Answers

Solving mathematical problems

Divisibility (p.8)

Challenge

Both Fabien and Sabah are right. If a number is divisible by 4, the last two digits of the number are divisible by 4, for example 116 and 732; and if a number is divisible by 3, the sum of its digits is divisible by 3, for example 87 and 651.

What if?

A number is divisible by 6 if the number is even, and the sum of its digits is divisible by 3, for example 96 and 858.

A number is divisible by 9 if the sum of its digits is divisible by 9, for example 396 and 855.

Ordering numbers (p.9)

Challenge

The numbers can be ordered in 12 different ways:

–6, –5, –3, –2, 0, 1, 4

–6, –5, –3, –2, 0, 2, 4

–6, –5, –3, –2, 0, 3, 4

–6, –4, –3, –2, 0, 1, 4

–6, –4, –3, –2, 0, 2, 4

–6, –4, –3, –2, 0, 3, 4

–6, –5, –3, –1, 0, 1, 4

–6, –5, –3, –1, 0, 2, 4

–6, –5, –3, –1, 0, 3, 4

–6, –4, –3, –1, 0, 1, 4

–6, –4, –3, –1, 0, 2, 4

–6, –4, –3, –1, 0, 3, 4

What if?

The numbers can be ordered in 32 different ways:

–10, –9, –5, –4, –2, –1, 3

–10, –9, –5, –4, –2, 0, 3

–10, –9, –5, –4, –2, 1, 3

–10, –9, –5, –4, –2, 2, 3

–10, –9, –5, –3, –2, –1, 3

–10, –9, –5, –3, –2, 0, 3

–10, –9, –5, –3, –2, 1, 3

–10, –9, –5, –3, –2, 2, 3

–10, –7, –5, –4, –2, –1, 3

–10, –7, –5, –4, –2, 0, 3

–10, –7, –5, –4, –2, 1, 3

–10, –7, –5, –4, –2, 2, 3

–10, –7, –5, –3, –2, –1, 3

–10, –7, –5, –3, –2, 0, 3

–10, –7, –5, –3, –2, 1, 3

–10, –7, –5, –3, –2, 2, 3

–10, –8, –5, –4, –2, –1, 3

–10, –8, –5, –4, –2, 0, 3

–10, –8, –5, –4, –2, 1, 3

–10, –8, –5, –4, –2, 2, 3

–10, –8, –5, –3, –2, –1, 3

–10, –8, –5, –3, –2, 0, 3

–10, –8, –5, –3, –2, 1, 3

–10, –8, –5, –3, –2, 2, 3

–10, –6, –5, –4, –2, –1, 3

–10, –6, –5, –4, –2, 0, 3

–10, –6, –5, –4, –2, 1, 3

–10, –6, –5, –4, –2, 2, 3

–10, –6, –5, –3, –2, –1, 3

–10, –6, –5, –3, –2, 0, 3

–10, –6, –5, –3, –2, 1, 3

–10, –6, –5, –3, –2, 2, 3

Nearest to 10, 100 and 1000 (p.10)

Challenge

Numbers that round to 350 when rounded to the nearest 10 and 400 when rounded to the nearest 100 are 350, 351, 352, 353 and 354.

Numbers that round to 6250 when rounded to the nearest 10 and 6200 when rounded to the nearest 100 are 6245, 6246, 6247, 6248 and 6249.

What if?

All the numbers from 6550 to 6649 (inclusive) round to 6600 when rounded to the nearest 100 and 7000 when rounded to the nearest 1000.

Making numbers (p.11)

Challenge/What if?

Results of the challenge will vary.

Dice calculations (p.12)

Challenge/What if?

Results of the challenge will vary.

Reverse, subtract, reverse, add (p.13)

Challenge

The final answer is always 1089, for example:

Step 1:	573	286
Step 2:	375	682

Step 3:

$$\begin{array}{r} 573 \\ -\,375 \\ \hline 198 \\ \hline \end{array} \qquad \begin{array}{r} 682 \\ -\,286 \\ \hline 396 \\ \hline \end{array}$$

Steps 4 and 5:

$$\begin{array}{r} 198 \\ +\,891 \\ \hline 1089 \\ \hline \end{array} \qquad \begin{array}{r} 396 \\ +\,693 \\ \hline 1089 \\ \hline \end{array}$$

What if?

Choosing 3-digit numbers where the ones and hundreds digits only have a difference of 1, such as 867, 425, 392 or 718, at Step 3 the answer is always 99. However, to continue and still end with a final answer of 1089, 99 needs to written as a 3-digit number, i.e. 099.

Step 1: 867 425

Step 2: 768 524

Step 3:

$$\begin{array}{r} 8\ 6\ 7 \\ -\ 7\ 6\ 8 \\ \hline 0\ 9\ 9 \\ \hline \end{array} \qquad \begin{array}{r} 5\ 2\ 4 \\ -\ 4\ 2\ 5 \\ \hline 0\ 9\ 9 \\ \hline \end{array}$$

Steps 4 and 5:

$$\begin{array}{r} 0\ 9\ 9 \\ +\ 9\ 9\ 0 \\ \hline 1\ 0\ 8\ 9 \\ \hline \end{array} \qquad \begin{array}{r} 0\ 9\ 9 \\ +\ 9\ 9\ 0 \\ \hline 1\ 0\ 8\ 9 \\ \hline \end{array}$$

Phone totals (p.14)

Challenge/What if?

Results of the challenge will vary.

Greatest domino calculation (p.15)

Challenge/What if?

Results of the challenge will vary.

Digit card calculations (p.16)

Challenge

Results of the challenge will vary. However, calculations may include 16 × 3 = 48 and 27 × 3 = 81.

What if?

Results of the challenge will vary. However, calculations may include 27 ÷ 3 = 9 and 52 ÷ 4 = 13 or 48 ÷ 3 = 16 and 81 ÷ 3 = 27.

Using factors (p.17)

Challenge

Pupils' calculations may vary but could include the following:

$42 \times 5 = 7 \times 6 \times 5 = 7 \times 30 = 210$

$18 \times 6 = 2 \times 9 \times 6 = 2 \times 54 = 108$

$35 \times 8 = 5 \times 7 \times 8 = 5 \times 56 = 280$ / $7 \times 40 = 280$

$24 \times 7 = 2 \times 12 \times 7 = 2 \times 84 = 168$

$36 \times 9 = 4 \times 9 \times 9 = 4 \times 81 = 324$

$28 \times 3 = 7 \times 4 \times 3 = 7 \times 12 = 84$

What if?

Pupils' checking methods, calculations and explanations will vary.

Fractions of amounts (p.18)

Challenge

Results of the challenge will vary. However, calculations may include $\frac{1}{3} \times 27 = 9$ and $\frac{1}{5} \times 30 = 6$.

What if?

Results of the challenge will vary. However, calculations may include:

$\frac{1}{3} \times 87 = 29$ and $\frac{1}{2} \times 68 = 34$

$\frac{2}{5} \times 10 = 4$ and $\frac{3}{4} \times 12 = 9$

$\frac{2}{3} \times 84 = 56$ and $\frac{3}{5} \times 70 = 42$

Making decimals (p.19)

Challenge/What if?

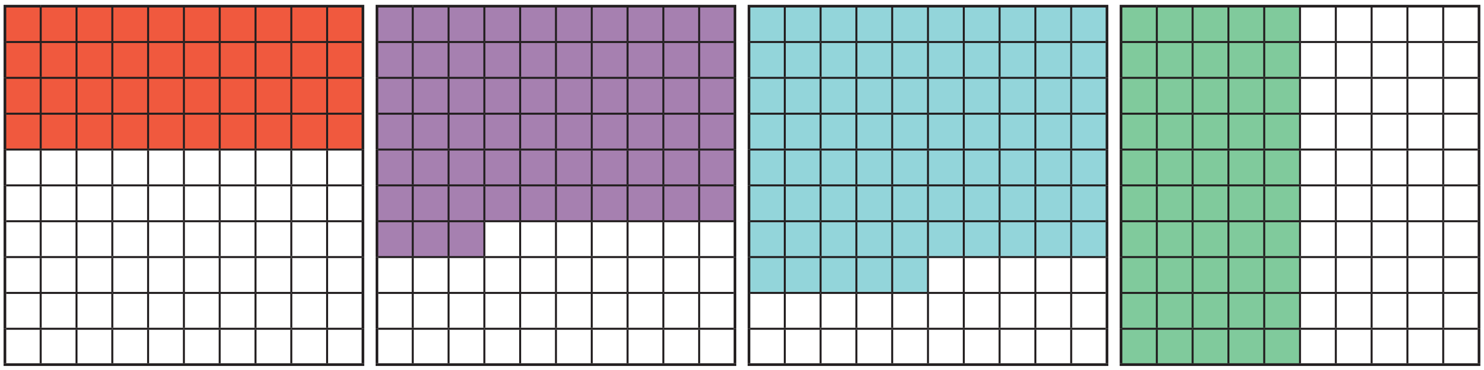
Results of the challenge will vary depending on the digit cards chosen by pupils. However, 12 different decimals are possible: six with one decimal place, and six with two decimal places. For example, using the digit cards in the illustration, i.e. 1, 5 and 8:

15·8	1·58
18·5	1·85
51·8	5·18
58·1	5·81
81·5	8·15
85·1	8·51

Fractions and decimals (p.20)

Challenge

Expressions will vary, but may include the following:



$\frac{40}{100}$	$\frac{63}{100}$	$\frac{75}{100}$	$\frac{50}{100}$
$\frac{4}{10}$	0·63	$\frac{3}{4}$	$\frac{5}{10}$
$\frac{2}{5}$		0·75	$\frac{1}{2}$
0·40			0·50
0·4			0·5

What if?

The following expressions describe the white squares in each of the 100 squares above:

$\frac{60}{100}$	$\frac{37}{100}$	$\frac{25}{100}$	$\frac{50}{100}$
$\frac{6}{10}$	0·37	$\frac{1}{4}$	$\frac{5}{10}$
$\frac{3}{5}$		0·25	$\frac{1}{2}$
0·60			0·50
0·6			0·5

For each 100 square, each fraction or decimal that describes the coloured squares and the white squares when added together totals 1, for example, for the orange square:

$\frac{40}{100} + \frac{60}{100} = \frac{100}{100}$ (= 1)

$\frac{4}{10} + \frac{6}{10} = \frac{10}{10}$ (= 1)

$\frac{2}{5} + \frac{3}{5} = \frac{5}{5}$ (= 1)

0·40 + 0·60 = 1

0·4 + 0·6 = 1

Shopping basket (p.21)

Challenge/What if?

Results of the challenge will vary.

Fruit juice (p.22)

Challenge

For a fruit drink containing two different types of fruit juice:

small: 140 ml of one juice and 60 ml of the other

medium: 210 ml of one juice and 90 ml of the other

large: 350 ml of one juice and 150 ml of the other

What if?

For a fruit drink containing three different types of fruit juice:

small: 100 ml of the first juice, 60 ml of the second juice and 40 ml of the third

medium: 150 ml of the first juice, 90 ml of the second juice and 60 ml of the third

large: 250 ml of the first juice, 150 ml of the second juice and 100 ml of the third

For a fruit drink containing all four different types of fruit juice:

small: 80 ml of the first juice, 50 ml of the second juice, 50 ml of the third juice and 20 ml of the fourth

medium: 120 ml of the first juice, 75 ml of the second juice, 75 ml of the third juice and 30 ml of the fourth

large: 200 ml of the first juice, 125 ml of the second juice, 125 ml of the third juice and 50 ml of the fourth

Same and different (p.23)

Challenge/What if?

Results of the challenge will vary.

Light bars (p.24)

Challenge

0123456789

Results of the challenge will vary.

What if?

Results of the challenge will vary.

Finding totals (p.25)

Challenge/What if?

10 different combinations are possible:

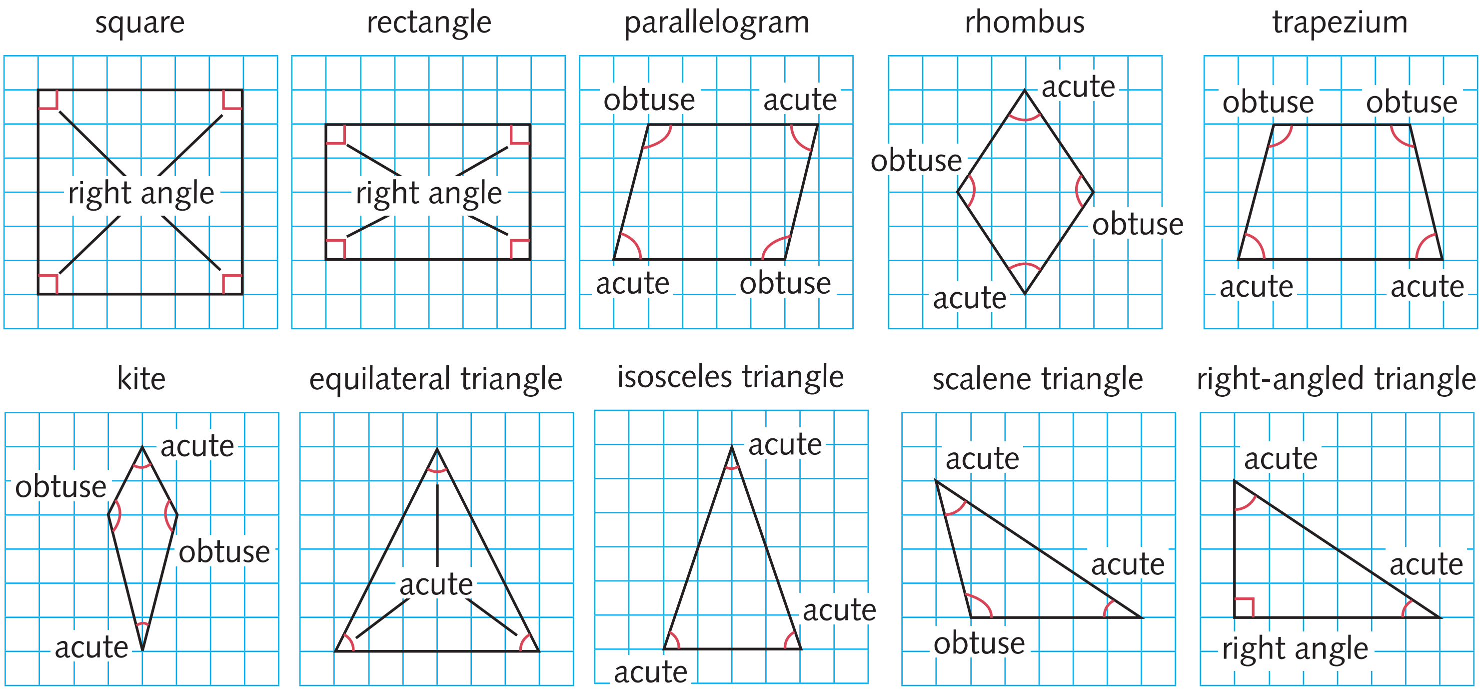
Total of pair of items	Change received	Possible notes and coins given as change*
watch and spaceship: £46.20	from £50: £3.80	£2, £1, 50p, 20p, 10p
watch and magic box: £33.45	from £40: £6.55	£5, £1, 50p, 5p
watch and telescope: £52.80	from £60: £7.20	£5, £2, 20p
watch and puzzle: £29.00	from £30: £1.00	£1
spaceship and magic box: £45.15	from £50: £4.85	£2, £2, 50p, 20p, 10p, 5p
spaceship and telescope: £64.50	from £70: £5.50	£5, 50p
spaceship and puzzle: £40.70	from £50: £9.30	£5, £2, £2, 20p, 10p
magic box and telescope: £51.75	from £60: £8.25	£5, £2, £1, 20p, 5p
magic box and puzzle: £27.95	from £30: £2.05	£2, 5p
telescope and puzzle: £47.30	from £50: £2.70	£2, 50p, 20p

* Other answers are possible. However, the amounts given are using the smallest number of coins.

Angles in shapes (p.26)

Challenge

NOTE: Pupils' shapes and labelling will vary.



What if?

All the angles in a regular pentagon, hexagon and octagon are obtuse angles.

Pupils' drawings and labelling of an irregular pentagon, hexagon and octagon will vary.

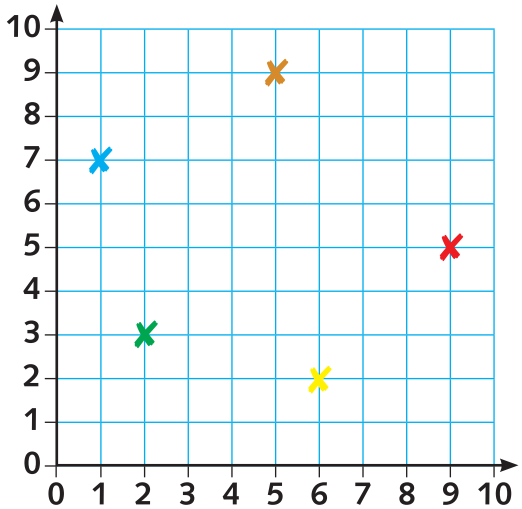
Symmetrical shapes (p.27)

Challenge/What if?

Pupils' shapes will vary.

Grid route (p.28)

Challenge



Pupils' sets of instructions will vary. One route for moving from the green cross to the blue cross, passing through the orange cross is:

- 3 squares to the right, 6 squares up, 4 squares to the left, 2 squares down.

What if?

Pupils' sets of instructions will vary. One route for moving from the red cross to the yellow cross, passing though all the other coloured crosses is:

- 4 squares up, 4 squares to the left (to reach the orange cross), 2 squares down, 4 squares to the left (to reach the blue cross), 4 squares down, 1 square to the right (to reach the green cross), 1 square down, 4 squares to the right (to reach the yellow cross).

Grid of quadrilaterals (p.29)

Challenge

What if?

Results of the challenge will vary.

Interpreting the weather (p.30)

Challenge/What if?

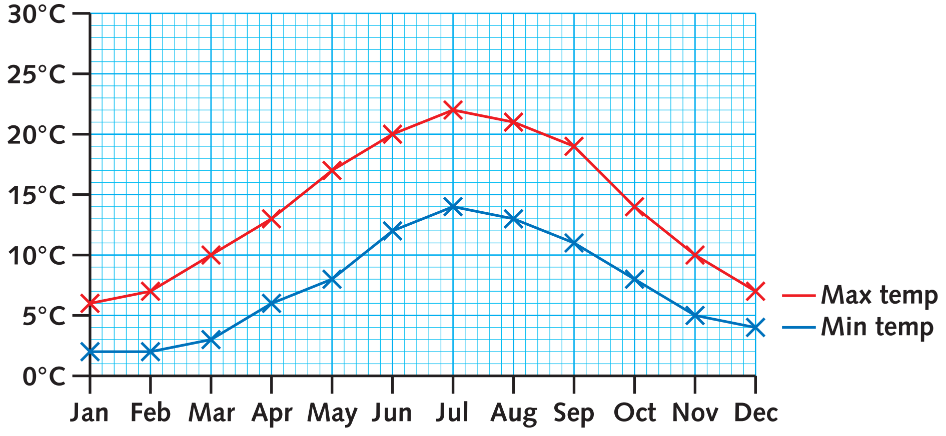
Pupils' statements will vary.

Presenting the weather (p.31)

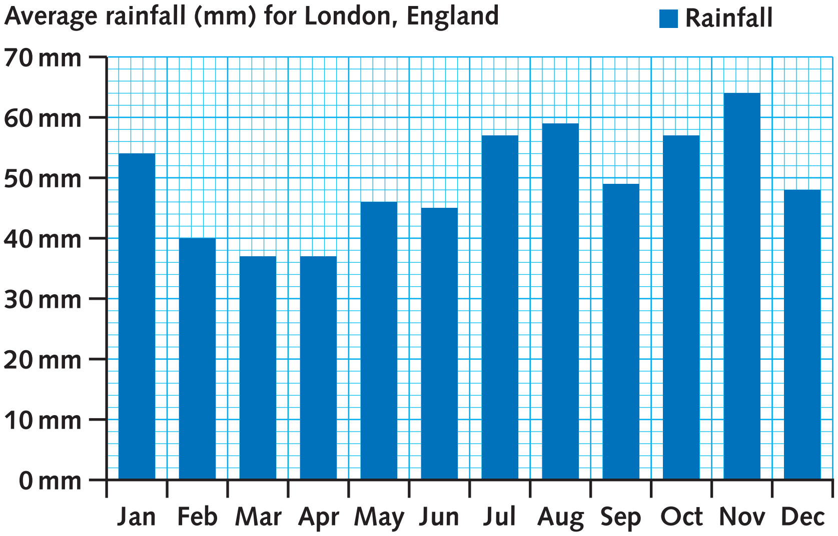
Challenge

Pupils' temperatures line graphs and rainfall bar charts will vary, especially in relation to the scales they use on the vertical axes. However, they may look similar to the following:

Average minimum and maximum temperatures (°C) for London, England



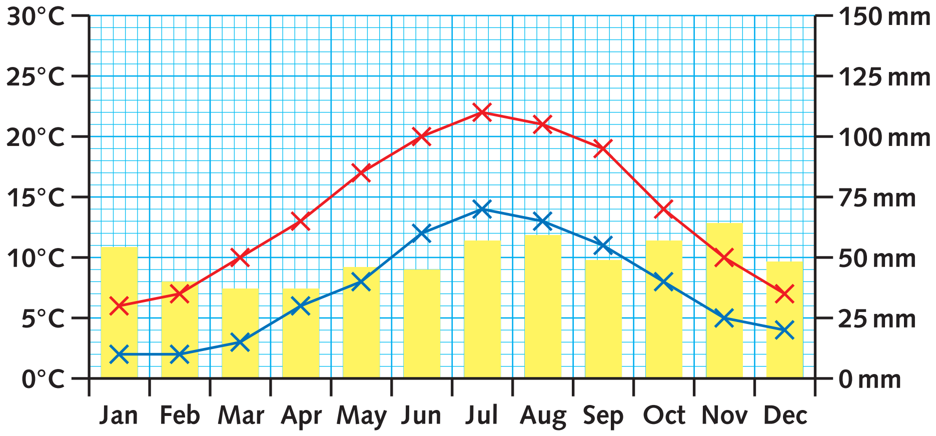
Average rainfall (mm) for London, England



Pupils' statements will vary.

What if?

Pupils' combined temperatures and rainfall graphs will vary, especially in relation to the scales they use on the vertical axes. However, they may look similar to the following.



Reasoning mathematically

Common multiples (p.32)

Challenge

Pupils' Venn diagrams will vary depending on the two numbers chosen, and their explanations will also vary depending on their depth of understanding. However, they should comment on the patterns that they notice in the common multiples (see below for some examples). Use your professional judgement when assessing pupils' reasoning.

- The numbers 15, 30, 45, 60, … are common multiples of 3 and 5. They are also all multiples of 15.
- The numbers 12, 24, 36, 48, … are common multiples of 3 and 4. They are also all multiples of 12.
- The numbers 20, 40, 60, 80, … are common multiples of 4 and 5. They are also all multiples of 20.
- The numbers 12, 24, 36, 48, … are common multiples of 4 and 6. They are also all multiples of 12.
- The numbers 40, 80, 120, … are common multiples of 5 and 8. They are also all multiples of 40.

What if?

Pupils' Venn diagrams will vary depending on the three numbers chosen, and their explanations will also vary depending on their depth of understanding. However, they should comment on the patterns that they notice in the common multiples (see the example below). Use your professional judgement when assessing pupils' reasoning.

The numbers 15, 30, 45, … are common multiples of 3 and 5. They are also all multiples of 15.

The numbers 12, 24, 36, 48, … are common multiples of 3 and 4. They are also all multiples of 12.

The numbers 20, 40, … are common multiples of 4 and 5. They are also all multiples of 10 and 20.

The number 60 is a common multiple of 3, 4 and 5.

Place value counters (p.33)

Challenge

Joseph is not right. You can make ten different four-digit numbers:

If the two place value counters in Lucy's hand are:	… then the following four-digit number can be made:
1000 and 1000	3213 (1213 + 2000)
1000 and 100	2313 (1213 + 1100)
1000 and 10	2223 (1213 + 1010)
1000 and 1	2214 (1213 + 1001)
100 and 100	1413 (1213 + 200)
100 and 10	1323 (1213 + 110)
100 and 1	1314 (1213 + 101)
10 and 10	1233 (1213 + 20)
10 and 1	1224 (1213 + 11)
1 and 1	1215 (1213 + 2)

What if?

Joseph is wrong. You can make 23 numbers less than 1000:

1 (1)

2 (1 + 1)

3 (1 + 1 + 1)

10 (10)

11 (10 + 1)

12 (10 + 1 + 1)

13 (10 + 1 + 1 + 1)

100 (100)

101 (100 + 1)

102 (100 + 1 + 1)

103 (100 + 1 + 1 + 1)

110 (100 + 10)

111 (100 + 10 + 1)

112 (100 + 10 + 1 + 1)

113 (100 + 10 + 1 + 1 + 1)

200 (100 + 100)

201 (100 + 100 + 1)

202 (100 + 100 + 1 + 1)

203 (100 + 100 + 1 + 1 + 1)

210 (100 + 100 + 10)

211 (100 + 100 + 10 + 1)

212 (100 + 100 + 10 + 1 + 1)

213 (100 + 100 + 10 + 1 + 1 + 1)

Making 6 and 4 equal 9 (p.34)

Challenge

You can make six different four-digit numbers where the tens digit is 6 and all four digits add up to 9: 1062, 1161, 1260, 2061, 2160, 3060

What if?

The 15 different 4-digit numbers where the tens digit is 4 and the four digits add up to 9 are:

1044, 1143, 1242, 1341, 1440, 2043, 2142, 2241, 2340, 3042, 3141, 3240, 4041, 4140, 5040

Rounding rules (p.35)

Challenge

Sabah is incorrect.

- If rounding to the nearest multiple of 10, the largest whole number that rounds to 600 is 604.
- If rounding to the nearest multiple of 100, the largest whole number that rounds to 600 is 649.

Fabien is incorrect.

- If rounding to the nearest multiple of 10, the smallest whole number that rounds to 3000 is 2995.
- If rounding to the nearest multiple of 100, the smallest whole number that rounds to 3000 is 2950.
- If rounding to the nearest multiple of 1000, the smallest whole number that rounds to 3000 is 2500.

What if?

597 rounds to 600 when rounded to the nearest 10 and also 600 when rounded to the nearest 100.

Other similar numbers include the numbers 595 to 604 (inclusive).

The following numbers also round in a similar way:

- The numbers 95 to 104 (inclusive) round to 100 when rounded to the nearest 10 and 100.
- The numbers 195 to 204 (inclusive) round to 200 when rounded to the nearest 10 and 100.
- The numbers 295 to 304 (inclusive) round to 300 when rounded to the nearest 10 and 100.
- The numbers 395 to 404 (inclusive) round to 400 when rounded to the nearest 10 and 100.

... and so on

Missing digits (p.36)

Challenge

Ten different calculations are possible:

$$\begin{array}{r} 3\,\boxed{0}\,2 \\ +\,\boxed{5}\,8\,7 \\ \hline 8\,\boxed{8}\,9 \\ \hline \end{array} \quad \begin{array}{r} 3\,\boxed{1}\,2 \\ +\,\boxed{5}\,8\,7 \\ \hline 8\,\boxed{9}\,9 \\ \hline \end{array} \quad \begin{array}{r} 3\,\boxed{2}\,2 \\ +\,\boxed{4}\,8\,7 \\ \hline 8\,\boxed{0}\,9 \\ \hline \end{array} \quad \begin{array}{r} 3\,\boxed{3}\,2 \\ +\,\boxed{4}\,8\,7 \\ \hline 8\,\boxed{1}\,9 \\ \hline \end{array} \quad \begin{array}{r} 3\,\boxed{4}\,2 \\ +\,\boxed{4}\,8\,7 \\ \hline 8\,\boxed{2}\,9 \\ \hline \end{array}$$

$$\begin{array}{r} 3\,\boxed{5}\,2 \\ +\,\boxed{4}\,8\,7 \\ \hline 8\,\boxed{3}\,9 \\ \hline \end{array} \quad \begin{array}{r} 3\,\boxed{6}\,2 \\ +\,\boxed{4}\,8\,7 \\ \hline 8\,\boxed{4}\,9 \\ \hline \end{array} \quad \begin{array}{r} 3\,\boxed{7}\,2 \\ +\,\boxed{4}\,8\,7 \\ \hline 8\,\boxed{5}\,9 \\ \hline \end{array} \quad \begin{array}{r} 3\,\boxed{8}\,2 \\ +\,\boxed{4}\,8\,7 \\ \hline 8\,\boxed{6}\,9 \\ \hline \end{array} \quad \begin{array}{r} 3\,\boxed{9}\,2 \\ +\,\boxed{4}\,8\,7 \\ \hline 8\,\boxed{7}\,9 \\ \hline \end{array}$$

Pupils' explanations will vary. Use your professional judgement when assessing pupils' reasoning.

What if?

Six different calculations are possible:

$$\begin{array}{r} 6\;3\;\boxed{8} \\ +\;2\;\boxed{4}\;4 \\ \hline 3\;\boxed{9}\;4 \\ \hline \end{array} \quad \begin{array}{r} 6\;3\;\boxed{8} \\ +\;2\;\boxed{5}\;4 \\ \hline 3\;\boxed{8}\;4 \\ \hline \end{array} \quad \begin{array}{r} 6\;3\;\boxed{8} \\ +\;2\;\boxed{6}\;4 \\ \hline 3\;\boxed{7}\;4 \\ \hline \end{array} \quad \begin{array}{r} 6\;3\;\boxed{8} \\ +\;2\;\boxed{7}\;4 \\ \hline 3\;\boxed{6}\;4 \\ \hline \end{array} \quad \begin{array}{r} 6\;3\;\boxed{8} \\ +\;2\;\boxed{8}\;4 \\ \hline 3\;\boxed{5}\;4 \\ \hline \end{array} \quad \begin{array}{r} 6\;3\;\boxed{8} \\ +\;2\;\boxed{9}\;4 \\ \hline 3\;\boxed{4}\;4 \\ \hline \end{array}$$

Pupils' explanations will vary. Use your professional judgement when assessing pupils' reasoning.

Estimating (p.37)

Challenge

The following two calculations have an answer between 500 and 600:

863 – 329 = 534

261 + 311 = 572

The following three calculations have an answer between 6000 and 6500:

4630 + 1605 = 6235

3500 + 2920 = 6420

7200 – 900 = 6300

Pupils' explanations will vary. Use your professional judgement when assessing pupils' reasoning.

What if?

Pupils' calculations will vary.

Making predictions (p.38)

Challenge

Pupils' calculations and predictions will vary. However, their conclusions should make reference to the following:

When pairs of even and/or odd numbers are added or subtracted, the following answers always apply:

E + E = E   E – E = E   O + O = E   O – O = E

O + E = O / E + O = O   O – E = O / E – O = O

What if?

Sabah's predictions are not good ones because when adding the hundreds digits, there may be carried over digits that need to be added, thereby increasing the total by 100, for example:

$$\begin{array}{r} 378 \\ +\ 454 \\ \hline 832 \\ \hline {\scriptstyle 1\,1\ } \end{array}$$

Similarly, when subtracting the hundreds digits, there may have been exchanging between the hundreds and tens digits in the minuend, thereby decreasing the hundreds digit by 1 and resulting in an answer 100 less, for example:

$$\begin{array}{rrrr} & \overset{3}{\cancel{4}} & \overset{14}{\cancel{5}} & \overset{14}{\cancel{4}} \\ - & 3 & 7 & 8 \\ \hline & & 7 & 6 \\ \hline \end{array}$$

Matching pairs (p.39)

Challenge

The matching pairs are:

5 × 5 × 7 = 25 × 7

4 × 8 × 9 = 32 × 9

9 × 2 × 7 = 18 × 7

3 × 9 × 8 = 27 × 8

4 × 4 × 8 = 16 × 8

What if?

Pupils' calculations will vary. However, they may include the following:

36 × 7 = 3 × 12 × 7, 2 × 18 × 7, 6 × 6 × 7 or 4 × 9 × 7

6 × 7 × 8 = 42 × 8, 6 × 56 or 48 × 7

3 × 8 × 6 = 24 × 6, 3 × 48 or 18 × 8

18 × 9 = 2 × 9 × 9 or 3 × 6 × 9

Pupils' explanations will vary. However, some mention should be made of the commutative law and also the use of factors.

Which do you prefer? (p.40)

Challenge/What if?

Pupils' preferred methods and explanations will vary. $743 \times 6 = 4458$

Letter values (p.41)

Challenge

Insert the following:

A = 5, B = 3, C = 7, D = 1, E = 4, F = 8, G = 6, H = 2, I = 9

$$\begin{array}{r} 53 \\ \times \quad 7 \\ \hline 371 \\ \hline \end{array} \qquad \begin{array}{r} 483 \\ \times \quad 6 \\ \hline 2898 \\ \hline \end{array}$$

Pupils' explanations will vary.

What if?

Several solutions are possible:

$2 \times 7 \times 8 = 112$ giving Q = 2, S = 8 and T = 1

$4 \times 7 \times 8 = 224$ giving Q = 4, S = 8 and T = 2

$6 \times 7 \times 8 = 336$ giving Q = 6, S = 8 and T = 3

Right or wrong? (p.42)

Challenge

Right calculations:

$\frac{4}{9} + \frac{3}{9} = \frac{7}{9}$

$\frac{5}{8} - \frac{3}{8} = \frac{1}{4}$

$\frac{3}{5} \times 20 = 12$

$\frac{5}{8} + \frac{1}{8} = \frac{3}{4}$

Wrong calculations:

$\frac{2}{3} \times 18 = 9$

$\frac{3}{10} + \frac{5}{10} = \frac{8}{20}$

$\frac{11}{12} - \frac{3}{12} = \frac{7}{12}$

Pupils' reasoning will vary. Use your professional judgement when assessing pupils' reasoning.

What if?

$$\begin{aligned}\frac{2}{3} \times 18 &= (18 \div 3) \times 2\\ &= 6 \times 2\\ &= 12\end{aligned}$$

$\frac{3}{10} + \frac{5}{10} = \frac{8}{10} = \frac{4}{5}$

$\frac{11}{12} - \frac{3}{12} = \frac{8}{12} = \frac{4}{6} = \frac{2}{3}$

Pupils' diagrams will vary.

Dividing by 10 and 100 (p.43)

Challenge

Sabah's statement is always true: dividing a one-digit number by 10 gives an answer with one decimal place, for example: 6 ÷ 10 = 0·6, 9 ÷ 10 = 0·9

Lucy's statement is always true: dividing a one-digit number by 100 gives an answer with two decimal places, for example: 3 ÷ 100 = 0·03, 7 ÷ 100 = 0·07

Fabien's statement is sometimes true: dividing a two-digit number by 10 gives an answer with one decimal place, for example: 34 ÷ 10 = 3·4, 62 ÷ 10 = 6·2. However, if the two-digit number is a multiple of 10, then the answer is a whole number, for example: 30 ÷ 10 = 3, 60 ÷ 10 = 6

Joseph's statement is sometimes true: dividing a two-digit number by 100 gives an answer with two decimal places, for example: 71 ÷ 100 = 0·71, 85 ÷ 100 = 0·85. However, if the two-digit number is a multiple of 10, then the answer has one decimal place, for example: 70 ÷ 100 = 0·7, 80 ÷ 100 = 0·8

What if?

Fabien is not correct: when ordering numbers with one decimal place the number with the greatest number of tenths is not always the largest number. The largest number will be the number with the largest most significant digits. So, for example, in the set of numbers: 0·9, 0·3 and 0·2, the number with the greatest number of tenths is the largest number, i.e. 0·9. However, in the set of numbers: 1·9, 2·3 and 2·2, the largest number is 2·3.

Comparing and ordering (p.44)

Challenge

Decimal numbers will vary.

What if?

Fractions will vary.

True or false? (p.45)

Challenge

1 km = 1000 m: True

1000 cm = 1 m: False (100 cm = 1 m)

1·6 cm = 160 mm: False (1·6 cm = 16 mm)

1000 ml = 1 *l*: True

250 ml = $\frac{1}{2}$ *l*: False (250 ml = $\frac{1}{4}$ *l*)

2·25 *l* = 2250 ml: True

1 kg = 100 g: False (1 kg = 1000 g)

1·3 kg = 130 g: False (1·3 kg = 1300 g)

750 g = $\frac{3}{4}$ kg: True

What if?

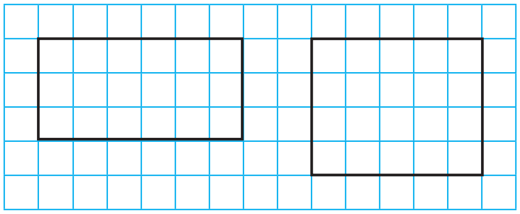
$\frac{3}{4}$ m (75 cm), 120 cm, $\frac{1}{2}$ of 3 m (150 cm), $\frac{1}{4}$ of 800 cm (200 cm)

Pupils' reasoning will vary. Use your professional judgement when assessing pupils' reasoning.

Perimeter and area (p.46)

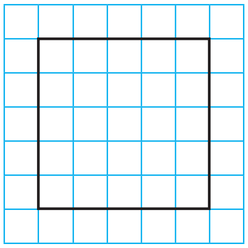
Challenge

The following are two different rectangles with a perimeter of 18 cm. Other rectangles (8 × 1 and 7 × 2) and orientations are possible.



6 × 3 5 × 4

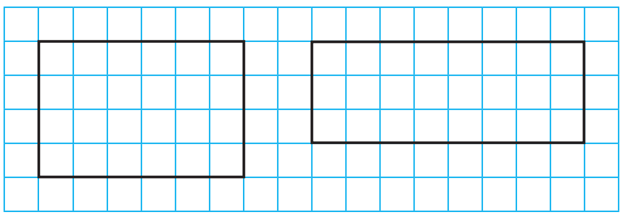
The following is the only square possible with a perimeter of 20 cm.



5 × 5

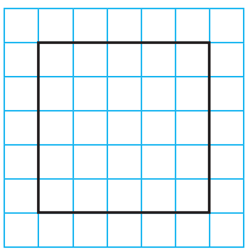
What if?

The following are two different rectangles with an area of 24 cm^2. Other rectangles (12 × 2 and 24 × 1) and orientations are possible.



6 × 4 8 × 3

The following is the only square possible with an area of 25 cm^2.



5 × 5

Digital time (p.47)

Challenge

There are 54 times on a 24-hour digital clock between midnight (00:00) and noon (12:00) when the minutes digits add up to the hours digits:

00:00

01:01, 01:10

02:02, 02:11, 02:20

03:03, 03:12, 03:21, 03:30

04:04, 04:13, 04:22, 04:31; 04:40

05:05, 05:14, 05:23, 05:32, 05:41, 05:50

06:06, 06:15, 06:24, 06:33, 06:42, 06:51

07:07, 07:16, 07:25, 07:34, 07:43, 07:52

08:08, 08:17, 08:26, 08:35, 08:44, 08:53

09:09, 09:18, 09:27, 09:36, 09:45, 09:54

10:19, 10:28, 10:37, 10:46, 10:55

11:29, 11:38, 11:47, 11:56

There are only six times on a 24-hour digital clock between noon (12:00) and midnight (00:00) when the minutes digits add up to the hours digits:

12:39, 12:48, 12:57

13:49, 13:58

14:59

What if?

There are 63 times on a 12-hour digital clock when the sum of the hours and minutes digits totals 12:

01:29, 01:38, 01:47, 01:56
02:19, 02:28, 02:37, 02:46, 02:55
03:09, 03:18, 03:27, 03:36, 03:45, 03:54
04:08, 04:17, 04:26, 04:35; 04:44, 04:53
05:07, 05:16, 05:25, 05:34, 05:43, 05:52
06:06, 06:15, 06:24, 06:33, 06:42, 06:51
07:05, 07:14, 07:23, 07:32, 07:41, 07:50
08:04, 08:13, 08:22, 08:31, 08:40
09:03, 09:12, 09:21, 09:30
10:29, 10:38, 10:47, 10:56
11:19, 11:28, 11:37, 11:46, 11:55
12:09, 12:18, 12:27, 12:36, 12:45, 12:54

There is just one time on a 24-hour digital clock when the sum of the hours and minutes digits totals 24: 19:59.

Which would you rather have? (p.48)

Challenge

Assuming that pupils would prefer to have the most money, then they would rather have:

Lucy: 620p (£6.20)

Fabien: 7 × 50p (£3.50)

Joseph: $\frac{1}{2}$ of £13 (£6.50)

Sabah: $\frac{1}{2}$ of £15 (£7.50)

What if?

£3, £3.20, £3.40, £3.50, £5.25, £5.50, £5.60, £5.70, £6, £6.20, £6.50, £7.50

Sorting shapes (p.49)

Challenge/What if?

Pupils' criteria and sorting diagrams will vary. However, criteria could include:

- polygon / not a polygon
- quadrilateral / triangle / other 2-D shape
- number of sides or angles
- types of angle
- shapes with parallel sides / shapes with perpendicular sides / shapes with parallel and perpendicular sides
- lines of symmetry

Who drew which angle? (p.50)

Challenge

Sabah: angle b

Lucy: angle a

Joseph: angle d

Fabien: angle c

Pupils' reasoning will vary. Use your professional judgement when assessing pupils' reasoning.

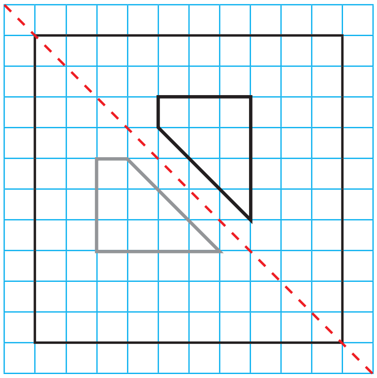
What if?

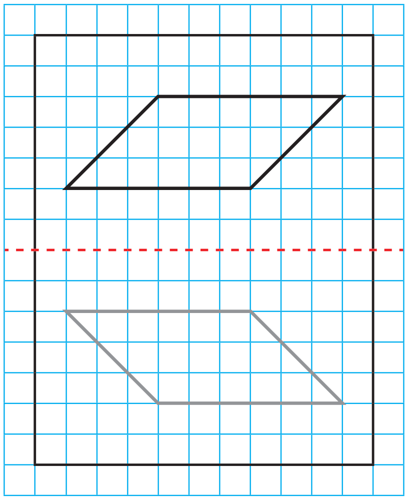
The diagonals of a square and a rhombus meet at right angles.

Pupils' justification will vary. Use your professional judgement when assessing pupils' reasoning.

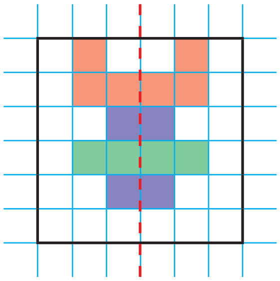
Symmetry (p.51)

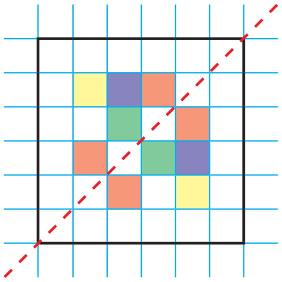
Challenge





What if?





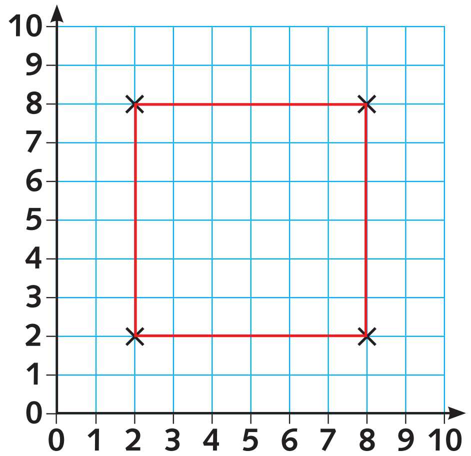
Pupils' explanations will vary.

Quadrilateral coordinates (p.52)

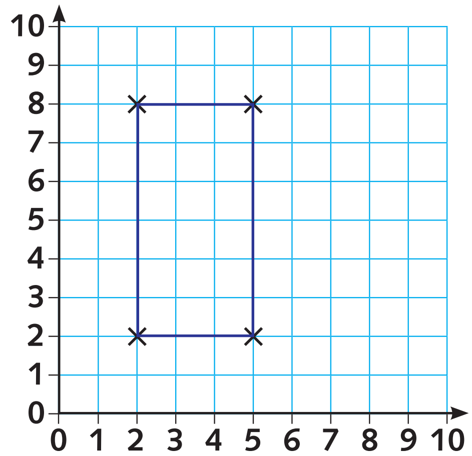
Challenge/What if?

It is possible to draw six different types of quadrilateral using the given coordinates (2, 2) (2, 8): a square, rectangle, parallelogram, rhombus, trapezium and kite. Pupils' drawings, orientations and subsequent coordinates of the six quadrilaterals will vary.

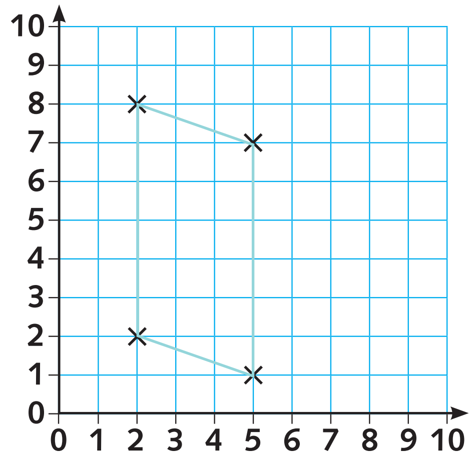
Square: (2,2) (2,8) (8,8) (8,2)



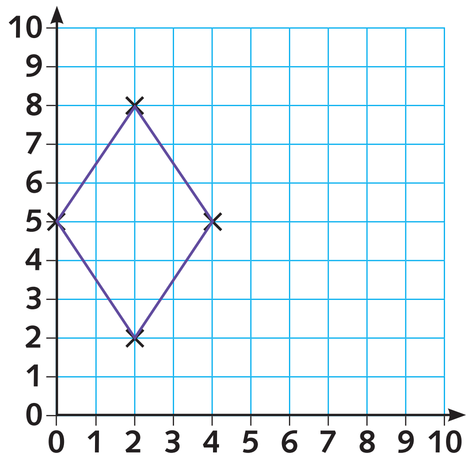
Rectangle: (2,2) (2,8) (5,8) (5,2)



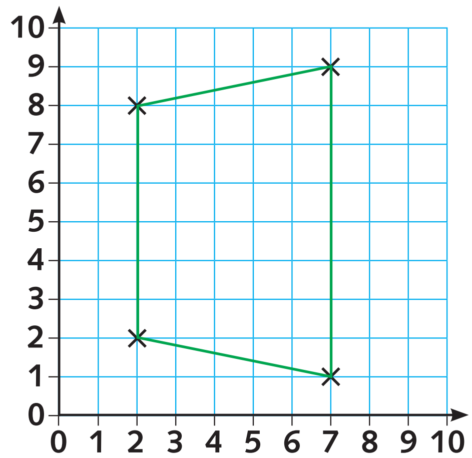
Parallelogram: (2,2) (2,8) (5,7) (5,1)



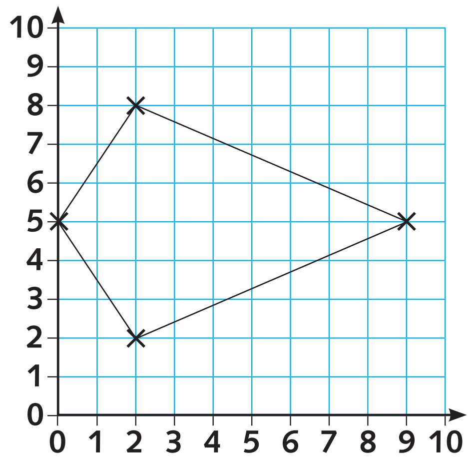
Rhombus: (2,2) (2,8) (0,5) (4,5)



Trapezium: (2,2) (2,8) (7,9) (7,1)



Kite: (2,2) (0,5) (2,8) (9,5)



Island travels (p.53)

Challenge

Lucy is right – there are more than two different ways to walk from the school to the shop, for example:

- up 3 squares, left 2 squares, up 3 squares, left 4 squares
  using coordinates: (10, 4), (10, 7), (8, 7), (8, 10) (4, 10)
- left 2 squares, up 5 squares, left 4 squares, up 1 square
  using coordinates: (10, 4), (8, 4), (8, 9), (4, 9), (4, 10)
- left 2 squares, up 6 squares, left 4 squares
  using coordinates: (10, 4), (8, 4), (8, 10), (4, 10)

Other routes are possible.

What if?

Pupils' routes will vary. However, they may include the following:

- caravan to light house via post box:
  up 2 squares, left 6 squares, up 2 squares, left 4 squares
  using coordinates: (12, 10), (12, 12), (6, 12), (6, 14), (2, 14)
- clump of trees to house:
  left 2 squares, down 8 squares, right 6 squares
  using coordinates: (10, 14) (8, 14) (8, 6) (14, 6)
- school to windmill via large tree:
  left 6 squares, up 2 squares, left 2 squares
  using coordinates: (10, 4), (4, 4), (4, 6), (2, 6)
- windmill to caravan:
  right 6 squares, up 4 squares, right 4 squares
  using coordinates: (2, 6), (8, 6), (8, 10), (12, 10)

Pupils' routes and coordinates will vary.

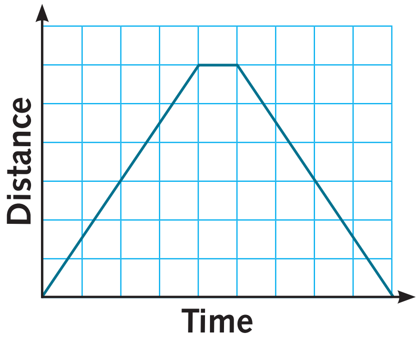
Time graph storytelling (p.54)

Challenge

Pupils' interpretations and conclusions of the line graphs will vary. Use your professional judgement when assessing pupils' reasoning.

What if?

Pupils' line graphs will vary. However, they should realise that the amount of time stationary in Nice is $\frac{1}{4}$ of the length of time for the flight. They should also realise that the return flight is also 2 hours.



All the same (p.55)

Challenge

Pupils' observations will vary.

What if?

Pupils' statements will vary.

Using and applying mathematics in real-world contexts

Ancient Greek numbers (p.56)

Challenge

- Using the Ancient Greek alphabet, 776 BC (the year of the first Ancient Olympic Games) is written as ψοϛ'.
- Using the Ancient Greek alphabet, 395 AD (the year of the last Ancient Olympic Games) is written as τϙε'.
- Using the Ancient Greek alphabet, 1896 AD (the year of the first Modern Olympic Games) is written as ͵αωϙϛ.

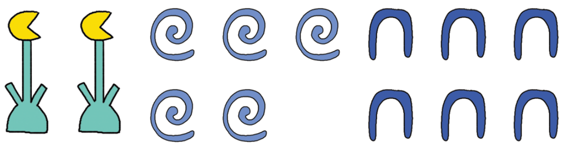
What if?

Pupils' numbers will vary.

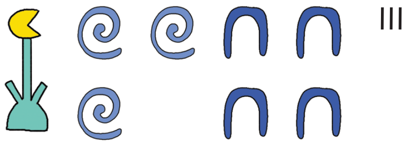
Ancient Egyptian number symbols (p.57)

Challenge

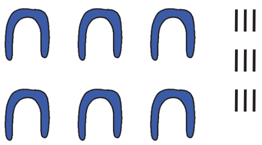
Using the Ancient Egyptian number symbols, 2560 BC (the year the Great Pyramid at Giza was built) is written as:



Using the Ancient Egyptian number symbols, 1343 BC (the year of Tutankhamun's death) is written as:



Using the Ancient Egyptian number symbols, 69 BC (the year of Cleopatra's birth) is written as:



What if?

Pupils' numbers will vary.

Hotel Kilo (p.58)

Challenge

10, 20, 30, 40, 50, 60, 70, 80, 90, 100, (11 zeros)

101, 102,103, 104, 105, 106, 107, 108, 109, (9 zeros)

110, 120, 130, 140, 150, 160, 170, 180, 190, 200, (11 zeros)

201, 202, 203, 204, 205, 206, 207, 208, 209, (9 zeros)

210, 220, 230, 240, 250, 260, 270, 280, 290, 300, (11 zeros)

301, 302, 303, 304, 305, 306, 307, 308, 309, (9 zeros)

310, 320, 330, 340, 350, 360, 370, 380, 390, 400, (11 zeros)

401, 402, 403, 404, 405, 406, 407, 408, 409, (9 zeros)

410, 420, 430, 440, 450, 460, 470, 480, 490, 500, (11 zeros)

501, 502, 503, 504, 505, 506, 507, 508, 509, (9 zeros)

510, 520, 530, 540, 550, 560, 570, 580, 590, 600, (11 zeros)

601, 602, 603, 604, 605, 606, 607, 608, 609, (9 zeros)

610, 620, 630, 640, 650, 660, 670, 680, 690, 700, (11 zeros)

701, 702, 703, 704, 705, 706, 707, 708, 709, (9 zeros)

710, 720, 730, 740, 750, 760, 770, 780, 790, 800, (11 zeros)

801, 802, 803, 804, 805, 806, 807, 808, 809, (9 zeros)

810, 820, 830, 840, 850, 860, 870, 880, 890, 900, (11 zeros)

901, 902, 903, 904, 905, 906, 907, 908, 909, (9 zeros)

910, 920, 930, 940, 950, 960, 970, 980, 990, 1000 (12 zeros)

Total: 192 zeros

What if?

9, 19, 29, 39, 49, 59, 69, 79, 89, 90, (10 nines)

91, 92, 93, 94, 95, 96, 97, 98, 99, (10 nines)

109, 119, 129, 139, 149, 159, 169, 179, 189, 190, (10 nines)

191, 192, 193, 194, 195, 196, 197, 198, 199, (10 nines)

209, 219, 229, 239, 249, 259, 269, 279, 289, 290, (10 nines)

291, 292, 293, 294, 295, 296, 297, 298, 299, (10 nines)

309, 319, 329, 339, 349, 359, 369, 379, 389, 390, (10 nines)

391, 392, 393, 394, 395, 396, 397, 398, 399, (10 nines)

409, 419, 429, 439, 449, 459, 469, 479, 489, 490, (10 nines)

491, 492, 493, 494, 495, 496, 497, 498, 499, (10 nines)

509, 519, 529, 539, 549, 559, 569, 579, 589, 590, (10 nines)

591, 592, 593, 594, 595, 596, 597, 598, 599, (10 nines)

609, 619, 629, 639, 649, 659, 669, 679, 689, 690, (10 nines)

691, 692, 693, 694, 695, 696, 697, 698, 699, (10 nines)

709, 719, 729, 739, 749, 759, 769, 779, 789, 790, (10 nines)

791, 792, 793, 794, 795, 796, 797, 798, 799, (10 nines)

809, 819, 829, 839, 849, 859, 869, 879, 889, 890, (10 nines)

891, 892, 893, 894, 895, 896, 897, 898, 899, (10 nines)

900, 901, 902, 903, 904, 905, 906, 907, 908, 909, (11 nines)

910, 911, 912, 913, 914, 915, 916, 917, 918, 919, (11 nines)

920, 921, 922, 923, 924, 925, 926, 927, 928, 929, (11 nines)

930, 931, 932, 933, 934, 935, 936, 937, 938, 939, (11 nines)

940, 941, 942, 943, 944, 945, 946, 947, 948, 949, (11 nines)

950, 951, 952, 953, 954, 955, 956, 957, 958, 959, (11 nines)

960, 961, 962, 963, 964, 965, 966, 967, 968, 969, (11 nines)

970, 971, 972, 973, 974, 975, 976, 977, 978, 979, (11 nines)

980, 981, 982, 983, 984, 985, 986, 987, 988, 989, (11 nines)

990, 991, 992, 993, 994, 995, 996, 997, 998, 999, (21 nines)

Total: 300 nines

Fabien is right. Albert will need 108 more nine digits than zero digits for all of the room numbers from 1 to 1000.

Dice imaginings (p.59)

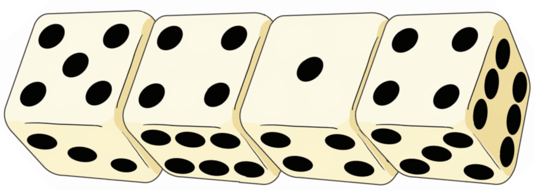
Challenge

Pupils' answers will vary.

However, once pupils realise that the opposite sides of a 1–6 dice total 7, then:

- by knowing the number of spots on the top of each dice, they can work out the number of spots on the bottom of each dice and therefore the total number of spots
- they will know that opposite faces of each of the two middle dice total 7
- it is possible to arrange four dice so that there is the same number of spots on the top of the wall as on the bottom of the wall, for example:

Top: 5 + 4 + 1 + 4 = 14



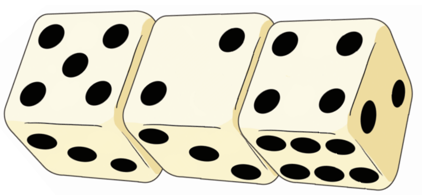
Bottom: 2 + 3 + 6 + 3 = 14

What if?

Pupils' answers will vary.

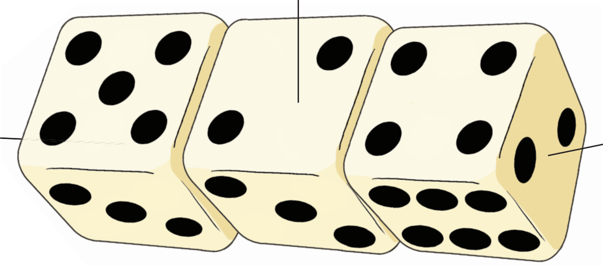
Knowing that opposite sides of a 1–6 dice total 7, then pupils can:

- calculate the product of the three sets of spots on the bottom of the wall, for example:



Bottom: 2 × 5 × 3 = 30

- calculate the product of the four sets of spots where the faces of the three dice touch, for example:



$6 \times 6 \quad \times \quad 1 \times 5 \quad = \quad 180$

Streets (p.60)

Challenge/What if?

Results of the challenge will vary.

Number plate calculations (p.61)

Challenge/What if?

Results of the challenge will vary. Be aware that, at this stage, most pupils will not understand the order of operations.

Carleton Harvest Festival (p.62)

Challenge

2 × 6-seater and 16 × 8-seater

6 × 6-seater and 13 × 8-seater

10 × 6-seater and 10 × 8-seater

14 × 6-seater and 7 × 8-seater

18 × 6-seater and 4 × 8-seater

22 × 6-seater and 1 × 8-seater

What if?

3 × 6-seater and 19 × 8-seater

7 × 6-seater and 16 × 8-seater

11 × 6-seater and 13 × 8-seater

15 × 6-seater and 10 × 8-seater

19 × 6-seater and 7 × 8-seater

23 × 6-seater and 4 × 8-seater

27 × 6-seater and 1 × 8-seater

Intercity travelling (p.63)

Challenge/What if?

There are 12 possible routes. Routes below are given in order of total distance: shortest first.

City	Distance (km)	City	Distance (km)	City	Distance (km)	City	Total (km)
London →	247	← Cardiff →	185	← Birmingham →	156	← Liverpool	588
Liverpool →	156	← Birmingham →	193	← London →	247	← Cardiff	596
London →	193	← Birmingham →	156	← Liverpool →	333	← Cardiff	682
London →	357	← Liverpool →	156	← Birmingham →	185	← Cardiff	698
London →	193	← Birmingham →	185	← Cardiff →	333	← Liverpool	711
Liverpool →	357	← London →	193	← Birmingham →	185	← Cardiff	735
London →	247	← Cardiff →	333	← Liverpool →	156	← Birmingham	736
Birmingham →	156	← Liverpool →	357	← London →	247	← Cardiff	760
Birmingham →	193	← London →	247	← Cardiff →	333	← Liverpool	773
Birmingham →	185	← Cardiff →	247	← London →	357	← Liverpool	789
London →	357	← Liverpool →	333	← Cardiff →	185	← Birmingham	875
Birmingham →	193	← London →	357	← Liverpool →	333	← Cardiff	883

Tides (p.64)

Challenge/What if?

Pupils' statements will vary. Use your professional judgment when assessing pupils' patterns and statements. However, pupils should realise some of the following patterns:

- The UK has two high tides and two low tides each day. (This is referred to as 'semi-diurnal tides'. Some parts of the world have 'diurnal tides' meaning there is only one high tide and one low tide each day.)
- Tides work in cycles and the exact time the high tide and low tide points occur changes every day.
- High tide and low tide occur at a later time every day as the tide cycle progresses. This is the same for every location in the UK, although the exact amount of time differs between locations. In some locations the time of tides moves along by over an hour every day, whereas in others the tides may take place only a short time later every day.

Money trail (p.65)

Challenge/What if?

Pupils' results will vary depending on the length of the classroom. However, the diameters of the coins are:

10p: 24·5 mm

£1: 23·43 mm

£2: 28·4 mm

50p: 27·3 mm

20p: 21·4 mm

5p: 18 mm

Visiting your local area (p.66)

Challenge/What if?

Results of the challenge will vary.

Going on holiday (p.67)

Challenge/What if?

Results of the challenge will vary.

School recycling and rubbish (p.68)

Challenge/What if?

Results of the challenge will vary.

Cooking (p.69)

Challenge/What if?

Variations on the quantities given below may occur.

* A class size of 30 has been assumed.

Italic text indicates the quantities given in the recipe.

	Serves 2	Serves 4	Serves 6	Serves 8	Serves 12	Serves 30*
Spanish omelette						
sliced potatoes	400 g	*800 g*	1·2 kg	1·6 kg	2·4 kg	6 kg
olive oil	30 ml	*60 ml*	90 ml	120 ml	180 ml	450 ml
eggs	3	*6*	9	12	18	45
chopped onions	75 g	*150 g*	225 g	300 g	450 g	1·125 kg
garlic cloves	1	*2*	3	4	6	15
tinned tomatoes	200 g	*400 g*	600 g	800 g	1·2 kg	3 kg
teaspoons chilli powder	$\frac{1}{2}$	*1*	$1\frac{1}{2}$	2	3	$7\frac{1}{2}$
Chocolate mousse						
dark chocolate	45 g	90 g	135 g	*180 g*	270 g	675 g
butter	30 g	60 g	90 g	*120 g*	180 g	450 g
eggs	1	2	3	*4*	6	15
tablespoons of coffee	$\frac{1}{2}$	1	$1\frac{1}{2}$	*2*	3	$7\frac{1}{2}$

Balanced menu (p.70)

Challenge/What if?

Results of the challenge will vary.

Healthy and unhealthy food (p.71)

Challenge/What if?

Results of the challenge will vary.

Timetables (p.72)

Challenge/What if?

Results of the challenge will vary.

School holidays (p.73)

Challenge

Results of the challenge will vary. However, in the UK pupils should attend school for 190 days a year.

What if?

Assuming a school year of 190 days, then Lucy is right. Pupils spend 190 days a year in school and 175 days a year out of school.

How fit? (p.74)

Challenge/What if?

Results of the challenge will vary. However, pupils should realise that they take approximately 1 breath for every 4 heartbeats.

Pizza anyone? (p.75)

Challenge/What if?

Results of the challenge will vary.

Angle time (p.76)

Challenge

The hands of the clock are at an acute angle at 1 o'clock, 2 o'clock, 10 o'clock and 11 o'clock.

The hands of the clock are at a right angle at 3 o'clock and 9 o'clock.

The hands of the clock are at an obtuse angle at 4 o'clock, 5 o'clock, 7 o'clock and 8 o'clock.

The order of the times according to the size of the angles, starting with the smallest angle is:

1 o'clock and 11 o'clock

2 o'clock and 10 o'clock

3 o'clock and 9 o'clock

4 o'clock and 8 o'clock

5 o'clock and 7 o'clock

What if?

- The hands of the clock are at an acute angle at $\frac{1}{2}$ past 3, $\frac{1}{2}$ past 4, $\frac{1}{2}$ past 5, $\frac{1}{2}$ past 6, $\frac{1}{2}$ past 7 and $\frac{1}{2}$ past 8.
- The hands of the clock are at an obtuse angle at $\frac{1}{2}$ past 12, $\frac{1}{2}$ past 1, $\frac{1}{2}$ past 2, $\frac{1}{2}$ past 9, $\frac{1}{2}$ past 10 and $\frac{1}{2}$ past 11.

- The hands of the clock are at an acute angle at $\frac{1}{4}$ past 12, $\frac{1}{4}$ past 1, $\frac{1}{4}$ past 2, $\frac{1}{4}$ past 3, $\frac{1}{4}$ past 4 and $\frac{1}{4}$ past 5.
- The hands of the clock are at an obtuse angle at $\frac{1}{4}$ past 6, $\frac{1}{4}$ past 7, $\frac{1}{4}$ past 8, $\frac{1}{4}$ past 9, $\frac{1}{4}$ past 10 and $\frac{1}{4}$ past 11.

- The hands of the clock are at an acute angle at $\frac{1}{4}$ to 7, $\frac{1}{4}$ to 8, $\frac{1}{4}$ to 9, $\frac{1}{4}$ to 10, $\frac{1}{4}$ to 11 and $\frac{1}{4}$ to 12.
- The hands of the clock are at an obtuse angle at $\frac{1}{4}$ to 1, $\frac{1}{4}$ to 2, $\frac{1}{4}$ to 3, $\frac{1}{4}$ to 4, $\frac{1}{4}$ to 5 and $\frac{1}{4}$ to 6.

Street map (p.77)

Challenge/What if?

Pupils' maps and directions will vary.

Rangoli (p.78)

Challenge/What if?

Pupils' patterns will vary.

Festivals (p.79)

Challenge/What if?

Results of the challenge will vary.