

## Solving mathematical problems

### Number sequences (p.8)

#### Challenge/What if?

Results of the challenge will vary.

### 1, 10 and 100 (p.9)

#### Challenge/What if?

Results of the challenge will vary.

Pupils' observations will vary depending on their depth of understanding. Use your professional judgement when assessing pupils' reasoning. However, pupils should realise that if a number crosses a tens or hundreds boundary when finding 1, 10 or 100 more or less, then more than one digit in the number will change, for example: 1 less than 510 is 509; 10 more than 498 is 508; 100 more than 945 is 1045.

### Numbers between (p.10)

#### Challenge

Using the digits 7, 4 and 8, six numbers lie between 70 and 700: 74, 78, 84, 87, 478 and 487.

Altogether, 12 different 2-digit and 3-digit numbers can be made using the digits 7, 4 and 8. These are: the six numbers listed above and 47, 48, 748, 784, 847 and 874.

#### What if?

Using the digits 9, 1 and 5, six numbers lie between 50 and 500: 51, 59, 91, 95, 159 and 195.

Altogether, 12 different 2-digit and 3-digit numbers can be made using the digits 9, 1 and 5. These are: the six numbers listed above and 15, 19, 519, 591, 915 and 951.

When using the digits 7, 4 and 8, six numbers lie between 70 and 700, two numbers are less than 70 and four numbers are greater than 700. The same applies when using the digits 9, 1 and 5: Six numbers lie between 50 and 500, two numbers are less than 50 and four numbers are greater than 500.

Pupils' chosen three digits and number range will vary.

## What are our numbers? (p.11)

### Challenge

Emily is thinking of the number 450.

Osaru is thinking of the number 599.

Alexander is thinking of the number 396.

Kate is thinking of the number 846.

### What if?

Results of the challenge will vary.

## Nearest to 500 (p.12)

### Challenge/What if?

Results of the challenge will vary.

## Consecutive reversals (p.13)

### Challenge

Seven different addition calculations are possible:

$$123 + 321 = 444$$

$$234 + 432 = 666$$

$$345 + 543 = 888$$

$$456 + 654 = 1110$$

$$567 + 765 = 1332$$

$$678 + 876 = 1554$$

$$789 + 987 = 1776$$

Pupils' observations will vary depending on their depth of understanding. Use your professional judgement when assessing pupils' reasoning. However, pupils should notice that for each calculation the hundreds and tens digits are the same.

## What if?

Seven different subtraction calculations are possible:

$$321 - 123 = 198$$

$$432 - 234 = 198$$

$$543 - 345 = 198$$

$$654 - 456 = 198$$

$$765 - 567 = 198$$

$$876 - 678 = 198$$

$$987 - 789 = 198$$

Pupils' observations will vary, depending on their depth of understanding. Use your professional judgement when assessing pupils' reasoning. However, pupils should notice that the difference is the same for all the calculations.

Two different addition calculations are possible when adding three consecutive even digits:

$$246 + 642 = 888$$

$$468 + 864 = 1332$$

Three different addition calculations are possible when adding three consecutive odd digits:

$$135 + 531 = 666$$

$$357 + 753 = 1110$$

$$579 + 975 = 1554$$

Pupils' observations will vary depending on their depth of understanding. Use your professional judgement when assessing pupils' reasoning. However, pupils should notice that these five answers are also five of the answers for the addition calculations in the Challenge.

## Making numbers (p.14)

### Challenge

$$315 + 300 = 615$$

$$315 - 9 = 306$$

$$247 + 300 = 547$$

$$315 - 70 = 245$$

$$247 - 70 = 177$$

$$682 + 9 = 691$$

$$682 - 300 = 382$$

$$682 + 70 = 752$$

$$247 - 9 = 238$$

The numbers 526 and 20 are not used to make any of the numbers on the cards.

### What if?

A further 23 different addition and subtraction calculations are possible using all nine numbers on the screen (i.e. including numbers 20 and 526):

$$247 + 9 = 256$$

$$247 + 20 = 267$$

$$247 - 20 = 227$$

$$300 - 247 = 53$$

$$247 + 70 = 317$$

$$682 - 9 = 673$$

$$682 + 20 = 702$$

$$682 - 20 = 662$$

$$682 + 300 = 982$$

$$682 - 70 = 612$$

$$315 + 9 = 324$$

$$315 + 20 = 335$$

$$315 - 20 = 295$$

$$315 - 300 = 15$$

$$315 + 70 = 385$$

$$526 + 9 = 535$$

$$526 - 9 = 517$$

$$526 + 20 = 546$$

$$526 - 20 = 506$$

$$526 + 300 = 826$$

$$526 - 300 = 226$$

$$526 + 70 = 596$$

$$526 - 70 = 456$$

## 1066 and 776 (p.15)

### Challenge

Addition calculations with an answer of 1066 will vary, but could include calculations such as:

$$\text{HTO} + \text{TO} / \text{HTO} + \text{T}, \text{ e.g. } 969 + 97 / 976 + 90$$

$$\text{HTO} + \text{HTO} / \text{HTO} + \text{H}, \text{ e.g. } 238 + 828 / 566 + 500$$

### What if?

Subtraction calculations with an answer of 776 will vary, but could include calculations such as:

$$\text{HTO} - \text{TO} / \text{HTO} - \text{T}, \text{ e.g. } 802 - 26 / 816 - 40$$

$$\text{HTO} - \text{HTO} / \text{HTO} - \text{H}, \text{ e.g. } 945 - 169 / 876 - 100$$

## Using tables (p.16)

### Challenge

The answers to the 6 times table are double the answers to the 3 times table.

The answers to the 12 times table are double the answers to the 6 times table.

### What if?

- The answers to the 8 times table are double the answers to the 4 times table.
- To multiply a number by 9, you multiply the number by 10, then subtract the number, e.g.  $9 \times 7 = (10 \times 7) - 7 = 63$ .  
To multiply a number by 11, you multiply the number by 10, then add the number, e.g.  $11 \times 7 = (10 \times 7) + 7 = 77$ .
- The 5 and 10 times tables can be used to work out the answer  $7 \times 15$  by multiplying 7 by 10, and 7 by 5 and adding the two products together, i.e.  $(7 \times 10) + (7 \times 5) = 70 + 35 = 105$ .

## Multiplication and division (p.17)

### Challenge

12 different multiplication calculations are possible (24 if including calculations that apply the commutative law):

$$2 \times 3 = 6 / 3 \times 2 = 6 \quad 3 \times 4 = 12 / 4 \times 3 = 12 \quad 4 \times 5 = 20 / 5 \times 4 = 20 \quad 5 \times 6 = 30 / 6 \times 5 = 30$$

$$2 \times 4 = 8 / 4 \times 2 = 8 \quad 3 \times 5 = 15 / 5 \times 3 = 15 \quad 4 \times 6 = 24 / 6 \times 4 = 24$$

$$2 \times 6 = 12 / 6 \times 2 = 12 \quad 3 \times 6 = 18 / 6 \times 3 = 18$$

$$2 \times 12 = 24 / 12 \times 2 = 24 \quad 3 \times 8 = 24 / 8 \times 3 = 24$$

$$2 \times 15 = 30 / 15 \times 2 = 30$$

## What if?

12 different division calculations are possible (24 if including calculations that swap the divisor and dividend):

$$6 \div 2 = 3 / 6 \div 3 = 2 \quad 12 \div 3 = 4 / 12 \div 4 = 3 \quad 20 \div 4 = 5 / 20 \div 5 = 4 \quad 30 \div 5 = 6 / 30 \div 6 = 5$$

$$8 \div 2 = 4 / 8 \div 4 = 2 \quad 15 \div 3 = 5 / 15 \div 5 = 3 \quad 24 \div 4 = 6 / 24 \div 6 = 4$$

$$12 \div 2 = 6 / 12 \div 6 = 2 \quad 18 \div 3 = 6 / 18 \div 6 = 3$$

$$24 \div 2 = 12 / 24 \div 12 = 2 \quad 24 \div 3 = 8 / 24 \div 8 = 3$$

$$30 \div 2 = 15 / 30 \div 15 = 2$$

Pupils' observations will vary depending on their depth of understanding. Use your professional judgement when assessing pupils' reasoning. However, pupils should notice the following relationships:

The inverse relationship between multiplication and division means that there is the same number of multiplication calculations as division calculations. There are also 12 sets of calculations, with each set containing four calculations (two multiplications and two divisions) where the same three numbers are used in all four calculations. For example:  $3 \times 4 = 12 / 4 \times 3 = 12 / 12 \div 3 = 4 / 12 \div 4 = 3$

## 2-digit numbers $\times$ 1-digit numbers (p.18)

### Challenge

The largest answer possible is 432 ( $54 \times 8$ ).

Apart from zero, the smallest answer possible is 20 ( $10 \times 2$  or  $20 \times 1$ ).

## What if?

There are 14 different calculations that have an answer between 250 and 350:

$$32 \times 8 = 256$$

$$34 \times 8 = 272$$

$$35 \times 8 = 280$$

$$84 \times 3 = 252$$

$$85 \times 3 = 255$$

$$40 \times 8 = 320 / 80 \times 4 = 320$$

$$81 \times 4 = 324$$

$$82 \times 4 = 328 / 41 \times 8 = 328$$

$$83 \times 4 = 332$$

$$42 \times 8 = 336$$

$$85 \times 4 = 340$$

$$43 \times 8 = 344$$

## Partition and multiply (p.19)

### Challenge

Partitioning 16 into 8 and 8 will result in the greatest product ( $8 \times 8 = 64$ ).

### What if?

Partitioning 19 into 10 and 9 will result in the greatest product ( $10 \times 9 = 90$ ).

The greatest product is created when the two numbers partitioned have the smallest difference. For example, 16 can be partitioned into:

15 + 1: The difference between 15 and 1 is 14. ( $15 \times 1 = 15$ )

14 + 2: The difference between 14 and 2 is 12. ( $14 \times 2 = 28$ )

13 + 3: The difference between 13 and 3 is 10. ( $13 \times 3 = 39$ )

12 + 4: The difference between 12 and 4 is 8. ( $12 \times 4 = 48$ )

11 + 5: The difference between 11 and 5 is 6. ( $11 \times 5 = 55$ )

10 + 6: The difference between 10 and 6 is 4. ( $10 \times 6 = 60$ )

9 + 7: The difference between 9 and 7 is 2. ( $9 \times 7 = 63$ )

8 + 8: The difference between 8 and 8 is 0. ( $8 \times 8 = 64$ )

Partitioning 23 into 12 and 11 will result in the greatest product ( $12 \times 11 = 132$ ).

Results will vary when pupils choose their own two-digit number.

## Fractions that total 1 (p.20)

### Challenge

There are five different ways to use two of the fractions to make a whole:

$$\frac{1}{10} + \frac{9}{10}$$

$$\frac{2}{10} + \frac{8}{10}$$

$$\frac{3}{10} + \frac{7}{10}$$

$$\frac{4}{10} + \frac{6}{10}$$

$$\frac{5}{10} + \frac{1}{2}$$

There are six different ways to use three of the fractions to make a whole:

$$\frac{7}{10} + \frac{2}{10} + \frac{1}{10}$$

$$\frac{6}{10} + \frac{3}{10} + \frac{1}{10}$$

$$\frac{5}{10} + \frac{4}{10} + \frac{1}{10}$$

$$\frac{1}{2} + \frac{4}{10} + \frac{1}{10}$$

$$\frac{5}{10} + \frac{3}{10} + \frac{2}{10}$$

$$\frac{1}{2} + \frac{3}{10} + \frac{2}{10}$$

### What if?

There are two different ways to use two of the fractions to make a half:

$$\frac{1}{10} + \frac{4}{10}$$

$$\frac{2}{10} + \frac{3}{10}$$

Pupils' other pairs of fractions that make a whole and a half will vary.

## Buying sweets (p.21)

### Challenge

Emily could have bought  $\frac{1}{8}$  of the sweets and Alexander  $\frac{7}{8}$ , and vice versa.

Emily could have bought  $\frac{2}{8}$  ( $\frac{1}{4}$ ) of the sweets and Alexander  $\frac{6}{8}$  ( $\frac{3}{4}$ ), and vice versa.

Emily could have bought  $\frac{3}{8}$  of the sweets and Alexander  $\frac{5}{8}$ , and vice versa.

Emily and Alexander could have bought  $\frac{4}{8}$  ( $\frac{1}{2}$ ) of the sweets each.



## What if?

Osaru, Emily and Alexander could have bought 12 sweets between them in the following ways:

$$\frac{1}{12}, \frac{1}{12} \text{ and } \frac{10}{12} \left(\frac{5}{6}\right)$$

$$\frac{1}{12}, \frac{2}{12} \left(\frac{1}{6}\right) \text{ and } \frac{9}{12} \left(\frac{3}{4}\right)$$

$$\frac{1}{12}, \frac{3}{12} \left(\frac{1}{4}\right) \text{ and } \frac{8}{12} \left(\frac{2}{3}\right)$$

$$\frac{1}{12}, \frac{4}{12} \left(\frac{1}{3}\right) \text{ and } \frac{7}{12}$$

$$\frac{1}{12}, \frac{5}{12} \text{ and } \frac{6}{12} \left(\frac{1}{2}\right)$$

$$\frac{2}{12} \left(\frac{1}{6}\right), \frac{2}{12} \left(\frac{1}{6}\right) \text{ and } \frac{8}{12} \left(\frac{2}{3}\right)$$

$$\frac{2}{12} \left(\frac{1}{6}\right), \frac{3}{12} \left(\frac{1}{4}\right) \text{ and } \frac{7}{12}$$

$$\frac{2}{12} \left(\frac{1}{6}\right), \frac{4}{12} \left(\frac{1}{3}\right) \text{ and } \frac{6}{12} \left(\frac{1}{2}\right)$$

$$\frac{2}{12} \left(\frac{1}{6}\right), \frac{5}{12} \text{ and } \frac{5}{12}$$

$$\frac{3}{12} \left(\frac{1}{4}\right), \frac{3}{12} \left(\frac{1}{4}\right) \text{ and } \frac{6}{12} \left(\frac{1}{2}\right)$$

$$\frac{3}{12} \left(\frac{1}{4}\right), \frac{4}{12} \left(\frac{1}{3}\right) \text{ and } \frac{5}{12}$$

$$\frac{4}{12} \left(\frac{1}{3}\right), \frac{4}{12} \left(\frac{1}{3}\right) \text{ and } \frac{4}{12} \left(\frac{1}{3}\right)$$

Kate, Osaru, Emily and Alexander could have bought 10 sweets between them in the following ways:

$$\frac{1}{10}, \frac{1}{10}, \frac{1}{10} \text{ and } \frac{7}{10}$$

$$\frac{1}{10}, \frac{1}{10}, \frac{2}{10} \left(\frac{1}{5}\right) \text{ and } \frac{6}{10} \left(\frac{3}{5}\right)$$

$$\frac{1}{10}, \frac{1}{10}, \frac{3}{10} \text{ and } \frac{5}{10} \left(\frac{1}{2}\right)$$

$$\frac{1}{10}, \frac{1}{10}, \frac{4}{10} \left(\frac{2}{5}\right) \text{ and } \frac{4}{10} \left(\frac{2}{5}\right)$$

$$\frac{1}{10}, \frac{2}{10} \left(\frac{1}{5}\right), \frac{2}{10} \left(\frac{1}{5}\right) \text{ and } \frac{5}{10} \left(\frac{1}{2}\right)$$

$$\frac{1}{10}, \frac{2}{10} \left(\frac{1}{5}\right), \frac{3}{10} \text{ and } \frac{4}{10} \left(\frac{2}{5}\right)$$

$$\frac{1}{10}, \frac{3}{10}, \frac{3}{10} \text{ and } \frac{3}{10}$$

$$\frac{2}{10} \left(\frac{1}{5}\right), \frac{2}{10} \left(\frac{1}{5}\right), \frac{2}{10} \left(\frac{1}{5}\right) \text{ and } \frac{4}{10} \left(\frac{2}{5}\right)$$

$$\frac{2}{10} \left(\frac{1}{5}\right), \frac{2}{10} \left(\frac{1}{5}\right), \frac{3}{10} \text{ and } \frac{3}{10}$$

## Equivalent fractions (p.22)

### Challenge

You can make ten equivalent statements using one set of 1–9 digit cards:

$$\frac{1}{2} = \frac{3}{6}$$

$$\frac{1}{2} = \frac{4}{8}$$

$$\frac{1}{3} = \frac{2}{6}$$

$$\frac{2}{3} = \frac{4}{6}$$

$$\frac{2}{3} = \frac{6}{9}$$

$$\frac{1}{4} = \frac{2}{8}$$

$$\frac{2}{4} = \frac{3}{6}$$

$$\frac{3}{4} = \frac{6}{8}$$

$$\frac{2}{6} = \frac{3}{9}$$

$$\frac{3}{6} = \frac{4}{8}$$

### What if?

Including the equivalent statements above, the following equivalent statements can be made using two sets of 1–9 digit cards:

$$\frac{1}{2} = \frac{2}{4}$$

$$\frac{1}{3} = \frac{3}{9}$$

$$\frac{2}{4} = \frac{4}{8}$$

Some pupils may use the digit cards to make equivalent statements similar to the following (where the numerator and/or denominator is a two-digit number). If so, check their statements.

$$\frac{1}{2} = \frac{6}{12}$$

$$\frac{1}{4} = \frac{12}{48}$$

## Measuring with ropes (p.23)

### Challenge

Kate is right. You cannot measure a length of 28 cm.

You can measure the following lengths using two pieces of rope:

$$3 \text{ cm} = 9 \text{ cm} - 6 \text{ cm}$$

$$15 \text{ cm} = 9 \text{ cm} + 6 \text{ cm}$$

$$4 \text{ cm} = 21 \text{ cm} - 17 \text{ cm}$$

$$17 \text{ cm} = 17 \text{ cm}$$

$$5 \text{ cm} = 17 \text{ cm} - 12 \text{ cm}$$

$$18 \text{ cm} = 12 \text{ cm} + 6 \text{ cm}$$

$$6 \text{ cm} = 6 \text{ cm}$$

$$21 \text{ cm} = 21 \text{ cm}$$

$$8 \text{ cm} = 17 \text{ cm} - 9 \text{ cm}$$

$$23 \text{ cm} = 17 \text{ cm} + 6 \text{ cm}$$

$$9 \text{ cm} = 9 \text{ cm}$$

$$26 \text{ cm} = 17 \text{ cm} + 9 \text{ cm}$$

$$11 \text{ cm} = 17 \text{ cm} - 6 \text{ cm}$$

$$27 \text{ cm} = 21 \text{ cm} + 6 \text{ cm}$$

$$12 \text{ cm} = 12 \text{ cm}$$

$$29 \text{ cm} = 17 \text{ cm} + 12 \text{ cm}$$

$$30 \text{ cm} = 21 \text{ cm} + 9 \text{ cm}$$

Other ways are possible.

You can measure the following lengths using three or more pieces of rope:

$$1 \text{ cm} = 12 \text{ cm} + 6 \text{ cm} - 17 \text{ cm}$$

$$16 \text{ cm} = (21 \text{ cm} + 12 \text{ cm}) - 17 \text{ cm}$$

$$2 \text{ cm} = (6 \text{ cm} + 17 \text{ cm}) - 21 \text{ cm}$$

$$19 \text{ cm} = (21 \text{ cm} + 9 \text{ cm} + 6 \text{ cm}) - 17 \text{ cm}$$

$$7 \text{ cm} = (21 \text{ cm} + 12 \text{ cm}) - (17 \text{ cm} + 9 \text{ cm})$$

$$20 \text{ cm} = (17 \text{ cm} + 9 \text{ cm}) - 6 \text{ cm}$$

$$10 \text{ cm} = (21 \text{ cm} - 17 \text{ cm}) + (12 \text{ cm} - 6 \text{ cm})$$

$$22 \text{ cm} = (21 \text{ cm} + 12 \text{ cm} + 6 \text{ cm}) - 17 \text{ cm}$$

$$13 \text{ cm} = (21 \text{ cm} + 9 \text{ cm}) - 17 \text{ cm}$$

$$24 \text{ cm} = (21 \text{ cm} + 12 \text{ cm}) - 9 \text{ cm}$$

$$14 \text{ cm} = (17 \text{ cm} + 12 \text{ cm}) - (9 \text{ cm} + 6 \text{ cm})$$

$$25 \text{ cm} = (21 \text{ cm} + 12 \text{ cm} + 9 \text{ cm}) - 17 \text{ cm}$$

Other ways are possible.

## What if?

Lengths of rope 1 cm, 2 cm, 5 cm, 10 cm and 20 cm would allow you to measure every length from 1 cm to 30 cm.

$$1 \text{ cm} = 1 \text{ cm}$$

$$2 \text{ cm} = 2 \text{ cm}$$

$$3 \text{ cm} = 2 \text{ cm} + 1 \text{ cm}$$

$$4 \text{ cm} = 5 \text{ cm} - 1 \text{ cm}$$

$$5 \text{ cm} = 5 \text{ cm}$$

$$6 \text{ cm} = 5 \text{ cm} + 1 \text{ cm}$$

$$7 \text{ cm} = 5 \text{ cm} + 2 \text{ cm}$$

$$8 \text{ cm} = 5 \text{ cm} + 2 \text{ cm} + 1 \text{ cm}$$

$$9 \text{ cm} = 10 \text{ cm} - 1 \text{ cm}$$

$$10 \text{ cm} = 10 \text{ cm}$$

$$11 \text{ cm} = 10 \text{ cm} + 1 \text{ cm}$$

$$12 \text{ cm} = 10 \text{ cm} + 2 \text{ cm}$$

$$13 \text{ cm} = 10 \text{ cm} + 2 \text{ cm} + 1 \text{ cm}$$

$$14 \text{ cm} = (10 \text{ cm} + 5 \text{ cm}) - 1 \text{ cm}$$

$$15 \text{ cm} = 10 \text{ cm} + 5 \text{ cm}$$

Other ways are possible.

$$16 \text{ cm} = 10 \text{ cm} + 5 \text{ cm} + 1 \text{ cm}$$

$$17 \text{ cm} = 10 \text{ cm} + 5 \text{ cm} + 2 \text{ cm}$$

$$18 \text{ cm} = 20 \text{ cm} - 2 \text{ cm}$$

$$19 \text{ cm} = 20 \text{ cm} - 1 \text{ cm}$$

$$20 \text{ cm} = 20 \text{ cm}$$

$$21 \text{ cm} = 20 \text{ cm} + 1 \text{ cm}$$

$$22 \text{ cm} = 20 \text{ cm} + 2 \text{ cm}$$

$$23 \text{ cm} = 20 \text{ cm} + 2 \text{ cm} + 1 \text{ cm}$$

$$24 \text{ cm} = (20 \text{ cm} + 5 \text{ cm}) - 1 \text{ cm}$$

$$25 \text{ cm} = 20 \text{ cm} + 5 \text{ cm}$$

$$26 \text{ cm} = 20 \text{ cm} + 5 \text{ cm} + 1 \text{ cm}$$

$$27 \text{ cm} = 20 \text{ cm} + 5 \text{ cm} + 2 \text{ cm}$$

$$28 \text{ cm} = 20 \text{ cm} + 10 \text{ cm} - 2 \text{ cm}$$

$$29 \text{ cm} = 20 \text{ cm} + 10 \text{ cm} - 1 \text{ cm}$$

$$30 \text{ cm} = 20 \text{ cm} + 10 \text{ cm}$$

## 50 g, 100 g and 500 g weights (p.24)

### Challenge

$$50 \text{ g} = 50 \text{ g}$$

$$100 \text{ g} = 100 \text{ g}$$

$$150 \text{ g} = 100 \text{ g} + 50 \text{ g}$$

$$200 \text{ g} = 100 \text{ g} + 100 \text{ g}$$

$$250 \text{ g} = 100 \text{ g} + 100 \text{ g} + 50 \text{ g}$$

$$300 \text{ g} = 100 \text{ g} + 100 \text{ g} + 100 \text{ g} \text{ or } 500 \text{ g} - 100 \text{ g} - 100 \text{ g}$$

$$350 \text{ g} = 500 \text{ g} - 100 \text{ g} - 50 \text{ g}$$

$$400 \text{ g} = 500 \text{ g} - 100 \text{ g}$$

$$450 \text{ g} = 500 \text{ g} - 50 \text{ g}$$

$$500 \text{ g} = 500 \text{ g}$$

### What if?

A Standard Metric Weight Set consists of: 500 g, 2 × 200 g, 100 g, 50 g, 2 × 20 g, 10 g and 5 g weights because by using these nine weights, all multiples of 5 g from 5 g to 1 kg can be made.

## Filling buckets (p.25)

### Challenge

$$1 \text{ litre} = 1 \text{ litre}$$

$$2 \text{ litres} = 2 \text{ litres}$$

$$3 \text{ litres} = 2 \text{ litres} + 1 \text{ litre}$$

$$4 \text{ litres} = 4 \text{ litres}$$

$$5 \text{ litres} = 4 \text{ litres} + 1 \text{ litre}$$

$$6 \text{ litres} = 4 \text{ litres} + 2 \text{ litres}$$

$$7 \text{ litres} = 4 \text{ litres} + 2 \text{ litres} + 1 \text{ litre}$$

$$8 \text{ litres} = 8 \text{ litres}$$

$$9 \text{ litres} = 8 \text{ litres} + 1 \text{ litre}$$

$$10 \text{ litres} = 8 \text{ litres} + 2 \text{ litres}$$

$$11 \text{ litres} = 8 \text{ litres} + 2 \text{ litres} + 1 \text{ litre}$$

$$12 \text{ litres} = 8 \text{ litres} + 4 \text{ litres}$$

$$13 \text{ litres} = 8 \text{ litres} + 4 \text{ litres} + 1 \text{ litre}$$

$$14 \text{ litres} = 8 \text{ litres} + 4 \text{ litres} + 2 \text{ litres}$$

$$15 \text{ litres} = 8 \text{ litres} + 4 \text{ litres} + 2 \text{ litres} + 1 \text{ litre}$$

$$16 \text{ litres} = 8 \text{ litres} + 8 \text{ litres}$$

$$17 \text{ litres} = 8 \text{ litres} + 8 \text{ litres} + 1 \text{ litre}$$

$$18 \text{ litres} = 8 \text{ litres} + 8 \text{ litres} + 2 \text{ litres}$$

$$19 \text{ litres} = 8 \text{ litres} + 8 \text{ litres} + 2 \text{ litres} + 1 \text{ litre}$$

$$20 \text{ litres} = 8 \text{ litres} + 8 \text{ litres} + 4 \text{ litres}$$

$$21 \text{ litres} = 8 \text{ litres} + 8 \text{ litres} + 4 \text{ litres} + 1 \text{ litre}$$

$$22 \text{ litres} = 8 \text{ litres} + 8 \text{ litres} + 4 \text{ litres} + 2 \text{ litres}$$

$$23 \text{ litres} = 8 \text{ litres} + 8 \text{ litres} + 4 \text{ litres} + 2 \text{ litres} + 1 \text{ litre}$$

$$24 \text{ litres} = 8 \text{ litres} + 8 \text{ litres} + 8 \text{ litres}$$

$$25 \text{ litres} = 8 \text{ litres} + 8 \text{ litres} + 8 \text{ litres} + 1 \text{ litre}$$

$$26 \text{ litres} = 8 \text{ litres} + 8 \text{ litres} + 8 \text{ litres} + 2 \text{ litres}$$

$$27 \text{ litres} = 8 \text{ litres} + 8 \text{ litres} + 8 \text{ litres} + 2 \text{ litres} + 1 \text{ litre}$$

$$28 \text{ litres} = 8 \text{ litres} + 8 \text{ litres} + 8 \text{ litres} + 4 \text{ litres}$$

$$29 \text{ litres} = 8 \text{ litres} + 8 \text{ litres} + 8 \text{ litres} + 4 \text{ litres} + 1 \text{ litre}$$

$$30 \text{ litres} = 8 \text{ litres} + 8 \text{ litres} + 8 \text{ litres} + 4 \text{ litres} + 2 \text{ litres}$$

## What if?

$$\frac{1}{4} \text{ litre} = \frac{1}{4} \text{ litre}$$

$$\frac{1}{2} \text{ litre} = \frac{1}{2} \text{ litre}$$

$$\frac{3}{4} \text{ litre} = \frac{1}{2} \text{ litre} + \frac{1}{4} \text{ litre}$$

$$1 \text{ litre} = 1 \text{ litre}$$

$$1\frac{1}{4} \text{ litres} = 1 \text{ litre} + \frac{1}{4} \text{ litre}$$

$$1\frac{1}{2} \text{ litres} = 1 \text{ litre} + \frac{1}{2} \text{ litre}$$

$$1\frac{3}{4} \text{ litres} = 1 \text{ litre} + \frac{1}{2} \text{ litre} + \frac{1}{4} \text{ litre}$$

$$3 \text{ litres} = 3 \text{ litres}$$

$$3\frac{1}{4} \text{ litres} = 3 \text{ litres} + \frac{1}{4} \text{ litre}$$

$$3\frac{1}{2} \text{ litres} = 3 \text{ litres} + \frac{1}{2} \text{ litre}$$

$$3\frac{3}{4} \text{ litres} = 3 \text{ litres} + \frac{1}{2} \text{ litre} + \frac{1}{4} \text{ litre}$$

$$4 \text{ litres} = 3 \text{ litres} + 1 \text{ litre}$$

$$4\frac{1}{4} \text{ litres} = 3 \text{ litres} + 1 \text{ litre} + \frac{1}{4} \text{ litre}$$

$$4\frac{1}{2} \text{ litres} = 3 \text{ litres} + 1 \text{ litre} + \frac{1}{2} \text{ litre}$$

$$4\frac{3}{4} \text{ litres} = 3 \text{ litres} + 1 \text{ litre} + \frac{1}{2} \text{ litre} + \frac{1}{4} \text{ litre}$$

$$8 \text{ litres} = 8 \text{ litres}$$

$$8\frac{1}{4} \text{ litres} = 8 \text{ litres} + \frac{1}{4} \text{ litre}$$

$$8\frac{1}{2} \text{ litres} = 8 \text{ litres} + \frac{1}{2} \text{ litre}$$

$$8\frac{3}{4} \text{ litres} = 8 \text{ litres} + \frac{1}{2} \text{ litre} + \frac{1}{4} \text{ litre}$$

$$9 \text{ litres} = 8 \text{ litres} + 1 \text{ litre}$$

$$9\frac{1}{4} \text{ litres} = 8 \text{ litres} + 1 \text{ litre} + \frac{1}{4} \text{ litre}$$

$$9\frac{1}{2} \text{ litres} = 8 \text{ litres} + 1 \text{ litre} + \frac{1}{2} \text{ litre}$$

$$9\frac{3}{4} \text{ litres} = 8 \text{ litres} + 1 \text{ litre} + \frac{1}{2} \text{ litre} + \frac{1}{4} \text{ litre}$$

$$11 \text{ litres} = 8 \text{ litres} + 3 \text{ litres}$$

$$11\frac{1}{4} \text{ litres} = 8 \text{ litres} + 3 \text{ litres} + \frac{1}{4} \text{ litre}$$

$$11\frac{1}{2} \text{ litres} = 8 \text{ litres} + 3 \text{ litres} + \frac{1}{2} \text{ litre}$$

$$11\frac{3}{4} \text{ litres} = 8 \text{ litres} + 3 \text{ litres} + \frac{1}{2} \text{ litre} + \frac{1}{4} \text{ litre}$$

$$12 \text{ litres} = 8 \text{ litres} + 3 \text{ litres} + 1 \text{ litre}$$

$$12\frac{1}{4} \text{ litres} = 8 \text{ litres} + 3 \text{ litres} + 1 \text{ litre} + \frac{1}{4} \text{ litre}$$

$$12\frac{1}{2} \text{ litres} = 8 \text{ litres} + 3 \text{ litres} + 1 \text{ litre} + \frac{1}{2} \text{ litre}$$

$$12\frac{3}{4} \text{ litres} = 8 \text{ litres} + 3 \text{ litres} + 1 \text{ litre} + \frac{1}{2} \text{ litre} + \frac{1}{4} \text{ litre}$$

## This year (p.26)

### Challenge/What if?

Results of the challenge will vary, depending on the year.

## 1 note and 3 coins (p.27)

### Challenge

Most pupils will probably record pounds and pence separately as the decimal recording of money is not introduced formally until Year 4, for example, '£5 and 3p' rather than '£5.03'.

$$£5 + 1\text{p} + 1\text{p} + 1\text{p} = £5.03$$

$$£5 + 2\text{p} + 2\text{p} + 2\text{p} = £5.06$$

$$£5 + 5\text{p} + 5\text{p} + 5\text{p} = £5.15$$

$$£5 + 10\text{p} + 10\text{p} + 10\text{p} = £5.30$$

$$£5 + 20\text{p} + 20\text{p} + 20\text{p} = £5.60$$

$$£5 + 50\text{p} + 50\text{p} + 50\text{p} = £6.50$$

$$£5 + £1 + £1 + £1 = £8$$

$$£5 + £2 + £2 + £2 = £11$$

$$£20 + 1\text{p} + 1\text{p} + 1\text{p} = £20.03$$

$$£20 + 2\text{p} + 2\text{p} + 2\text{p} = £20.06$$

$$£20 + 5\text{p} + 5\text{p} + 5\text{p} = £20.15$$

$$£20 + 10\text{p} + 10\text{p} + 10\text{p} = £20.30$$

$$£20 + 20\text{p} + 20\text{p} + 20\text{p} = £20.60$$

$$£20 + 50\text{p} + 50\text{p} + 50\text{p} = £21.50$$

$$£20 + £1 + £1 + £1 = £23$$

$$£20 + £2 + £2 + £2 = £26$$

$$£10 + 1\text{p} + 1\text{p} + 1\text{p} = £10.03$$

$$£10 + 2\text{p} + 2\text{p} + 2\text{p} = £10.06$$

$$£10 + 5\text{p} + 5\text{p} + 5\text{p} = £10.15$$

$$£10 + 10\text{p} + 10\text{p} + 10\text{p} = £10.30$$

$$£10 + 20\text{p} + 20\text{p} + 20\text{p} = £10.60$$

$$£10 + 50\text{p} + 50\text{p} + 50\text{p} = £11.50$$

$$£10 + £1 + £1 + £1 = £13$$

$$£10 + £2 + £2 + £2 = £16$$

$$£50 + 1\text{p} + 1\text{p} + 1\text{p} = £50.03$$

$$£50 + 2\text{p} + 2\text{p} + 2\text{p} = £50.06$$

$$£50 + 5\text{p} + 5\text{p} + 5\text{p} = £50.15$$

$$£50 + 10\text{p} + 10\text{p} + 10\text{p} = £50.30$$

$$£50 + 20\text{p} + 20\text{p} + 20\text{p} = £50.60$$

$$£50 + 50\text{p} + 50\text{p} + 50\text{p} = £51.50$$

$$£50 + £1 + £1 + £1 = £53$$

$$£50 + £2 + £2 + £2 = £56$$

### What if?

Most pupils will probably record pounds and pence separately as the decimal recording of money is not introduced formally until Year 4, for example, '£15 and 1p' rather than '£15.01'.

$$1\text{p} + £5 + £5 + £5 = £15.01$$

$$2\text{p} + £5 + £5 + £5 = £15.02$$

$$5\text{p} + £5 + £5 + £5 = £15.05$$

$$10\text{p} + £5 + £5 + £5 = £15.10$$

$$20\text{p} + £5 + £5 + £5 = £15.20$$

$$50\text{p} + £5 + £5 + £5 = £15.50$$

$$£1 + £5 + £5 + £5 = £16$$

$$£2 + £5 + £5 + £5 = £17$$

$$1\text{p} + £10 + £10 + £10 = £30.01$$

$$2\text{p} + £10 + £10 + £10 = £30.02$$

$$5\text{p} + £10 + £10 + £10 = £30.05$$

$$10\text{p} + £10 + £10 + £10 = £30.10$$

$$20\text{p} + £10 + £10 + £10 = £30.20$$

$$50\text{p} + £10 + £10 + £10 = £30.50$$

$$£1 + £10 + £10 + £10 = £31$$

$$£2 + £10 + £10 + £10 = £32$$

$$1p + \pounds20 + \pounds20 + \pounds20 = \pounds60.01$$

$$2p + \pounds20 + \pounds20 + \pounds20 = \pounds60.02$$

$$5p + \pounds20 + \pounds20 + \pounds20 = \pounds60.05$$

$$10p + \pounds20 + \pounds20 + \pounds20 = \pounds60.10$$

$$20p + \pounds20 + \pounds20 + \pounds20 = \pounds60.20$$

$$50p + \pounds20 + \pounds20 + \pounds20 = \pounds60.50$$

$$\pounds1 + \pounds20 + \pounds20 + \pounds20 = \pounds61$$

$$\pounds2 + \pounds20 + \pounds20 + \pounds20 = \pounds62$$

$$1p + \pounds50 + \pounds50 + \pounds50 = \pounds150.01$$

$$2p + \pounds50 + \pounds50 + \pounds50 = \pounds150.02$$

$$5p + \pounds50 + \pounds50 + \pounds50 = \pounds150.05$$

$$10p + \pounds50 + \pounds50 + \pounds50 = \pounds150.10$$

$$20p + \pounds50 + \pounds50 + \pounds50 = \pounds150.20$$

$$50p + \pounds50 + \pounds50 + \pounds50 = \pounds150.50$$

$$\pounds1 + \pounds50 + \pounds50 + \pounds50 = \pounds151$$

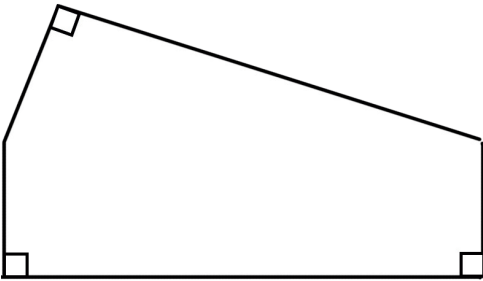
$$\pounds2 + \pounds50 + \pounds50 + \pounds50 = \pounds152$$

## Right angle shapes (p.28)

### Challenge

Many different pentagons can be drawn with one, two or three right angles.

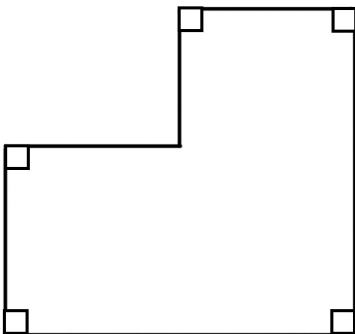
It is not possible to draw a pentagon with more than three right angles. The maximum number of right angles a pentagon can have is three, for example:



### What if?

Many different hexagons can be drawn with one, two, three, four or five right angles.

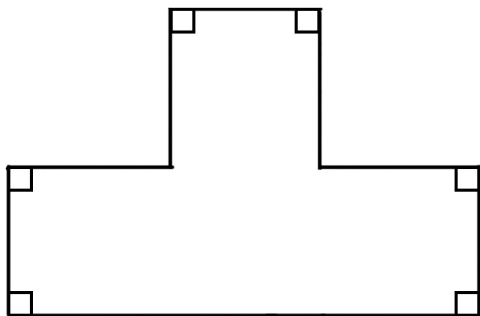
The maximum number of right angles a hexagon can have is five, for example:





Many different octagons can be drawn with one, two, three, four, five or six right angles.

The maximum number of right angles an octagon can have is six, for example:



## Tessellating shapes (p.29)

### Challenge

Squares, rectangles, triangles and hexagons all tessellate.

### What if?

Results of the challenge will vary.

## Brothers and sisters (p.30)

### Challenge/What if?

Results of the challenge will vary.

## Television (p.31)

### Challenge/What if?

Results of the challenge will vary.

## Reasoning mathematically

### 10 and 100 more and less (p.32)

#### Challenge

456 → 466 → 476 → 486 → 496 → 506 → 516 → 526

647 → 747 → 847 → 947 → 1047 → 1147 → 1247 → 1347

851 → 841 → 831 → 821 → 811 → 801 → 791 → 781

757 → 657 → 547 → 457 → 357 → 257 → 157 → 57

Pupils' explanations will vary depending on their depth of understanding. However, they should refer to the following. Use your professional judgement when assessing pupils' reasoning.

Kate and Alexander's statements are sometimes true.

In most cases, only the tens digit changes when finding 10 more or less than a three-digit number, for example: 10 more than 457 is 467; 10 less than 362 is 352. However, if a three-digit number has 9 tens, or 9 tens and 9 hundreds when adding 10, or no tens when subtracting 10, then the tens and hundreds digits in the number change, for example: 10 more than 496 is 506; 10 more than 996 is 1006, 10 less than 801 is 791.

In most cases, only the hundreds digit changes when finding 100 more or less than a three-digit number, for example: 100 more than 358 is 458; 100 less than 759 is 659. However, if a three-digit number has 9 hundreds when adding 100, then the number becomes a four-digit number with 1 thousand and the hundreds digit becomes zero (a place holder), for example: 100 more than 947 is 1047.

#### What if?

In most cases, only the tens digit changes when finding 10 more or less than a two-digit number, for example: 10 more than 46 is 56; 10 less than 59 is 49. However, if a two-digit number has 9 tens when adding 10, then the number becomes a three-digit number with 1 hundred and the tens digit becomes zero (a place holder), for example: 10 more than 97 is 107.

In most cases, only the tens digit changes when finding 10 more or less than a four-digit number, for example: 10 more than 2716 is 2726; 10 less than 5323 is 5313. However, if a four-digit number has 9 tens or 9 tens and 9 hundreds when adding 10, or no tens when subtracting 10, then the tens and hundreds digits in the number change, for example: 10 more than 1296 is 1306; 10 more than 1996 is 2006; 10 less than 3401 is 3391.

In most cases, only the hundreds digit changes when finding 100 more or less than a four-digit number, for example: 100 more than 7452 is 7552; 100 less than 8452 is 8352. However, if a four-digit number has 9 hundreds, or 9 hundreds and 9 thousands when adding 100, or no hundreds when subtracting 100, then the hundreds and thousands digits in the number change, for example: 100 more than 2965 is 3065; 100 more than 9986 is 10086; 100 less than 3046 is 2946.

## Same and different numbers (p.33)

### Challenge

Pupils' explanations will vary depending on their depth of understanding. However, they should refer to the place value of each of the three digits in each number, and the value of the digit 4 in each representation.

Use your professional judgement when assessing pupils' reasoning.

### What if?

Other ways of expressing 738 may include:

- 6 hundreds, 13 tens and 8 ones
- 6 hundreds, 12 tens and 18 ones
- 63 tens and 108 ones
- 738 ones

Pupils' chosen three-digit number will vary.

## Greatest numbers possible (p.34)

### Challenge

Pupils' explanations will vary depending on their depth of understanding. However, they should refer to the fact that in order to make the three greatest three-digit numbers possible, they are aiming to put the digits 9, 8 and 7 in the hundreds place value, the digits 6, 5 and 4 in the tens place value, and the digits 3, 2 and 1 in the ones place value.

Use your professional judgement when assessing pupils' reasoning.

### What if?

As above. Pupils are aiming to put the largest digits in the most significant place values.

## Numbers and statements (p.35)

### Challenge

The numbers 50, 75 and 25 match only one statement.

The numbers 24, 15 and 250 match two statements.

The number 700 matches the most statements.

The following statements only match one of the numbers:

- more than 500
- less than 20
- between 30 and 40

24: multiple of 8 / multiple of 4

50: multiple of 50

300: multiple of 50 / 3-digit number / multiple of 100 / multiple of 4\*

75: odd number

15: odd number / less than 20

32: multiple of 8 / between 30 and 40 / multiple of 4

250: multiple of 50 / 3-digit number

700: multiple of 50 / more than 500 / 3-digit number / multiple of 100 / multiple of 4\*

25: odd number

\* NOTE: At this stage, many pupils will probably not recognise 300 and 700 as being multiples of 4.

### What if?

Statements and numbers will vary.

### In your head or on paper? (p.36)

#### Challenge/What if?

Results of the challenge will vary. However, answers to the eight calculations are given below. Although pupils' methods will vary, with the exception of c) and f), most pupils should be able to work out the answers mentally.

a)  $475 + 60 = 535$

b)  $942 - 9 = 933$

c)  $734 - 268 = 466$

d)  $283 + 5 = 288$

e)  $706 - 400 = 306$

f)  $358 + 576 = 934$

g)  $838 - 70 = 768$

h)  $216 + 300 = 516$

## Missing digits (p.37)

### Challenge

Pupils' explanations of how they worked out the missing digits will vary. However, the answers to the two calculations are:

$$\begin{array}{r} 427 \\ + 396 \\ \hline 823 \end{array} \qquad \begin{array}{r} 918 \\ - 563 \\ \hline 355 \end{array}$$

### What if?

Calculations will vary. One possible solution for each calculation is:

$$\begin{array}{r} 519 \\ + 264 \\ \hline 783 \end{array} \qquad \begin{array}{r} 846 \\ - 517 \\ \hline 329 \end{array}$$

## Spot the mistake (p.38)

### Challenge

Explanations of the errors will vary, however they may be similar to the following. Use your professional judgement when assessing pupils' reasoning.

Character	Error	Correct answer
Kate	Subtracted 4 ones from 7 ones, giving 3 ones, instead of subtracting 7 ones from 14 ones (after exchanging 5 tens into 4 tens and 1 ten).	$\begin{array}{r} 954 \\ - 437 \\ \hline 517 \end{array}$
Emily	Didn't carry the 1 hundred after adding 6 tens and 8 tens (i.e. 14 tens) into the hundreds place value column. Instead, wrote the 14 tens on the answer line and wrote the sum of the hundreds column in the thousands place without adding the carried 1 hundred.	$\begin{array}{r} 361 \\ + 582 \\ \hline 943 \end{array}$
Alexander	Didn't carry the 1 ten into the tens column after adding 9 ones and 8 ones (i.e. 1 ten and 7 ones).	$\begin{array}{r} 629 \\ + 358 \\ \hline 987 \end{array}$

Osaru	Exchanged 7 hundreds into 6 hundreds and 10 tens in order to subtract 2 tens from 10 tens (i.e. 8 tens), but then didn't subtract 5 hundreds from 6 hundreds, instead subtracted 5 hundreds from 7 hundreds.	$\begin{array}{r} 704 \\ - 522 \\ \hline 182 \end{array}$
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## What if?

Tips will vary but may include the following:

Kate: To look carefully at the digits in each of the numbers. For each place value, if the top digit (part of the minuend) is smaller than the digit you are subtracting (in the subtrahend), you have to 'exchange' the minuend digit in the place value column to the left, decreasing that digit by one.

Emily: Make sure to carry over any digits into the place value column to the left if the total is more than 10 ones/tens/hundreds...

Alexander: Make sure to add any carry over digits.

Osaru: When you decrease a digit in the minuend by one after 'exchanging', remember to cross out the original digit and write the new value of the minuend above in smaller handwriting.

## Estimating and checking (p.39)

### Challenge

Pupils' calculations will vary. However, for Osaru's addition calculation, the answer should be near to 600 and for the subtraction calculation, the answer should be near to 500, for example:

$$\begin{array}{r} 382 \\ + 243 \\ \hline 625 \end{array} \quad \begin{array}{r} 659 \\ - 147 \\ \hline 512 \end{array}$$

## What if?

The answers to Emily's calculations are:

$$\begin{array}{r} 455 \\ + 368 \\ \hline 823 \end{array} \quad \begin{array}{r} 613 \\ - 287 \\ \hline 326 \end{array}$$

Emily's checking calculations (if using the inverse operations) will be:

For the addition calculation:

$$\begin{array}{r} 823 \\ - 455 \\ \hline 368 \end{array} \quad \text{or} \quad \begin{array}{r} 823 \\ - 368 \\ \hline 455 \end{array}$$

For the subtraction calculation:

$$\begin{array}{r} 326 \\ + 287 \\ \hline 613 \end{array} \quad \text{or} \quad \begin{array}{r} 613 \\ - 326 \\ \hline 287 \end{array}$$

The answers and checking calculations for Osaru's calculations will vary and be dependent on pupils' calculations in the Challenge above.

## Making links (p.40)

### Challenge

Pupils' calculations and explanation will vary. However, they should in some way relate to the known fact  $4 \times 8 = 32$ , and may be similar to the following:

$$40 \times 8 = \boxed{320}$$

Given that  $4 \times 8 = 32$ , and as 40 is 10 times larger than 4, then the answer to  $40 \times 8$  must be 10 times larger than the answer to  $4 \times 8$ .

$$41 \times 8 = \boxed{328}$$

This calculation can be worked out by first finding the answer to  $40 \times 8$ , then adding another 8, i.e.  $41 \times 8 = (40 \times 8) + 8$ .

$$32 \div \boxed{4} = 8$$

The inverse relationship between multiplication and division can be applied to finding the value of the missing number.

$$4 \times 16 = \boxed{64}$$

Given that  $4 \times 8 = 32$ , and as 16 is twice (double) 8, then the answer to  $4 \times 16$  must be twice (double) the answer to  $4 \times 8$ .

$$64 \div 4 = \boxed{16}$$

As 64 is twice (double) 32, and applying the inverse relationship between multiplication and division, if  $32 \div 4 = 8$ , then  $64 \div 4 = 16$  (twice/double 8).

$$80 \times 4 = \boxed{320}$$

By applying the commutative law for multiplication (i.e.  $4 \times 8 = 8 \times 4$ ) and given that  $4 \times 8 = 32$ , and as 80 is 10 times larger than 8, then the answer to  $80 \times 4$  must be 10 times larger than the answer to  $8 \times 4$  (or  $4 \times 8$ ).

$$320 \div 8 = \boxed{40}$$

Using the inverse relationship between multiplication and division, and as 320 is 10 times larger than 32, then the answer to  $320 \div 8$  must be 10 times larger than the answer to  $32 \div 8$ .

## What if?

Calculations will vary but may include the following:

$30 \times 6 = 180$

$18 \div 3 = 6$

$3 \times 60 = 180$

$18 \div 6 = 3$

$30 \times 60 = 1800$

$180 \div 3 = 60$

$31 \times 6 = 186$

$180 \div 6 = 30$

$32 \times 6 = 192$

$180 \div 30 = 6$

$3 \times 61 = 183$

$180 \div 60 = 3$

$3 \times 62 = 186$

## Missing numbers and digits (p.41)

### Challenge

×	60	7
4	240	28

	2	4	0
+		2	8
	2	6	8

×	50	6
8	400	48

	4	0	0
+		4	8
	4	4	8

Pupils' explanations for finding the missing numbers will vary.

## What if?

	6	<input type="text" value="5"/>	
×		<input type="text" value="3"/>	
	1	9	5

	<input type="text" value="4"/>	9	
×		<input type="text" value="8"/>	
	3	9	2

	<input type="text" value="6"/>	<input type="text" value="3"/>	
×		4	
	2	5	2

Pupils' explanations for finding the missing digits will vary.



## True or false? (p.42)

### Challenge

Emily's statement is true: all the answers to the 8 times table are also answers to the 2 times table, i.e.

8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, ...

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, ...

Osaru's statement is true: all the answers to the 4 times table are also answers to the 2 times table, i.e.

4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, ...

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, ...

Alexander's statement is false: not all the answers to the 2 times table are also answers to the 4 times table, i.e.

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, ...

4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, ...

Kate's statement is true: all the answers to the 8 times table are also answers to the 4 times table, i.e.

8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, ...

4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, ...

### What if?

Emily's statement is true: all the answers to the 2, 4 and 8 times tables are even numbers.

2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, ...

4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, ...

8, 16, 24, 32, 40, 48, 56, 64, 72, 80, 88, 96, ...

## Double multiplications (p.43)

### Challenge

If you double the first number, using the calculation given, i.e.  $4 \times 2 = 8$ , the pattern continues:

$$8 \times 2 = 16$$

$$16 \times 2 = 32$$

$$32 \times 2 = 64$$

$$64 \times 2 = 128$$

$$128 \times 2 = 256$$

New answers are always twice (double) the previous answer.

### What if?

If you double the second number, using the calculation given, i.e.  $4 \times 2 = 8$ , the pattern continues:

$$4 \times 4 = 16$$

$$4 \times 8 = 32$$

$$4 \times 16 = 64$$

$$4 \times 32 = 128$$

$$4 \times 64 = 256$$

New answers are always twice (double) the previous answer. This is the same as when the first number is doubled (see above).

If you double both numbers, using the calculation given, i.e.  $4 \times 2 = 8$ , the pattern continues:

$$8 \times 4 = 32$$

$$16 \times 8 = 128$$

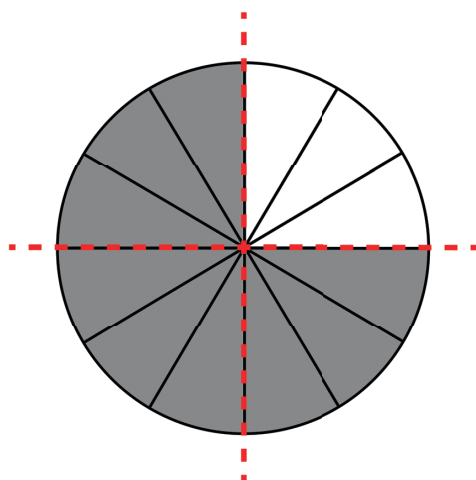
$$32 \times 16 = 512$$

New answers are always four times the previous answer.

## Fractions of numbers (p.44)

### Challenge

Pupils' observations and pictures/diagrams will vary. However, they may be similar to the following. Use your professional judgement when assessing pupils' reasoning.



Osaru is right – 9 is  $\frac{3}{4}$  of 12.

Other pairs of numbers where one number is three-quarters of the other may include:

- 3 is  $\frac{3}{4}$  of 4
- 6 is  $\frac{3}{4}$  of 8
- 12 is  $\frac{3}{4}$  of 16
- 15 is  $\frac{3}{4}$  of 20

The first numbers are all multiples of 3; the second numbers are all multiples of 4.

### What if?

Pairs of numbers where one number is two-thirds of the other may include:

- 2 is  $\frac{2}{3}$  of 3
- 4 is  $\frac{2}{3}$  of 6
- 6 is  $\frac{2}{3}$  of 9
- 8 is  $\frac{2}{3}$  of 12

The first numbers are all multiples of 2; the second numbers are all multiples of 3.

Pairs of numbers where one number is three-fifths of the other may include:

- 3 is  $\frac{3}{5}$  of 5
- 6 is  $\frac{3}{5}$  of 10
- 9 is  $\frac{3}{5}$  of 15
- 12 is  $\frac{3}{5}$  of 20

The first numbers are all multiples of 3; the second numbers are all multiples of 5.

Pairs of numbers where one number is four-fifths of the other may include:

- 4 is  $\frac{4}{5}$  of 5
- 8 is  $\frac{4}{5}$  of 10
- 12 is  $\frac{4}{5}$  of 15
- 16 is  $\frac{4}{5}$  of 20

The first numbers are all multiples of 4; the second numbers are all multiples of 5.

Pairs of numbers where one number is seven-tenths of the other may include:

- 7 is  $\frac{7}{10}$  of 10
- 14 is  $\frac{7}{10}$  of 20
- 21 is  $\frac{7}{10}$  of 30
- 28 is  $\frac{7}{10}$  of 40

The first numbers are all multiples of 7; the second numbers are all multiples of 10.

## Missing numerators and denominators (p.45)

### Challenge

Pupils' explanations will vary. Use your professional judgement when assessing pupils' reasoning.

$$\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$$

$$\frac{5}{6} - \frac{2}{6} = \frac{3}{6}$$

$$\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$$

$$\frac{6}{8} - \frac{1}{8} = \frac{5}{8}$$

$$\frac{4}{10} + \frac{3}{10} = \frac{7}{10}$$

$$\frac{7}{9} - \frac{5}{9} = \frac{2}{9}$$

### What if?

$$\frac{1}{9} + \frac{8}{9} = 1$$

$$1 - \frac{1}{7} = \frac{6}{7}$$

$$\frac{2}{9} + \frac{7}{9} = 1$$

$$1 - \frac{2}{7} = \frac{5}{7}$$

$$\frac{3}{9} + \frac{6}{9} = 1$$

$$1 - \frac{3}{7} = \frac{4}{7}$$

$$\frac{4}{9} + \frac{5}{9} = 1$$

$$1 - \frac{4}{7} = \frac{3}{7}$$

$$\frac{5}{9} + \frac{4}{9} = 1$$

$$1 - \frac{5}{7} = \frac{2}{7}$$

$$\frac{6}{9} + \frac{3}{9} = 1$$

$$1 - \frac{6}{7} = \frac{1}{7}$$

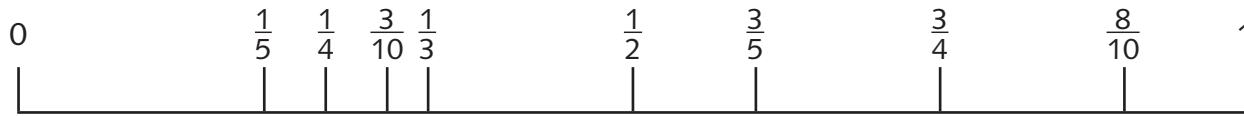
$$\frac{7}{9} + \frac{2}{9} = 1$$

$$\frac{8}{9} + \frac{1}{9} = 1$$

## Another, another and another (p.46)

### Challenge

Pupils' fractions and number lines will vary. However, they may look something similar to the following:



### What if?

Pupils' equivalent fractions will depend on the fractions chosen in the main part of the Challenge. However, equivalent fractions for those fractions used above may include:

$$\frac{1}{5} = \frac{2}{10}$$

$$\frac{1}{4} = \frac{3}{12}$$

$$\frac{3}{10} = \frac{6}{20}$$

$$\frac{1}{3} = \frac{2}{6}$$

$$\frac{1}{2} = \frac{5}{10}$$

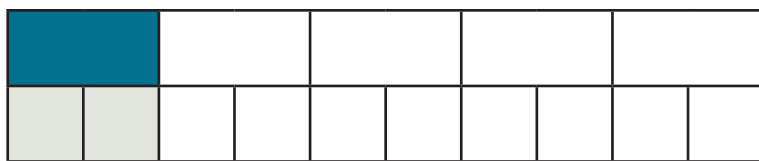
$$\frac{3}{5} = \frac{6}{10}$$

$$\frac{3}{4} = \frac{6}{8}$$

$$\frac{8}{10} = \frac{4}{5}$$

Pupils' pictures or diagrams will also vary. However, they may be represented similar to the following:

$$\frac{1}{5} = \frac{2}{10}$$



## Same and different lengths (p.47)

### Challenge

Pupils' observations about the similarities and differences between the lengths will vary. However, they may include the following:

- All metre lengths: 3.5 m and  $3\frac{1}{2}$  m
- All centimetre lengths: 35 cm and 350 cm
- All millimetre lengths: 35 mm and 350 mm

- Centimetre and millimetre length: 3 cm 50 mm
- 3.5 m,  $3\frac{1}{2}$  m and 350 cm are all the same measurement expressed in different ways/units.
- 35 cm and 350 mm are the same measurement expressed in different units.
- 35 mm and 3 cm 5 mm are the same measurement expressed in different units.

## What if?

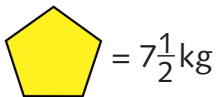
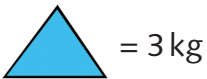
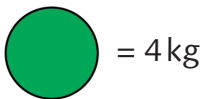
The order of the measures, starting with the smallest is: 40 mm, 4.5 cm, 5 cm and 300 mm.

Pupils' suggestions for measures (including units of measure) that are between pairs of measures will vary. One example is:

40 mm, 4.2 cm, 4.5 cm, 48 mm, 5 cm, 20 cm, 300 mm

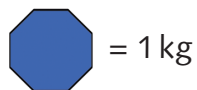
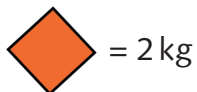
## Balance problems (p.48)

### Challenge



Pupils' explanations will vary. Use your professional judgement when assessing pupils' reasoning.

### What if



Pupils' explanations will vary. Use your professional judgement when assessing pupils' reasoning.

## Volume order (p.49)

### Challenge

2 litres, 1.75 litres,  $1\frac{1}{2}$  litres, 1000 ml,  $\frac{3}{4}$  litre, 0.5 litre, 330 ml, 300 ml,  $\frac{1}{4}$  litre, 150 ml

Pupils' explanations will vary. Use your professional judgement when assessing pupils' reasoning.

### What if?

Pupils' statements will vary.

## All the same time? (p.50)

### Challenge

Alexander is right. Without a.m. or p.m. notation, it is not possible to know whether the first (12-hour analogue clock with Roman numerals), second (12-hour digital clock) and third (12-hour analogue clock with Hindu-Arabic numerals) clocks show 25 minutes past 10 in the morning or at night, or whether the last clock (24-hour analogue clock) is also displaying time in the morning or at night.

### What if?

Pupils' suggestions will vary. However, it should include some reference to showing a.m. or p.m. notation, or in some way indicating that the time is either morning or evening.

## Seaside shopping (p.51)

### Challenge

Assuming that each character did not buy more than one of the same item then:

Kate could have bought a rubber tube and a pair of sunglasses, or she could have bought a fishing net and a beach ball.

Emily bought a bucket, spade, fishing net and a pair of sunglasses.

Alexander bought a fishing net, rubber tube and a beach ball.

Osaru could have bought a bucket, spade and a beach ball, or he could have bought a spade, fishing net and a pair of sunglasses.

The characters could have bought more than one of the same item. If pupils have based their answers on one or more of the characters buying more than one of the same item, then check their answers and explanations.

Explanations will vary. Use your professional judgement when assessing pupils' reasoning.

## What if?

- Assuming that each character did not buy more than one of the same item, then Kate and Osaru could have bought the same number of items, and spent the same amount of money, but have bought different items (see above).
- Many different answers are possible for the characters buying a different number of items but still spending the same amount of money. Check pupils' answers and explanations.

As above, for both 'What if?' questions, the characters could have bought more than one of the same item. If pupils have based their answers on one or more of the characters buying more than one of the same item, then check their answers and explanations.

## Angles in shapes (p.52)

### Challenge

Shapes that only have right angles are: square and rectangle.

Shape that only has angles less than a right angle: equilateral triangle.

Shapes that only have angles greater than a right angle are: pentagon, hexagon, heptagon and octagon.

Shape that has right angles, and angles less or greater than a right angle: right-angled triangle.

Shape with no angles: circle.

## What if?

Emily's statement is true: in a polygon, the number of its sides is always the same as the number of its angles.

Alexander's statement is false: the angles in the two green shapes are not the same, despite the fact that both shapes are triangles. For the right-angled triangle (second row), one angle is a right angle ( $90^\circ$ ), and the other two angles are less than a right angle ( $45^\circ$ ). For the equilateral triangle (top row), all the angles are the same and less than a right angle ( $60^\circ$ ).

NOTE: At this stage, pupils are not expected to know that angles are measured in degrees, nor identify and name acute and obtuse angles. However, they should be able to identify right angles and whether an angle is greater than or less than a right angle.

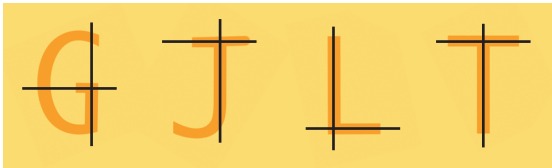


## Perpendicular and parallel letters (p.53)

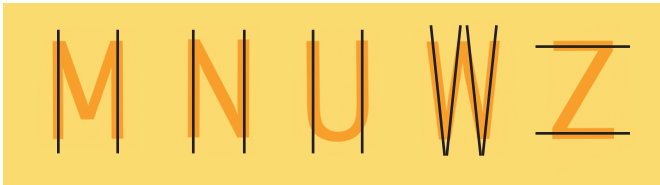
### Challenge

Answers will vary depending on the font used or the child's handwriting, however, generally:

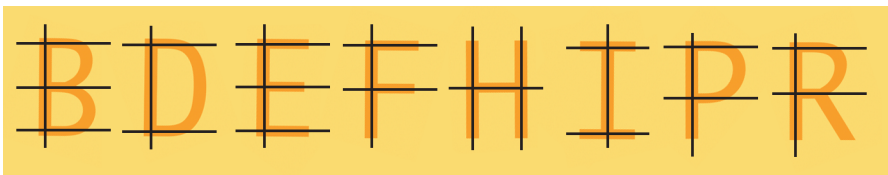
- the following letters have only perpendicular lines: **G, J, L, T**



- the following letters have only parallel lines: **M, N, U, W, Z**



- the following letters have both perpendicular and parallel lines: **B, D, E, F, H, I, P, R**

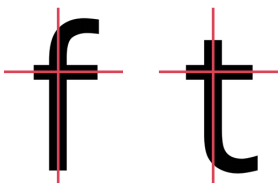


- the following letters have neither perpendicular nor parallel lines: **A, C, K, O, Q, S, V, X, Y**

### What if?

Once again, answers will vary depending on the font used or the child's handwriting, however, generally:

- the following letters have only perpendicular lines: **f, t**



- the following letters have only parallel lines: **h, m, n, u, w, z**



- the following letters have both perpendicular and parallel lines: None
- the following letters have neither perpendicular nor parallel lines: **a, b, c, d, e, g, i, j, k, l, o, p, q, r, s, v, x, y**

## Who belongs where? (p.54)

### Challenge

Jessica: A or D

Lucy: C or F

Matt: A or D

Oscar: B or G

Pari: B or G

Sophie: C or F

Vihaan: E

	Has brown eyes	Does not have brown eyes
Is less than 20 years old	E Vihaan	A D Matt / Jessica
Is more than 20 years old	B / G Pari / Oscar	C F Lucy / Sophie

### What if?

Jessica: K

Lucy: H, J or N

Matt: I or L

Oscar: M

Pari: H, J or N

Sophie: H, J or N

Vihaan: I or L

	Female	Male
Is less than 20 years old	K Jessica	I L Matt / Vihaan
Is more than 20 years old	H J N Lucy / Pari / Sophie	M Oscar

Jessica: P, R or U

Lucy: P, R or U

Matt: O

Oscar: Q or T

Pari: S

Sophie: P, R or U

Vihaan: Q or T

	Has brown eyes	Does not have brown eyes
Female	S Pari	P R U Lucy / Sophie / Jessica
Male	Q T Vihaan / Oscar	O Matt

## Chart story telling (p.55)

### Challenge/What if?

Pupils' interpretations and conclusions of the bar chart and pictogram will vary. Use your professional judgement when assessing pupils' reasoning.

# Using and applying mathematics in real-world contexts

## Numbers in newspapers (p.56)

### Challenge/What if?

Results of the challenge will vary.

## Binary numbers (p.57)

### Challenge

Hindu-Arabic number	Binary number
11	1011
12	1100
13	1101
14	1110
15	1111
16	10000
17	10001
18	10010
19	10011
20	10100

512	256	128	64	32	16	8	4	2	1	Hindu-Arabic number
									1	1
								1	0	2
								1	1	3
							1	0	0	4
							1	0	1	5
							1	1	0	6
							1	1	1	7
						1	0	0	0	8
						1	0	0	1	9
						1	0	1	0	10
						1	0	1	1	11
						1	1	0	0	12
						1	1	0	1	13
						1	1	1	0	14
						1	1	1	1	15
					1	0	0	0	0	16
					1	0	0	0	1	17
					1	0	0	1	0	18
					1	0	0	1	1	19
					1	0	1	0	0	20
binary number										

### What if?

Results of the challenge will vary.

## Codes (p.58)

### Challenge

There are many different possibilities for designing a pattern where the secret code is 4 9 3 5 4 6, for example:

1	2	3	4	5	6	
						4
						9
						3
						5
						4
						6

1	2	3	4	5	6	
						4
						9
						3
						5
						4
						6

1	2	3	4	5	6	
						4
						9
						3
						5
						4
						6

Pupils' patterns will vary.

### What if?

Pupils' codes will vary.

## Half-time scores (p.59)

### Challenge

Firstly, pupils need to realise that the half-time scores and the full-time scores could be exactly the same, i.e. either team may not have scored any further goals in the second half of the match.

For a full-time score of 3 – 3, there are 16 different possible half-time scores:

0 – 0	1 – 0	2 – 0	3 – 0
0 – 1	1 – 1	2 – 1	3 – 1
0 – 2	1 – 2	2 – 2	3 – 2
0 – 3	1 – 3	2 – 3	3 – 3

## What if?

For a full-time score of 5 – 4, there are 30 different possible half-time scores:

0 – 0	1 – 0	2 – 0	3 – 0	4 – 0	5 – 0
0 – 1	1 – 1	2 – 1	3 – 1	4 – 1	5 – 1
0 – 2	1 – 2	2 – 2	3 – 2	4 – 2	5 – 2
0 – 3	1 – 3	2 – 3	3 – 3	4 – 3	5 – 3
0 – 4	1 – 4	2 – 4	3 – 4	4 – 4	5 – 4

For a full-time score of 2 – 1, there are 6 different possible half-time scores:

0 – 0	1 – 0	2 – 0
0 – 1	1 – 1	2 – 1

The rule for working out how many different half-time scores there are using just the final score is to add 1 to each full-time score and multiply the two totals together, i.e.

For a full-time score of 3 – 3:  $(3 + 1) \times (3 + 1) = 16$

For a full-time score of 5 – 4:  $(5 + 1) \times (4 + 1) = 30$

For a full-time score of 2 – 1:  $(2 + 1) \times (1 + 1) = 6$

## Dice totals (p.60)

### Challenge

#### Two 1–6 dice

The most common total when rolling two 1–6 dice is 7. This is because there are six different combinations that total 7 (i.e. 1 + 6, 2 + 5, 3 + 4, ... 6 + 1) – more than for any other total.

The least common totals when rolling two 1–6 dice are 2 and 12. This is because there is just one combination that totals 2 (i.e. 1 + 1) and one combination that totals 12 (i.e. 6 + 6) – less than for any other totals.

#### Two 1–10 dice

The most common total when rolling two 1–10 dice is 11. This is because there are ten different combinations that total 11 (i.e. 1 + 10, 2 + 9, 3 + 8, ... 10 + 1) – more than for any other total.

The least common totals when rolling two 1–10 dice are 2 and 20. This is because there is just one combination that totals 2 (i.e. 1 + 1) and one combination that totals 20 (i.e. 10 + 10) – less than for any other totals.

**Two 0–9 dice**

The most common total when rolling two 0–9 dice is 9. This is because there are 10 different combinations that total 9 (i.e.  $0 + 9$ ,  $1 + 8$ ,  $2 + 7$ , ...  $9 + 0$ ) – more than for any other total.

The least common totals when rolling two 0–9 dice are 0 and 18. This is because there is just one combination that totals 0 (i.e.  $0 + 0$ ) and one combination that totals 18 (i.e.  $9 + 9$ ) – less than for any other totals.

**What if?****Two 1–12 dice**

The most common total when rolling two 1–12 dice is 13. This is because there are 12 different combinations that total 13 (i.e.  $1 + 12$ ,  $2 + 11$ ,  $3 + 10$ , ...  $12 + 1$ ) – more than for any other total.

The least common totals when rolling two 1–12 dice are 2 and 24. This is because there is just one combination that totals 2 (i.e.  $1 + 1$ ) and one combination that totals 24 (i.e.  $12 + 12$ ) – less than for any other totals.

**Three 1–6 dice**

The most common totals when rolling three 1–6 dice are 10 and 11. This is because there are 27 different combinations that total 10 and 27 different combinations that total 11 – more than for any other total.

The least common totals when rolling three 1–6 dice are 3 and 18. This is because there is just one combination that totals 3 (i.e.  $1 + 1 + 1$ ) and one combination that totals 18 (i.e.  $6 + 6 + 6$ ) – less than for any other totals.

**Dice operations (p.61)****Challenge/What if?**

Results of the challenge will vary.

**Fractions of a newspaper (p.62)****Challenge/What if?**

Results of the challenge will vary.

**Tailor made (p.63)****Challenge/What if?**

Results of the challenge will vary.

### Walking (p.64)

#### Challenge/What if?

Results of the challenge will vary.

### In a minute (p.65)

#### Challenge/What if?

Results of the challenge will vary. However, pupils should realise that their estimates for 1 minute (interlocking cubes) and 5 minutes (water) will be fairly accurate, but their estimates for 10 minutes will be less accurate.

### Water usage (p.66)

#### Challenge/What if?

Results of the challenge will vary.

### Workout (p.67)

#### Challenge/What if?

Results of the challenge will vary.

### Youngest and oldest (p.68)

#### Challenge/What if?

Results of the challenge will vary.



## Calendar events (p.69)

### Challenge

The design of the pupils' calendars will vary, however the information contained in it should be as follows:

July, 2017						
Monday	Tuesday	Wednesday	Thursday	Friday	Saturday	Sunday
24	25	26	27	28	29 Kate's and Sarah's birthday	30
August, 2017						
31	1	2 Arrived in Nice	3 Went to the beach	4 Wrote letter to Aunty Joan and went to the beach	5 Picnic	6
7	8	9	10	11	12 Go to the mountains	13 Sarah and her parents arrive
14	15	16	17	18 Leave the mountains for Nice	19	20
21 Fly back home	22	23	24	25	26	27
28	29	30	31	1	2	3
September, 2017						
4 Back to school	5	6	7	8	9	10

### What if?

Kate was born on 29 July 2009.

Sarah was born on 29 July 2005.

Pupils' explanations as to how they worked out on which date Kate and Sarah were born will vary.

### School road (p.70)

#### Challenge/What if?

Results of the challenge will vary.

### Public transport (p.71)

#### Challenge/What if?

Results of the challenge will vary.

### Clothes 4-U (p.72)

#### Challenge/What if?

Results of the challenge will vary.

### School perimeter (p.73)

#### Challenge/What if?

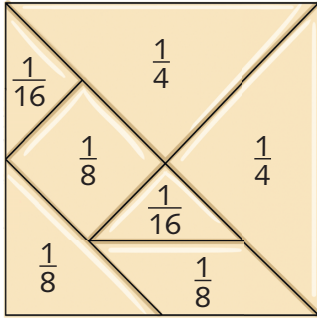
Pupils' plans and perimeters will vary. Use your professional judgement when assessing a child's accuracy in drawing and calculating the perimeter.

### Investigating tangrams (p.74)

#### Challenge

Results of the challenge will vary.

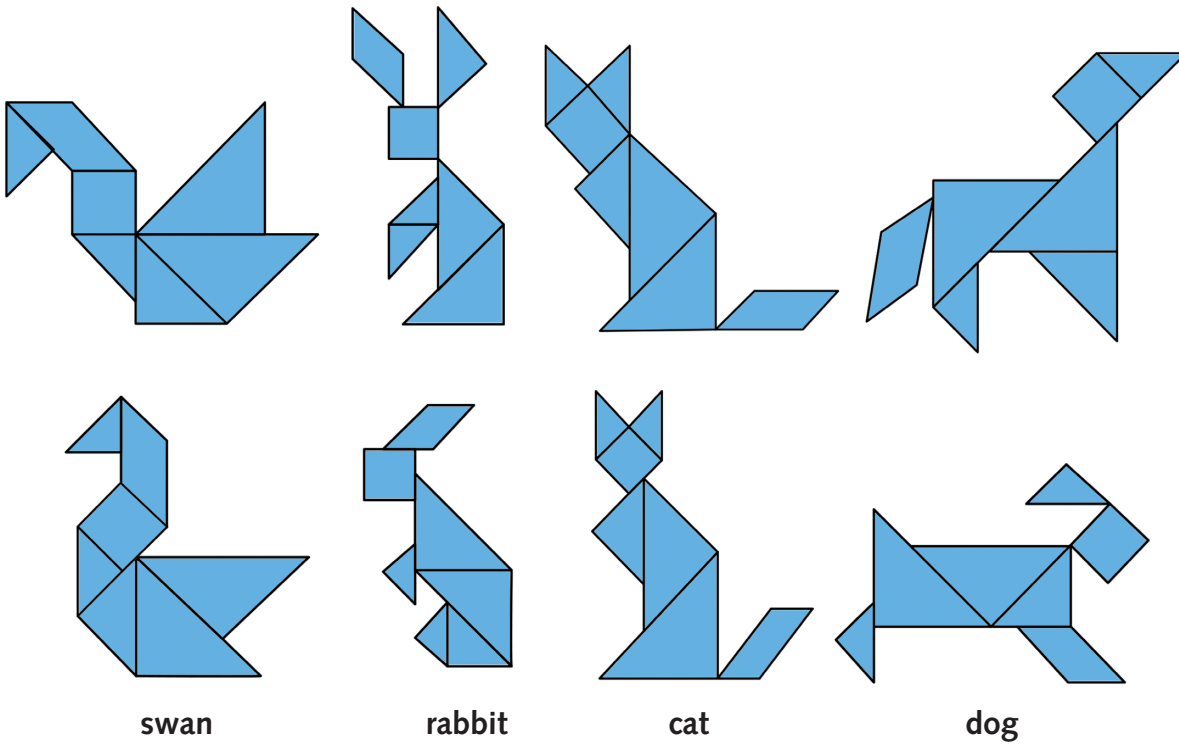
What if?



Tangram animals (p.75)

Challenge

Pupils' animals may vary slightly, for example:



Pupils' other animals and objects will vary.

What if?

Pupils' tangrams will vary.

### Mazes (p.76)

#### Challenge/What if?

Pupils' sets of directions will vary.

### Right angle programmes (p.77)

#### Challenge/What if?

Results of the challenge will vary.

### Home-school directions (p.78)

#### Challenge/What if?

Results of the challenge will vary.

### Animal classification (p.79)

#### Challenge/What if?

Results of the challenge will vary.