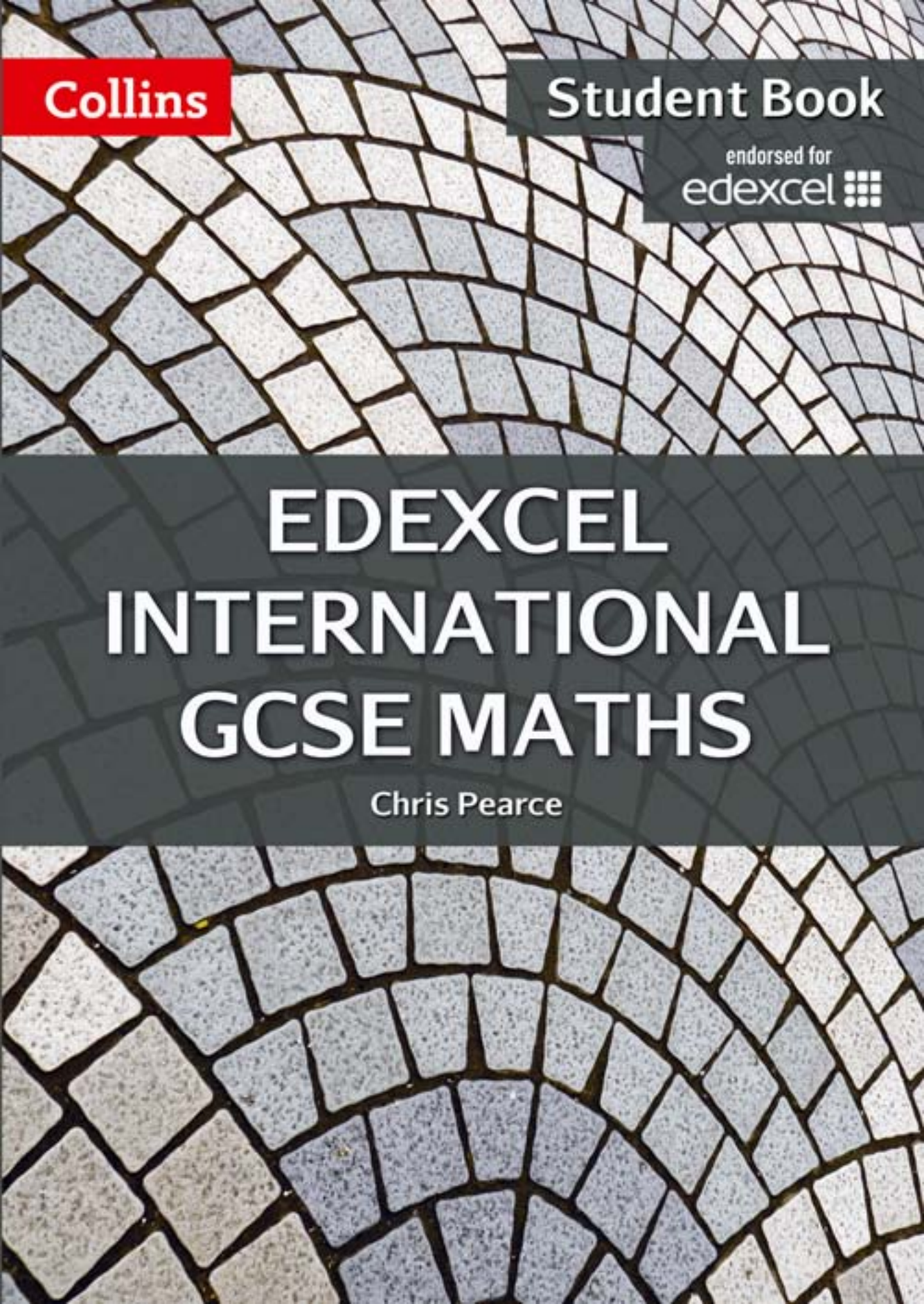
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Chris Pearce

Why this chapter matters

We use percentages and fractions in many situations in our everyday lives.

Why use fractions and percentages?

Because:

- basic percentages and simple fractions are easy to understand
- they are a good way of comparing quantities
- fractions and percentages are used a lot in everyday life.

Who uses them?

Here are some examples of what you might see:

- Shops and businesses
  - Everything at half price in the sales!
  - Special offer — 10% off!
- Banks
  - Interest rates on loans 6.25%.
  - Interest rates on savings 2.5%.
- Salespeople
  - Earn 7.5% commission on sales.
- Government
  - Half of government workers are over 55.
  - Unemployment has fallen by 1%.
- Workers
  - My pay rise is 2.3%.
  - My income tax is 20%.
- Teachers
  - Your test result is 67%.
  - Three-fifths of our students gain a grade C or above in IGCSE mathematics.

Can you think of other examples?

You will find many everyday uses of fractions and percentages in this chapter.

Chapter

2 Fractions and percentages

Topics	Level	Key words
1 Equivalent fractions	FOUNDATION	numerator, denominator, cancel, lowest terms, simplest form, proper fraction, vulgar fraction, mixed number, top-heavy fraction
2 Fractions and decimals	FOUNDATION	decimal, fraction, recurring decimal, terminating decimal
3 Recurring decimals	HIGHER	
4 Percentages, fractions and decimals	FOUNDATION	percentage, decimal equivalent
5 Calculating a percentage	FOUNDATION	quantity, multiplier
6 Increasing or decreasing quantities by a percentage	FOUNDATION	
7 Expressing one quantity as a percentage of another	FOUNDATION	percentage change, percentage increase, percentage decrease, percentage profit, percentage loss
8 Reverse percentage	FOUNDATION	unitary method
9 Interest and depreciation	FOUNDATION	compound interest, depreciation
10 Compound interest problems	HIGHER	
11 Repeated percentage change	HIGHER	

What you need to be able to do in the examinations:

FOUNDATION	HIGHER
• Understand and use equivalent fractions, mixed numbers and vulgar fractions and simplify a fraction by cancelling common factors. • Express a given number as a fraction or percentage of another number. • Convert a fraction to a decimal or percentage and vice versa. • Recognise that a terminating decimal is a fraction. • Understand percentages and their multiplicative nature as operators. • Solve simple percentage problems, including percentage increase and decrease. • Use reverse percentages. • Use compound interest and depreciation.	• Convert recurring decimals into fractions. • Use repeated percentages. • Solve compound interest problems.

2.1 Equivalent fractions

Equivalent fractions are two or more fractions that represent the same part of a whole.

EXAMPLE 1

Complete the following.

a $\frac{3}{4} \rightarrow \frac{\times 4}{\times 4} = \frac{\square}{16}$ **b** $\frac{2}{5} = \frac{\square}{15}$

a Multiplying the **numerator** by 4 gives 12. This means $\frac{12}{16}$ is an equivalent fraction to $\frac{3}{4}$.

b To convert the **denominator** from 5 to 15, you multiply by 3. Do the same thing to the numerator, which gives $2 \times 3 = 6$. So, $\frac{2}{5} = \frac{6}{15}$.

The fraction $\frac{3}{4}$, in Example 1a, is in its **lowest terms** or **simplest form**.

This means that the only number that is a factor of both the numerator and denominator is 1.

A fraction with the numerator (top number) smaller than the denominator (bottom number) is called a **proper fraction**. An example of a proper fraction is $\frac{4}{5}$.

A **vulgar fraction** has a bigger numerator (top number) than the denominator (bottom number). An example of an vulgar fraction is $\frac{9}{5}$. It is sometimes called a **top-heavy fraction**.

A **mixed number** is made up of a whole number and a proper fraction. An example of a mixed number is $1\frac{3}{4}$.

EXAMPLE 2

Convert $\frac{14}{5}$ into a mixed number.

$\frac{14}{5}$ means $14 \div 5$.

Dividing 14 by 5 gives 2 with a remainder of 4 (5 fits into 14 two times, with 4 left over).

This means that there are 2 whole ones and $\frac{4}{5}$ left over.

So, $\frac{14}{5} = \frac{5}{5} + \frac{5}{5} + \frac{4}{5}$

$= 2\frac{4}{5}$

EXAMPLE 3

What fraction of 25 is 10?

The fraction we want is $\frac{10}{25}$

We can simplify this:

$\frac{10}{25} = \frac{2}{5}$

because both numbers are divisible by 5.

So 10 is $\frac{2}{5}$ of 25

EXERCISE 2A

1 Copy and complete the following.

a $\frac{2}{5} \rightarrow \frac{\times 4}{\times 4} = \frac{\square}{20}$ **b** $\frac{1}{4} \rightarrow \frac{\times 3}{\times 3} = \frac{\square}{12}$ **c** $\frac{3}{8} \rightarrow \frac{\times 5}{\times 5} = \frac{\square}{40}$

d $\frac{2}{3} \rightarrow \frac{\times \square}{\times \square} = \frac{\square}{18}$ **e** $\frac{3}{4} \rightarrow \frac{\times \square}{\times \square} = \frac{\square}{12}$ **f** $\frac{5}{8} \rightarrow \frac{\times \square}{\times \square} = \frac{\square}{40}$

2 Copy and complete the following.

a $\frac{10}{15} \rightarrow \frac{\div 5}{\div 5} = \frac{\square}{\square}$ **b** $\frac{12}{15} \rightarrow \frac{\div 3}{\div 3} = \frac{\square}{\square}$ **c** $\frac{20}{28} \rightarrow \frac{\div 4}{\div 4} = \frac{\square}{\square}$

d $\frac{12}{18} \rightarrow \frac{\div \square}{\div \square} = \frac{\square}{\square}$ **e** $\frac{15}{25} \rightarrow \frac{\div 5}{\div \square} = \frac{\square}{\square}$ **f** $\frac{21}{30} \rightarrow \frac{\div \square}{\div \square} = \frac{\square}{\square}$

3 **Cancel** each of these fractions to its simplest form.

a $\frac{4}{6}$ **b** $\frac{5}{15}$ **c** $\frac{12}{18}$ **d** $\frac{6}{8}$ **e** $\frac{3}{9}$

f $\frac{5}{10}$ **g** $\frac{14}{16}$ **h** $\frac{28}{35}$ **i** $\frac{10}{20}$ **j** $\frac{4}{16}$

4 Put the fractions in each set in order, with the smallest first.

a $\frac{1}{2}, \frac{5}{6}, \frac{2}{3}$ **b** $\frac{3}{4}, \frac{1}{2}, \frac{5}{8}$ **c** $\frac{7}{10}, \frac{2}{5}, \frac{1}{2}$ **d** $\frac{2}{3}, \frac{3}{4}, \frac{7}{12}$ **e** $\frac{1}{6}, \frac{1}{3}, \frac{1}{4}$ **f** $\frac{9}{10}, \frac{3}{4}, \frac{4}{5}$

5 What fraction of 20 is:

a 10 **b** 5 **c** 4 **d** 15 **e** 6?

Write your answers in the lowest terms.

6 Write your answers to this question as simply as possible.

a What fraction of 16 is 12?

b What fraction of 45 is 30?

c What fraction of 35 is 21?

d What fraction of 48 is 16?

e What fraction of 40 is 15?

7 Convert each of these vulgar fractions into a mixed number.

a $\frac{7}{3}$ **b** $\frac{8}{3}$ **c** $\frac{9}{4}$ **d** $\frac{10}{7}$ **e** $\frac{12}{5}$ **f** $\frac{7}{5}$

8 Convert each of these mixed numbers into an vulgar fraction.

a $3\frac{1}{3}$ **b** $5\frac{5}{6}$ **c** $1\frac{4}{5}$ **d** $5\frac{2}{7}$ **e** $4\frac{1}{10}$ **f** $5\frac{2}{3}$

g $2\frac{1}{2}$ **h** $3\frac{1}{4}$ **i** $7\frac{1}{6}$ **j** $3\frac{5}{8}$ **k** $6\frac{1}{3}$ **l** $9\frac{8}{9}$

9 Check your answers to questions **1** and **2**, using the fraction buttons on your calculator.

FOUNDATION

10 Which of these vulgar fractions has the largest value?

$\frac{27}{4}$ $\frac{31}{5}$ $\frac{13}{2}$

Show your working to justify your answer.

11 Find a mixed number that is greater than $\frac{85}{11}$ but smaller than $\frac{79}{10}$.

2.2 Fractions and decimals

Here are three **decimals**:

0.6 0.62 0.615.

Which is the largest?

Put them in a place value table:

Units	.	Tenths	Hundredths	Thousandths
0	.	6		
0	.	6	2	
0	.	6	1	5

They all have 6 tenths. The largest is 6.2 because it has 2 hundredths. The smallest is 0.6 because it has no hundredths.

EXAMPLE 4

Express 0.32 as a **fraction**.

$0.32 = \frac{32}{100}$

This cancels to $\frac{8}{25}$

So, $0.32 = \frac{8}{25}$

You can convert a fraction into a decimal by dividing the numerator by the denominator.

EXAMPLE 5

a Express $\frac{3}{8}$ as a decimal.

$\frac{3}{8}$ means $3 \div 8$. This is a division calculation.

So, $\frac{3}{8} = 3 \div 8 = 0.375$

b Express $\frac{5}{9}$ as a decimal.

$\frac{5}{9} = 5 \div 9 = 0.555\ldots$

The decimal expression does not stop. The dots show that the sequence of 5s could continue forever. We call this a **recurring decimal**. It can be written as $0.\dot{5}$.

0.375 is called a **terminating decimal**. The decimal expression stops after three digits in this case.

EXERCISE 2B

1 Convert each of these decimals to fractions, cancelling where possible.

a 0.7 **b** 0.4 **c** 0.5 **d** 0.03 **e** 0.06

f 0.13 **g** 0.25 **h** 0.38 **i** 0.55 **j** 0.64

2 Convert each of these fractions to decimals.

a $\frac{1}{2}$ **b** $\frac{3}{4}$ **c** $\frac{3}{5}$ **d** $\frac{9}{10}$

e $\frac{1}{8}$ **f** $\frac{5}{8}$ **g** $\frac{7}{8}$ **h** $\frac{7}{20}$

3 Put each of the following sets of numbers in order, with the smallest first.

a 0.6, 0.3, $\frac{1}{2}$ **b** $\frac{2}{5}$, 0.8, 0.3

c 0.35, $\frac{1}{4}$, 0.15 **d** $\frac{7}{10}$, 0.72, 0.71

e 0.8, $\frac{3}{4}$, 0.7 **f** 0.08, 0.1, $\frac{1}{20}$

g 0.55, $\frac{1}{2}$, 0.4 **h** $1\frac{1}{4}$, 1.2, 1.23

HINTS AND TIPS

Convert the fractions to decimals first.

4 Write these fractions as recurring decimals:

a $\frac{1}{3}$ **b** $\frac{2}{3}$ **c** $\frac{1}{9}$ **d** $\frac{4}{9}$ **e** $\frac{1}{11}$ **f** $\frac{8}{11}$

5 Say whether these fractions can be written as terminating or recurring decimals:

a $\frac{5}{8}$ **b** $\frac{5}{9}$ **c** $\frac{5}{10}$ **d** $\frac{5}{11}$ **e** $\frac{5}{12}$

6 Which is bigger, $\frac{7}{8}$ or 0.87?

Show your working.

7 Which is smaller, $\frac{2}{3}$ or 0.7?

Show your working.

FOUNDATION

2.3 Recurring decimals

H

Writing fractions as recurring decimals is easy.

Writing recurring decimals as fractions is more difficult.

Suppose we want to write $0.\dot{8} = 0.888...$ as a fraction:

Write	$f = 0.888...$
Multiply by 10	$10f = 8.888...$

Now subtract the top row from the bottom.

$9f = 8$ (When you subtract, the digits after the decimal point cancel out).

$\Rightarrow \quad f = \frac{8}{9}$

If there are two recurring digits, multiply by 100. If there are three recurring digits, multiply by 1000, and so on.

For example, to write $0.\dot{3}\dot{6} = 0.363636...$ as a fraction:

Let	$f = 0.3636...$
Multiply by 100	$100f = 36.3636$
Subtract	$99f = 36$

$\Rightarrow \quad f = \frac{36}{99}$ which cancels to $\frac{4}{11}$

So $0.3636... = \frac{4}{11}$

EXERCISE 2C

HIGHER

1 Write $\frac{2}{3}$ as a recurring decimal.

2 **a** Write 0.222... as a fraction.

b Write 0.7777... as a fraction.

c Write 0.4444... as a fraction.

d Look at your answers above. What do they suggest 0.9999… is as a fraction?

3 **a** Write 0.272727... as a fraction.

b Write 0.090909... as a fraction.

c Write 0.636363... as a fraction.

d What do your answers to **a**, **b** and **c** suggest about other fractions which give recurring decimals? Check your suggestions.

HIGHER

4 Write these recurring decimals as fractions:

a $0.\dot{5}$ **b** $0.\dot{2}\dot{4}$ **c** $0.\dot{4}\dot{8}$

5 **a** You know that $\frac{1}{3} = 0.3333...$

What fraction is 0.03333...?

b What fraction is 0.06666...?

2.4 Percentages, fractions and decimals

100% means the *whole* of something. So if you want to, you can express *part* of the whole as a **percentage**.

Per cent means 'out of 100'.

So, any percentage can be converted to a fraction with denominator 100.

For example:

$32\% = \frac{32}{100}$ which can be simplified by cancelling to $\frac{8}{25}$

Also, any percentage can be converted to a decimal by dividing the percentage number by 100. This means moving the digits two places to the right.

For example:

$65\% = 65 \div 100 = 0.65$

Any decimal can be converted to a percentage by multiplying by 100%.

For example:

$0.43 = 0.43 \times 100\% = 43\%$

Any fraction can be converted to a percentage by converting the denominator to 100 and taking the numerator as the percentage.

For example:

$\frac{2}{5} = \frac{40}{100} = 40\%$

Fractions can also be converted to percentages by dividing the numerator by the denominator and multiplying by 100%.

For example:

$\frac{2}{5} = 2 \div 5 \times 100\% = 40\%$

Knowing the percentage and **decimal equivalents** of common fractions is extremely useful.

$\frac{1}{2} = 0.5 = 50\%$ $\frac{1}{4} = 0.25 = 25\%$ $\frac{3}{4} = 0.75 = 75\%$ $\frac{1}{8} = 0.125 = 12.5\%$

$\frac{1}{10} = 0.1 = 10\%$ $\frac{1}{5} = 0.2 = 20\%$ $\frac{1}{3} = 0.33 = 33\frac{1}{3}\%$ $\frac{2}{3} = 0.67 = 67\%$

The following table shows how to convert from one to the other.

Convert from percentage to:	
Decimal	**Fraction**
Divide the percentage by 100, for example $52\% = 52 \div 100$ $= 0.52$	Make the percentage into a fraction with a denominator of 100 and simplify by cancelling down if possible, for example $52\% = \frac{52}{100} = \frac{13}{25}$

Convert from decimal to:	
Percentage	**Fraction**
Multiply the decimal by 100%, for example $0.65 = 0.65 \times 100\%$ $= 65\%$	If the decimal has 1 decimal place put it over the denominator 10. If it has 2 decimal places put it over the denominator 100, etc. Then simplify by cancelling down if possible, for example $0.65 = \frac{65}{100} = \frac{13}{20}$

Convert from fraction to:	
Percentage	**Decimal**
Write the fraction as an equivalent with a demonimator of 100 if possible, then the numerator is the percentage, for example $\frac{3}{20} = \frac{15}{100} = 15\%$ or convert to a decimal and change the decimal to a percentage, for example $\frac{7}{8} = 7 \div 8 = 0.875 = 87.5\%$	Divide the numerator by the denominator, for example $\frac{9}{40} = 9 \div 40 = 0.225$

EXAMPLE 6

Convert the following to decimals: **a** 78% **b** 35% **c** $\frac{3}{25}$ **d** $\frac{7}{40}$.

a $78\% = 78 \div 100 = 0.78$

b $35\% = 35 \div 100 = 0.35$

c $\frac{3}{25} = 3 \div 25 = 0.12$

d $\frac{7}{40} = 7 \div 40 = 0.175$

EXAMPLE 7

Convert the following to percentages: **a** 0.85 **b** 0.125 **c** $\frac{7}{20}$ **d** $\frac{3}{8}$.

a $0.85 = 0.85 \times 100\% = 85\%$

b $0.125 = 0.125 \times 100\% = 12.5\%$

c $\frac{7}{20} = \frac{35}{100} = 35\%$

d $\frac{3}{8} = 3 \div 8 \times 100\% = 0.375 \times 100\% = 37.5\%$

EXAMPLE 8

Convert the following to fractions: **a** 0.45 **b** 0.4 **c** 32% **d** 15%.

a $0.45 = \frac{45}{100} = \frac{9}{20}$

b $0.4 = \frac{4}{10} = \frac{2}{5}$

c $32\% = \frac{32}{100} = \frac{8}{25}$

d $15\% = \frac{15}{100} = \frac{3}{20}$

EXERCISE 2D

1 Write each percentage as a fraction in its simplest form.

a 8% **b** 50% **c** 25%

d 35% **e** 90% **f** 75%

2 Write each percentage as a decimal.

a 27% **b** 85% **c** 13%

d 6% **e** 80% **f** 32%

3 Write each decimal as a fraction in its simplest form.

a 0.12 **b** 0.4 **c** 0.45

d 0.68 **e** 0.25 **f** 0.625

4 Write each decimal as a percentage.

a 0.29 **b** 0.55 **c** 0.03

d 0.16 **e** 0.6 **f** 1.25

5 Write each fraction as a percentage.

a $\frac{7}{25}$ **b** $\frac{3}{10}$ **c** $\frac{19}{20}$

d $\frac{17}{50}$ **e** $\frac{11}{40}$ **f** $\frac{7}{8}$

6 Write each fraction as a decimal.

a $\frac{9}{15}$ **b** $\frac{3}{40}$ **c** $\frac{19}{25}$

d $\frac{5}{16}$ **e** $\frac{1}{20}$ **f** $\frac{1}{8}$

FOUNDATION

FOUNDATION

7 **a** Convert each of the following test scores into a percentage. Give each answer to the nearest whole number.

Subject	Result	Percentage
Mathematics	38 out of 60	
English	29 out of 35	
Science	27 out of 70	
History	56 out of 90	
Technology	58 out of 75	

b If all the tests are of the same standard, which was the highest result?

8 Copy and complete the table.

Percentage	Decimal	Fraction
34%		
	0.85	
		$\frac{3}{40}$
45%		
	0.3	
		$\frac{2}{3}$
84%		
	0.45	
		$\frac{3}{8}$

2.5 Calculating a percentage

To calculate a percentage of a **quantity**, you multiply the quantity by the percentage. The percentage may be expressed as either a fraction or a decimal. When finding percentages without a calculator, base the calculation on 10% (or 1%) as these are easy to calculate.

EXAMPLE 9

Calculate: **a** 10% of 54 kg **b** 15% of 54 kg.

a 10% is $\frac{1}{10}$ so $\frac{1}{10}$ of 54 kg = 54 kg ÷ 10 = 5.4 kg

b 15% is 10% + 5% = 5.4 kg + 2.7 kg = 8.1 kg

Using a percentage multiplier

You have already seen that percentages and decimals are equivalent so it is easier, particularly when using a calculator, to express a percentage as a decimal and use this to do the calculation.

For example, 13% is a **multiplier** of 0.13, 20% a multiplier of 0.2 (or 0.20) and so on.

EXAMPLE 10

Calculate 45% of 160 cm.

45% = 0.45, so 45% of 160 = 0.45 × 160 = 72 cm

Find 52% of $460.

52% = 0.52

So, 0.52 × 460 = 239.2

This gives $239.20

Remember to always write a money answer with 2 decimal places.

EXERCISE 2E

1 What multipliers are equivalent to these percentages?

a 88% **b** 30% **c** 25%

d 8% **e** 115%

2 What percentages are equivalent to these multipliers?

a 0.78 **b** 0.4 **c** 0.75

d 0.05 **e** 1.1

3 Calculate the following:

a 15% of $300 **b** 6% of $105

c 23% of 560 kg **d** 45% of 2.5 kg

e 12% of 9 hours **f** 21% of 180 cm

g 4% of $3 **h** 35% of 8.4 m

i 95% of $8 **j** 11% of 308 minutes

k 20% of 680 kg **l** 45% of $360

4 An estate agent charges 2% commission on every house he sells. How much commission will he earn on a house that he sells for $120 500?

FOUNDATION

5 A store had 250 employees. During one week of a flu epidemic, 14% of the store's employees were absent.

a What percentage of the employees went into work?

b How many of the employees went into work?

6 It is thought that about 20% of fans at a soccer match are women. For one match there were 42 600 fans. How many of these do you think were women?

7 At a Paris railway station, in one week 350 trains arrived. Of these trains, 5% arrived early and 13% arrived late. How many arrived on time?

8 A school estimates that for a school play 60% of the students will attend. There are 1500 students in the school. The caretaker is told to put out one seat for each person expected to attend plus an extra 10% of that number in case more attend. How many seats does he need to put out?

HINTS AND TIPS

It is not 70% of the number of students in the school.

9 A school had 850 pupils and the attendance record in one week was:

Monday 96% Tuesday 98% Wednesday 100% Thursday 94% Friday 88%

How many pupils were present each day?

10 Calculate the following.

a 12.5% of \$26 **b** 6.5% of 34 kg

c 26.8% of \$2100 **d** 7.75% of \$84

e 16.2% of 265 m **f** 0.8% of \$3000

11 Air consists of 80% nitrogen and 20% oxygen (by volume). A man's lungs have a capacity of 600 cm^3. How much of each gas will he have in his lungs when he has just taken a deep breath?

12 A factory estimates that 1.5% of all the garments it produces will have a fault in them. One week the factory produces 850 garments. How many are likely to have a fault?

13 An insurance firm sells house insurance and the annual premiums are usually set at 0.3% of the value of the house. What will be the annual premium for a house valued at \$90 000?

14 Average prices in a shop went up by 3% last year and 3% this year. Did the actual average price of items this year rise by more, the same amount, or less than last year? Explain how you decided.

2.6 Increasing or decreasing quantities by a percentage

Increasing by a percentage

There are two methods for increasing a quantity by a percentage.

Method 1

Work out the increase and add it on to the original amount.

EXAMPLE 11

Increase $6 by 5%.

Work out 5% of $6: (5 ÷ 100) × 6 = $0.30

Add the $0.30 to the original amount: $6 + $0.30 = $6.30

Method 2

Use a multiplier. An increase of 6% is equivalent to the original 100% *plus* the extra 6%. This is a total of 106% and is equivalent to the multiplier 1.06

EXAMPLE 12

Increase $6.80 by 5%.

A 5% increase is a multiplier of 1.05

So $6.80 increased by 5% is $6.80 × 1.05 = $7.14

EXERCISE 2F

1 What multiplier is used to increase a quantity by:

a 10% **b** 3% **c** 20% **d** 7% **e** 12%?

2 Increase each of the following by the given percentage. (Use any method you like.)

a $60 by 4%
b 12 kg by 8%
c 450 g by 5%
d 545 m by 10%
e $34 by 12%
f $75 by 20%
g 340 kg by 15%
h 670 cm by 23%
i 130 g by 95%
j $82 by 75%
k 640 m by 15%
l $28 by 8%

FOUNDATION

3 Azwan, who was on a salary of $27 500, was given a pay rise of 7%. What is his new salary?

4 In 2005 the population of a city was 1 565 000. By 2010 it had increased by 8%. What was the population of the city in 2010?

5 A small firm made the same pay increase of 5% for all its employees.

a Calculate the new pay of each employee listed below. Each of their salaries before the increase is given.

Caretaker, $16 500	Supervisor, $19 500
Driver, $17 300	Manager, $25 300

b Explain why the actual pay increases are different for each employee.

6 A bank pays 7% interest on the money that each saver keeps in the bank for a year. Allison keeps $385 in the bank for a year. How much will she have in the bank after the year?

7 In 1980 the number of cars on the roads of a town was about 102 000. Since then it has increased by 90%. Approximately how many cars are there on the roads of the town now?

8 An advertisement for a breakfast cereal states that a special-offer packet contains 15% more cereal for the same price as a normal 500 g packet. How much breakfast cereal is in a special-offer packet?

9 A headteacher was proud to point out that, since he had arrived at the school, the number of students had increased by 35%. How many students are now in the school, if there were 680 when the headteacher started at the school?

10 At a school concert there are always about 20% more girls than boys. If at one concert there were 50 boys, how many girls were there?

11 A government adds a sales tax to the price of most goods in shops. One year it is 17.5% on all electrical equipment.

Calculate the price of the following electrical equipment when sales tax of 17.5% is added.

Equipment	*Pre-sales tax price*
TV set	$245
Microwave oven	$72
CD player	$115
Personal stereo	$29.50

12 A television costs $400 before sales tax at 17.5% is added.

If the rate of sales tax goes up from 17.5% to 20%, by how much will the cost of the television increase?

Decreasing by a percentage

There are two methods for decreasing by a percentage.

Method 1

Work out the decrease and subtract it from the original amount.

EXAMPLE 13

Decrease $8 by 4%.

Work out 4% of $8: (4 ÷ 100) × 8 = $0.32

Subtract the $0.32 from the original amount: $8 – $0.32 = $7.68

Method 2

Use a multiplier. A 7% decrease is equivalent to 7% less than the original 100%, so it represents 100% – 7% = 93% of the original. This is a multiplier of 0.93

EXAMPLE 14

Decrease $8.60 by 5%.

A decrease of 5% is a multiplier of 0.95

So $8.60 decreased by 5% is $8.60 × 0.95 = $8.17

EXERCISE 2G

1 What multiplier is used to decrease a quantity by:

a 8% **b** 15% **c** 25% **d** 9% **e** 12%?

2 Decrease each of the following by the given percentage. (Use any method you like.)

a $10 by 6%
b 25 kg by 8%
c 236 g by 10%
d 350 m by 3%
e $5 by 2%
f 45 m by 12%
g 860 m by 15%
h 96 g by 13%
i 480 cm by 25%
j 180 minutes by 35%
k 86 kg by 5%
l $65 by 42%

3 A car valued at $6500 last year is now worth 15% less. What is its value now?

FOUNDATION

4 A new diet guarantees that you will lose 12% of your mass in the first month. What mass should the following people have after one month on the diet?

a Gracia, who started at 60 kg **b** Pierre, who started at 75 kg

c Greta, who started at 52 kg

5 A motor insurance firm offers no-claims discounts off the full premium, as follows.

1 year with no claims	15% discount off the full premium
2 years with no claims	25% discount off the full premium
3 years with no claims	45% discount off the full premium
4 years with no claims	60% discount off the full premium

Mr Patel and his family are all offered motor insurance from this firm.

Mr Patel has four years' no-claims discount and the full premium would be $440.

Mrs Patel has one year's no-claims discount and the full premium would be $350.

Sandeep has three years' no-claims discount and the full premium would be $620.

Priyanka has two years' no-claims discount and the full premium would be $750.

Calculate the actual amount each member of the family has to pay for the motor insurance.

6 A large factory employed 640 people. It had to streamline its workforce and lose 30% of the workers. How big is the workforce now?

7 On the last day of term, a school expects to have an absence rate of 6%. If the school population is 750 students, how many students will the school expect to see on the last day of term?

8 Most speedometers in cars have an error of about 5% from the true reading. When my speedometer says I am driving at 70 km/h,

a what is the lowest speed I could be doing?

b what is the highest speed I could be doing?

9 Kerry wants to buy a sweatshirt ($19), a tracksuit ($26) and some running shoes ($56). If she joins the store's premium club which costs $25 to join she can get 20% off the cost of the goods.

Should she join or not? Use calculations to support your answer.

10 A biscuit packet normally contains 300 g of biscuits and costs $1.40.

There are two special offers.

Offer A: 20% more for the same price

Offer B: Same amount for 20% off the normal price

Which is the better offer?

a Offer A **b** Offer B **c** Both the same **d** Cannot tell

Justify your choice.

2.7 Expressing one quantity as a percentage of another

You find one quantity as a percentage of another by writing the first quantity as a fraction of the second, making sure that the *units of each are the same*. Then you can convert the fraction into a percentage by multiplying by 100%.

EXAMPLE 15

Express $6 as a percentage of $40.

Set up the fraction and multiply by 100%.

$$\frac{6}{40} \times 100\% = 15\%$$

EXAMPLE 16

Express 75 cm as a percentage of 2.5 m.

First, convert 2.5 m to 250 cm to get a common unit.

So, the problem now becomes: Express 75 cm as a percentage of 250 cm.

Set up the fraction and multiply by 100%.

$$\frac{75}{250} \times 100\% = 30\%$$

Percentage change

A **percentage change** may be a **percentage increase** or a **percentage decrease**.

$$\text{Percentage change} = \frac{\text{change}}{\text{original amount}} \times 100\%$$

Use this to calculate **percentage profit** or **percentage loss** in a financial transaction.

EXAMPLE 17

Jake buys a car for $1500 and sells it for $1800. What is Jake's percentage profit?

Jake's profit is $300, so his percentage profit is:

$$\text{percentage profit} = \frac{\text{profit}}{\text{original amount}} \times 100\% = \frac{300}{1500} \times 100\% = 20\%$$

EXERCISE 2H

1 Express each of the following as a percentage. Give suitably rounded figures (see page 116) where necessary.

a \$5 of \$20 **b** \$4 of \$6.60 **c** 241 kg of 520 kg

d 3 hours of 1 day **e** 25 minutes of 1 hour **f** 12 m of 20 m

g 125 g of 600 g **h** 12 minutes of 2 hours **i** 1 week of a year

j 1 month of 1 year **k** 25 cm of 55 cm **l** 105 g of 1 kg

2 Liam went to school with his pocket money of \$2.50. He spent 80 cents at the shop. What percentage of his pocket money had he spent?

3 In Greece, there are 3 654 000 acres of agricultural land. Olives are grown on 237 000 acres of this land. What percentage of the agricultural land is used for olives?

4 During one year, it rained in Detroit on 123 days of the year. What percentage of days were wet?

5 Find the percentage profit on the following. Give your answers to one decimal place.

	Item	*Retail price* (selling price)	*Wholesale price* (price the shop paid)
a	CD player	\$89.50	\$60
b	TV set	\$345.50	\$210
c	Computer	\$829.50	\$750

6 Before Anton started to diet, his mass was 95 kg. His mass is now 78 kg. What percentage of his original mass has he lost?

7 In 2009 a city raised \$14 870 000 in local tax. In 2010 it raised \$15 597 000 in tax. What was the percentage increase?

8 When Ziad's team won the soccer league in 1995, they lost only four of their 42 league games. What percentage of games did they *not* lose?

9 In one year Britain's imports were as follows.

British Commonwealth	\$109 530 000
USA	\$138 790 000
France	\$53 620 000
Other countries	\$221 140 000

a What percentage of the total imports came from each source? Give your answers to 1 decimal place.

b Add up your answers to part **a**. What do you notice? Explain your answer.

10 Imran and Nadia take the same tests. Both tests are out of the same mark.

Here are their results.

	Test A	Test B
Imran	12	17
Nadia	14	20

Whose result has the greater percentage increase from test A to test B? Show your working.

11 A supermarket advertises its cat food as shown.

A government inspector is checking the claim.

She observes that over one hour, 46 people buy cat food and 38 buy the store's own brand.

Based on these figures, is the store's claim correct?

12 Aya buys antiques and then sells them on the internet.

Find her percentage profit or loss on each of these items:

Item	Aya bought for:	Aya sold for:
Vase	$105	$84
Radio	$72	$90
Doll	$15	$41.25
Toy train	$50	$18

2.8 Reverse percentage

Reverse percentage questions involve working backwards from the final amount to find the original amount when you know, or can work out, the final amount as a percentage of the original amount.

Method 1: The unitary method

The **unitary method** has three steps.

Step 1: Equate the final percentage to the final value.

Step 2: Use this to calculate the value of 1%.

Step 3: Multiply by 100 to work out 100% (the original value).

EXAMPLE 18

The price of a car increased by 6% to $9116. Work out the price before the increase.

106% represents $9116.

Divide by 106. *1% represents $9116 ÷ 106*

Multiply by 100. *100% represents original price: $9116 ÷ 106 × 100 = $8600*

So the price before the increase was $8600.

Method 2: The multiplier method

The multiplier method involves fewer steps.

Step 1: Write down the multiplier.

Step 2: Divide the final value by the multiplier to give the original value.

EXAMPLE 19

In a sale the price of a freezer is reduced by 12%. The sale price is $220. What was the price before the sale?

A decrease of 12% gives a multiplier of 0.88

Dividing the sale price by the multiplier gives $220 ÷ 0.88 = $250

So the price before the sale was $250.

EXERCISE 2I

FOUNDATION

1 Find what 100% represents in these situations.

a 40% represents 320 g

b 14% represents 35 m

c 45% represents 27 cm

d 4% represents $123

e 2.5% represents $5

f 8.5% represents $34

2 A group of students go on a training course. Only 28 complete the course. This represented 35% of the original group. How large was the original group?

3 Sales tax is added to goods and services. With sales tax at 17.5%, what is the pre-sales tax price of the following goods?

T-shirt	$9.87	Tights	$1.41	Shorts	$6.11
Sweater	$12.62	Trainers	$29.14	Boots	$38.07

4 Howard spends $200 a month on food. This is 24% of his monthly pay. How much is his monthly pay?

5 Tina's weekly pay is increased by 5% to $315. What was Tina's pay before the increase?

6 The number of workers in a factory fell by 5% to 228. How many workers were there originally?

7 In a sale the price of a TV is reduced to $500. This is a 7% reduction on the original price. What was the original price?

8 If 38% of plastic bottles in a production line are blue and the remaining 7750 plastic bottles are brown, how many plastic bottles are blue?

9 I pay $385 sales tax on a car. Sales tax is 17.5% of the purchase price. How much did I pay for the car?

10 A company asks their workers to take a 10% pay cut.

Rob works out that his pay will be $1296 per month after the cut. How much is his pay now?

11 Manza buys a car and sells it for $2940. He made a profit of 20%.

What was the original price of the car?

12 When a suit is sold in a shop the selling price is $171 and the profit is 80%.

What was the original price?

13 Oliver buys a chair. He sells it for $63 in an auction and makes a loss of 55%.

What did he pay for the chair?

14 A woman's salary increased by 5% in one year. Her new salary was $19 845.

How much was the increase in dollars?

15 After an 8% increase, the monthly salary of a chef was $1431. What was the original monthly salary?

16 Cassie invested some money at 4% compound interest per annum for two years. After two years, she had $1406.08 in the bank. How much did she invest originally?

17 A teacher asked her class to work out the original price of a cooker for which, after a 12% increase, the price was 291.20 dollars.

This is Lee's answer: 12% of 291.20 = 34.94 dollars

Original price = 291.2 – 34.94 = 256.26 ≈ 260 dollars

When the teacher read out the answer Lee ticked his work as correct.
What errors has he made?

2.9 Interest and depreciation

When you put money in a bank you are paid interest each year.

EXAMPLE 20

Boris puts $600 in a bank and leaves it there for 2 years.

He is paid 5% interest every year.

How much does he have after **a** one year **b** two years?

It is best to use the multiplier method. To increase by 5% you multiply by 1.05

a After one year he has $600 × 1.05 = $630

b After two years he has $630 × 1.05 = $661.50

In the example Boris was paid $30 interest in the first year and $31.50 in the second year.

The amount of interest increases each year as his money increases.

This is an example of **compound interest**.

If you buy a new car or a computer or a washing machine, the value goes down each year. This is called **depreciation**.

Depreciation is often expressed as a percentage.

EXAMPLE 21

A businessman buys new machinery for $9000.

The value goes down by 20% in the first year and by 10% in the second year.

What is the value after 2 years?

a To decrease by 20% the multiplier is 0.8

After 1 year the value of the machinery is $9000 × 0.8 = $7200

b To decrease by 10% the multiplier is 0.9

After 2 years the value is $7200 × 0.9 = $6480

EXERCISE 2J

1 Samir puts $2000 in a bank and leaves it there for 2 years.

She is paid 3% per year interest.

Work out how much she has after a 1 year b 2 years

2 Repeat question 1 if the rate of interest is 6%

FOUNDATION

3 Luis puts $750 in a bank.

He is paid 5% interest in the first year and 4% in the second year.

a Work out how much he has after two years.

b How much interest is paid to him?

4 Carla puts $6500 in a bank.

She is paid 2% interest each year for three years.

a How much does she have after 3 years?

b How much interest does she receive?

5 Hamid buys a car for $15 000.

The value depreciates by 25% in the first year and 15% in the second year.

Work out the value of the car after **a** 1 year **b** 2 years.

6 A factory owner buys a machine for $35 000.

The value decreases by 12% each year.

Find the value after **a** 1 year **b** 2 years **c** 3 years.

7 Marta puts $10 000 in a bank. She is paid interest of 10% a year.

a How much does she have after **i** 1 year **ii** 2 years **iii** 3 years?

b Work out the interest she is she paid **i** in the first year **ii** in the second year **iii** in the third year.

8 Eric buys a car for $25 000.

The value of the car depreciates by 20% a year.

a Find the value of the car after **i** 1 year **ii** 2 years **iii** 3 years.

b Work out the fall in value **i** in the first year **ii** in the second year **iii** in the third year.

9 Yasmin has $5000 to put in a bank for 2 years.

Axel Bank offers 2% interest in the first year and 12% in the second year.

Barco Bank offers 7% interest each year.

Yasmin says "2 + 12 = 14 and 7 + 7 = 14 so they will both give the same amount of interest".

Is Yasmin correct? Give a reason for your answer.

10 Rory buys a boat for $6000.

The value depreciates by 25% a year.

Rory says "After 2 years it will be worth $3000". Is Rory correct? Give a reason for your answer.

2.10 Compound interest problems H

When you are working out compound interest for a number of years, you can use the power button on your calculator to find the answer more efficiently.

EXAMPLE 22

Elspeth put $4000 in a bank account. She is paid 6% compound interest.
Work out how much interest she has after 5 years.

The multiplier for an increase of 6% is 1.06
After 5 years she has $4000 \times 1.06 \times 1.06 \times 1.06 \times 1.06 \times 1.06$
You can write this as $4000 \times 1.06^5 = \$5352.90$
You should be able to use your calculator to find this.
Round your answer to 2 decimal places.
The interest is $\$5352 - \$4000 = \$1352$

EXERCISE 2K

HIGHER

1 Aglaya has $650 in a bank account. The rate of interest is 8%.
Work out how much she has after 4 years.

2 Rahul puts $3000 in a bank. He is paid compound interest of 4%.
Work out:

a the amount he has after 5 years **b** the interest he is paid

3 Lee put $1000 in a bank. Find the total value in the following cases:

a 3% interest is paid for 7 years

b 7% interest is paid for 3 years

c 5% interest is paid for 5 years

4 Rania puts $2000 in a bank where she is given 5% interest. Work out the interest she has

a after 1 year **b** after 5 years **c** after 10 years

5 A man puts $500 in a bank for 4 years. How much is it worth if the interest rate is

a 2.5% **b** 5% **c** 7.5% **d** 10%?

6 A woman puts $4000 in a bank account. The rate of interest is 5%.
How many years will it be until she has more than $6000?

2.11 Repeated percentage change H

Repeated percentage changes include compound interest and depreciation but there are many other examples.

EXAMPLE 23

In one year the population of a town increases by 5%.
The next year the population increases by 10%.
Work out the overall percentage change.

You have not been told the initial population of the town. You can work out the overall percentage change without it.
The multiplier for a 5% increase is 1.05
The multiplier for a 10% increase is 1.1
The combined multiplier is $1.05 \times 1.1 = 1.155$
This is the multiplier for an increase of 15.5%.

If you want to check that is correct you can choose any population to start with.
Suppose the population is 10 000.
After one year it is $10\,000 \times 1.05 = 10\,500$
After 2 years it is $10\,500 \times 1.1 = 11\,550$
The increase is 1550 and the percentage increase is $\frac{1550}{10\,000} \times 100\% = 15.5\%$
This is the same answer. It is quicker just to use the multipliers.

EXAMPLE 24

The number of birds of one species in a wood increases by 12% in one year.
The next year it decreases by 15%.
Find the overall percentage change.

The multipliers are 1.12 and 0.85
The combined multiplier is $1.12 \times 0.85 = 0.952$
$1 - 0.952 = 0.048$ so this is the multiplier for a 4.8% decrease.

EXERCISE 2L

1 The mass of a baby is 3.00 kg.

One month the mass increases by 5%. The next month the mass increases by 8%.

a Work out the mass of the baby after two months.

b Work out the overall percentage increase in mass.

HIGHER

HIGHER

2 One year the average attendance at a football ground is 8000.

The next year the attendance increases by 20%.

The year after that the attendance increases by 30%.

Find the total percentage increase over the two years.

3 Marsha puts some money in a bank.

The money earns 6% interest for three years.

What is the overall percentage increase in the value of her money?

4 The price of a coat is $750.

In a sale the price is reduced by 30% and then by a further 30%.

a Work out the price of the coat after the two reductions.

b Work out the total percentage decrease.

5 Mathias gets a 15% pay increase every year for three years.

Find his total percentage pay increase.

6 The value of a car decreases by 25% in its first year and then by 20% in each subsequent year.

Work out the total percentage decrease in value after

a 2 years **b** 4 years

7 A newspaper says the price of property in the centre of a city is increasing by 15% each year.

Work out the total percentage increase after

a 2 years **b** 5 years

8 A tree is 2 m tall and the height is increasing by 10% a year.

Show that if the tree continues to grow at this rate it will be over 8 m tall in 15 years' time.

9 From 1920 to 1960 the population of a town increased by 31%.

From 1960 to 2000 the population decreased by 17%.

Work out the percentage change from 1920 to 2000.

10 **a** Show that a 20% increase followed by another 20% increase is equivalent to a total increase of 44%.

b Show that a 20% decrease followed by another 20% decrease is equivalent to a total decrease of 36%.

c Find the total percentage change after an increase of 20% followed by a decrease of 20%.

11 Find the total percentage change in the following cases:

a An increase of 10% followed by a decrease of 10%

b An increase of 25% followed by a decrease of 25%

c An increase of 75% followed by a decrease of 75%

12 Look at these two changes:

X a decrease of 13% followed by an increase of 42%

Y an increase of 42% followed by a decrease of 13%

Which has the greater total percentage change? Give a reason for your answer.