Adding and subtracting integers

Adding a **negative** number **or** subtracting a **positive** number will have the **same result**.

Adding a positive number **or** subtracting a negative number will have the **same result**.

$$3 + –5 = –2$$
$$3 – +5 = –2$$

Go down by 5.

Use a number line to visualise the answer.

Adding a negative number means subtract.

Subtracting a negative number means add.

$$–1 + +4 = +3$$
$$–1 – –4 = +3$$

Go up by 4.

+ +	means	+
+ –	means	–
– +	means	–
– –	means	+

Multiplying and dividing integers

Look at these examples.

Multiplying a negative number by a positive number always gives a negative answer.

$$–5 \times +3 = –15$$
$$+5 \times –3 = –15$$

Multiplying two positive numbers **or** multiplying two negative numbers always gives a positive answer.

$$+4 \times +3 = +12$$
$$–4 \times –3 = +12$$

The same rules work for division.

$$+10 \div –5 = –2$$
$$–10 \div –5 = +2$$

This table summarises the rules:

+	× or ÷	+	=	+
+	× or ÷	–	=	–
–	× or ÷	+	=	–
–	× or ÷	–	=	+

A positive number multiplied by a negative number gives a negative answer.

A negative number multiplied by a negative number gives a positive answer.

KEYWORDS

Integer ➤ An integer is a whole number; it can be positive, negative or zero.

Positive ➤ A number above zero.

Negative ➤ A number below zero.

Use of symbols

Look at the following symbols and their meanings.

Symbol	Meaning	Examples
$>$	Greater than	$5 > 3$ (5 is greater than 3)
$<$	Less than	$-4 < -1$ (-4 is less than -1)
$\geqslant$	Greater than or equal to	$x \geqslant 2$ (x can be 2 or higher)
$\leqslant$	Less than or equal to	$x \leqslant -3$ (x can be -3 or lower)
$=$	Equal to	$2 + {+3} = 2 - {-3}$
$\neq$	Not equal to	$4^2 \neq 4 \times 2$ (16 is not equal to 8)

Place value

Look at this example.

Given that $23 \times 47 = 1081$, work out 2.3×4.7

The answer to 2.3×4.7 must have the digits 1 0 8 1 ← Do a quick estimate to find where the decimal point goes.

2.3 is about 2 and 4.7 is about 5. Since $2 \times 5 = 10$, the answer must be about 10.

Therefore $2.3 \times 4.7 = 10.81$

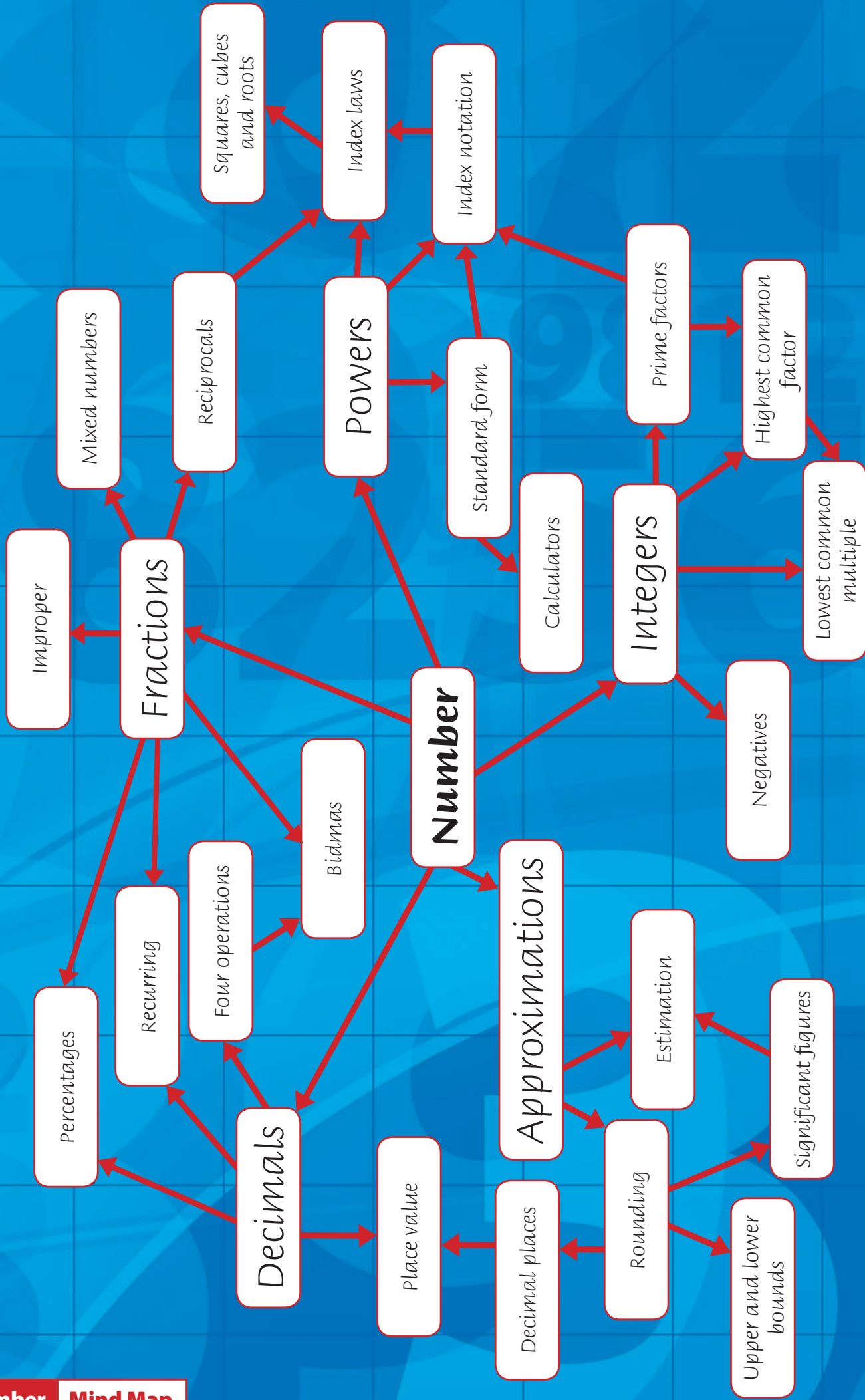Write the following symbols and numbers on separate pieces of paper.

| + | − | × | ÷ | = | 0 |

| +2 | −2 | +4 | −4 | +8 | −8 |

Arrange them to form a correct calculation.

How many different calculations can you make? For example:

| +2 | − | −2 | = | +4 |

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1. Calculate the following:
   (a) $-5 - -8$
   (b) $-2 + -6$
   (c) $-7 + -3 - -5$
2. Calculate the following:
   (a) -12×-4
   (b) $24 \div -3$
   (c) $-3 \times -4 \times -5$
3. State whether these statements are true or false.
   (a) $6 < 3$
   (b) $-4 > -5$
   (c) $2 + -3 = 2 - +3$
4. Given that $43 \times 57 = 2451$, calculate the following:
   (a) 4.3×0.57
   (b) 430×570
   (c) $2451 \div 5.7$

Number

Squares, cubes and roots

Index laws

Index notation

Mixed numbers

Reciprocals

Powers

Standard form

Prime factors

Highest common factor

Improper

Fractions

Calculators

Integers

Lowest common multiple

Bidmas

Negatives

Percentages

Recurring

Four operations

Approximations

Estimation

Significant figures

Decimals

Place value

Decimal places

Rounding

Upper and lower bounds

1. Work out the following. 📱

   (a) 4×5.8 **[2]**   (b) 2.3×42.7 **[3]**

   (c) $24 \div 0.08$ **[2]**   (d) $46.8 \div 3.6$ **[2]**

   (e) $-3 + -4$ **[1]**   (f) $-2 - -5 + -6$ **[1]**

   (g) $45 \div -9$ **[1]**   (h) $-4 \times -5 \times -7$ **[1]**

   (i) $4 + 3^2 \times 7$ **[1]**   (j) $(5 - 6^2) - (4 + \sqrt{25})$ **[2]**

2. (a) Write 36 as a product of its prime factors. Write your answer in index form. 📱 **[2]**

   (b) Write 48 as a product of its prime factors. Write your answer in index form. 📱 **[2]**

   (c) Find the highest common factor of 48 and 36. 📱 **[2]**

   (d) Find the lowest common multiple of 48 and 36. 📱 **[2]**

3. (a) Work out the following.

   $$\frac{4}{3} \times \frac{5}{6}$$

   Write your answer as a mixed number in its simplest form. 📱 **[2]**

   (b) The sum of three mixed numbers is $7\frac{11}{12}$. Two of the numbers are $2\frac{3}{4}$ and $3\frac{5}{6}$.

   Find the third number and give your answer in its simplest form. 📱 **[3]**

   (c) Calculate $3\frac{1}{5} \div 2\frac{1}{4}$.

   Give your answer as a mixed number in its simplest form. 📱 **[3]**

4. Simplify the following. 📱

   (a) $5^3 \div 5^{-5}$ **[1]**   (b) $\sqrt{98}$ **[1]**   (c) $\sqrt{7}(4 - 3\sqrt{7})$ **[2]**

5. Evaluate the following. 📱

   (a) 2.3^0 **[1]**   (b) $9^{-\frac{1}{2}}$ **[2]**   (c) $27^{\frac{4}{3}}$ **[2]**

6. (a) Write the following numbers in standard form. 📱

   (i) $43\,600$ **[1]**   (ii) $0.008\,03$ **[1]**

   (b) Calculate the following, giving your answer in standard form. 📱

   $(1.2 \times 10^8) \div (3 \times 10^4)$ **[2]**

7. Prove the following. You must show your full working out. 📱

   (a) $0.4\dot{8} = \frac{16}{33}$ **[2]**   (b) $0.1\dot{2}\dot{3} = \frac{61}{495}$ **[2]**

8. Rationalise the following surds. 📱

   (a) $\frac{7}{\sqrt{5}}$ **[2]**   (b) $\frac{3}{1 + \sqrt{2}}$ **[2]**

9. Paul's garage measures 6m in length to the nearest metre. His new car measures 5.5m in length to 1 decimal place.

   Is Paul's garage definitely long enough for his new car to fit in? Show your working. 📱 **[3]**

10. $a = 4.3$ and $b = 2.6$ to 1 decimal place.

   Find the minimum value of $\frac{a}{b}$ and give your answer to 3 decimal places. **[3]**

1 If Shabir has 250ml of soup for her lunch, how many kilocalories of energy will she get?

SOUP
100ml = 59kcal

.... [2 marks]

2 Leon changes £500 to euros at the rate shown and goes to France on holiday.

| £1 = 1.29 euros | £1 = 187.99 Japanese yen |
| £1 = 1.56 US dollars | £1 = 97.04 Indian rupees |

(a) How many euros does he take on holiday? .... [1 mark]

Leon spends €570.

(b) He changes his remaining euros on the ferry where the exchange rate is £1 : €1.33

How much in pounds sterling does he take home? .... [2 marks]

3 James' dairy herd of 80 cattle produces 1360 litres of milk per day.

(a) If James buys another 25 cattle and is paid 30p/litre, what will his annual milk income be? .... [4 marks]

(b) If 6 tonnes of hay will last 80 cattle for 10 days, how long will the same amount of hay last the increased herd? [2 marks]

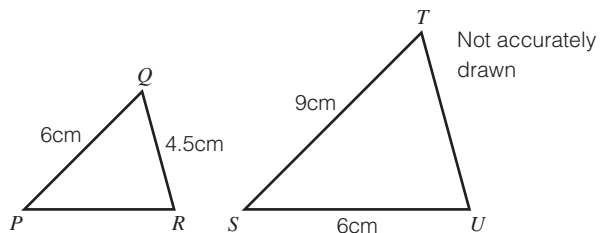..

4 Triangles *PQR* and *STU* are similar.

Find the missing lengths *PR* and *TU*. [4 marks]

PR = ...

TU = ...

Q
6cm
4.5cm
P
R

T
Not accurately drawn
9cm
S
6cm
U

5 Two similar cylinders P and Q have surface areas of 120cm² and 270cm².

If the volume of Q is 2700cm³, what is the volume of P? [3 marks]

Q

P

Surface area: 120cm²

Surface area: 270cm²

Volume: 2700cm³
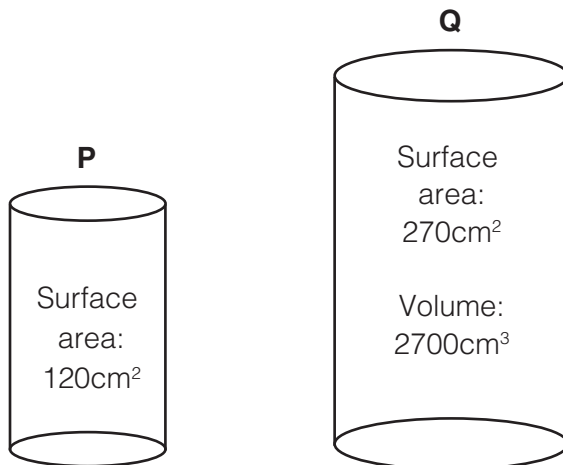
.... cm³

Score **/18**

For more help on this topic, see Letts GCSE Maths Higher Revision Guide pages 50–51.

1 **(a)** Peter invests £10 000 in a savings account which pays 2% compound interest per annum.

How much will his investment be worth after four years? [2 marks]

....

(b) Paul invests £10 000 in company shares.
In the first year the shares increase in value by 15%.
In the second year they increase by 6%.
In the third year they lose 18% of their value.
In the fourth year the shares increase by 1%.

What is his investment worth after four years? .... [3 marks]

2 Lazya invests £6500 at 3% compound interest for three years. She works out the first year's interest to be £195. She tells her family she will earn £585 over three years.

Is she right? Show working to justify your decision. [3 marks]

....

....

3 This graph shows a tank being filled with water. 🖩

(a) How deep is the water when the tank is full?

.... [1 mark]

(b) Between what times is the tank filling fastest?

.... [1 mark]

(c) Work out the rate of decrease of water level as the tank empties.
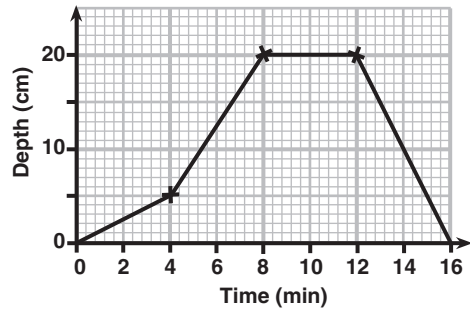
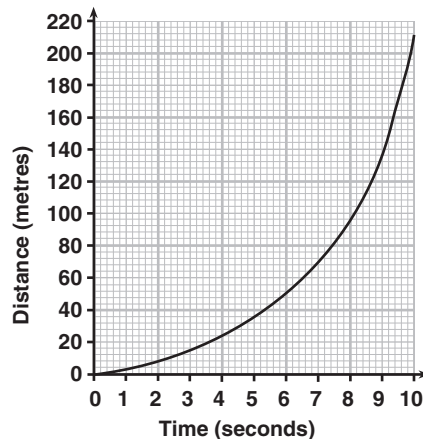.. [1 mark]

4 This graph shows the distance travelled by a cyclist for the first 10 seconds of a race.

(a) Work out the cyclist's average speed for the first 10 seconds.

.... [2 marks]

(b) Estimate the actual speed at 5 seconds.

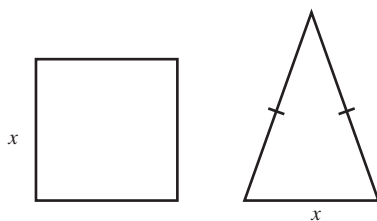.. [3 marks]

Score **/16**

For more help on this topic, see Letts GCSE Maths Higher Revision Guide pages 52–53.

14. Here is a square and an isosceles triangle.

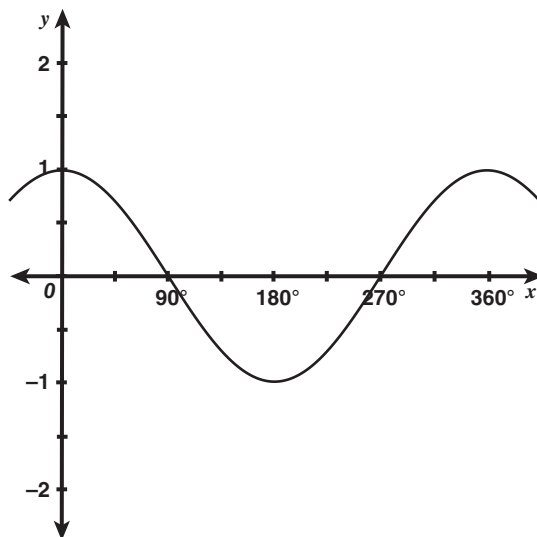The length of each of the equal sides of the triangle is 3cm greater than the side of the square.

(a) If the perimeters of the two shapes are equal, find the value of x. **[3]**

....

(b) Show that the height of the triangle is equal to the diagonal of the square. **[3]**

....

....

....

....

15. The graph of $y = \cos x$ is shown for $0° \leqslant x \leqslant 360°$
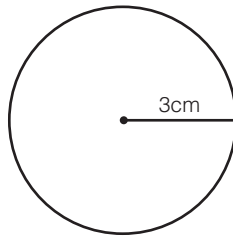

On the same grid, sketch and label the graphs of

(a) $y = -\cos x$ **[2]**

(b) $y = \cos x + 1$ **[2]**
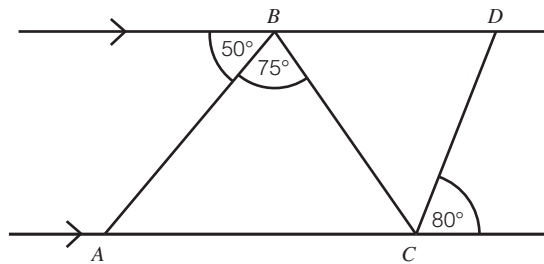
16. Calculate the circumference of this circle.

Leave your answer in terms of π. **[2]**

.... cm

17. Calculate angle BCD, giving your reasons. **[3]**

....

....

....

....

18. Work out the next term of this quadratic sequence:

−2   3   14   31 **[2]**

....