700, 24 700, 35 000

9 **a** 35 min **b** 55 min **c** 15 min
d 50 min **e** 10 min **f** 15 min
g 45 min **h** 35 min **i** 5 min
j 0 min

10 **a** 375 **b** 350 to 374 inclusive

11 A number between 75 and 84 inclusive added to a number between 45 and 54 inclusive with a total not equal to 130, for example 79 + 49 = 128

Exercise 6B

1 **a** 4.8 **b** 3.8 **c** 2.2 **d** 8.3
e 3.7 **f** 46.9 **g** 23.9 **h** 9.5
i 0.1 **j** 0.6 **k** 65.0 **l** 51.0

2 **a** 5.78 **b** 2.36 **c** 0.98 **d** 33.09
e 23.57 **f** 91.79 **g** 8.00 **h** 2.31
i 6.00 **j** 3.51 **k** 96.51 **l** 0.01

3 **a** 4.6 **b** 0.08 **c** 45.716 **d** 94.85
e 602.1 **f** 671.76 **g** 7.1 **h** 6.904
i 13.78 **j** 0.1 **k** 4.002 **l** 60.0

4 **a** 9 **b** 9 **c** 7 **d** 3
e 8 **f** 8 **g** 6 **h** 4
i 2 **j** 47 **k** 154 **l** 343

5 £1 + £7 + £4 + £1 = £13

6 3, 3.46, 3.5

7 4.7275 or 4.7282

8 6.445 or 6.446 or 6.447 or 6.448 or 6.449

9 $8.75 \leq \text{value} < 8.85$

10 £8.45 to £8.49 inclusive

11 **a** $6.45 \text{ cm} \leq \text{value} < 6.55 \text{ cm}$
b Smallest value of perimeter =
$2 \times (6.45 + 3.25) = 19.4$ cm
Largest value of perimeter =
$2 \times (6.55 + 3.35) = 19.8$ cm
Therefore $19.4 \leq \text{perimeter} < 19.8$
c Smallest area = $6.45 \times 3.25 = 20.96 \text{ cm}^2$

12 **a** 29.83 cm **b** 32.991 cm

13 3.645

Exercise 6C

1 **a** 50 000 **b** 60 000 **c** 30 000
d 90 000 **e** 90 000 **f** 50
g 90 **h** 30 **i** 100
j 200 **k** 0.5 **l** 0.3
m 0.006 **n** 0.05 **o** 0.0009
p 10 **q** 90 **r** 90
s 200 **t** 1000

2 **a** 56 000 **b** 27 000 **c** 80 000
d 31 000 **e** 14 000 **f** 5900
g 1100 **h** 850 **i** 110
j 640 **k** 1.7 **l** 4.1
m 2.7 **n** 8.0 **o** 42
p 0.80 **q** 0.46 **r** 0.066
s 1.0 **t** 0.0098

3 **a** 64 500 **b** 19 300 **c** 4.55
d 4.08 **e** 14.4 **f** 6.69
g 1.07 **h** 9430 **i** 10.4
j 9.68

4 **a** 65, 74 **b** 95, 149 **c** 950, 1499

5 Ayton: 750, 849; Beeville: 950, 1499; Charlestown: 150 000, 249 999

6 **a** 64.4 **b** 63.5

7 15, 16 or 17

8 1, because there could be 450 then 449

9 Between 1945 and 1954

Exercise 6D

1 **a** 35 000 **b** 15 000 **c** 960
d 12 000 **e** 1050 **f** 4000
g 5 **h** 20 **i** 1200

2 **a** £4000 **b** £3000 **c** £2000 **d** £800

3 **a** £15 000 **b** £20 000 **c** £15 000

4 £21 000

5 **a** 14 **b** 10 **c** 1.1 **d** $\frac{3}{4}$
e 5 **f** $\frac{2}{3}$ **g** 3 **h** 0.3
i 6 **j** 400 **k** 2 **l** **24**

6 **a** 500 **b** 200 **c** 90
d 50 **e** 50 **f** 500

7 8

8 **a** 200 **b** 2800 **c** 10 **d** 1000

9 **a** 40 **b** 10 **c** £70

10 1000 or 1200

11 Yes, the approximation for both distances is 120 km

12 400

13 **a** 3 kg **b** 200

14 **a** \$210 000 000 **b** \$42 000 000

15 $7.5 \leq N < 8.5$

16 **a** $9.5 \leq L < 10.5$ **b** $4.5 \leq W < 5.5$
c $28.0 \leq P < 32.0$ **d** $42.75 \leq A < 57.75$

Review questions

1 **a** 6000 **b** 790

2 **a** 3200 **b** 5000

3 **a** **i** 36 000 000 **ii** 3 600
b **i** 1000 **ii** 100

4 **a** $259 \approx 260$ m **b** 1340 – 1090 = 250
c No, one value rounds up, the other down

5 **a** 7650 **b** 7749

6 **a** 2450 **b** 2549 **c** **i** 2475 **ii** 2524

7 **a** 2 **b** 3 **c** 3 **d** 4

8 **a** 2 **b** 1 **c** 2 **d** 3
e 3 **f** 4 **g** 3 **h** 1

9 **a** 67.5 **b** 31 **c** 3000 **d** 4.0
e 20 **f** 12.4 **g** 13.57 **h** 29.4

10 No, as there could be 44 people on the bus and 24 at the bus stop. That makes 68 people altogether and the bus can only have 52 + 10 = 62 passengers maximum.

11 Width wise: Yes, as error interval of the envelope is 8.5 ≤ width < 9.5 and for the postcard the error interval is 7.75 ≤ width < 8.25

Length wise: No, as error interval of the envelope is 12.5 ≤ length < 13.5 and for the postcard the error interval is 12.25 ≤ length < 12.75, so largest postcard is larger than the smallest envelope.

12 1.850 or 1.851 or 1.852 or 1.853 or 1.854

13 235 ≤ number < 245

14 **a** 49.5 ≤ side < 50.5
b No, as the error interval for the peg is 48 ≤ diameter < 50

15 **a** 10.5 ≤ perimeter < 13.5
b 4.375 ≤ area < 7.875

Chapter 7 – Number: Decimals and fractions

Exercise 7A

1 **a** 0.48 **b** 2.92 **c** 1.12 **d** 0.12
e 0.028 **f** 0.09 **g** 0.192 **h** 2.91

2 **a** 8 **b** 9 **c** 4 **d** 8.7
e 7.8 **f** 9.7 **g** 12.1 **h** 9.7

3 Yes, with 25p left over

4 Alistair 0.90 Shane 70 Charlotte 66

5 27 (three 0.7 m lengths from each 2.4 m length)

6 **a** **i** 5 × 1 = 5 **ii** 3.36, 1.64
b **i** 2 × .5 = 1 **ii** 1.44, 0.44
c **i** 15 × 1= 15 **ii** 13.77, 1.23
d **i** 20 × 1 = 20 **ii** 16.08, 3.92

7 **a** 572 **b** **i** 5.72 **ii** 1.43 **iii** 22.88

8 **a** 29 **b** 24 **c** 0.12 **d** 0.08

9 **a** 33.25 ≤ *ab* < 47.25
b Because it is the highest value of a divided by the lowest value of b
c 2.11

Exercise 7B

1 **a** 0.5 **b** $0.\dot{3}$ **c** 0.25 **d** 0.2
e $0.1\dot{6}$ **f** $0.\dot{1}42857\dot{}$ **g** 0.125 **h** $0.\dot{1}$
i 0.1 **j** $0.\dot{0}76\ 92\dot{3}$

2 **a** **i** 0.571428571428571428571428 ...
ii 0.714285714285714285714285 …
iii 0.857142857142857142857142 …
b The recurring digits are all in the same sequence but they start in a different place each time.

3 **a** **i** 0.4 **ii** 0.2 **iii** 0.1
b Halve the answers to get 1 ÷ 20 = 0.05 and 1 ÷ 40 = 0.025

4 $0.\dot{1}$, $0.\dot{2}$, $0.\dot{3}$, $0.\dot{4}$, $0.\dot{5}$, $0.\dot{6}$, $0.\dot{7}$, $0.\dot{8}$
The recurring digit is the numerator of the fraction.

5 $0.\dot{0}\dot{9}$, $0.\dot{1}\dot{8}$, $0.\dot{2}\dot{7}$, $0.\dot{3}\dot{6}$, $0.\dot{4}\dot{5}$, $0.\dot{5}\dot{4}$, $0.\dot{6}\dot{3}$, $0.\dot{7}\dot{2}$, $0.\dot{8}\dot{1}$, $0.\dot{9}\dot{0}$
The recurring digits follow the nine times table.

6 $\frac{9}{22} = 0.40\dot{9}$, $\frac{3}{7} = 0.\dot{4}28\ 57\dot{1}$,
$\frac{16}{37} = 0.\dot{4}3\dot{2}$, $\frac{4}{9} = 0.\dot{4}$,
$\frac{5}{11} = 0.4\dot{5}$, $\frac{6}{13} = 0.\dot{4}61538\dot{}$,

7 $\frac{7}{24} = \frac{35}{120}$, $\frac{3}{10} = \frac{36}{120}$, $\frac{19}{60} = \frac{38}{120}$,
$\frac{2}{5} = \frac{48}{120}$, $\frac{5}{12} = \frac{50}{120}$

8 **a** $\frac{1}{8}$ **b** $\frac{17}{50}$ **c** $\frac{29}{40}$ **d** $\frac{5}{16}$
e $\frac{89}{100}$ **f** $\frac{1}{20}$ **g** $2\frac{7}{20}$ **h** $\frac{7}{32}$

9 **a** $0.08\dot{3}$ **b** 0.0625 **c** 0.05
d 0.04 **e** 0.02

10 **a** $\frac{4}{3} = 1\frac{1}{3}$ **b** $\frac{6}{5} = 1\frac{1}{5}$ **c** $\frac{5}{2} = 2\frac{1}{2}$
d $\frac{10}{7} = 1\frac{3}{7}$ **e** $\frac{20}{11} = 1\frac{9}{11}$ **f** $\frac{15}{4} = 3\frac{3}{4}$

11 **a** 0.75, $1.\dot{3}$ **b** $0.8\dot{3}$, 1.2 **c** 0.4, 2.5
d $0.7, 1.\dot{4}28\,57\dot{1}$ **e** $0.55, 1.8\dot{1}$ **f** $0.2\dot{6}, 3.75$
b No, look at **c** above for example.

12 The answer is always 1.

13 $1 \div 0$ is infinite so there is no finite answer.

14 **a** 10 **b** 2
c The reciprocal of a reciprocal is always the original number.

15 The reciprocal of x is greater than the reciprocal of y.
For example, reciprocal of 2 is 0.5, reciprocal of 10 is 0.1.

Exercise 7C

1 **a** $\frac{1}{3}$ **b** $\frac{1}{5}$ **c** $\frac{2}{5}$ **d** $\frac{5}{24}$
e $\frac{2}{5}$ **f** $\frac{1}{6}$ **g** $\frac{2}{7}$ **h** $\frac{1}{3}$

2 $\frac{3}{5}$

3 $\frac{2}{5}$

4 $\frac{2}{3}$

5 $\frac{1}{5}$

6 $\frac{1}{2}$

7 Jon saves $\frac{30}{90} = \frac{1}{3}$
Matt saves, $\frac{35}{100} = \frac{7}{20}$ which is greater than $\frac{1}{3}$, so Matt saves the greater proportion of his earnings.

8 $\frac{13}{20} = \frac{65}{100}$ and $\frac{16}{25} = \frac{64}{100}$ so 13 out of 20 is the better mark.

9 $\frac{3}{8}$

10 $\frac{11}{24}$

11 26 out of 63 because 13 out of 32 is the same as 26 out of 64.

12 The price is the same at both shops: £432

Exercise 7D

1 **a** $\frac{8}{15}$ **b** $\frac{7}{12}$ **c** $\frac{11}{12}$
d $\frac{1}{10}$ **e** $\frac{1}{8}$ **f** $\frac{1}{12}$

2 Three-quarters of 68

3 **a** $4\frac{47}{60}$ **b** $\frac{41}{72}$ **c** $1\frac{109}{120}$ **d** $1\frac{23}{30}$

4 **a** $\frac{1}{6}$ **b** 30

5 No, one eighth is left, which is 12.5 cl, so enough for one cup but not two cups.

6 He has added the numerators and added the denominators instead of using a common denominator. Correct answer is $3\frac{7}{12}$.

7 Possible answer: The denominators are 4 and 5. I first find a common denominator. The lowest common denominator is 20 because 4 and 5 are both factors of 20. So I am changing the fractions to twentieths. One-quarter is the same as five-twentieths (multiplying numerator and denominator by 5). Two-fifths is the same as eight-twentieths (multiplying numerator and denominator by 4). Five-twentieths plus eight-twentieths = thirteen-twentieths.

8 $\frac{11}{20}$ of 900 = 495, $\frac{2}{11}$ of 495 = 90 left-handed boys. 900 − 495 = 405 girls. $\frac{2}{9}$ of 405 = 90 left-handed girls. 180 left-handed students altogether so 180 out of 900 = $\frac{1}{5}$.

9 $\frac{1}{5} + \frac{3}{8} = \frac{23}{40}$, so $\frac{17}{40}$ of the counters are yellow. $\frac{17}{40}$ of 600 = 255

10 **a** because $\frac{27}{40} + \frac{2}{5} = 1\frac{3}{40}$ which is greater than 1.

b $\frac{2}{5}$ of 200 = 80. $\frac{5}{8}$ of 80 = 50 women at least 40. $\frac{27}{40}$ of 200 = 135 members at least 40. 135 − 50 = 85 men at least 40. $\frac{3}{5}$ of 200 = 120, so 120 − 85 = 35 men under 40.

Exercise 7E

1 **a** $\frac{1}{6}$ **b** $\frac{3}{8}$ **c** $\frac{7}{20}$ **d** $\frac{3}{5}$
e $\frac{5}{12}$ **f** $2\frac{11}{12}$ **g** $3\frac{9}{10}$ **h** $3\frac{1}{3}$

2 **a** $\frac{3}{4}$ **b** $1\frac{1}{15}$ **c** 5
d $\frac{4}{9}$ **e** $1\frac{3}{5}$

3 $\frac{3}{8}$

4 $\frac{1}{8}$

5 40

6 $\frac{2}{5}$ of $6\frac{1}{2}$

7 £10.40

Exercise 7F

1 **a** $1\frac{11}{20}$ **b** $1\frac{1}{4}$ **c** $1\frac{63}{80}$
d $\frac{11}{30}$ **e** $\frac{61}{80}$ **f** $\frac{167}{240}$

2 **a** $12\frac{1}{4}$ miles **b** $3\frac{1}{4}$ miles

3 **a** $6\frac{11}{20}$ **b** $8\frac{8}{15}$ **c** $11\frac{63}{80}$
d $3\frac{11}{30}$ **e** $7\frac{61}{80}$ **f** $1\frac{277}{396}$

4 **a** $-\frac{77}{1591}$ **b** The answer is negative

5 $\frac{5}{12}$ (anticlockwise) or $\frac{7}{12}$ (clockwise)

6 **a** $\frac{3}{5}$ **b** $\frac{27}{128}$ **c** $5\frac{2}{5}$ **d** $5\frac{1}{7}$
e $3\frac{9}{32}$ **f** $\frac{11}{18}$

7 **a** $8\frac{11}{20}$ **b** $65\frac{91}{100}$ **c** $52\frac{59}{160}$ **d** $2\frac{17}{185}$
e $2\frac{22}{103}$ **f** $7\frac{881}{4512}$

8 3 rolls

9 $22 \div (2 \times \frac{22}{7}) = \frac{7}{2}$, $\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} = 38\frac{1}{2}$ cm^2

Review questions

1 0.3 < decimal < 0.33… for example 0.32

2 17

3 **a** 0.4 **b** $\frac{9}{20}$

4 $\frac{22}{75}$

5 **a** $0.\dot{5}$ **b** $0.\dot{3}$

6 28 ÷ 40 = 0.7; 33 ÷ 50 = 0.66. Zac did better in English

7 $\frac{4}{5}$

8 900 × 12 = 10 800 miles per year. 10 800 ÷ 45 = 240 gallons. 240 × 4.55 = 1092 litres. 1092 × 1.4 = £1528.80 per year.

9 **a** 5 **b** $\frac{3}{2}$ **c** $\frac{7}{10}$ **d** $\frac{1}{8}$

10 Starbuys 10 × 0.8 = £8. Shoprite 12 × 0.8 = £9.60. 20% off = £7.68, so Shoprite is best value.

11 **a** 1672 **b** **i** 167.2 **ii** 83.6 **iii** 16.72

12 **a** 3.2 **b** 360 **c** 0.32

13 **a** $\frac{7}{20}$ **b** $\frac{2}{5}$ **c** $1\frac{1}{2}$

14 40.96

15 $2\frac{1}{24}$

16 $18\frac{11}{12}$ cm

17 $18\frac{5}{12}$

18 **a** 0.5 (T), $0.\dot{6}$ (R), 0.75 (T), 0.8 (T), $0.8\dot{3}$ (R), $0.\dot{8}57142\dot{8}$ (R), 0.875 (T), $0.\dot{8}$ (R), 0.9 (T)
b 2, 2 × 2, 5, 2 × 2 × 2, 2 × 5
c If the prime factors of the denominator of a fraction in its simplest form are only 2 and/or 5 its decimal will terminate.

19

	Blue	Red	Total
Squares	136	74	210
Triangles	95	95	190
Total	231	169	400

Chapter 8 – Algebra: Linear graphs

Exercise 8A

1 **a** A(1, 2), B(3, 0), C(0, 1), D(–2, 4), E(–3, 2), F(–2, 0), G(–4, –1), H(–3, –3), I(1, –3), J(4, –2)
b **i** (2, 1) **ii** (–1, –3) **iii** (1, 1)

2 **a** $x = -3$, $x = 2$, $y = 3$, $y = -4$
b **i** $x = -\frac{1}{2}$ **ii** $y = -\frac{1}{2}$

3 Values of y: 2, 3, 4, 5, 6

4 Values of y: –2, 0, 2, 4, 6

5 Values of y: 1, 2, 3, 4, 5

6 Values of y: –4, –3, –2, –1, 0

7 **a** Values of y: 0, 4, 8, 12, 16 and 6, 8, 10, 12, 14
b (6, 12)

8 Points could be (0, –1), (1, 4), (2, 9), (3, 14), (4, 19), (5, 24), etc

b £25.00, £28.50, £32.00, £35.50, £39.00, £42.50;
(£20.00), £23.50, £27.00, £30.50, £34.00, £37.50;
(£15.00), £18.50, £22.00, £25.50, £29.00, £32.50;
(£10.00), (£13.50), £17.00, £20.50, £24.00, £27.50;
(£5.00), (£8.50), (£12.00), £15.50, £19.00, £22.50;
(£0.00), (£3.50), (£7.00), (£10.50), (£14.00), £17.50

c Yes, they had 3 rolls and 4 baguettes.

10

b $y = 3x + 1$

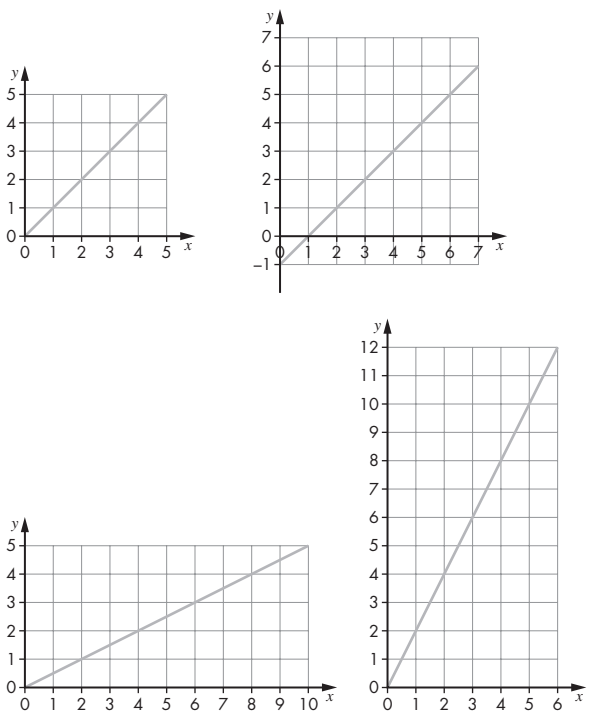
c Graph from (0, 1) to (5, 16)

d Read from 13 on the y-axis across to the graph and down to the x-axis. This should give a value of 4.

11 0, 8, 16, 24, 32, 40, 48

Exercise 8B

1



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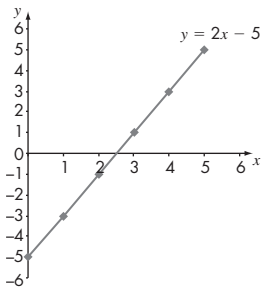

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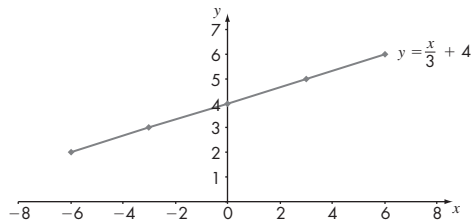
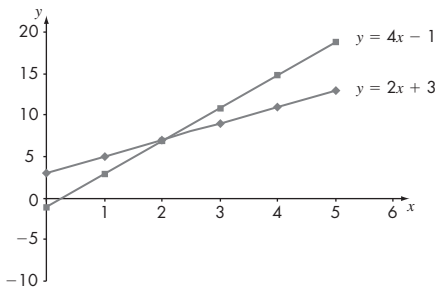
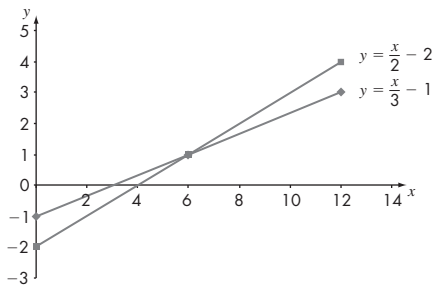


4 a

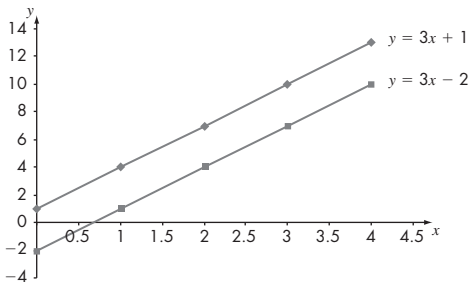


b (2, 7)

5 a



b



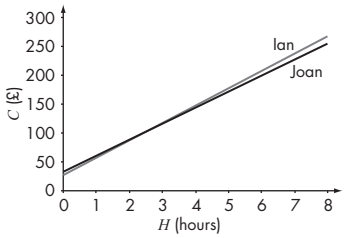
c Lines in part a intersect at (6, 1). Lines in part **b** don't intersect because they are parallel.

6 **a** Line isn't straight

b

x	–3	–2	–1	0	1	2	3
y	–5	–3	–1	1	3	5	7

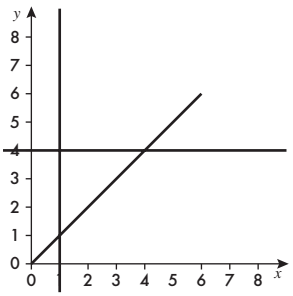
7 **a**



b Ian, as Ian only charges £85, whilst Joan charges £90 for a 2-hour job

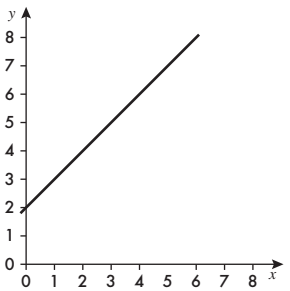
8 Jada's

9 **a**



b 4.5 units squared

10



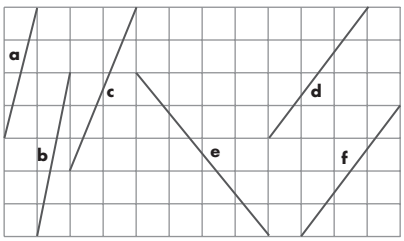
Exercise 8C

1 **a** 2 **b** $\frac{1}{3}$ **c** –3 **d** 1

e –2 **f** $-\frac{1}{3}$ **g** 5 **h** –5

i $\frac{1}{5}$ **j** $-\frac{3}{4}$

2



3 **a** Both answers are correct

b generally the bigger the triangle the more accurate the answer, so Brianna

4 **a** Slide won't be steep enough for children to slide down.

b If it is too steep children might slide down too quickly.

c A, B, E and F satisfy the safety regulations; C and D do not

5 **a** 0.5 **b** 0.4 **c** 0.2 **d** 0.1 **e** 0

6 **a** 1.4 **b** 2 **c** 3.3 **d** 10 **e** ∞

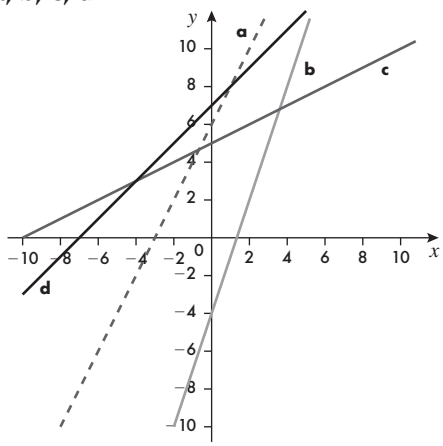
7 Raisa has misread the scales. The second line has four times the gradient (2.4) of the first (0.6).

8 4 horizontal 6 vertical; 5 horizontal 8 vertical; 3 horizontal 5 vertical; 6 horizontal 11 vertical; 1 horizontal 2 vertical; 2 horizontal 5 vertical

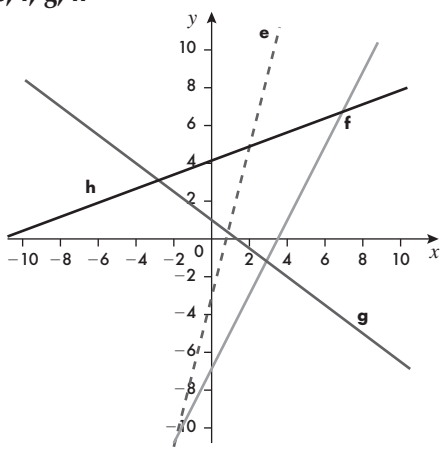
9 0, 2, –1, $\frac{1}{2}$, $-\frac{3}{2}$

Exercise 8D

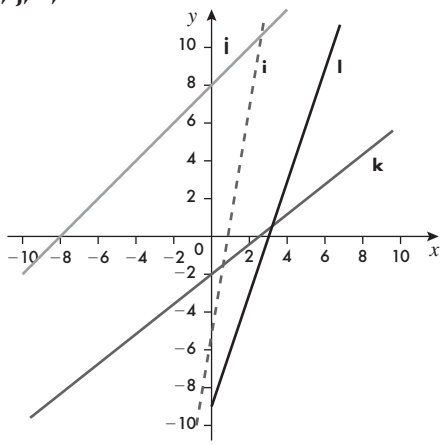
1 **a, b, c, d**



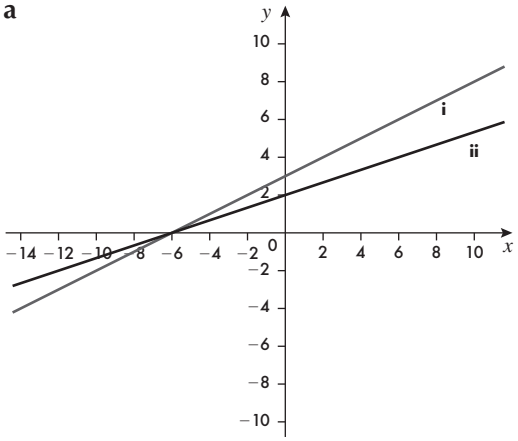
e, f, g, h



I, j, k, l



2 a



b (–6, 0)

3 a $y = 3x - 2$

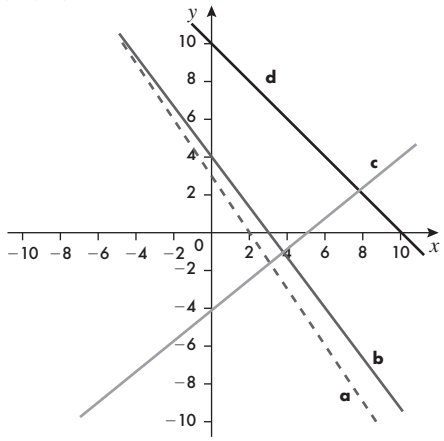
b They intercept the y-axis at the same point (0, –2).

c (–1, –4)

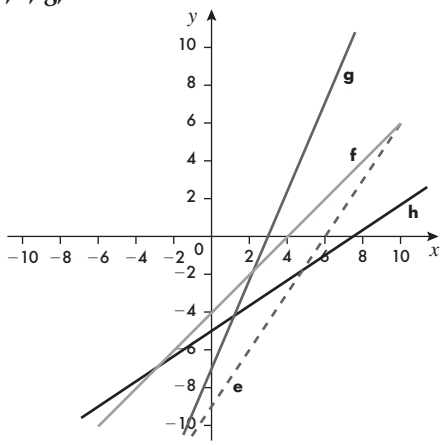
4 a –2 **b** $\frac{1}{2}$

Exercise 8E

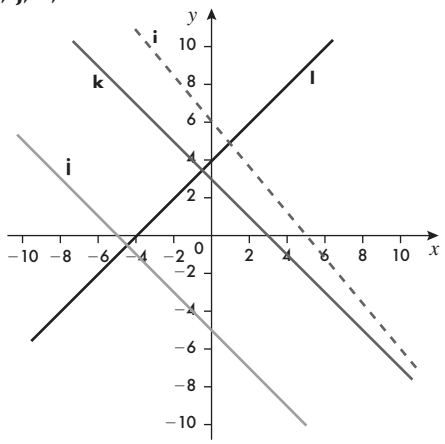
1 a, b, c, d



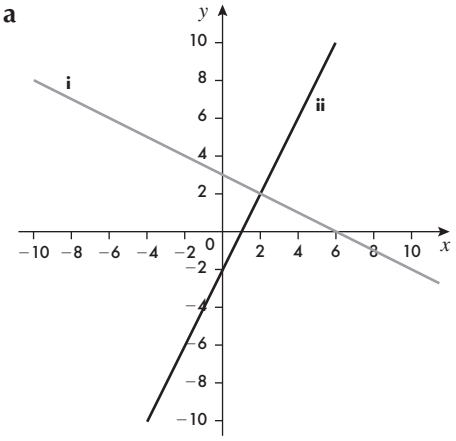
e, f, g, h



I, j, k, l



2 a



b (2, 2)

3 a Intersect at (6, 0) **b** Intersect at (0, –3)
c Parallel **d** $-2x + 9y = 18$

4 a vi **b** iii **c** v **d** ii
e i **f** iv

5 a i $x = 3$ **ii** $x - y = 4$ **iii** $y = -3$
iv $x + y = -4$ **v** $x = -3$ **vi** $y = x + 4$

b i –3 **ii** $\frac{1}{3}$ **iii** $-\frac{1}{3}$

6 Cover-up method for $2x + y = 10$ and gradient-intercept method for $y = 1 - 2x$, but actually the lines are the same

Exercise 8F

1 **a** $y = \frac{4}{3}x - 2$ or $3y = 4x - 6$
b $y = 2x$ **c** $2y = x + 6$

2 **a** **i** $y = 2x + 1$, $y = -2x + 1$
ii Reflection in y-axis (and $y = 1$)
iii Different sign
b **i** $5y = 2x - 5$, $5y = -2x - 5$
ii Reflection in y-axis (and $y = -1$)
iii Different sign
c **i** $y = x + 1$, $y = -x + 1$
ii Reflection in y-axis (and $y = 1$)
iii Different sign

3 **a** x-coordinates go from $2 \to 1 \to 0$ and y-coordinates go from $5 \to 3 \to 1$.
b x-step between the points is 1 and y-step is 2.
c $y = 3x + 2$

4 **a** $y = -x + 1$
b $5y = -2x - 5$
c $y = -\frac{3}{2}x - 3$ or $2y = -3x - 6$

5 **a** **i** $y = \frac{1}{2}x$, $y = 2x$
ii Reflection in $x = y$
iii Reciprocal of each other
b **i** $y = 2$, $x = 2$ **ii** Reflection in $x = y$
iii Reciprocal of each other (reciprocal of zero is infinity)

6 All of the lines except $y = \frac{1}{4}x + 9$

7 **a** $y = -3x + 5$ **b** $y = 2x - 4$
c $y = 8x - 3$ **d** $y = 25 - 2x$
e $y = \frac{2}{3}x - 1$

8 $5x + 6y = 30$

9 Chris is correct. The equation of the line is $y = \frac{1}{2}x + 2$ and (12, 8) satisfies the rule

10 **a** **i** $x + y = 100$ **ii** $k = 1$
b **i** $x = 46$ **ii** $k = 46$
c **i** $y = 2x + 1$ **ii** $k = 60$
d **i** $y = x + 19$ **ii** $k = -17$

Exercise 8G

1 A and B are parallel because they have the same gradient ($m = 3$)

2 $x = 8$ and $x = -2$
$y = 3 - x$ and $y = 10 - x$
$y = 3x - 5$ and $y = 3x + 9$
$y = -\frac{1}{8}x$ and $y = -\frac{1}{8}x + 6$
$2x + y = 9$ and $2x + y = 13$
$y = 15 - 4x$ and $y = 7 - 4x$

3 **a** $y = \frac{1}{2}x - 2$
b $y = -x + 3$
c $y = 3x - 8$
d $y = -\frac{1}{3}x + 10$

4 **a** $y = 2x + 4$ **b** $y = 17 - 3x$
c $y = 3x - 1$

Exercise 8H

1 **a** **i** $8\frac{1}{4}$ kg **ii** $2\frac{1}{4}$ kg **iii** 9 lb **iv** 22 lb
b 2.2 lb
c Read off the value for 12 lb (5.4 kg) and multiply this by 4 (21.6 kg)
d $m = \frac{5}{11}$
e $\frac{5}{11}$ kilograms per pound

2 **a** **i** £100 **ii** £325
b **i** 500 **ii** 250

3 **a** **i** 95 °F **ii** 68 °F **iii** 10 °C **iv** 32 °C
b 32 °F
c $m = \frac{5}{9}$, $\frac{5}{9}$ °C per °F

4 **a** Check student's graph
b 2:15 pm
c $m = 48$, 48 cups of tea per hour

5 **a** Check student's graph, £50

6 **a** Anya: CabCo £8.50, YellaCabs £8.40, so YellaCabs is best Bettina: CabCo £11.50, YellaCabs £11.60, so CabCo is best Calista: CabCo £10, YellaCabs £10, so either
b If they shared a cab, the shortest distance is 16 km, which would cost £14.50 with CabCo and £14.80 with Yellacabs.

7 **a** \$642.86
b **i** \$7500 **ii** £3666.67

Exercise 8I

1 (4, 1)

2 (2, 3)

3 (3, 10)

4 (–2, 6)

5 (–6, –9)

6 (1, –1)

7 (2, 6)

8 (2, 8)

9 $x + 2y = 9.5$, $2x + y = 8.5$
Graphs intersect at (2.5, 3.5), so a bun costs £2.50 and a cake costs £3.50.

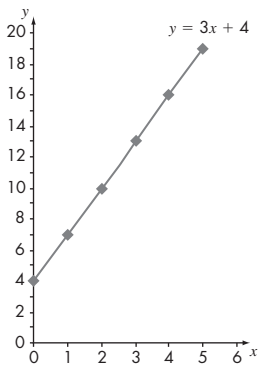
10 **a** P and R
b R and S
c P and Q
d Q and S

11 (0, 0), (–3, 3), (–3, –3), (–3, 2), (–2, 2), (2, 2)

12 **a** no solutions
b infinite solutions
c one solution
d **a** Parallel **b** Same line **c** Not parallel

Review questions

1



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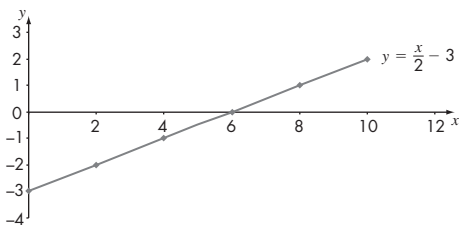
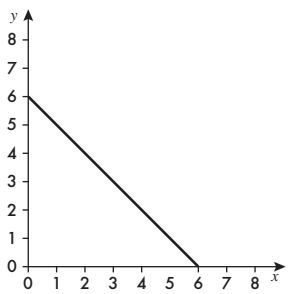


3



4 **a** $\frac{1}{10}$
b 24.5 cm
c 0.1 cm or 1 mm
d $l = 24.5 + \frac{W}{10}$ or Length = 24.5 + 1 mm/kg

5 A Q, B T, C P, D R, E S

6 **a** (5, 5)
b (1, 5)
c (3, 16)

7 72 square units

8 **a** 2 **b** $y = 2x + 2$
c $y = 2x - 5$

9 **a** $x + y = 50$ (or $y = 50 - x$)
b –3
c $x + y = 40$ (or $y = 40 - x$)

Chapter 9 – Algebra: Expressions and formulae

Exercise 9A

1 **a** $x + 2$ **b** $x - 6$ **c** hj
d $y \div t$ or $\frac{y}{t}$ **e** a^2 **f** g^2

2 **a** Dad: $(72 + x)$ years, me: $(T + x)$ years **b** 31

3 For example, $2 \times 6m$, $1 \times 12m$, $6m + 6m$, etc

4 Any values picked for l and w and substituted into the formulae to give the same answers

5 13 years

6 15p

7 **a** expression, **b** formula,
c identity, **d** equation

Exercise 9B

1 **a** 3 **b** 11 **c** 43

2 **a** 9 **b** 5 **c** –1

3 **a** £4.00 **b** 13 km
c Yes, the journey will cost £5.00

4 **a** 1.4 **b** 1.4 **c** –0.4

5 **a** 2.5 **b** –20 **c** 2.5

6 **a** $\frac{150}{n}$ **b** £925

7 **a** $2 \times 8 + 6 \times 11 - 3 \times 2 = 76$
b $5 \times 2 - 2 \times 11 + 3 \times 8 = 12$

8 **a** One odd, one even value, different from each other.

b Any valid combination, e.g. $x = 1$, $y = 2$

9 **a** **i** Odd **ii** Odd **iii** Even **iv** Odd

b Any valid expression such as $xy + z$

10 **a** £20 **b** –£40 **c** 40 miles

11 **a** First term is cost of petrol, each mile is a tenth of £1.20. Second term is the hire cost divided by the miles.

b 32 p per mile

Exercise 9C

1 **a** $6 + 2m$ **b** $10 + 5l$ **c** $12 - 3y$
d $20 + 8k$ **e** $6 - 12f$ **f** $10 - 6w$
g $10k + 15m$ **h** $12d - 8n$ **i** $t^2 + 3t$
j $k^2 - 3k$ **k** $4t^2 - 4t$ **l** $8k - 2k^2$
m $8g^2 + 20g$ **n** $15h^2 - 10h$ **o** $y^3 + 5y$
p $h^4 + 7h$ **q** $k^3 - 5k$ **r** $3t^3 + 12t$
s $15d^3 - 3d^4$ **t** $6w^3 + 3tw$ **u** $15a^3 - 10ab$
v $12p^4 - 15mp$ **w** $12h^3 + 8h^2g$ **x** $8m^3 + 2m^4$

2 **a** $5(t - 1)$ and $5t - 5$

b Yes, as $5(t - 1)$ when $t = 4.50$ is $5 \times 3.50 =$ £17.50

3 **a** He has worked out 3×5 as 8 instead of 15 and he has not multiplied the second term by 3.

b $15x - 12$

4 **a** $3(2y + 3)$

b $2(6z + 4)$ or $4(3z + 2)$

Exercise 9D

1 **a** $7t$ **b** $3y$ **c** $9d$ **d** $3e$
e $3p$ **f** $2t$ **g** $5t^2$ **h** $5ab$
i $3a^2d$

2 **a** $22 + 5t$ **b** $21 + 19k$ **c** $22 + 2f$
d $14 + 3g$

3 **a** $2 + 2h$ **b** $9g + 5$ **c** $17k + 16$
d $6e + 20$

4 **a** $4m + 3p + 2mp$ **b** $3k + 4h + 5hk$
c $12r + 24p - 13pr$ **d** $19km + 20k - 6m$

5 **a** $9t^2 + 13t$ **b** $13y^2 + 5y$
c $10e^2 - 6e$ **d** $14k^2 - 3kp$

6 **a** $17ab + 12ac + 6bc$ **b** $18wy + 6ty - 8tw$
c $14mn - 15mp - 6np$ **d** $8r^3 - 6r^2$

7 **a** $5(f + 2s) + 2(2f + 3s) = 9f + 16s$

b **i** £$270f$ **ii** £$480s$

c £42 450 – £30 000 = £12 450

8 For x-coefficients, 3 and 1 or 1 and 4; for y-coefficients, 5 and 1 or 3 and 4 or 1 and 7

9 $5(3x + 2) - 3(2x - 1) = 9x + 13$

Exercise 9E

1 **a** $6(m + 2t)$ **b** $3(3t + p)$ **c** $4(2m + 3k)$
d $4(r + 2t)$ **e** $m(n + 3)$ **f** $g(5g + 3)$
g $2(2w - 3t)$ **h** $y(3y + 2)$ **i** $t(4t - 3)$
j $3m(m - p)$ **k** $3p(2p + 3t)$ **l** $2p(4t + 3m)$
m $4b(2a - c)$ **n** $5bc(b - 2)$ **o** $2b(4ac + 3de)$
p $2(2a^2 + 3a + 4)$ **q** $3b(2a + 3c + d)$
r $t(5t + 4 + a)$ **s** $3mt(2t - 1 + 3m)$
t $2ab(4b + 1 - 2a)$ **u** $5pt(2t + 3 + p)$

2 **a** Mary has taken out a common factor.

b Because the bracket adds up to £10 **c** £30

3 **a**, **d**, **f** and **h** do not factorise
b $m(5 + 2p)$ **c** $t(t - 7)$
e $2m(2m - 3p)$ **g** $a(4a - 5b)$
i $b(5a - 3bc)$

4 **a** Bernice

b Aidan has not taken out the largest possible common factor. Craig has taken m out of both terms but there isn't an m in the second term.

5 There are no common factors.

6 Perimeter $= 2x + 8 + x + 5 + 5x + 4 + 9x - 3 + 10 - x = 16x + 24 = 8(2x + 3)$

7 $\frac{4x^2 - 12x}{2x - 6} = \frac{4x(x - 3)}{2(x - 3)}$

Exercise 9F

1 **a** $x^2 + 5x + 6$ **b** $t^2 + 7t + 12$
c $w^2 + 4w + 3$ **d** $m^2 + 6m + 5$

2 **a** $p^2 + 3p - 70$ **b** $u^2 - 12u + 32$
c $k^2 + 2k - 15$ **d** $z^2 - 12z + 27$

3 **a** should be 35 on the end
b should be – 80 **c** should be $-10x$
d should be $12y$ **e** should be $-9z$

Exercise 9G

1 **a** $k^2 + 8k + 15$ **b** $a^2 + 5a + 4$
c $x^2 + 2x - 8$ **d** $t^2 + 2t - 15$
e $w^2 + 2w - 3$ **f** $f^2 - f - 6$

2 **a** $r^2 - 10r + 16$ **b** $s^2 - 17s + 70$
c $d^2 - 17d + 16$ **d** $m^2 - 9m + 18$
e $q^2 - 20q + 99$ **f** $y^2 - 13y + 40$

3 **a** $20a$ **b** $3b$ **c** 200
d $-11d$ **e** $12e$, 28

Exercise 9H

1 **a** $g^2 - 3g - 4$ **b** $y^2 + y - 12$
c $x^2 + x - 12$ **d** $p^2 - p - 2$
e $k^2 - 2k - 8$ **f** $y^2 + 3y - 10$
g $a^2 + 2a - 3$

2 **a** $x^2 - 9$ **b** $t^2 - 25$ **c** $m^2 - 16$
d $t^2 - 4$ **e** $y^2 - 64$ **f** $p^2 - 1$
g $25 - x^2$ **h** $49 - g^2$ **i** $x^2 - 36$

3 $(x + 2)$ and $(x + 3)$

4 **a** B: $1 \times (x - 2)$ C: 1×2 D: $2 \times (x - 1)$
b $(x - 2) + 2 + 2(x - 1) = 3x - 2$
c Area A = $(x - 1)(x - 2)$ = area of square minus areas (B + C + D) = $x^2 - (3x - 2) = x^2 - 3x + 2$

5 **a** $x^2 + 2x + 1$ **b** $x^2 - 2x + 1$ **c** $x^2 - 1$

Exercise 9I

1 **a** $6x^2 + 11x + 3$ **b** $12y^2 + 17y + 6$
c $6t^2 + 17t + 5$ **d** $8t^2 + 2t - 3$
e $10m^2 - 11m - 6$ **f** $12k^2 - 11k - 15$
g $6p^2 + 11p - 10$ **h** $10w^2 + 19w + 6$
i $6a^2 - 7a - 3$ **j** $8r^2 - 10r + 3$
k $15g^2 - 16g + 4$ **l** $12d^2 + 5d - 2$
m $8p^2 + 26p + 15$ **n** $6t^2 + 7t + 2$
o $6p^2 + 11p + 4$ **p** $6 - 7t - 10t^2$
q $12 + n - 6n^2$ **r** $6f^2 - 5f - 6$
s $12 + 7q - 10q^2$ **t** $3 - 7p - 6p^2$
u $4 + 10t - 6t^2$

2 **a** $(3x - 2)(2x + 1) = 6x^2 - x - 2$,
$(2x - 1)(2x - 1) = 4x^2 - 4x + 1$,
$(6x - 3)(x + 1) = 6x^2 + 3x - 3$,
$(4x + 1)(x - 1) = 4x^2 - 3x - 1$
$(3x + 2)(2x + 1) = 6x^2 + 7x + 2$

3 **a** $4x^2 - 1$ **b** $9t^2 - 4$ **c** $25y^2 - 9$
d $16m^2 - 9$ **e** $4k^2 - 9$ **f** $16h^2 - 1$
g $4 - 9x^2$ **h** $25 - 4t^2$ **i** $36 - 25y^2$
j $a^2 - b^2$ **k** $9t^2 - k^2$ **l** $4m^2 - 9p^2$
m $25k^2 - g^2$ **n** $a^2b^2 - c^2d^2$ **o** $a^4 - b^4$

4 **a** $a^2 - b^2$
b Dimensions: $a + b$ by $a - b$; Area: $a^2 - b^2$
c Areas are the same, so $a^2 - b^2 = (a + b) \times (a - b)$

5 First shaded area is $(2k)^2 - 1^2 = 4k^2 - 1$
Second shaded area is $(2k + 1)(2k - 1) = 4k^2 - 1$

Exercise 9J

1 **a** $x^2 + 10x + 25$ **b** $m^2 + 8m + 16$
c $t^2 + 12t + 36$ **d** $p^2 + 6p + 9$
e $m^2 - 6m + 9$ **f** $t^2 - 10t + 25$
g $m^2 - 8m + 16$ **h** $k^2 - 14k + 49$

2 **a** $9x^2 + 6x + 1$ **b** $16t^2 + 24t + 9$
c $25y^2 + 20y + 4$ **d** $4m^2 + 12m + 9$
e $16t^2 - 24t + 9$ **f** $9x^2 - 12x + 4$
g $25t^2 - 20t + 4$ **h** $25r^2 - 60r + 36$
i $x^2 + 2xy + y^2$ **j** $m^2 - 2mn + n^2$
k $4t^2 + 4ty + y^2$ **l** $m^2 - 6mn + 9n^2$
m $x^2 + 4x$ **n** $x^2 - 10x$
o $x^2 + 12x$ **p** $x^2 - 4x$

3 **a** Bernice has just squared the first term and the second term. She hasn't written down the brackets twice.
b Pete has written down the brackets twice but has worked out $(3x)^2$ as $3x^2$ and not $9x^2$.
c $9x^2 + 6x + 1$

4 Whole square is $(2x)^2 = 4x^2$.
Three areas are $2x - 1$, $2x - 1$ and 1.
$4x^2 - (2x - 1 + 2x - 1 + 1) = 4x^2 - (4x - 1)$
$= 4x^2 - 4x + 1$

Exercise 9K

1 **a** $(x + 2)(x + 3)$ **b** $(t + 1)(t + 4)$
c $(m + 2)(m + 5)$ **d** $(k + 4)(k + 6)$

2 **a** $(p + 2)(p + 12)$ **b** $(r + 3)(r + 6)$
c $(w + 2)(w + 9)$ **d** $(x + 3)(x + 4)$

3 **a** $(a + 2)(a + 6)$ **b** $(k + 3)(k + 7)$
c $(f + 21)(f + 1)$ **d** $(b + 8)(b + 12)$

4 **a** $(t - 2)(t - 3)$ **b** $(d - 4)(d - 1)$
c $(g - 2)(g - 5)$ **d** $(x - 3)(x - 12)$

5 **a** $(c - 2)(c - 16)$ **b** $(t - 4)(t - 9)$
c $(y - 4)(y - 12)$ **d** $(j - 6)(j - 8)$

6 **a** $(p - 3)(p - 5)$ **b** $(y + 6)(y - 1)$
c $(t + 4)(t - 2)$ **d** $(x + 5)(x - 2)$

7 **a** $(m + 2)(m - 6)$ **b** $(r + 1)(r - 7)$
c $(n + 3)(n - 6)$ **d** $(m + 4)(m - 11)$

8 **a** $(w + 4)(w - 6)$ **b** $(t + 9)(t - 10)$
c $(h + 8)(h - 9)$ **d** $(t + 7)(t - 9)$

9 **a** $(d + 1)^2$ **b** $(y + 10)^2$
c $(t - 4)^2$ **d** $(m - 9)^2$

10 **a** $(x - 12)^2$ **b** $(d + 3)(d - 4)$
c $(t + 4)(t - 5)$ **d** $(q + 7)(q - 8)$

11 $(x + 2)(x + 3)$, giving areas of $2x$ and $3x$, or $(x + 1)(x + 6)$, giving areas of x and $6x$.

12 **a** $x^2 + (a + b)x + ab$
b **i** $p + q = 7$
ii $pq = 12$
c 7 can only be 1×7 and $1 + 7 \neq 12$

Exercise 9L

1 **a** $(x+3)(x-3)$ **b** $(t+5)(t-5)$
c $(m+4)(m-4)$ **d** $(3+x)(3-x)$
e $(7+t)(7-t)$ **f** $(k+10)(k-10)$
g $(2+y)(2-y)$ **h** $(x+8)(x-8)$
i $(t+9)(t-9)$

2 **a** x^2
b **i** $(x-2)$ **ii** $(x+2)$ **iii** x^2 **iv** 4 **v** x^2-4
c A + B − C = x^2-4, which is the area of D, which is $(x+2)(x-2)$.

3 **a** $x^2+4x+4-(x^2+2x+1)=2x+3$
b $(a+b)(a-b)$
c $(x+2+x+1)(x+2-x-1)=(2x+3)(1)=2x+3$
d The answers are the same.
e $4x$

Exercise 9M

1 $k=\frac{T}{3}$

2 $y=X+1$

3 $p=3Q$

4 $r=\frac{A-9}{4}$

5 **a** $m=p-t$
b $t=p-m$

6 $m=gv$

7 $m=\sqrt{t}$

8 $l=\frac{P-2w}{2}$

9 $p=\sqrt{m-2}$

10 **a** $-40-32=-72$, $-72\div 9=-8$, $5\times -8=-40$
b $68-32=36$, $36\div 9=4$, $4\times 5=20$
c $f=\frac{9}{5}C+32$

11 Average speeds: outward journey = 72 km/h, return journey = 63 km/h, taking 2 hours. He was held up for 15 minutes.

12 **a** $y=\frac{5x-75}{9}$
b Check by substitution

13 **a** $a=\frac{v-u}{t}$
b $t=\frac{v-u}{a}$

14 $d=\sqrt{\frac{4A}{\pi}}$

15 **a** $y=\frac{x+w}{5}$
b $w=5y-x$

16 $p=\sqrt{\frac{k}{2}}$

17 **a** $t=u^2-v$
b $u=\sqrt{v+t}$

18 **a** $w=k-5n^2$
b $n=\sqrt{\frac{K-w}{5}}$

19 **a** $D=\frac{P+Y(K-U)}{3(K-U)}$ or $\frac{\frac{P}{K-U}+Y}{3}$ or $D=\frac{P}{3(K-U)}+\frac{Y}{3}$
b 16

Review questions

1 **t** $-xy$

2 **a** 180 − p − q
b 59°

3 **a** $r=\frac{C}{2\pi}$
b $r=\sqrt{\frac{A}{\pi}}$

4 **a** $20x+16$ **b** $5x+4$

5 **a** $c=\frac{R-3d-5}{7}$ **b** 6

6 $12x^2-xy-35y^2$

7 **a** **i** 3.5 ml **ii** 3.7 ml **iii** 3.84
b **i** 22 **ii** 38 **iii** 90

8 13.5 m^2

9 **a** $6r(r+h)$ **b** $h=\frac{A-6r^2}{6r}$ **c** 5 cm

10 **b** $6x^2-51x+90$

11 **a** $x=\frac{A-y}{0.01y}$ **b** $y(1+0.01x)$ **c** 22 g

12 **a** **i** $15x^2-19x-56$ **ii** $16x-2$
b 162.25 cm^2

13 **a** $4x^2+4x+1$ **b** 441

14 20 m

15 **a** $2(x-8)$ **b** $x(x-16)$ **c** $(x-4)(x+4)$
d $(x-7)(x-9)$

16 **a** $(x-10)(x+10)$
b $(x-20)(x+20)$
c $(x-200)(x+200)$

17 −3 + 7 is not −20
(x − 21)(x + 1)

Chapter 10 – Ratio and proportion and rates of change: Ratio, speed and proportion

Exercise 10A

1 **a** 1 : 3 **b** 2 : 3 **c** 2 : 5 **d** 5 : 8
e 3 : 2 **f** 7 : 3 **g** 1 : 6 **h** 5 : 3

2 **a** 1 : 3 **b** 5 : 12 **c** 17 : 15 **d** 4 : 1
e 1 : 24 **f** 5 : 2 **g** 2 : 1
h 31 : 200

3 $\frac{7}{10}$

4 $\frac{2}{5}$

5 **a** $\frac{2}{5}$ **b** $\frac{3}{5}$

6 2 : 1

7 **a** Fruit crush $\frac{1}{6}$, lemonade $\frac{5}{6}$

Fruit Crush	1.25	1	0.2	0.4	0.5
Lemonade	6.25	5	1	2	2.5

b 0.4 litres **c** 2.5 litres

8 Sugar $\frac{5}{22}$, flour $\frac{3}{11}$, margarine $\frac{2}{11}$, fruit $\frac{7}{22}$

9 4

10 1 : 4

11 **a** 5 : 3 : 2 **b** 20 years old

12 $13\frac{1}{2}$ litres

Exercise 10B

1 **a** 160 g, 240 g **b** 80 kg, 200 kg
c 175 min, 125 min **d** £20, £30, £50
e £36, £60, £144 **f** 50 g, 250 g, 300 g

2 **a** 175 women **b** 30%

3 **a** 40% **b** 300 kg

4 21 horses

5 **a** 1 : 400 000 **b** 1: 125 000
c 1 : 250 000 **d** 1 : 25 000
e 1 : 20 000 **f** 1 : 40 000

6 **a** Map A 1 : 250 000, Map B 1 : 1 000 000
b 2 km **c** 1.2 cm or $\frac{6}{5}$ cm
d 4.8 cm

7 **a** 1 : 1.6 **b** 1: 3.25 **c** 1: 1.125
d 1 : 1.44 **e** 1 : 5.4 **f** 1 : 1.5
g 1 : 4.8 **h** 1 : 42 **i** 1 : 1.25

8 Diesel : Petrol = 60 : 90. $\frac{1}{5}$ of 60 = 12. $\frac{4}{9}$ of 90 = 40. Total red cars = 52 which is more than 150 ÷ 3 = 50, so yes.

9 13 – 9 = 4. 4 ÷ 5 = 0.8. 2 × 0.8 = 1.6, 9 + 1.6 = 10.6

10 Athos has 3 more parts than Zena. 24 ÷ 3 = 8, so 1 part is 8. Zena has 8 marbles.

Exercise 10C

1 **a** 3 : 2 **b** 32 **c** 80

2 1000 g or 1 kg

3 8100

4 **a** 14 minutes **b** 75 minutes

5 Kevin £2040, John £2720

6 **a** Lemonade 20 litres, ginger 0.5 litres
b The one in **b**. In part **a** there are 50 parts in the ratio 40 : 9 : 1, so ginger is $\frac{1}{50}$ of total amount; in part **b** there are 13 parts in the ratio 10 : 2 : 1, so ginger is $\frac{1}{13}$ of total amount.
$\frac{1}{13} > \frac{1}{50}$

7 296 seats

8 100 students

9 40 ml

10 **a** Will as his multiple of 10 is also a multiple of 9
b Zeke has rounded off to 1dp and and Yoko has rounded off to 2 dp. They have not used a recurring decimal notation.

11 43 people (20 boys, 8 girls, 15 adults)

12 $\frac{6}{11}$

13 $\frac{1}{2}$

Exercise 10D

1 18 mph

2 280 miles

3 52.5 mph

4 11.50 am

5 **a** 75 mph **b** 6.5 h
c 175 miles **d** 240 km
e 64 km/h **f** 325 km
g 4.3 h (4 h 18 min)

6 **a** 500 m or 0.5 km
b 12:06 p.m.
c His actual speed and distance may not be the same as his estimate, so he may not be back at the estimated time.

7 **a** 2.25 h **b** 99 miles

8 **a** 1.25 h **b** 1 h 15 min

9 **a** Sheffield to London via Midland mainline 74.38 mph. Sheffield to London via East Coast mainline 78.26
b Doncaster to London 94.12 mph

10 **a** 120 km **b** 48 km/h

11 **a** 10 m/s **b** 3.3 m/s **c** 16.7 m/s

12 **a** 90 km/h **b** 43.2 km/h
c 1.8 km/h

13 18 m/s is 64.8 km/h. 40 km at 64.8 km/h is 0.617 hours ≈ 37 minutes so train arrives at 8.07 am

14

Time	Cyclist A	Cyclist B
10	16	0
10.15	20	6
10.30	24	12
10.45	28	18
11	32	24
11.15	36	30
11.30	40	36
11.45	44	42
12	48	48
12.15	52	54

Cyclist B catches cyclist A at 12 noon

15 Rebecca: 10 minutes at 50 mph covers 8.333 miles, 10 minutes at 70 mph covers 11.666 miles, so total distance is 20 miles in 20 minutes which is 60 mph, so Rebecca is correct.
Nick: 10 miles at 40 mph takes 15 minutes, 10 miles at 60 mph takes 10 minutes, so total distance is 20 miles in 25 minutes, which is 48 mph, so Nick is wrong.

16 Josh should take 40 minutes. Nell should take $50 \div 70 \times 60 = 43$ minutes, but Josh is likely to meet traffic through town so is unlikely to travel at anywhere near 30 mph. Nell is likely to be able to travel at 70 mph on the motorway.

Exercise 10E

1 60 g

2 £5.22

3 45 trees

4 £6.72

5 **a** £312.50 **b** 8 textbooks

6 **a** 56 litres **b** 350 miles

7 **a** **i** 100 g, 200 g, 250 g, 150 g
ii 150 g, 300 g, 375 g, 225 g
iii 250 g, 500 g, 625 g, 375 g
b 24 biscuits

8 **a** Peter: £2.30 ÷ 6 = 38.33p each; I can buy four packs (24 sausages) from him (£9.20)
Paul: £3.50 ÷ 10 = 35p each; I can only buy two packs (20 sausages) from him (£7)
I should use Peter's shop to get the most sausages for £10.
b Buy one pack from Peter and two packs from Paul. That would amount to 26 sausages for £9.30.

9 400 ÷ 10 = 40 loaves. 40 ÷ 3 = 13.333, so 13.333 × 1.8 = 24 kg of flour.

10 4 buns and 5 cakes

Exercise 10F

1 **a** £4.50 for a 10-pack
b £1.08 for 6
c £2.45 for 1 litre
d Same value

2 **a** Large jar as more g per £
b 75 ml tube as more ml per £
c Large box as more g per £
d 400 ml bottle as more ml per £

3 **a** £5.11
b Large tin (small £5.11/l, medium £4.80/l, large £4.47/l)

4 **a** 95p **b** Family size

5 Mary

6 Kelly

7 12-pack $360 \div 12 = 30$ p per sachet. 40-pack $1150 \div 40 = 28.75$ p per sachet. 4 sachets cost $4 \times 35 = £1.40$ but you get 5, so $140 \div 5 = 28$ p per sachet, so the offer is the best value.

8 **a** Abe uses $10 \times 0.75 = 7.5$ litres to do 100 km. Caryl uses $100 \div 14 = 7.14$ litres to do 100 km and Des uses $100 \div 55 \times 4.55 = 8.27$ litres to do a 100 km, so Caryl's car is the most economical.
b It does not give a 'unit' value, i.e. miles per gallon or litres per mile.

9 Yes. Let the initial price be x. 50% off means the cost is $0.5x$ each. Two for the price of one means two will cost x, so one will cost $0.5x$.

10 $\$2.09 \div 1.51 = £1.38$ per US gallon.

$£1.38 \div 3.8 = £0.36$ per litre in US.

$£1.09 \div £0.36 = 3.02$.

Petrol is therefore approximately 3 times more expensive in UK than in the USA.

11 $£42 \times 1.2 = £50.40$

$65 \div 50.4 = 1.28$

$€1.28 = £1$

12 **a** Shirts 4 U: $75 + 400 \times 3.45 = £1455$
Top Tees: $100 + 200 \times 3.99 + 200 \times 2.75 = £1448$
Therefore Top Tees is cheaper.
b Shirts 4 U: $75 + 3.45 \times 180 = £1386$
Top Tees: $100 + 200 \times 3.99 + 180 \times 2.75 = £1393$
Therefore Shirts 4 U is cheaper.

13 **a** Yogurt $3 \times 250 = 750$, Banana $5 \times 150 = 750$ Kiwi $17 \times 90 = 1530$
Ratio $1530 : 750 : 750 \approx 1500 : 750 : 750 = 2 : 1 : 1$
This is a sensible recipe because the ratio means they can use whole fruits and whole cartons of yoghurt.
b 15 Kiwi 1350 g, 2 banana 300 g, 4 yogurt 1000 g
$1350 : 300 : 1000 \approx 1333 : 333 : 1000 = 4 : 1 : 3$
Other answers are possible.

Review questions

1 £3.20

2 **a** 150 g **b** 472.5 ml

3 48 mph

4 225 km

5 72 km/h

6 Totals are 40 and 60 giving 2 : 3 and a total of 100. 9 : 11 is a ratio of 45 : 55 so swap 10 and 15

7 **a** $1.73 \div 0.04 = 43.25$ so 43 horses, $2.64 \div 0.065 = 40.61$ so 40 cattle and $0.95 \div 0.01 = 95$. Total $43 + 40 + 95 = 178$ animals.
b Horses in field A = 43, Sheep in field B = $2.64 \div 0.01 = 264$, Cattle in field C, $0.95 \div 0.065 = 14.61$, so 14 cattle.
Total $43 + 264 + 14 = 321$ animals.

8 3 : 2

9 65 blue counters

10 **a** 22.5 kg **b** 30 kg **c** £19.80

11 **a** $42 : 63 = 2 : 3$ **b** 60 out of 200 = 30%
c

	Men	Women
40 years or over	70	54
Under 40 years	54	72

Chapter 11 – Geometry and measures: Perimeter and area

Exercise 11A

1 **a** 24 cm, 35 cm^2 **b** 28 cm, 33 cm^2
c 36 cm, 45 cm^2 **d** 34 cm, 70 cm^2
e 28 cm, 48 cm^2 **f** 14 cm, 10 cm^2
g 26 cm, 42 cm^2 **h** 36 cm, 77 cm^2

2 **a** 20 cm, 24 cm^2 **b** 36 cm, 80 cm^2
c 3 cm, 26 cm **d** 7 cm, 20 cm
e 3 cm, 18 cm^2 **f** 6 cm, 30 cm^2

3 **a** 530 m **b** 17400 m^2

4 $160 \times 70 = 11200$, $11200 \div 10000 = 1.12$ hectares

5 19 bags

Exercise 11B

1 **a** 10 cm **b** 12 cm **c** 12 cm
d 14 cm **e** 12 cm **f** 12 cm

2 18 m

3 No, the perimeter is 30 cm as two sides will touch.

4 False, it is 28 cm. The sum of the lengths of all the sides is 28 cm.

5 **a** 30 cm^2 **b** 40 cm^2 **c** 51 cm^2
d 35 cm^2 **e** 43 cm^2 **f** 51 cm^2
g 48 cm^2 **h** 33 cm^2

6 24

7 The correct answer is 44 cm^2, the length of the bottom rectangle is 6 cm (10 – 4).

8 72 cm^2

9 48 cm

10 Yes, the area to paint is 9.1 m^2

Exercise 11C

1 **a** 6 cm^2, 12 cm **b** 120 cm^2, 60 cm
c 30 cm^2, 30 cm

2 40 cm^2

3 84 m^2

4 **a** 21 cm^2 **b** 55 cm^2 **c** 165 cm^2

5 **c** 75 cm^2

6 May is correct with 32 cm, but Maggie is incorrect, the area should be 36 cm^2, it looks like she has forgotten to halve her calculations.

7 Area of rectangle = $5 \times 12 = 60$ cm^2. The two triangles are the same with one side 6 cm (half of 12) and the other half of (21 – 5) which is 8 cm. Area of one triangle = $\frac{1}{2} \times 6 \times 8 = 24$ cm^2. Area of two triangles = 48 cm^2, so total area = $60 + 48 = 108$ cm^2.

8 **a** 5.5 m^2 **b** 4

Exercise 11D

1 **a** 21 cm^2 **b** 12 cm^2 **c** 14 cm^2
d 55 cm^2 **e** 90 cm^2 **f** 140 cm^2

2 64 cm^2

3 **a** 40 cm^2 **b** 65 m^2 **c** 80 cm^2

4 **a** 65 cm^2 **b** 50 m^2

5 For example: height 10 cm, base 10 cm; height 5 cm, base 20 cm; height 25 cm, base 4 cm; height 50 cm, base 2 cm

6 **a** 1500 cm^2 **b** 1800 cm^2

7 Triangle c; a and b each have an area of 15 cm^2 but c has an area of 16 cm^2.

8 For example: Let vertical side be T, then area = $\frac{1}{2} \times 2 \times T = T$ cm^2. The perimeter is $2 + T +$ other side, which is bigger than T. So Tim is correct.

Exercise 11E

1 **a** 96 cm^2 **b** 70 cm^2 **c** 20 cm^2
d 125 cm^2 **e** 10 cm^2 **f** 112 m^2

2 No, it is 24 cm^2, she used the slanting side instead of the perpendicular height.

3 For example: If $x = 20$, then the base of each parallelogram is $\frac{1}{2}(20 - 4) = 8$. This would make the area of each parallelogram $8 \times 5 = 40$, making the whole shape $4 \times 40 = 160$, which is incorrect. The correct value of x is 16.

4 **a** 500 cm^2 **b** 15

Exercise 11F

1 **a** 30 cm^2 **b** 77 cm^2 **c** 24 cm^2
d 42 cm^2 **e** 40 m^2 **f** 6 cm
g 3 cm **h** 10 cm

2 **a** 27.5 cm, 36.25 cm^2 **b** 33.4 cm, 61.2 cm^2
c 38.6 m, 88.2 m^2

3 Any pair of lengths that add up to 10 cm
For example: 1 cm, 9 cm; 2 cm, 8 cm; 3 cm, 7 cm; 4 cm, 6 cm; 4.5 cm, 5.5 cm

4 Shape c. Its area is 25.5 cm^2

5 Shape a. Its area is 28 cm^2

6 45 cm^2

7 2 cm

8 1.4 m^2

9 **b** 64 cm^2

Exercise 11G

1 Top of head-tangent, eyebrows-arcs, nose-sector, mouth-segment, face-circle

2

Diameter	The width of a circle
Radius	The distance from the centre to the edge of the circle
Circumference	The perimeter of a circle
Chord	A line joining two points on a circumference
Tangent	A line that touches the circumference of the circle at one point only
Arc	A part of the circumference of a circle
Sector	A part of a circle lying between two radii and an arc
Segment	A part of the area of a circle, lying between a chord and an arc

3 Students' own drawings

4 Students' own drawings

Exercise 11H

1 **a** 25.1 cm **b** 15.7 cm **c** 44.0 cm
d 22.0 cm **e** 18.8 cm **f** 47.1 cm
g 28.9 cm **h** 14.8 cm

2 **a** 6.3 cm **b** 8.2 cm **c** 5.3 cm
d 7.5 cm

3 **a** 31.4 cm **b** 18.8 cm **c** 9.4 cm
d 25.1 cm **e** 5.7 cm **f** 15.7 cm
g 81.7 cm **h** 39.6 cm

4 **a** 198 cm **b** 505

5 **a** A 188.5 m, B 194.8 m, C 201.1 m, D 207.3 m
b 18.7 or 18.8 m

6 879.6 or 880 cm

7 **a** 37.7 cm **b** 3770 cm **c** 37.7 m
d 37.7 km

8 100 cm

9 24.2 cm

10 15.9 cm

11 25.7 cm

12 **a** Fir: 62.8 cm, ash: 75.4 cm, elm: 69.1 cm, oak: 81.7 cm
b The difference between the distances round the circumferences of two trees is 2π times the difference between their radii, or π times the difference between their diameters.
c 6.28 m

13 **a** Perimeters of shapes A and B are both 25.1 cm
b 25.1 cm

14 $4a = 2\pi r$, so $2a = \pi r$, therefore $r = \frac{2a}{\pi}$

15 7

16 **a** $4\pi = 12.6$cm **b** New radius of small circle will be 3, giving new perimeter of the whole shape of 12π, which is three times larger than 4π.

Exercise 11I

1 **a** 78.5 cm^2 **b** 28.3 cm^2 **c** 7.1 cm^2
d 50.3 cm^2 **e** 2.5 cm^2 **f** 19.6 cm^2
g 530.9 cm^2 **h** 124.7 cm^2

2 **a** 3.1 cm^2 **b** 5.3 cm^2 **c** 2.3 cm^2
d 4.5 cm^2

3 **a** 50.3 cm^2 **b** 19.6 cm^2 **c** 153.9 cm^2
d 38.5 cm^2 **e** 28.3 cm^2 **f** 176.7 cm^2
g 66.5 cm^2 **h** 17.3 cm^2

4 **a** 9.1 cm² **c** 138 **b** 2000 cm²
d 1255.8 cm² or 1252.9 cm² using unrounded answer from **a**
e 744.2 cm² or 747.1 cm², using unrounded answer from **a**

5 3848.5 m²

6 **a** **i** 56.5 cm **ii** 254.5 cm²
b **i** 69.1 cm **ii** 380.1 cm²
c **i** 40.8 cm **ii** 132.7 cm²
d **i** 88.0 cm **ii** 615.8 cm²

7 962.9 cm² (or 962.1 cm²)

8 **a** 56.5 cm² **b** 19.6 cm²

9 **a** 50.3 m² **b** 44.0 cm²

10 $a^2 = \pi r^2$, so $r^2 = \frac{a^2}{\pi}$, therefore $r = \frac{a}{\sqrt{\pi}}$

11 Area of the square is 100cm², Area of circle is $\pi \times 5^2 = 78.54$, leaving the shaded area as $100 - 78.54 = 21.46$cm². Percentage of shaded area $= \frac{21.46}{100} \times 100 = 21.46\%$, which rounds to 21%, so Lewis is correct.

12 **a** 4π or 12.6 cm²
b **i** 16π or 50.3 cm² **ii** 36π or 113 cm²
iii 64π or 201 cm²
c $a = 4\pi r^2$

Exercise 11J

1 **a** 4 π cm **b** 20 π cm **c** 15 π cm
d 4 π cm

2 **a** 16 π cm² **b** 25 π cm² **c** 9 π cm²
d 81 π cm²

3 25 cm

4 10 cm

5 Rory is correct. Area $= \pi \times \left(\frac{100}{\pi}\right)^2$, so Area $= \pi \times \frac{100^2}{\pi^2} = \frac{10\,000}{\pi}$, so Alice was incorrect. She needed to put an extra zero in the numerator.

6 **a** 12.5π cm² **b** 16π cm² **c** $(16\pi + 80)$ cm²
d $(50\pi + 100)$ cm²

7 **a** 32π cm² **b** 16π cm²
c 8π cm² **d** 4π cm²

8 $18\pi - 9\pi = 9\pi$

9 **c**; $80 + \frac{1}{4}(\pi \times 8^2) = 80 + 16\pi$

Review questions

1 30 cm²

2 48 tiles

3 Area $= \frac{1}{2} \times 12 \times 9 = 54$cm², Tilly has forgotten the $\frac{1}{2}$ in her calculation.

4 8 cm

5 209 sheep

6 $\pi d = 30$, so $d = \frac{30}{\pi}$, so radius $= \frac{15}{\pi}$, so area $= \pi \times \left(\frac{15}{\pi}\right)^2 = \frac{225}{\pi} = 71.6$

7 Circle area $= \pi \times 25 = 78.5$, Parallelogram area $= 9 \times 8 = 72$, difference $= 78.5 - 72 = 6.5$, Andy is correct.

8 **a** 2.5 cm
b Area of large circle $= 10^2 \times \pi = 100\pi$, area of two small circles is $2 \times \pi \times 2.5^2 = 12.5\pi$, the shaded part $= (100 - 12.5)\pi = 87.5\pi$.
The percentage of shaded $= \frac{87.5\,\pi}{100\,\pi} \times 100 = 87.5\%$

9 $\frac{20}{\pi}$

10 25.7 cm

Chapter 12 – Geometry and measures: Transformations

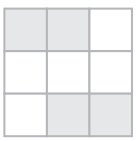
Exercise 12A

1 **a** 4 **b** 5 **c** 6
d 4 **e** 6

2 **a** 2 **b** 2 **c** 2
d 2 **e** 2

3 A, B, C, D, E, F, G, J, K, L, M, P, Q, R, T, U, V, W, Y

4 for example:



5

		Number of lines of symmetry			
		0	1	2	3
Order of rotational symmetry	1	D	A		
	2	E		B	
	3				C

6 She is correct since the angle sum around the centre point is 360 and 360 ÷ 3 = 120

7 Yes she is correct. A triangle can only have 1 or 3 lines of symmetry. If a triangle has 3 lines of symmetry it also has rotational symmetry of order 3, so this triangle must only have 1 line of symmetry. This will mean it has two angles identical and two equal sides, and hence an isosceles triangle.

Exercise 12B

1 Check students' translations.

2 Check students' translations.

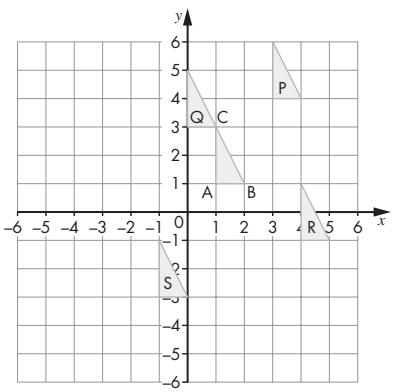
3 **a** **i** $\begin{pmatrix}1\\3\end{pmatrix}$ **ii** $\begin{pmatrix}4\\2\end{pmatrix}$ **iii** $\begin{pmatrix}2\\-1\end{pmatrix}$ **iv** $\begin{pmatrix}5\\1\end{pmatrix}$

b **i** $\begin{pmatrix}-1\\-3\end{pmatrix}$ **ii** $\begin{pmatrix}3\\-1\end{pmatrix}$ **iii** $\begin{pmatrix}-2\\3\end{pmatrix}$ **iv** $\begin{pmatrix}3\\3\end{pmatrix}$

c **i** $\begin{pmatrix}-2\\-3\end{pmatrix}$ **ii** $\begin{pmatrix}1\\-1\end{pmatrix}$ **iii** $\begin{pmatrix}-5\\4\end{pmatrix}$ **iv** $\begin{pmatrix}0\\4\end{pmatrix}$

d **i** $\begin{pmatrix}3\\2\end{pmatrix}$ **ii** $\begin{pmatrix}-4\\2\end{pmatrix}$ **iii** $\begin{pmatrix}5\\-4\end{pmatrix}$ **iv** $\begin{pmatrix}-2\\-7\end{pmatrix}$

4



5 **a** $\begin{pmatrix}-3\\-1\end{pmatrix}$ **b** $\begin{pmatrix}4\\-4\end{pmatrix}$ **c** $\begin{pmatrix}-5\\-2\end{pmatrix}$

d $\begin{pmatrix}4\\7\end{pmatrix}$ **e** $\begin{pmatrix}-1\\5\end{pmatrix}$ **f** $\begin{pmatrix}1\\6\end{pmatrix}$

g $\begin{pmatrix}-4\\4\end{pmatrix}$ **h** $\begin{pmatrix}-4\\-7\end{pmatrix}$

6 $10 \times 10 = 100$ (including $\begin{pmatrix}0\\0\end{pmatrix}$

7 **a** Check students' designs for a Snakes and ladders board.

b because the ladders always mean moving up the board

ii the snakes always mean moving down the board

8 $\begin{pmatrix}-x\\-y\end{pmatrix}$

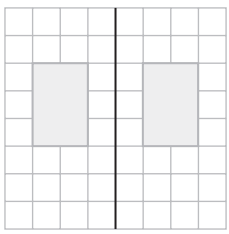
9 **a** Check student's diagram **b** $\begin{pmatrix}-300\\-500\end{pmatrix}$

10 **a** Check student's diagram **b** $\begin{pmatrix}-1\\4\end{pmatrix}$

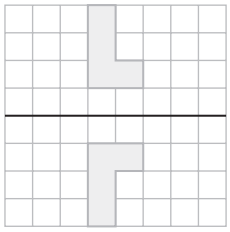
11 Multiply each part of the vector describing the move by –1

Exercise 12C

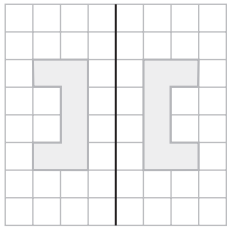
1 **a**



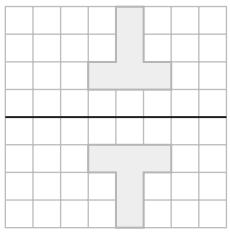
b



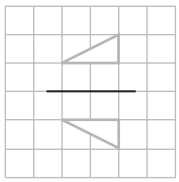
c



d



2 a



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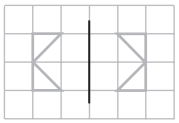

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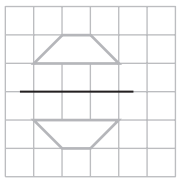

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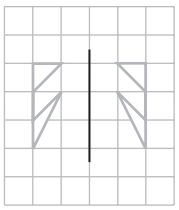

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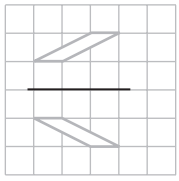

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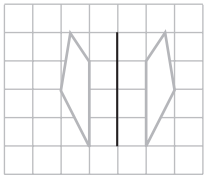

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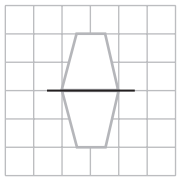
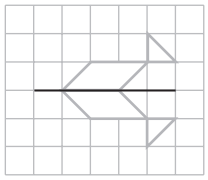
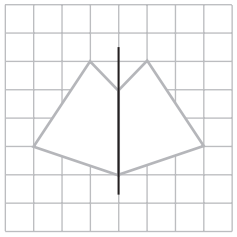


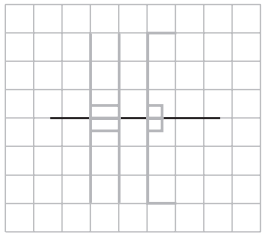
3 a



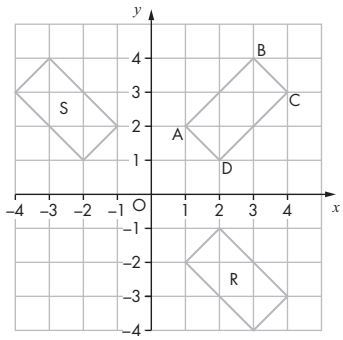
b



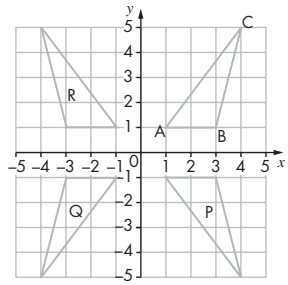
c



4 a–b

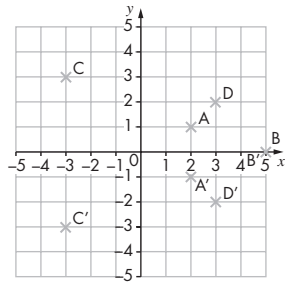


5 a–e



f Reflection in the y-axis

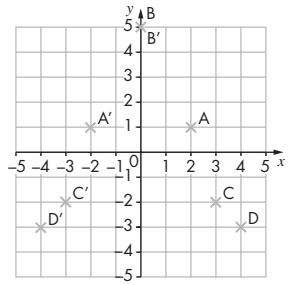
6 a–b



c y-value changes sign

d $(a, -b)$

7 a–b



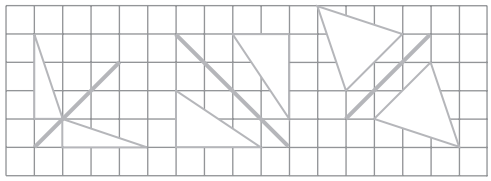
c x-value changes sign

d $(-a, b)$

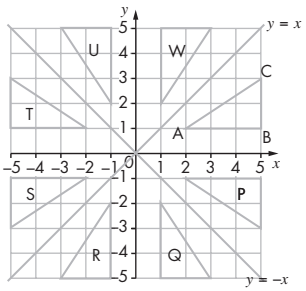
8 Possible answer: Take the centre square as ABCD then reflect this square each time in the line, AB, then BC, then CD and finally AD.

9 $x = -1$

10

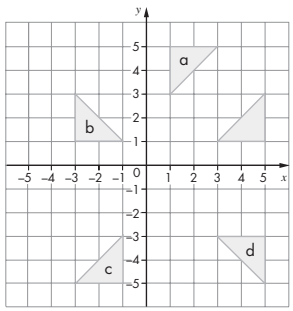


11 a–i

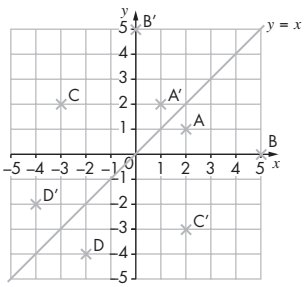


j A reflection in $y = x$

12 a–c



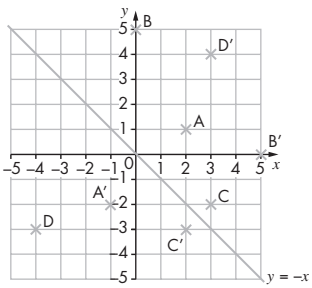
13 a–c



d Coordinates are reversed: x becomes y and y becomes x

e (b, a)

14 a–c



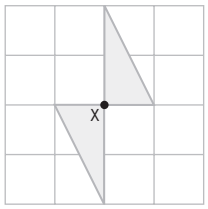
d Coordinates are reversed and change sign, x becomes $-y$ and y becomes $-x$

e $(-b, -a)$

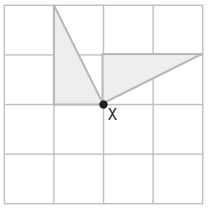
15 A reflection in the same line

Exercise 12D

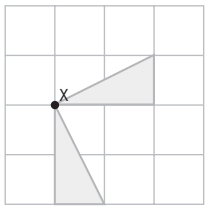
1 a



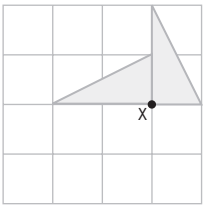
b



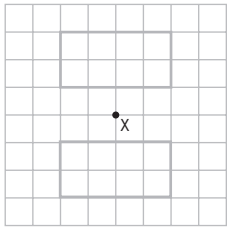
c



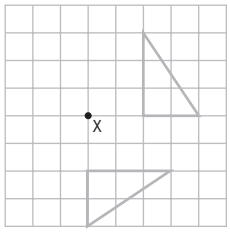
d



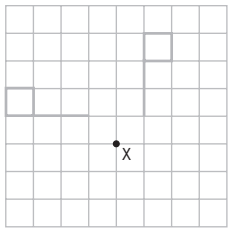
2 a



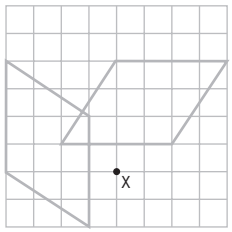
b



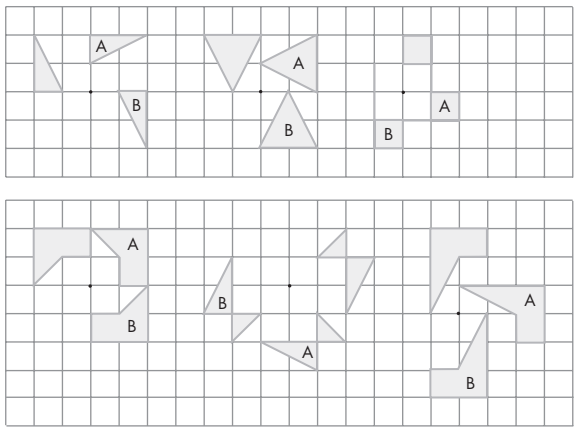
c



d

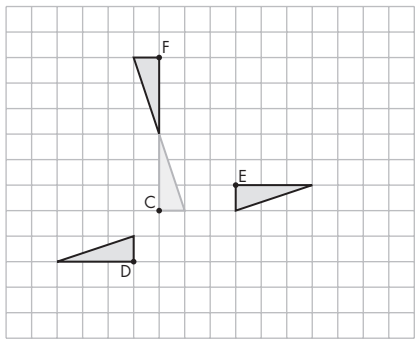


3 a

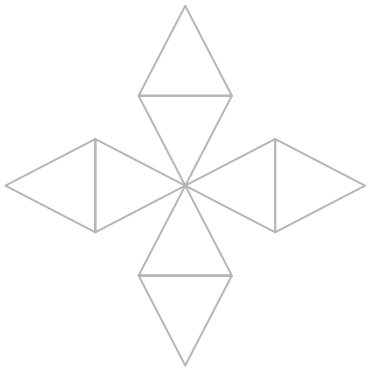


b i Rotation 90°anticlockwise
ii Rotation 90° clockwise

4 a-d



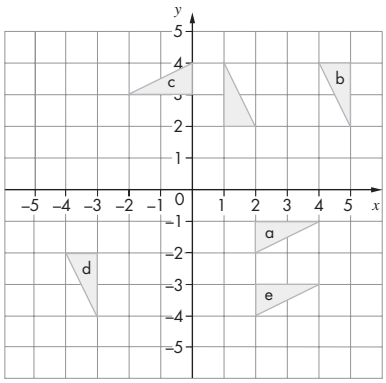
5



b Rotate shape 60° around point B, then repeat another four times.

6 Possible answer: If ABCD is the centre square, rotate about A 90° anticlockwise, then rotate about A 180°, now rotate about new B 180°, and finally rotate about new C 180°.

7

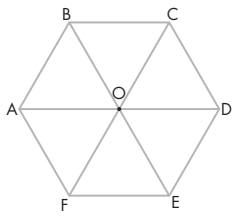


8 a Because the opposite rotations equal a total of 360°, which is the same point
b i 90° anticlockwise
ii 270° anticlockwise
iii 300° clockwise
iv 260° clockwise

9 Show by drawing a shape or use the fact that (a, b) becomes $(a, -b)$ after reflection in the x-axis, and $(a, -b)$ becomes $(-a, -b)$ after reflection in the y-axis, which is equivalent to a single rotation of 180°.

10 She is correct.

11 a



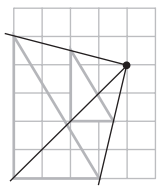
b i Rotation 60°clockwise about O
ii Rotation 120°clockwise about O
iii Rotation 180°about O
iv Rotation 240°clockwise about O
c i Rotation 60°clockwise about O
ii Rotation 180°about O

12 Rotation 90° anticlockwise about (3, 22)

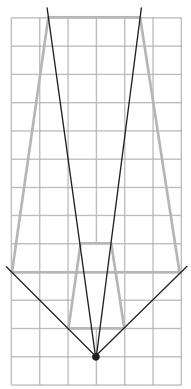
13 A rotation of the same angle but in the opposite direction. In this case the direction would be in the clockwise direction.

Exercise 12E

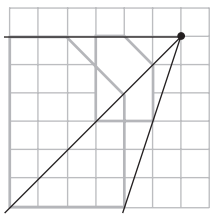
1 a



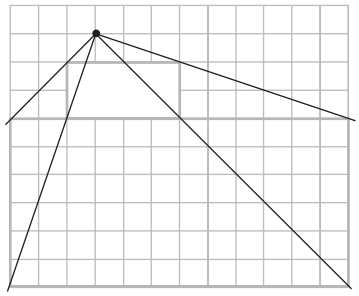
b



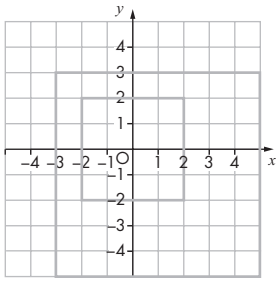
c



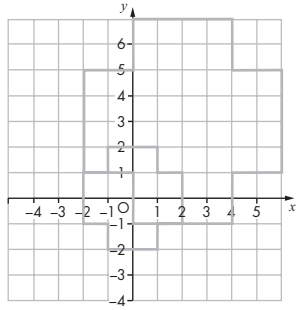
d



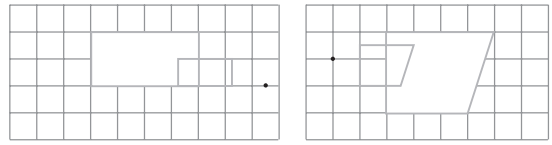
2 a



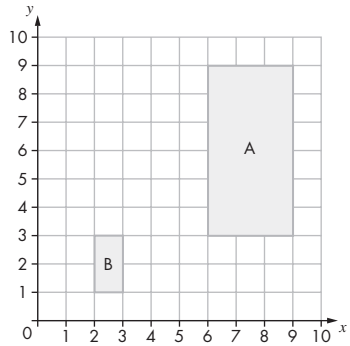
b



3



4 a



b 3 : 1
c 3 : 1
d 9 : 1

5 (1, –5), (3, –3), (–5, –5)

6 b Rotation 180° around (–1, 2)

7 a Enlargement by scale factor –2 from centre of enlargement (1, 3)
b Enlargement by scale factor $-\frac{1}{2}$ from centre of enlargement (1, 3)

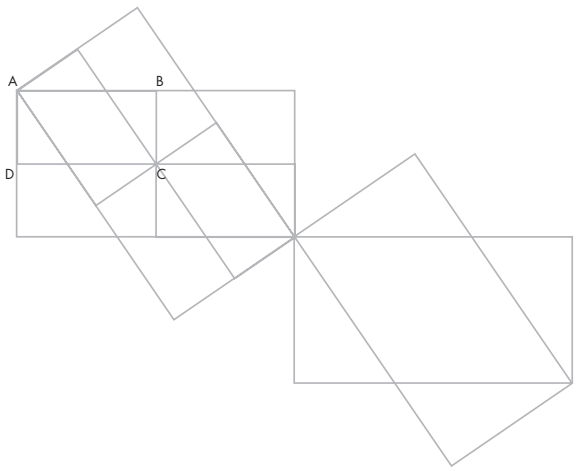
Exercise 12F

1 a (–4, –3) and (–2, –1)

2 a (–5, 2) **b** (–5, –2)
c Rotation of 180° about the origin.

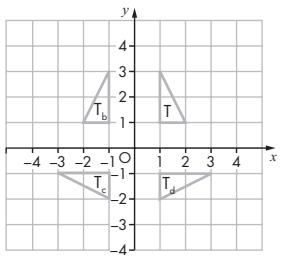
3 A: translation $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$, B: reflection in y-axis, C: rotation 90°clockwise about (0, 0), D: reflection in $x = 3$, E: reflection in $y = 4$, F: enlargement by scale factor 2, centre (0, 1)

4



5 **a** T_1 to T_2: rotation 90°clockwise about (0, 0)
b T_1 to T_6: rotation 90°anticlockwise about (0, 0)
c T_2 to T_3: translation $\begin{pmatrix} 2 \\ 2 \end{pmatrix}$
d T_6 to T_2: rotation 180°about (0, 0)
e T_6 to T_5: reflection in y-axis
f T_5 to T_4: translation $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$

6 **a–d**



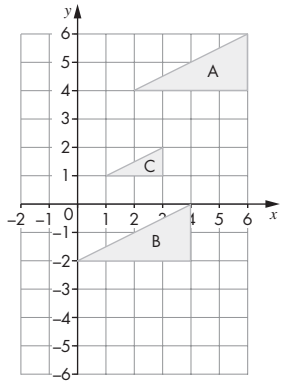
e T_d to T: rotation 90° anticlockwise about (0, 0)

7 (3, 1)

8 For example: Reflection in x-axis, translation $\begin{pmatrix} 0 \\ -5 \end{pmatrix}$, rotation 90°clockwise about (0, 0)

9 For example: Translation $\begin{pmatrix} 0 \\ -8 \end{pmatrix}$, reflection in x-axis, rotation 90°clockwise about (0, 0)

10 **a**

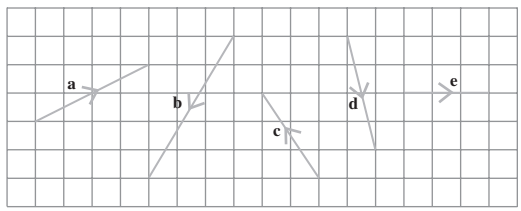


b enlargement scale factor 2 from (0, -2)

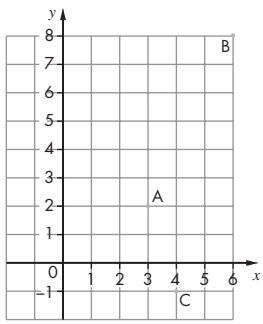
11 No, the two reflections will result in the original orientation.

Exercise 12G

1



2 **a**



b **i** $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$ **ii** $\begin{pmatrix} -2 \\ -9 \end{pmatrix}$ **iii** $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

3 **a** (0, 0) and (3, 5) **b** $\begin{pmatrix} 3 \\ 5 \end{pmatrix}$

4 **a** (2, 2), (6, 1), (6, 4)
b (–6, –1), (–2, –2), (–2, 1)

5 (1, 5)

6 **a** (3, 8) **b** (6, –6)

Exercise 12H

1 **a** Any three of: $\overrightarrow{AC}$, $\overrightarrow{CF}$, $\overrightarrow{BD}$, $\overrightarrow{DG}$, $\overrightarrow{GI}$, $\overrightarrow{EH}$, $\overrightarrow{HJ}$, $\overrightarrow{JK}$

b Any three of: $\overrightarrow{BE}$, $\overrightarrow{AD}$, $\overrightarrow{DH}$, $\overrightarrow{CG}$, $\overrightarrow{GJ}$, $\overrightarrow{FI}$, $\overrightarrow{IK}$

c Any three of: $\overrightarrow{AO}$, $\overrightarrow{CA}$, $\overrightarrow{FC}$, $\overrightarrow{IG}$, $\overrightarrow{GD}$, $\overrightarrow{DB}$, $\overrightarrow{KJ}$, $\overrightarrow{JH}$, $\overrightarrow{HE}$

d Any three of: $\overrightarrow{BO}$, $\overrightarrow{EB}$, $\overrightarrow{HD}$, $\overrightarrow{DA}$, $\overrightarrow{JG}$, $\overrightarrow{GC}$, $\overrightarrow{KI}$, $\overrightarrow{IF}$

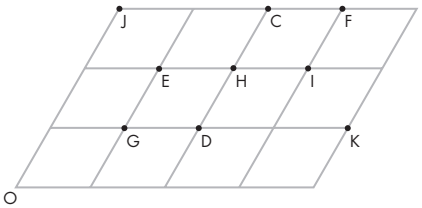
2 **a** $2a$ **b** $2b$ **c** $a + b$
d $2a + b$ **e** $2a + 2b$ **f** $a + 2b$
g $a + b$ **h** $2\boldsymbol{a} + 2\boldsymbol{b}$ **i** $3a + b$
j $2\boldsymbol{a}$ **k** b **l** $2a + b$

3 **a** Equal **b** $\overrightarrow{AI}$, $\overrightarrow{BJ}$, $\overrightarrow{DK}$

4 **a** $\overrightarrow{OJ} = 2\overrightarrow{OD}$ and parallel **b** $\overrightarrow{AK}$

c $\overrightarrow{OF}$, $\overrightarrow{BI}$, $\overrightarrow{EK}$

5



6 **a** Lie on same straight line
b All multiples of $a + b$ and start at O
c $4\boldsymbol{a} + 2\boldsymbol{b}$

7 **a** $-\boldsymbol{b}$ **b** $3\boldsymbol{a} - \boldsymbol{b}$ **c** $2\boldsymbol{a} - \boldsymbol{b}$
d $\boldsymbol{a} - \boldsymbol{b}$ **e** $\boldsymbol{a} + \boldsymbol{b}$ **f** $-\boldsymbol{a} - \boldsymbol{b}$
g $2\boldsymbol{a} - \boldsymbol{b}$ **h** $-\boldsymbol{a} - 2\boldsymbol{b}$ **i** $\boldsymbol{a} + 2\boldsymbol{b}$
j $-\boldsymbol{a} + \boldsymbol{b}$ **k** $2\boldsymbol{a} - 2\boldsymbol{b}$ **l** $\boldsymbol{a} - 2\boldsymbol{b}$

8 **a** Equal but in opposite directions

b Any three of: $\overrightarrow{DA}$, $\overrightarrow{EF}$, $\overrightarrow{GJ}$, $\overrightarrow{FI}$, $\overrightarrow{AH}$

9 **a** Opposite direction and $\overrightarrow{AB} = -\frac{1}{2}\overrightarrow{CK}$

b $\overrightarrow{BJ}$, $\overrightarrow{CK}$

c $\overrightarrow{EB}$, $\overrightarrow{GO}$, $\overrightarrow{KH}$

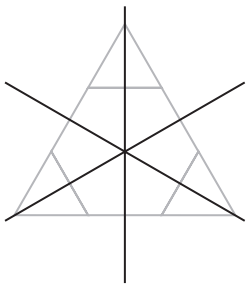
10 **a** **i** $3\boldsymbol{a} + 2\boldsymbol{b}$ **ii** $3\boldsymbol{a} + \boldsymbol{b}$
iii $2\boldsymbol{a} - \boldsymbol{b}$ **iv** $2\boldsymbol{b} - 2\boldsymbol{a}$

b $\overrightarrow{DG}$ and $\overrightarrow{BC}$

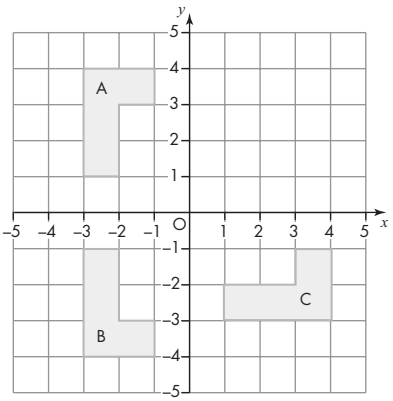
Review questions

1 **a** 3

b There will be 3 lines all through a vertex and the opposite side's midpoint.



2 **a**

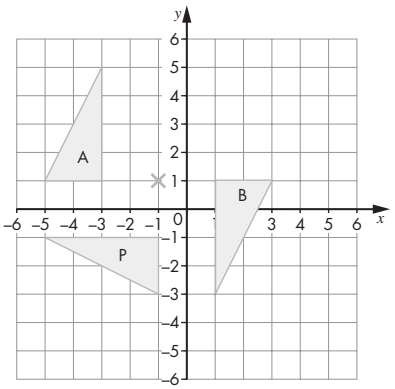


b 90° anticlockwise rotation about (0, 0)

3 180° rotation about (0, 0)

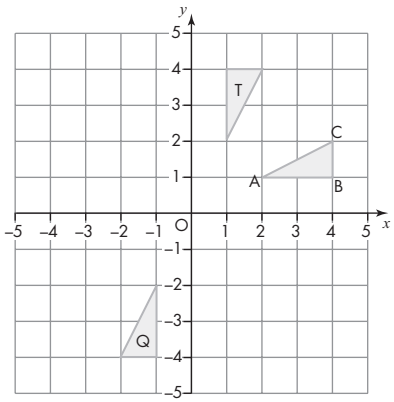
4 Translation with vector $\begin{pmatrix} 4 \\ 1 \end{pmatrix}$

5 **a**



c Rotation of 180° about (–1, 1)

6 **a – b**



c Rotation of half a turn around the origin

7 (9, 0), (11, –3), (2, –1)

8 **i** –**b** **ii** –**a** **iii** **b** **iv** **a** **v** **a** + **b** **vi** **a** – **b**

9 Students' diagram.

10 For example: Reflection in the y axis, reflection in the line $y = 1$

Chapter 13 – Probability: Probability and events

Exercise 13A

1 **a** $\frac{1}{6}$ **b** $\frac{1}{6}$ **c** $\frac{1}{2}$ **d** $\frac{1}{13}$ **e** $\frac{1}{4}$
f $\frac{1}{2}$ **g** 1 **h** $\frac{1}{26}$ **i** $\frac{1}{13}$ **j** 0

2 **a** 0 **b** 1

3 **a** $\frac{1}{10}$ **b** $\frac{1}{2}$ **c** $\frac{2}{5}$ **d** $\frac{1}{5}$ **e** $\frac{3}{5}$

4 **a** $\frac{1}{2}$ **b** $\frac{1}{3}$ **c** $\frac{1}{6}$

5 $\frac{1}{50}$

6 **a** $\frac{1}{13}$ **b** $\frac{1}{13}$ **c** $\frac{1}{2}$ **d** $\frac{1}{26}$

7 **a** **i** $\frac{12}{25}$ **ii** $\frac{7}{25}$ **iii** $\frac{6}{25}$

b They add up to 1.
c All possible outcomes are used.

8 **a** AB, AC, AD, AE, BC, BD, BE, CD, CE, DE
b 1 **c** $\frac{1}{10}$ **d** 6 **e** $\frac{3}{5}$ **f** $\frac{3}{10}$

9 There might not be the same number of boys as girls in the class.

10 Class U

Exercise 13B

1 **a** $\frac{19}{20}$ **b** 55% **c** 0.2

2 **a** **i** $\frac{3}{13}$ **ii** $\frac{10}{13}$
b **i** $\frac{1}{4}$ **ii** $\frac{3}{4}$
c **i** $\frac{2}{13}$ **ii** $\frac{11}{13}$

3 **a** $\frac{1}{2}$ **b** 1 **c** $\frac{1}{3}$

4 **a** $\frac{6}{11}$ **b** $\frac{5}{11}$ **c** $\frac{6}{11}$

5 **a** $\frac{1}{2}$ **b** $\frac{1}{5}$ **c** $\frac{4}{5}$ **d** $\frac{7}{10}$

6 **a** $\frac{7}{15}$ **b** $\frac{2}{15}$ **c** $\frac{13}{15}$ **d** $\frac{8}{15}$
e $\frac{7}{15}$ **f** 0

7 P(Noah will lose) = $\frac{1}{2}$ and P(Phoebe will lose) = $\frac{1}{3}$, so P(Noah will not lose) = $\frac{1}{2}$ and P(Phoebe will not lose) = $\frac{2}{3}$. Phoebe has a better chance of not losing as $\frac{2}{3} > \frac{1}{2}$

8 Because it might be possible for the game to end in a draw.

9 **a** **i** $\frac{1}{4}$ **ii** $\frac{3}{4}$
b **i** $\frac{3}{11}$ **ii** $\frac{8}{11}$

10 35%

11 0.5

Exercise 13C

1 **a** $\frac{1}{6}$ **b** $\frac{1}{6}$ **c** $\frac{1}{3}$

2 **a** $\frac{1}{4}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$

3 **a** $\frac{2}{11}$ **b** $\frac{4}{11}$ **c** $\frac{6}{11}$

4 **a** $\frac{1}{3}$ **b** $\frac{2}{5}$ **c** $\frac{11}{15}$ **d** $\frac{11}{15}$ **e** $\frac{1}{3}$

5 **a** 0.8 **b** 0.2

6 **a** $\frac{17}{20}$ **b** $\frac{2}{5}$ **c** $\frac{3}{4}$

7 Because these are not mutually exclusive events and probability cannot exceed 1.

8 $\frac{3}{4}$

9 The probability for each day stays the same, at $\frac{1}{4}$.

10 **a** The choices of drink and snack are not mutually exclusive.
b (C, D), (T, G), (T, R), (T, D), (H, G), (H, R), (H, D)
c **i** (C, D) = 0.12, (T, G) = 0.15, (T, R) = 0.05, (T, D) = 0.3, (H, G) = 0.09, (H, R) = 0.03, (H, D) = 0.18
ii The total is 1 as all the outcomes are exhaustive.

Exercise 13D

1 **a** B **b** B **c** C **d** A
e B **f** a **g** B **h** B

2 **a** 0.2, 0.08, 0.1, 0.105, 0.148, 0.163, 0.1645
b 6 **c** 1 **d** $\frac{1}{6}$

3 **a** 0.095, 0.135, 0.16, 0.265, 0.345
b 40
c No; all numbers should be close to 40.

4 **a** 0.2, 0.25, 0.38, 0.42, 0.385, 0.3974
b 79

5 **a** Caryl, threw the greatest number of times.
b 0.39, 0.31, 0.17, 0.14
c Yes; all the relative frequencies should be close to 0.25.

6 Thursday; the day with the highest relative frequency

7 The missing top numbers are 4 and 5, the bottom two numbers are both likely to be close to 20.

8 Although he might expect the probability to be close to $\frac{1}{2}$, giving 500 heads, the actual number of heads is unlikely to be exactly 500, but should be close to it.

9 Roza is correct, as the expected numbers are: 50, 12.5, 25, 12.5. Sam has not taken into account the fact that there are four red sectors.

Exercise 13E

1 **a** $\frac{1}{6}$ **b** 25

2 **a** $\frac{1}{2}$ **b** 1000

3 **a** **i** $\frac{1}{2}$ **ii** $\frac{1}{13}$ **iii** $\frac{1}{4}$ **iv** $\frac{1}{52}$
b **i** 260 **ii** 40 **iii** 130 **iv** 10

4 **a** $\frac{1}{37}$ **b** 5

5 **a** 150 **b** 100 **c** 250 **d** 0

6 **a** 167 **b** 833

7 **a** 0.6 **b** 120

8 **a**

Score	1	2	3	4	5	6
Expected occurrences	10	10	10	10	10	10

b $\frac{1\times10+2\times10+3\times10+4\times10+5\times10+6\times10}{60}=\frac{210}{60}=3.5$
c Find the average of the scores, which is $\frac{21}{6}=3.5$

9 **a** 0.111 **b** 40

10 281 days

11 Multiply the number of plants by 0.997.

12 400

Exercise 13F

1 **a** 48 **b** 12

2 **a** 40

3 **a** (BS, BT), (BS, RT), (BS, GT), (WS, BT), (WS, RT), (WS, GT), (GS, BT), (GS, RT), (GS, GT)
b $\frac{7}{9}$

4 $10^9 = 1\,000\,000\,000$ (1 billion)

5 **a** $\frac{1}{2}$
b HHH, HHT, HTH, HTT, THH, THT, TTH, TTT $\frac{3}{8}$

6 20

7 $2^5 = 32$

8 56

9 $26^2 \times 10^2 = 67\,600 > 50\,000$

10 **a** $2 \times 26 \times 999\,999 \times 4 = 207\,999\,792$

b $\frac{1}{4}$

11 **a** 24 **b** $\frac{1}{6}$

12 $\frac{11}{21}$ Number of possible permutations is 21, £2 + any coins =6, £1 + any coin other than £2 = 5, etc, so total is 6 + 5 + 4 + 3 + 2 + 1 = 21, Only the ones with a £2 or a £1 in will give a total greater than £1 = 11 ways

13 **a** 6, 16, etc.. up to 196 which is 20 plus 60, 61 up to 69, which is 9 (66 already counted) plus 160, 162 up to 169 which is 9 (166 already counted) giving a total of 38

b $\frac{38}{200} = \frac{19}{100}$

c $\frac{11}{200}$

14 **a** Key pad A, $10^4 = 10000$ Keypad B $13^4 = 28561$

b $3 \times 10^3 = 3\,000$ **c** $\frac{1}{24}$

15 **a** **i** 5040 **ii** 2.43×10^{18} (3 sf)

b This depends on your calculator but $69! = 1.71 \times 10^{98}$, which is about the number of atoms in 1 quintillion universes. (Look this term up if you're not sure what it means).

Review questions

1 **a** April and May

b Daffodil

c February

d Crocus

e **i** $\frac{1}{5}$

ii

2 $\frac{5}{12}$

3 **a** $\frac{13}{50}$ **b** $\frac{1}{5}$ **c** $\frac{1}{2}$ **d** $\frac{7}{10}$

4 **a** 1

b There are five possible outcomes and only two of them are 2, so the probability is $\frac{2}{5}$.

5 **a** 0.5

b yes, as you would expect all the probabilities to be close to 0.25

6 **a** $\frac{3}{8}$ **b** 12

7 **a** **i** 0.8 **ii** 12

b No, as P(six) = $\frac{1}{6}$, so Tom is likely to throw 10 sixes

8 **a**

Colour	White (W)	Green (G)	Blue (B)
Relative frequency	$\frac{1}{4}$	$\frac{9}{20}$	$\frac{3}{10}$

b The second table, as there were more trials

9 **a** (H, 1), (H, 2), (H, 3), (H, 4), (H, 5), (H, 6), (T, 1), (T, 2), (T, 3), (T, 4), (T, 5), (T, 6)

b $\frac{2}{12} = \frac{1}{6}$ as (H, 3) scores 6 and (T, 6) scores 6

10 $\frac{3}{6} = \frac{1}{2}$. There are 6 combinations, 2 + 14 = 16, 11 + 14 = 25 and 14 + 22 = 36 give square numbers

11 **a** $10^3 = 1000$ **b** $\frac{1}{100}$

c Trial and improvement $9^5 = 59\,049$

Chapter 14 – Geometry and measures: Volumes and surface areas of prisms

Exercise 14A

1 **a**

Name	Number of faces (*F*)	Number of vertices (*V*)	Number of edges (*E*)
Cuboid	6	8	12
Square-based pyramid	5	5	8
Tetrahedron	4	4	6
Octahedron	8	6	12
Triangular prism	5	6	9
Hexagonal prism	8	12	18
Hexagon based pyramid	7	7	12

b $F + V = E + 2$

2 **a** **i** 6, 12 **ii** 24, 96
b **i** 2 **ii** 4 **iii** 8
c **ii** 2, 4 **ii** 18, 108
d **i** 3 **ii** 9 **iii** 27

3 She might have calculated this by multiplying 5 by 5 by 4 to get 100.

4 65 cm^2

5 6

6 David is correct. A pentagon has 5 sides, so a pentagonal prism has 5 + 5 + 5 edges, and 2 + 5 faces.
5 + 5 + 5 + 2 + 5 = 22.

Exercise 14B

1 **a** **i** 198 cm^3 **ii** 234 cm^2
b **i** 90 cm^3 **ii** 146 cm^2
c **i** 1440 cm^3 **ii** 792 cm^2
d **i** 525 cm^3 **ii** 470 cm^2

2 24 litres

3 **a** 160 cm^3 **b** 480 cm^3 **c** 150 cm^3

4 **a** **i** 64 cm^3 **ii** 96 cm^2
b **i** 343 cm^3 **ii** 294 cm^2
c **i** 1000 mm^3 **ii** 600 mm^2
d **i** 125 m^3 **ii** 150 m^2
e **i** 1728 m^3 **ii** 864 m^2

5 86

6 **a** 180 cm^3 **b** 5 cm **c** 6 cm
d 10 cm **e** 81 cm^3

7 32 000 litre is equivalent to 32 000 × 1000 cm^3 = 32 000 000 cm^3. $V = lwh$, so 650 × 310 × h = 32 000 000, so $h = \frac{32\,000\,000}{(650 \times 310)} = 158.8$ cm.

8 Volume = base area × height, so base area = volume/height = $\frac{168}{3.5} = 48$ m^2. So Peter is only correct if he has rounded to the nearest ten.

9 **a** 3 cm **b** 5 m **c** 2 mm **d** 1.2 m

10 **a** 148 cm^3 **b** 468 cm^3

11 If this was a cube, the side length would be 5 cm, so total surface area would be 5 × 5 × 6 = 150 cm^2; so this particular cuboid is not a cube.

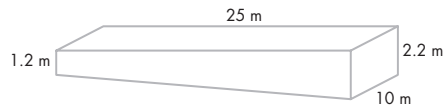
12 600 cm^2

Exercise 14C

1 **a** **i** 21 cm^2 **ii** 63 cm^3
b **i** 48 cm^2 **ii** 432 cm^3
c **i** 36 m^2 **ii** 324 m^3

2 **a** 432 m^3 **b** 225 m^3 **c** 1332 m^3

3 **a**



b About $3\frac{1}{2}$ hours

4 $V = (1.5 + 3) \div 2 \times 1.7 \times 2 = 7.65$ m^3

5 27 = 3 × 3 × 3, 27 + 37 = 64 = 4^3, 4 − 3 = 1. Hence the side length is 1 small cube longer, hence 2 cm longer

6 **a** **i** 21 cm^3 **ii** 54 cm^2
b **i** 210 cm^3 **ii** 270 cm^2

7 10.2 tonnes

8 She was silly because 26 880 is simply all the dimensions multiplied together. The volume is 672 cm^3.

Exercise 14D

1 **a** **i** 226 cm^3 **ii** 207 cm^2
b **i** 14.9 cm^3 **ii** 61.3 cm^2
c **i** 346 cm^3 **ii** 275 cm^2
d **i** 1060 cm^3 **ii** 636 cm^2

2 **a** 251 cm^3 **b** 445 cm^3
c 2150 cm^3 **d** 0.25 m^3

3 Volume = $\pi \times (0.3)^2 \times 4.2 = 0.378\pi$
Cost = 0.378 × π × £67.50 = £80.16 which is £80 to 2 sf

4 1.23 tonnes

5 665 cm^3

6 Label should be less than 10.5 cm wide so that it fits the can and does not overlap the rim and more than 23.3 cm long to allow an overlap.

7 Volume = $\pi \times 32.5^2 \times 1000 = 331\,831$ cm^3
1 litre = 1000 cm^3
volume = 331 831 ÷ 1000 = 331.831 litres = 332 litres (3 sf)

8 There is no right answer. Students could start with the dimensions of a real can. Often drinks cans are not exactly cylindrical. One possible answer is height of 6.6 cm and diameter of 8 cm.

9 About 127 cm

10 Volume of A = $2^2 \times \pi \times 5 = 20\pi$, volume of B = $1^2 \times \pi \times 21 = 21\pi$, B has the larger volume.

11 146 cm^3

12 78 m^3 (78.3 m^3)

13 327 litres

Review questions

1 **a** 60 cm^3 **b** 104 cm^2

2 38 cm^2

3 Arrange small boxes so that the 8 by 8 part fills one layer in the 32 by 32 box, giving 4×4 boxes. Then this will give 5 layers of 4×4 boxes giving $5 \times 4 \times 4 = 80$ boxes.

4 Area of circular end = $\pi r^2 = \pi \times 4.5^2 = 63.617251$, curved surface area = $\pi dh = \pi \times 9 \times 21 = 593.76101$, total area = $2 \times 63.617251 + 593.76101 = 720.99551 = 721$ cm^2 (nearest whole number)

5 5740 cm^3 (to 3 sf)

6 610 g (2 sf)

7 600 cm^3

8 360 g

Chapter 15 – Algebra: Linear equations

Exercise 15A

1 **a** $x = 4$ **b** $w = 14$
c $x = 5$ **d** $x = 6$
e $z = 24$ **f** $x = 2.5$

2 Any valid equation such as $4x = 16$ or $x + 3 = 7$

3 **a** 6 **b** 9 **c** 7
d 3 **e** 1 **f** 5
g 4 **h** 2 **i** 8

Exercise 15B

1 **a** $x = 2$ **b** $x = 13$
c $x = 13$ **d** $y = 6$
e $x = 9$ **f** $x = 5$
g $t = 18$ **h** $y = 24$
i $k = 18$ **j** $h = 40$

2 2

3 27p

Exercise 15C

1 **a** $x = 56$ **b** $y = 2$ **c** $x = 6$ **d** $y = 3$
e $w = 21$ **f** $x = 72$ **g** $m = 56$ **h** $x = 0$
i $h = -7$ **j** $f = 60$

2 **a** –4 **b** 15

3 Any valid equation such as $3x + 8 = 2$, $6x + 20 = 8$

4 **a** Betsy
b Second line: Amanda subtracts 1 instead of adding 1; fourth line: Amanda subtracts 2 instead of dividing by 2.

Exercise 15D

1 **a** $k = 5$ **b** $h = 28$ **c** $w = 5$
d $x = 35$ **e** $y = 33$ **f** $f = 23$

2 **a** 33 **b** 48

3 **a** $x = 2$ **b** $x = 7$ **c** $y = 2\frac{4}{5}$
d $y = 1$ **e** $x = 11\frac{1}{2}$ **f** $t = \frac{1}{5}$

4 Any valid equations

5 **a** Anwar

b First line: Brody adds 4 instead of multiplying by 5.

Second line: Brody adds 5 instead of multiplying by 5.

Fourth line: Brody subtracts 2 instead of dividing by 2.

6 **a** $\frac{x + 10}{5} = 9.50$

b £37.50

7 **a** $\frac{8}{3}$ **b** Student's own working

Exercise 15E

1 **a** $\frac{1}{2}$ **b** $1\frac{1}{5}$ **c** 2 **d** –2
e –1 **f** –2 **g** –2 **h** –1

2 Any values that work, e.g. $a = 2$, $b = 3$ and $c = 30$.

3 3 cm

4 5

5 Multiplying out the brackets and collecting like terms gives $4x - 24 = 0$ which has the solution $x = 6$.

6 168°

Exercise 15F

1 **a** $x = 2$ **b** $y = 1$ **c** $a = 7$
d $t = 4$ **e** $p = 2$ **f** $k = -1$
g $m = 3$ **h** $s = -2$

2 $3x - 2 = 2x + 5$, $x = 7$

3 **a** $d = 6$ **b** $x = 11$ **c** $y = 1$
d $h = 4$ **e** $b = 9$ **f** $c = 6$

4 **a** $6x + 3 = 6x + 10$; $6x - 6x = 10 - 3$; $0 = 7$, which is obviously false. Both sides have $6x$, which cancels out.
b Multiplying out the brackets gives $12x + 18 = 12x + 18$ which is true for all values of x.

5 $8x + 7 + x + 4 = 11x + 5 - x - 4$, $x = 10$, so the total value of silver coins is £1.90

6 **a** Both are expressions for the length of the rectangle so they must be equal
b 70 cm^2

7 **a** 15
b –1
c $2(n + 3)$, $2(n + 3) - 5$
d $2(n + 3) - 5 = n$, $2n + 6 - 5 = n$, $2n + 1 = n$, $n = -1$

8 $4x + 18 = 3x + 1 + 50$, $x = 33$. Large bottle 1.5 litres, small bottle 1 litre

Review questions

1 **a** $x = 6$ **b** $x = 9$ **c** $x = \frac{2}{3}$
d $x = -5$ **e** $x = 72$

2 3 years

3 $x = -6$

4 **a** $x = -3$ **b** $x = -3$ **c** $x = -3$

5 **c**

6 8

7 Length is 5.5 m, width is 2.5 m and area is 13.75 m^2. Carpet costs £123.75

8 **a** All the sides of a square are equal
b $42\frac{1}{4}$ m^2

9 $x = 10$

10 14 cm

11 **a** B: $\frac{3}{8}x$, C: $\frac{3}{8}x$, D: $\frac{1}{4}x$
b $\frac{3}{8}x = 300$, $x = 800$ cars
c : $\frac{1}{4}x = 500$, $x = 2000$ cars

12 No, as $x + x + 2 + x + 4 + x + 6 = 360$ gives $x = 87°$ so the consecutive numbers (87, 89, 91, 93) are not even but odd

Chapter 16 – Ratio and proportion and rates of change: Percentages and compound measures

Exercise 16A

1 **a** $\frac{2}{25}$ **b** $\frac{7}{20}$ **c** $\frac{9}{10}$

2 **a** 0.27 **b** 0.85 **c** 0.13

3 **a** $\frac{3}{25}$ **b** $\frac{2}{5}$ **c** $\frac{9}{20}$

4 **a** 29% **b** 60% **c** 125%

5 **a** 28% **b** 30% **c** 95%
d 34% **e** 27.5% **f** 87.5%

6 **a** 0.6 **b** 0.075 **c** 0.76
d 0.3125 **e** 0.05 **f** 0.125

7 none

8 **a** 77% **b** 39% **c** 63%

9 27%

10 **a** 50% **b** 20% **c** 80%

11 **a** 87.5%, 75%, 62.5%, 50%, 25%
b 37.5%

12 **a** $\frac{3}{5}$ **b** 0.6 **c** 60% **d** 60%

13 **a** 63%, 83%, 39%, 62%, 77%
b English

14 10%

15 34%, 0.34, $\frac{17}{50}$; 85%, 0.85, $\frac{17}{20}$; 7.5%, 0.075, $\frac{3}{40}$; 45%, 0.45, 9/20; 30%, 0.3, 3/10; 67%, 0.67, 2/3; 84%, 0.84, 21/25; 45%, 0.45, 9/20; 37.5%, 0.375, 3/8

16 $\frac{9}{10}$ or 90%

Exercise 16B

1 **a** 0.88 **b** 0.3 **c** 0.25 **d** 0.08 **e** 1.15

2 **a** 78% **b** 40% **c** 75% **d** 5% **e** 110%

3 **a** £45 **b** £6.30 **c** 128.8 kg
d 1.125 kg **e** 1.08 h **f** 37.8 cm
g £0.12 **h** 2.94 m **i** £7.60
j 33.88 min **k** 136 kg **l** £162

4 **a** 504 **b** 48

5 £2410

6 **a** 86% **b** 215

7 8520

8 287

9 A total of 91% were allocated, leaving 9% or 8100 tickets for celebrities

10 990

11 Mon: 816, Tue: 833, Wed: 850, Thu: 799, Fri: 748

12 **a** £3.25 **b** 2.21 kg **c** £562.80
d £6.51 **e** 42.93 m **f** £24

13 480 cm^3 nitrogen, 120 cm^3 oxygen

14 13

15 £270

16 More in 2014 as it was 3% of a higher amount than in 2013

Exercise 16C

1 **a** 1.1 **b** 1.03 **c** 1.2 **d** 1.07 **e** 1.12

2 **a** 0.92 **b** 0.85 **c** 0.75 **d** 0.91 **e** 0.88

3 **a** 391 kg **b** 824.1 cm **c** 253.5 g
d £143.50 **e** 736 m **f** £30.24

4 **a** 731 m **b** 83.52 g **c** 360 cm
d 117 min **e** 81.7 kg **f** £37.70

5 448

6 No, as the total is £101. She will save £20.20, which is less than the £25 it would cost to join the club.

7 A 7% pay rise. 7% is £1925 per year; £150 per month is £ 1800 per year.

8 **a** £6.13 (£6.125)
b $x \times 0.025$
c $y \div 1.175 \times 1.2$

9 Offer B is the better offer:

Offer A gives 360 grams for £1.40, i.e. 0.389 pence per gram.

Offer B gives 300 grams for £1.12, i.e 0.373 pence per gram.

Or: Offer A is 360 for 1.40 = 2.6 g/p, offer B is 300 for 1.12 = 2.7 g/p.

10 **c** Both the same as $1.05 \times 1.03 = 1.03 \times 1.05$

11 **a** Shop A as $1.04 \times 1.04 = 1.0816$, so an 8.16% increase.

12 £425.25

13 $1.10 \times 0.9 = 0.99$ (99%)

Exercise 16D

1 **a** 25% **b** 60.6% **c** 46.3% **d** 12.5%
e 41.7% **f** 60% **g** 20.8% **h** 10%
i 1.9% **j** 8.3% **k** 45.5% **l** 10.5%

2 32%

3 6.5%

4 33.7%

5 **a** 49.2% **b** 64.5% **c** 10.6%

6 4.9%

7 90.5%

8 Stacey had the greater percentage increase.

Stacey: $(20 - 14) \times 100 \div 14 = 42.9\%$

Calum: $(17 - 12) \times 100 \div 12 = 41.7\%$

9 Yes, as 38 out of 46 is over 80% (82.6%)

10 Let $z = 100$. $y = 75$, $x = 0.6 \times 75 = 45$, so x is 45% of z

11 Let z be 100, $x = 60$. If x is 75% of y, $y = 80$, so y is 80% of z.

12 30% of 4800 = 1440. $1.2 \times 4800 = 5760$. 70% of 5760 = 4032. 4032 − 1440 = 2592.

2592 ÷ 1440 = 1.8, so the increase in numbers owning a mobile phone is 180%.

13 31 ÷ 26 = 1.19 which is a 19% increase. 31% is 5% more of the total votes cast than 26%

Exercise 16E

1 **a** £260 **b** £307.50 **c** £323.75 **d** £289

2 **a** £7.50 **b** £9.05 **c** £5.80 **d** £10.75

3 **a** 38 h **b** $41\frac{1}{2}$ h **c** 35 h **d** 40 h

4 **a** Fewer hours **b** More pay per week

5 **a** £540 **b** £702

6 £6.90

7 375 − 330 = 45, 45 ÷ 6 = £7.50.
$(375 - 12 \times 7.50) \div 7.50 = 38$ hours

8 £$1\frac{1}{4}x$ or £$\frac{5x}{4}$

9 Pay is £442.50 tax is £88.50, NI is 442.50 − 88.50 − 327.45 = 26.55, 26.55 ÷ 442.5 = 0.06, so the NI rate is 6%

10 407 factorises to 1×407 or 11×37, so Antoine works 37 hours a week at £11 per hour.

Exercise 16F

1 0.75 g/cm^3

2 4 Pa

3 8.33 g/cm^3

4 2.5 N

5 32 g

6 5 m^2

7 120 cm^3

8 156.8 g

9 The 30 cm × 20 cm face

10 The handles exert greater pressure as there is less area in contact with your hands.

11 To reduce the pressure they exert, which helps prevent them sinking into soft sand.

12 **a** 19.3 kg
b 19.3 kg. The scale shows the mass, which is the same
c (a) 9650 Pa, (b) 38 600 Pa

13 The first statue is a fake

14 The 400 g piece has a bigger volume by 1 cm^3

15 If the soles of the shoes and slippers are the same size he will exert the same pressure

16 They are smooth and narrow so cut through snow. They have a big area so they don't push into the snow

17 Lay down and slide. Less pressure is exerted on the ice

18 Each wheel takes some of the mass so the pressure on the road ora weak bridge is less.

19 A, D, C, B. If they have the same mass, the one with the largest area will exert the least pressure.

20 **a** Same, as the area is the same
b Cone, as all the mass is concentrated on the point which has negligible area.

21 0.34 m^3

22 1 m × 0.8 m × 0.5 m

23 **a** True **b** False **c** False

Review questions

1 £144.00

2 £208.25

3 £317.55

4 £8.90

5 France. Their speed limit is equivalent to 75 mph.

6 **a** 1.73 ÷ 0.04 = 43.25 so 43 horses, 2.64 ÷ 0.065 = 40.61 so 40 cattle and 0.95 ÷ 0.01 = 95. Total 43 + 40 + 95 = 178 animals.

b Horses in field A = 43, Sheep in field B = 2.64 ÷ 0.01 = 264, Cattle in field C, 0.95 ÷ 0.065 = 14.61, so 14 cattle. Total 43 + 264 + 14 = 321 animals

7 25%

8 £8.98

9 8 g/cm^3

10 0.88

11 240 Pa

12 Shop B

13 **a** £9.00
b £13.20

14 £322

15 Yes, the council exceeded the promise which was for a maximum of 1200 unemployed people.

16 The 20% reduction will reduce the prices by more than the original VAT because they will be reducing prices by 20% of a bigger amount.

17 **a** 150 men, 100 women **b** 12%

18 13%

Chapter 17 – Ratio and proportion and rates of change: Percentages and variation

Exercise 17A

1 **a** £400 **b** £90 **c** £480 **d** £12.80

2 **a** 10.5 g **b** 11.03 g **c** 12.16 g **d** 14.07 g

3 **a** 2550 **b** 2168 **c** 1331

4 **a** £6800 **b** £5440 **c** £3481.60

5 **a** **i** 51 980 **ii** 84 752 **iii** 138 186

6 **a** 8 years **b** 12 years

7 12 years

8 **a** £14 272.27 **b** 20 years

9 **a** **i** 1.9 million litres
ii 1.6 million litres
iii 1.2 million litres
b 10th August

10 **a** 21 years **b** 21 years

Exercise 17B

1 **a** 800 g **b** 250 m **c** 60 cm
d £3075 **e** £200 **f** £400

2 80

3 T-shirt £8.40, Tights £1.20, Shorts £5.20, Sweater £10.75, Trainers £24.80, Boots £32.40

4 £833.33

5 £300

6 240

7 £350

8 4750 blue bottles

9 £22

10 **a** £1600
b Yes. With 10% cut each year he earns £1440 × 12 + £1296 × 12 = £17 280 + £15 552 = £32 832. With an immediate 14% cut he earns £1376 × 24 = £33 024, so he made the correct decision.

Exercise 17C

1 **a** £3.40 **b** 17 **c** 12

2 **a** 100 cm **b** 20 inches
c 82 inches is longer

3 **a** 150 **b** 6

4 **a** 22.5 **b** 12

5 **a** 175 miles **b** 8 hours

6 **a** £66.50 **b** 175 kg

7 **a** 44 **b** 84 m^2

8 **a** 50

9 17 minutes 30 seconds

10 **a** l = 4.55G **b** No, 6 gallons is 27.3 litres
c £5.55 **d** 11.9 miles per litre

Exercise 17D

1 **a** 12.5 watts **b** 2.5 m

2 8 hours

3 **a** 5 cm **b** 10 cm

4 **a** 625 cm^3 **b** 0.625 bar

5 **a** 6 **b** 12.5

6 **a** 27 min 31.2 sec **b** 34.4 km/h

7 **a** £150 **b** 36

8 **a** 3 °C **b** 12 km

9

x	8	24	16
y	1	$\frac{1}{3}$	$\frac{1}{2}$

10 **a** 112.5 miles **b** 7.5 miles

Review questions

1 £2120

2 £19 728

3 £112.50

4 **a** 37p **b** 3 hours
c 20 minutes **d** 36 kg
e 1 hour **f** 16 hours
g Three days **h** 21 minutes
i 2 hours **j** 600 p.m.

5 She should choose A

6 £8357.35

7 **a** 3 days **b** 2 days

8 £8.60

9 £38.50

10 **a** 6 **b** 100

11 **a** 4 **b** 32

12 £375

13 **a** 66 **b** 1.75

14 **a** 8 **b** 0.5

15

x	25	**50**	400
y	10	20	**160**

16 **a** 125 **b** $\frac{1}{2}$

17 $P = k_1Q$, $k_1 = \frac{P}{Q} = \frac{4}{10} = 0.4$. So $P = 0.4Q$.
$QR = k_2 = 2 \times 8$. So $QR = 16$. When $R = 0.8$, then $Q = 16 \div 0.8 = 20$.
So $P = 0.4Q = 0.4 \times 20 = 8$

Chapter 18 – Statistics: Representation and interpretation

Exercise 18A

1 **a** secondary data
b primary data
c primary or secondary data
d primary or secondary data
e primary data
f primary data

2 Plan the data collection: Choose a random sample of 30 boys and 30 girls from Year 11.

Collect the data: Ask each student to spell the same 10 words. This will avoid bias. Pick words that are often misspelt, eg accommodation, necessary, separate

Choose the best way to process and represent the data: Calculate the mean number and range for the number of correct spellings for the boys and for the girls. Draw a dual bar chart to illustrate the data.

Interpret the data and make conclusions: Compare the mean and range to arrive at a conclusion. Is there a clear conclusion or do you need to change any of the 10 words or take a larger sample?

3 Plan the data collection: Choose a random sample of 20 boys and 20 girls from Year 11.

Collect the data: Ask each student, on average, how many hours of sport they play and how many hours of TV they watch each week.

Choose the best way to process and represent the data: Calculate the mean number of hours for the number of hours playing sport and the number of hours watching TV. Draw dual bar chart illustrate the data.

Interpret the data and make conclusions: Compare the means and arrive at a conclusion. Is there a clear conclusion or do you need to take a larger sample?

4 One more girl

5 **a** 3:1

b It has been assumed that the ratio of males to females in the sample is the same as the ratio of males to females attending the match.

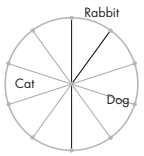
6 28.5%

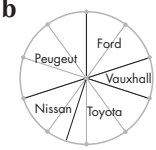
7 **a**

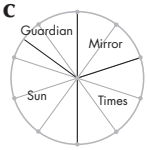
	Male	Female	Total
Full time	13	7	20
Part time	4	6	10
Total number of employees			30

b This is not true as the values have to be rounded to the nearest person.

Exercise 18B

1 **a** 

b 

c 

2 Pie charts with the following angles:
a 36°, 90°, 126°, 81°, 27°
b 90°, 108°, 60°, 78°, 24°
c 168°, 52°, 100°, 40°

3 **a** Pictogram with suitable key
b Bar chart correctly labelled
c Vertical line chart correctly labelled
d Pie chart with these angles: 60°, 165°, 45°, 15°, 75° and correctly labelled
e Vertical line chart. It shows the frequencies, the easiest one to draw and comparisons can be made.

4 **a** 36
b Pie charts with these angles: 50°, 50°, 80°, 60°, 60°, 40°, 20° and correctly labelled
c Bar chart correctly labelled
d Bar chart, because it is easier to make comparisons

5 **a** Pie charts with these angles: 124°, 132°, 76°, 28° and correctly labelled
b Split of parties can be seen at a glance

6 **a** Pie charts with these angles:
Strings: 36°, 118°, 126°, 72°, 8°
Brass: 82°, 118°, 98°, 39°, 23°
b Overall, the strings candidates did better, as a smaller proportion scored the lowest grade. a higher proportion of brass candidates scored the highest grade.

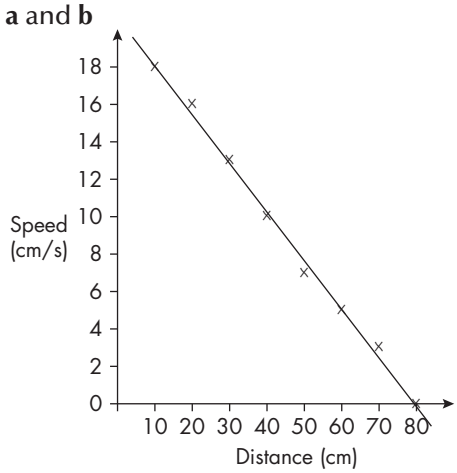
7 The missing angle is 40°, so P(Don't know) $= \frac{40}{360} = \frac{1}{9}$

8 **a** 55° **b** 22 **c** $33\frac{1}{3}\%$

Exercise 18C

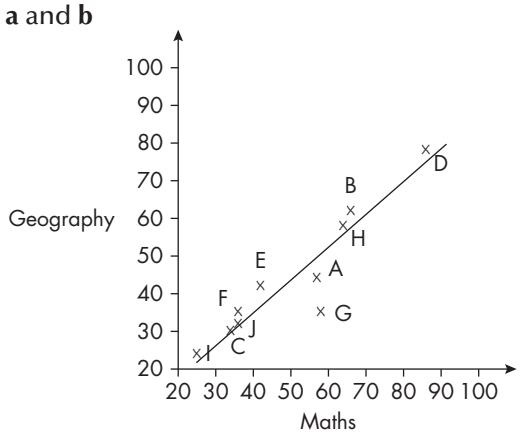
1 **a** **i** Good positive correlation
ii Time increases with the number of press-ups
b **i** Strong negative correlation
ii You complete a crossword quicker as you get older
c **i** No correlation
ii Speed of cars on M1 is not related to the temperature
d **i** Weak positive correlation
ii Older people generally have more money saved in the bank

2 **a** and **b**



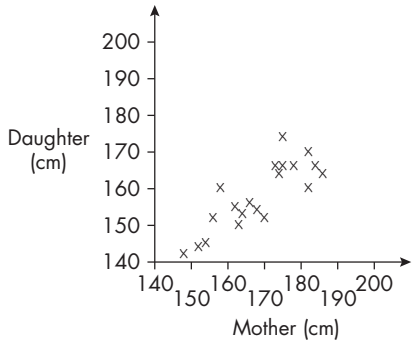
c about 19 cm/s
d about 34 cm

3 **a** and **b**



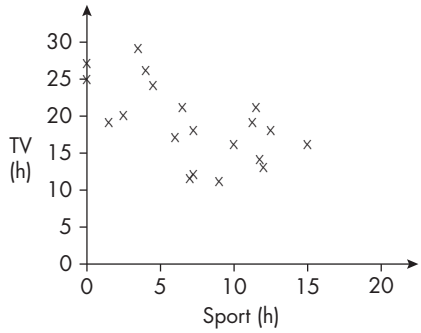
c Greta
d about 70
e about 70

4 **a**



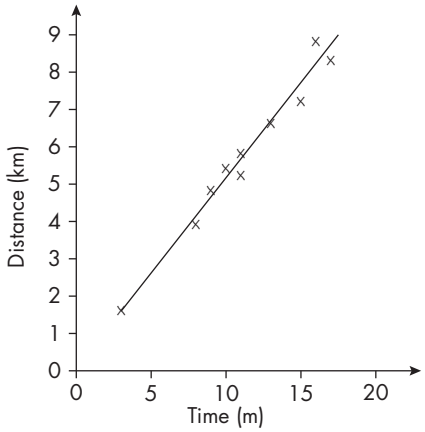
b Yes, as good positive correlation

5 **a**



b Little correlation, so cannot draw a line of best fit

6 **a** and **b**



c about 2.4 km
d about 8 minutes
e you cannot extrapolate values from a scatter diagram or the data may change for longer journeys

7 about 23 mph

8 Points showing a line of best fit sloping down from top left to bottom right

Exercise 18D

1 **a** **i** $30 < x \leq 40$ **ii** 29.5 **iii** $30 < x \leq 40$
b **i** $0 < y \leq 100$ **ii** 158.3 **iii** $100 < y \leq 200$
c **i** $5 < z \leq 10$ **ii** 9.43 **iii** $5 < z \leq 10$
d **i** 7–9 **ii** 8.41 **iii** 7–9

2 **a** $100 < m \leq 120$ **b** 10.86 kg **c** 108.6 g

3 **a** $175 < h \leq 200$
b 31%
c 193.3 hours
d No, the mean was under 200.

4 **a** Yes, average distance is 11.7 miles per day.
b Because shorter runs will be done faster which will affect the average
c Yes, because the shortest could be 1 mile, the longest 25 miles.

5 24

6 Soundbuy; average increases are Soundbuy 17.7p, Springfields 18.7p, Setco 18.2p

7 **a** 160 **b** 52.6 minutes
c Modal group **d** 65%

8 The first 5 and the 10 are the wrong way round.

9 Find the midpoint of each group, multiply that by the frequency and add those products. Divide that total by the total frequency.

Review questions

1 **a** 18 **b** 80°

2 **a** **i** Diagram C **ii** Diagram A **iii** Diagram B
b Diagram A: strong negative correlation, diagram B: no correlation, diagram C: strong positive correlation

3 **a/b** Time on horizontal axis from 0 to 20 and Distance (km) on vertical axis from 0 to 10 with the following points plotted: (3, 1.7) (17, 8.3) (11, 5.1) (13, 6.7) (9, 4.7) (15, 7.3) (8, 3.8) (11, 5.7) (16, 8.7) (10, 5.3) and with line of best fit drawn.
c/d Answers depend on student's plotting

4 **a** $10 < t \leq 20$ **b** $10 < t \leq 20$ **c** 19 minutes

5 **a** Because over half the students have more than £10 pocket money, so the mean must be more than £10
b £11.00

6 17.15 mph

Chapter 19 – Geometry and measures: Constructions and loci

Exercise 19A

1 **a** BC = 2.9 cm, ∠B = 53°, ∠C = 92°
b EF = 7.4 cm, ED = 6.8 cm, ∠E = 50°
c ∠G = 105°, ∠H = 29°, ∠I = 46°
d ∠J = 48°, ∠L = 32°, JK = 4.3 cm
e ∠N = 55°, ON = OM = 7.0 cm
f ∠P = 51°, ∠R = 39°, QP = 5.7 cm

2 **a** Students can check one another's triangles.
b ∠ABC = 44°, ∠BCA = 79°, ∠CAB = 57°

3 **a** 5.9 cm
b 18.8 cm^2

4 BC = 2.6 cm, 7.8 cm

5 **a** 4.3
b 10.8 cm^2

6 **a** 4.3 cm
b 34.4 cm^2

7 An equilateral triangle of side 4 cm

8 Even with all three angles, you need to know at least one length

Exercise 19B

1–9 Practical work; check students' constructions

10 The centre of the circle

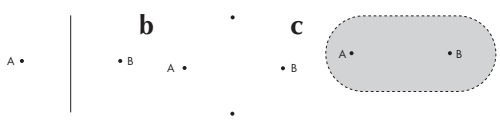
11 Start with a base line AB; then construct a perpendicular to the line from point A. At point B, construct an angle of 60°. Ensure that the line for this 60° angle crosses the perpendicular line; where they meet will be the final point C.

12–14 Practical work; check students' constructions

Exercise 19C

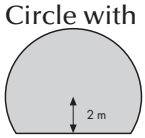
1 Circle with radius:
a 2 cm **b** 4 cm **c** 5 cm

2 **a** **b** **c**



3 **a** Circle with radius 4 m

b



4 **a**

b

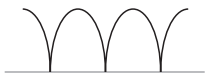
c

d

e

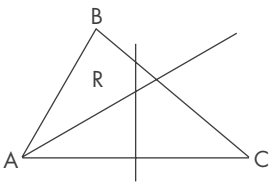
f

5



6

7 Construct the bisector of angle BAC and the perpendicular bisector of the line AC.



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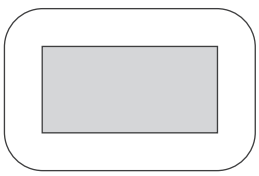
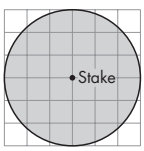


9 Start with a base line, AB, 3cm long. At point A, draw a few points all 3cm away from a towards the upper right side. Lightly join these dots with an arc. You can now find the point that is 3cm away from point B and draw the equilateral triangle.

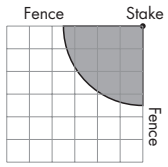
10 Gary is correct about the triangle inside, but not a triangle outside as there will be three straight lines, parallel to each side of the triangle, then these straight lines will be joined with arcs centred on the vertex between the lines.

Exercise 19D

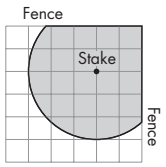
1



2 **a**



b



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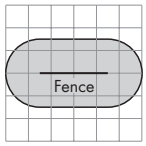
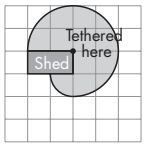


4



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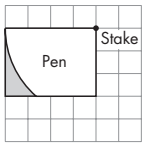

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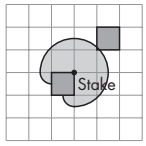


7 **a** Sketch should show a circle of radius 6 cm around London and one of radius 4 cm around Glasgow.
b No
c Yes

8 **a** Yes
b Sketch should show a circle of radius 4 cm around Leeds and one of radius 4 cm around Exeter. The area where they overlap should be shaded.

9 **a** This is the perpendicular bisector of the line from York to Birmingham. It should pass just below Manchester and just through the top of Norwich.
b Sketch should show a circle of radius 7 cm around Glasgow and one of radius 5 cm around London.
c The transmitter can be built anywhere on line constructed in part **a** that is within the area where the circles shown in part **b** overlap.

10 Sketch should show two circles around Birmingham, one of radius 3 cm and one of radius 5 cm. The area of good reception is the area between the two circles.

11 Sketch should show a circle of radius 6 cm around Glasgow, two circles around York, one of radius 4 cm and one of radius 6 cm and a circle around London of radius 8 cm. The small area in the Irish Sea that is between the two circles around York and inside both the circle around Glasgow and the circle around London is where the boat can be.

12 Sketch should show two circles around Newcastle upon Tyne, one of radius 4 cm and one of radius 6 cm, and two circles around Bristol, one of radius 3 cm and one of radius 5 cm. The area that is between both pairs of circles is the area that should be shaded.

13 Sketch should show the perpendicular bisector of the line running from Newcastle upon Tyne to Manchester and that of the line running from Sheffield to Norwich. Where the lines cross is where the oil rig is located.

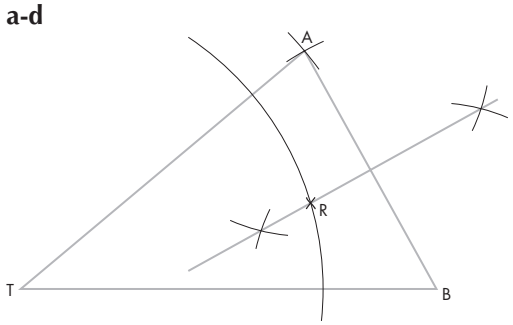
14 Sketch should show the perpendicular bisector of the line running from Glasgow to Norwich and that of the line running from Norwich to Exeter. Where the lines cross is where Fred's house is.

15 Leeds

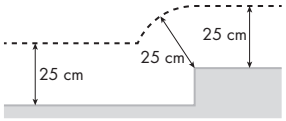
16 No, if you accurately draw a diagram showing the path of each boat, you will find the boat leaving from point B meets the path of the other boat in much shorter time as it's a smaller distance than from a to the cross over point.

17 On a map, draw a straight line from Newcastle to Bristol, construct the line bisector, then the search will be anywhere on the sea along that line.

18 a-d

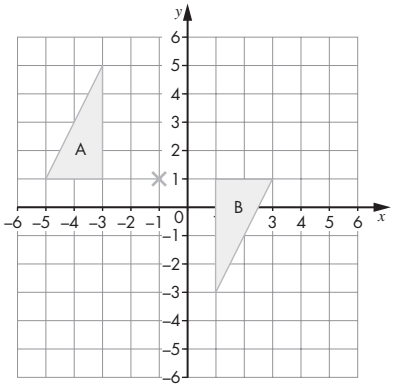


19



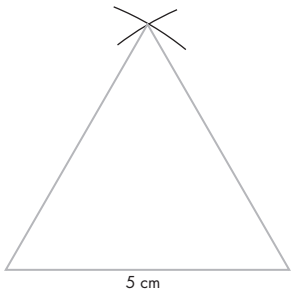
Review questions

1 a - b



c You should have measured the error of the angle size and converted that to a percentage error.

2



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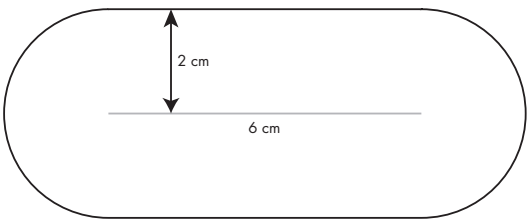

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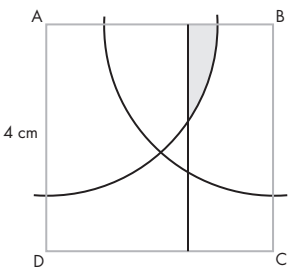
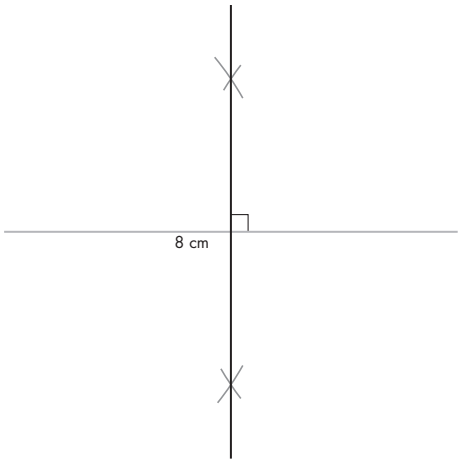
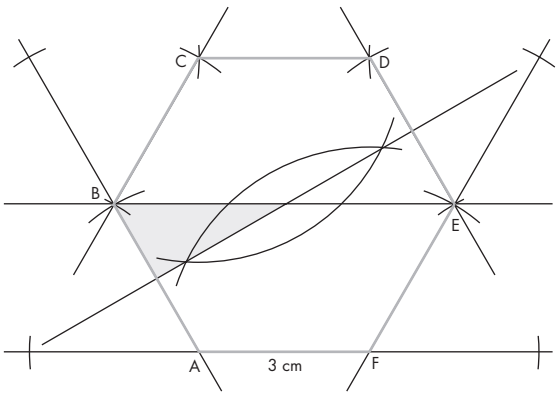


5 **a**



b ….. an equal distance from a and B, or equidistant from a and B.

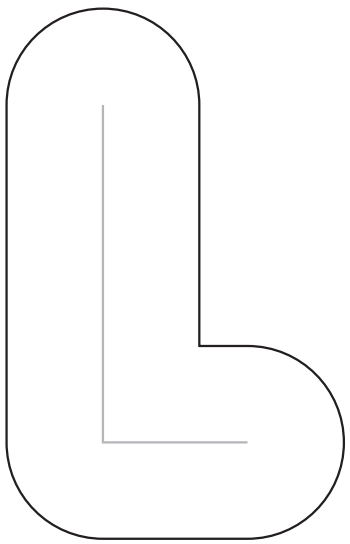
6



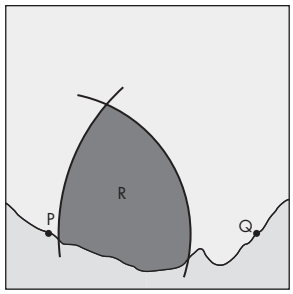
7 **a** Student's construction
b 20 cm^2

8 **a** Student's construction
b 24 cm^2

9 Oliver is incorrect as the correct shape is:



10



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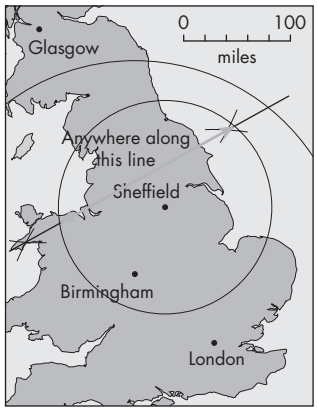


Chapter 20 – Geometry and measures: Curved shapes and pyramids

Exercise 20A

1 **a** **i** 5.86 cm **ii** 22.3 cm^2
b **i** 5.24 cm **ii** 13.1 cm^2
c **i** 10.99 cm **ii** 19.24 cm^2
d **i** 16.34 cm **ii** 98.02 cm^2

2 **a** **i** 6.8 cm **ii** 18.8 cm
b **i** 7.9 cm **ii** 25.9 cm
c **i** 8.4 cm **ii** 16.4 cm
d **i** 17.5 cm **ii** 37.5 cm
e **i** 3.3 cm **ii** 20.3 cm

3 Arc length = 2 π, area of sector = 6π

4 **a** 270° **b** 51.8 cm **c** 73.8 cm

5 area of sector = $\frac{1}{4} \times \pi \times 8^2 = 16\,\pi$, area of circle = $\pi \times 4^2 = 16\,\pi$

6 **a** 28.6° **b** 66.8° **c** 54.9°

7 Area B = 107 cm^2, area B = 173 cm^2. Half of B is 86.5 which is less than 107, so Gabby is wrong, the area of a is more than half the area of B.

8 43.6 cm

9 **a** 66.8° **b** 10 cm^2

Exercise 20B

1 **a** 56 cm^3 **b** 168 cm^3
c 1040 cm^3 **d** 84 cm^3
e 160 cm^3

2 $\frac{1}{3}$ base area $\times h = \frac{1}{3} \times 9 \times 9 \times 10 = 270$ cm^3

3 **a** 73.3 m^3 **b** 45 m^3 **c** 3250 cm^3

4 208 g

5 1.5 g/cm^3

6 $\frac{1}{3} \times 6.4 \times 6.4 \times H = 81.3$
so $h = \frac{3 \times 81.3}{6.4 \times 6.4} = 5.59 = 6.0$ cm (2 sf)

7 14.4 cm

Exercise 20C

1 **a** 72 cm^2 **b** 172.5 cm^2

2 Area of base = $9^2 = 81$ cm^2, area of one triangle = $\frac{1}{2} \times 9 \times 10 = 45$ cm^2, total surface area = $(4 \times 45) + 81 = 261$ cm^2.

3 Total area = 32.46 m^2, so 3 tins are needed

4 Surface areas are a = 1200 cm^2, b = 576 cm^2, c = 864 cm^2, d = 526.5 cm^2 , Volumes are a = 960 cm^3, b = 512 cm^3, c = 960 cm^3, d = 540 cm^3. This does not show that the larger the volume, the larger the surface area, so Andrew is incorrect.

5 Yes, the area is 218.7 m^2 and will cost £2515.05 to retile.

Exercise 20D

1 **a** 17.5 cm^2 b 103 cm^2 c 94.2 cm^2

2 c Radii are 1.1 cm, 4.7 cm, 5 cm

3

Sector	Area of sector	Radius of cone, *r*	Slant height, *l*	$\pi \times r \times l$
a	17.5 cm^2	1.1 cm	5 cm	17.3 cm^2
b	103 cm^2	4.7 cm	7 cm	103 cm^2
c	94.2 cm^2	5 cm	6 cm	94.2 cm^2

4 **a** They are the same
b Curved surface area = πrl

Exercise 20E

1 **a** **i** 151 cm^3 **ii** 176 cm^2
b **i** 75 cm^3 **ii** 108 cm^2
c **i** 872 cm^3 **ii** 550 cm^2
d **i** 3560 cm^3 **ii** 1426 cm^2
e **i** 314 cm^3 **ii** 283 cm^2
f **i** 1018 cm^3 **ii** 679 cm^2

2 935 g

3 $\pi rl + \pi r^2 = 3\pi l + 9\pi = 75.4$
$3\pi l = 75.4 - 28.27 = 47.13$
$l = \frac{47.3}{3\pi} = 5$ cm

4 **a** 816π cm^3 **b** 720π mm^3

5 The total surface area, to 3sf, is 75.4 cm^2, so Mandy is correct if the figure was rounded to 1 sf. Neil is incorrect as he has cut off his final answer instead of rounding it.

Exercise 20F

1 **a** 113 cm^3 **b** 905 cm^3 **c** 4189 cm^3

2 **a** 113 cm^2 **b** 314 cm^2 **c** 616 cm^2

3 Volume = 4500π cm^3, surface area = 900π cm^2

4 **a** 1960 cm^2 (to 3 sf)
b 7444 cm^3 (to nearest unit)

5 125 cm

6 6232

Review questions

1 **a** 43.9 cm **b** 112 cm^2

2 49 000 m^3

3 Volume of top cube = $30^3 = 27\,000$, volume of bottom pyramid = $\frac{1}{3} \times 30^2 \times 10 = 3000$ cm^3, total volume = top + bottom = 27 000 + 3 000 = 30 000cm^3

4 Bottle will hold = $\pi r^2 h = \pi \times 4.5^2 \times 25 = 1590.4$ cm^3. One glass will hold $\frac{1}{3}\pi r^2 h = \frac{1}{3} \times \pi \times 2.5^2 \times 6 = 39.27$ cm^3. So $1590.4 \div 39.27 = 40.5$ glasses can be filled. David is wrong as just over 40 glasses will be filled.

5 One bearing has volume = $\frac{4}{3}\pi r^3 = \frac{4}{3} \times \pi \times 0.75^3 = 1.767$ cm^3, so one ball bearing has a mass of $1.767 \times 7.3 = 12.900$ g. 20 ball bearings have a mass of 258.00 g or 258 g (nearest whole number).

6 0.5 cm

7 34.6 m^2

8 **a** A is largest at 1018 cm^2
b B is smallest at 1003 cm^2
Surface area of C is 1016 cm^2.

9 0.825 m^3

10 7.25 cm

11 Neither of them is correct as the surface area of the sphere is 1257 cm^2, surface area of the cube is 1262 cm^2 but the surface area of the cone is 1282 cm^2, so the cone has the greatest surface area

12 91.5 cm

Chapter 21 – Algebra: Number and sequences

Exercise 21A

1 **a** $11111 \times 11111 = 123454321$,
$111111 \times 111111 = 12345654321$
b $99999 \times 99999 = 9999800001$,
$999999 \times 999999 = 999998000001$

2 **a** $7 \times 8 = 7^2 + 7$, $8 \times 9 = 8^2 + 8$
b $50 \times 51 = 2550$, $60 \times 61 = 3660$

3 **a** $1 + 2 + 3 + 4 + 5 + 4 + 3 + 2 + 1 = 25 = 5^2$,
$1 + 2 + 3 + 4 + 5 + 6 + 5 + 4 + 3 + 2 + 1 = 36 = 6^2$
b $21 + 23 + 25 + 27 + 29 = 125 = 5^3$,
$31 + 33 + 35 + 37 + 39 + 41 = 216 = 6^3$

4 **a** $1 + 6 + 15 + 20 + 15 + 6 + 1 = 64$,
$1 + 7 + 21 + 35 + 35 + 21 + 7 + 1 = 128$
b $12\,345\,679 \times 45 = 555\,555\,555$,
$12\,345\,679 \times 54 = 666\,666\,666$

5 **a** $1^3 + 2^3 + 3^3 + 4^3 = (1 + 2 + 3 + 4)^2 = 100$,
$1^3 + 2^3 + 3^3 + 4^3 + 5^3 = (1 + 2 + 3 + 4 + 5)^2 = 225$
b $36^2 + 37^2 + 38^2 + 39^2 + 40^2 = 41^2 + 42^2 + 43^2 + 44^2$,
$55^2 + 56^2 + 57^2 + 58^2 + 59^2 + 60^2 = 61^2 + 62^2 + 63^2 + 64^2 + 65^2$

6 **a** 12345678987654321
b 999999998000000001
c $12^2 + 12$
d 8190
e $81 = 9^2$
f $512 = 8^3$
g 512
h 999 999 999
i $(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9)^2 = 2025$

7 $1 + 500 = 501$, $2 + 499 = 501$, …. $250 + 251 = 501$, $250 \times 501 = 125250$

Exercise 21B

1 **a** 21, 34: add previous 2 terms
b 49, 64: next square number
c 47, 76: add previous 2 terms

2 15, 21, 28, 36

3 61, 91, 127

4 6, 11, 16, 21, 26 …

5 $\frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{5}{7}, \frac{3}{4}$

6 **a** 6, 10, 15, 21, 28
b It is the sums of the natural numbers, or the numbers in Pascal's triangle or the triangular numbers.

7 **a** 2, 6, 24, 720 **b** 69!

8 *X*. There are 351 ($1 + 2 + ... + 25 + 26$) letters from a to Z. $3 \times 351 = 1053$. $1053 - 26 = 1027$, $1027 - 25 = 1002$, so, as *Z* and *Y* are eliminated, the 1000th letter must be *X*.

9 364: Daily totals are 1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78 (these are the triangular numbers). Cumulative totals are: 1, 4, 10, 20, 35, 56, 84, 120, 165, 220, 286, 364.

10 29 and 41

11 No, because in the first sequence, the terms are always one less than in the 2nd sequence

12 $4n - 2 = 3n + 7$ rearranges as $4n - 3n = 7 + 2$, so $n = 9$

Exercise 21C

1 **a** 13, 15, $2n + 1$ **b** 25, 29, $4n + 1$
c 33, 38, $5n + 3$ **d** 32, 38, $6n - 4$
e 20, 23, $3n + 2$ **f** 37, 44, $7n - 5$
g 21, 25, $4n - 3$ **h** 23, 27, $4n - 1$
i 17, 20, $3n - 1$ **j** –8, –18, –10n + 42
k 4, 0, –4n + 24 **l** –1, –6, –5n + 29

2 **a** $3n + 1$, 151 **b** $2n + 5$, 105
c $5n - 2$, 248 **d** $4n - 3$, 197
e $8n - 6$, 394 **f** $n + 4$, 54
g $5n + 1$, 251 **h** $8n - 5$, 395
i $3n - 2$, 148 **j** $3n + 18$, 168
k –7n + 47, –303 **l** –8n + 41, –359

3 **a** 33rd **b** 30th
c 100th = 499

4 **a** **i** $4n+1$ **ii** 401 **iii** 101, 25th
b **i** $2n+1$ **ii** 201 **iii** 99 or 101, 49th and 50th
c **i** $3n+1$ **ii** 301 **iii** 100, 33rd
d **i** $2n+6$ **ii** 206 **iii** 100, 47th
e **i** $4n+5$ **ii** 405 **iii** 101, 24th
f **i** $5n+1$ **ii** 501 **iii** 101, 20th
g **i** $3n-3$ **ii** 297 **iii** 99, 34th
h **i** $6n-4$ **ii** 596 **iii** 98, 17th
i **i** $-8n+205$ **ii** -595 **iii** 101, 13th
j **i** $-2n+227$ **ii** 27 **iii** 99 or 101, 63rd and 64th

5 **a** $\frac{2n+1}{3n+1}$
b Getting closer to $\frac{2}{3}$ $(0.\dot{6})$
c **i** 0.667 774 (6dp) **ii** 0.666 778 (6dp)
d 0.666 678 (6dp), 0.666 667 (6dp)

6 **a** $\frac{4n-1}{5n+1}$
b Getting closer to $\frac{4}{5}$ (0.8)
c **i** 0.796 407 (6dp) **ii** 0.799 640 (6dp)
d 0.799 964 (6dp), 0.799 9996 (7dp)

7 **a** £305 **b** £600 **c** 3 **d** 5

8 **a** $\frac{3}{4}$, $\frac{5}{7}$, $\frac{7}{10}$
b **i** 0.666 666 777 8 **ii** $\frac{2}{3}$
c For n, $\frac{2n+1}{3n+1} \approx \frac{2n}{3n} = \frac{2}{3}$

9 **a** $8n+2$ **b** $8n+1$ **c** $8n$ **d** £8

10 **a** Sequence goes up in 2s; first term is $2+29$
b $n+108$
c Because it ends up as $2n \div n$
d 79th

11 The expressions show that the terms in the sequence 2n – 1 are always one less than those in the 2n sequence, so they will have no numbers in common.

12 Difference is $19-10=9$. $9 \div 3 = 3$ so a = 3. $3 \times 5 + b = 10$, $b = -5$

Exercise 21D

1 **a** Even,

+	Odd	Even
Odd	Even	Odd
Even	Odd	Even

b Odd,

×	Odd	Even
Odd	Odd	Even
Even	Even	Even

2 **a** $1+3+5+7=16=4^2$, $1+3+5+7+9=25=5^2$
b **i** 100 **ii** 56

3 **a** 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144
b Because odd + odd = even, odd plus even = odd and even + odd = odd
c **i** a+2b, 2a+3b, 3a+5b, 5a+8b, 8a+13b
ii The coefficients of both a and b each form a Fibonacci sequence

4 **a** Even **b** Odd **c** Odd
d Odd **e** Odd **f** Odd
g Even **h** Odd **i** Odd

5 **a** Odd or even **b** Odd or even
c Odd or even **d** Odd
e Odd or even **f** Even

6 **a** **i** Odd **ii** Even **iii** Even
b Any valid answer, e.g. $x(y+z)$

7 **a** 64, 128, 256, 512, 1024
b **i** 2^n-1 **ii** 2^n+1 **iii** 3×2^n

8 **a** The number of zeros equals the power.
b 6
c **i** 10^n-1 **ii** 2×10^n

9 **a** 125, 216
b $1+8=9$, $1+8+27=36$, $1+8+27+64=100$… the answers are square numbers

10 **a** 28, 36, 45, 55, 66
b **i** 210 **ii** 5050
c You get the square numbers.

11 **a** **i** If n is odd, $n+1$ is even.
If n is even, $n+1$ is odd.
Even times odd is always even.
ii $2n$ must be even, so $2n+1$ must be odd.
b Odd
Odd
Even
Even
Odd
c $(2n+1)^2 = 4n^2+4n+1$
or $(2n)^2 = 4n^2$
$4n^2+4n$ is even so adding 1 makes it odd
$4n^2$ is $2 \times 2n^2$ which is even

12 11th triangular number is 66, 18th triangular number is 171

13 **a** 36, 49, 64, 81, 100
b **i** n^2+1 **ii** $2n^2$ **iii** n^2-1

14 **a** **i** 4, 16, 64, 256, 1024
ii 7, 49, 343, 2401, 16807
iii 5, 25, 125, 625, 3125
iv 9, 81, 729, 6561, 59 049
v 8, 64, 512, 4096, 32 768
b **i** 60 **ii** 40

15 2. Only 2 plus a prime will give another prime, but not always. This is because after 2, all primes are odd, and odd + odd is always an even number. No even numbers after 2 are prime.

16 **a** There are many answers, $5 + 31 = 36$, $7 + 29 = 36$, $2 + 47 = 49$ etc.
b There are many answers, $49 - 36 = 13$, $81 - 64 = 17$

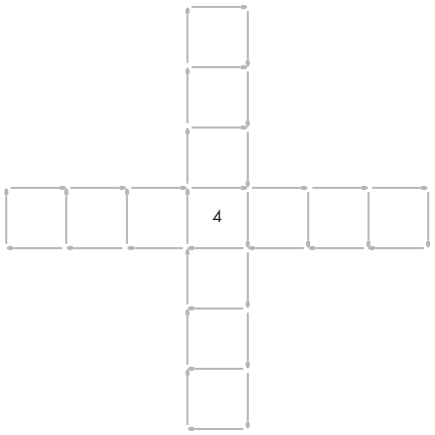
17 **a** 3, 6, 11, 18, 27
b 102
c 20th
d No. Because the terms of the sequence n^2 alternate between even and odd, and adding 2 to each term will not change this even-odd pattern.

Exercise 21E

1 **a**

b $4n - 3$
c 97
d 50th diagram

2 **a**



b $2n + 1$ **c** 121 **d** 49th set

3 **a** 18 **b** $4n + 2$ **c** 12

4 **a** 5 cm, 8 cm, 11 cm, 14 cm
b **i** 20 cm **ii** $(3n + 2)$ cm **iii** 152 cm
c 332

5 **a** **i** 24 **ii** $5n - 1$ **iii** 224
b 25

6 **a** **i** 20 **ii** 162
b 79.8 km

7 **a** **i** 14 **ii** $3n + 2$ **iii** 41
b 66

8 **a** **i** 5 **ii** n **iii** 18
b 20 tins

9 $2n - 1$

10 $(n + 1)^2$

11 **a** 2^n
b **i** $100 \times 2^{n-1}$ ml **ii** 1600 ml
c Next sizes after super giant are 3.2 l, 6.4 l and 12.8 l with weights of 3.2 kg, 6.4 kg and 12.8 kg, so the largest size is 6.4 litres.

12 The nth term is $\left(\frac{3}{4}\right)^n$, so as n gets very large, the unshaded area gets smaller and smaller and eventually it will be zero; so the shaded area will eventually cover the triangle.

13 Yes, as the number of matches is 12, 21, 30, 39, … which is $9n + 3$; so he will need $9 \times 20 + 3 = 183$ matches for the 20th step and he has $5 \times 42 = 210$ matches.

14 **a** 20 **b** 120

15 Alex's answer gives $4(n + 2) = 4n + 8$
Colin's method gives $4n + 4$
Ed's method gives $4(n + 1) = 4n + 4$
Gail's method gives $2 \times n + 2(n + 2) = 2n + 2n + 4 = 4n + 4$
Linear sequence is 8, 12, 16 , 20 , …. Which has an nth term of $4n + 4$ so they are all valid methods except for Alex who forgot that the corners overlap and should have taken the 4 overlapping corners away to get $4n + 8 - 4 = 4n + 4$

16 **a** £128 **b** £31 **c** £2048

Review questions

1 No. Sequence is 7, 10, 13, 16, 19, 22, 25, 28, … so the first 3 odd terms are prime but 25 is not prime.

2 **a** 13, 9
b Subtract 4
c All the terms are odd and –4 is even

3 **a** **i** 40, 34 **ii** 32, 64
b 4, 16, 64

4 **a** Always even **b** Odd or even

5 0, 3, 8

6 **a** 5, 3
b 15, 14.5
c He is right as when it gets nearly to 14, the next term will be $\frac{14+14}{2} = 14$

7 **a** **i** 6, 11, 16 **ii** All terms end in 1 or 6
b No. When $n = 1$ then $(1+2)^2 - 9 = 9 - 9 = 0$ and after that $(n+2)^2$ is always greater than 9

8 **a** $4n + 1$
b All the terms are odd and 112 is even
c 28th term is 113

9 nth term is $5n + 1$. $5 \times 150 + 1 = 751$

10 **a** $6n + 3$
b No, $3n + 2$ generates the sequence 5, 8, 11, 14, 17, 20, 23, … so the even terms of this sequence are always 1 less than the terms of the original sequence

11 **a** Always odd **b** Always odd

12 For example: $0.5^2 = 0.25$, $0.25 < 0.5$

13 **a** $2 \times n$ is always even, any even number plus 1 is odd
b Odd × odd = odd, even × even = even

Chapter 22 – Geometry and measures: Right-angled triangles

Exercise 22A

1 **b** 90° **c** 9, 16, 25 **d** 9 + 16=25

2 **b** 90° **c** 25, 144, 169 **d** 25 + 144 = 169

3 **b** 90° **c** 36, 64, 100 **d** 36 + 64 = 100

4 $c^2 = a^2 + b^2$

5 **b** 7.8 cm **c** 7.8 cm

Exercise 22B

1 **a** 10.3 cm **b** 5.9 cm **c** 8.5 cm
d 20.6 cm **e** 18.6 cm **f** 17.5 cm
g 32.2 cm **h** 2.4 m **i** 500 m
j 5 cm **k** 13 cm **l** 10 cm

2 14.4 cm

3 Diagonal$^2 = 6^2 + 6^2 = 72$, diagonal = $\sqrt{72}$ = 8.485284 = 8.5 cm to 1 decimal place

4 50 cm, 1.2 m and 1.3 m or 1.5 m, 2 m and 2.5 m

5 Because $6^2 + 7^2 = 85$ and $\sqrt{85} = 9.2$ not 10

6 **a** 15 **b** 13 **c** 7.3 **d** 14.2

7 **a** 9, 12, 15
b 25, 60, 65
c Any six more Pythagorean triples, for example 10, 24, 26

Exercise 22C

1 **a** 15.0 cm **b** 14.7 cm **c** 6.3 cm
d 18.3 cm **e** 5.4 cm **f** 217.9 m
g 0.4 cm **h** 8.0 m

2 **a** 20.8 m **b** 15.5 cm **c** 15.5 m
d 12.4 cm **e** 22.9 m **f** 19.8 m
g 7.1 m **h** 0.6 m

3 **a** 5.0 m **b** 6.0 m **c** 3.0 m **d** 50.0 cm

4 The 12×30 has diagonal length of 32.31 cm, the 11×30 has a diagonal length of 31.95 cm, so the 11×30 is the closest to 32cm.

5 Many different combinations are possible, such as: 8 cm and 11.5 cm, 10 cm and 9.8 cm, 12 cm and 7.2 cm.

6 $80^2 + 60^2 = 6400 + 3600 = 10\,000 = 100^2$

7 **a** Student's own diagram
b **i** The large square is 17 by 17 giving 289 square units.
ii The red and yellow triangles all have shorter lengths of 5 and 12, with an area of 30 square units each. The four red triangles have an area of $4 \times 30 = 120$ square units
iii The area of the inner square (green and yellow) must be $289 - 4 \times 30 = 169$, so the hypotenuse of the red and yellow triangles must be $\sqrt{169} = 13$
c The above working shows that $5^2 + 12^2 = 13^2$

Exercise 22D

1 No. The foot of the ladder is about 6.6 m from the wall.

2 2.06 m

3 11.3 m

4 About 17 minutes

5 27.4 m

6 4.58 m

7 **a** 3.87 m **b** 1.74 m

8 3.16 m

9 This creates a right-angled triangle with two short sides of 5 and 12. Use Pythagoras' theorem to show length of line $= \sqrt{(5 + 12^2)} = 13$

10 **a** 4.85 m
b 4.83 m (There is only a small difference.)

11 Yes, because $24^2 + 7^2 = 25^2$

12 6 cm

13 He is partly correct. The perimeter must be larger than 20 cm or the rectangle has no width, and the area is largest when it's a square, giving a perimeter of 28.3 cm (3sf). So he should have said the perimeter is between 20 and 28.3 cm.

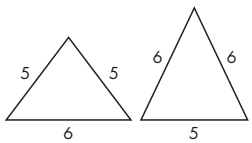
Exercise 22E

1 **a** 32.2 cm^2 **b** 2.83 cm^2 **c** 50.0 cm^2

2 22.2 cm^2

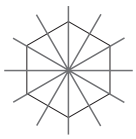
3 15.6 cm^2

4 **a**



b The areas are 12 cm^2 and 13.6 cm^2 respectively, so triangle with 6 cm, 6 cm, 5 cm sides has the greater area.

5 **a**



b 166.3 cm^2

6 259.8 cm^2

7 **a** No, areas vary from 24.5 cm^2 to 27.7 cm^2
b No, the equilateral triangle gives the largest area.
c The closer the isosceles triangle gets to an equilateral triangle the larger its area becomes.

8 The right-angled triangle made with hypotenuse 6.5 m and base $7.4 \div 2 = 3.7$ gives the height of the triangle as 5.344 cm. Use area $= \frac{1}{2} \times 7.4 \times 5.344$ to give 19.7733 which rounds to 19.8 m^2 (3sf)

9 48 cm^2

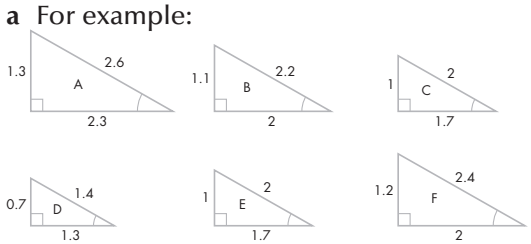
10 **a** 10 cm **b** 26 cm **c** 9.6 cm

11 6 or 8 cm

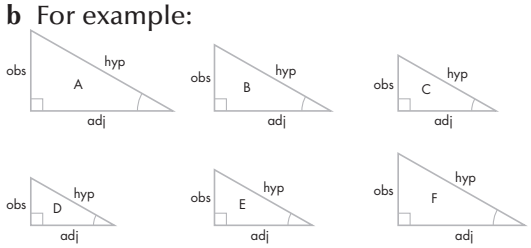
12 Andrew didn't round off any answers until the last calculation, and Olly used a rounded off value to find an intermediate result.

Exercise 22F

1 **a** For example:



b For example:



c **i**

Triangle	Opposite	Adjacent	Hypotenuse	Opposite / Hypotenuse	Adjacent / Hypotenuse	Opposite / Adjacent
A	1.3	2.3	2.6	0.5	0.88	0.57
B	1.1	2	2.2	0.5	0.91	0.55
C	1	1.7	2	0.5	0.85	0.59
D	0.7	1.3	1.4	0.5	0.93	0.54
E	1	1.7	2	0.5	0.85	0.59
F	1.2	2	2.3	0.5	0.87	0.6

ii Regardless of the size of the triangle the ratios are similar in each case.

2 **b** Yes, the pattern is the same.

3 **a** The answer the calculator gives is almost the same as the $\frac{\text{opposite}}{\text{hypotenuse}}$.

b The answer the calculator gives is almost the same as the $\frac{\text{adjacent}}{\text{hypotenuse}}$.

c The answer the calculator gives is almost the same as the $\frac{\text{opposite}}{\text{adjacent}}$.

d Yes

e Match up calculator result with student table result from above.

Exercise 22G

1 **a** x is opp, y is hyp, w is adj
b x is adj, y is opp, w is hyp
c x is hyp, y is opp, w is adj
d x is adj, y is hyp, w is opp
e x is hyp, y is opp, w is adj
f x is adj, y is hyp, w is opp

2 **b** $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = \frac{5}{12}$

c $\sin\theta = \frac{9}{41}$, $\cos\theta = \frac{40}{41}$, $\tan\theta = \frac{9}{40}$

3 **a** $\sin\theta = \frac{12}{20}$, $\cos\theta = \frac{16}{20}$, $\tan\theta = \frac{12}{16}$

b $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$, $\tan\theta = \frac{5}{12}$

c $\sin\theta = \frac{24}{26}$, $\cos\theta = \frac{10}{26}$, $\tan\theta = \frac{24}{10}$

d $\sin\theta = \frac{20}{25}$, $\cos\theta = \frac{15}{25}$, $\tan\theta = \frac{20}{15}$

e $\sin\theta = \frac{36}{39}$, $\cos\theta = \frac{15}{39}$, $\tan\theta = \frac{36}{15}$

f $\sin\theta = \frac{40}{50}$, $\cos\theta = \frac{30}{50}$, $\tan\theta = \frac{40}{30}$

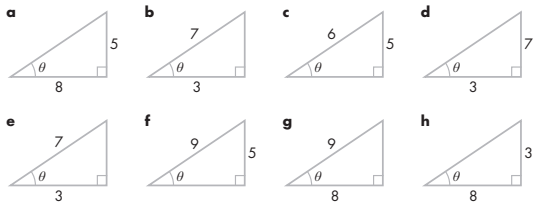
4 **b** $\cos\theta = \frac{adj}{hyp} = \frac{5}{7}$ **c** $\tan\theta = \frac{opp}{adj} = \frac{7}{10}$

d $\sin\theta = \frac{opp}{hyp} = \frac{8}{11}$ **e** $\cos\theta = \frac{adj}{hyp} = \frac{7}{12}$

f $\tan\theta = \frac{opp}{adj} = \frac{5}{13}$

5 If the triangle is right-angled, then the sides are 3, 4 and 5. This is a Pythagorean triple and so he could be correct.

6



7 **a** **i** $\tan x = \frac{3}{4}$ **ii** $\sin x = \frac{3}{5}$ **iii** $\cos x = \frac{4}{5}$

b **i** $\tan y = \frac{4}{3}$ **ii** $\sin y = \frac{4}{5}$ **iii** $\cos y = \frac{3}{5}$

c **i** the same **ii** the same

d One is the reciprocal of the other

Exercise 22H

1 **a** 0.682 **b** 0.829 **c** 0.922
d 1 **e** 0.707 **f** 0.342
g 0.375 **h** 0

2 **a** 0.731 **b** 0.559 **c** 0.388
d 0 **e** 0.707 **f** 0.940
g 0.927 **h** 1

3 45°

4 **a** **i** 0.574 **ii** 0.574
b **i** 0.208 **ii** 0.208
c **i** 0.391 **ii** 0.391
d Same
e **i** $\sin 15°$ is the same as $\cos 75°$
ii $\cos 82°$ is the same as $\sin 8°$
iii $\sin x$ is the same as $\cos(90° - x)$

5 **a** 0.933 **b** 1.48 **c** 2.38
d Infinite **e** 1 **f** 0.364
g 0.404 **h** 0

6 **a** 0.956 **b** 0.899 **c** 2.16
d 0.999 **e** 0.819 **f** 0.577
g 0.469 **h** 0.996

7 Tan has some values > 1

8 **a** 10.7 **b** 5.40
c Infinite **d** 0

9 **a** 3.56 **b** 8.96
c 28.4 **d** 8.91

10 $(4.5315389 + 4.458869) - (6 + 0) = 8.9904079 - 6 = 2.990408 = 3.0$ (2 sf)

11 0.6

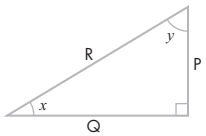
Exercise 22I

1 **a** 9.5cm **b** 3.8cm
c 4.4cm **d** 8.3 cm
e 12.0cm **f** 4.2cm
g 10.3cm **h** 19.6cm
i 5.1cm **j** 9.2cm
k 6.1cm **l** 13.3cm

2 Yes because one side = 10 sin 60° = 8.7 cm (1 dp) and the other 10 cos 60° = 5 cm

3 Opposite = 8 tan 38° = 6.25 cm (2 dp)

4 Sin $x = \frac{opp}{hyp} = \frac{P}{Q}$, cos $y = \frac{adj}{hyp} = \frac{P}{Q}$ so where x and y are the two acute angles in a right-angled triangle, the sine of one is equal to the cosine of the other.



Exercise 22J

1 **a** 30.0° **b** 51.7° **c** 39.8°
d 61.3° **e** 87.4° **f** 45.0°

2 **a** 60.0° **b** 50.2° **c** 2.6°
d 45.0° **e** 78.5° **f** 45.6°

3 **a** 31.0° **b** 20.8° **c** 41.8°
d 46.4° **e** 69.5° **f** 77.1°

4 **a** 53.1° **b** 41.8° **c** 44.4°
d 56.4° **e** 2.4° **f** 22.6°

5 **a** 36.9° **b** 48.2° **c** 45.6°
d 33.6° **e** 87.6° **f** 67.4°

6 **a** 31.0° **b** 37.9° **c** 15.9°
d 60.9° **e** 57.5° **f** 50.2°

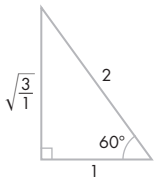
7 **a** His statement is true.
b It is true for cosine for the same reason as sine. It is not true for tangent because tan x is larger than 1 when the opposite is larger than the adjacent.
c Largest value 1

8 **a** **i** 17.5° **ii** 72.5° **iii** 90°
b Yes

9 **a** 61.0 **b** 26.4 **c** 36.9 **d** 45.6
e 43.3 **f** 54.3 **g** 51.3 **h** 49.4
i 36.9 **j** 41.4 **k** 39.5 **l** 69.4

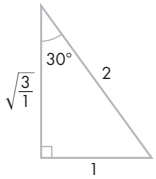
Exercise 22K

1



a The opposite side is $\sqrt{3}$ from Pythagoras. Tan 60° = $\frac{O}{A} = \frac{\sqrt{3}}{1} = \sqrt{3}$

b Sin 60° = $\frac{O}{H} = \frac{\sqrt{3}}{2}$



2 **a** The adjacent side is $\sqrt{3}$ from Pythagoras. Tan 30 = $\frac{O}{A} = \frac{1}{\sqrt{3}}$

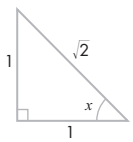
b Cos 30° = $\frac{A}{H} = \frac{\sqrt{3}}{2}$

3 **a** 4.5 cm **b** 5 cm **c** 3.5 cm

4 The ratio to use is the cos θ = $\frac{A}{H}$. cos 60 = $\frac{3.5}{AC}$. Therefore AC = 7 cm. Iain is correct.

5 **i-iv**
In each of the diagrams the smaller the side at the bottom the more each of sin 90° tends towards 1, cos 0° tends towards 1, sin 0° tends towards 0, and cos 90° tends towards 0.

6



a From the diagram above, tan x = $\frac{O}{A} = \frac{1}{1} = 1$. As the triangle is isosceles each acute angle is equal and therefore x = 45 degrees.

b The hypotenuse is $\sqrt{2}$ due to Pythagoras' theorem. Therefore cos x = $\frac{A}{H} = \frac{1}{\sqrt{2}}$ and sin $x = \frac{O}{H} = \frac{1}{\sqrt{2}}$

Exercise 22L

1 65°

2 The safe limits are between 1.04 m and 2.05 m. The highest the ladder will reach is 5.9 m up the wall.

3 44°

4 6.82 m

5 31°

6 **a** 25°
b 2.10 m
c Thickness of wood has been ignored

7 **a** 20° **b** 4.78 m

8 She would calculate 100 tan 23°. The answer is about 42.4 m

9 21.1 m

10 $\frac{Height}{3000}$ = sin5.1, so height = 3000sin5.1 = 266.68 = 267 (to 3 sf)

11 8.2 m (2 sf)

Exercise 22M

1 10.1 km

2 22°

3 429 m

4 **a** 156 m
b No, the new angle of depression is $\tan^{-1}\left(\frac{200}{312}\right) = 33°$ and half of 52° is 26°

5 **a** 222 m **b** 42°

6 **a** 21.5 m **b** 17.8 m

7 13.4 m. We don't know if the angle of elevation is from Sunil's feet or head. This would make a difference to the answer as we would need to add Sunil's height if the angle was from his head.

8 **Sin** $\theta = \frac{1}{3}$ so $\sin^{-1} 0.3333 = 19.5°$ (3sf)

9 The angle is 16° so Cara is not quite correct.

Exercise 22N

1 **a** 73.4 km **b** 15.6 km

2 **a** 14.7 miles **b** 8.5 miles

3 Draw a diagram representing the relative places. Your diagram will show the angle of the bearing to fly is $\tan^{-1}\left(\frac{70}{120}\right) + 90° = 120.256°$ which rounds to 120° (3 sf).

4 **a** 59.4 km **b** 8.4 km

5 **a** 15.9 km **b** 24.1 km
c 31.2 km **d** 052°

6 2.28 km

7 235°

8 The distance North is 120cos35 = 98.298245 km, so he hasn't gone as far as 100 km, but rounded to the nearest ten he is correct.

Exercise 22O

1 **a** 5.79 cm **b** 48.2°
c 7.42 cm **d** 81.6 cm

2 9.86 m

3 **a** 36.4 cm^2 **b** 115 cm^2
c 90.6 cm^2 **d** 160 cm^2

4 473 cm^2

Review questions

1 8.6 cm

2 6.7 cm

3 5.7 cm (1 dp)

4 Height = 5sin75 = 4.8296 m, which is just higher than 4.8 m so Michael is correct.

5 13.6 cm^2

6 2pm

7 237°

8 52.3°

9 **a** 5.9 m **b** 82°

10 **a** 75.5° **b** 47.4 cm^2

11 The areas are parallelogram 85.8 cm^2, triangle 82.8 cm^2 and circle 84.9 cm^2. So all answers are close, but Joe is correct, the parallelogram does have the greatest area.

12 **a** 32.60 m **b** 10.07 m
c 4.10 m **d** 9.34 m

Chapter 23 – Geometry and measures: Congruency and similarity

Exercise 23A

1 **a** SAS **b** SSS **c** RHS
d SSS **e** ASA **f** ASA

2 **a** **i** SSS **ii** A to R, B to P, C to Q
b **i** SAS **ii** A to R, B to Q, C to P

3 **a** 60° **b** 80° **c** 40° **d** 5 cm

4 The triangle is made of parallel lines so each triangle has a set of corresponding angles with each other. Because the squares share sides with the triangles, each triangle has the same-sized sides. Hence each triangle is congruent, ASA.

5 SSS or RHS

6 SSS or SAS or RHS

7 **a** always
b always
c always
d sometimes; it's true if the scale factor is 1

8 **a** The square has rotated through angle x, so each side has rotated the same angle.
b **i** ED and DG are both sides of the same square **ii** CD and AD are both sides of the same square
c SAS, where CD corresponds to AD, DG corresponds to DE and the angle between them is x in each case.

9 AB and PQ are the corresponding sides to the 42° angle, but they are not equal in length.

Exercise 23B

1 **c** Each triangle has the same angles as the other.
d They are all twice as large.
e 2

2 **c** Each triangle has the same angles as the other
d They are all three times as large.
e 3

3 **c** Each triangle has the same angles as the other.
d yes
e 1.5

4 **d** Each triangle has the same angles as the other.
e 2

5 **d** Each triangle has the same angles as the other.
e 3

Exercise 23C

1 **a** similar, scale factor 3
b similar, scale factor 2
c similar, scale factor 1.5
d not similar as two corresponding sides have a scale factor of 5, the other one doesn't

2 **a** 3 **b** angle R **c** BA

3 **a** angle B = 180 – (75 + 50) = 55 and angle P = 180 – (55 + 50) = 75, so both triangles have the same angles of 50, 55 and 75
b PR **c** BC **d** **i** 28 cm **ii** 6 cm

4 **a** 40°
b 1.5
c Multiply length AB by 1.5

5 **a** 8 cm
b 7.5 cm

6 **a** **i** ABC with DEF
ii 3
iii $x = 3$ cm $y = 18$ cm
b **i** GHI with LKJ
ii 0.5
iii $x = 8$ cm $y = 3.5$ cm
c **i** PQR with TUS
ii 1.5
iii $x = 5.33$ cm $y = 12$ cm
d **i** MNL with WUV
ii 5
iii $x = 9$ cm $y = 40$ cm

7 **a** Sides in same ratio
b 3 **c** 13 cm **d** 39 cm

8 **a** The interior angles of all regular hexagons are 120°, so all the angles of each hexagon will correspond, hence similar shapes.
b 12 cm
c 6.928 cm

Exercise 23D

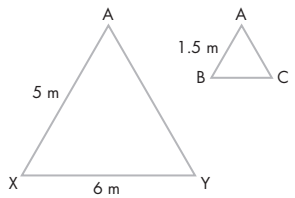
1 82 m

2 $\frac{\text{pole}}{330} = \frac{400}{600}$, pole $= 330 \times \frac{4}{6} = 220$ feet

3 15 m

4 3.3 m

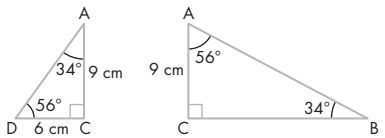
5 **a**



b 1.8 m

6 **a** **i** ABC and ADE **ii** 9 cm
b **i** ABC and ADE **ii** 12 cm

7 **a**



b $\frac{CD}{9} = \frac{9}{6}$, $CD = 9 \times \frac{9}{6} = 13.5$ cm

8 **c** 4 cm

Review questions

1 **a** B, C and D
b **i** rotation
ii reflection
iii reflection
iv enlargement
v no transformation
c It is not true because E, the enlarged triangle, is not congruent, and nor is F.

2 Ellie is correct:
a both triangles have an identical set of angles: 90, 65 and 25
b both shapes are rectangles and the lengths of the large rectangle are four times larger than the small one, so the large rectangle is an enlargement of the small one.

3 **a** always
b always
c sometimes
d sometimes
e always

4 **a** congruent SAS
b congruent RHS
c not congruent
d congruent SSS

5 Scale factor between similar triangles is $1000 \div 40 = 25$, hence tree height $= 140 \times 25 = 3500$ cm $= 35$ m

6 Construction of a triangle with sides exactly the same size as the given triangle

7 ABX with DCX and ADX with BCX

8 Andrew is correct. Eve has calculated the length of AD (8 cm) so ED should be 2 cm.

9 4.5 cm^2

Chapter 24 – Probability: Combined events

Exercise 24A

1 **a** 7 **b** 2 and 12

c

Score	Probability
2	$\frac{1}{36}$
3	$\frac{1}{18}$
4	$\frac{1}{12}$
5	$\frac{1}{9}$
6	$\frac{5}{36}$
7	$\frac{1}{6}$
8	$\frac{5}{36}$
9	$\frac{1}{9}$
10	$\frac{1}{12}$
11	$\frac{1}{18}$
12	$\frac{1}{36}$

d **i** $\frac{1}{12}$ **ii** $\frac{1}{3}$ **iii** $\frac{1}{2}$ **iv** $\frac{7}{36}$ **v** $\frac{5}{12}$ **vi** $\frac{5}{18}$

2 **a** $\frac{1}{12}$ **b** $\frac{11}{36}$ **c** $\frac{1}{6}$ **d** $\frac{5}{9}$

3 **a** $\frac{1}{36}$ **b** $\frac{11}{36}$ **c** $\frac{5}{18}$

4 **a** $\frac{1}{4}$ **b** $\frac{1}{2}$ **c** $\frac{3}{4}$ **d** $\frac{1}{4}$

5 **a** $\frac{1}{12}$ **b** $\frac{1}{4}$ **c** $\frac{1}{6}$

6 **a**

Score on second dice \ Score on first dice	1	2	3	4	5	6
6	5	4	3	2	1	0
5	4	3	2	1	0	1
4	3	2	1	0	1	2
3	2	1	0	1	2	3
2	1	0	1	2	3	4
1	0	1	2	3	4	5

b **i** $\frac{5}{18}$ **ii** $\frac{1}{6}$ **iii** $\frac{1}{9}$ **iv** 0 **v** $\frac{1}{2}$

7 **a**

Score on second spinner \ Score on first spinner	1	2	3	4	5
5	6	7	8	9	10
4	5	6	7	8	9
3	4	5	6	7	8
2	3	4	5	6	7
1	2	3	4	5	6

b 6

c **i** $\frac{4}{25}$ **ii** $\frac{13}{25}$ **iii** $\frac{1}{5}$ **iv** $\frac{3}{5}$

8 $\frac{18}{64} = \frac{1}{8}$

9 It will show all the possible products.

10 **a** HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

b **i** $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{1}{8}$ **iv** $\frac{7}{8}$

11 **a** 16 **b** 32 **c** 1024 **d** 2^n

12 **a**

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b $\frac{1}{18}$ **c** 18 **d** twice

Exercise 24B

1 a

		Shaded	Unshaded
Shape	Circle	3	3
	Triangle	2	2

b i $\frac{1}{5}$ **ii** $\frac{1}{2}$

2 a 40 **b** 45% **c** $\frac{3}{40}$

3 a

	England	Scotland	Wales	Total
Female	26	12	4	42
Male	24	8	6	38
Total	50	20	10	80

b $\frac{1}{4}$ **c** $\frac{3}{19}$

4 a

	Blackpool	Chester Zoo	Total
Boys	32	28	60
Girls	26	14	40
Total	58	42	100

b $\frac{29}{50}$ **c** $\frac{7}{20}$

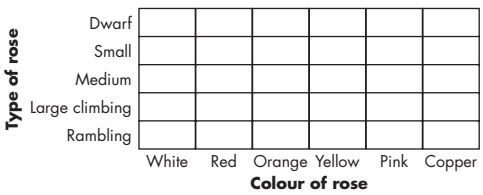
5 a 23 **b** 20% **c** $\frac{3}{50}$ **d** 520

b 9 **c** 0 **d** $\frac{5}{12}$

e The 0 on the first spinner cannot be taken as odd.

6 a 15% **b** $\frac{9}{40}$ **c** 75

7 a



b i $\frac{1}{5}$ **ii** $\frac{4}{5}$

8 a $\frac{2}{45}$ **b** 40% c 45%

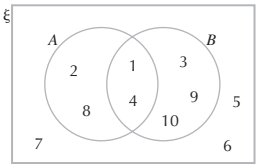
d No, as you don't know how much the people who get over £350 actually earn

Exercise 24C

1 a 0.9 **b** 0.7

2 a 0.75 **b** 0.45

3 a

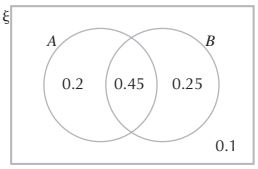


b i $\frac{2}{5}$ **ii** $\frac{3}{5}$ **iii** $\frac{1}{2}$ **iv** $\frac{1}{2}$ **v** $\frac{7}{10}$ **vi** $\frac{1}{5}$

4 a i 0.52 **ii** 0.48 **iii** 0.65 **iv** 0.35
v 0.82 **vi** 0.35

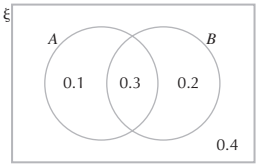
b 0.3

5 a



b i 0.5 **ii** 0.6 **iii**0.3

6 a



b i 0.65 **ii** 0.7 **iii** 0.9 **iv** 0.45

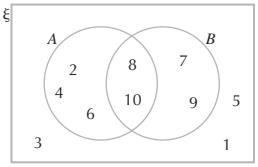
7 a 130

b i $\frac{8}{13}$

ii The probability that a student chosen at random walks to and from school

c $\frac{5}{26}$

8



a $\frac{1}{2}$ **b** $\frac{2}{5}$ **c** $\frac{7}{10}$ **d** $\frac{1}{5}$

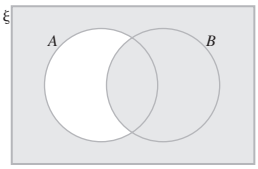
9 0.4

10 0.5

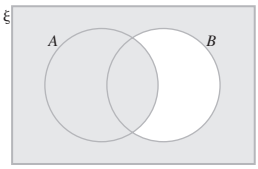
11 a $(A \cup B)'$ **b** $(A \cap B)'$

12

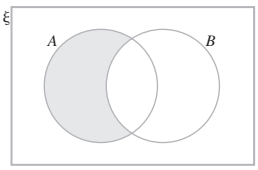
a



b



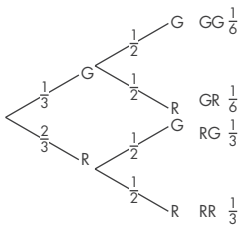
c



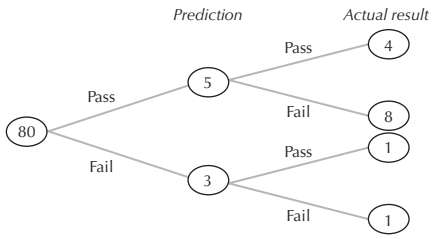
13 $\frac{37}{80}$

Exercise 24D

1



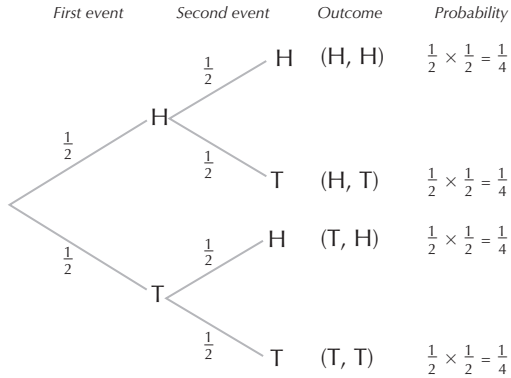
2 a



b i $\frac{1}{4}$ **ii** $\frac{1}{2}$ **iii** $\frac{3}{4}$

3 a $\frac{1}{13}$ **b** $\frac{12}{13}$

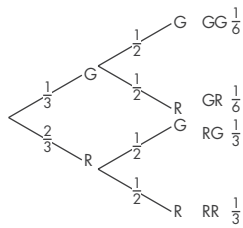
c



d i $\frac{1}{169}$ **ii** $\frac{24}{169}$ **iii** $\frac{25}{169}$

4 a $\frac{2}{3}$ **b** $\frac{1}{2}$

c

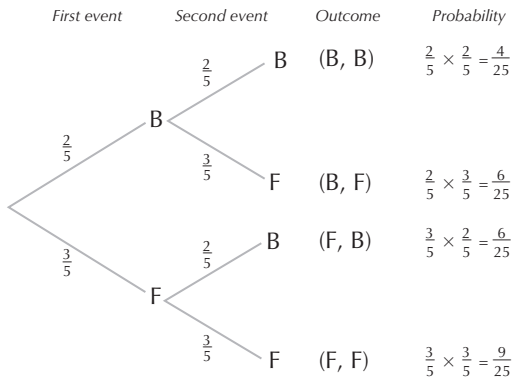


d i $\frac{1}{6}$ **ii** $\frac{1}{2}$ **iii** $\frac{5}{6}$

e 15 days

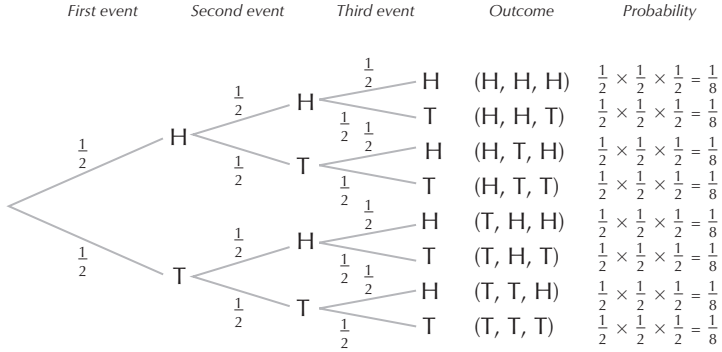
5 a $\frac{2}{5}$

b



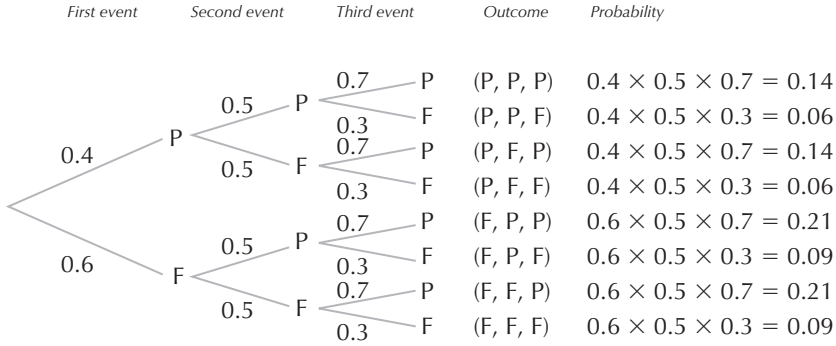
i $\frac{4}{25}$ **ii** $\frac{12}{25}$

6 a



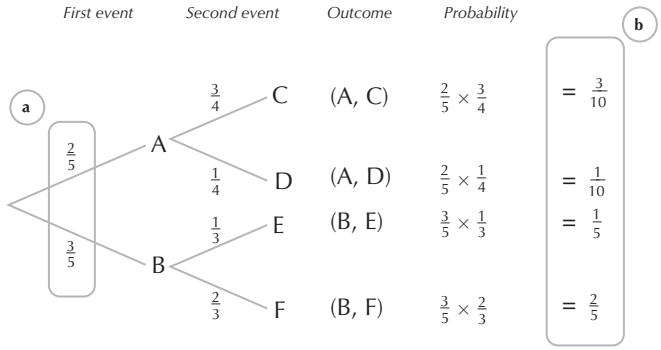
b i $\frac{1}{8}$ **ii** $\frac{3}{8}$ **iii** $\frac{7}{8}$

7 a



b 0.14 **c** 0.41 **d** 0.09

8 a 1 **b** 1 **c**



9 a 27 **b** 3

10 a $\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}\times\frac{1}{2}=\frac{1}{16}$ **b** $\frac{1}{2}n$

Review questions

1 a

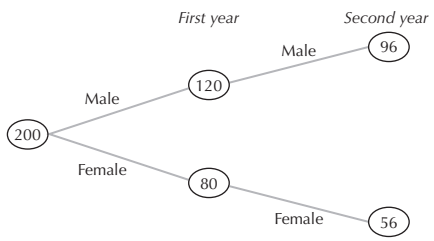
1		Spinner				
		2	3	4	5	
Coin	H	1, H	2, H	3, H	4, H	5, H
	T	1, T	2, T	3, T	4, T	5, T

b $\frac{3}{10}$

2 **a** 23 **b** 20% **c** $\frac{4}{25}$ **d** 480

3 **a** 110 **b** $\frac{19}{55}$ **c** $\frac{3}{22}$ **d** $\frac{36}{55}$

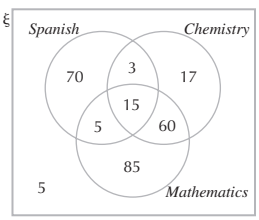
4



5 **a** 0.1

b **i** 0.3 **ii** 0.5 **iii** 0.6 **iv** 0.3

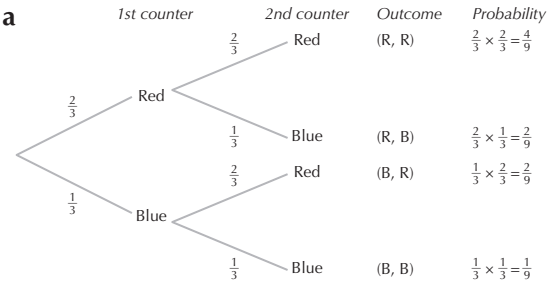
6 **a**



b **i** $\frac{70}{260} = 0.269$ **ii** $\frac{60}{260} = 0.231$

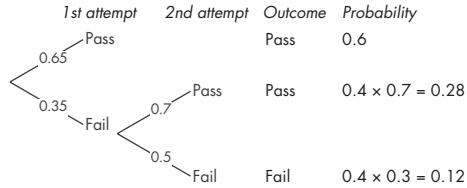
iii $\frac{5}{260} = 0.0192$ **iv** $\frac{20}{165} = 0.121$

7 **a**



b $\frac{5}{9}$ **c** $\frac{4}{9}$

8 **a**



b **i** 0.88 **ii** 0.12

c calculate 0.88^2

Chapter 25 – Number: Powers and standard form

Exercise 25A

1 **a** 2^4 **b** 3^5 **c** 7^2 **d** 5^3
e 10^7 **f** 6^4 **g** 4^1 **h** 1^7
i 0.5^4 **j** 100^3

2 **a** $3 \times 3 \times 3 \times 3$
b $9 \times 9 \times 9$
c 6×6
d $10 \times 10 \times 10 \times 10 \times 10$
e $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$
f 8
g $0.1 \times 0.1 \times 0.1$
h 2.5×2.5
i $0.7 \times 0.7 \times 0.7$
j 1000×1000

3 **a** 16 **b** 243
c 49 **d** 125
e 10 000 000 **f** 1296
g 4 **h** 1
i 0.0625 **j** 1 000 000

4 **a** 81 **b** 729
c 36 **d** 100 000
e 1024 **f** 8
g 0.001 **h** 6.25
i 0.343 **j** 1 000 000

5 125 m^3

6 **b** 10^2 **c** 2^3 **d** 5^2

7 **a** 1 **b** 4 **c** 1
d 1 **e** 1

8 Any power of 1 is equal to 1.

9 10^6

10 10^6

11 **a** 1 **b** –1 **c** 1
d 1 **e** –1

12 **a** 1 **b** –1 **c** –1
d 1 **e** 1

13 2^{24}, 4^{12}, 8^8, 16^6

14 $x = 7$

Exercise 25B

1 **a** 5^4 **b** 5^{10} **c** 5^5 **d** 5^3
e 5^{15} **f** 5^9 **g** 5^6 **h** 5^9
i 5^8

2 **a** x^8 **b** x^9 **c** x^8 **d** x^5
e x^{12} **f** x^{13} **g** x^{11} **h** x^{10}
i x^{16}

3 **a** 6^3 **b** 6^5 **c** 6^1 **d** 6^0
e 6^1 **f** 6^3 **g** 6^2 **h** 6^1
i 6^2

4 **a** x^4 **b** x^5 **c** x^3 **d** x^3
e x^6 **f** x^5 **g** x^2 **h** x^6
i x^9

5 **a** 1 **b** 6^0 **c** 1

6 **a** 1 **b** 5^0 **c** 1

7 Answers for power 0 are always 1.

8 Two values with a sum of 7, for example $a = 5$ and $b = 2$

9 **a** $a = 1$ and $x = 2$
b $x = 1$

10 **a** $6a^5$ **b** $9a^2$ **c** $8a^6$
d $-6a^4$ **e** $8a^8$ **f** $-10a^{-3}$

11 **a** $3a$ **b** $4a^3$ **c** $3a^4$
d $6a$ **e** $4a^3$ **f** $5a^{-4}$

12 **a** Possible answer: $6x^2 \times 2y^5$ and $3xy \times 4xy^4$
b Possible answer: $24x^2y^7 \div 2y^2$ and $12x^6y^8 \div x^4y^3$

13 **a** $3a^3b^2$ **b** $3a^2c^4$ **c** $8a^2b^2c^3$

14 **a** $8a^5b^4$ **b** $10a^3b$ **c** $30a^{-2}b^{-2}$
d $2ab^3$ **e** $8a^{-5}b^7$ **f** $4a^5b^{-5}$

15 **a** 4^6 **b** 4^{15} **c** 4^6
d 4^{-6} **e** 4^6 **f** 4^0

16 12 ($a = 2$, $b = 1$, $c = 3$)

Exercise 25C

1 **a** 60 000 **b** 120 000 **c** 150
d 42 000 **e** 1400 **f** 300
g 400 **h** 8000 **i** 160 000
j 4500 **k** 8000 **l** 250 000

2 **a** 5 **b** 50 **c** 25 **d** 30
e 7 **f** 300 **g** 6 **h** 30
i 1 **j** 15 **k** 40 **l** 5
m 40 **n** 320

3 **a** 54 400 **b** 16 000

4 $30 \times 90\,000 = 2\,700\,000$
$600 \times 8000 = 4\,800\,000$
$5000 \times 4000 = 20\,000\,000$
$200\,000 \times 700 = 140\,000\,000$

5 1400 million

6 **a** 31 **b** 310 **c** 3100 **d** 31 000

7 **a** 65 **b** 650 **c** 6500 **d** 65 000

8 **a** 0.31 **b** 0.031 **c** 0.0031 **d** 0.000 31

9 **a** 0.65 **b** 0.065 **c** 0.0065 **d** 0.000 65

10 **a** 250 **b** 34.5 **c** 4670
d 346 **e** 207.89 **f** 56 780
g 246 **h** 0.76 **i** 999 000
j 23 456 **k** 98 765.4 **l** 43 230 000
m 345.78 **n** 6000 **o** 56.7
p 560 045

11 **a** 0.025 **b** 0.345
c 0.004 67 **d** 3.46
e 0.207 89 **f** 0.056 78
g 0.0246 **h** 0.0076
i 0.000 000 999 **j** 2.3456
k 0.098 765 4 **l** 0.000 043 23
m 0.000 000 034 578 **n** 0.000 000 000 06
o 0.000 000 567 **p** 0.005 600 45

12 **a** 230 **b** 578 900 **c** 4790
d 57 000 000 **e** 216 **f** 10 500
g 0.000 32 **h** 9870

13 They are all true.

Exercise 25D

1 **a** 0.31 **b** 0.031 **c** 0.0031 **d** 0.000 31

2 **a** 0.65 **b** 0.065 **c** 0.0065 **d** 0.000 65

3 **a** $9999999999 \times 10^{99}$
b $0.000000001 \times 10^{-99}$(depending on number of digits displayed)

4 **a** 31 **b** 310 **c** 3100 **d** 31 000

5 **a** 65 **b** 650 **c** 6500 **d** 65 000

6 **a** 250 **b** 34.5 **c** 0.004 67
d 34.6 **e** 0.020 789 **f** 5678
g 246 **h** 7600 **i** 897 000
j 0.008 65 **k** 60 000 000 **l** 0.000 567

7 **a** 2.5×10^2 **b** 3.45×10^{-1}
c 4.67×10^4 **d** 3.4×10^9
e 2.078×10^{10} **f** 5.678×10^{-4}
g 2.46×10^3 **h** 7.6×10^{-2}
i 7.6 3 10^{-4} **j** 9.99×10^{-1}
k 2.3456×10^2 **l** 9.876 54 × 10
m 6×10^{-4} **n** 5.67×10^{-3}
o 5.600 45 × 10

8 2.7797×10^4

9 $2.815\,81 \times 10^5$, 3×10, $1.382\,101 \times 10^6$

10 1.298×10^7, 2.997×10^9, 7.4×10^4

11 **a** 5.67×10^3 **b** 6×10^2 **c** 3.46×10^{-1}
d 7×10^{-4} **e** 5.6×10^2 **f** 6×10^5
g 7×10^3 **h** 1.6 **i** 2.3×10^7
j 3×10^{-6} **k** 2.56×10^6 **l** 4.8×10^2
m 1.12×10^2 **n** 6×10^{-1} **o** 2.8×10^6

12 **a** 1.08×10^8 **b** 4.8×10^6 **c** 1.2×10^9
d 1.08 **e** 6.4×10^2 **f** 1.2×10
g 2.88 **h** 2.5×10^7 **i** 8×10^{-6}

13 2×10^{13}, 1×10^{-10}, mass = 2×10^3 g (2 kg)

14 5×10^4

15 7800, $3.7 \times 2.1 \times 10^4$, 7.85×10^4, 79×10^3

16 0.0005, 4 x 10^{-3}, 0.52 x 10^{-2}, 6 x 10^{-2}

17 **a** 1.8 x 10^4 **b** 1.7 x 10^7 **c** 8.8 x 10^7
d 1.9 x 10^5 **e** 6.9 x 10^6 **f** 7.1 x 10^{-9}
g 1.2 x 10^{-7} **h** 5.3 x 10^8

18 **a** 1 x 10^0
b 1.0 x 10
c It is easier to use ordinary numbers in this case

Review questions

1 **a** 64 **b** 64 **c** 10 000 000

2 **a** 62 **b** 4900 **c** 3.47 **d** 0.123

3 **a** 6 000 **b** 300 000 **c** 80 **d** 40

4 **a** 196 **b** units digits is $5 \times 5 = 25$ so it should end in 5

5 **a** 7^9
b x^4
c He multiplied the powers of x, but he should have added them. $15x^9$

6 **a** t^8 **b** m^5 **c** $9x^6$ **d** $10a^7h^5$

7 **a** x^{11} **b** m^{-5} **c** $8k^5m^3$

8 **a** 7.5×10^4 **b** 0.009

9 **a** $2y$ **b** $8m^9p^{12}$

10 **a** 1200

b $1 \times 10^b = 1^b \times 10^b$ or $a \times 10^1 = a^1 \times 10^1$

11 3

Chapter 26 – Algebra: Simultaneous equations and linear inequalities

Exercise 26A

1 **a** $x = 4\frac{1}{2}$, $y = 1\frac{1}{2}$ **b** $x = -2$, $y = 4$
c $x = 2\frac{1}{2}$, $y = -1\frac{1}{2}$ **d** $x = 3$, $y = 10$
e $x = 9$, $y = -2$ **f** $x = 0.5$, $y = 4$

2 **a** $a = 7$, $b = 10$ **b** $c = 4$, $d = 11$
c $e = 5$, $f = 3$ **d** $g = -1$, $h = 6$
e $j = 13$, $k = 2$ **f** $m = 1.5$, $n = -7$

3 $x = 12$, $y = 2$

Exercise 26B

1 **a** $x = 9$, $y = -2$ **b** $x = \frac{1}{2}$, $y = 5$
c $x = -3$, $y = -10$ **d** $x = -12$, y = –6
e $x = 3$, $y = 5$ **f** $x = 0.5$, $y = 6.5$

2 **a** $x = 2\frac{1}{4}$, $y = 6\frac{1}{2}$ **b** $x = 4$, $y = 3$
c $x = 5$, $y = 3$ **d** $x = 4$, $y = 5$
e $x = 11$, $y = -3$ **f** $x = -7$, $y = 2$

3 **a** $x = 1$, $y = 3$ **b** $x = 5$, $y = 9$

Exercise 26C

1 **a** $x = 2$, $y = 5$ **b** $x = 4$, $y = -3$
c $x = 1$, $y = 7$ **d** $x = \frac{1}{2}$, $y = -\frac{3}{4}$
e $x = -1$, $y = 5$ **f** $x = 1\frac{1}{2}$, $y = \frac{3}{4}$

2 **a** $x = 5$, $y = 1$ **b** $x = 3$, $y = 8$
c $x = 9$, $y = 1$ **d** $x = 7$, $y = 3$
e $x = 4$, $y = 2$ **f** $x = 6$, $y = 5$

3 **a** $a = 10$, $b = \frac{1}{2}$, **b** $c = -3$, $d = 5$,
c $e = 1$, $f = 14$, **d** $g = -2$, $h = -5$,
e $m = 8$, $n = 0$, **f** $p = 3\frac{1}{2}$, $q = -2$,
g $r = 7$, $s = 7$, **h** $t = \frac{2}{3}$, $u = -1$,
i $v = 56$, $w = 32$

Exercise 26D

1 **a** $14a + b = 47$ **b** $a = 3$, $b = 5$
c 146, 443

2 **a** $3t + 5b = 810$, $3t + 3b = 630$
b **i** £1.20 **ii** 90p
c £10.20

3 **a** $10x + 5y = 840$, $8x + 10y = 1044$
b £4.07

4 (1, –2) is the solution to equations A and C; (–1, 3) is the solution to equations A and D; (2, 1) is the solution to B and C; (3, –3) is the solution to B and D.

5 84p

6 10.3 kg

7 $p = 36$, $c = 22$. Total weight for Baz is 428 pounds so he can carry the load safely on his trailer.

8 £4.40

9 b = £3.50, p = £1.75. Camilla needs £35 so she will not have enough money.

10 **a** My age minus 6 equals 2 x (my son's age minus 6)
b $x = 46$ and $y = 26$

11 Amul £7.20, Kim £3.50

12 **a** They are the same equation. Divide the first by 2 and it is the second, so they have an infinite number of solutions.
b Double the second equation to get $6x + 2y = 14$ and subtract to get $9 = 14$. The left-hand sides are the same if the second is doubled so they cannot have different values.

Exercise 26E

1 **a** $x < 3$ **b** $t > 8$ **c** $p > 10$ **d** $x < 5$
e $y < 3$ **f** $t > 5$ **g** $x < 6$ **h** $y < 15$
i $t > 18$ **j** $x < 7$ **k** $x < 3$ **l** $t > 5$

2 **a** 7 **b** 6 **c** 16
d 3 **e** 7

3 **a** 11 **b** 16 **c** 16
d 3 **e** 7

4 $2x + 3 < 20$, $x < 8.50$, so the most each could cost is £8.49

5 **a** Because $3 + 4 = 7$, which is less than the third side of length 8
b $x + x + 2 > 10$, $2x + 2 > 10$, $2x > 8$, $x > 4$, so smallest value of x is 5

6 **a** $x = 6$ and $x < 3$ scores –1 (nothing in common), $x < 3$ and $x > 0$ scores +1 (1 in common for example), $x > 0$ and $x = 2$ scores +1 (2 in common), $x = 2$ and $x > 4$ scores –1 (nothing in common), so we get $-1 + 1 + 1 - 1 = 0$

b $x > 0$ and $x = 6$ scores +1 (6 in common), $x = 6$ and $x > 4$ scores +1 (6 in common), $x > 4$ and $x = 2$ scores −1 (nothing in common), $x = 2$ and $x < 3$ scores +1 (2 in common). $+1 + 1 - 1 + 1 = 2$

c Any acceptable combination, e.g. $x = 2$, $x < 3$, $x > 0$, $x > 4$, $x = 6$

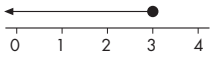
7 **a** $y \leq 4$ **b** $x \geq -2$ **c** $x \leq \frac{14}{5}$
d $x > 38$ **e** $x < 6\frac{1}{2}$ **f** $y \leq \frac{7}{5}$

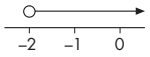
8 **a** $3 < x < 6$ **b** $2 < x < 5$ **c** $-1 < x \leq 3$
d $1 \leq x < 4$ **e** $2 \leq x < 4$ **f** $0 \leq x \leq 5$

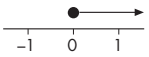
9 6 bottles

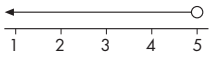
Exercise 26F

1 **a** $x > 1$ **b** $x \leq 3$ **c** $x < 2$
d $x \geq -1$ **e** $1 < x \leq 4$ **f** $-2 < x < 4$

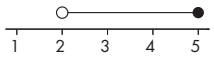
2 **a**


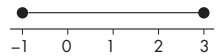
b


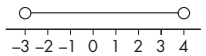
c


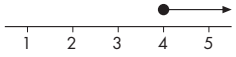
d


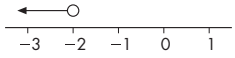
e

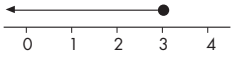
f


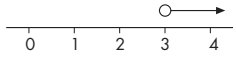
g


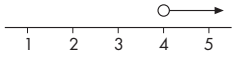
h


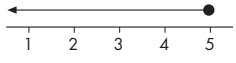
3 **a** $x \geq 4$


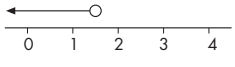
b $x < -2$


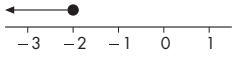
c $x \leq 3$


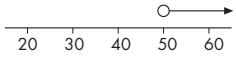
d $x > 3$


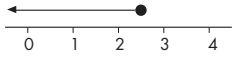
e $x > 4$


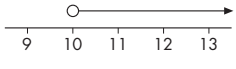
f $x \leq 5$


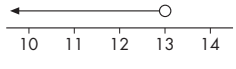
g $x < 1.5$


h $x \leq -2$


i $x > 50$


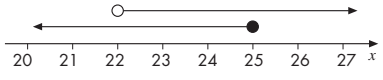
j $x \leq 2.5$


k $x < 10$


l $x > 13$


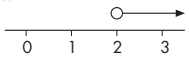
4 **a** $x > 22$

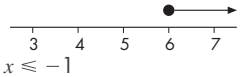
b $x \leq 25$

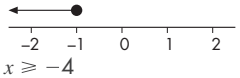
c


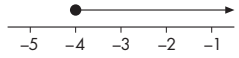
d Apples could cost 23p, 24p or 25p.

5 1 and 4

6 **a** $x > 2$


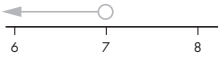
b $x \geq 6$


c $x \leq -1$


d $x \geq -4$


Review questions

1 **a** $x > 23$, **b** $x \leq 15$, **c** $x < 48$,
d $x \geq 6$, **e** $5 \leq x \leq 25$

2 **a** $x < 7$


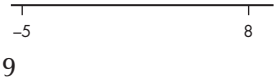
b $x > 15$

c $x \leqslant 2$

d $x \geqslant 4$

e $x \geqslant -12$

f $x < 8$

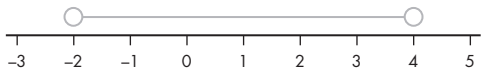
3


4 9

5 £340

6 2 hr 10 min

7 £62

8 **i** $-3 < x < 1$, number line B; **ii** $-2 < x < 4$, number line below; **iii** $-1 < x < 2$, number line A


9 $x = 36$, $y = 216$

10 £195

11 **a** $x = 4$, $y = 3$
b **i** $1000x + 1000y = 7000 \rightarrow x + y = 7$
ii $984x - 984y = 984 \rightarrow x - y = 1$
c $a = 9$, $b = 5$

12 $x = 5$, $y = -1$

13 Let straight part of track = D, inner radius of end = r, outer radius = $r + x$
x being the width of the track
Length of inner track = $2D + 2\pi r = 300$
(Call this equation **i**)
Length of outer track = $2D + 2\pi(r + x) = 320$
(Call this equation **ii**)
Subtract equation **i** from **ii** to give

$$2\pi(r + x) - 2\pi r = 20$$
$$2\pi r + 2\pi x - 2\pi r = 20$$
$$2\pi x = 20$$
$$x = 3.2 \text{ 2sf}$$

Chapter 27 – Algebra: Non-linear graphs

Exercise 27A

1 **a** 1 hour **b** $1\frac{1}{2}$ hours
c $1\frac{1}{2}$ hours **d** 45 km per hour
e 30 km per hour

2 **a** **i** 2 h **ii** 3 h **iii** 5 h
b **i** 40 km/h **ii** 120 km/h **iii** 40 km/h

3 **a** $2\frac{1}{2}$ km/h **b** 3.75 m/s **c** $2\frac{1}{2}$ km/h

4 **a** 30 km **b** 40 km **c** 50 km
d 100 km/h

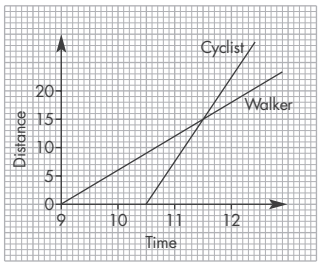
5 **a** **i** 263 m/min (3 sf) **ii** 15.8 km/h (3 sf)
b **i** 500 m/min
c Paul by 1 minute

6 **a** Patrick ran quickly at first, then had a slow middle section but he won the race with a final sprint. Araf ran steadily all the way and came second. Sean set off the slowest, speeded up towards the end but still came third.
b **i** 1.67 m/s **ii** 6 km/h

7 There are three methods for doing this question. This table shows the first, which is writing down the distances covered each hour.

Time	Walker	Cyclist
9am	0	0
9:30	3	0
10:00	6	0
10:30	9	0
11:00	12	7.5
11:30	15	15
12:00	18	22.5
12:30	21	30

The second method is algebra:
Walker takes T hours until overtaken, so $T = \frac{D}{6}$;
Cyclist takes T - 1.5 to overtake, so $T - 1.5 = \frac{D}{15}$.
Rearranging gives $15T - 22.5 = 6T$, $9T = 22.5$, $T = 2.5$.
The third method is a graph:



All methods give the same answer of 11:30 when the cyclist overtakes the walker.

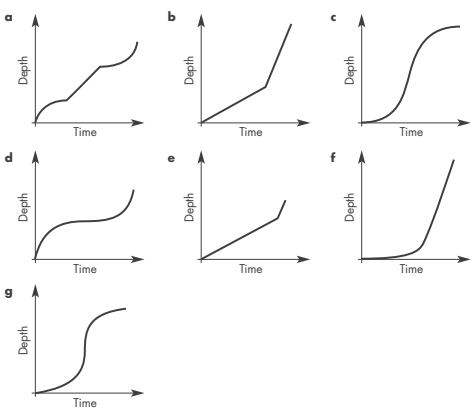
8 **a** Vehicle 2 overtook Vehicle 1
b Vehicle 1 overtook Vehicle 2
c Vehicles passed travelling in different directions
d Vehicle 2 overtook Vehicle 1
e 17:15
f 11.3 mph if you only count travelling time, or 5.1 mph if you count total time.

Exercise 27B

1 **a** Two taps on
b One tap on
c Maureen gets in the bath
d Maureen has a bath
e Maureen takes the plug out, water leaves the bath
f Maureen gets out of the bath
g Water continues to leave the bath until the bath is empty

2 **a** Graph C
b

3



Exercise 27C

1 **a** 6 m/s^2 **b** 3 m/s^2 **c** 20 seconds

2 First half hour: acceleration of 40 km/hour^2
Next hour: acceleration of 30 km/hour^2
Last half hour: deceleration of 100 km/hour^2

3 $0.3v$ m/s^2

4 **a** Bus **b** Car **c** 20 min, 75 min

Exercise 27D

1

x	–5	–4.5	–4	–3.5	–3	–2.5
y	25	20.25	16	12.25	9	6.25
		–2	–1.5	–1	–0.5	0
		4	2.25	1	0.15	0

2

x	–3	–2	–1	0	1	2	3
y	27	12	3	0	3	12	27

3 **a**

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
y	27	18	11	6	3	2	3	6	11	18	27

b $y = 2$

4 **a**

x	–5	–4	–3	–2	–1	0	1	2	3	4	5
x^2	25	16	9	4	1	0	1	4	9	16	25
$-3x$	15	12	9	6	3	0	–3	–6	–9	–12	–5
y	40	28	18	10	4	0	–2	–2	0	4	10

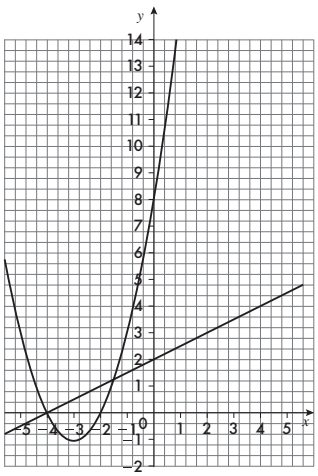
b 1.8 **c** –1.2, 4.2 **d** $y = 0$

5 **a** Values of y: 7, 0, –5, –8, –9, –8, –5, 0, 7
b –6.7
c –0.45 or 4.45

6 **a** Values of y: 15, 8, 3, 0, –1, 0, 3, 8, 15

b –0.75 **c** –1.6, –4.4

d



e (–4, 0) and (–1.5, 1.25)

7 **a** x: –2, –1, 0, 1, 2, 3, 4
y: 8, 3, 0, –1, 0, 3, 8
b (0, 0) and (3, 3)

8 **a** They have the same value, $x = 4$
b

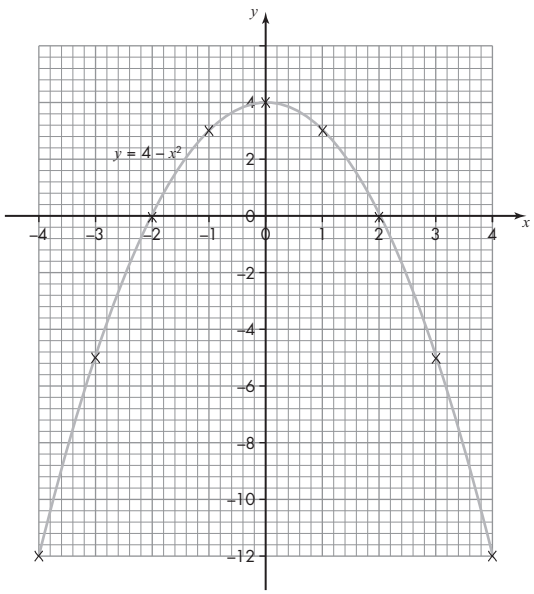
x	–3	–2	–1	0	1	2	3	4	5	6	7
y	28	19	12	7	4	3	4	7	12	19	28

Since the quadratic graph has a vertical line of symmetry and the y-values for $x = 1$ and $x = 3$ are the same, this means that the y-values will be symmetric about $x = 2$. Hence the y-values will be the same for $x = 0$ and $x = 4$, and so on.

9 **a** x: –4, –3, –2, –1, 0, 1, 2, 3, 4
y: 12, 5, 0, –3, –4, –3, 0, 5, 12
b (2,0) and (–2,0)
c –1.75
d $x = 3.5$ or –3.5

10 **a**

x	–4	–3	–2	–1	0	1	2	3	4
y	–12	–5	0	3	4	3	0	–5	–12



b (2,0) and (–2,0)
c 1.75
d $x = 3.5$ or –3.5
e The curves are reflections in the x-axis.

Exercise 27E

1 **a** –2, –5 **b** 4, 9 **c** –6, 3
d –3, –1 **e** –6, –4 **f** –3, 2
g –1, 3 **h** –4, 5 **i** 1, –2
j 2, –5 **k** 7, –4 **l** 3, 2
m 1, 5 **n** 4, 3 **o** –7, 5

2 **a** –2, –3 **b** –4, –1 **c** 2, 4
d –2, 5 **e** –3, 5 d: –3, 5 **f** –6, 3
g –1, 2 **h** –2, –5 **i** 7, 7
j –9, –2 **k** 3, 5 **l** –6, 2
m –2, –2 **n** 4 **o** –2, –6

3 **a** $x^2 + 6x - 55 = 25 + 30 - 55 = 0$
b $x = -11$

4 **a** 5, –10 **b** 7, –8 **c** 1, 18
d –2, –11 **e** 2, –15 **f** 3, 10
g –1, –999 **h** 1 **i** –10, 1

5 $x^2 + 8x + 16 = (x + 2)(x + 6) = 0$ so the answers are –2 and –6

6 $x^2 + 5x - 6 = (x + 6)(x - 1) = 0$ so the answers are 1 and –6

7 **a** Area = $(x + 1)(x + 2) = x^2 + 3x + 2 = 42$, therefore $x^2 + 3x - 40 = 0$

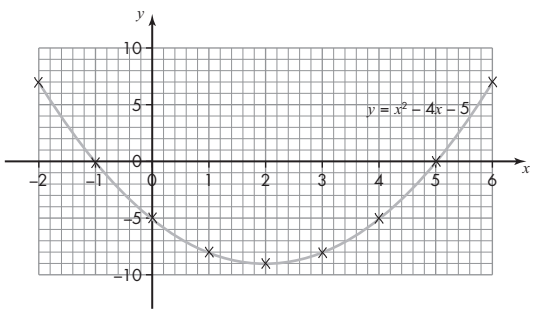
b $x = 5$ cm (discard the negative value as x is a length)

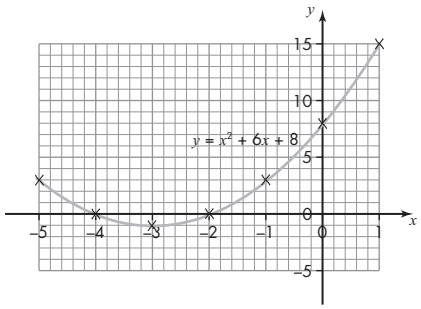
c Perimeter = 26 cm

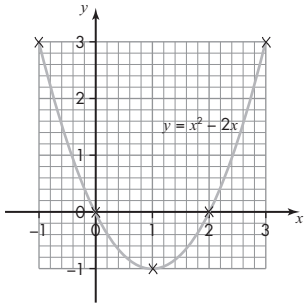
8 $x = 9$ cm

Exercise 27F

1 **a** (0, –5), (0, 8) and (0, 0).
b Each equation is written in the form $x^2 + ax + b$. You should find that the y-intercept is the value of b.







2 **a** $x = -1$ or 5; $x = -2$ or –4; $x = 0$ or 2
b (–1, 0), (5, 0); (–4, 0), (–2, 0); (0, 0), (2, 0)

3 (2, –9), (–3, –1), (1, –1)

Exercise 27G

1 **i** (0, 5) **ii** (–5, 0) and (1, 0)
iii (–2, 9)

2 **i** (0, –3) **ii** (–1, 0) and (3, 0)
iii (1, –4)

3 **a** roots: (–2, 0) and (2, 0); y-intercept (0, –4)
b roots: (0, 0) and (6, 0); y-intercept (0, 0)
c roots: (–1, 0) and (3, 0); y-intercept (0, –3)
d roots: (–11, 0) and (–3, 0); y-intercept (0, 33)

4 (2, –1)

5 **a** (3, 0) and (–3, 0) **b** (0, 9)

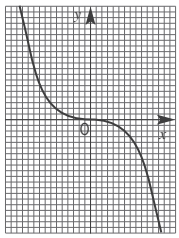
6 **i** (0, 16) **ii** (2, 0), (8, 0) **iii** (5, –9)

7 **i** (0, –15)
ii (3, 0), (–5, 0)
iii (–1, –16)

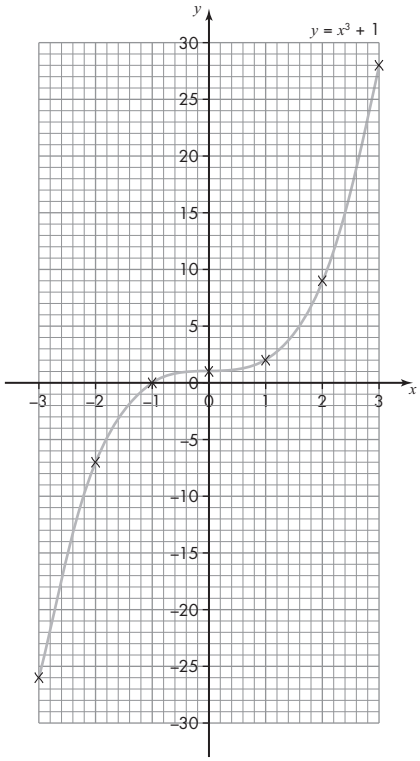
8 **a** (–2.6, 0), (1.6, 0)
b (–0.5, 8.5)

Exercise 27H

1



2 **a** Values of y: –26, –7, 0, 1, 2, 9, 28

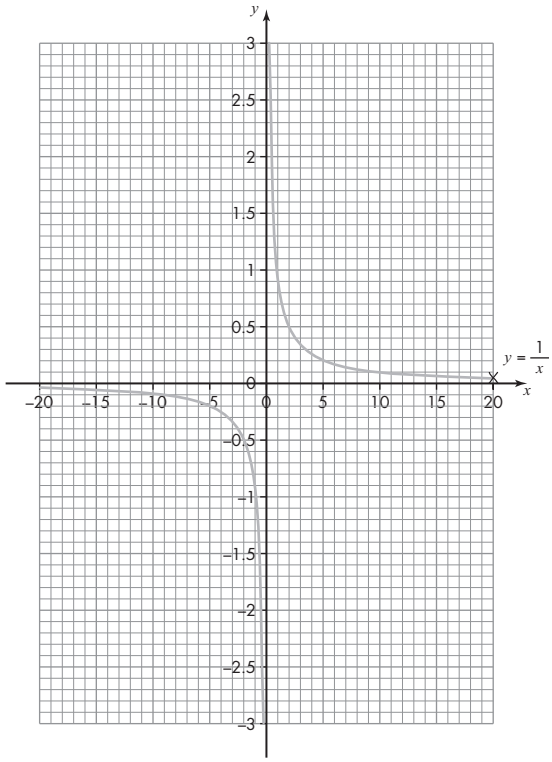


b 16.6

3 **a** Values of y: –24, –12.63, –5, –0.38, 2, 2.9, 3, 3.13, 4, 6.38, 11, 18.63, 30

b 4.7 **c** (–1.45, 0)

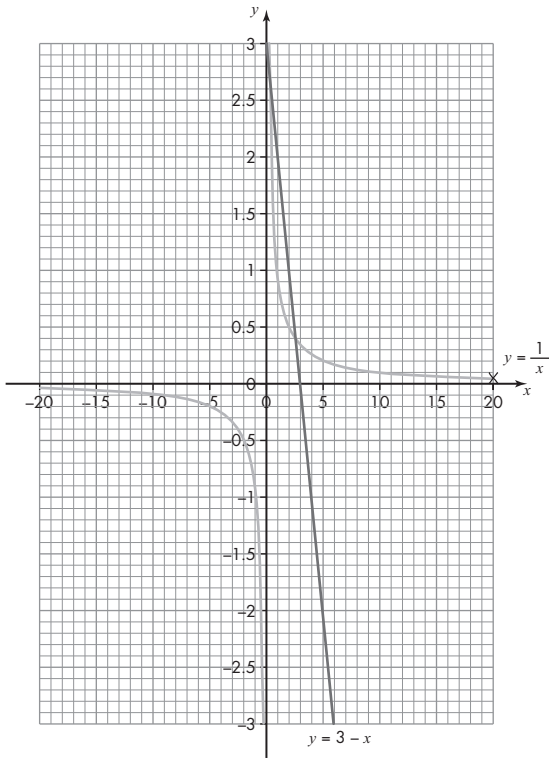
4 **a**



Values of y: –0.05, –0.1, –0.2, –0.25, –0.5, –1, –2, –2.5, –5, 5, 2.5, 2, 1, 0.5, 0.2, 0.1, 0.067, 0.05

b $\frac{1}{0}$ is undefined

c



d 0.4 and 2.6

5 **a** Quadratic **b** Linear **c** None
d Reciprocal **e** None **f** Cubic
g Linear **h** None **i** Quadratic

Review questions

1 **a**

b Answers will vary

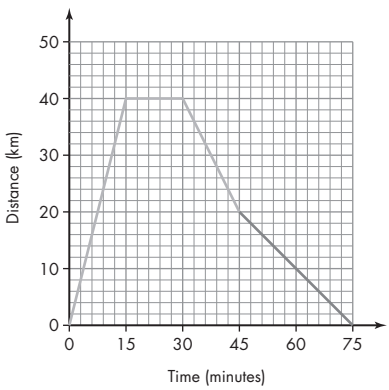
2 **a** The horizontal line from 30 min to 80 min

b Only the second statement: the car is slowing down

3 **a** 160 km/h

b 80 km/h

c



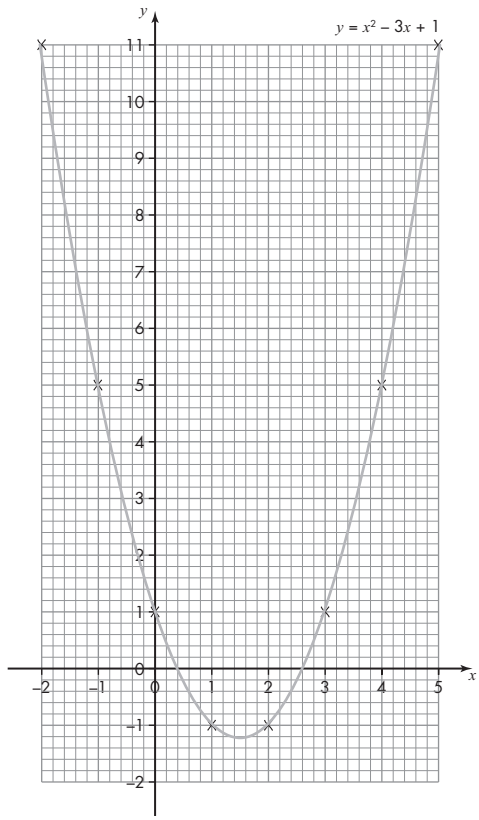
d 14:41

4 $x = 9$ or 10

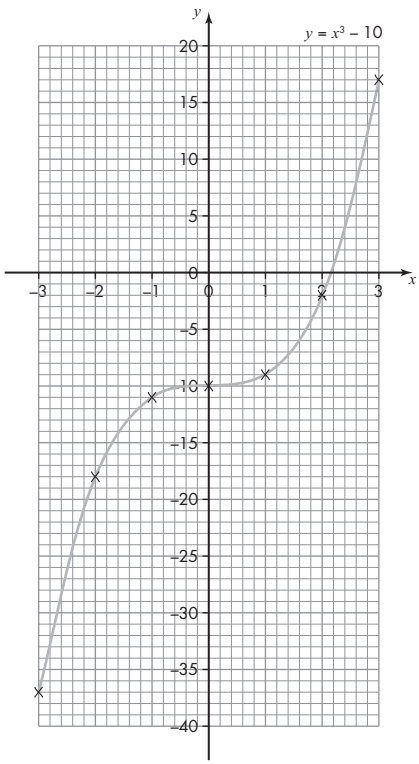
5 23, 10, 5, 2

6 2m/s^2

7 **a**



b **i** 0.4, 2.6
ii –0.3, 3.3
iii –1.4, 4.4

8 **a** $x = 6$ **b** 38 cm

9