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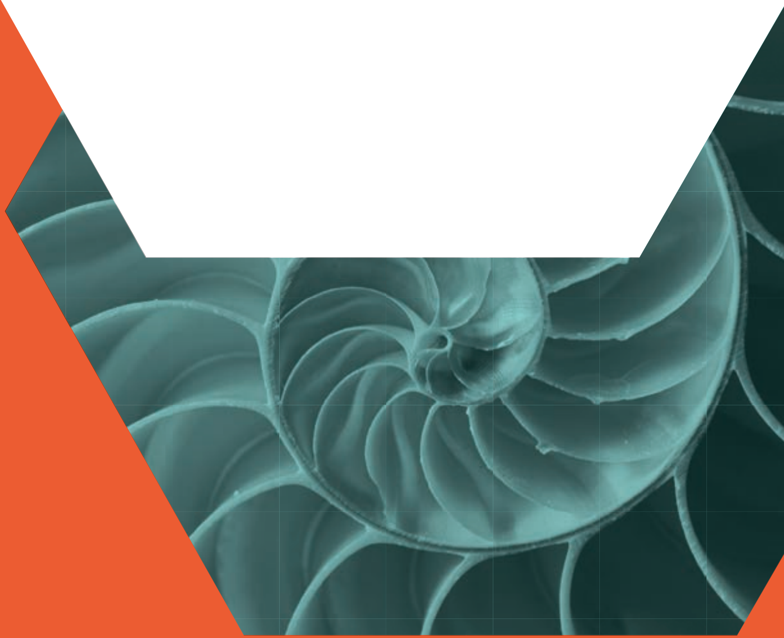

Maths

4th Edition

GCSE

Higher Skills Book

Reason, interpret and communicate mathematically and solve problems

Sandra Wharton

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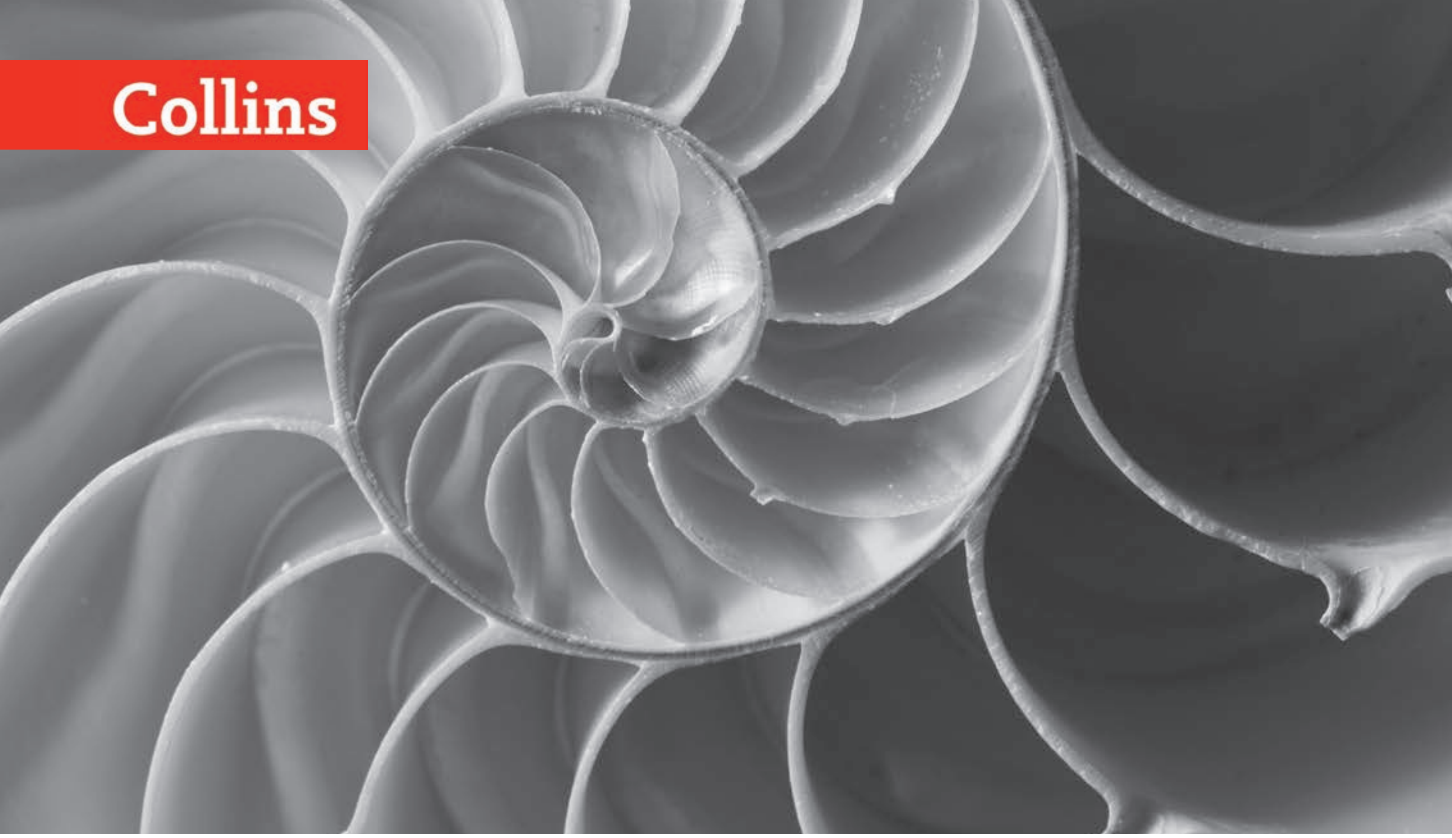

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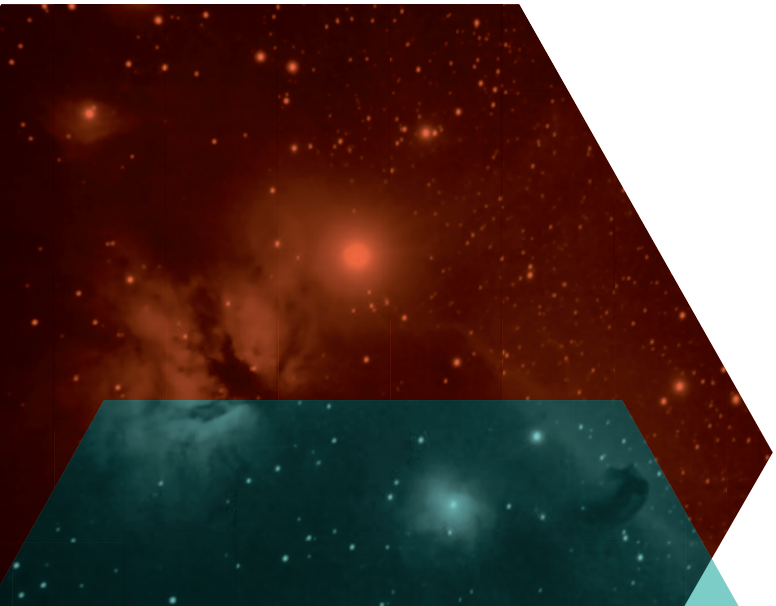

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How to use this book

Focused on the new assessment objectives AO2 and AO3, this book is full of expertly written practice questions to help students succeed in mathematical reasoning and problem-solving.

This book is ideal to be used alongside the Practice Book or Student Book. It is structured by strand to encourage students to tackle questions without already knowing the mathematical context in which they sit.

The longer questions can be tried in class to generate discussions, and new questions require students to think and analyse their work, encouraging independence. Students should take their time with the longer and multi-step questions.

Some questions have a hint ? at the back of the book to get you started, but students should be encouraged to try to answer the question first, before looking at the hint.

Full mark schemes will be provided online – a brief sample is included in this Evaluation Pack.

The questions in this book are differentiated with colours to show progression. Green is the most accessible, moving through blue to the pink questions, which are the most challenging.

1 Algebraic fractions and functions

1 Explain the difference between each of these expressions.

a $2n$ and $n + 2$ **b** $3(c + 5)$ and $3c + 5$ **c** n^2 and $2n$ **d** $2n^2$ and $(2n)^2$

2 Explain how you know if a letter symbol represents an unknown or a variable.

3 Look at each of the examples below. Identify the errors in each one and explain how they should be corrected. Provide some written feedback to the person who did them that does not give them the answer but would help them to see their mistake.

a $5(c + 4) = 5c + 4$ **b** $6(t – 2) = 6t – 4$ **c** $–3(4 – s) = –12 – 2s$

d $15 – (n – 4) = 11 – n$

4 Look at the list of formulae below.

i $V = lwh$ **ii** $s = ut + \frac{1}{2}at^2$ **iii** $x = 3y – 2$ **iv** $\left(\frac{u + v}{t}\right)t$

v $V^2 = u^2 + 2ad$ **vi** $A = 2\pi r^2 + 2\pi rh$

a Which of these examples might be easy/difficult to substitute in to? Explain what makes them easy/difficult and identify what the classic mistakes might be.

b Now imagine you are going to change the subject of the formula in each case. Again explain which of the examples might be easy/difficult to rearrange. Explain what makes them easy/difficult and identify what the classic mistakes might be.

5 Look at the list of formulae below.

i $y = 2x + 3$ **ii** $z = 2(x + 3)$ **iii** $t = –2(3 – x)$ **iv** $\frac{-2(z + 2)}{x}$

If you are substituting a negative value for the variable in each case, explain which of these might be difficult and why. What typical mistakes might people make? Do you have any suggestions to help them avoid making these mistakes?

6 Explain the similarities and differences between rearranging a formula and solving an equation.

7 When you substitute $s = 6$ and $t = 2$ into the formula: $z = \frac{3(s + 2t)}{12}$ you get 2.5.
Can you make up two more formulae that also give $z = 2.5$ when $s = 6$ and $t = 2$ are substituted? Explain what you are doing and try to use two different methods as well.

Example 1

An identity can be described as two expressions that mean exactly the same thing. Alternatively they could be described as an equation that is TRUE no matter the value of the variable.
For example:

$\frac{a}{4} = a \div 4$ or $2n(n + 4) = 2n^2 + 8n$ or $2^a \times 2^b = 2^{a+b}$

8 Could this expression be written in a different way, but still be the same? $\frac{2n + 6}{2}$

9 The height of an isosceles triangle is three times its base. The area is 6 cm^2. What is its height?

10 **a** Write an expression that is equivalent to $\frac{z}{2} - q(q + 1) - 4$

b Write an expression that would simplify to:

i $\frac{3x + 2y}{5}$ **ii** $5x\left(\frac{4x}{5} + 2\right)$

11 **a** When working with a quadratic expression explain what it means to factorise.

b Explain what information you need to complete this.

c Explain what it tells you about the factors if the constant term of the quadratic is negative.

d Explain what difference it makes if the constant term is zero.

12 **a** Write an expression which can be written as the difference of two squares. Explain how you know it can be written as the difference of two squares.

b Why must 1000×998 give the same result as $999^2 - 1$?

13 Give two examples of algebraic fractions that can be cancelled and two that cannot be cancelled. Explain how you decided on your examples.

14 An expression of the form $\frac{ax^2 - b}{cx^2 + dx^2 - e}$ simplifies to $\frac{3x + 4}{x + 2}$.
What was the original expression?

15 For homework, a teacher asks her class to simplify the expression $\frac{x^2 - 9}{x^2 + 2x - 3}$

This is Phillips' answer:

$$\frac{x^2 - 9}{x^2 + 2x - 3}$$

$$\frac{\cancel{x}^2 - \cancel{9}^3}{\cancel{x}(x + 2) - \cancel{3}_1}$$

$$= \frac{x - 3}{x + 2 - 1}$$

$$= \frac{x - 3}{x - 1}$$

Phillip checked the answer and it was correct. However, when the teacher marked the homework, she found that Phillip had in fact made several mistakes.

Explain the mistakes that Phillip made.

16 Explain how the product of two linear expressions of the form $(2a \pm b)$ are different from the product of two linear expressions of the form $(a \pm b)$?

17 Six friends agreed to buy each other a chocolate Easter egg.

Four of the friends are girls and two of them are boys.

Each girl gives each boy a red egg.

Each boy gives each girl a blue egg.

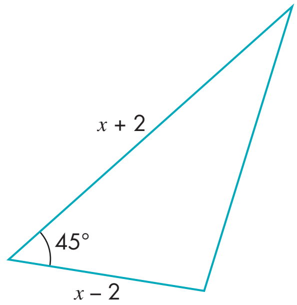
Each girl gives each of the other girls a yellow egg.

And each boy gives each of the other boys a green egg.

How many eggs of each colour do the friends buy between them? Make sure you show your working.

18 **a** Using the sine rule and the general formula for the area of a triangle show that the area of any triangle is equal to $\frac{1}{2}ab\sin C$.

b Show that if $x = 4$ the area of this triangle is equal to $3\sqrt{2}$.



19 **a** Explain why:

i 'I think of a number and double it' is different from

ii 'I think of a number and double it. The answer is 12.'

b $6 = 2p - 8$

i How many solutions does this equation have?

ii Write another equation with the same solution.

iii Why do they have the same solution? How do you know?

iv Can you write a rule for generating an infinite number of solutions?

20 **a** Imagine you are teaching a short session on expressions, equations, formulae and identities. Explain carefully the similarities and differences between them.

b Produce a short activity that will help your class to become confident and fluent when using expressions, equations, formulae and identities.

21 The functions f and g are such that

$f(x) = 3 - 7x$ $\qquad$ $g(x) = 7x + 3$

Prove that $f^{-1}(x) + g^{-1}(x) = 0$ for all values of x.

22 Without drawing the graphs, compare and contrast features of the following pairs of graphs.

i $y = 2x$ $y = 2x + 6$	**ii** $y = x + 5$ $y = x - 6$	**iii** $y = 4x - 5$ $y = -4x + 6$	**iv** $y = 2x$ $y = \frac{1}{2}x$

Make sure your explanations use as much mathematical vocabulary as possible.

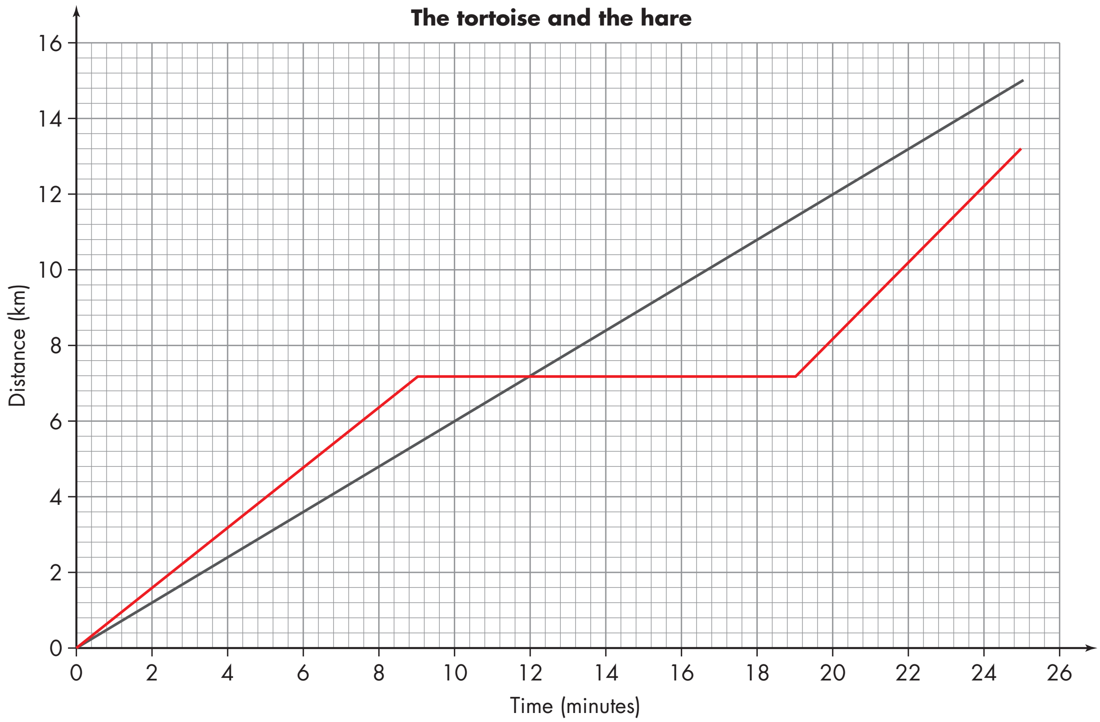
23 For real-life problems that generate linear functions:

a Explain what the gradient means in terms of the original problem.

b Do the intermediate points have any practical meaning?

c What does the intercept mean in terms of the original problem?

24



Do you know the story of the race between the hare and the tortoise? This graph shows information about the race between these two creatures. The hare is red and the tortoise is grey.

a Who was ahead after five minutes?

b What happened at nine minutes?

c After how long did the tortoise draw level with the hare?

d Who won the race and by how much?

e If they both continued at the same speed they were travelling at the end of the race how much further would the race need to be run over for the hare to retake the lead?

f Can you write your own question similar to this one that uses the graph of a linear function?

25

a What is special about the two linear expressions that, when expanded, have:

i a positive x coefficient?

ii a negative x coefficient?

iii no x coefficient?

b Give an example of an expression in the form $(x + a)(x + b)$ which when expanded has:

i the x coefficient equal to the constant term

ii the x coefficient greater than the constant term.

iii What does the sign of the constant term tell you about the original expression?

This example will remind you about completing the square. You will then try a couple of questions on completing the square before using this technique to deduce the turning points for quadratic equations.

Example 2

What number should you add to $x^2 + 3x$ to complete the square?

The general formula for completing the square is $x^2 + bx + \left(\frac{b}{2}\right)^2$.

So in this case you will need to add $\left(\frac{3}{2}\right)^2 = \frac{9}{4}$.

26 a What number should be added to $x^2 + 5x$ to complete the square?

b Solve the quadratic equation $2x^2 + 10x - 5 = 0$ by completing the square.

c Adam tried to solve a quadratic by completing the square but he made a number of mistakes. Look at his working below. What mistakes did he make?

$$x^2 + 5x - 5 = 0$$

$$\left(x + \frac{5}{2}\right)^2 = \left(\frac{5}{2}\right)^2$$

$$\left(x + \frac{5}{2}\right)^2 = \frac{25}{2}$$

$$x = \frac{5}{2} + 5\sqrt{2}$$

$$x + \frac{5}{2} = \sqrt{\frac{25}{2}}$$

d Copy his working and write some comments which show Adam where he has gone wrong. Write them in a way that does not correct his answer for him but will help make sure he does not make the same mistake again. Your answer should include guidance on why he should have known he had made a mistake this time as well as how to make sure he doesn't make the same mistake again.

27 For each part of this question think of a quadratic equation of the type $y = (x + a)^2 + b$. Give an example of a quadratic that fits the description. Remember to justify your example in each case, which means giving an explanation as well as an example.

a The turning point has a positive x-value.

b The turning point has a positive y-value.

c The y-intercept is positive.

28 a Sketch the graph of $f(x) = x^2 + x + \frac{9}{4}$

b Hence determine whether $f(x + 3) - 2 = 0$ has any real roots. Give a clear justification for your answer.

Hints and tips

1, 2, 6	You may find a couple of examples will help you explain this.
9	You may find it useful to draw a diagram and make sure you read the question carefully.
11	Explain how you work out the two linear factors.
18	First use what you did in part 1 to show that the area of this triangle can be written as $\frac{\sqrt{2}}{4}(x^2 - 4)$. Also sin 45 is equal to $\frac{\sqrt{2}}{2}$
20	You may want to use some examples to support your explanation.
23	It might help your explanation if you use some real life examples.
25	In each case consider what it means when you expand two linear functions of the type $(ax + b)(bx + c)$.

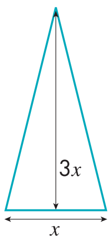
Solutions

Guidance on the use of codes for this mark scheme

M1	Method mark	E1	Explanation or justification mark
A1	Accuracy mark	oe	Or equivalent
B1	Working mark	ft	Follow through
C1	Communication mark		

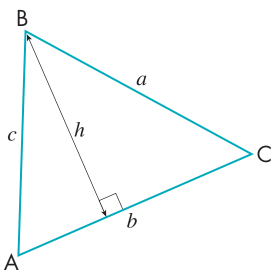
Question	Working	Answer	Mark	Notes
1(a)	Example $2 \times 3 = 6$ $2 + 3 = 5$		C1 C1	C1 for explanation that $2n$ means $2 \times n$ which is different from $n + 2$. An example could be given to support the argument. An additional mark can be given for identifying the exception which is when $n = 2$.
(b)	$3(c + 5) = 3c + 15$		M1	M1 for multiplying out the brackets to show that the two expressions are not equivalent.
(c)	Example $3^2 = 3 \times 3 = 9$ $2 \times 3 = 6$		C1	C1 for explanation that n^2 means $n \times n$ which is different from $2n$. An example could be given to support the argument. An additional mark can be given for identifying the exception which is when $n = 2$.
(d)	Example $2n^2 = 2 \times (3^2) = 2 \times 9 = 18$ $(2 \times 3)^2 = 6 \times 6 = 36$		C1 C1	C1 for an explanation that using BIDMAS for $2n^2$ tells you to calculate the power first. BIDMAS tells you that $(2n)^2$ tells you that you do the calculation inside the bracket first. An example could be given to support the argument.
			6	
2			C1 C1 C1 C1	An explanation that a variable is a category of number that has a range of values. With an example such as $F = \frac{9}{5}C + 32$ An unknown is a discrete quantity that you can solve for. With an example such as $3x + 2 = 8$
			4	

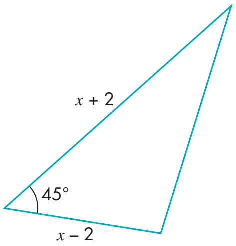
Question	Working	Answer	Mark	Notes
3 (a)	$5(c + 4) = 5c + 20$		M1	M1 for correctly multiplying out the brackets.
			C1	Comment such as "Don't forget to multiply out both terms in the brackets.
(b)	$6(t - 2) = 6t - 12$		M1	M1 for correctly multiplying out the brackets.
			C1	Comment such as "Don't forget 6(……..) means MULTIPLY both terms by 6.
(c)	$-3(4 - s) = -12 + 3s$		M1	M1 for correctly multiplying out the brackets.
			C1	Comment such as "Don't forget –3(……..) means MULTIPLY both terms by 6 and a minus × minus = ……..
(d)	$15 - (n - 4) = 15 - n + 4 = 15 + 4 - n = 19 - n$		M1	M1 for correctly multiplying out the brackets.
			C1	Comment such as "Don't forget –1(n – 4) means ………
			C1	Extra communication mark for giving the advice in the form of a question.
			9	
4 (a)			M1	M1 for identifying at least 2 examples
			E1	E1 for justifications such as: • Examples with brackets are more difficult with negative numbers • Divisions and powers can be more difficult when substituting non integer values.
(b)			M1	M1 for identifying at least 2 examples
			E1	E1 for justifications such as: • Multi step problems are more difficult. • Examples that include all the possible variations of BIDMAS are more difficult
			E1	Identification of a classic mistake such as 2^2 as 2 × 2. Incorrect use of BIDMAS.
			5	

Question	Working	Answer	Mark	Notes
5			M1 E1 C2 **5**	Identify that examples iii or iv might be more difficult because they have negative numbers outside the brackets. The mistake would be especially common if there is a negative number inside the bracket as people would either forget or not understand that a negative × negative will give a positive. C2 for a comment such as $-3(n - \ldots\ldots)$ Will the second term be positive or negative when you multiply out the brackets? 1 mark if the answer is given in the comment.
6			C1 C1 **2**	Explanation such as: SAME - When rearranging a formula or solving an equation you will be moving variables or numbers across the equals using inverse operations. DIFFERENT when rearranging a formula you are not looking for an exact value for the variable. Whilst when solving an equation you are looking to identify the value of the unknown in the equation.
7			M1 C1 M1 **3**	M1 for correct example. C1 for accurate explanation of how the examples were found. Extra method mark for having a systematic approach to finding examples.
8	$n + 3$	Yes	M2 **2**	M2 for simplest form. M1 for other equivalent example $\frac{1}{2}(2n + 6)$
9	 $\frac{1}{2}(3x \times x) = \frac{4x}{2}$ $\frac{4x}{2} = 6$ $4x = 12$	$x = 3$ so height is 6 cm	C1 M1 A1 **3**	C1 for drawing a suitable diagram A1 cao

Question	Working	Answer	Mark	Notes
10 (a)			M1 C1	M1 for correct example. C1 for accurate explanation of how the examples were found.
b (i)			M1 C1	M1 for correct example. C1 for accurate explanation of how the examples were found.
(ii)			M1 C1	M1 for correct example. C1 for accurate explanation of how the examples were found.
			M1	Extra method mark for having a systematic approach to finding examples.
			5	
11 (a)			M2	M1 for suitable explanation, for example "it is finding what we can multiply to get the quadratic." M2 for also identifying that the factors identify the roots or the x intercepts of the quadratic.
(b)			E2	Identifying that you need to know the coefficients of x^2 and x and the constant term of the quadratic. E1 for identify 2 of these.
(c)			E2	Identifying in words that it tells you that the brackets are of the form $(x + a)(x - b)$
(d)			E1	Identifying in words that it tells you that the brackets are of the form $x(x \pm a)$
			C1	Extra C1 for generalising using algebra to support explanation.
			8	

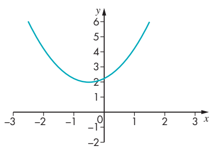
Question	Working	Answer	Mark	Notes
12 (a)			M1	M1 for a correct example.
			E2	E1 for explaining that the sign of the constant term is negative and it is a square number. Need both for E2
			C1	$x^2 - b^2$ C1 for generalising using algebra to support explanation.
(b)		$999^2 - 1$ Completing the square gives: $(999 + 1)(999 - 1)$ This is the same as 1000×998 As required.	M2	M1 with no explanation
			6	
13			M1	M1 for correct examples.
			C2	C1 for accurate explanation of how the examples were found. Extra method mark for having a systematic approach to finding examples.
			3	
14	Using the difference of two squares for the numerator gives $(3x + 4)(3x - 4)$. Assuming the new term in the numerator is also in the denominator gives: $\frac{(3x+4)(3x-4)}{(x+2)(3x-4)}$ If you then multiply out the numerator you will get: $9x^2 - 12x + 12x - 16$ $= 9x^2 - 16$ Expand the brackets in the denominator to give: $3x^2 - 4x + 6x - 8$ $= 3x^2 + 2x - 8$	The original expression: $\frac{9x^2 - 16}{3x^2 + 2x - 8}$	B2 C1 A1 M1	B2 for correct working B1 for ft C1 for explanation of process A1 cao M1 extra method mark for using a value to check answer. For example substituting $x = 1$ into both the simplified and the original equation gives $\frac{7}{3}$
			5	

Question	Working	Answer	Mark	Notes
15		In line 2 Phillip has initially rearranged $x^2 + 2x - 3$ to $x(x + 2) - 3$ and then he has incorrectly simplified in line 3 $\frac{\cancel{x}^2 - \cancel{9}^3}{\cancel{x}(x + 2) - \cancel{3}_1}$	M2 C1 **3**	M1 for each mistake C1 for clear explanation
16		The coefficient of a^2 for the product of $(2a \pm b)$ is four times the coefficient of a^2 for the product of $(a \pm b)$. The coefficient of a for the product of $(2a \pm b)$ is twice the coefficient of a for the product of $(a \pm b)$. The constant term is the same for both.	M3 C1 **3**	M1 for each coefficient and one for the constant. C1 for effect use of algebraic notation to support explanation.
17		$4 \times 2 = 8$ blue eggs $4 \times 2 = 8$ red eggs $4 \times 3 = 12$ yellow eggs $2 \times 1 = 2$ green eggs	B3 A1 **4**	B3 for correct working B1 if don't realise that when calculating the totals for girls to girls and boys to boys it is $n \times (n \times 1)$ A1 cao
18 (a)		Using trigonometric function $\sin A = \frac{h}{c}$ $\sin C = \frac{h}{a}$ Therefore: $h = c \sin A$ $A = a \sin C$ Then using the basic formula for area of a triangle Area $\Delta = \frac{1}{2}hb$ Substituting for h gives: Area $\Delta = \frac{1}{2}a \sin C \times b$ Area $\Delta = \frac{1}{2}ab \sin C$ as required	M4 C1	M1 for drawing diagram M3 for correct method C1 extra mark for good explanation possibly using connectives effectively. 

Question	Working	Answer	Mark	Notes
(b)		 Area = $\frac{1}{2}ab \sin 45$ $= \frac{1}{2}ab\frac{\sqrt{2}}{2}$ $= \frac{\sqrt{2}}{2}(x+2)(x-2)$ $= \frac{\sqrt{2}}{2}(x^2-4)$ Therefore if $x = 4$ $= \frac{\sqrt{2}}{2}(4^2-4)$ $= \frac{\sqrt{2}}{2} = \times 12$ $= 3\sqrt{2}$ as required	M3 C1 5	M3 for correct method C1 extra mark for good explanation possibly using connectives effectively.
19 (a)			E1	An explanation such as: Number (i) is an expression whilst number (ii) is an equation and therefore can be solved for *n*.
(b)		(i) One (ii) Any suitable example (iii) Something similar to – Both sides have been doubled. (iv) $6n = n(2p-8)$ or equivalent	A1 A1 E1 A2 6	
20 (a)			C2	At least 3 differences and 2 similarities well explained.
(b)			C3 5	Suitable activity deduct a marks for any inaccuracies.

Question	Working	Answer	Mark	Notes
21		$f^{-1}(x) = -\left(\frac{x-3}{7}\right)$	M2	
		$g^{-1}(x) = \left(\frac{x-3}{7}\right)$	C1	C1 for clear layout and use of connectives.
		Therefore: $f^{-1}(x) + g^{-1}(x)$ $= -\left(\frac{x-3}{7}\right) + \left(\frac{x-3}{7}\right)$ $= 0$ for all values of x as required.		
			3	
22 (i)			E1	Both have a gradient of 2 but the first crosses the y axis at 0 the other at 6
(ii)			E1	Both have a gradient of 1 but the first crosses the y axis at 5 the other at –6
(iii)			E2	The magnitude of the gradient is the same but the first is positive whilst the second is negative. The first crosses the y axis at –5 the other at 6
(iv)			E1	The first has a gradient of 2 the second has a gradient of $\frac{1}{2}$. They both cross the y axis at the origin.
			C1	C1 for good use of mathematical vocabulary.
			7	
23 (a)			E1	The gradient describes how much the dependent variable changes for a given change in the independent variable, oe
(b)			E2	A straight line graph should describe a functional relationship that is defined for all values of x within the stated domain, oe
(c)			E1	The intercept is the value of the dependent variable when the independent variable is equal to 0, oe
			C1	C1 extra mark for good use of mathematical vocabulary.
			5	

Question	Working	Answer	Mark	Notes
24 (a)		Hare	A1	
(b)		The Hare stops	A1	
(c)		12 minutes	A1	
(d)		The Tortoise won the race by 2 km	A2	
(e)		About 4 km	M1 A1	Calculate approximate gradient for each line and use this to work out when the 2 lines are equivalent.
(f)			B2	B1 for suitable question B2 for quality of question
			9	
25 (a) (i)			E1	The constant terms in the linear expressions are either both negative OR both positive, oe
(ii)			E1	One of the constant terms in the linear expressions is negative the other is positive, oe.
(iii)			E2	The constant terms in the linear expressions are either both 0. When multiplied out this will give ax^2, oe.
			C1	C1 extra mark for good use of mathematical vocabulary.
b (i)		$(x + a)(x + b)$ As the coefficient for x needs to be 1	E1	
(ii)		$(x + 1)^2$		
(iii)		$ab < 1$	E1	
		If the constant term is positive the signs are the same. If negative the signs are different.	E1	
			5	
26 (a)		$\left(\frac{5}{2}\right)^2$	A1	
(b)		To solve $2x^2 + 10x - 5 = 0$	M2	
		Divide through by 2 $x^2 + 5x - \frac{5}{2} = 0$ Rearrange	B4	Award working marks for clarity of presentation. Deduct marks if not aware of ± or if not simplified.
		$x^2 + 5x + \frac{5}{2}$ $\left(x + \frac{5}{2}\right)^2 - \left(\frac{5}{2}\right)^2 = \frac{5}{2}$	C1	C1 extra mark for good use of mathematical vocabulary and/or commentary

Question	Working	Answer	Mark	Notes
		Completing the square $\left(x+\frac{5}{2}\right)^2 - \frac{25}{4} = \frac{5}{2}$ Rearrange $\left(x+\frac{5}{2}\right)^2 = \frac{5}{2} + \frac{25}{4}$ $\left(x+\frac{5}{2}\right)^2 = \frac{35}{4}$ Square root both sides $x + \frac{5}{2} = \pm\sqrt{\frac{35}{4}}$ $x = \pm\sqrt{\frac{35}{4}} - \frac{5}{2}$ $x = \pm\frac{1}{2}\left(\sqrt{35} - 5\right)$		
(c)			A2	Award marks for identifying error
(d)			C2	Award marks for suitable formative comments
			10	
27 (a)			A1	Suitable quadratic of the form $(x - a)^2 \pm b$
			E1	Explanation to refer to a positive translation of x^2 in the x axis
(b)			A1	Suitable quadratic of the form $(x \pm a)^2 + b$
			E1	Explanation to refer to a positive translation of x^2 in the y axis
			A1	Any suitable example for which $y > 1$ when $x = 0$
(c)			E1	E1 for explanation of why example chosen.
			C1	C1 extra mark for good use of mathematical vocabulary.
			6	
28 (a)			A2	
(b)	One real root at (–7/2,0)		E2 A1	Explanation that $f(x + 3) - 2$ is a translation 3 left and 2 down of $f(x)$. Therefore as the turning point moves 2 down it will turn on the x axis giving one real root. (–7/2,0), actual value not necessary as long as accurate explanation of why one repeated root.
			5	