Gradients & areas under a curve

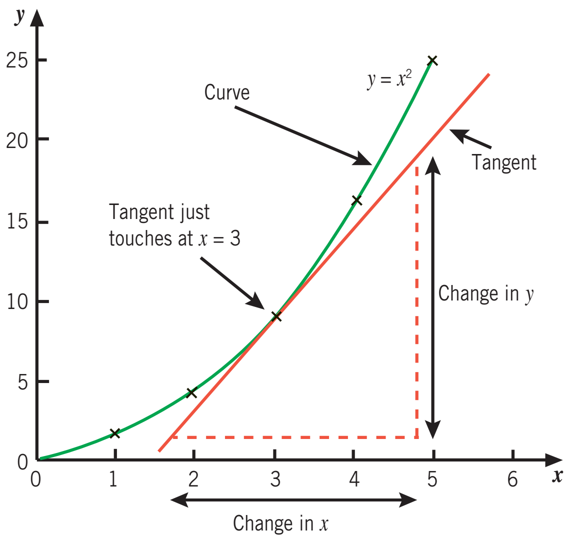
Tangents and gradients

For a curved graph, the **gradient** is constantly changing.

In order to find the gradient of a particular point on a curved graph, a **tangent** needs to be drawn at that point.

The gradient of the tangent drawn is then worked out.

For example:



1. Draw the tangent so that it just touches the curve at that point.
2. Choose two points on the tangent and draw a triangle.
3. Find the gradient of the triangle (do not count the squares as the scales may be different).

   $$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x}$$

4. Decide whether the gradient is uphill (+) or downhill (–).

For the tangent shown above:

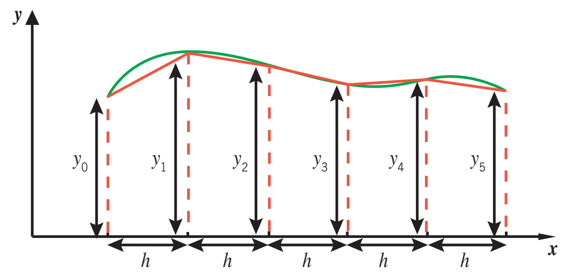
$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x} = \frac{19 - 1}{4.8 - 1.7} = \frac{18}{3.1} = 5.806$$

Area under a curve – the trapezium rule

For a curved graph, you can find an approximate area by splitting up the graph into trapeziums. The area of each trapezium is then calculated and then added together. Another way is to use the trapezium rule, which is given by

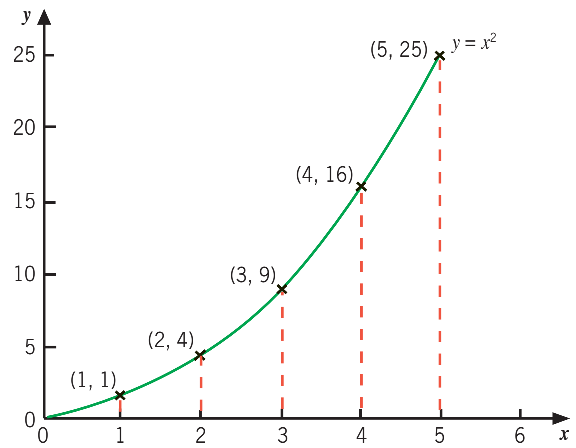
$$\text{Area} = \frac{h}{2}(y_0 + 2(y_1 + y_2 + y_3 + \ldots y_{n-1}) + y_n)$$

where h is the width of the trapeziums and y_1, y_2 … y_n are the lengths of the parallel sides of the trapeziums. (Note: the trapeziums must be of equal width).



Example

Find the area under the graph $y = x^2$ for the values of x between 0 and 5.



Using the trapezium rule gives:

$h = 1$

$A = \frac{1}{2}(0 + 2(1 + 4 + 9 + 16) + 25)$

$= \frac{1}{2}(0 + 2(30) + 25) = \frac{1}{2} \times (85)$

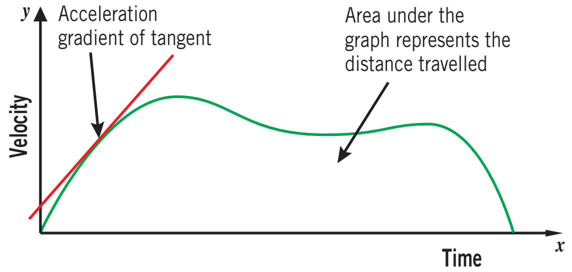
Area = 42.5 squared units

Interpreting area and gradients

The **gradient** represents the **rate of change**. The area under a graph is the **total amount**.

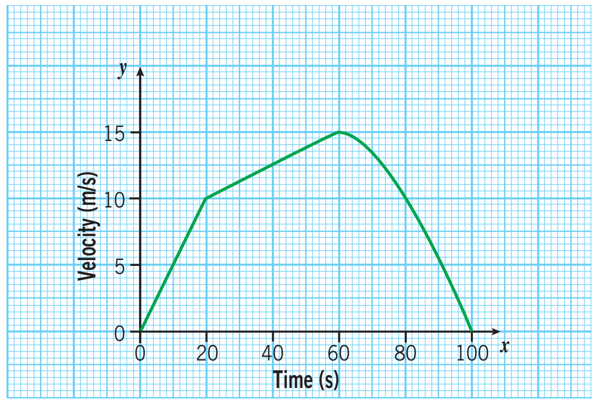
Velocity-time graphs often appear on the exam paper. The acceleration or deceleration can be found by drawing a tangent at that point.

The area under the graph represents the total distance travelled.



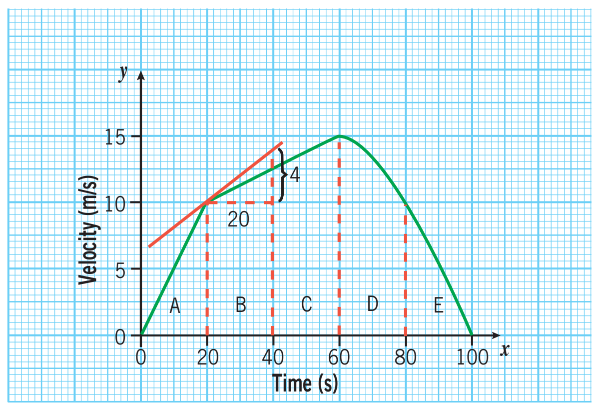
Example

Here is the velocity-time graph of a skier:



a) Calculate an estimate for the acceleration of the skier 20 seconds after the start.

Acceleration is the gradient of the curve. So draw the tangent at 20s, then find its gradient.



Gradient $= \frac{4}{20} = 0.2\ \text{ms}^{-1}$ ← Care needs to be taken when finding the gradient since the scales on both axes are different.

b) Estimate the total distance travelled by the skier.

This is the area under the graph. Use triangles and trapeziums to estimate this area.

$$\begin{aligned}
\text{Distance} &= A + B + C + D + E \\
&= [\tfrac{1}{2} \times 20 \times 10] + [\tfrac{1}{2} \times 20 \times (10 + 12\tfrac{1}{2})] + [\tfrac{1}{2} \times 20 \times (12\tfrac{1}{2} + 15)] + [\tfrac{1}{2} \times 20 \times (15 + 10)] + [\tfrac{1}{2} \times 20 \times 10] \\
&= 100 + 225 + 275 + 250 + 100 \\
&= 950\text{m}
\end{aligned}$$