1 Numbers and indices

1.1 Fractions

What you should already know:

- how to add, subtract, multiply and divide by fractions.
- how to convert between mixed numbers and top-heavy fractions.

In this section you will learn:

- how to substitute a fraction into a formula.

Revisiting GCSE

In GCSE Maths you performed calculations with fractions. The numbers involved may have been **top-heavy** (or **improper**) fractions, such as $\frac{11}{7}$ or **mixed numbers** such as $2\frac{3}{5}$.
You should be confident in converting mixed numbers into top-heavy fractions.

For example, $2\frac{3}{5} = 2 + \frac{3}{5}$

Use a common denominator of 5: $= \frac{2 \times \mathbf{5}}{1 \times \mathbf{5}} + \frac{3}{5}$

$= \frac{10}{5} + \frac{3}{5}$

Add the numerators: $= \frac{13}{5}$

So $2\frac{3}{5} = \frac{13}{5}$

GCSE C Example 1

Work out $\frac{7}{3} \div \frac{4}{9}$. Give your answer in its simplest form.

Working

You multiply $\frac{7}{3}$ by the **reciprocal** of $\frac{4}{9}$.

$$\frac{7}{3} \div \frac{4}{9} = \frac{7}{3} \times \frac{\mathbf{9}}{\mathbf{4}}$$

Cancel the common factor 3: $= \frac{7}{{}_1\cancel{3}} \times \frac{{}^3\cancel{9}}{4}$

Bring the numerators together:
Bring the denominators together: $= \frac{7 \times 3}{1 \times 4}$

So $\frac{7}{3} \div \frac{4}{9} = \frac{21}{4}$

> *Handy hint*
> 'Reciprocate' means 'invert' or 'turn upside down'.
> The reciprocal of $\frac{4}{9}$ is $\frac{\mathbf{9}}{\mathbf{4}}$.
> Make sure you only invert the fraction you are dividing by.

> *Handy hint*
> Common factors reduce the size of the numbers you are working with.
> Without cancellation, this calculation is $\frac{7 \times 9}{3 \times 4} = \frac{63}{12}$.

Key point

The rules for fraction arithmetic are:

$\frac{a}{b} \times \frac{c}{d} = \frac{ac}{bd}$ $\qquad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$

$\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$ $\qquad \frac{a}{b} - \frac{c}{b} = \frac{a-c}{b}$

> *Handy hint*
> You **must** make the denominators equal before you can add or subtract fractions.

In AS Maths, you will be expected to work confidently with fractions. They are often needed when simplifying, or finding the value of, an expression. You will generally find it easier to work with top-heavy fractions rather than mixed numbers.

For example, it is not easy to work out the value of $\frac{4}{1\frac{1}{2}}$ without a calculator.

However, by writing $1\frac{1}{2}$ as $\frac{3}{2}$, you can easily work out this answer.

$$\frac{4}{\left(\frac{3}{2}\right)} = \frac{4}{1} \times \frac{\mathbf{2}}{\mathbf{3}}$$
$$= \frac{4 \times 2}{1 \times 3}$$
$$= \frac{8}{3}$$

Moving on to AS Level

AS Level Example 2

Using the formula $y = \frac{3}{4}x + \frac{5}{2}$, find the value of y when $x = 2$.

Working

You substitute $x = 2$ into the formula.

Write 2 as the top-heavy fraction $\frac{2}{1}$.

Formula: $y = \frac{3}{4}x + \frac{5}{2}$

$= \frac{3}{4} \times \frac{2}{1} + \frac{5}{2}$

Cancel the common factor 2: $= \frac{3}{{}_2\cancel{4}} \times \frac{{}^1\cancel{2}}{1} + \frac{5}{2}$

Simplify the product: $= \frac{3}{2} + \frac{5}{2}$

Add the numerators: $= \frac{3+5}{2}$

$= \frac{8}{2}$

So $y = 4$ when $x = 2$

> *Common error*
> $\frac{3}{4} \times 2$ does **not** equal $\frac{3 \times 2}{4 \times 2}$.
> To avoid this type of error, write 2 as $\frac{2}{1}$.

AS Level Example 3

Using the formula $y = \frac{5x}{3} - \frac{1}{2}$, find the value of y when $x = \frac{2}{3}$.

Working

You can write $\frac{5x}{3}$ as $\frac{5}{3} \times \frac{x}{1}$, or simply as $\frac{5}{3}x$.

So the formula is $y = \frac{5}{3}x - \frac{1}{2}$.

You substitute $x = \frac{2}{3}$ into this formula.

Formula: $y = \frac{5}{3}x - \frac{1}{2}$

$= \frac{5}{3} \times \frac{2}{3} - \frac{1}{2}$

$= \frac{10}{9} - \frac{1}{2}$

Subtract the fractions by using a common denominator 18:

$= \frac{10 \times \mathbf{2}}{9 \times \mathbf{2}} - \frac{1 \times \mathbf{9}}{2 \times \mathbf{9}}$

$= \frac{20}{18} - \frac{9}{18}$

Subtract the fractions: $= \frac{11}{18}$

So $y = \frac{11}{18}$ when $x = \frac{2}{3}$

> *Handy hint*
>
> The coefficient of x is $\frac{5}{3}$
>
> You can separate the coefficient from the x-term.

Key point

The expression $\frac{ax}{b}$ can be written as $\frac{a}{b}x$, where a and b are any numbers, $b \neq 0$.

Taking it further

You will need to be confident in using fractions as you progress through your course. In particular, you use them when working with straight-line equations.

1.2 Surds

What you should already know:

- how to re-write a surd such as $\sqrt{8}$.
- how to rationalise the denominator of a fraction such as $\frac{10}{\sqrt{5}}$

In this section you will learn:

- how to use the rules of surds to simplify more complex expressions.
- how to rationalise more complex denominators in fractions.
- how to calculate with numbers and expressions involving surds.
- the meaning of 'exact form'.

Revisiting GCSE

A surd is a number of the form $\sqrt{a}$ where a is not a square number.

For example, $\sqrt{8}$ is a surd. A decimal approximation for $\sqrt{8}$ is 2.828427125 but you can never exactly describe a surd using decimals.

However, $\sqrt{9}$ is *not* a surd because you can write down its exact value, 3.

> *Handy hint*
>
> You should be familiar with the square numbers: 1, 4, 9, 16, 25, 36…

> *Handy hint*
>
> You can assume $\sqrt{9}$ means the positive square root of 9.

You need to know the rules of surds:

Key point

The rules of surds are: $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$, $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$ (where b is not zero).

These rules allow you to break down a surd. For example,

$\sqrt{8} = \sqrt{4 \times 2}$

$= \sqrt{4} \times \sqrt{2}$

$= 2 \times \sqrt{2}$

It is not possible to break down $\sqrt{2}$ any further using integers.

$\sqrt{8}$ has been expressed in **simplified surd form** or **exact form**.

GCSE A Example 4

a Express $\sqrt{12} + \sqrt{48}$ in the form $k\sqrt{3}$ where k is an integer.

b Simplify $\frac{21}{\sqrt{3}}$ by rationalising the denominator.

> *Common error*
>
> $\sqrt{12} + \sqrt{48}$ is NOT the same as $\sqrt{12 + 48}$ – check this on a calculator.

Working

a You need to re-write $\sqrt{12}$ using factors of 12.

$\sqrt{12} = \sqrt{4 \times 3}$

$= \sqrt{4} \times \sqrt{3}$

$= 2\sqrt{3}$

Re-write $\sqrt{48}$ using factors of 48.

48 has lots of factors. Look for the *greatest* square number which is a factor of 48 (i.e. 16).

$\sqrt{48} = \sqrt{16 \times 3}$

$= \sqrt{16} \times \sqrt{3}$

$= 4\sqrt{3}$

So, $\sqrt{12} + \sqrt{48} = 2\sqrt{3} + 4\sqrt{3}$

$= 6\sqrt{3}$ (so $k = 6$)

> *Handy hint*
>
> Use a square number which is a factor of 12 (i.e. 4).

> *Handy hint*
>
> Before answering a surd question, make a list of the first few square numbers (i.e. 1, 4, 9, **16**, 25…).

> *Handy hint*
>
> You can add 'like' surds.

b The denominator of $\frac{21}{\sqrt{3}}$ is the surd $\sqrt{3}$.

You need to find a fraction with the same value as $\frac{21}{\sqrt{3}}$ which does not have a surd in its denominator.

Since $\sqrt{3} \times \sqrt{3} = 3$, you multiply the numerator and denominator of this fraction by $\sqrt{3}$.

$\frac{21}{\sqrt{3}} = \frac{21 \times \sqrt{\mathbf{3}}}{\sqrt{3} \times \sqrt{\mathbf{3}}}$

$= \frac{21\sqrt{\mathbf{3}}}{3}$

$= \frac{21}{3}\sqrt{3}$

$= 7\sqrt{3}$

> *Handy hint*
>
> You must multiply top **and** bottom by $\sqrt{3}$.

Key point

$\sqrt{a} \times \sqrt{a} = a$ for any number $a \geqslant 0$. Exact form is the same as giving and answer in surd form.

Moving on to AS Level

In AS Maths you will need to be able to simplify expressions involving surds. Also, you sometimes need to give answers in **exact form**, rather than using decimals.

AS Level Example 5

a Show that $(3 + \sqrt{7})(3 - \sqrt{7})$ is an integer, stating its value.

b Hence express $\frac{1 + \sqrt{7}}{3 + \sqrt{7}}$ in the form $a + b\sqrt{7}$ where a and b are integers to be stated.

> *AS Alert!*
> Part **b** does not tell you how to start.

Working

a You need to expand the brackets and then simplify the numbers.

$$\begin{aligned}(3 + \sqrt{7})(3 - \sqrt{7}) &= 3 \times 3 \cancel{-3\sqrt{7}} \cancel{+3\sqrt{7}} - \sqrt{7} \times \sqrt{7} \\ &= 9 - 7 \\ &= 2, \text{ which is an integer.}\end{aligned}$$

> *Handy hint*
> Write $\sqrt{7} \times 3$ as $3\sqrt{7}$.
> Use $\sqrt{7} \times \sqrt{7} = 7$.

> *AS Alert!*
> Part **b** is a 'Hence' question – this means you should make use of part **a**.

b You need to rationalise the denominator of the fraction $\frac{1 + \sqrt{7}}{3 + \sqrt{7}}$.

This can be done by multiplying the numerator and denominator by $(3 - \sqrt{7})$.

So, $$\frac{1 + \sqrt{7}}{3 + \sqrt{7}} = \frac{(1 + \sqrt{7}) \times (\mathbf{3} - \sqrt{\mathbf{7}})}{(3 + \sqrt{7}) \times (\mathbf{3} - \sqrt{\mathbf{7}})}$$

$$= \frac{3 - \sqrt{7} + 3\sqrt{7} - 7}{2}$$

> *Handy hint*
> You should use the result of part **a** to simplify the denominator.

Combine numbers and like surds: $$= \frac{-4 + 2\sqrt{7}}{2}$$

$$= \frac{-4}{2} + \frac{2\sqrt{7}}{2}$$

$$= -2 + \sqrt{7}$$

Compare this answer to the form $a + b\sqrt{7}$ given in the question: $-2 + \sqrt{7}$ looks like $a + b\sqrt{7}$ where $a = -2$, $b = 1$.

> *AS Alert!*
> You must write down the value of a and the value of b, as directed by the question.

Key point

- These are some definitions for negative and fractional indices: $a^{\frac{1}{2}} = \sqrt{a}$ for any number $a \geqslant 0$.
  $a^{\frac{1}{3}} = \sqrt[3]{a}$ for any number a.
  $a^0 = 1$ for any number a, $a^{-1} = \frac{1}{a}$ and $a^{-2} = \frac{1}{a^2}$ for any non-zero number a.
- If a and b are integers, then $(a + \sqrt{b})(a - \sqrt{b})$ is also an integer, and equals $a^2 - b$.

  To rationalise the denominator of a fraction of the form $\frac{N}{a + \sqrt{b}}$, multiply top and bottom by $(a - \sqrt{b})$.
  To rationalise the denominator of a fraction of the form $\frac{N}{a - \sqrt{b}}$, multiply top and bottom by $(a + \sqrt{b})$.

> *Handy hint*
> $\sqrt[3]{a}$ is the cube root of a. This is the number b such that $b^3 = a$.

Taking it further

Surds appear in many areas of AS Maths, such as solving quadratic equations and calculating distances between points.

1.3 Indices

What you should already know:

- how to simplify expressions such as $(a^2b)^2$.
- how to calculate values such as 3^{-2} or $4^{\frac{1}{2}}$.

In this section you will learn:

- how to work out the value of more complicated expressions using the rules of indices.

Revisiting GCSE

In GCSE Maths you would have used various rules of indices to simplify an expression. For example, in the expression $a^2 \times a^3$ you can add the indices together so that $a^2 \times a^3$ simplifies to a^5.

This works because $a^2 \times a^3 = (a \times a) \times (a \times a \times a)$
$= a^5$

Key point

Here are some results involving indices:

1. $a^m \times a^n = a^{m+n}$
2. $\frac{a^a}{a^n} = a^{m-n}$ for a not zero
3. $(a^m)^n = (a^n)^m = a^{mn}$
4. $a^n \times b^n = (ab)^n$
5. $\frac{a^n}{b^n} = \left(\frac{a}{b}\right)^n$ for b not zero

> *Handy hint*
> Rule **3** means $(a^m)^n$ and $(a^n)^m$ are equal to each other and are also equal to a^{mn}.

GCSE B Example 6

Simplify $(2p^3q^2)^2$.

Working

You can write rule **4** as $(ab)^n = a^n \times b^n$

So $(2p^3q^2)^2 = 2^2 \times (p^3)^2 \times (q^2)^2$

Use rule 3 on each bracket: $= 4 \times (p^{3\times2}) \times (q^{2\times2})$

Simplify the indices: $= 4p^6q^4$

So $(2p^3q^2)^2$ simplifies to $4p^6q^4$.

> *Handy hint*
> Rule **4** can be extended to three or more terms.

You have also met the **definition** of negative and fractional indices.

For example, 2^{-3} means $\frac{1}{2^3}$ and so 2^{-3} has value $\frac{1}{8}$.

Similarly, $4^{\frac{1}{2}}$ means the positive square root of 4, and so $4^{\frac{1}{2}}$ has value 2.

GCSE A Example 7

Find the value of $3^{-2} \times 8^{\frac{1}{3}}$.

Working

You apply the definitions to each expression.

$$3^{-2} = \frac{1}{3^2} \qquad 8^{\frac{1}{3}} = \sqrt[3]{8}$$

$$= \frac{1}{9} \qquad = 2$$

So $3^{-2} \times 8^{\frac{1}{3}} = \frac{1}{9} \times 2$

$$= \frac{2}{9}$$

Moving on to AS Level

Key point

Here are some general definitions which you need to know for AS Maths.

6 $a^{-n} = \frac{1}{a^n}$ where n is any integer.

7 $a^{\frac{1}{n}} = \sqrt[n]{a}$ where n is any positive integer.

In AS Maths you will be expected to evaluate expressions involving indices, possibly without the use of a calculator.

AS Level Example 8

a Find the value of $16^{-\frac{3}{2}}$.

b Simplify $\frac{12x^2}{2\sqrt{x}}$.

c Express $\frac{4x^3 - 3}{x}$ in the form $ax^2 - bx^n$, stating the value of the constants a, b and n.

AS Alert!

You can see that the numbers and expressions are more complex at this level. You need to combine several rules of indices to answer these questions.

Working

a Using rule **6** you can write $16^{-\frac{3}{2}}$ as $\frac{1}{16^{\frac{3}{2}}}$.

Now concentrate on the denominator.

You can write $16^{\frac{3}{2}}$ as $\left(16^{\frac{1}{2}}\right)^3$ by using rule **3**.

Now use rule **7**: $\left(16^{\frac{1}{2}}\right)^3 = (\sqrt{16})^3$

$$= (4)^3$$

$$= 64$$

So $-16^{\frac{3}{2}} = \frac{1}{64}$

Handy hint

Deal with the negative sign on the index first.

Handy hint

Using rule **3**, you could also work out $16^{\frac{3}{2}}$ as $(16^3)^{\frac{1}{2}}$, but this would mean having to find 16^3.

Handy hint

Although $2 - \frac{1}{2} = 1\frac{1}{2}$, you should avoid writing an index using a mixed number.

b You can write $\frac{12x^2}{2\sqrt{x}}$ as $\frac{12}{2} \times \frac{x^2}{\sqrt{x}}$ which simplifies to $6\frac{x^2}{\sqrt{x}}$.

Now express the denominator as a power of x.

So $6\frac{x^2}{\sqrt{x}} = 6\frac{x^2}{x^{\frac{1}{2}}}$

Use rule **2**: $= 6x^{-\frac{1}{2}}$

$$= 6x^{\frac{3}{2}}$$

So $\frac{12x^2}{2\sqrt{x}}$ simplifies to $6x^{\frac{3}{2}}$.

c You must split the fraction up into two parts:

$$\frac{4x^3 - 3}{x} = \frac{4x^3}{x} - \frac{3}{x}$$

Separate the numbers from the x-terms: $= 4 \times \frac{x^3}{x} - 3 \times \frac{1}{x}$

Use rules of indices: $= 4x^{3-1} - 3x^{-1}$

$$= 4x^2 - 3x^{-1}$$

So $\frac{4x^3 - 3}{x} = 4x^2 - 3x^{-1}$

Compare $4x^2 - 3x^{-1}$ with $ax^2 - bx^n$: $a = 4$, $b = 3$, $n = -1$

Common error

b is **not** -3. Comparing terms, $-b = -3$ so $b = 3$.

Key point

$$a^{\frac{m}{n}} = (\sqrt[n]{a})^m$$

$$= \sqrt[n]{a^m}$$

When calculating $a^{\frac{m}{n}}$ it is usually easier to use the result $(\sqrt[n]{a})^m$ rather than $\sqrt[n]{a^m}$

Taking it further

Indices appear in many AS Maths topics, but in particular you will need to be confident in using them when studying differentiation and integration.

1 Practice: Numbers and indices

1.1 Fractions

1 Work out these. Give each answer in its simplest form.

a $\frac{2}{3} \times \frac{5}{6}$ **b** $\frac{3}{4} \div \frac{4}{5}$

c $\frac{3}{8} \times \left(\frac{2}{3}\right)^2$ **d** $\frac{6}{25} \div \frac{9}{10}$

2 Work out these. Give each answer in its simplest form.

a $\frac{1}{3} + \frac{2}{5}$ **b** $\frac{3}{5} - \frac{1}{10}$

c $\frac{4}{5} + \frac{1}{4} - \frac{1}{2}$ **d** $2 - \frac{1}{2} - \frac{3}{5}$

3 Convert these mixed numbers into top-heavy fractions.

a $2\frac{2}{3}$ **b** $4\frac{3}{5}$

c $5\frac{7}{11}$ **d** $\left(1\frac{2}{3}\right)^2$

4 Simplify these. Give each answer as a top-heavy fraction in its simplest form.

a $\frac{7}{\left(\frac{2}{5}\right)}$ **b** $\frac{\left(-\frac{8}{3}\right)}{2}$

c $\frac{\left(\frac{15}{4}\right)}{-3}$ **d** $\frac{\left(\frac{5}{6}\right)}{\left(\frac{4}{9}\right)}$

5 By converting mixed numbers to top-heavy fractions, find the value of these. Give each answer as a fraction in its simplest form.

a $\frac{6}{2\frac{1}{2}}$ **b** $\frac{8}{1\frac{1}{4}}$

c $\frac{\left(\frac{9}{2}\right)}{2\frac{2}{5}}$ **d** $\frac{3\frac{3}{5}}{6\frac{3}{4}}$

6 Use these formulae to find the value of y for the given value of x. Give each answer for y as a fraction in its simplest terms.

a $y = \frac{2}{3}x + 1, x = 4$

b $y = \frac{4x}{3} - \frac{3}{2}, x = \frac{5}{2}$

c $y = 3 - \frac{5x}{4}, x = -\frac{2}{3}$

d $y = \frac{1}{4}x^2, x = \frac{2}{3}$

e $y = \frac{4x^3}{5} + \frac{3}{4}, x = \frac{1}{2}$

f $y = \frac{6}{5x}, x = \frac{3}{2}$

7 A curve has equation $y = \frac{x}{x+1}$. Find the y-coordinate of the point on this curve where

a $x = \frac{1}{2}$ **b** $x = -\frac{4}{3}$

8 The formula for converting the temperature C degrees Celsius into the temperature F degrees Fahrenheit is

$F = \frac{9C}{5} + 32.$

a What is 15 °C in degrees Fahrenheit?

b The minimum overnight temperature in a village was −7.5 °C. What was this temperature in degrees Fahrenheit?

9 The formula for converting the distance K kilometres into the distance M miles is

$M = \frac{8K}{5}$

a How far in miles is the distance $9\frac{1}{3}$ km?

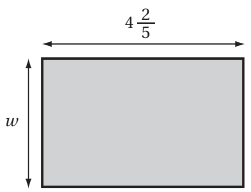
b How far in km is the distance $3\frac{3}{4}$ miles?

10 Solve these equations. Give each answer as a fraction in its simplest form.

a $\frac{3}{4}x = 2$ **b** $-\frac{5}{3}x = \frac{1}{2}$

c $1\frac{4}{7}x = -1$ **d** $3\frac{1}{4}x = 2\frac{3}{5}$

11 A small rectangular garden lawn has length $4\frac{2}{5}$ metres and width w metres. The lawn has area 11 m².



a Show that $w = \frac{5}{2}$.

b Find the perimeter of the lawn in metres. Give your answer as a mixed number.

1.2 Surds

1 Simplify these.

a $3\sqrt{5} + 4\sqrt{5}$ **b** $2\sqrt{3} \times 3\sqrt{2}$

c $\frac{4\sqrt{10}}{2\sqrt{5}}$ **d** $(2\sqrt{5})^2$

e $(-2\sqrt{2})^3$ **f** $8\sqrt{\frac{7}{16}}$

2 Express these in simplified surd form.

a $\sqrt{45} + \sqrt{20}$ **b** $\sqrt{32} - \sqrt{18}$

c $2\sqrt{48} - 4\sqrt{27}$

3 By simplifying each surd, find the value of $\frac{\sqrt{50} + \sqrt{32}}{\sqrt{72} - \sqrt{18}}$.

4 Simplify these expressions.

a $(1 + \sqrt{2})(2 + \sqrt{2})$

b $(4 + \sqrt{3})(1 - \sqrt{3})$

c $(\sqrt{2} + 3)(2\sqrt{2} - 1)$

d $(2 + \sqrt{6})^2$

e $(5 - 2\sqrt{3})(4 + 3\sqrt{3})$

f $(\sqrt{2} + \sqrt{3})^2$

5 Express these fractions in the form $a + b\sqrt{3}$, where a and b are integers.

a $\frac{1}{2 + \sqrt{3}}$ **b** $\frac{12}{3 - \sqrt{3}}$

c $\frac{4\sqrt{3}}{\sqrt{3} + 1}$

6 Simplify these fractions.

a $\frac{5 + \sqrt{7}}{3 - \sqrt{7}}$ **b** $\frac{4 - \sqrt{3}}{2 - \sqrt{3}}$

c $\frac{3 + \sqrt{3}}{3 - 2\sqrt{3}}$

7 Show that $\frac{\sqrt{24} - 6}{3 - \sqrt{6}}$ is an integer, stating its value.

8 Given that $D = b^2 - 4ac$

a find the value of $\sqrt{D}$ when

i $a = 2, b = 4, c = 1$

ii $a = 3, b = -4, c = -2$

Give each answer in simplified surd form.

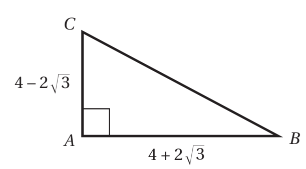
b Explain why $\sqrt{D}$ does not have a real value when $a = b = c$ and $b > 0$.

9 Express these in simplified surd form.

a $\sqrt{75}$ **b** $\sqrt{180}$

c $\sqrt{192}$ **d** $\sqrt{150} + \sqrt{96}$

10 ABC is a right-angled triangle. $AB = 4 + 2\sqrt{3}$, $AC = 4 - 2\sqrt{3}$.



a Find the area of this triangle.

b Show that $BC^2 = 56$.

c Hence find the exact perimeter of this triangle. Give your answer in the form $a + b\sqrt{14}$, where a and b are integers to be stated.

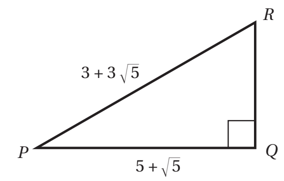
11 **a** Find the greatest of these numbers. You may use a calculator if you wish.

$1 + \sqrt{3}, 2 + 2\sqrt{3}, 3 + \sqrt{3}$

b Show that these three numbers are sides of a right-angled triangle.

c Find the area of this triangle, giving your answer in the form $a + b\sqrt{3}$, where a and b are integers to be stated.

12 PQR is a right-angled triangle. $PQ = 5 + \sqrt{5}$, $PR = 3 + 3\sqrt{5}$.



a Expand and then simplify $(1 + \sqrt{5})^2$.

b Show that $QR^2 = 24 + 8\sqrt{5}$.

c Using your answer to part **a**, find the *exact* length QR.

d Show that the area and perimeter of this triangle are numerically equal.

1.3 Indices

1 Express each of these in the form 2^n where n is an integer.

a $2^3 \times 2^4$ **b** $(2^3)^3$

c 4^5 **d** $(2^4 \times 4^2)^3$

2 Express these as fractions in their simplest terms.

a 4^{-2} **b** 2^{-4}

c 5^{-3} **d** $2^{-1} \times 3^{-3}$

3 Evaluate these.

a $9^{\frac{1}{2}}$ **b** $4^{\frac{1}{2}} \times 27^{\frac{1}{3}}$

c $\frac{64^{\frac{1}{3}}}{16^{\frac{1}{4}}}$ **d** $32^{\frac{1}{5}} \times 17^0$

4 Evaluate these. Where appropriate, give answers as fractions in their simplest form.

a $4^{\frac{3}{2}}$ **b** $27^{\frac{2}{3}}$

c $8^{\frac{5}{3}}$ **d** $81^{\frac{3}{4}}$

e $16^{-\frac{1}{2}}$ **f** $8^{-\frac{1}{3}}$

g $125^{-\frac{2}{3}}$ **h** $64^{-\frac{4}{3}}$

5 By writing 16 as a power of 2, or otherwise, solve the equation $16^x = 32$.

6 Solve these equations.

a $8^x = 16$ **b** $16^x = 64$

c $9^x \times 3^x = 9$ **d** $\frac{8^x}{4^{x+1}} = 32$

7 Express these terms in the form ax^n where a is a real number.

a $\frac{4x}{2x^2}$ **b** $\frac{1}{2x^3}$

c $3x\sqrt{x}$ **d** $\frac{\sqrt[3]{x^2}}{4}$

e $\frac{2}{\sqrt{x}}$ **f** $\frac{3x}{\sqrt[3]{x}}$

g $\frac{3\sqrt{x^3}}{6x^2}$ **h** $\frac{10x}{\sqrt[4]{x^3}}$

8 Determine whether each of these statements is true or false. Use rules of indices to prove those which you think are true. For those statements that you think are false, give an example to show that it is incorrect.

a $a^n \times a^n = a^{2n}$ for all numbers a and positive integers n

b $a^n \times b^n = ab^n$ for all numbers a and b and positive integers n

c $a^{mn} = a^m \times a^n$ for all numbers a and positive integers m and n

d $a^n \times a^{-n} = 1$ for all non-zero numbers a and positive integers n

e $(a^n)^2 = a^{2n}$ for all numbers a and positive integers n

9 Evaluate these expression using the given value of x. Give answers as top-heavy fractions where appropriate.

a $3x^{\frac{3}{2}}$ when $x = 4$

b $x^{\frac{1}{2}} - x^{-\frac{1}{2}}$ when $x = 9$

c $\frac{1}{2}x^{\frac{1}{3}} + 16x^{-2}$ when $x = 8$

d $12x^{-\frac{2}{3}} - 6x^{-\frac{1}{2}}$ when $x = 64$

e $4x^{\frac{1}{2}} + x^{-\frac{1}{2}}$ when $x = \frac{1}{4}$

f $4x^{-1} + \sqrt{2}\,x^{\frac{1}{2}}$ when $x = \frac{8}{9}$

10 **a** Express $\frac{3x^3 + 2}{x^2}$ in the form $ax + bx^n$, where a, b and n are constants.

b Express $\frac{2x^2 - 3x + 1}{2x^2}$ in the form $a + bx^{-1} + cx^{-2}$, where a, b and c are constants.

11 Express these as sums of powers of x.

a $\frac{(2x + 1)(x - 1)}{x}$

b $\frac{(3x + 2)^2}{x^3}$

c $\frac{x^2 + 3x - 6}{\sqrt{x}}$

d $\frac{(2 + \sqrt{x})^2}{x^2}$

12 A curve C has equation $y = \frac{(3x + 2)(2x + 3)}{x^2}$ where $x > 0$.

a Express y in the form $a + bx^{-1} + cx^{-2}$, where a, b and c are constants.

b Explain why, as x increases, the value of y approaches 6.

c Is there a point on this curve with y-coordinate 6?