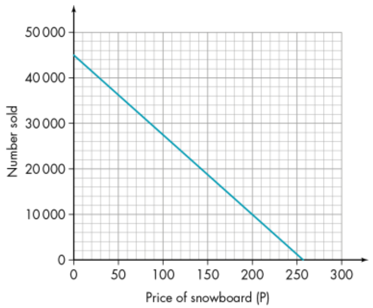
Guidance on the use of codes for this mark scheme	
M	Method mark
A	Accuracy mark
B	Working mark
cao	Correct answer only
oe	Or equivalent
ft	Follow through

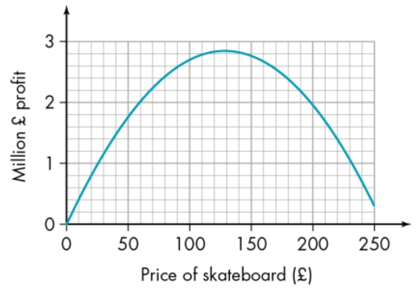
Question	Working	Answer	Mark	AO	Notes	Grade
1 a	Example $2 \times 3 = 6$ $2 + 3 = 5$	$2n$ means $2 \times n$ which is different from $n + 2$.	B1 B1	2	B1 for explanation; an example could be given to support the argument An additional mark can be given for identifying the exception, which is when $n = 2$	B
b	$3(c + 5) = 3c + 15$		M1		M1 for multiplying out the brackets to show that the two expressions are not equivalent	
c	Example $3^2 = 3 \times 3 = 9$ $2 \times 3 = 6$	n^2 means $n \times n$ which is different from $2n$.	B1 B1		B1 for explanation; an example could be given to support the argument An additional mark can be given for identifying the exception which is when $n = 2$	
d	Example $2n^2 = 2 \times (3^2) =$ $2 \times 9 = 18$ $(2 \times 3)^2 = 6 \times 6 = 36$	BIDMAS for $2n^2$ tells you to calculate the power first. BIDMAS for $(2n)^2$ tells you that you do the calculation inside the bracket first.	B1 **6**		B1 for an explanation; an example could be given to support the argument	
2		A letter, say f, stands for an unknown if it is in an equation such as $3f + 2 = 14$. Then $f = 4$ is the only number that satisfies this equation. A letter stands for a variable if it is part of an equation that has more than two letters, e.g. $A = \pi r^2$, where both A and r are variables that will be different for different values of A or r.	B1 B1 B1 B1 **4**	2	B1 for a clear explanation B1 for an example alongside the explanation B1 for a clear explanation B1 for an example alongside the explanation	B

3 **a**	$5(c + 4) = 5c + 20$ Feedback: Don't forget to multiply out both terms in the brackets.		M1 A1	2	M1 for correctly expanding the brackets A1 for suitable feedback	B
b	$6(t – 2) = 6t – 12$ Feedback: Don't forget 6(…..) means **multiply** both terms by 6.		M1 A1		M1 for correctly expanding the brackets A1 for suitable feedback	
c	$–3(4 – s) = –12 + 3s$ Feedback: Don't forget –3(……) means **multiply** both terms by 6 and minus × minus = …		M1 A1 M1		M1 for correctly expanding the brackets A1 for suitable feedback M1 for correctly expanding the brackets	
d	$15 – (n – 4) = 15 – n + 4 = 15 + 4 – n$ $= 19 – n$ Feedback: Don't forget $–(n – 4)$ means multiply each term inside the brackets by –1 and that the – inside the brackets belongs to the 4 to make it – 4.		A1 **8**		A1 for suitable feedback	
4		Start with numbers that work. $\frac{(6\ -\ 1)}{2} = 2.5$ So $z = \frac{(s\ -\ 1)}{t}$ will satisfy conditions. Start with a formula. e.g. $z = \frac{(3s - 4t + x)}{2}$ Substitute $z = 2.5$, $s = 6$, $t = 2$ to find x. $5 = 18 – 8 + x$ $x = –5$ so $z = \frac{(3s - 4t - 5)}{2}$ satisfies conditions.	M1 A1 M1 A1 B1 **5**	2 3	M1 for first method, e.g. starting with numbers A1 for an example that works M1 for second method, e.g. starting with a formula A1 for an example that works B1 for a clear, complete solution showing two different methods and two examples	B
5		$\frac{(2n + 6)}{2}$ $= \frac{2(n + 3)}{2}$ $= n + 3$	M1 A1 **2**	2	M1 for factorising A1 for any correct expression	B

6		Let the base length be b, then the height will be $3b$. Area of triangle = $\frac{1}{2}$ × base × height = $\frac{1}{2} \times b \times 3b$ = $\frac{3}{2}b^2$ Where $A = 6$ $\frac{3}{2}b^2 = 6$ $b^2 = \frac{2 \times 6}{3} = 4$ $b = 2$ so height is 3 × 2 which is 6 cm.	B1 B1 B1 M1 A1 A1 **6**	3	B1 for stating variables B1 for stating triangle formula B1 for correct expression M1 for equating 6 with found expression A1 for $b = 2$ A1 for 6 cm	B
7		Boys get: one red egg each from each of 4 girls = 4 red one green egg from each other = 2 green Girls get: one blue egg from each of the 2 boys = 8 blue one yellow egg from each other = 3 yellow eggs each = 12 yellow altogether.	B1 B1 B1 B1 B1 **5**	3	B1 for explanation of 4 red B1 for explanation of 2 green B1 for explanation of 8 blue B1 for explanation of 12 yellow B1 for complete clear solution	B
8 a **b** **c** **d** **e** **f**		Abi Abi stopped 12 minutes Bryn By 1.8 km 4.5 km Another suitable question	B1 B1 B1 B1 B1 B1 B1 B1 **8**	3	B1 cao B1 cao B1 cao B1 cao B1 cao B1 cao B1 for suitable question using a linear function B1 for a suitable graph	B

9 a	Need to find both times when $h = 0$. Substitute $u = 16$ m/s into the equation. $16t - 5t^2 = 0$ $t(16 - 5t) = 0$ so $t = 0$ or $(16 - 5t) = 0$ $t = 0$ or $5t = 16$ $t = 3.2$	 3.2 s	B1 M1 A1 A1	3	B1 for clear explanation M1 for setting $h = 0$ A1 for 0 and 3.2 A1 for 3.2 seconds	
b	Maximum height = $16 \times 1.6 - 5 \times 1.6^2$ $= 25.6 - 12.8$ $= 12.8$ m		B1 B1		M1 for substituting $t = 1.6$ B1 for 12.8 m	
c	$h = ut + 5t^2 + 1$		B1 **7**		B1 cao	
10 a	Use $s = ut - \frac{1}{2}gt^2$.	Parabola/quadratic equation	B1	3	B1 for either of these	B
b	Assuming $g = 10$, given $u = 8 \sin \theta$ and assuming suitable value for θ, for example, 30° sin 30° = 0.5 So $s = -5t^2 + 4t$ Complete the square to give: $(t - \frac{4}{10})^2 - (\frac{4}{10})^2 = 0$ Comparing this to the equation of $y = x^2$. Then the horse will reach its maximum at $t = 0.4$. Substitute this into $s = -5t^2 + 4t$ to give $s = 0.8$ m.	 $s = 0.8$ m	B1 M1 M1 A1		EC for stating suitable assumptions for the starting point M1 for a suitable method for finding greatest height, could also be sketch graph M1 for suitable comparison A1 for ft from the initial assumption	
c		Suitable justification, e.g. Yes it does, the horse is in the air for 0.8 s and jumps 0.8 m into the air.	A1 **6**		A1 cao	

11 a		Minimum demand = 1 So 1 = 45 000 – 175P 175P = 44 999 $P = \dfrac{44\,999}{175}$ = £257	B1 M1	3	B1 for a clearly graph drawn M1 for method of solving equation when demand = 1	B
			A1		A1 for answer £257	
	This graph shows expected sales for the different prices charged. If he prices his snowboard at more than £257 demand will be 0.		B1		B1 for a clear explanation	
b	The graph also shows that the cheaper the snowboards, the more he will sell. But he needs to consider his charges to make sense of it all.		B2		B1 for explaining the relationship B1 for commenting that there will be other costs to take into account	
c	Number sold = 450 00 – 175P	Demand = 45 000 + 95P				
	Sales = number sold × P = (45 000 – 175P)P Costs = set-up fees + manufacturing costs per board		M1		M1 for setting up the sales equation	
		Profit = sales – costs Profit = (45 000 – 175P)P – (45 000 + 95P) = 45 000P – 175P^2 – 45 000 – 95P	M1		M1 for setting up the profit equation	
		= 44 905P – 175P^2 – 45 000	A1		A1 for cao	
d		From a graph of this quadratic function, his maximum profit would be approximately £2 850 000 if he sold his boards at £135.	B2		B1 for profit between £2 800 000 and £2 900 000 B1 for cost of boards between £120 and £140	
			B1		B1 for a drawn graph of the quadratic equation found	

			**10**			
12 a		ii, v and vi might be difficult as they all involve squaring a term. The typical error made in ii will be to calculate half of at and then to square that. The same error can be found in vi where $2\pi r$ can be calculated first and then squared.	B2	2	B1 for identifying some examples with a valid reason B1 for clear identification and explanation of classic errors	M
b		ii and vi are also difficult to rearrange as they involve a quadratic element and it's not easy to make each variable the subject of the formula. Typical errors in rearranging the equation $s = ut + \frac{1}{2}at^2$ to make a the subject include: • incorrect sign when changing sides, e.g. $s + ut = \frac{1}{2}at^2$ • incorrect removal of fraction, e.g. $\frac{1}{2}$ to leave $(s + t) = at^2$.	B2		B1 for identifying some examples with a valid reason B1 for clear identification and explanation of classic errors	
			4			

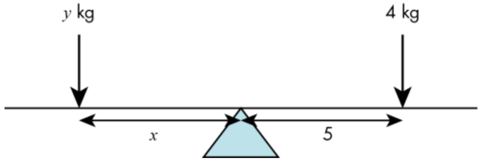
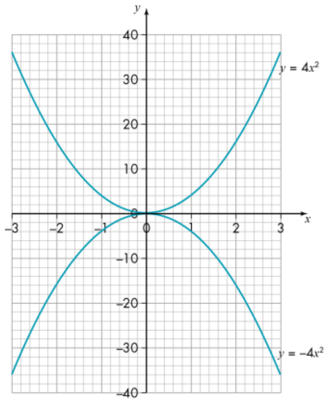
13		c and d can be difficult because they contain minus signs; errors are often made when combining minus signs. In substituting $x = -3$ into $t = -2(3 - x)$, a common error is to assume 3 – –3 is 0. In substituting $x = -3$ into $z = \frac{-2(x+2)}{x}$ a typical error is to assume a negative divided by a negative gives a negative answer. A suggestion to avoid these errors is to remember that when multiplying or dividing with positive and negative numbers, same signs means positive, different signs means negative.	B1 B2 B1 **4**	2	B1 for identifying some examples with a valid reason B1 for clear identification of one typical error with one equation. B1 for another typical error B1 for a satisfactory suggestion	M
14		The similarities are that both include an equals sign and both require the manipulation of terms. The difference is that in solving an equation you reach a numerical answer, but in rearranging you still have a formula.	B1 B1 **2**	2	B1 for clear explanation of similarities B1 for clear explanation of differences	M
15		In line 2 Phillip has initially rearranged $x^2 + 2x - 3$ to $x(x + 2) - 3$ when he should have factorised it as $(x + 3)(x - 1)$. He has incorrectly simplified in line 3. He should have factorised $(x^2 - 9)$ to $(x + 3)(x - 3)$. Philip has cancelled incorrectly just by looking at the different numbers and not realising that you can only cancel a number on both numerator and denominator if it is a factor of the complete expression.	B1 B1 B1 **3**	2	B1 for identifying the first error B1 for identifying the second error B1 for a clear explanation of the errors made	M

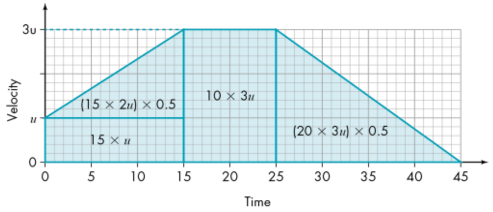
16 a		‘I think of a number and double it’ just means an expression of $2x$, where x is the number I thought of – still unknown at the moment. ‘I think of a number and double it – the answer is 12’ has a solution that I know is 6.	B1	2	B1 for clear explanation of the difference	M
bi **ii** **iii**		One e.g. $10 = p + 3$ Because each solution is $p = 7$.	B1 B1 B1		B1 cao B1 a correct example B1 a clear explanation	
			4			

17 a		An expression is any combination of letters and numbers, e.g. $3x + 5y$. An equation contains an equals sign and at least one variable, e.g. $3x + 5y = 10$. A formula is like an equation, but it is a rule for working out a particular value, such as the area of a rectangle or the cost of cleaning windows, e.g. $A = lb$, where A is area, l is length and b is breadth. An identity looks like a formula but it is true for **all** values, e.g. $(x + 1)^2 = x^2 + 2x + 1$ is true for all values of x.	B4	2	B1 for explanation of expression B1 for explanation of equation B1 for explanation of formula B1 for explanation of identity	M
b		For example: State whether each item is an expression, equation, formula or identity. Explain why. **a** $x + y$ $2x + y = 6$ **b** $m^2 = m \times m$ $5x^2 - 3$ $10 = x - 7$ $A = \frac{1}{2}bh$ **c** $v = ut + \frac{1}{2}at^2$ $10 - 5t$ $x^2 = 16$ $5p = 5 \times p$ **d** $y = x^2 - 1$ $v = \frac{b^2h}{3}$ $x^2 - 1 = (x + 1)(x - 1)$ $\frac{6}{y}$	B3		B1 for an activity that works B1 for plenty of practice B1 for quality of activity	
			7			

18 a		The two straight-line graphs will be parallel, with the same gradient of 2, $y = 2x$ crosses the y-axis at the origin, and $y = 2x + 6$ crosses the y axis at $y = 6$	B2	2	B1 for explanation of parallel B1 for explanation containing points of intersection of axes	M
b		The two straight-line graphs will be parallel, having the same gradient of 1, $y = x + 5$ crosses the y axis at $y = 5$, and $y = x – 6$ crosses the y axis at $y = –6$	B2		B1 for explanation of parallel B1 for explanation containing points of intersection of axes	
c		The two straight-line graphs will cross each other at $(\frac{11}{8}, \frac{1}{2})$ and each one is a reflection of the other in a vertical mirror line.	B2		B1 for explanation containing point of intersection B1 for explanation of symmetry	
d		The two straight-line graphs will both cross the y-axis at the origin, one with gradient 2, the other with gradient $\frac{1}{2}$.	B2		B1 for explanation of passing through origin B1 for explanation about gradient	
			8			
19 a		The gradients represent how quickly the variable on the y-axis changes as the variable on the x-axis changes.	B1	3	B1 for clear explanation	M
b		The intermediate points will only have any meaning for continuous data, such as mass or height. If the data is discrete then the points will only have values when they coincide with actual data.	B2		B1 for clarity of continuous data B1 for clarity of discrete data	
c		The intercept indicates a value that must be added to a variable value, such as a standing charge of £3.50 for a taxi fare, being included before adding on a rate per km.	B1		B1 for clear explanation, using an example	
			4			

20 a i		The highest power will be 2 with no negative powers, e.g. $y = x^2 + 3x - 1$, where 2 is the highest power.	B1 B1	2	B1 for clear explanation B1 for also using an example	M
ii		The highest power will be 3 with no negative powers, e.g. $y = x^3 + 5x^2 - 6$, the 3 being the highest power.	B1 B1		B1 for clear explanation B1 for also using an example	
b		Find points that cross the axes where possible and then create a table of values including for the turning point and the axis intercepts so that you have sufficient points to plot the curve.	B1		B1 for clear explanation	
c		1: x^2 is always positive for positive or negative values of x, hence $2x^2$ will also be positive as positive multiplied by positive is positive. The +5 moves the graph up 5, so $y = 2x^2 + 5$ will also always be positive for all values of x. 2: Draw the graph and illustrate all the points on the graph are above the x axis.	B1 B1		B1 for first explanation B1 for second explanation	
			7			
21 a		A quadratic function always has line symmetry because x^2 and $(-x)^2$ have the same y value.	B1	2	B1 for clear explanation	M
b		A cubic equation does not have a line of symmetry as A^3 will have different values depending on whether A is negative or positive.	B1		B1 for clear explanation	
c		Rotational symmetry of order 2 about the point of inflection.	B1		B1 for clear description	
			3			
22 i		D, $y = 5x^2$	B1	3	B1 correct letter with correct example	M
ii		B, $y = \frac{12}{x}$	B1		B1 correct letter with correct example	
iii		A, $y = 0.5x + 2$	B1		B1 correct letter with correct example	
iv		C, $d = \sqrt{A}$	B1		B1 correct letter with correct example	
			4			

23		As they are balanced $12 = xy$ Therefore rearranging $y = \frac{12}{x}$, they are inversely proportional.	B1 B1 B1 **3**	3	B1 for a good diagram accompanying the explanation. B1 for the equation B1 for the correct explanation of the relationship	M
24 a **b**		Any two examples of the form $y = x^3 + c$, e.g. $y = x^3 + 7$, $y = x^3 - 4$. They are all quadratics. All of them pass through the origin except $y = 4x^2 + 3$. $y = 4x^2$ and $y = 4x^2 + 3$ have the same shape, but the latter is moved up 3 units.	B1 B1 B3 **5**	2	B1 for the first example B1 for the second example B1 for similarities B2 for differences	M
25		Sometimes true. It is only true when $a > 0$.	B1 B1 **2**	2	B1 for sometimes B1 for explanation	M
26		When you draw the graphs of $y = 4x^2$ and $y = -4x^2$ you get the graphs shown here.  It can be seen that $y = 4x^2$ is a reflection of the graph of $y = -4x^2$ in the x-axis.	B1 B1 B1 **3**	2	B1 for explanation of drawing a graph of each on the same axes B1 for an accurate diagram of both graphs on the same pair of axes B1 for clear explanation bringing everything together.	M

27		Distance travelled = $\frac{1}{2}(15 \times (u + 3u))$ + $(10 \times 3u)$ + $\frac{1}{2}(20 \times 3u)$ = $30u + 30u + 30u = 90u$ The assumption is that acceleration is at a steady rate when the motorbike speeds up and slows down.	B1 M1 M1 A1 A1 **5**	3	B1 for a good diagram illustrating the journey M1 for method of using $\frac{v}{t}$ diagram M1 for correct area equation A1 cao A1 for clear explanation of assumption.	M
28 a **b** **c**		$x = 0$ The circle has a radius of 5 and its centre is at (0, 0), halfway between D and E. Find the distance of F(–3, 4) from the origin. If it is 5 units from the origin, it is on the circumference of the circle. Using Pythagoras' theorem: distance = $\sqrt{3^2 + 4^2} = \sqrt{25} = 5$ So F(–3, 4) is on the circumference of the circle. The tangent at F will be at right angles to the line joining the point to the origin. Gradient of the line is $-\frac{4}{3}$. The product of the gradient of this line and the tangent is –1. Use this to work out the gradient of the tangent. Substitute the gradient and the coordinates of the point into the general equation of a straight line ($y = mx + c$) to find the y-intercept c.	B1 B2 B3 **6**	2	B1 cao B1 for explanation referring to a right angle triangle B1 for explaining how using Pythagoras' theorem helps in finding the length 5 B1 for explaining the gradient is tangent of the angle B1 for showing $y = mx + c$ B1 for complete explanation showing how to work out the equation of the line	M

29 a		Use the method of elimination, in which you combine the equations to eliminate one of the variables leaving an equation in the other variable. Solve this equation then substitute the value into one of the original equations to work out the other value.	B3	2	B1 for each explanation	M
		Use the substitution method, in which you make one of the variables the subject of one equation and substitute this into the other equation. Solve this equation and then substitute the value into one of the original equations to work out the other value. Use the graphical method, in which you draw a graph of both equations on the same axes and the solution is the point of intersection.	B3		B1 for each explanation	
b		Use the elimination method when you can eliminate one variable easily by either adding or subtracting the two equations. Use the substitution method when it is easy to make one of the variables in one of the equations the subject of the equation. Use the graphical method to solve equations where there is a quadratic. You might use more than one method: if you used the graphical method and didn't get integer values for the solution, you might then use the elimination method to find the fractional answers.	B1		B1 for clear explanation of why you might use two methods	
			7			

30		$3x - 4y = 13$ (1) $2x + 3y = 20$ (2) Using elimination: Multiply (1) by 2 and (2) by 3: $6x - 8y = 26$ $6x + 9y = 60$ Subtract the first equation from the second equation: $9y - -8y = 60 - 26$ $17y = 34$ $y = 2$ Substitute $y = 2$ into (2): $2x + 3 \times 2 = 20$ $2x = 14$ $x = 7$ Check by substituting both values into (1) $3 \times 7 - 4 \times 2 = 21 - 8 = 13$ Correct. Using substitution: Rearrange one of the equations: $2x + 3y = 20$ $y = \frac{20-2x}{3}$ Substitute into the other equation and rearrange. $3x - 4(\frac{20-2x}{3}) = 13$ $3x - \frac{80}{3} + \frac{8x}{3} = 13$ $3x + \frac{8x}{3} = 13 + \frac{80}{3}$ $\frac{9x+8x}{3} = \frac{39+80}{3}$ $\frac{17x}{3} = \frac{119}{3}$	M1 A1 M1 A1 M1 A1 M1 A1 M1 A1	3	M1 for method of changing equations in order to be able to eliminate A1 for correct equations M1 for subtracting equations A1 cao M1 for substitution A1 cao M1 for rearrangement to get one variable as a subject A1 cao M1 for substitution of the one variable into the other equation A1 cao	M

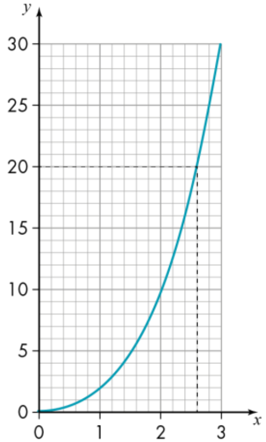
		$17x = 119$ $x = 7$ Substitute this into other equation as before to give $y = 2$.	M1 A1 **12**		M1 for substitution A1 cao	
31 a		The equations look awkward. The method of eliminating one variable by multiplying the first equation by 3 and the second equation by 5 will give new equations with $15y$ and $-15y$ in both of them. These terms can be eliminated by adding the two new equations. Rearranging one equation to make one variable the subject will give awkward fractions and so is not desirable. It is not obvious whether drawing a graph of each equation will produce an integer solution.	B3	2	B1 for explanation of advantage of first B1 for explanation of first disadvantage B1 for explanation of second disadvantage	M
b		The first equation already has y as the subject and the substitution method is ideal, substituting for y into the second equation. The elimination method would mean unnecessary work to eliminate one of the variables. Drawing a graph would give integer values, but it would take more time than the simple substitution method.	B3		B1 for explanation of advantage of first B1 for explanation of first disadvantage B1 for explanation of second disadvantage	
c		As one of the equations is a quadratic drawing graphs could the best method; this would only be justified if an integer solution was found. It seems straightforward to make x the subject of the first equation and then substitute it into the second equation to get a quadratic equation that could then be solved for two solutions. Because one equation is a quadratic it is not suitable to use the elimination method.	B3 **9**		B1 for explanation of advantage of first B1 for explanation of first disadvantage B1 for explanation of second disadvantage	

32 a		If one of the equations is a multiple of the other then there will be an infinite number of solutions, e.g. $x + y = 5$ $2x + 2y = 10$ Every point on the line $x + y = 5$ is a solution, giving an infinite number of solutions. If the equations have graphs that are parallel to each other then there will be no intersection and so no solution, e.g. $x + y = 5$ $x + y = 6$	B1 B1 B1 B1	2	B1 for clear explanation B1 for use of a good example accompanying the explanation B1 for clear explanation B1 for use of a good example accompanying the explanation	M
b i		One solution, as one equation is not a multiple of the other and they are not parallel.	B1		B1 for clear explanation	
ii		None –the gradient is the same but the intercepts are different so they are parallel.	B1		B1 for clear explanation	
iii		Infinite number of solutions as the first equation is a multiple of the second, so drawing graphs gives the same line.	B1		B1 for clear explanation	
c i		$y - 2x = -5$ $y = 0.5x + 1$ Substitute for y in the first equation: $0.5x + 1 - 2x = -5$ $-1.5x = -6$ $x = 4$ Substitute into second equation: $y = 0.5 \times 4 + 1 = 3$ Check by substituting both values in the first equation $3 - 2 \times 4 = 3 - 8 = -5$ Correct.	M2 A1		M1 for arranging equations in a suitable format M1 equating both equations A1 cao	
ii		No solution.				
iii		Infinite number of solutions.				
d		You can see how many times the graphs cross each other.	B1 **11**		B1 clear explanation	

33 a		They are the same equation. Multiply the first equation by 3 and it is the same as the second equation, so they have an infinite number of solutions.	B1	3	B1 for clear explanation	M
b		Treble the first equation to get $15x - 3y = 27$ They have the same coefficients of x and y but a different constant so they are parallel lines with no intersections and so no solutions.	B1 **2**		B1 for clear explanation	
34 a	$x^2 + 2x - 5 = 6x - 9$ $x^2 - 4x + 4 = 0$ $(x - 2)(x - 2) = 0$ $x = 2$ $y = 6 \times 2 - 9 = 3$ $y = 3$	$x = 2$ $y = 3$	M1 M1 M1 A1 A1	2 3	M1 for equating both equations M1 for arranging to equal 0 M1 for factorising A1 for $x = 2$ cao A1 for $y = 3$ cao	M
b	There is just one intersection of the two graphs, so it has to be sketch **iii**, as the straight line touches the curve once.		B1 B1 **7**		B1 for sketch iii B1 for clear explanation	

35 a	Let the cost of a second class stamp be x.		M1	3	M1 for clear explanation of variables chosen	M
	Let the cost of a first class stamp be y.					
	$10x + 6y = 902$ …….(1)		M1		M1 for first equation created	
	$8x + 10y = 1044$ ……(2)		M1		M1 for second equation created	
	5 × (1) $50x + 30y = 4510$ ….(3)		M1		M1 for multiplying equation in order to be able to eliminate a variable	
	3 × (2) $24x + 30y = 3132$ ….(4)					
	Subtract (4) from (3):		M1		M1 for subtracting	
	$26x = 1378$					
	$x = 53$		A1		A1 cao	
	Substitute for x in (1):		M1		M1 for substituting x into an equation	
	$10 \times 53 + 6y = 902$					
	$6y = 902 - 530$					
	$6y = 372$					
	$y = 62$		A1		A1 cao	
	So 3 second-class plus 4 first-class will cost:					
	$3 \times 53 + 4 \times 62 = 407$					
	Cost will be £4.07.	£4.07	A1		A1 cao	
b	Let the cost of a can of cola be c.		M1		M1 for clear explanation of variables chosen	
	Let the cost of a chocolate bar be b.					
	Then:		M1		M1 for first equation created	
	$6c + 5b = 437$ …….(1)		M1		M1 for second equation created	
	$3c + 2b = 200$ ……(2)		M1		M1 for multiplying an equation in order to be able to eliminate a variable	
	2 × (2) .. $6c + 4b = 400$ ….(3)		M1		M1 for subtracting	
	Subtract (3) from (1):		A1		A1 cao	
	$b = 37$					
	Substitute for b in (2):		M1		M1 for substituting b into an equation	
	$3c + 74 = 200$					
	$3c = 126$					
	$c = 42$		A1		A1 cao	
	So three cans of cola and a chocolate bar will cost:					
	$2 \times 42 + 37 = 121$					
	Cost will be £1.21.	£1.21	A1		A1 cao	
			18			

36 a	$S = \frac{5B}{4}$ $B = \frac{2}{5}C$		B1 B1 B1	3	B1 for first equation B1 for second equation B1 for third equation	M
	$S + C = 75$ $C = \frac{5}{2}B$ $\frac{5}{2}B + \frac{5B}{4} = 75$ $\frac{15B}{4} = 75$ $B = 75 \times \frac{4}{13}$		M1		M1 for creating single equation with one unknown	
	$= 20$	B = £20	A1		A1 for £20 cao	
	$S = 1.25B$ $S = 1.25 \times 20 =$ £25 $C = 75 - 25 =$ £50	S = £25 C = £50	A1 A1		A1 for £25 cao A1 for £50 cao	
b	The method used was to find equations linking each two persons at a time, and then use those to create one equation that could be solved. Once you had one solution you could find the rest.		B2		B1 for clear explanation. B1 for matching explanation with the work done	
	Answer checked by going back to beginning statements and ensuring each one works. They do.		B1		B1 for clear explanation	
			10			

37 ai	Draw the graph of $y = x + x^3$. Then solve for $y = 20$.		B2	3	B1 for explanation about drawing a graph B1 for using it to solve for $y = 20$	M
ii	Let the width be x, then the length is $x + 2$. Hence the area is $x(x + 2) = 67.89$. Solve the quadratic equation to find x.		B2		B1 for showing how equation is created B1 for explaining there will be a quadratic equation that needs solving	
bi	Create a table of values to assist in drawing the graph of $y = x + x^3 = 20$. x: 0, 1, 2, 3 y: 0, 2, 10, 30		M1 A1		M1 for creating a suitable table of values A1 for at least 4 correct, useful values	
	Plot the points and draw the graph. 		B1		B1 for a good, accurate graph drawn	
	Follow the line from $y = 20$ to the graph and read down to the x-axis to find $x = 2.6$.	$x = 2.6$	M1 A1		M1 for drawing the line $y = 20$ A1 cao	
ii	$x^2 + 2x = 67.89$ $x^2 + 2x - 67.89 = 0$ Solve with the formula: $x = \frac{-2 \pm \sqrt{(2^2 - 4 \times 1 \times -67.89)}}{2}$ $x = \frac{-2 \pm 16.6}{2}$ so $x = 7.3$ or -9.3 Width, x, cannot be negative hence solution is width = 7.3 cm.	Width = 7.3 cm	M1 M1 M1 A1 A1 A1 A1 **16**		M1 for setting up initial equation M1 for amending it to arrive at equation = 0 M1 for using the formula A1 for correct intermediate values A1 for both possible solutions A1 for picking out 7.3 A1 for explanation of why this solution was selected	

38		Area of large square = $7^2 = 49$ To find the length (z) of the side of the small square: $z^2 = \sqrt{3^2+4^2}$ $z^2 = \sqrt{25}$ $z^2 = 5$ So the area of the small square is 25. Shaded area is equal to area large square – area small square. 49 – 25 = 24 So less than half is shaded, as required.	B1 M1 A1 A1 A1 B2 **7**	2 3	B1 for area of large square M1 for use of Pythagoras' theorem to help find the side length of inner square A1 cao A1 cao A1 for area of shaded part B1 for clear explanation B1 for complete explanation with correct mathematical notation throughout	M
39		Set up two simultaneous equations, using the information given. 5 is the first term; from the rule for the sequence, the next term is: $5 \times a - b$, which equals 23 Hence $5a - b = 23$ …… (1) Doing the same to the next term gives: $23a - b = 113$ ……. (2) Subtract (1) from (2) to give: $18a = 90$ $a = \frac{90}{18} = 5$ Substitute in (1): $25 - b = 23$ $b = 2$ *So* $a = 5$, $b = 2$ Hence, the next 2 terms are 563 and 2813.	M1 B1 B1 M1 A1 M1 A1 B1 **8**	2	M1 for clear initial explanation B1 for first equation B1 for second equation M1 for clear method of elimination A1 cao M1 for substitution A1 cao B1 for next two terms correctly found	M

40 a		If the sum of the whole numbers from 1 to 50 is 1275 the sum from 2 to 51 will be 1275 + 51 – 1 = 1325.	B1	2	B1 for clear explanation.	M
b		Check method, using simple examples.	B1		B1 for clear explanation	
c		Using $S_n = \frac{n(n+1)}{2}$ $S_{52} = \frac{52(52+1)}{2} = 1326$ 1326 – 1 + 1325 as above.	B2		B1 for clear use of the formula B1 for showing same answer as above	
d		If 1 is the first term then the nth term will be n, so their sum is 1 + n. Because they come in pairs, there will be $\frac{n}{2}$ of these pairs adding to the total. So the total = (n + 1) × $\frac{n}{2} = \frac{n(n+1)}{2}$, the formula given.	B2		B1 for clear explanation. B1 for use of the generalization and good mathematical language.	
			6			

41 a		Ali receives: £1000 + £2000 + … + £20 000.		2		M
		This amount, in pounds, is: 1000 × (sum of the numbers 1 to 20)	B1		B1 for clear explanation	
		The sum of the first n natural or whole numbers is $\frac{1}{2}n(n + 1)$.	B1		B1 for explaining how the formula for sum of integers helps	
		So the amount Ali receives after n years is: $1000 \times \frac{1}{2}n(n + 1)$ or $500n(n + 1)$.	B1		B1 for explaining the given sum	
b		The amounts Ben receives each year are £1, £2, £4. The general term is £2^{n-1}. Adding these amounts gives a running total, in pounds, of: 1, 3, 7, 15,…	B1		B1 for explaining how annual totals are found	
		Looking for a pattern linking the number of years with the amount, we see that after 2 years, it is: $3 = 4 - 1 = 2^2 - 1$ After 3 years it is: $7 = 8 - 1 = 2^3 - 1$ After 4 years it is: $15 = 16 - 1 = 2^4 - 1$ So after n years it is $2^n - 1$.	B1		B1 for explaining how to look for a pattern	
c			B1		B1 for showing the pattern building up	
		After 20 years, Ali will have 500 × 20 ×19 = £190 000	B1		B1 for showing how the generalisation is found	
		Ben will have $2^{20} - 1$ = £1 048 575	B1		B1 for showing both totals	
		Ben will have more than five times the amount that Ali has.	B1		B1 for explanation linking both totals with formula found	
			9			

42 a **b** **c** **d**		$2n + 1$ $3n + 4$ $\frac{2001}{3004}$ No, because $\frac{2n}{3n}$ will always equal $\frac{2}{3}$ no matter what n is, and the denominator increase of 4 will always give a larger increase than the numerator increase of 1, hence the fraction can never be larger than $\frac{2}{3}$.	B1 B1 B1 B1 B1 5	2	B1 cao B1 cao B1 cao B1 for no B1 for a clear concise explanation	M
43 a i **ii** **iii** **b** **c**		Neither – it's the Fibonacci series, where each term is found by adding the previous two. Geometric – because each term is multiplied by 2 to find the next term. Arithmetic – because to find the next term you add 4 to the previous term. Arithmetic. Arithmetic.	B1 B1 B1 B1 B1 B1 B1 B1 8	2	B1 for neither B1 for clear explanation B1 for geometric B1 for clear explanation B1 for arithmetic B1 for clear explanation B1 for arithmetic B1 for arithmetic	M

44 a		a is the first term. d is the amount added each time.	B1 B1	2	B1 for explaining a B1 for explaining d	M
b		For example: use a = 2 and d = 3 to generate the sequence in part a as: 2, 5, 8, 11, 14, … The 5th term is 14. Using the given $X_n = a + (n - 1)d$: the 5th term will be 2 + 4 × 3 = 14, the same value.	B1 B1 B1		B1 for creating a suitable example B1 for generating a value higher than the third term B1 for showing the X_n formula gives the same value	
c		$X_n = ar^{n-1}$	B1		B1 cao	
d		It is a quadratic sequence because it contains a term in n^2.	B1		B1 cao	
			7			
45 a		Evidence of reproducing proof as given in question.	B1	2	B1 for correct proof clearly explained	M
b		$S_n = \frac{n}{2}(2a + (n - 1)d)$ for the set of integers, a = 1 and d = 1. Hence $S_n = \frac{n}{2}(2 + (n - 1))$ $= \frac{n}{2}(n + 1)$	B1 B1 B1		B1 for using a and d equal to 1 B1 for correct substitution into the formula B1 for showing how this simplifies to the desired formula	
			4			

<table>
<tr><td>

46 a

B

c

d

e</td><td></td><td>

$S_n = a + ar + ar^2 + ar^3 \dots + ar^n$

$rS_n = ar + ar^2 + ar^3 + ar^4 \dots + ar^n + ar^{n+1}$
<table>
<tr><td>S_n</td><td>a</td><td>$+ ar$</td><td>$+ ar^2$</td><td>…</td><td>$+ ar^n$</td><td></td></tr>
<tr><td>rS_n</td><td></td><td>ar</td><td>$+ ar^2$</td><td>…</td><td>$+ ar^n$</td><td>$+ ar^{n+1}$</td></tr>
<tr><td>$S_n - rS_n$</td><td>a</td><td></td><td></td><td></td><td></td><td>$- ar^{n+1}$</td></tr>
</table>

Therefore:
$S_n - rS_n = a - ar^{n+1}$

$S_n(1 - r) = a(1 - r^{n+1})$
$S_n = \dfrac{a(1-r^{n+1})}{(1-r)}$</td><td>B1

B1

B2

B1

B1

B1

7</td><td>2
3</td><td>B1 for equation showing at least up to ar^3 and the generalisation
B1 for equation showing at least up to ar^4 and the two generalisations

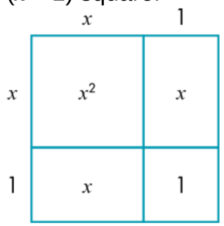
B1 for top two rows shown correctly
B1 for the bottom row shown correctly

B1 cao

B1 cao

B1 cao</td><td>M</td></tr>
<tr><td>

47</td><td></td><td>Considering the area of an $(x + 1)$ by $(x + 1)$ square:



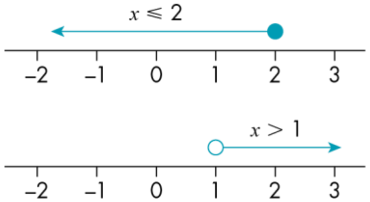
The area of each rectangle created above is shown inside that rectangle, so it can be seen that:
$(x + 1)^2 = x^2 + x + x + 1$
$= x^2 + 2x + 1$ as required</td><td>B1

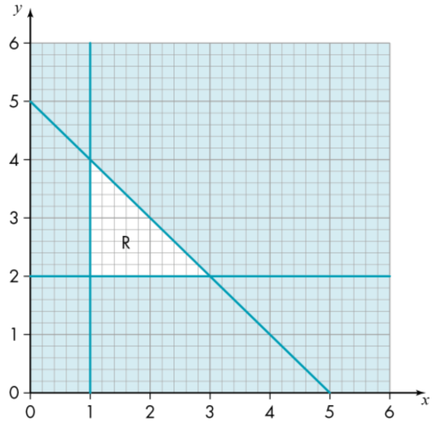
B2

B1
4</td><td>2</td><td>B1 for explaining the sides of each square are $(x + 1)$

B1 for creating the square divided into the rectangles, using the x and the 1
B1 for areas of each rectangle indicated in the rectangles

B1 for clear explanation of required result</td><td>M</td></tr>
</table>

48 a		For example, $2x > 10$ Divide both sides by 2 to get $x > 5$.	B1 B1	2	B1 for a correct inequality B1 for clear explanation of how to solve the inequality chosen	M
b		We show the solution with a line and a circle at each end point. A solid circle means that the solution includes the end point; an open circle means that the solution does not include the end point. For example:  The top diagram shows $x \leq 2$. It has a solid circle at the end point $x = 2$ because that is part of the solution. The bottom diagram shows $x > 1$, it has an open circle at the end point $x = 1$ because $x = 1$ is not part of the solution.	B1 B1 B1 B1		B1 for explanation about circles at the end of each line B1 for clear explanation differentiating between solid and open circles B1 for use of a clear diagram to support the explanation B1 for a full, clear explanation showing both aspects of the circles	
c		Starting with an equation $10 - x > 4$ and solving by adding x to both sides gives the solution $6 > x$. This can also be given as $x < 6$ ….. (1) Consider again $10 - x > 4$. This time multiply throughout by –1. Keeping the inequality sign the same gives: $-10 + x > -4$ Add 10 to each side to give $x > 10 - 4$. This gives the solution as $x > 6$ ….. (2) But comparing this with equation (1) we see that the signs are the other way round, this illustrates that when we multiplied through by a negative number, we should have changed the sign from >	B2 8		B1 for clear explanation B1 for using an example in a way that illustrates the principle	

49 a		In this example, x and y are values satisfying the conditions: $x + y \leq 5 \quad x > 1 \quad y > 2$ These are drawn on the diagram.	B1	3 2	B1 for choosing three inequalities that will define a region	M
			B1		B1 for a clear diagram illustrating the chosen inequalities	
		Any region needs a minimum of three straight lines to enclose it. The region R above is where the solutions satisfying all three inequalities lie.	B1		B1 for clear explanation linking the chosen inequalities with the diagram	
bi		Point (x, y) is inside the region if the point satisfies all three inequalities.	B1		B1 for clear correct explanation	
		For example, (1.5 , 3) is inside the region since $1.5 + 3 \leq 5$, $1.5 > 1$ and $3 > 2$.	B1		B1 for use of an example to illustrate this	
ii		Point (x, y) is outside the region if it does not satisfy at least one of the inequalities.	B1		B1 for clear correct explanation	
		For example, (2, 4) satisfies two of the conditions ($x > 1$ and $y > 2$) but does not satisfy $x + y \leq 5$.	B1		B1 for use of an example to illustrate this	
iii		Point (x,y) is on the boundary of the region if the point satisfies one of the inequalities but only as an equality.	B1		B1 for clear correct explanation	
		For example, (2, 3) is on the boundary of $x + y \leq 5$ as $2 + 3 = 5$.	B1		B1 for use of an example to illustrate this	
			9			

50 a		The total number of games cannot be greater than 4, hence $w + d \le 4$. The number of points must be 8 or more, they score 3 for a win, 1 for a draw, hence $3w + d \ge 8$. The shaded area is the region that satisfies these two inequalities.	B3	3	B1 for explaining $w + d \le 4$ B1 for explaining $3w + d \ge 8$ B1 for explaining what the shaded region is	M
b		In four games, they need to score at least 8 points. The graph shows that to do this they can win at least 3 games or win 2 games and draw two games.	B2		B1 for explaining they need at least 8 points B1 for showing all the possible ways this could happen	
c		The team would still need to score at least 8 points, but now they have five games in which to do it. The inequality $w + d \le 4$ would change to $w + d \le 5$. The other inequality is unchanged. The line for $w + d = 4$ would move up to go through (0, 5) and (5, 0).The other line would be unchanged.	B2 7		B1 for explanation of how this affects both equations B1 for complete solution, clearly showing what the new line(s) are	

<table>
<tr><td>51 a</td><td></td><td>For example, n^2 can generate a quadratic sequence:
1, 4, 9, 16, 25 ……. n^2</td><td>B1</td><td>2</td><td>B1 for a valid quadratic sequence</td><td>M</td></tr>
<tr><td>b</td><td></td><td>Similarities: you find the nth term for both types of sequence by looking at the differences between terms.</td><td>B1</td><td></td><td>B1 for a clear explanation</td><td></td></tr>
<tr><td></td><td></td><td>Differences: in a linear sequence you find the first term by subtracting the difference from the second term.
In a quadratic sequence you also have to look at the second differences. This allows you to extend the differences backwards to find the values of a, b and c in the nth term of $an^2 + bn + c$.</td><td>B1</td><td></td><td>B1 for a clear explanation</td><td></td></tr>
<tr><td>c</td><td></td><td>For a linear sequence, just keep on adding 6 each time to give:</td><td>B1</td><td></td><td>B1 for explaining how you woud find the sequence</td><td></td></tr>
<tr><td></td><td></td><td>2, 8, 14, 20 ………..($6n – 4$)
The nth term includes $6n$ because we add 6 each time, $6n – 4$ because $(2 – 6) = –4$.</td><td>B1</td><td></td><td>B1 for cao</td><td></td></tr>
<tr><td></td><td></td><td>For a quadratic equation, we build up the series by again having the first differences as 6, then choosing a second difference, say 2.
This will give a table such as:
<table><tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>nth term</td><td>2</td><td>8</td><td>16</td><td>26</td></tr><tr><td>1st difference</td><td>6</td><td>8</td><td>10</td><td></td></tr><tr><td>2nd difference</td><td>2</td><td>2</td><td></td><td></td></tr></table></td><td>B3</td><td></td><td>B1 for explaining second differences
B1 for use of a table or equivalent
B1 for correctly finding a quadratic equation with 2 and 8 as starting terms</td><td></td></tr>
<tr><td></td><td></td><td>Start with the two terms in positions n = 1 (2) and n = 2 (8) in the sequence.
Put each second difference as 2.
Then complete the first differences as shown.
Finally the nth terms can be completed</td><td>B2</td><td></td><td>B1 for explaining how you would find the nth term</td><td></td></tr>
</table>

<table>
<tr><td></td><td></td><td>as shown.
<table><tr><td>n</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>c</td><td>–2</td><td>2</td><td>8</td><td>16</td><td>26</td></tr><tr><td>$a + b$</td><td>4</td><td>6</td><td>8</td><td>10</td><td></td></tr><tr><td>$2a$</td><td>2</td><td>2</td><td>2</td><td></td><td></td></tr></table>
Extending the table backwards will allow us to find the values of a, b and c in the nth term $an^2 + bn + c$.
$2a = 2 \rightarrow a = 1$
$a + b = 4 \rightarrow b = 3$
$c = –2$
Hence nth term is $n^2 + 3n – 2$.</td><td>
10</td><td></td><td>B1 for correctly finding the nth term</td><td></td></tr>
<tr><td>52</td><td></td><td>$(m^2 - n^2)^2 + (2mn)^2 =$
$m^4 - 2m^2n^2 + n^4 + 4m^2n^2$
$= m^4 + 2m^2n^2 + n^4$
$= (m^2 + n^2)^2$</td><td>M1

A1
A1
A1
4</td><td>2
3</td><td>M1 choosing two smallest terms, squaring and adding

A1 for cao
A1 for cao
A1 for showing the factorisation leads to the given result</td><td>M</td></tr>
<tr><td>53</td><td></td><td>Because the differences between consecutive terms are 2, 3, 4, 5, 6, etc. (even and odd alternately); when generating the triangular number sequence starting with the odd 1, add even to odd to generate odd …3; add odd to odd to generate even …6; add even to even to generate even …10. Add odd to even to generate odd …15. We are now back again where we add even to odd to generate odd, and the whole sequence continues in the same way, continually giving two odd, two even, etc.</td><td>B3

3</td><td>2</td><td>B1 for explaining the about the differences of the terms being the set of integers

B1 for explaining the pattern is odd, even, odd, even and so on

B1 for explaining the complete sequence of combining odd and even to generate the final sequence</td><td>M</td></tr>
</table>

54		The assumption is that p and q are integers. $10p + q = 7n$ where n is also an integer. $7p + 3p + q = 7n$ $3p + q = 7n - 7p$ $= 7(n - p)$ As n and p are integers then $n - p$ is also an integer hence $7(n - p)$ is a multiple of 7 and so $3p + q$ must be as well.	M1 A1 M1 A1 A1 **5**	2	M1 for giving the assumption about p and q A1 for expressing $10p + q$ as a multiple of 7 M1 for expressing $3p + q$ in terms of $10p + q$ A1 for similar expression A1 for clear full explanation	M
55		$2(5(x - 2) + y) = 2(5x - 10 + y)$ $= 10x - 20 + 2y$ …….(1) $10(x - 1) + 2y - 10 = 10x - 10 + 2y - 10$ $= 10x - 20 + 2y$ ….(2) Equation (1) = equation (2) Hence the two expressions are equal.	M1 A1 M1 A1 A1 **5**	2	M1 for expanding A1 cao M1 for expanding A1 cao A1 for explaining the two expressions are the same	M

56 a		Take two numbers x and y where $x > y$. First step: $5(x - 2) = 5x - 10$ Second step: $2(5x - 10 + y)$ $= 10x - 20 + 2y$ Third step: $10x - 20 + 2y + 9 - y$ $= 10x - 11 + y$ Fourth step: $10x - 11 + y + 11 = 10x + y$ Hence where the first two numbers might have been 7 and 3, the final outcome would be 70 + 3 = 73.	B1 B1 B1 B1 B1 M1	2	B1 for initial explanation B1 cao B1 cao B1 cao B1 cao M1 for explanation of how this shows the final result	M
b		Let the single-digit number be x and the two-digit number be $10a + b$. First step: $10(10a + b) - 9x = 100a + 10b - 9x$ $= 100a + 10b + x - 10x$ $= 100a + 10b - 10x + x$ $= 100a + 10(b - x) + x$ The hundreds unit is a. The tens unit is $(b - x)$. The units term is x, the same as the single digit we started with. The split then becomes $10a + (b - x)$ and x. Adding these two gives $10a + b - x + x$ which is $10a + b$, the two-digit term.	A1 M1 A1 A1 A1 A1 B1 **13**		A1 for defining each digit M1 for expressing the first manipulation algebraically A1 for showing it in terms of hundreds, tens and units A1 for correct hundreds A1 for correct tens A1 for correct unit explanation B1 for clear explanation as to how the second manipulation gives the two-digit term	
57		Expand each square and add: $n^2 - 2n + 1 + n^2 + n^2 + 2n + 1$ $= 3n^2 + 2n - 2n + 2$ $= 3n^2 + 2$ as given.	M1 A1 B1 B1 **4**	2	M1 for expanding brackets A1 for correct expansion of brackets B1 for showing how the n terms cancel B1 for complete solution with no incorrect notation or terminology	M

<table>
<tr>
<td>58</td>
<td></td>
<td>The difference is 5 so nth term is $5n + c$
where c = first term – 5
$= 4 - 5 = -1$
So nth term is $5n - 1$.
Check the 4th term gives 19.
When $n = 4$, $4n - 1 = 20 - 1 = 19$, correct.</td>
<td>B1

M1
A1

A1

B1
5</td>
<td>2</td>
<td>B1 for obtaining the difference of 5

M1 for finding c
A1 cao

A1 cao

B1 for showing a check works</td>
<td>M</td>
</tr>
<tr>
<td>59</td>
<td></td>
<td>Triangular numbers are 1, 3, 6, 10, 15 …..
This will give a table as:
<table>
<tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>nth term</td><td>1</td><td>3</td><td>6</td><td>10</td></tr>
<tr><td>1st diff'nce</td><td>2</td><td>3</td><td>4</td><td></td></tr>
<tr><td>2nd diff'nce</td><td>1</td><td>1</td><td></td><td></td></tr>
</table>
Extending the table backwards will allow us to find the values of a, b and c in the nth term $an^2 + bn + c$.
<table>
<tr><td>n</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>c</td><td>0</td><td>1</td><td>3</td><td>6</td><td>10</td></tr>
<tr><td>$a + b$</td><td>1</td><td>2</td><td>3</td><td>4</td><td></td></tr>
<tr><td>$2a$</td><td>1</td><td>1</td><td>1</td><td></td><td></td></tr>
</table>
$2a = 1 \rightarrow a = \frac{1}{2}$
$a + b = 1 \rightarrow b = \frac{1}{2}$
$c = 0$
Hence nth term is $\frac{1}{2}n^2 + \frac{1}{2}n = \frac{1}{2}(n^2 + n)$
$= \frac{1}{2}n(n + 1)$</td>
<td>B1

M1
A1

B1

M1
A1

B1
B1
B1

B1
10</td>
<td>2
3</td>
<td>B1 for showing triangular numbers

M1 for method of finding differences
A1 for correct table

B1 for explanation of extending table backwards

M1 for method of extending table
A1 for correct table

B1 correct a
B1 correct b
B1 correct c

B1 for correct evaluation of generalisation to show given result</td>
<td>M</td>
</tr>
</table>

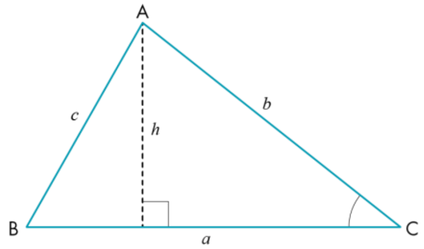
60		$T_n = \frac{1}{2}n(n+1)$ $T_{2n+1} = \frac{1}{2}(2n+1)(2n+1+1)$ $= \frac{1}{2}(2n+1)(2n+2)$ $= \frac{1}{2}(2n+1)(n+1)$ $= 2n^2 + 3n + 1$ …….. (1) $T_{n+1} = \frac{1}{2}(n+1)(n+1+1)$ $= \frac{1}{2}(n+1)(n+2)$ $= \frac{1}{2}(n^2+3n+2)$ …….(2) So $T_{2n+1} - T_{n+1}$ $= 2n^2 + 3n + 1 - \frac{1}{2}n^2 - \frac{3}{2}n - 1$ $= \frac{3}{2}n^2 + \frac{3}{2}n$	B1 M1 A1 M1 A1 B1 B1 **7**	2	B1 for correct T_n formula M1 for substituting $2n + 1$ A1 cao M1 for substituting $n + 1$ A1 cao B1 for subtracting each equation B1 for clear full explanation of proving the final connection	M
61		$T_n = \frac{1}{2}n(n+1)$ $\frac{T_n - 1}{T_n} = \frac{\frac{1}{2}n(n+1)-1}{\frac{1}{2}n(n+1)}$ $= \frac{\frac{1}{2}(n^2+n-2)}{\frac{1}{2}n(n+1)} = \frac{(n^2+n-2)}{n(n+1)}$ But $n^2 + n - 2$ factorises to $(n-1)(n+2)$ So final expression is $\frac{(n-1)(n+2)}{n(n+1)}$	B1 B3 B1 B1 **6**	2	B1 for T_n formula B1 for numerator expansion B1 for denominator expansion B1 for cancelling $\frac{1}{2}$ B1 for correct factorisation B1 for fully clear correct proof with no mathematical notational errors	M

62		Let the first number be x, then the next four are $x + 1$, $x + 2$, $x + 3$ and $x + 4$. The sum of these is $5x + 1 + 2 + 3 + 4$ which is $5x + 10 = 5(x + 2)$, a multiple of 5.	B1 B1 B1 **3**	2	B1 for stating each term in algebraic form B1 for adding all 5 terms B1 for showing they are multiple of 5	M
63		$10^x = \frac{a}{b}$ Hence $b \times 10^x = a$ Substitute this into $10^y = \frac{b}{a}$ to give $10^y = \frac{b}{(b \times 10^x)}$ Hence $10^y = \frac{1}{10^x}$ So $10^y \times 10^x = 1$ So $10^{(x+y)} = 1$ But $10^0 = 1$ And so $x + y = 0$.	M1 M1 A1 A1 M1 A1 B1 **7**	2	M1 for expressing a as subject M1 for substituting into other expression A1 for showing the correct substitution A1 for showing the product of the two terms equal to 1 M1 for showing combination of indices A1 for $10^0 = 1$ B1 for complete, clear proof with clear mathematical statements	M

64		Where two terms are x and $x + 1$ the expression required is: $(x + x + 1)^2 - (x^2 + (x + 1)^2)$ $= (2x + 1)^2 - (x^2 + x^2 + 2x + 1)$ $= 4x^2 + 4x + 1 - 2x^2 - 2x - 1$ $= 2x^2 + 2x$ $= 2x(x + 1)$ …..(1) Where $T_x = \frac{1}{2}x(x + 1)$ $4T_x = 2x(x + 1)$, which is same as the result in equation (1).	M1 A1 A1 A1 A1 B1 **6**	2	M1 for identifying the two terms A1 for correct expression as asked A1 for correct expansion of all brackets A1 for correct simplification A1 for correct factorisation B1 for clear complete proof with correct mathematical notation	M
65		Let $p = x$ Then $q = x + 1$ and $r = x + 2$ so $pr = x(x + 2)$ $= x^2 + 2x$ $q^2 - 1 = (x + 1)^2 - 1$ $= x^2 + 2x + 1 - 1$ $= x^2 + 2x = pr$	 B1 B1 B1 B1 B1 B1 **6**	2	 B1 for q expressed algebraically B1 for r expressed algebraically B1 for product pr expressed algebraically B1 for $q^2 - 1$ expressed algebraically B1 for simplification B1 for complete clear proof	M
66 a **bi** **ii**		There are many equivalent expressions. For example, expand the bracketed term: $\frac{z}{2} - q^2 - q - 4$ For example: $\frac{(6x+4y)}{10}$ For example: $4x^2 + 10x$	B1 B1 B1 **3**	2 3	B1 for a correct example B1 for a correct example B1 for a correct example	H
67 a **b** **c** **d**		To make it a product of two linear expressions. The quadratic expression. That the signs of the numbers in the brackets are different. One factor of the constant term is zero. There is only one set of brackets.	B1 B1 B1 B1 **4**	2 3	B1 for clear explanation B1 for clear explanation B1 for clear explanation B1 for clear explanation	H

68 a		For example: $x^2 - 1$. Because each part is a square, x^2 and 1^2, one is subtracted from the other.	B1	3	B1 for clear explanation	H
b		Because: $1000 \times 998 = (999 + 1) \times (999 - 1)$ $= 999^2 - 1$	B1		B1 for clear explanation	
			2			
69		Two that can be cancelled, for example: $\frac{2x}{x}$ and $\frac{5(x+1)}{10(x+1)}$	B1	2 3	B1 for two examples that cancel	
		I chose two straightforward ones, one that would cancel by a single letter and one that would cancel by an algebraic term.	B1		B1 for a clear explanation	
		Two that cannot be cancelled, for example: $\frac{x}{3}$ and $\frac{x^2}{x+1}$	B1		B1 for two examples that don't cancel	
		I chose two straightforward examples, one being a single term as numerator and denominator, the other one where the denominator was more than a single term.	B1		B1 for a clear explanation	
			4			
70		To get such a term on the top this must be the difference of two squares, hence the two expressions both need multiplying by $(3x - 4)$ to	B1 B1	2	B1 for clear explanation B1 for $(3x - 4)$	H
		give: $\frac{(3x+4)(3x-4)}{(x+2)(3x-4)}$	B1		B1 for setting up the expression	
		This expands to: $\frac{9x^2-16}{3x^2+2x-8}$	B1		B1 for showing how to find the final expression in suitable format	
			4			

71		$(2a + b)(2a + b) = 4a^2 + 4ab + b^2$ $(2a + b)(2a - b) = 4a^2 - b^2$ $(2a - b)(2a - b) = 4a^2 - 4ab + b^2$ $(a + b)(a + b) = a^2 + 2ab + b^2$ $(a + b)(a - b) = a^2 - b^2$ $(a - b)(a - b) = a^2 - 2ab + b^2$ The difference in the two is that in the $(2a \pm b)$ product, both the a^2 and ab terms have a coefficient of 4 (when the ab term is not zero), but in the $(a \pm b)$ product, the a^2 term has a coefficient of 1 and the ab term has a coefficient of 2 (when the ab term is not zero).	B1 B1 B1 **3**	2	B1 for showing all the possibilities B1 for showing all the possibilities B1 for clear explanation	H

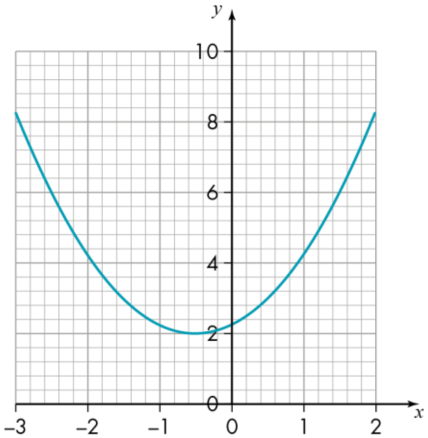
72 a		Draw a triangle ABC.  	B1	2	B1 for drawing diagram correctly labelled	H
		Using trigonometric functions: $\sin C = \frac{h}{b}$	B1		B1 for trigonometric expression linking C, h and b	
		Therefore: $h = b \sin C$	B1		B1 for $h = b \sin$ C	
		Then using the basic formula for area of a triangle: Area = $\frac{1}{2}a \times h$ Substituting for h gives:	B1 M1		B1 formula for area of triangle M1 substitution of h	
		Area = $\frac{1}{2}a \times b \sin C$ Area = $\frac{1}{2}ab \sin C$ as required.	A1 M1		A1 cao M1 for complete correct proof with correct mathematical notation throughout	
b		Using the given triangle: $C = 45°$, $a = x + 2 = 6$, $b = x - 2 = 2$ Area = $\frac{1}{2}ab \sin C$	M1		M1 for expressing a, b and C	
		$= \frac{1}{2} \times 6 \times 2 \times \sin 45°$	M1		M1 for substituting for a, b and C	
		$= 6 \times \frac{1}{\sqrt{2}}$ $= \frac{6}{\sqrt{2}}$	A1		A1 cao	
		Multiply numerator and denominator by $\sqrt{2}$. This gives $\frac{6 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{6 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$ $= 3\sqrt{2}$ as required.	M1 A1		M1 for dealing with the $\sqrt{2}$ in denominator A1 for full explanation showing given result	
			12			

73		$f(x) = 3 - 7x$ Find $f^{-1}(x)$ from $y = 3 - 7x$: $7x = 3 - y$ $x = \frac{3-y}{7}$ So $f^{-1}(x) = \frac{3-x}{7}$ Find $g^{-1}(x)$ from $y = 7x + 3$: $7x = y - 3$ $x = \frac{y-3}{7}$ So $g^{-1}(x) = \frac{x-3}{7}$ So $f^{-1}(x) + g^{-1}(x) = \frac{3-x}{7} + \frac{x-3}{7}$ $= \frac{3-x+x-3}{7} = \frac{0}{7}$ $= 0$ as required.	M1 A1 M1 A1 M1 B1 **6**	2	M1 for method of finding inverse A1 cao M1 for method of finding inverse A1 cao M1 for showing how the two functions can be added together B1 for complete explanation of how they sum to 0	H

74 ai		It means that the constant terms in both expressions are positive, or if one is positive and one is negative, their sum is positive.	B1 B1	2	B1 for first condition B1 for second condition	H
ii		It means that the constant terms in both expressions are negative, or if one is positive and one is negative their sum is negative.	B1 B1		B1 for first condition B1 for second condition	
iii		The expression is the difference of two squares.	B1		B1 for clear explanation	
bi		For example, $(4x + 2)(x - 1)$ $= 4x^2 - 2x - 2$ as required.	B1		B1 for correct example explained	
ii		For example, $(3x + 1)(x + 1)$ $= 3x^2 + 4x + 1$ as required.	B1		B1 for correct example explained	
c		If it is positive then both expressions have the same sign. If it is negative then the expressions have different signs.	B1		B1 for complete clear explanation	
			8			

75 a		$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$	M1 A1	2	M1 for squaring half of 5 A1 cao	H
b		$2x^2 + 10x - 5 = 0$ Divide through by 2: $x^2 + 5x - \frac{5}{2} = 0$ Completing the square: $(x + \frac{5}{2})^2 - \frac{5}{2} - \frac{25}{4} = 0$ $(x + \frac{5}{2})^2 = \frac{(10+25)}{2} = \frac{35}{2}$ Taking the square root of each side: $x + \frac{5}{2} = \frac{\pm\sqrt{35}}{2} = \pm 2.958$ $x = 2.958 - 2.5$ or $x = -2.958 - 2.5$ $x = 0.458$ or $x = -5.458$	M1 M1 A1 A1 A1 A1		M1 correctly simplifying equation M1 completing the square A1 cao A1 for 2.958 (3 dp or more) A1 for showing the two possible solutions A1 for two correct solutions (3 dp or more)	
c		Consider the 4 lines of his working. Line 1: he has forgotten the –5 in the equation. Line 2: he has squared the right-hand side incorrectly. Line 3: the $\frac{5}{2}$ should be $\frac{-5}{2}$ and the $5\sqrt{2}$ should be $\frac{\pm\sqrt{45}}{2}$ Line 4: there should be two solutions.	B1 B1 B1 B1		B1 for line 1 comments B1 for line 2 comments B1 for line 3 comments B1 for line 4 comments	
d		$x^2 + 5x - 5 = 0$ $(x + \frac{5}{2})^2 - 5 = (\frac{5}{2})^2$ Don't forget the –5 in the original equation. $(x + \frac{5}{2})^2 - 5 = \frac{25}{4}$ Square the whole of the bracket on the right-hand side. $(x + \frac{5}{2})^2 = 5 + \frac{25}{4}$ Remember, when you add a term on one side, you must also add it on the other side. $(x + \frac{5}{2}) = \pm\sqrt{\frac{45}{4}} = \pm\frac{\sqrt{45}}{2}$ Be careful when taking square roots of fractions. Don't forget that when you find square root there is a positive and a negative root.	B2		B1 for commenting on each line B1 for clear positive comments that would be deemed helpful	
			14			

76 a		If the coefficient of x^2 is positive, the turning point is between the two roots, so choose an equation with two positive x roots, say $x = 1$ and $x = 3$. The quadratic that has these roots is $y = (x – 1)(x – 3)$, which is $y = x^2 – 4x + 3$. Complete the square to get $y = (x – 2)^2 – 2^2 + 3$. Hence the turning point of $y = (x – 2)^2 – 1$ will have a positive x-value.	B1 B1	2	B1 for clear explanation of what equation to look for. B1 for choosing a suitable equation with these characteristics.	H
b		If the equation has no roots, then the turning point will be above the x-axis, hence a positive value of y. From the general form of the quadratic equation, $y = ax^2 + bx + c$, the value of b^2 is less than $4ac$ so, keeping a as 1, we could choose c as 6 and b as 2, giving $y = x^2 + 2x + 6$ Complete the square to get $y = (x + 1)^2 – 1 + 6$. Hence the turning point of $y = (x + 1)^2 + 5$ will have a positive y value.	B1 B1 B1		B1 for a suitable equation with complete justification B1 for clear explanation of what equation to look for. B1 for choosing a suitable equation with these characteristics.	
c		The y-intercept will be positive if y is positive when $x = 0$. for example: $y = (x + 2)^2 + 3$, when $x = 0$ $y = 7$, positive so $y = (x + 2)^2 + 3$ has a y-intercept that is positive.	B1 B1		B1 for a suitable equation with complete justification B1 for complete clear explanation	
			7			

<table>
<tr><td>77 a</td><td></td><td>A table of values for the graph will be:
<table><tr><td>x</td><td>–3</td><td>–2</td><td>–1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>y</td><td>8.25</td><td>4.25</td><td>2.25</td><td>2.25</td><td>4.25</td><td>8.25</td></tr></table></td><td>B1

B1</td><td>2
3</td><td>B1 for finding suitable points to assist sketch the graph

B1 for a suitable sketch of the graph</td><td>H</td></tr>
<tr><td>b</td><td></td><td>f(x + 3) – 2 is a translation 3 left and 2 down of f(x). Therefore, as the turning point moves 2 down, it will now turn on the x-axis, giving one real root at the point (–3.5, 0).</td><td>B1

B1

4</td><td></td><td>B1 for explaining how the function will change the graph

B1 for explanation about turning point being now on the x-axis</td><td></td></tr>
</table>