|  |  |
| --- | --- |
| **Guidance on the use of codes for this mark scheme** | |
| M | Method mark |
| A | Accuracy mark |
| B | Working mark |
| C | Communication mark |
| P | Proof, process or justification mark |
| cao | Correct answer only |
| oe | Or equivalent |
| ft | Follow through |

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Question** | | **Working** | **Answer** | **Mark** | | **AO** | **Notes** | **Grade** | |
| **1 a i**  **ii**  **iii**  **iv**  **b i**    **ii**    **iii**    **iv**  **c** | |  | No  Yes  Yes  No  Assume third angle is 90°. 180° – 90° = 90°. So both the remaining angles must be acute. If the third angle is bigger than 90° both remaining must also be acute. If the third angle is acute you would need to make one of the other angles at least 90°.  Example with 2 acute angles.  Example with 1 obtuse angle.  The obtuse angles will be (90° + x) and (90° + y), adding these together you get 180° + x + y, which is more than the sum of the angles in a triangle, so it’s impossible.  If you draw a line between two parallel lines, the two allied angles formed add up to 180°, which leaves nothing for a third angle. | B1  B1  B1  B1  C1  C1  B1  B1  C1  C1  C1  C1 | | 2 | C1 for full explanation  C1 for clarity of explanation  B1 for giving an example that works  B1 for giving an example that works  C1 for a correct explanation  C1 for clear communication  C1 for clear explanation  C1 for clarity of the communication | B | |
| **12** | |
| **2** | |  | Sometimes. Here are examples, one of when it is not true and on of when it is true.  Shape A: perimeter of 14 cm  area 10 cm2  Shape B; perimeter of 16 cm  area 12 cm2  Shape C; perimeter of 18 cm  area 8 cm2  Statement is true for A and B, but false for B and C. | B1  B2  C1 | | 2 | B1 for sometimes  B1 for example that shows it can be true  B1 for example that shown it can be false  C1 for clear communication of both | B | |
| **4** | |
| **3** | |  | True. Demonstration of proof of area triangle equal to half area of rectangle true also for non-right angled triangle.  Area of ABT =  of AEBT   =  of 36 cm2 = 18 cm2  Area of CTB =  of CTBF   =  of 12 cm2 = 6 cm2  Area of triangle ABC = 18 + 6 = 24 cm2  =  × 4 × 12  =  × area AEFC | B1  C1  C1 | | 2  3 | B1 for true  C1 for clear explanation  C1 for concise communication with clear diagrams | B | |
| **3** | |
| **4** | | A rotation of 90°anticlockwise around point (2, 2). |  | P1  A1  A1  C1 | | 3 | P1 for a process of finding the centre of rotation  A1 for indicating 90° anticlockwise (or 270°clockwise)  A1 for indicating centre of rotation as (2, 2)  C1 for full, clear description | B |
| **4** | |
| **5** | Area of front and back =  2 × 12 × 25 = 600 m2  Area of sides = 2 × 12 × 12 = 288 m2  Area of openings = 40 × 2 × 1 = 80 m2  Total area to be painted = 600 + 288 – 80  = 808 m2  For 2 coats of paint, area = 2 × 808  = 1616 m2  Number of litres of paint needed = 1616 ÷ 16 = 101 litres  Number of cans of paint = 101 ÷ 10 = 10.1  11 cans are needed.  Cost of paint = 11 × £25 = £275  Assume painters work 5 days per week.  Number of days = 2 × 5 = 10  Cost of painters = 10 × 3 × 120 = £3600  Total cost = £275 + £3600 + £500 = £4375  Add 10%: £4375 × 1.1 = £4812.50  Add 20% VAT: £4812.50 × 1.2 = £5775 | | The builder should charge the council £5775. | | M1  M1  A1  B1  M1  A1  M1  A1  M1  A1  A1 M1  M1  A1  C2 | 2  3 | M1 for correct formula for area of rectangle  M1 for correct method of finding total surface area  A1 for 808 cao  M1 for correct method of finding number of tins  A1 for correct number of tins used  M1 for method of finding cost of tins  A1 for 275 cao  M1 for method of calculating cost for two days  A1 for 3600 cao  A1 for 4375 cao  M1 for correct calculation of 10%  M1 for correct calculation of 20%  A1 for correct total cost £5775  C1 for clear explanation marks with structure and technical use of language in explanation and  C1 for stating any necessary assumptions | B |
| **16** |
| **6** **a**  **b**  **c** | Area of face = 42 = 16 m2  Area of circle = πr2  Using π = 3.142  area of circle = π1.22  = 4.524 48 m2  Remaining SA of front face = 16 – 4.524 48  = 11.475 52 m2  Total remaining surface area:  front and back = 2 × 11.47552  = 22.95104 m2  Area of other four sides = 4 × 16 = 64 m2  Total = 64 + 22.951 04 = 86.951 04 m2  Volume of original cuboid = 43 = 64 m3  Volume of cylinder = πr2h  = πr24  = 4.524 48 × 4  = 18.097 92 m3  Remaining volume = 64 – 18.097 92  = 45.902 08 m3  Amount of light blue paint  = outside area ÷ coverage of 1 litre of paint  = 87 ÷ 9 = 9.666  Surface area inside cylinder = 2πrh  = 2 × 3.142 × 1.2 × 4  = 30.1632 m2  30.1632 ÷ 9 = 3.3515 | | 87.0 m2  45.9m3  Amount of light blue = 9.7 litres  Amount of dark blue = 3.4 litres | | M1  M1  A1  A1  A1  A1  A1  M1  A1  M1  A1  A1  M1  A1  M1  A1  A1 | 3 | M1 for the correct method of finding area of a rectangle  M1 for correct method of finding area of a circle  A1 for correct area of circle  A1 for correct area of face with circle.  A1 for correctly combining front and back  A1 for correct area of the other 4 sides  A1 for correct total area, rounded to 2,3 or 4 sf  M1 for correct method for finding volume of cube  A1 for 64  M1 for correct method for finding volume of cylinder  A1 for a correct volume of cylinder (any rounding)  A1 for correct total volume, rounded to 2,3 or 4 sf  M1 for dividing total outside surface by 9  A1 for correct answer rounded to 1,2,3 or 4sf  M1 for correct method of finding curved surface area  A1 for a correct surface area (any rounding)  A1 for correct answer to 2,3 or 4 sf | B |
| **17** |
| **7** |  | | Yes, he is correct.  This is one of the conditions for being able to draw a triangle (SAS). | | C1 | 3 | C1 for clear communication that he is correct | M |
| **1** |
| **8** |  | |  | | B4 | 3 | B1 for each different possible triangle shown and clearly labelled | M |
| **4** |
| **9** |  | | The locus is none of these as it is a point, so d. | | B1  C1  C1 | 3 | B1 for stating d is the only correct option  C1 for a clear explanation of why  C1 for clear communication, using diagrams to illustrate answer | M |
| **3** |
| **10 a**    **b** | Angles in a triangle add up to 180°.  You can split any quadrilateral into two triangles.  Therefore, the sum of the interior angles of any quadrilateral = 2 × 180o.  For the two outside triangles, use the sum of the angles in triangle = 180° and the interior angle of a regular pentagon = 108°.  108° + 2x = 180°  2x = 180° – 108°  2x = 72°  x = 36°  For the middle triangle, use interior angle of regular pentagon = 108°.  y = 108° – x  y = 108° – 36°  y = 72°  Using the sum of angles in triangle = 180°:  z = 180 – 2y  z = 180 – 144  z = 36o  Or 2x + z = 108°  x = 36°  so 2 × 36° + z = 108°  z + 72° = 108°  z = 36° | | Two triangles with one angle = 108° and two other equal angles of 36°.  One triangle with one angle = 36° and two angles = 72°. | | C1  C1  B1  C1  C1  C1  M1  A1  M1  A1  M1  A1  C1  C1 | 2 | C1 for clear explanation  C1 for communication with clear diagram  B1 for showing inter angles of quadrilateral = 2 × 180  C1 for a clear diagram correctly showing the three triangles  C1 for identifying each angle of the triangles with symbols showing which angles are equal  C1 for clear explanation  M1 for correct method of finding angle x  A1 for 36°  M1 for correct method of finding angle y  A1 for 72°  M1 for correct method of finding angle z  A1 for 36°  C1 for clear argument and stating assumptions used  C1 for use of diagram with clarity of explanation | M |
| **14** |
| **11** |  | | A line of symmetry has the same number of vertices on each side of the line, so there is an even number of vertices and therefore an even number of sides. | | C2  C1 | 2 | C1 for line of symmetry and number of vertices link  C1 for reference to even number of vertices  oe  C1 for use of diagram to illustrate the answer | M |
| **3** |
| **12 a**    **b**  **c** |  | | Suitable diagram, e.g.  Suitable diagram, e.g. as part a  In a parallelogram opposite sides are equal.  In a trapezium at least one set of opposite sides are parallel.  Therefore every parallelogram is also a trapezium. | | B1  B1  B1  C1 | 2  3 | B1 for a correct diagram  B1 for a correct diagram  B1 for a correct diagram of a parallelogram  C1 for a correct explanation to support the diagram | M |
| **4** |
| **13** |  | | Always true. To follow a path around the perimeter of any polygon, you must turn through a total of 360° to get back to where you started. Therefore the external angles of every polygon sum to 360o. | | B1  C1  P1 | 2 | B1 for always true  C1 for a satisfactory explanation  P1 for use of diagram to illustrate answer | M |
| **3** |
| **14** | Ratio = 6 : 5 : 7  6 + 5 + 7 = 18  Sum of the angles in a triangle = 180°  and 180° ÷ 18 = 10°  Therefore the angles are:  6 x 10° = 60°  5 x 10° = 50°  7 x 10° = 70°  Check:  60 + 50 + 70 = 180 | | 60°, 50°, 70° | | M1  C1  M1  B3  P1 | 2 | M1 for summing parts of ratio  C1 for clear statement regarding angle sum of triangle  M1 for dividing 180° by 18  B1 for each correct angle found  P1 for showing the check that the answers sum to 180° | M |
| **7** |
| **15** | Assume the height of one large triangle is equal to the radius of the large circle, 6 cm, and its base is equal to the diameter of the small circle, 3 cm.  Consider one shaded triangle.  Its height is (6 – 1.5) = 4.5 cm  The shaded triangle and the large triangle shown are similar triangles, where  =  Hence shaded base = 3 ×  = 2.25  The area of one shaded triangle will be  area =  × 2.5 × 4.5 = 5.0625 cm2.  As an estimate, call this 5 cm2.  So a reasonable estimate for the area of the five shaded triangles could be:  5 × 5 = 25 cm2 | | 25 cm2 | | C1  B1  M1  A1  M1  A1  M1  A1  C1  P1 | 2 | C1 for stating assumptions clearly  B1 for correct triangle height  M1 for a sensible way to estimate triangle base length  A1 for an accurate calculation at this point  M1 for correct method of finding area of triangle  A1 for an suitably estimated area (1 or 2 sf)  M1 for multiplying one area by 5  A1 for correct final estimation (integer value)  C1 for clear explanation supporting the working  P1 for clear diagrams illustrating the approach | M |
| **10** |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **16 a**  **b** |  | The interior angle of an equilateral triangle is 60°.  The interior angle of a square is 90°.  The interior angle of a regular hexagon is 60°.  All three are factors of 360° so these shapes will tessellate around a point.  This is not true for other regular polygons as their interior angles are not factors of 360°.  The interior angle of a regular octagon is 135°.  The interior angle of a square is 90°.  Using a similar argument to part a:  2 × 135° + 90° = 270° + 90° = 360° | C1  C1  C1  C1  P1 | 2 | C1 for clear explanation of all three shapes  C1 for use of clear diagrams to support the explanation  C1 for clear explanation.  C1 for clear explanation  P1 for use of clear diagrams to support the explanation | M |
| **8** |
| **17** |  | All three sides (SSS).  Two sides and the included angle (SAS).  Two sides and a non-included angle (SSA).  Two angles and a side (ASA or AAS). | B4 | 3 | B1 for each correct statement | M |
| **4** |
| **18 a**  **b**  **c**  **d** |  | True. In a parallelogram opposite sides are parallel.  In a rhombus, opposite sides are parallel and all sides are the same length. So a rhombus is a type of parallelogram.  In a square all sides are the same length.  So a rhombus with right angles must be a square.  True. A rhombus must be a parallelogram (part a) but a parallelogram does not all sides the same length so it does not have to be a rhombus.  True. Using the diagram of a trapezium above, you see each pair of angles between the parallel sides are allied angles, adding up to 180°, so each pair can only have a maximum of one acute angle, hence the whole shape can have no more than two acute angles.  True. A quadrilateral can have three acute angles, e.g. 80°, 80°, 80° and 120°. | B1  C1  C1  B1  C1  P1  B1  C1  P1  B1  B1  M1 | 3 | B1 for true  C1 for clear explanation  C1 for clear explanation  B1 for true  C1 for clear explanation  P1 for clear use of diagrams to support explanation  B1 for true  C1 for clear explanation  P1 for clear use of diagrams to support the explanation  B1 for true  B1 for clear explanation supported by a clear diagram  M1 for clear use of a correct diagram: a diagram is essential | M |
| **12** |
|  |  | Look at the sides and/or angles you have been given and what you need to calculate.  When the triangle has a right angle, use Pythagoras’ theorem when you need to work out one side length and you know the other two side lengths.  Otherwise, when the triangle does not have a right angle, use sine, cosine or tangent when you need to work out an angle or a side.  Use the cosine rule to find angles when all sides of any triangle are known or to find the third side when two sides and the included angle are known.  Use the sine rule when two sides and one angle other than the included angle are known, or two angles and one side are known. | C1  C1  C1  C1 | 3 | C1 for clear Pythagoras explanation  C1 for clear right angled trig explanation  C1 for clear cosine rule explanation  C1 for clear sine rule explanation | M |
| **4** |
| **20 a i**  **ii**  **b i**  **ii** |  | A suitable simple reflection.  A suitable reflection with a mirror line that is parallel to one of the sides of the shape.  A suitable simple rotation.  A suitable rotation with centre not on an extension of one of the sides of the shape. | B1  C1  B1  C1 | 2  3 | B1 for a diagram of a simple reflection  C1 for a clear explanation  B1 for a diagram of a simple rotation  C1 for a clear explanation | M |
| **4** |
| **21 a**  **b**  **c**  **d i**  **ii** | Diagram to help explanation:  The image is smaller than the object. | The lengths of the sides change by the scale factor. Angles in the shape stay the same.  The **scale factor** and **centre of enlargement**.  Suitable explanation of enlargements with use of diagram to help explanation. For example: Draw lines connecting corresponding vertices on the shape and its enlargement. The centre of enlargement is where these lines cross.  Centre of enlargement outside the shape: the shape will move up and down a line that passes through the shape.  Centre inside the original shape: the enlargement is either inside or around the shape depending on whether the scale factor is whole or fractional.  When the centre is on a vertex the shape and enlargement share part of two sides.  When the centre is on a side, the shape and enlargement shape part of the side.  The image is smaller than the object. | C2  B2  C1  C1  C2  P1  C1  C1 | 2  3 | C1 for correct statement about the lengths  C1 for correct statement about the angles  B1 for scale factor B1 for centre of enlargement  C1 for a clear explanation  C1 for a good accurate diagram to support the explanation  C1 for correct explanation of centre outside the shape  C1 for correct explanation of centre within the shape  P1 for clear diagrams used in both explanations  C1 for clear explanation.  C1 for clear explanation. | M |
| **11** |
| **22 a**  **b** |  | When a shape has been translated the orientation is the same.  When it has been reflected its orientation is different.  Rotating a rectangle about its centre: all the vertices move and the image is superimposed over the object.  Rotating the rectangle about one of its vertices: all the other vertices move and, as the angle increases, the image is no longer superimposed over the object. | C2  P1  C1  P1  C1  P1 | 2 | C1 for comment about orientation staying the same in translation  C1 for comment about orientation being different in rotation  P1 for a clear diagram to support the explanation  C1 for clear explanation  P1 for good diagram to support explanation  C1 for clear explanation  P1 for use of diagram to illustrate explanation | M |
| **7** |
| **23** | Cross-sectional area is a quarter of circle with radius 1.5 cm and a rectangle 1.5 cm by 6.5 cm.  area of quarter circle= π1.52  = 1.767 145 9 cm2  Area of rectangle 1.5 × 6.5 = 9.75 cm2  Total area = 1.767 145 9 + 9.75   = 11.517 146 cm2  Total volume of wood is  11.517 146 × 12 000 = 138 205.75 cm2.  Convert this to m2 by dividing by 1 000 000.  Total volume = 0.138 205 75 m2 | 138 000 cm2  or  0.14 m2 | M1  A1  B1  B1  M1  A1 | 2  3 | M1 for method of finding area of the quadrant  A1 for any rounding to 4 or more sf  B1 for 9.75  B1 for any rounding to 4 or more sf  M1 for method of finding volume  A1 for correct answer rounded to either 2 or 3 sf  Accept alternative answer in cubic metres given correctly to 2 or 3 sf | M |
| 6 |
| **24** |  | Area of the sector = πr2  Area of segment   = area of sector – area of triangle  = πr2 –ab sin θ  As *a* and *b* are both equal to r, this becomes:  πr2 – r2 sin θ  Factorising gives:  r2(π –sin θ)  as required. | B1  B1  M1  M1  M1  A1  C1 | 3 | B1 for correct formula of area of sector  B1 for correct formula of area of triangle  M1 for correctly stating the combined equation for segment area  M1 for correct use of *r* in triangle formula  M1 for factorising  A1 for correct factorisation  C1 for clear use of diagram to support explanation | M |
| **7** |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **25** | Using trigonometry:  cos *α* =  α = cos–1  *α* = 33.557 31°  Area of segment of one circle  = πr2 – r2sinθ  = r2(π – sin θ)  where θ = 2 × 33.6° = 67.2°and r = 6 cm.  Area = 62(π –sin67.2°)  = 36(0.586 431 –sin 67.2°)  = 36(0.586 431 – 0.460 931 6)  = 4.517 979 cm2  Area of overlap = 2 × 4.517 979 cm2  = 9.035 959 cm2 | 9.0 cm2 | M1  A1  M1  M1  A1  A1  M1  A1  C1 | 2 | M1 for correct trigonometric statement for angle  A1 for any rounded answer to 2 or more sf  M1 for correct segment formula  M1 for correct factorisation  A1 for correct substitution of radius and a correct angle  A1 for correct answer to 2 or more sf  M1 for multiplying by 2  A1 for correct answer to either 1 or 2 sf  C1 for use of mathematical language and diagrams to support solution | M |
| **9** |
| **26 a**  **b**  **c** | Triangle 11 will move round to sit next to face 13, square 4 will move round to be next to face 12, leaving face 2 opposite to face 13. | 14 faces: the same as the number of polygons in the net.  13  I would create the shape first then draw what I see from above as the plan and from the side as the elevation.  Once I have created the shape, I can measure the lengths and angles concerned. | B1  C1  B1  C1  P1  C2 | 2 | B1 for the 14 faces  C1 for clear explanation  B1 for face 13  C1 for clear explanation  P1 for use of diagrams to support the explanation  C1 for an explanation of the plan  C1 for explanation of elevation | M |
| **7** |
| **27** | Circumference of wheel = πD  = π × 68  = 213.6283 cm  10 km = 10 × 1000 × 100 cm  = 1 000 000 cm  Number of revolutions in 10 km  = 1 000 000 cm ÷ 213.6283 cm  = 4681.028 | 4681 complete rotations | M1  A1  B1  M1  A1 | 2 | M1 for method of calculating circumference of wheel  A1 for full unrounded answer  B1 for use of 1 000 000 as a conversion factor either way round  M1 for correct division with common units  A1 for cao | M |
| **5** |
| **28** | = 5.830 951 9 | x = 5.8 km | C1  C1  M1  M1  A1 | 2 | C1 for use of a correct diagram  C1 for explanation of how and why using Pythagoras  M1 for correct application of Pythagoras’ theorem  M1 for correct method of finding hypotenuse  A1 for correct rounding to 2 or 3 sf | M |
| **5** |
| **29** | Let *c* = the height of the chimney.  = tan 53°  x = *c* tan 53°  = tan 62°  30 + x = c tan 62°  x = c tan 62° – 30  Combining equations 1 and 2 to eliminate x:  c tan 53° = c tan 62° – 30  Rearrange to get *c* on one side of the equation.  30 = c tan 62° – c tan 53°  30 = c(tan 62° – tan 53°)  c =  = 54.182 761 m | 54.2 m | C1  M1  A1  M1  A1  M1  M1  A1 | 2 | C1 for clear correct diagram used  M1 for correct use of trigonometry with x, c and angle 53° or 37°  A1 for correct equation having x as subject  M1 for correct use of trigonometry with x, c and angle 62° or 28°  A1 for correct equation in format to combine with equation 1  M1 for correctly eliminating x  M1 for correct equation with c as subject  A1 for correct rounding to 2 or 3 sf | M |
| **8** |
| **30** | Angle at A is 90° + (360° – 330°) = 120°  Angle at B is 290° – 270° = 20°  Angle at S is 180° – (120° + 20°) = 40°  Use the sine rule.  =  x = 15 ×  = 7.981 33 | 8.0 km | C2  B1  B1  B1  M1  A1 | 2 | C1 for diagram illustrating how angles found at A and B  C1 for complete triangle drawn, showing all relevant data  B1 for 120°  B1 for 20°  B1 for 40°  M1 for use of sine rule  A1 for correct answer rounded to 1, 2 or 3 sf | M |
| **7** |
| **31 a**  **b**  **c** |  | Sometimes true.  It is not true if the number of individual cubes has fewer than 3 factors, including 1 and itself, for example, you cannot do it with 7 cubes (factors 1 and 7)  You can only make one cuboid if the number of factors, including 1 is equal to 3, for example, with 21 cubes (factors 1, 3 and 7).    You can make more than one cuboid if the number of cubes has more than 3 factors not including itself, for example, 30 (factors 1, 2, 3 and 5). | C1  C1  C1  C1  C1 | 2 | C1 for sometimes true  C1 for clear explanation for when not true  C1 for clear explanation for when only 1 cuboid could be made  C1 for clear explanation for when more than 1 cuboid could be made  C1 for use of examples to illustrate the explanations | M |
| **5** |
| **32 a**  **b**  **c**  **d** |  | Yes.  Yes.  Yes.  Yes. | B1  P1  B1  P1  B1  P1  B1  P1 | 2 | B1 for yes  P1 for a clear diagram or explanation  B1 for yes  P1 for a clear diagram or explanation  B1 for yes  P1 for a clear diagram or explanation  B1 for yes  P1 for a clear diagram or explanation | M |
| **8** |
| **33** | Assume the cuboid has dimensions x, y and t.  The surface area = 2(xy + xt + yt)  Volume = xyt  Doubling the lengths gives dimensions as 2x, 2y and 2t.  So surface area   = 2(2x × 2y + 2x × 2t + 2y × 2t)  = 2(4xy + 4xt + 4yt)  = 8(xy + xt + yt)  which is 4 times the first area. and  V = 2x × 2y × 2t  = 8xyt  which is 8 times the first volume. | False. | B1  C1  C1  P1  C1  B1  C1  B1 | 2 | B1 for false  C1 for surface area with either specific lengths or a generalisation  C1 for volume with either specific lengths or a generalisation  P1 for showing correct follow through of double the lengths  C1 for a correct statement of SA with their data  B1 for 4 times area  C1 for a correct statement of volume with their data  B1 for 8 times volume | M |
| **8** |
| **34** | Consider just half the shape, where x is the length of the string.  Use Pythagoras’ theorem.  x2 = 102 + 22.52  = 606.25  x =  = 24.622 145  Two lengths of string will be  49.244 289 cm  Subtract the original 45 cm  Gives extension as 4.244 289 | 4.2 cm | C1  M1  M1  A1  A1  A1 | 2 | C1 for clear diagram  M1 for correct Pythagoras statement  M1 for correct method of applying Pythagoras’ theorem  A1 for full answer  A1 for double the initial x  A1 for rounded answer to either 2 or 3 sf | M |
| **6** |
| **35** | Let AC = x, the length of the new road.  Use Pythagoras’ theorem.  x2 = 4.92 + 6.32  = 63.7  x =  = 7.981 228  Current distance = 4.9 + 6.3 = 11.2 km  Saving = 11.2 – 7.981 228  = 3.218 772 km | 3.22 km | C1  M1  M1  A1  B1  M1  A1 | 2 | C1 for use of a diagram to assist the explanation  M1 for clear statement of Pythagoras  M1 for correctly applying Pythagoras’ theorem  A1 for full answer  B1 for 11.2  M1 for subtracting lengths  A1 for correct rounding to 2 or 3 sf | M |
| **7** |
| **36** |  | Yes. *θ* = sin–1  = 53.13°  = 53°to the nearest degree  = 50° to 1 sf  12 cm has range of 11.5 to 12.5 cm  15 cm has range of 14.5 to 15.5 cm  The smallest value for sin *θ* is  which gives *θ* = sin–1 0.7419   *θ* = 47.9°  So there are values that round to 12 cm and 15 cm which will give an angle that rounds to 50°. | B1  P1  P1  B1  C1 | 2 | B1 for yes  P1 for showing that using trigonometry and rounding can give 50°  P1 for showing the ranges of lengths of the sides  B1 for showing the least possible value of the angle given the ranges  C1 for final summary explaining that it is possible | M |
| **5** |
| **37** |  | Use Pythagoras’ theorem.  AC2 = 42 – (2)2  = 16 – 8 = 8  BC2 = 8 = (2)2  = 8  Hence BC = AC, an isosceles triangle. | M1  A1  M1  A1  C1 | 2 | M1 for correct Pythagoras statement  A1 for correct value of AC2  M1 for finding BC2  A1 for correct value of BC2  C1 for clear explanation of sides being the same length | M |
| **5** |
| **38** | AB2 = 22 – 12  = 4 – 1 = 3  AB = | cm | M1  A1  C1 | 2 | M1 for correct Pythagoras statement  A1 for 3  C1 for a clear communication of the method used | M |
| **3** |
| **39** | cos 68° = –cos 112° = –cos 248° = 0.3746  cos 338° = 0.9271 | cos 338° is the odd one out.  All the others have the same numerical value (ignoring signs). | B1  C1 | 2 | B1 for cos 338°  C1 for a clear explanation | M |
| **2** |
| **40 a i**  **ii**  **b** | sin x + 1 = 2  sin x = 1  x = sin–11 = 90°    2 + 3cos x = 1  3cos x = 1 – 2 = –1  cos x = –  x = 109.5°  and 360° – 109.5° = 250.5°  cos 320° = 0.766 044 4  sin–1 0.766 044 4 = 50°  and 180° – 50° = 130° | x = 90°  x = 109.5°, 250.5°  x = 50° and 130° | M1  A1  M1  A1  A1  A1  M1  A1  A1 | 2 | M1 for sin x = 1  A1 for 90°  M1 for first step of solving equation  A1 for correct statement of cos x  A1 for correct angle to 1 dp  A1 for correct angle to 1dp  M1 for method of getting to sin–1  A1 for 50°  A1 for 130° | M |
| **9** |
| **41 a**    **b** | tan x = sin x ÷ cos x  ÷  =  =  = = 1  tan–1 1 = 45°  Alternatively, use Pythagoras’ theorem to work out the length of the third side, e.g.  (side)2 = 16 – 8 = 8  side =  tan x = = 1  tan-1 1 = 45° | tan x = 1  45° | C1  M1  A1  A1  M1  A1 | 2 | C1 for communicating effectively how the sine and cos can be used to find tan (This could be tan x = )  M1 for correct use of tan  A1 for correct combination of surds  A1 for tan *x* = 1  M1 for use of inverse tan or recognising an isosceles triangle  A1 for 45° | M |
| **6** |
| **42** |  | Using Pythagoras’ theorem,  the hypotenuse =  = = 4  Then sin x=    and cos x=  Hence (sin x*)*2+ (cos x*)*2=  +  = = 1 | M1  A1  B1  B1  C1 | 2 | M1 for correct Pythagoras statement  A1 for 4  B1 for correct sin x  B1 for correct cos x  C1 for correct explanation | M |
| **5** |
| **43** | Use the cosine rule.  c2 = a2 + b2 - 2ab cos *C*  = 9 + 16 – 2 × 3 × 4 × cos 60°  = 25 – 24 ×  = 13  c = | cm | C1  M1  A1  B1  M1  A1 | 2 | C1 for clear use of diagram  M1 for correct cosine rule statement  A1 for correct substitution  B1 for cos 60° =  M1 for taking square root  A1 for correct surd form | M |
| **6** |
| **44** |  |  | B1  B1 | 2 | B1 for correct diagram  B1 for correct vector | M |
| **2** |
| **45** |  | No.  To work out the return vector, multiply each component by –1.  The return vector is . | B1  B1  C1 | 2 | B1 for No  B1 for correct vector  C1 for a clear explanation of what Joel should have done | M |
| **3** |
| **46 a**  **b** |  |  | C3  C3 | 2 | C1 for showing construction of 60°  C1 for showing bisection of 60°  C1 for showing bisection of 30° to show 15°  C1 for showing construction of 90°  C1 for showing construction of 60° to leave a 30° angle  C1 for bisecting that 30° angle to leave 75° | H |
| **6** |
| **47** |  | Each angle bisector is the locus of points equidistant from the two arms or sides of the angle.  Hence, where they all meet will be the only point that is equidistant from each of the three sides.  Hence a circle can be drawn inside the triangle, with this centre, that just touches each side of the triangle. | C2  P1 | 2 | M1 for explanation referring to the bisector being equidistant  M1 for the interpretation of the intersections  P1 for the use of a diagram to help interpret the explanation | H |
| **3** |
| **48 a–d**    **e**    **f** |  | A circle, centre P, should pass through each of the nine labelled points. | M1  A1  M1  A1  A1  M1  A1  M1  A1  C1 | 2 | M1 for method of constructing any midpoint  A1 for all three midpoints correct and labelled  M1 for constructing any perpendicular from a vertex to opposite face  A1 for all three correct  A1 all 3 feet correctly labelled  M1 for constructing a midpoint of AO or BO or CO  A1 for all midpoints correct and labelled  M1 for constructing a bisector of LM or LN or MN  A1 for all three bisectors correct and point of intersection labelled P  C1 for correct explanation | H |
| **10** |
| **49** |  | AB = CD (given)  ∠ABD = ∠CDB (alternate angles)  ∠BAC = ∠DCA (alternate angles)  so ΔABX ≡ ΔCDX (ASA) | C3  C1 | 2 | C1 for correct statement with justification  C1 for correct statement with justification  C1 for correct statement with justification  C1 for stating ASA within correct explanation | H |
| **4** |
| **50** |  | AB and PQ are the corresponding sides opposite the 50° angle but they are not equal in length. | C2 | 2 | C1 for stating the corresponding side link  C1 for complete clear statement | H |
| **2** |
| **51 a**  **b** | Area of original rectangle = x2  If the length of the side of the inscribed square is h, then:  h2 =  h2 =  h2 =  Area of inscribed square is h2  = | Joining adjacent midpoints forms a right-angled isosceles triangle, with the perpendicular sides each half the length of the side of the square. All four triangles are congruent. At each midpoint, there are two angles of 45°, leaving the vertex of the new shape being 90°. Since all the sides are equal and all the angles are 90°< the new shape is a square.  The area of the inscribed square is half the area of the original square. | C2  C1  M1  A1  A1  C1 | 2 | C1 for explaining all sides are the same length  C1 for explaining how all angles are 90°  C1 for communicating a given original side length and finding the length of a side of the inscribed square  M1 for correctly using Pythagoras’ theorem to find the length of the side of the new square  A1 for correct expression of h2  A1 for correctly stating area  C1 for clear communication of complete solution | H |
| **7** |
| **52** |  | Each internal angle of an octagon is 135°.  Each internal angle of a hexagon is 120°.  The sum of these two angles is 255°.  The sum of the angles in a quadrilateral is 360° so the sum of the remaining angles is:  360° – 255°  = 105°.  The two remaining angles are equal, as the line joining through the vertices C, J, G and L (and thus through the obtuse angles) is a line of symmetry.  JFG = 52.5° | C2  B1  M1  A1  C2  B1 | 2 | C1 for explanation of 135°  C1 for explanation of 120°  B1 for 255°  M1 for subtraction from 360°  A1 for 105°  C1 for explaining two angles are equal  C1 for clear reasons given as to why  B1 for correct 52.5° | H |
| **8** |
| **53** |  | Use the sine rule in both triangles.        As *A* = *P*, then sin *P* = sin *A*  So  Hence =  Hence =  So p = | M1  A1  A1  M1  B1  P1 | 2 | M1 for use of sine rule  A1 for correct equation  A1 for correct equation  M1 for equating both known angles  B1 for correct statement linking p, a, b and c  P1 for clear communication of the full solution | H |
| **6** |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **54** |  | Use Pythagoras’ theorem to write expressions for z2.  Triangle ABC: z2 = r2 + (s + y)2  Triangle ABD: z2 = w2 + (x + t)2  So r2 + (s + y)2 = w2 + (x + t)2  Now r2 = x2 – s2  and w2 = y2 – t2  Substitute for *r*2 and *w*2 in the equation:  x2 – s2 + (s + y)2 = y2 – t2 + (x + t)2  Multiply out the brackets:  *x*2 – *s*2 + *s*2 + 2*sy* + *y*2   = *y*2 – *t*2 + *x*2 + 2*xt* + t2  Simplify:  x2 + 2sy + y2 = y2 + x2 + 2xt  2sy = 2xt  sy = xt  Alternatively:  Assume T is the intersection of AD and BC. ACT is similar to BDT as both contain the same angles. (Angles ATC and BTD are vertically opposite and so are equal.)  Hence by similarity =  So xt = sy | P1  P1  P1  P1  P1  C1  C1 | 2 | P1 for combining expressions  P1 for substitution  P1 for multiplying out to get individual terms  P1 for simplifying  P1 for clear explanation  C1 for  =  C1 for clear communication of proof | H |
| **7** |
| **55 a**  **b** |  | Take the smallest two sides, square each of them, add the squares together then find the square root of the sum.  If this root is equal to the length of the longest side, then the triangle is right angled.  If the root is smaller than the length of the longest side, then the angle is greater than 90°.  If the root is larger than the length of the longest side then there is no angle greater than 90°.  In the first triangle the hypotenuse is = 13  In the second triangle, the hypotenuse is 12 with the unknown side , which is between 10 and 11 cm.  So they both have a short side of 5 cm.  But the lengths of the other short side and the hypotenuse are different. | C1  C1  C1  C1  B1  B1  C1  C1 | 2 | C1 for correct Pythagoras statement  C1 for clear explanation  C1 for clear explanation  C1 for clear explanation  B1 for 13  B1 for side length between 10 and 11 or more accurate.  C1 for explanation of what is the same  C1 for clear explanation of what is different | H |
| **8** |
| **56** |  | In the top triangle ABM:  (m + t + n)2 = (2x)2 + x2 = 5x2  So m + t + n = x  Putting together shape P and Q:  area = (m + n)t  So the area, A, of the whole shape is:  top triangle ABM + bottom triangle DCN + (P + Q) + *t*2  Area of ABM =  × 2x × x = x2  Area of DCN =  × 2x × x = x2  So total area = x2 + x2 + (m + n)t + t2  = 2x2 + t(m + n + t)  But m + n + t = x  So A = 2x2 + tx  But the sides are of length 2x,  so A = 4x2  Then 2x2 + tx= 4x2  and tx= 2x2  *t*= 2x  Squaring each side gives:  5t2 = 4x2  t2 =  Whole area A = 4x2  Hence the middle square, *t*2, is of A. | C1  M1  A1  M1  A1  M1  A1  M1  B1  M1  A1  M1  A1  C1  C1 | 3 | C1 for clear diagram  M1 for using a large triangle to identify parts  A1 for correct equation  M1 for method of adding P and Q  A1 for (m + n)t  M1 for adding all separate components  A1 for correct statement of A  M1 for substituting (m + n + t)  B1 for whole area 4x2  M1 for equating the two equations  A1 for equation enabling *t* to be identified  M1 for squaring  A1 for correct expression for t2  C1 for clear explanation showing  idea  C1 for complete clear solution | H |
| **15** |
| **57 a**  **b**  **c** |  | Not true.  For example these are both right-angled triangles but their sides are not in proportion.    True.  With an enlargement the final shape will be in the same *proportion* as the original so it will be similar.  True.  All circles are in proportion to each other and so will be similar. | B1  C1  B1  C1  B1  C1 | 2 | B1 for not true  C1 for clear explanation with an example drawn to illustrate  B1 for true  C1 for clear explanation  B1 for true  C1 for clear explanation | H |
| **6** |
| **58 a**  **b** |  | Any two regular polygons with the same number of sides will have the same angles so the ratio of lengths of the sides will be the same. This means the shapes will be similar.  As the corresponding sides of triangle A and triangle B are the same, the two triangles are congruent, SSS. Therefore equivalent angles are the same. This demonstrates the corresponding angles between parallel lines are the same. | C1  P1  C1  P1 | 2 | C1 for clear example  P1 for use of diagrams to support the explanation  C1 for clear explanation  P1 for use of diagram to clarify explanation | H |
| **4** |
| **59 a**  **b**  **c** |  | Enlargement with scale factor – about (–6, 2). | B1  B2  B2 | 2 | B1 for accurate image B drawn  B1 for enlargement of scale factor –2  B1 for correct enlargement to image C  B1 for enlargement scale factor –  B1 for centre of enlargement (–6, 2) | H |
| **5** |
| **60** |  | Enlargement with sf –3 about (1, 2).  Drawing lines from the two vertices at the base of triangle A to their image points in B.  The intersection shows the centre of enlargement.  The base of A is 2 squares wide, the base of B is 6 squares wide, and 6 is 3 times 2, so the sf is 3. | B1  C1  C1 | 2 | B1 for correct statement  C1 for explanation of how centre of enlargement is found  C1 for explanation how sf is found | H |
| **3** |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **61 a**  **b**  **c**  **d** |  | Examine the four possible starting points for the stamp. These are at the top right and bottom left of each side, allowing for 180° rotation of each side. Four rotations mean that each of these points is covered once.  No, the machine would not detect the stamp placed on the top left-hand corner because none of the rotations will put the stamp in the top right-hand corner.  Four corners on each side could possibly be the ‘top right’.  One way is to rotate about H and then rotate about one of the diagonals (call it D). Keep repeating the sequence H, D, H, D, … to check all eight corners. | C1  B1  C1  C1 | 2 | C1 for clear explanation  B1 for No  C1 for clear explanation  C1 for clear explanation | H |
| **4** |
| **62** |  | The hypotenuses (OA and OB) are the same, as each is a radius of the circle.  OM is common to both triangles.  OMA and OMB are both right angles.  Triangles OAM and OBM are congruent, therefore AM = MB.  Therefore, M is the midpoint of AB and the chord has been bisected, as required. | C4  C2 | 2 | C1 for hypotenuse same  C1 for OM common  B1 for right angles  C1 for congruency  C1 for clear explanation and good use of mathematical language.  C1 for use of diagram to support proof | H |
| **6** |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **63** |  | Circle Theorem 1 states that the angle at the centre of a circle is twice the angle at the circumference when they are both subtended by the same arc.  Theorem 1 tells you that any angle subtended from arc AB at a point C on the circumference is always half the angle subtended from arc AB at the centre.  Therefore, every possible angle subtended from arc AB at the circumference will correspond to the same angle at the centre.  Hence all the angles subtended at the circumference will be equal. | P2  C1  C1 | 2 | P1 for stating Theorem 1  P1 for extending this to this proof  C1 for clear overall explanation and clarity  C1 for use of diagram and mathematical language | H |
| **4** |
| **64** |  | Draw triangle BCP, where P is on the circumference in the opposite segment .  Then angle BPC is 180° – *x* as BAC and BPC are opposite angles in a cyclic quadrilateral.  Angle BOC is double BPC, which is  360° – 2x.  Angle BCO will also be y, as BCO is an isosceles triangle.  Angles in triangle OBC sum to 180°,  so 2y + 360° – 2x = 180°  and 2y = 2x – 180°  Divide both sides by 2 to give:  y = x – 90°. | C1  M1  A1  A1  B1  M1  A1  C1 | 2 | C1 for clear use of diagram  M1 for using point P  A1 for 180 – x  A1 for identifying 360° – 2x  B1 for identifying BCO  M1 for adding known angles together  A1 for simplifying  C1 for a clear, well presented solution | H |
| **8** |
| **65** | As ABCD is a cyclic quadrilateral, the opposite angles will sum to 180°.  So 2x – 5° + 5y – 20° = 180°  2x + 5y = 205° ………(1)  And 3y + 5 + 2x + 20° = 180°  2x + 3y = 155° ……..(2)  Subtract (2) from (1):  2y = 50°  y = 25°  Substitute y into (2):  2x + 75° = 155°  2x = 80°  x = 40° | y= 25°  x= 40° | C1  M1  A1  A1  M1  A1  M1  A1 | 2 | C1 for explanation of cyclic quadrilateral  M1 for adding opposite angles  A1 for first correct equation  A1 for second correct equation  M1 for method of eliminating one variable  A1 for 25°  M1 for method of substitution  A1 for 40° | H |
| **8** |
| **66** | Drop perpendiculars down to Y and X and complete the trapezium, with the top triangle being ABT, as shown.  Length TB will be the same as YZ.  Use Pythagoras’ theorem on triangle ABT.  TB2 = 92 – 32  = 81 – 9  = 72  TB =  = 8.485 281 4 | 8.49 cm | C1  C1  C1  M1  A1  A1 | 3 | C1 for good clear diagram showing perpendiculars to Y and Z  C1 for clear diagram of ABT  C1 for clear explanation of how the triangle ABT has been formed  M1 for use of Pythagoras’ theorem  A1 for correct statement  A1 for correct answer to 2 or 3 sf | H |
| **6** |
| **67** | Circle Theorem 1 states The angle at the centre of a circle is twice the angle at the circumference when they are both subtended by the same arc.Theorem 1 tells you that any angle subtended from arc AB at a point C on the circumference is always half the angle subtended from arc AB at the centre.  Therefore, every possible angle subtended from arc AB at the circumference will correspond to the same angle at the centre.  Hence angles subtended at the circumference are equal. |  | P2  C1  C1 | 3 | P1 for stating Theorem 1  P1 for extending this to this proof  C1 for clear overall explanation and clarity  C1 for use of diagram and mathematical language. | H |
| **4** |
| **68 a**  **b** | The angle PQS is the same as PRS, 3x, as they are angles in the same quadrant (subtended by the same arc).  The angle sum of triangle PQT is 180°.  Hence 2x + 3x + 5x = 180°  10x = 180°  x = 18°  Complete the diagram with all the known angles.  You can now see that: angle P = 54°, angle Q = 72°, angle R = 126°, angle S = 108° and angle STP = 90°.  The numbers 54, 72, 90, 108 and 126 form a sequence with a difference of 18. | x = 18 | C1  M1  A1  C1  C1  C1  C1 | 2 | C1 for clear explanation  M1 for using angles in a triangle  A1 for 18  C1 for clear use of a diagram marked with all the angles  C1 for clear explanation  C1 for finding all the angles and showing them clearly  C1 for clear explanation of the number sequence found | H |
| **7** |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **69** |  | As it is shown in the diagram, the proportion of the height of the container that is filled is =.  The current volume of water is  90 × 40 × 80 = 288 000 cm3.  Assume you move the container so that the base is 40 × 120. The volume of water must stay the same and take its height as *C* cm. Then:  40 × 120 × C = 288 000  so 4800 × C = 288 000  C = 60  Comparing this with the height, 90 cm:  =  Now assume you move the container so the base is 90 × 120 then  90 × 120 × C = 288 000  10 800C = 288 000  C =  Comparing this with the height, 40 cm:  = = =  So the proportion of the height of the container that is filled with water remains constant. | M1  A1  B1  C2  C2  C1 | 3 | M1 for method of finding proportion of height  A1 for  B1 for 288 000  C1 for clear explanation of changing to a different base  C1 for structure and clarity of explanation  C1 for clear explanation of changing to a different base  C1 for structure and clarity of explanation  C1 for clear explanation  A1 for clear complete explanation of solution | H |
| **8** |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **70 a**  **b** | Assume the sides all have integer values.  The formula for the volume of the prism will be:  a × b × 10 = 160 cm3  but if the base is a square, then a = b.  a2 = 16 cm3  This has only one solution: a = 4. | | The end face or cross-section of a right prism is a polygon and is perpendicular to the vertical sides all through its length. The end faces (polygons) are the same shape and size. The volume is equal to the area of cross-section × the height of the prism. So if you know the height and the volume you know the area of the cross-section but not the dimensions or the shape of the base of the prism.  Only 1  4 × 4 × 10 | C3  B1  B1  C1 | 2 | C1 for perpendicular sides  C1 for same base as top  C1 for statement about base area  B1 for answer 1  B1 for identifying the 4 × 4 × 10  C1 for clarity and detail of explanation | H |
| **6** |
| **71 a**  **b**    **c** |  | The formula for the circumference of circle is 2πr. This is just a length, r, multiplied by a number, so the answer will still be a length.  The formula for the area of a circle is πr2. This is a number multiplied by a length squared so this will be an area.  A formula for a surface area will include the product of two lengths, but a formula for a volume will include the product of three lengths.  b2h includes the produce of three lengths, b × b × h and so must be the volume.  4πr2 includes the product of two lengths, r × r, and so must be the area. | | C2  C1  C1  C1  P1 | 2 | C1 for explanation about circumference  C1 for explanation about area  C1 for clear of explanation  C1 for clear explanation  C1 for clear explanation  P1 for demonstrating an understanding of how to help other people to understand this concept. | H |
| **6** |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **72** |  | Identify the third arrangement.  The single row of 8 cubes shows the cuboid must have a volume of 8 cubes.  The missing arrangement is a 1 × 2 × 4 cuboid.  Find the amount of string for each of the arrangements.  For the cuboid above, 1 × 2 × 4:  *L* = 4 × 15, *W* = 2 ×15, *H* = 1 × 15 and  S= 2 × 60 + 2 × 30 + 4 × 15 + 20  = 260 cm  For the 1 × 1 × 8 cuboid:  *L* = 8 × 15, *W* = 1×15, *H* = 1×15 and S= 2 × 120 + 2 × 15 + 4 × 15 + 20  = 350 cm  For the 2 × 2 × 2 cuboid:  *L* = 2 × 15, *W* = 2 × 15, *H* = 2 × 15 and  S= 2 × 30 + 2 × 30 + 4 × 30 + 20  = 260 cm  So the least amount of string is used in the 1 × 2 × 4 and the 2 × 2 × 2 arrangements. | B1  B1  P1  M1  A1  A1  A1  B1  C1 | 2 | B1 for volume being 8 cubes  B1 for missing dimensions  P1 for clearly showing how to find the missing arrangement  M1 for correct method of using formula  A1 for 260  A1 for 350  A1 for 260  B1 for identifying which uses least string  C1 for clear complete solution. | H |
| **9** |
| **73** |  | Surface area of end = 1000 ÷ 20  = 50 cm2  Consider a cuboid with a square end.  Side length of square =  = 7.0711  Surface area (SA)  = 50 + 50 + 4 × 7.0711 × 20  = 665.6 cm2  Consider a triangular prism with a triangular end.  Area of triangular end = = 50  So *a*2 = = 115.470 054  *a* = 10.745 70 cm  SA = 50 + 50 + 3 × 10.745 70 × 20  = 745 cm2  Consider a cylinder with a circular end.  Then π*r*2 = 50  r = = 3.989 42  SA = 50 + 50 + π × 2 × 3.989 42 × 20  = 601.4 cm2  Differences between the three surface areas:   * surface area of the triangular prism is 79.4 cm2 larger than the cuboid and 143.6 cm2 larger than the cylinder * surface area of cuboid is 64.2 cm2 larger than the cylinder.   The larger the surface area, the more packaging material is required therefore the higher the production costs. | B1  B1  B1  M1  A1  A1  M1  A1  A1  C1  C1  C1  C1 | 3 | B1 for 50  B1 for 7.07 any rounding  B1 for 665.6 any rounding 2sf or more  M1 for use of given formula  A1 for 10.75 any rounding 2sf or more  A1 for 745 any rounding 2sf or more  M1 for use of formula  A1 for correct answer to any rounding  A1 for 601.4 any rounding  C1 for clear explanation  C1 for clear explanation  C1 for clear explanation  C1 for a clear complete solution | H |
| **13** |
| **74** | Use the cosine rule.  AB2 = 2.12 + 1.82 – 2 × 2.1 × 1.8 × cos 70°  = 5.064 328  AB = = 2.250 406  Extra distance = (2.1 + 1.8) – 2.25 km | 1.65 km | M1  A1  A1  A1  A1 | 3 | M1 for use of cosine rule  A1 for correct substitution into cosine rule  A1 for 5.06 rounded to 3 or more sf  A1 for 2.25 rounded to 2 or 3 sf  A1 1.65 rounded to 2 or 3 sf | H |
| **5** |
| **75** | Use Pythagoras’ theorem to find BY.  BY =  = 3.605 551 3  Use the right-angled triangle XYB to find XY.  = tan 6°  XY = BY × tan 6°  = 0.378 958 7 km  = 378.9587 m | 379 m | C1  M1  A1  M1  A1  A1 | 3 | C1 for clear diagram showing all data  M1 for use of Pythagoras’ theorem  A1 for BY unrounded  M1 for use of tangent  A1 for XY unrounded  A1 for 379 in metres and rounded to 2 or 3 sf | H |
| **6** |
| **76 a**  **b**  **c**  **d** | PM = =  PM = 8.944 271 9 cm  VM = =  VM = 10.770 33  cos P =  = 0.415 227 4  Angle VPM = 65.466 362°  Vertical height of V above face PRQ is given by VT in diagram above.  VT = =  =  = 11.661 904  Add the 15 cm of the base to give  26.661 904 cm. | 8.9 cm  10.8 cm  65.5°  26.7 cm | C1  M1  A1  C1  M1  A1  C1  M1  A1  C1  M1  A1  A1 | 3 | C1 for use of clear diagram  M1 for use of Pythagoras’ theorem  A1 for 8.9 rounded to 2 or 3 sf  C1 for clear diagram  M1 for use of Pythagoras’ theorem  A1 for 10.8 rounded to 2 or 3 sf  C1 for use of clear diagram  M1 for use of correct cosine  A1 for 65.5 rounded to 2 or 3 sf  C1 for explanation of how the height is to be found  M1 for use of Pythagoras’ theorem  A1 for 11.619 04 and any rounding 3 or more sf  A1 for 26.7 rounded to 2 or 3 sf | H |
| **13** |
| **77** | He needs to find half of AC (not AC) to make a right-angled triangle.  i.e. cos *x* == 0.721 110  = cos–1 0.721 110  = 43.853 779° | 43.9 | C1  M1  A1 | 2 | C1 for clear explanation  M1 for correct cosine method  A1 for 43.9 rounded to either 2 or 3 sf | H |
| **3** |
| **78** | Using angles on a straight line.  θ = 180° – 58° = 122°  Angle ADB = *α* = 180° – (32° + 122°)  = 180° – 154° = 26°  Use the sine rule.    e = sin 32° ×  e = sin 32° × 63.872 816 9  = 63.847 436  Using trigonometry:  sin 58° =  c = e sin 58°  = 28.704 253 7  So the height of the tower is 29 m to the nearest metre. |  | M1  B1  M1  B1  M1  M1  A1  C1 | 3 | M1 for method for finding angle ABD  B1 for 26°  M1 for use of sine rule  B1 for e and any rounding  M1 for correct use of trigonometry  M1 for correct expression for c  A1 for 29 answer rounded to 2 or 3 sf  A1 for complete accurate solution well laid out. | H |
| **8** |
| **79** | sin 24° =  t = 4.7 sin 24°  cos 24° =  x= 4.7 cos 24°  sin 52° =  y = 8.6 sin 52°  cos 52° =  *s* = 8.6 cos 52°  e = x + y = 4.7cos 24° + 8.6sin 52°  = 11.070 556 km  and  f = s – t = 8.6 cos 52° – 4.7sin 24°  = 3.383 0264 km | Use Pythagoras’ theorem to find *d*.    =  = 11.575 93 km  = 11.6 km  Use trigonometry to find β.  tan *β* = =  = 3.272382  β = tan–1 3.272 382 = 73.007 489°  The required bearing = 360° – β  = 286.992 510 8°  = 287° | C1  M1  A1  M1  A1  M1  A1  M1  A1  M1  A1  M1  A1  M1  A1  A1  M1  A1  A1  C1 | 3 | C1 for a clear diagram with all relevant lengths and angles marked  M1 for method of finding t  A1 for correct expression of t  M1 for method of finding x  A1 for correct expression of x  M1 for method of finding y  A1 for correct expression of y  M1 for method of finding s  A1 for correct expression of s  M1 for method of finding e  A1 for correct e, rounded to 4 or more sf  M1 for method of finding f  A1 for correct f rounded to 4 or more sf  M1 for using Pythagoras’ theorem  A1 for correct d any value 3 sf or more  A1 for correct d rounded to 3sf  M1 for method of finding β  A1 for β to 4 or more sf  A1 for final three-figure bearing  C1 for effective use of diagrams and good use of mathematical language | H |
| **20** |
| **80** | Use the sine rule to work out angle B.    sin *B* =  *B* = sin–1 = 62°  But we know ABC is obtuse, so:  *B* = 180° – 62° = 118°  Then *C* = 180° – (118° + 32°) = 30°  Use the sine area rule.  Area =  =  × 10 × 6 × sin 30°  = 15 cm2 | 15 cm2 | M1  A1  A1  B1  M1  M1  A1  C1 | 3 | M1 for use of sine rule  A1 for 62°, accept any rounding to 2sf or more  A1 for 118° and any rounding to 3sf or more  B1 for 30° with any rounding to 2sf or more  M1 for use of area sine rule  M1 for correct substitution  A1 for 15 rounded to 2 or 3 sf  C1 for a complete solution clearly set out with correct mathematical language and symbols | H |
| **8** |
| **81 a**  **b** | Use Pythagoras’ theorem to work out *H*.  *H* =  = 26  Use the cosine rule to work out angle D.  cos *D* =  = 0.275  *D* = 74.037 986°  Use the area sine rule to work out the area of triangle ADC.  Area = × 20 × 23 × sin 74.037 986°  = 221.13217 m2  Area of triangle ABC =  × 24 × 10  = 120 m2  Total area = 341.132 17 m2  341.1321 ÷ 5 = 68.226 434 | 341 m2  68 trees | C1  M1  A1  M1  A1  M1  A1  M1  A1  C1  M1  A1  A1 | 3 | C1 for use of clear diagram  M1 for use of Pythagoras’ theorem  A1 for 26 cao  M1 for use of cosine rule  A1 for *D* and any rounding to 2 or more sf  M1 for use of area sine rule  A1 for area correct to 3 or more sf  M1 for triangle area rule  A1 for 120 cao  C1 for 341 correct to 2 or 3 sf  M1 for dividing by 5  A1 for 68 cao  A1 for complete clear solution with correct mathematical notation. | H |
| **13** |
| **82** |  | Let the vertical height of the parallelogram be *H*.  = sin θ  *H* = a sin θ  Area of parallelogram = base × height  = b × a sin θ  = ab sin θ | C1  M1  A1  M1  A1  C1 | 2 | C1 for clear diagram showing vertical height.  M1 for correct use of trigonometry to find the height  A1 for correct height expression  M1 for correct method of finding area of a parallelogram  A1 for correct expression that will simplify to *ab* sin θ  C1 for complete, clear explanation with good mathematical notation | H |
| **6** |
| **83** | Use Pythagoras’ theorem to work out DB.  DB =  = 10 cm  Use the cosine rule to work out angle A.  cos *A* =  = 0.702 380 9  *A* = 45.381 658°  Use the area sine rule to work out the area of triangle ADB.  Area =  × 9 × 14 × sin 45.381 658°  = 44.843 478 cm2  Area of triangle BDC =  × 6 × 8  = 24 cm2  Total area = 68.843 478 cm2 | 69 cm2 | M1  A1  M1  A1  M1  A1  M1  A1  A1  C1 | 3 | M1 for use of Pythagoras’ theorem  A1 for 10 cao  M1 for use of cosine rule  A1 for *A* with any rounding to 2 or more sf  M1 for use of area sine rule  A1 for area correct to 3 or more sf  M1 for triangle area rule  A1 for 24 cao  A1 for 69 correct to 2 or 3 sf  C1 for clear, complete solution with mathematical language | H |
| **10** |
| **84 a**  **b**  **c**  **d** | A possible triangle is one with sides 3, 4 and 5 (check, using Pythagoras’ theorem).  Then s = 6 and the area is  =  = 6  Also, area = × base × height  =  × 3 × 4 = 6  which gives the same answer.  Suppose the triangle has a side of 10 cm (you could use any number you like).  The formula gives:  = = 43.3 cm2  You could also use the area sine rule:  area =   =  × 10 ×10 × sin 60°  = 43.3 cm2  which gives the same answer.  The sides of the triangle are 18 metres,  22 metres and 24 metres. So:  s = 32  area =  =  = 189.314 55  A diagonal will divide the field into two triangles. Measure the four sides and a diagonal. Use the formula to find the area of each triangle separately and then add the answers together. | 189 m2 | C2  C2  B1  M1  A1  C1 | 2  3 | C1 for showing an example that works  C1 for clarity of explanation to support the example  C1 for showing an example that works  C1 for clarity of explanation to support the example  B1 for 32  M1 for correct substitution  A1 for 189 to 2 or 3 sf  C1 for clear explanation | H |
| **8** |
| **85** | One example is:Start at coordinates (1, 1), then move through translation  then through translation  and finish with  to get back to (1, 1). |  | B1  C3 | 3 | Note there are a few different correct answers. Check all vectors sum to  B1 for having just 3 translations  C1 for the describing first translation  C1 for the describing second translation  C1 for the describing final translation | H |
| **4** |
| **86 a i**  **ii**  **iii**  **iv**  **b** |  | **b** – **a**  –2**a**  2**b** – **a**  2**b – a**  Parallel and equal in length. | B1  B1  B1  B1  B1  B1 | 2 | B1 cao  B1 cao  B1 cao  B1 cao  B1 for parallel  B1 for equal in length | H |
| **6** |
| **87 a**  **b** |  | They lie in a straight line,  AC = ×  =  –  9**a** + 6**b** – (6**a** + 4**b**)  = 3**a** + 2**b**  is 2 ×  So AB : BC = 2 : 1 | C1  C1  M1  A1  B1 | 2 | C1 for clearly stating they lie in a straight line  C1 for explaining one is a multiple of the other  M1 for finding  A1 forcao  B1 for correct ratio | H |
| **5** |
| **88 a i**  **ii**  **iii**  **b** |  | = **c** – **b**  = **b** + (**c** – **b**)  = **c**  = **a** + AC  = **c** – **a**  = **a** + (**c** – **a**)  = **c**  It is a parallelogram, because   =  = **c**  and hence they are parallel and equal in  length. | B1  M1  A1  M1  B1  A1  B1  C2 | 2  3 | B1 cao  M1 method of finding NQ  A1 cao  M1 method of finding MP  B1 cao  A1 cao  B1 for parallelogram  C1 for stating vectors are parallel  C1 for stating vectors will be same length | H |
| **9** |
| **89** |  | = **b** – **a**,  =  +  = –**a** + 3**b** – 2**a**  **= 3b – 3a**  so  = 3  and hence ABC is a straight line. | B1  M1  A1  C1  C1 | 2 | B1 cao  M1 for adding vectors  A1 cao  C1 for explaining the two vectors are multiples of each other  C1 for explaining this gives a straight line | H |
| **5** |
| **90** |  | The knight can get to all the squares shown.  Do not forget that you can use –**a** and  –**b** as well as **a** and **b**.  The starting position must match the question (bottom left white square).  The lines show all the possible paths of the Knight, using **a**, **b**, **–a** and **–b**.  There are many ways to reach the king. However, there are three ways to get to the king in the minimum of five moves. | C1  C4 | 3 | C1 is for a clear diagram to support the explanation, showing all the possible places the knight can move to  C1 for explaining there are numerous ways to get to the king  C1 for explaining there is a minimum of five moves to get to the king  C1 for explaining there are only three ways to get to the king with these 5 minimum moves  C1 for a clear cohesive explanation | H |
| **5** |