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Engineering physics AQA A-level Year 2

Chris Gidzewicz

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This optional topic is part of the Collins AQA A-Level Physics Year 2 Student Book.

10 9 8 7 6 5 4 3 2 1

ISBN 978-0-00-759764-2

A catalogue record for this book is available from the British Library

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Proofread by Mitch Fitton and Sue Glover
Artwork and typesetting by Jouve
Cover design by We are Laura

The publisher would like to thank Sue Glover and Peter Robinson.

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ENGINEERING PHYSICS

Why do cats fall on their feet? They seem to have a special ability to orient themselves in a fall, which allows them to avoid serious injury. In cities, there is a good chance of cats falling from high-rise flats when they are distracted by passing birds. According to vets in New York, cats have a 90% survival rate if they fall from a height of six storeys. A person who falls from that height certainly would not have that chance of survival. Cats have a unique skeletal structure – their backbone is very flexible and they have no collarbone. This allows a falling cat to change its shape by stretching out its legs and tail (see figure). This position reduces the cat's terminal velocity, like a parachute. But the ability to alter its shape also, crucially, allows the cat to alter its rotational motion.

Angular momentum is conserved for an object on which no external turning force acts. The angular momentum is the product of the object's angular velocity and its 'moment of inertia' – which is an inertial resistance to rotational motion, just as mass is an inertial resistance to linear motion. As the cat falls, spinning, it can increase its moment of inertia by 'spreading out' its body mass. The conservation of angular momentum means that this will reduce its angular velocity. Together with the lower terminal linear velocity, this allows the cat time to control which parts will touch the ground first.

The final 'twist' is that the time to fall six or seven storeys allows the cat to 'unwind' and relax just before landing – being relaxed is the best way to avoid injury. So, if your cat falls from a height, use your knowledge of motion to calm yourself!

An understanding of rotational motion is essential in engineering. Engineers constantly need to improve the efficiency of their production and use of energy, and minimise the impact on our environment. Improved designs for engines, power station rotors and turbines are vital (see figure).

Cats have unusual control over their rotational motion.

Background *The technology of wind turbines relies on experts in rotational dynamics.*

1 ROTATIONAL DYNAMICS

PRIOR KNOWLEDGE

In earlier studies (*in Chapter 9 of Year 1 Student Book*), you learned about the motion of objects in straight lines. You applied the physics of linear dynamics to situations and learned how to analyse the motion graphically. You solved problems to obtain quantities such as displacement, velocity and acceleration using the equations of linear motion. You have also (*in Chapter 11 of Year 1 Student Book*) applied Newton's laws of motion to moving bodies and learned about the conservation of momentum. In Chapter 1 of this book, you learned about the motion of an object in a circle, angular measure and angular velocity.

LEARNING OBJECTIVES

In this chapter you will consider objects that are rotating (spinning). You will learn about the significance of the moment of inertia of an extended object or a system, and how to calculate rotational kinetic energy, analyse rotational motion graphically and apply the mathematics of angular measure in the use of the equations of rotational motion. You will consider the relationship between torque and angular acceleration, and calculate the power developed by rotating objects. You will learn that angular momentum, like linear momentum, is conserved, and consider applications of this.

(Specification 3.11.1.1 to 3.11.1.6)

1.1 THE MATHEMATICS OF CIRCULAR MOTION

In order to build up the physics of a rotating object, we first need to consider the motion of a point particle of that object, which will be moving in a circle. When a particle P moves in a circular path (Figure 1), it has an **angular velocity** ω given by the change in **angular displacement** θ (in **radians**) of the radius per unit time:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

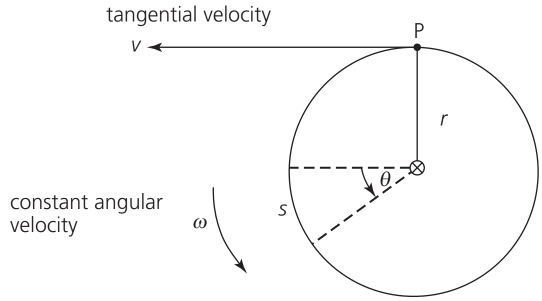

Figure 1 *A particle P moving in a circular path*

The unit of ω is rad s^{-1}. One radian (see section 1.1 of Chapter 1) is the angle θ at the centre of a circle when the arc length s is equal to the radius r (and so $s = r[\theta]$). There are 2π radians in a complete circle, that is, when the arc length is equal to the circumference. So 2π radians corresponds to 360°.

Another common angular measure in rotational dynamics is the **revolution**, or rev. One revolution is equivalent to 2π radians. As we are dealing with rotating objects, as well as engines later on (Engineering Physics Chapter 2), they are usually expressed in **rpm** (revolutions per minute) rather than per second. Converting rpm to rad s^{-1} is often required in problems.

The period of rotation (T) can be determined from the angular velocity by the relationship

$$T = \frac{2\pi}{\omega}$$

Worked example

Convert an angular velocity of 100 rpm to radians per second. What is the period of rotation?

Angular velocity: 100 rpm $= \dfrac{100 \times 2\pi}{60} = 10.5\,\text{rad s}^{-1}$

Period: $T = \dfrac{2\pi}{\omega}$ gives us $T = \dfrac{2\pi}{10.5} = 0.60\,\text{s}$

QUESTIONS

1. A spinning top rotates 30 times in 1 s. Express this in rpm and also in rad s^{-1}. If it spun at the same rate for 1 minute, how many radians would it have turned through?

A particle moving in a circular path also has a linear velocity, or **tangential velocity**. This tangential velocity v of the particle P is related to the angular velocity ω by the expression

$$v = r\omega$$

where r is the radius of the circle. The units for the velocity and radius will be related through this equation. If r is measured in m, then the velocity will be in m s^{-1}. However, if r is expressed in cm, then the tangential velocity will be in cm s^{-1}.

The fact that, for a particular angular velocity, the tangential velocity is proportional to the radius can be seen quite clearly by spinning a CD or vinyl record on a pencil (Figure 2). By drawing a series of dots along a radius, you can see the motion of each one separately. The further out, the faster they move.

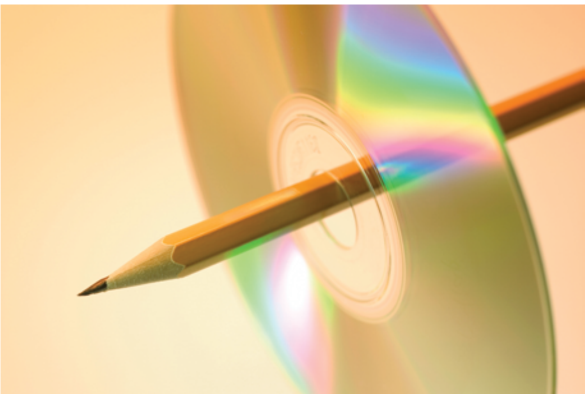

Figure 2 *The further the distance from the axis, the faster is the linear velocity.*

Angular acceleration

An **angular acceleration**, symbol α, arises when a particle moving in a circular path increases or decreases its speed of rotation. It is defined as the rate of change of angular velocity:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

The unit of α is rad s^{-2}.

Note that angular acceleration is *not* to be confused with the concept of **centripetal acceleration**, which you covered in Chapter 1. In section 1.1 of Chapter 1, the angular velocity was constant (an object orbiting at a constant speed, for example). However, in all circular motion, as the (tangential) velocity vector is continually changing direction, there is an acceleration, which is directed *towards the centre of the circle*. This is the centripetal acceleration, a linear acceleration, in m s^{-2} (such as the acceleration due to gravity, which allows a satellite to orbit the Earth). Here, we are considering the case where the angular velocity of a particle is *not* constant, so there is an *angular* acceleration, in rad s^{-2}.

An increase in angular velocity, giving rise to an angular acceleration, must be accompanied by a corresponding increase in the *tangential* velocity (because the edge velocity is also increasing), and hence a **tangential acceleration** a (because the edge velocity is also increasing). The relation between these two quantities is given by

$$a = r\alpha$$

As before, r is the radius of the circle. In the same way, the units for both the tangential acceleration and the radius will use the same measure. If r is measured in m, then the tangential acceleration will also be in m s^{-2}.

QUESTIONS

2. Vinyl records have been making a comeback over the past few decades (Figure 3). These have a diameter of 30.0 cm and rotate at $33\frac{1}{3}$ rpm, which is fixed, unlike for a CD or DVD. The stylus is placed on the record at the edge and works its way inwards on a spiral track.

Figure 3 A vinyl record. The stylus tracks the groove inwards from the edge.

a. Calculate the angular velocity of any point on the record in rad s^{-1}.

b. What is the tangential velocity of a point on the outer edge of the record?

c. The stylus will lift up when 6.0 cm from the centre. What is the tangential velocity of a point at that distance?

3. A record is placed on a turntable and reaches $33\frac{1}{3}$ rpm in 2.0 s. When turned off, it comes to a halt in 5.0 s. Use some of your answers from question 2 to calculate the following:

a. the initial angular acceleration of the record

b. the tangential acceleration at the edge

c. the angular deceleration of the record when the power to the motor is cut and it slows

d. the tangential deceleration on the record at the point where the stylus is raised up.

KEY IDEAS

- Angular velocity is defined as the rate of change of angular displacement:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

Its unit is rad s^{-1}.

- The tangential and angular velocities are related by $v = r\omega$, where v is in m s^{-1}.

- Angular acceleration is the rate of change of angular velocity:

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

Its unit is rad s^{-2}.

- The tangential and angular accelerations are related by $a = r\alpha$, where a is in m s^{-2}.

1.2 THE CONCEPT OF MOMENT OF INERTIA

Consider a rigid solid body that is free to rotate (see Figures 4 and 5). The force F is *not* acting through the centre of mass in either case. As a consequence, there will be a torque (or moment) causing the solid body to rotate around the fixed axis.

A **torque** (symbol T) is the product of a force and the perpendicular distance of the line of action of the force from a particular point or axis of rotation (see section 10.4 in Chapter 10 in Year 1 Student Book). In Figure 4, torque $T = Fs$. The unit of torque is newton metre (N m). The convention adopted is that positive torques produce a clockwise rotation.

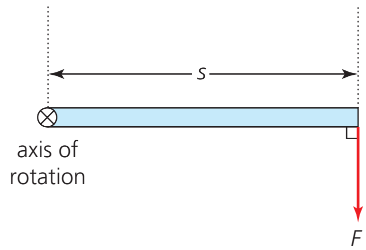

Figure 4 A force producing a turning effect

Now let us look at the effect of this torque on one of the point masses (m), a distance r from the axis that make up the extended rigid object in Figure 5.

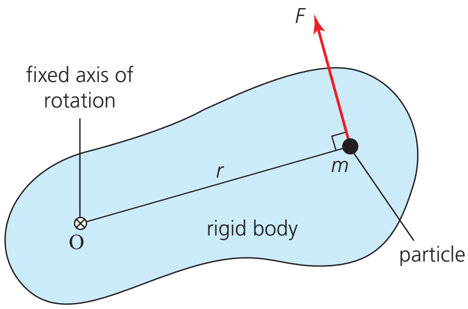

Figure 5 Newton's second law as applied to a particle in a rigid body

The force F produces a linear acceleration of the particle:

$$F = m \times a$$

As $a = r \times \alpha$, where α is the angular acceleration, substituting gives

$$F = m \times r \times \alpha$$

The torque acting on the particle is $F \times r$, so torque is given by

$$T = m \times r \times \alpha \times r$$

$$T = m \times r^2 \times \alpha$$

This equation is the angular version of Newton's second law, $F = ma$.

Linear version: $F = m \times a$

Rotational version: $T = mr^2 \times \alpha$

You can see that force is replaced by torque, linear acceleration by angular acceleration, but notice that the mass term m is replaced by mr^2. In rotational motion, it is necessary to include as significant the *distance* of the particle from the axis of rotation.

The term **inertia** is used to describe the resistance of a physical object to a change in its state of motion. When a force is applied, acceleration will occur, but the magnitude will be controlled by its mass ($a = F/m$). On the Moon, although objects weigh less (because of the Moon's lower gravitational field strength, see Chapter 4), they have the same mass and so would be equally difficult to start (or stop) moving, because $F = ma$ still applies.

Similarly, an object has inertial resistance when it is being forced to *rotate*. The object's **moment of inertia** (symbol I) is a measure of the extent to which the object resists being *rotationally accelerated* about a *certain axis*.

For one particle, of mass m, that is a distance r from an axis of rotation, the moment of inertia is

$$I = mr^2$$

The unit of I is kg m^2.

Real objects are made up of a large number of such particles, all at different distances from the axis of rotation, so a complete description of the rotational motion requires an analysis of the distribution of the mass of all the particles. In effect, we need to add together all the individual moments of inertia around the particular axis.

Consider a two-dimensional rigid body (a lamina) that is rotating about some fixed axis, say O, at a constant angular velocity ω, as shown in Figure 6. The rigid body is composed of many point masses, such as the one, P, shown with mass m_1, which is at a distance r_1 from the axis of rotation.

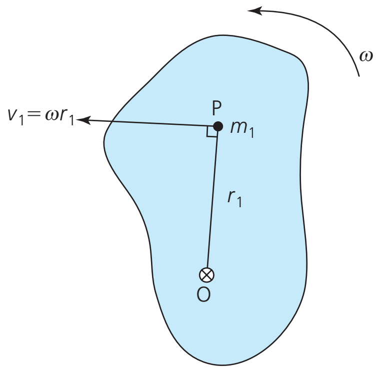

Figure 6 *A rigid lamina that is rotating about an axis at O perpendicular to the page*

We need to sum the product of each individual point mass with the square of its distance from the point of rotation:

$$m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + \cdots = \sum m_ir_i^2$$

where $i = 1, 2, 3, \ldots$; the mass of the elemental particle i is m_i, and r_i is its distance from the axis of rotation. The summation (represented by the symbol $\sum$) is taken over the whole of the body. This composite quantity is known as the moment of inertia, I, of the whole object:

$$I = \sum mr^2$$

Figure 7 shows the resultant equations for the moment of inertia, derived using the summation method above, for some three-dimensional objects of regular shape. Remember that the precise location of the axis of rotation will affect the value of the moment of inertia, as the mass distribution around it may change considerably, so altering the moment of inertia.

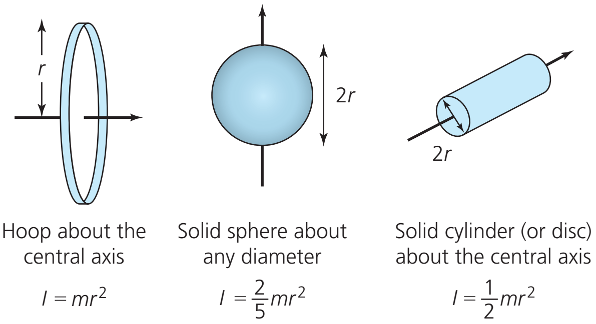

Figure 7 *Some examples of moments of inertia. Here m denotes the total mass and r is the dimension shown.*

A common mechanical device used is a **flywheel**, which is basically a disc or wheel that can rotate rapidly. As the flywheel spins, it stores energy (kinetic energy of rotation, see Engineering Physics section 1.6), which can be released and used subsequently. The greater the moment of inertia of a rotating object, the more energy is stored – in much the same way that the greater the mass of an object, the greater its translational kinetic energy. So a hoop ($I = mr^2$) will be better than a disc ($I = \frac{1}{2}mr^2$) at storing more rotational kinetic energy when they are both rotating at the same angular velocity.

Worked example

A hoop has a diameter of 70 cm and a mass of 300 g. What is the moment of inertia of the hoop about the central axis?

From Figure 7, the equation for the moment of inertia of a hoop is $I = mr^2$. So

$$I = 0.3 \times 0.35^2 = 0.04\,\text{kg}\,\text{m}^2 \text{ to 2 s. f.}$$

QUESTIONS

Refer to Figure 7 in answering these questions.

4. Calculate the moment of inertia for a hoop of mass 100 g, and 20 cm diameter. Compare this with the moment of inertia of a disc of the same mass and diameter.

5. a. Calculate the moment of inertia for a metal disc of mass 500 g, 15 cm diameter.

b. Calculate the moment of inertia for a metal sphere of mass 5 kg, 20 cm diameter.

Rotating systems

When two rotating objects share a common axis of rotation, then their individual moments of inertia are added together to give a moment of inertia for the combined system. For example, two flywheels A and B with radii R_A and R_B, respectively, have moments of inertia $\frac{1}{2}M_A R_A^2$ and $\frac{1}{2}M_B R_B^2$. So, the moment of inertia of the combination is given by

$$I = \frac{1}{2}M_A R_A^2 + \frac{1}{2}M_B R_B^2$$

This holds for any combination provided they rotate about a common axis.

QUESTIONS

In these questions, there are four small objects, each of mass 0.5 kg, attached by light (that is, negligible mass) rods of length 0.3 m to a common axis of rotation. Treat the objects, for calculation purposes, as point masses.

6. What is the total moment of inertia if the structure is spun as shown in Figure 8?

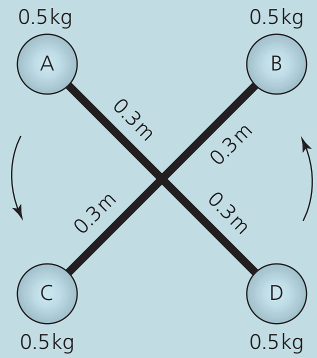

Figure 8

7. The structure is now spun along the DB axis in Figure 9. What is the new moment of inertia?

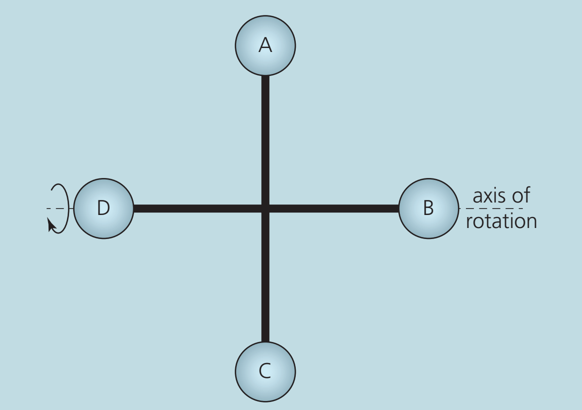

Figure 9

KEY IDEAS

- A torque (or moment) is the turning effect of a force. It is the product of a force and the perpendicular distance of the line of action of the force from a particular point of rotation:

  $$T = F \times r$$

  The unit of torque is newton metre (N m). Positive torques produce a clockwise rotation.
- An object's moment of inertia I is a measure of the object's resistance to angular acceleration about a certain axis.
- For a particle of mass m a distance r from an axis of rotation, the moment of inertia is

  $$I = mr^2$$

  The unit of I is kg m^2.
- The moment of inertia of an extended object is calculated by summation of the moments of inertia of all its particles:

  $$I = \sum mr^2$$

1.3 TORQUE AND ANGULAR ACCELERATION

In section 1.2, by considering a torque T acting on one particle, of mass m, a distance r from an axis of rotation, we found that

$$T = mr^2 \times \alpha$$

where α is the angular acceleration. By comparing this with Newton's second law, $F = ma$, we arrived at the idea of the moment of inertia ($I = mr^2$) as the rotational equivalent of the inertial mass m in linear motion.

We can extend this to an entire solid object. The external torque on an extended rigid object can be considered as the sum of the individual torques T_i on each particle, causing the whole object to experience an angular acceleration α:

$$\text{total } T = \sum T_i = \sum (m_i r_i^2 \times \alpha)$$

The angular acceleration α must be the same for the whole object (as it is a rigid solid), so

$$T = \sum (m_i r_i^2) \times \alpha$$

The summation $\sum (m_i r_i^2)$ is the total moment of inertia (I) of the object. Newton's second law, for rotational motion, then becomes simply, for the total torque,

$$T = I \times \alpha$$

So, if there is an applied torque T (unit of N m) acting on a solid object, whose moment of inertia is I (unit kg m^2), there will be an angular acceleration α (unit rad s^{-2}) given by $\alpha = T/I$.

It is useful to point out that the unit of I from $I = T/\alpha$ would be (N m)/(rad s^{-2}). How does this relate to kg m^2, the unit we had initially for I?
Since 1 N is the force required to give an object of mass 1 kg an acceleration of 1 m s^{-2}, we can substitute for N, giving (kg m s^{-2} m)/(rad s^{-2}).
A rad has no unit as it is a ratio of lengths, so we are left with kg m^2.

Worked example

A torque of 10 N m is applied to an object whose moment of inertia is 5.0 kg m^2. What is the angular acceleration?

$$\alpha = T/I = 10/5.0 = 2.0 \text{ rad s}^{-2}$$

QUESTIONS

8. A force of 30 N is applied tangentially at the edge of a disc of mass 5.0 kg. Its radius is 50 cm. Determine the moment of inertia, the torque applied and the angular acceleration.

9. A park roundabout (Figure 10) has a diameter of 2.2 m. When a small child provides a tangential force of 30 N at the edge of the roundabout, there is an angular acceleration of 0.30 rad s^{-2}. Calculate the combined moment of inertia of the roundabout and the children on it.

Figure 10

KEY IDEAS

- A torque T on an object will produce an angular acceleration α in rad s^{-2}, the value of which is controlled by the moment of inertia I:

$$T = I \times \alpha$$

- This is Newton's second law for rotational motion.

1.4 EQUATIONS OF ROTATIONAL DYNAMICS

From section 1.3, you can see a similarity between the expression for angular acceleration, $\alpha = T/I$, and the expression for linear acceleration, $a = F/m$.

Is there a similar equivalent of the equation $a = (v - u)/t$ and the other equations of linear motion? From Table 1, we can see that the equations of motion of linear dynamics (*see section 9.5 in Chapter 9 in Year 1 Student Book*) have an exact analogy when dealing with rotational dynamics.

Linear equation of motion	Rotational equation of motion
$v = u + at$	$\omega_2 = \omega_1 + \alpha t$
$s = \frac{1}{2}(u + v)t$	$\theta = \frac{1}{2}(\omega_1 + \omega_2)t$
$s = ut + \frac{1}{2}at^2$	$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$
$v^2 = u^2 + 2as$	$\omega_2^2 = \omega_1^2 + 2\alpha\theta$

Table 1 *The similarity between the linear equations of motion for constant acceleration and the rotational equations of motion for constant angular acceleration*

Just as with linear dynamics, the rotational equations of motion can only be used when we are dealing with constant (uniform) acceleration. A graphical method needs to be employed when α is not constant (see Engineering Physics section 1.5).

When dealing with problems involving rotational motion, it is helpful to think of the variables used in the linear motion equations and consider what their rotational 'counterparts' are – see Table 2.

Linear variable	Unit	Rotational variable	Unit
Displacement, s	m	Angle turned through, θ	rad
Initial velocity, u	m s^{-1}	Initial angular velocity, ω_1	rad s^{-1}
Final velocity, v	m s^{-1}	Final angular velocity, ω_2	rad s^{-1}
Time, t	s	Time, t	s
Acceleration, a	m s^{-2}	Angular acceleration, α	rad s^{-2}

Table 2 *Variables in linear and rotational motion*

Then it is straightforward to write down the rotational equations by remembering the linear versions and 'mapping' the variables across, as in Table 1.

The rotational equations are set up in the same way as their linear counterparts. For example, we define angular acceleration (α) as the change of angular velocity ($\Delta\omega$) divided by the time taken (Δt):

$$\alpha = \frac{\Delta\omega}{\Delta t}$$

So over a time interval t we have

$$\alpha = \frac{\omega_2 - \omega_1}{t}$$

By rearranging, we get

$$\omega_2 = \omega_1 + \alpha t$$

which is the first equation on the right-hand side in Table 1. The other equations are derived in a similar way.

Worked example 1

A stationary wheel undergoes an angular acceleration of $8.0\,\text{rad}\,\text{s}^{-2}$. If this acceleration lasts 30 s, calculate the final angular velocity and how many radians it will have moved through. How many revolutions is that equivalent to?

Initially, we need to find ω_2, so we choose $\omega_2 = \omega_1 + \alpha t$, where $\omega_1 = 0$. Thus

$$\omega_2 = 8.0 \times 30 = 240\,\text{rad}\,\text{s}^{-1}$$

The angular displacement is θ, so we choose $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$, where $\omega_1 t = 0$ again. So

$$\theta = \frac{1}{2} \times 8.0 \times 30^2 = 3600\,\text{rad}$$

As there are 2π rad in one revolution, it will have turned $3600/2\pi \approx 570$ rev.

Worked example 2

A flywheel has a moment of inertia $2.4 \times 10^{-2}\,\text{kg}\,\text{m}^{-2}$ and is rotating with an angular velocity of $20\,\text{rad}\,\text{s}^{-1}$. Calculate the torque that is required to bring the flywheel to rest in five revolutions.

We are given that $I = 2.4 \times 10^{-2}\,\text{kg}\,\text{m}^{-2}$, $\omega_1 = 20\,\text{rad}\,\text{s}^{-1}$ and $\theta = 5.0\,\text{rev} = 5.0 \times 2\pi = 10\pi$ rad. So, we can use the following equation of rotational motion to calculate the angular acceleration:

$$\omega_2^2 = \omega_1^2 + 2\alpha\theta$$

Hence,

$$0 = 400 + 2\alpha(10\pi)$$

giving

$$\alpha = -\frac{20}{\pi} = -6.37\,\text{rad}\,\text{s}^{-2}$$

which is a deceleration.

The torque is then

$$T = I\alpha = -2.4 \times 10^{-2} \times 6.37 = -0.153$$
$$= -0.15\ \text{N m (to 2 s.f.)}$$

So a torque of 0.15 N m opposing the motion is needed.

QUESTIONS

10. A flywheel is mounted on a horizontal axle of diameter 0.1 m. A constant force of 100 N is applied tangentially to the axle (Figure 11). If the moment of inertia of the whole system (flywheel and axle) is $8.0\,\text{kg}\,\text{m}^2$, determine

- **a.** the angular acceleration of the flywheel
- **b.** the number of revolutions the flywheel makes in 30 s (assume the flywheel starts from rest).

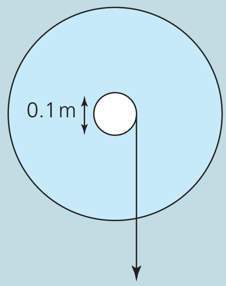

Figure 11

11. A park roundabout has been left rotating at 30 rpm. A passer-by slows it down and stops it by applying a tangential braking force of 200 N. The moment of inertia of the roundabout is $500\,\text{kg}\,\text{m}^2$ and its radius is 2.0 m. Calculate

- **a.** the initial angular velocity
- **b.** the braking torque being applied
- **c.** the angular deceleration of the roundabout
- **d.** the time taken to stop and how many revolutions this will take.

12. In a hammer throw (Figure 12), the athlete spins around six times in roughly 5 s. The release velocity is $30\,\text{m}\,\text{s}^{-1}$, the mass of the hammer is 7.00 kg, and the length of the wire is 1.3 m. Assume the length of the athlete's arm is 0.7 m. In practice, the hammer's velocity is built up in stages, but here assume a constant angular acceleration.

Figure 12 *The hammer thrower builds up the angular velocity by spinning several times*

a. Calculate the moment of inertia of the hammer on the end of the wire.
b. Calculate the average angular acceleration.
c. Calculate the average torque being applied by the athlete.
d. Are we justified in ignoring the mass of the athlete's arm in the above calculation? A typical human arm has a mass of 9 kg. Assume that a reasonable estimate for the moment of inertia of an arm can be gauged by $\frac{1}{3}mL^2$, where L is the length. Calculate the moment of inertia of the athlete's arm and compare it to the value obtained in part **a**. Comment.

KEY IDEAS

- For uniform angular acceleration, the equations of rotational motion can be used to solve problems:

$$\omega_2 = \omega_1 + \alpha t$$
$$\theta = \frac{1}{2}(\omega_1 + \omega_2)t$$
$$\theta = \omega_1 t + \frac{1}{2}\alpha t^2$$
$$\omega_2^2 = \omega_1^2 + 2\alpha\theta$$

- Each of these equations is analogous to one of the linear equations of motion for uniform acceleration.

1.5 GRAPHICAL METHODS

The equations of rotational motion introduced in section 1.4 are only applicable when the angular acceleration is constant. In the real world, this seldom happens. Graphs are often needed to solve problems.

The approach is like that in linear motion (*see section 9.4 in Chapter 9 in Year 1 Student Book*), except the object is spinning. There are analogous rules:

1. The angular displacement is angular velocity × time. So the area under an angular velocity–time graph is equal to the displacement in radians. To calculate the number of revolutions, divide by 2π.
2. The gradient of an angular velocity–time graph is the angular acceleration at that time.

Worked example

Figure 13 shows the angular velocity–time graph for a spinning object. Calculate the angular acceleration and the total angular displacement at 120 s. How many complete revolutions has it made?

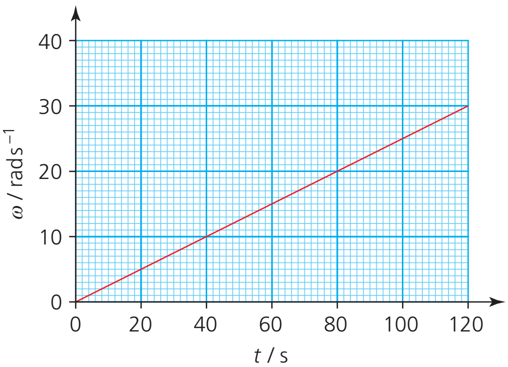

Figure 13

The angular acceleration is the gradient of the graph. By drawing the biggest triangle possible, the gradient is 30/120 or 0.25 rad s^{-2}.

The total angular displacement is the area under the graph up to 120 s, which is $\frac{1}{2} \times 120 \times 30 = 1800$ rad. As there are 2π radians in a circle, this is about 286 revolutions.

QUESTIONS

13. The graph in Figure 14 is for a spinning object.

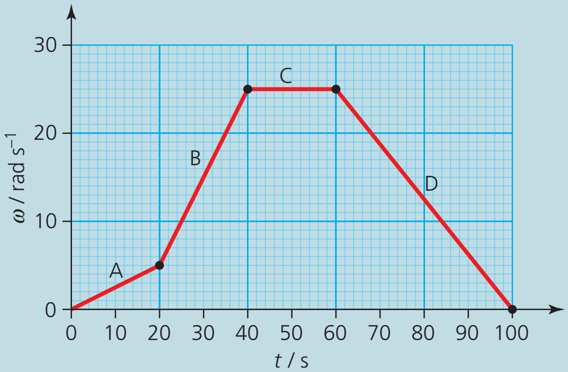

Figure 14

a. Describe the motion for the four sections labelled A, B, C and D.

b. Calculate the angular acceleration in each section.

c. Calculate the total angular displacement. How many revolutions has the object made overall?

14. Figure 15 shows the increase in angular velocity of an object with time.

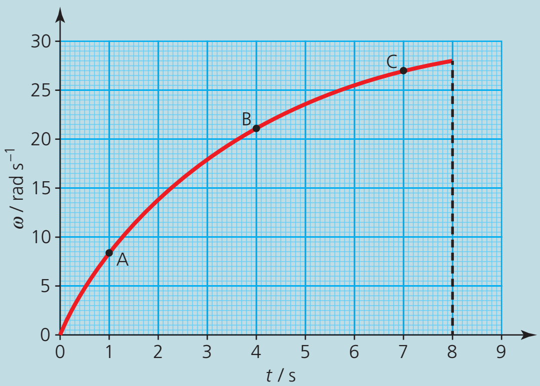

Figure 15

a. Calculate the angular acceleration at points A, B and C.

b. Estimate the total angular displacement and therefore the number of revolutions.

KEY IDEAS

- The area under an angular velocity–time graph is equal to the displacement in radians.
- The gradient of an angular velocity–time graph at any point is equal to the angular acceleration at that time.

1.6 ROTATIONAL KINETIC ENERGY

Any object rotating has kinetic energy due to its rotational motion. Consider first a point mass m_1 moving in a circle about some point O a distance r_1 away. The particle is moving with a linear speed v_1. The kinetic energy of the particle is given by

$$E_k = \frac{1}{2}m_1v_1^2$$

The linear velocity is connected to the angular velocity via the equation $v_1 = r_1\omega$. The kinetic energy of the particle can therefore be rewritten as

$$E_k = \frac{1}{2}m_1(r_1\omega)^2$$

This can be rearranged to give

$$E_k = \frac{1}{2}(m_1r_1^2)\omega^2$$

If we compare this expression with that for kinetic energy of linear motion, angular velocity has replaced linear velocity, and the term that has replaced the mass is the moment of inertia of the point mass about the point of rotation. This is just as we found in Engineering Physics section 1.3, in deriving a rotational version of Newton's second law.

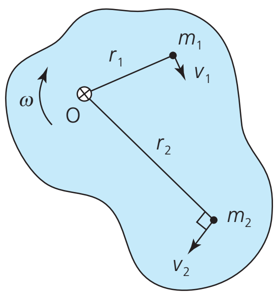

Figure 16 *Calculating the rotational kinetic energy of an extended rotating body*

An extended body that is rotating about an axis through a point O comprises many such point masses (see Figure 16). The total kinetic energy of rotation is given by

$$E_k = \frac{1}{2}(m_1r_1^2)\omega^2 + \frac{1}{2}(m_2r_2^2)\omega^2 + \\ = \frac{1}{2}\omega^2(m_1r_1^2 + m_2r_2^2 + \quad)$$

with r_1, r_2, … being the perpendicular distances from the axis of rotation. For a given body rotating about a given axis, the term $m_1r_1^2 + m_2r_2^2 + \cdots$ is the moment of inertia, I, so that the **rotational kinetic energy** is simply given by

$$E_k = \frac{1}{2}I\omega^2$$

A flywheel is a simple device used for storing rotational kinetic energy. As this rotational kinetic energy depends on two key factors – the angular velocity (ω) and the moment of inertia (I) – both of these must be as large as possible to maximise the **energy storage capacity** of the flywheel. Large I is achieved by ensuring that, as far as possible, most of the mass is far from the axis. The limitation is dependent on maintaining the structural integrity of the flywheel as it spins.

The expression for rotational kinetic energy shows that E_k increases by the *square* of ω, just as linear E_k depends on v^2, and so therefore does the energy required to decelerate and stop the motion, with a consequent effect on the stopping distance.

Worked example 1

A disc is spun up initially to 100 rpm and then later to 300 rpm. It has a radius of 50 cm and a mass of 10 kg. Find E_k in each case.

The same mass is now made into a hoop, with everything else being kept the same. Calculate the new value for E_k in each case. Comment on the answers.

For the disc, the moment of inertia is $\frac{1}{2}mr^2$, which is $\frac{1}{2} \times 10 \times (0.5)^2 = 1.25\,\text{kg}\,\text{m}^2$.

At 100 rpm = $100 \times 2\pi/60 = 10.5\,\text{rad}\,\text{s}^{-1}$, we have

$E_k = \frac{1}{2} \times 1.25 \times (10.5)^2 = 68.9\,\text{J}$.

At 300 rpm = $300 \times 2\pi/60 = 31.4\,\text{rad}\,\text{s}^{-1}$, we have

$E_k = \frac{1}{2} \times 1.25 \times (31.4)^2 = 616\,\text{J}$.

That is quite a dramatic increase.

For the hoop, the moment of inertia is mr^2, so it will be $2.5\,\text{kg}\,\text{m}^2$.

At 100 rpm ($10.5\,\text{rad}\,\text{s}^{-1}$), $E_k = \frac{1}{2} \times 2.5 \times (10.5)^2$
$= 138\,\text{J}$.

At 300 rpm ($31.4\,\text{rad}\,\text{s}^{-1}$), $E_k = \frac{1}{2} \times 2.5 \times (31.4)^2$
$= 1230\,\text{J}$.

As can be seen, the combination that gives the maximum amount of stored energy is the largest angular velocity and the largest moment of inertia.

QUESTIONS

15. A solid steel cylindrical rotor is being tested. The rotor has a mass of 272 kg and a radius of 38.0 cm. During the test, the rotor reaches an angular speed of 14 000 rev min^{-1} before breaking.

a. Calculate the angular velocity when this happens.

b. How much energy had the rotor stored? What happened to this energy when the rotor broke?

16. a. Assuming the Earth to be a sphere with moment of inertia $\frac{2}{5}MR^2$, where M is the mass of the Earth (6.0×10^{24} kg) and R is the radius of the Earth (6.4×10^6 m), calculate the moment of inertia of the Earth.

b. What is its rotational kinetic energy?

Objects such as balls that are rolling (rather than sliding) have two forms of kinetic energy: linear as well as rotational (Figure 17).

Figure 17 A bowling ball is rotating as well as moving down the lane towards the pins.

Worked example 2

A uniform disc has mass 500 g and diameter 30.0 cm. It is rolled on its edge along a horizontal table. It rolls a distance of 45.0 cm in 1 s. Calculate the total energy of the disc.

Since the disc is rotating as well as moving horizontally, its energy will be the sum of the two forms of E_k: linear plus rotational.

Linear velocity $v = 0.450\,\text{m s}^{-1}$

Linear $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 0.5 \times 0.450^2$
$= 0.0506\,\text{J}$

To calculate the rotational $E_k\left(\frac{1}{2}I\omega^2\right)$, we need to calculate the angular velocity ω of the disc. Imagine first that the disc is just rotating, with angular velocity ω and tangential velocity v_t (Figure 18a), so $\omega = v_t/r$, where r is its radius.

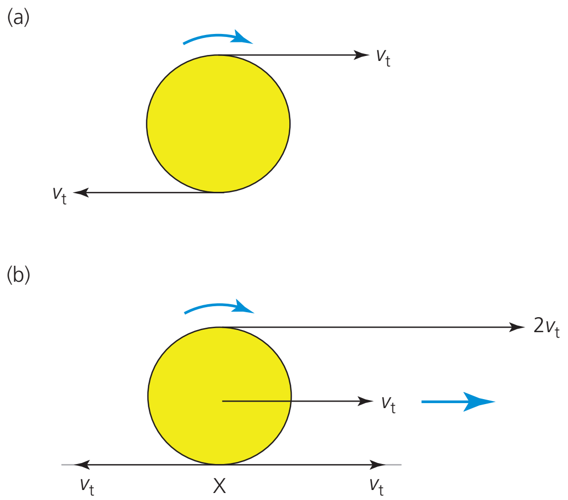

Figure 18 *The tangential velocity of the rotating disc must be equal and opposite to the linear velocity.*

Now imagine the rotating disc placed on the table surface. The disc rolls and its centre of mass moves at linear velocity v_t. Why? The tangential velocity v_t at the table surface (point X in Figure 18b) must be equal and opposite to the linear velocity, as otherwise the disc would be sliding along the surface. The top will therefore be moving at twice this tangential velocity.

So, $\omega = v_t/r = 0.450/0.150$
$= 3.00\,\text{rad s}^{-1}$.

Moment of inertia $I = \frac{1}{2}mr^2$

$= \frac{1}{2} \times 0.500 \times 0.150^2 = 0.0056\,\text{kg m}^2$

Rotational $E_k = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.0056 \times 3.00^2$
$= 0.0252\,\text{J}$

Total energy of the disc $= 0.0506 + 0.0252$
$= 0.0758\,\text{J}$

QUESTIONS

17. Each wheel of a bicycle is 60.0 cm in diameter and has a mass of 1800 g. Assuming that each wheel is a hoop, calculate the total kinetic energy of each wheel when the bicycle is travelling at 30 km per hour.

Stretch and challenge

18. Two identical tins of fruit juice, one of which is frozen, are placed at the top of a slope and released at the same time, so that they roll straight down the slope (Figure 19). Which will get to the bottom first? Explain your answer.

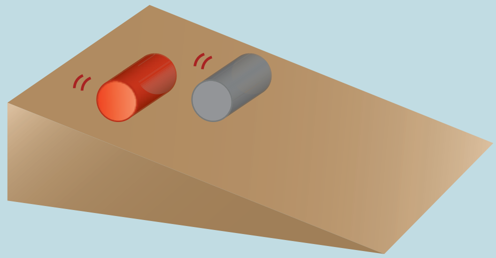

Figure 19

KEY IDEAS

- Rotational kinetic energy is given by $E_k = \frac{1}{2}I\omega^2$.
- A flywheel is a mechanical device that is made to spin in order to store rotational kinetic energy.
- For maximum energy storage capacity, the flywheel's moment of inertia needs to be as large as possible, and its angular velocity as large as possible.

ASSIGNMENT 1: DETERMINING THE MOMENT OF INERTIA OF A FLYWHEEL BY A PRACTICAL METHOD

(PS 1.1, PS 1.2, PS 2.3, PS 3.3, MS 0.1, MS 1.1, MS 1.2, MS 1.5, MS 2.2, MS 2.3)

The moment of inertia of a large flywheel can be determined by using a hanging mass to produce a torque on the flywheel (Figure A1).

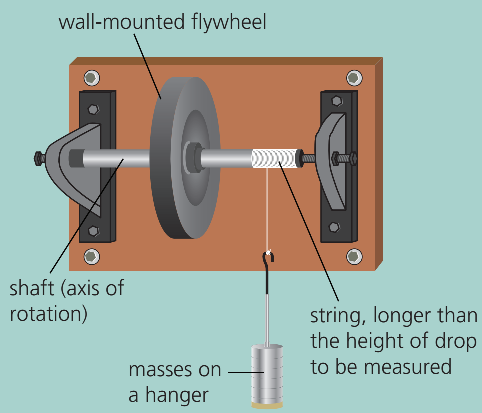

Figure A1 *The experimental set-up*

The weight of the mass exerts a torque on the central shaft, causing it to rotate. The flywheel is physically attached to the shaft, so it also rotates.

The basis of the experiment is the principle of conservation of energy. As the mass descends, its gravitational potential energy ($E_p = mg\Delta h$) is transferred as linear kinetic energy of the masses and rotational kinetic energy of the flywheel:

$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

The measurements shown in Table 3 were made.

Measurement	Value	Uncertainty
Mass m descending	700 g	Assumed to be zero
Drop h over which time is measured	100 cm	Measured using a ruler, precise to 1 mm
Mass of the flywheel	7.26 kg	Assumed to be zero
Mass of the axle	1.13 kg	Assumed to be zero
Diameter of the flywheel	20.0 cm	Measured using a ruler, precise to 1 mm
Diameter of the axle	25.0 mm	Measured using Vernier callipers, precise to 0.1 mm
Times to drop	11.44 s 10.94 s 11.21 s	Measured with a stopwatch, precise to 0.01 s

Table 3

Questions

A1 **a.** Calculate the mean of the dropping times.
b. By using the range of the times, calculate the spread and therefore the uncertainty as a percentage of the time.
c. How does the percentage uncertainty compare with the precision of the timer? Why is there a difference?

A2 **a.** We can assume that the acceleration of the falling mass is constant. Will it be $9.81\ \mathrm{m\,s^{-2}}$?
b. Calculate the final velocity of the dropping mass.

A3 **a.** Calculate the E_p lost by the mass and the linear E_k of the mass gained after falling through 100 cm.
b. Use conservation of energy to calculate the rotational E_k of the flywheel.

A4 **a.** Calculate the final angular velocity of the flywheel. [*Hint:* This is the same as the final angular velocity of the shaft.]
b. From your answers to part **b** of question **A3** and part **a** of this question, calculate a value for the moment of inertia of the flywheel.
c. Calculate the percentage and absolute uncertainty in this value of I.

A5 Assuming the flywheel to be a ring of material, calculate the theoretical moment of inertia, from its mass and radius.

A6 **a.** There is some disagreement between the experimental value and the theoretical value. Can these two answers be reconciled by considering the assumptions that have been made?
b. Using the initial data, calculate the moment of inertia for the axle and comment.

1.7 APPLICATIONS OF FLYWHEELS

A flywheel, for optimum performance, needs to have most of its mass as far as possible from the axis of rotation. Because a hoop is not a practical shape (how could it be attached?), in practice a large flywheel is often designed as a rim linked by spokes to the axis (Figure 20).

Figure 20 *In a flywheel, the spokes are symmetric, for stability. Although the majority of the mass ideally needs to be at the rim, there are also structural and safety considerations to take into account.*

In industry and in transport applications, flywheels have three main uses:

1. They can smooth out torque or speed variations in vehicles.
2. They can recover and re-use some of the kinetic energy that would otherwise be wasted, through braking, for example.
3. They are an essential component of machinery in some production processes.

Smoothing torque and speed

In vehicle engines, power is not produced continuously, but only in the 'power stroke' or combustion part of the engine cycle. (This will be covered in more detail in section 2.1 of Engineering Physics Chapter 3.) As a consequence, the engine produces a torque that fluctuates (see Figure 21).

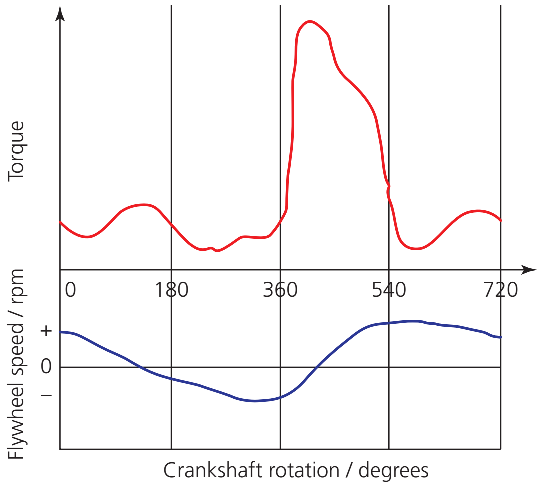

Figure 21 *The upper curve shows the variation of torque in a standard car engine during one cycle of the engine. Notice the erratic nature of the curve, peaking in the third section. This would make for an uncomfortable ride. The effect of a flywheel (lower curve) is to smooth the large changes.*

The torque is what makes the wheels rotate, thereby moving the vehicle forwards. Uneven torque will cause a jerky motion in the vehicle and unwanted vibration. This will be uncomfortable for the occupants and a waste of energy. The flywheel that is added will speed up and slow down over a period of time because of its inertia, and hence sharp fluctuations in torque are flattened, or 'smoothed'.

In practice, to increase total power and to further control these fluctuations, real engines usually have four or more combustion cylinders. These are staggered in their operation and are all attached collectively to a large flywheel (see Figure 22), which will effectively smooth or average out the torques being produced by the separate cylinders. The greater the number of cylinders working in this way, the smoother the operation of the whole engine will be. But more cylinders lead to greater cost, weight and complexity.

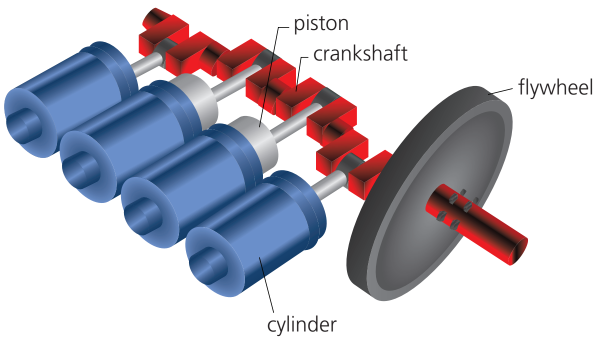

Figure 22 *A flywheel is used to smooth out the rotation of a crankshaft in a four-cylinder car.*

Kinetic energy recovery system (KERS)

The idea of **regenerative braking** has been around for a while – rather than have a car's kinetic energy (KE) wasted by transferring to heat through conventional braking, methods are used to 'collect' and store energy from a car's braking motion for re-use. Kinetic energy recovery system (KERS) is the expression for any system that does this.

For example, in **hybrid** cars, there is a combination of conventional combustion engines and an electric motor. This involves generators, which are spun as braking occurs. This produces electricity, which is stored in batteries. When needed, these batteries power an electric motor (the generator working backwards) to contribute to the driving torque of the conventional engine. However, a drawback is that the batteries have a large mass, so the overall fuel efficiency is not as good as might be expected.

A recent development is the **flybrid** design, the word being a combination of 'flywheel' and 'hybrid'. In this system, when the vehicle brakes, some of the KE is used to spin a flywheel. This can be achieved in two ways: either through a generator/motor method, similar to the hybrid, or by a direct mechanical linkage via gears between the rear driveshaft and the flywheel (Figure 23).

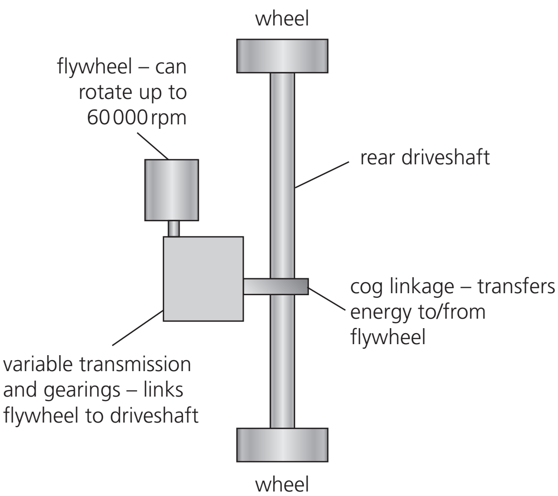

Figure 23 *The KERS layout for a car. Rotational energy is transferred to the flywheel, where it is stored until needed, then transferred back as required.*

When accelerating, the situation is reversed: the flywheel rotational energy is transferred back to the driveshaft as required. Again, this is done either by using a direct mechanical method, or by producing electricity, which is then used to spin motors attached to the wheels of the car. Control of the flow of power to and from the flywheel is computerised.

The future may lie in combinations of batteries and flywheels, as the cost of batteries used in hybrids is falling.

QUESTIONS

19. A flywheel used in a flybrid car is a ring of carbon fibre on a steel hub. It rotates in a vacuum inside a steel container. The whole flybrid assembly has a mass of 60 kg.

a. What is the advantage of the flywheel spinning in a vacuum?

b. The batteries used in a hybrid are 300 kg. List two advantages the flybrid has over the hybrid.

c. The flywheel can store sufficient energy to deliver about 60 kW for 10 s. How much energy can it then deliver?

d. Under what driving conditions would 10 s extra power be sufficient?

Production applications

Many industrial manufacturing applications, such as forming and piercing sheet metal, rely on continuous, non-fluctuating action (Figure 24). Traditionally, leather belts, driven by electric motors, were used to drive presses. Flywheels were essential because the belts could stretch or warp with differing atmospheric conditions; they could also become inflexible or slip momentarily. The flywheel helped to combat the consequences of these problems. Later, belts were replaced by motors directly attached to the presses. But motors can have non-regularities, so the best solution is to have motors spin large flywheels. These reduce problems occurring in the production process as a result of fluctuations.

Figure 24 *Punch presses are designed to cut (very regular) holes in material, so precision and consistency are essential. A flywheel, as can be seen in this old punch press, ensures that this happens.*

When ores, extracted from mines, need to be transferred to other processing sections, conveyor belts are used (Figure 25). Some of these can be very long, extending up to a kilometre, lifting up ore through a vertical height of a quarter of a kilometre or so. They are made of one continuous piece of material. There are usually two main pulleys: one at the top (the 'head') and one at the bottom (the 'tail'). There will be a motorised drive as well as a take-up pulley, which maintains the correct tension (see Figure 26).

Figure 25 *A long conveyor belt used in the mining industry*

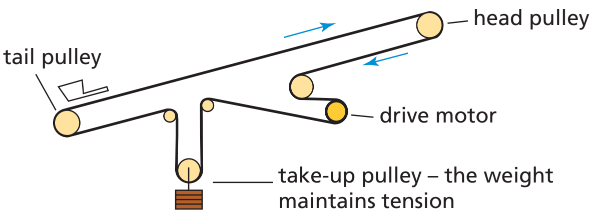

Figure 26 *A conveyor belt used for transporting materials normally has two main pulleys, at either end of the carrying section.*

If there is a power fluctuation, a failure or an emergency stop, the inertia of the belt will try to keep it moving. But different parts of the belt will be under different tension, so each part will react differently. Some parts will stop quickly, others will not be able to. Stress waves will be created, which will move along the belt, leading to regions of sag and over-stretch, and potential damage on a large scale.

This is solved by the addition of a flywheel to the conveyor system.

QUESTIONS

20. **a.** Where in the conveyor belt system in Figure 26 would you expect the flywheel to be placed?

b. Explain how the addition of a flywheel will have a dramatic impact. In your answer, discuss any change in the stopping time. Why and how is this important?

KEY IDEAS

- Flywheels are used to smooth out torque variations in vehicle engines.
- Flywheels can be used to store kinetic energy transferred on the braking of a vehicle.
- Flywheels are an essential component to ensure smooth running of machinery in some production processes.

1.8 ANGULAR MOMENTUM

The **angular momentum** L of a particle about an axis is the product of its linear momentum and the perpendicular distance of the particle from the axis:

$$L = m_1 v_1 \times r_1$$

For a particle P rotating about an axis at O with an angular velocity ω (where $v_1 = r_1\omega$), this leads to the expression

$$L = m_1 \omega r_1^2$$

For a rigid body composed of many such point masses $m_1, m_2, m_3, \ldots$ at distances $r_1, r_2, r_3, \ldots$ from the axis of rotation, the total angular momentum is given by

$$\begin{aligned} L &= m_1\omega r_1^2 + m_2\omega r_2^2 + \cdots \\ &= \omega(m_1 r_1^2 + m_2 r_2^2 + \cdots) \end{aligned}$$

leading to the expression

$$L = I\omega$$

Angular momentum is a vector quantity and has an associated direction, which is along the axis of rotation, and hence perpendicular to the plane of rotation (Figure 27). By convention, the direction of the angular momentum vector is towards an observer if the direction of rotation is anticlockwise.

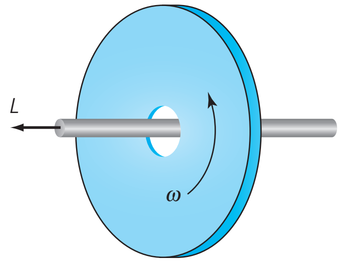

Figure 27 *The angular momentum vector is perpendicular to the plane of rotation.*

As angular momentum is defined in terms of the product of the moment of inertia and the angular velocity, the associated SI unit will be $\text{kg m}^2\,\text{s}^{-1}$. This emphasises the link to moment of inertia. However, since $T = I\alpha$, angular momentum $L = (T/\alpha \times \omega)$ and so we can also use the unit $((\text{N m})/(\text{rad s}^{-2})) \times (\text{rad s}^{-1})$, or N m s. A similar argument was used in an earlier section, when we derived the unit for moment of inertia.

Worked example

The London Eye has a diameter of 135 m and makes a complete rotation at a constant angular speed ω in 30 minutes. If the total moment of inertia of the Eye is $8.2 \times 10^9\,\text{kg m}^2$, calculate the magnitude of the angular momentum.

To find the angular speed, we use the fact that the wheel goes through an angular displacement of $\theta = 2\pi$ rad in a time period $T = 30$ min. Hence,

$$\omega = \frac{2\pi}{30 \times 60} = 0.0035\,\text{rad s}^{-1}$$

Using the expression for angular momentum gives

$$L = I\omega = 8.2 \times 10^9 \times 0.0035 = 2.9 \times 10^7\,\text{kg m}^2\,\text{s}^{-1}$$

Conservation of angular momentum

When dealing with linear momentum, we used the principle of conservation of linear momentum, which was valid provided no external forces were applied. There is an analogous **principle of conservation of angular momentum**:

The total angular momentum of a system at some initial time is equal to the total angular momentum at some later time, provided no external torque acts upon the system.

$$L_1 = L_2$$
$$I_1\omega_1 = I_2\omega_2$$

Imagine an ice skater spinning about a vertical axis (Figure 28). When she brings her arms much closer to her body, her mass is much closer to her axis of rotation. This results in a lowering of her moment of inertia I. As angular momentum ($I\omega$) is conserved, this results in a much larger value of ω, the angular velocity, so the skater spins faster. There is a corresponding increase in the rotational kinetic energy $\frac{1}{2}I\omega^2$. This energy has come from the skater, who has had to do work to pull her arms in.

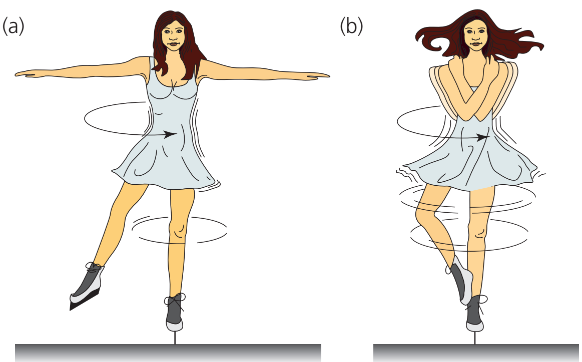

Figure 28 *As the skater draws her arms in, she spins faster.*

A further example is with high-board diving, where the diver changes his or her distribution of mass when descending, in order to speed up or slow down the spin rate, so that manoeuvres are completed effectively (just like the cat in the figure in the introduction to this option).

This principle holds, no matter what takes place *within* the system. In a collision between two objects, the conservation of angular momentum may result in the objects rotating in opposite directions. The law has no known exceptions – it holds true for situations ranging from subatomic particles to planets, and even for objects that travel close to the speed of light.

The Earth–Moon system has its own angular momentum. Owing to tidal drag, the Earth's spin on its axis is slowing down. As a direct consequence, angular momentum is transferred to the Moon, and hence its orbital radius is increasing, at the rate of about 3 cm each year.

QUESTIONS

21. **a.** All stars rotate (as a consequence of their original formation). After a star explodes in a supernova, the central core will shrink until its radius is about the size of a city and it becomes a neutron star. By considering the angular momentum, state what will happen to its rate of rotation. [Assume no mass loss of the star at this point.]

b. Our Sun's radius is 7×10^8 m. It takes about 30 days to spin once, on average.

i. Estimate the angular velocity of the Sun.

ii. If the Sun became a neutron star of radius 20 km, what would be its approximate angular velocity?

iii. Estimate how many times the neutron star would revolve in 1 s.

22. Two children are on a roundabout, which has a moment of inertia of 500 kg m^2. It is rotating at 20 rpm. The children, both of 40 kg mass, are standing at the edge on opposite sides, a distance of 2.0 m from the axis of rotation. Both let go together and jump off.

a. Calculate the total moment of inertia with the children on board. What assumptions do you need to make?

b. Calculate the angular momentum of the roundabout plus children.

c. What will be the new angular velocity of the roundabout after the children jump off?

Gyroscopes

A **gyroscope** is a device consisting of a wheel or disc that spins rapidly about an axis that is also free to change direction (Figure 29). As a consequence of the conservation of angular momentum, if it is spun pointing in a certain direction (for example, vertically), then *as long as it stays spinning quite fast*, it will continue to point that same way. The orientation of its axis will be *unaffected* by tilting or rotating the mounting.

Figure 29 A simple spinning gyroscope of the sort that has intrigued children of all ages

However, friction will cause the gyroscope to lose energy and it will begin to slow down. As it is already in an unstable position, balancing on a small base, there are a variety of reasons why the axis will soon move slightly away from its initial position (friction with the surface, the effect of air pressure or some non-uniform distribution of the mass of the gyroscope to start with). As soon as this starts to occur, there will be an external torque due to gravity. This causes a change in the angular momentum. The result is a continuously changing direction of the axis – the gyroscope, as well as spinning on its axis, rotates slowly around the vertical. This is an effect called **precession** (Figure 30). The direction of precession will be in the same sense (clockwise or anticlockwise) as the spin of the gyroscope itself.

This effect will only occur as long as the gyroscope is spinning and, the slower the spin, the bigger the angle it will make to the vertical, eventually toppling over as it becomes too unstable.

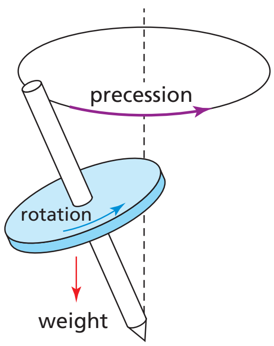

Figure 30 The precession of a gyroscope

Stretch and challenge

Applications of gyroscopes

If loss of energy can be prevented, the orientation of a gyroscope's axis will be maintained, and this means they have some useful applications.

- The motion of an aircraft away from a gyroscope axis can be measured and then this information can be used either to determine location or to adjust the position to get it back 'on track'.
- Satellites use gyroscopes that measure changes in all three dimensions, to navigate by.
- A ship can use a gyroscope to enable it to stay more upright, especially in stormy seas. The gyroscope is fixed to the hull and is spun up to 10 000 rpm.
- Segway scooters and two-legged robots use an assembly of gyroscopes in order to maintain their vertical position.
- When a racing car is driven around a bend, the spinning engine gives rise to a gyroscopic effect, which causes the car's nose to be forced either up or down, affecting the friction with the ground.
- Some computer devices have gyroscopes inside them, so that the movements of a mouse can be input without it resting on a surface.
- In virtual reality headsets, the motion of your head is detected by miniature gyroscopic sensors, and this information is fed back to the computer, which will then change the display, so creating a seamless perspective.

QUESTIONS

Stretch and challenge

23. Choose one of the gyroscope applications mentioned in the previous list. Explain briefly how it works, making reference to angular momentum. You may need to do some research.

Angular impulse

As you will recall from *section 11.2 of Year 1 Student Book, linear momentum* is only conserved when *no external force* is acting. Then, the change of momentum is equal to the force × time (known as the *impulse*).

In exactly the same way, we can say that *angular momentum* is only conserved if *no external torque* acts. If a torque is applied, then there *will* be a change of angular momentum. This is called the **angular impulse**:

$$\Delta L = L_2 - L_1 = I\omega_2 - I\omega_1 = I(\omega_2 - \omega_1)$$

Angular impulse therefore has the same unit as angular momentum: $\text{kg m}^2\,\text{s}^{-1}$.

The torque T applied can be related to the angular impulse ΔL. Now, from Engineering Physics section 1.3, we know that the torque is equal to the rate of change of angular momentum:

$$T = I\alpha = I\frac{\Delta\omega}{\Delta t} = \frac{\Delta L}{\Delta t}$$

The consequent expression for angular impulse is

$$\Delta L = T\Delta t$$

This is the angular version of linear impulse: average force F times the duration of the impact Δt (*see section 11.3 in Chapter 11 in Year 1 Student Book*).

Since ΔL could be either $\Delta I \times \omega$ or $I \times \Delta\omega$, we can therefore write

$$\Delta(I\omega) = T\Delta t$$

as the most generalised version, which should be used where the moment of inertia may not be constant.

QUESTIONS

24. An exercise bike has a flywheel whose moment of inertia is $40\,\text{kg m}^2$. How big an angular impulse is needed to bring the rotational speed up to $0.8\,\text{rev s}^{-1}$ from a standing start?

25. a. A bicycle wheel, which is free to rotate, has a diameter of 60 cm and a mass of 1.8 kg. It is initially stationary. When a person applies a tangential force to the edge, the wheel sped up to 9.0 revolutions per second. What was the angular impulse applied?

b. The person applied the force to the wheel for 3.0 s. What torque was applied and what tangential force was applied?

KEY IDEAS

- The angular momentum L of a particle about an axis is the product of its linear momentum and the perpendicular distance of the particle from the axis.
- This leads to $L = I\omega$, where I is the moment of inertia of an object or system, and the ω is the angular velocity.
- Angular momentum is always conserved, provided there is no external torque acting on the system.
- When a torque T acts, the impulse is equal to the change in angular momentum. This is given by $\Delta(I\omega) = T\Delta t$.

1.9 WORK AND POWER IN ROTATING SYSTEMS

Consider a rigid body that turns through an angle θ about an axis when a force F is applied. The perpendicular distance from the axis to the line of action of the force is r. The force produces a constant torque $T = Fr$. As the rigid body rotates about the axis, the force moves along an arc of length $s = r\theta$. The **work done** is the product of the force and the distance moved:

$$W = Fr\theta$$

which is

$$W = T\theta$$

Here, the work done, W, is in joules when the constant torque, T, is in N m and the angle moved, θ, is given in radians.

Work done may increase the rotational kinetic energy of the rigid body, but it must also overcome resistive forces that may be present. Friction cannot be completely avoided when two moving objects are in contact. Attempts are made to minimise this by varying techniques such as lubricants or by ball bearings between such surfaces. The frictional force in the case of an object rotating on an axle – like a flywheel on an axle – is at the edge of the rotating object, so there will be a torque about the axis, called **frictional torque**.

A frictional torque may act to resist the motion of a rotating body, and so reduce its rotational kinetic energy. Alternatively, a frictional torque may be *applied* – such as when you grip and turn a screwdriver – in which case the rotational kinetic energy will increase. Whenever a torque turns an object through an angle, work is done.

Power is the rate of doing work:

$$P = \frac{\Delta W}{\Delta t} = \frac{\Delta(T\theta)}{\Delta t} = T\frac{\Delta\theta}{\Delta t}$$

where $\Delta\theta/\Delta t$ is the change in angular displacement with respect to time, that is, the angular velocity. Hence, power is given by

$$P = T\omega$$

This equation is the angular version of the linear equation $P = Fv$ (*see section 11.5 in Chapter 11 in Year 1 Student Book*), which enables calculation of the power needed to overcome resistive forces, thereby maintaining a constant velocity v.

Worked example

A spinning disc is being kept at a constant angular velocity of 2700 rpm by a motor. There is friction present, which is producing a (braking) frictional torque of 45 N m. Calculate the power needed to maintain the angular velocity of the disc.

The angular velocity is kept constant, so the power from the motor must be equal to the rate of work done against the frictional torque.

We need to convert the 2700 rpm to radians per second. This is $(2700/60) \times 2\pi = 280\ \text{rad s}^{-1}$.

Power $P = T \times \omega = 45 \times 280 = 13\,000\ \text{W}$

QUESTIONS

26. Calculate the power needed to maintain an angular velocity of $15\ \text{rad s}^{-1}$ in a disc, where the frictional torque is 25 N m.

27. A braking force of 200 N is applied to the edge of a disc of diameter 20 cm. Calculate the braking torque. The power supplied by a motor is able to keep the disc rotating at $45\ \text{rad s}^{-1}$. Calculate the power delivered by the motor.

28. This question concerns a potter's wheel, with a disc of clay on it (Figure 31). Relevant data are:

Diameter of potter's wheel = 70.0 cm

Mass of potter's wheel = 40.0 kg

Wheel's rotation rate = $3.50\ \text{rev s}^{-1}$

Diameter of clay disc = 20.0 cm

Mass of clay disc = 2.40 kg

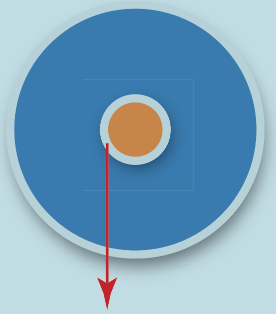

Figure 31

The potter applies a frictional force of 20 N for 10 s on one side of the clay, which slows the wheel down.

a. Calculate the torque the potter applies to the clay and wheel.

b. Calculate the angular velocity of the wheel after 10 s. Assume there is no slippage of the clay.

c. Calculate the work done by the potter in slowing the wheel down.

KEY IDEAS

- The work done W by a torque T in moving a system through an angular displacement θ is $W = T\theta$.
- Real machines are subject to frictional torque, which resists the rotation.
- The power needed to overcome a frictional torque T in order to maintain a constant angular momentum ω is given by $P = T\omega$.

1.10 ANALOGY BETWEEN LINEAR AND ROTATIONAL MOTION

The correspondences between quantities in linear and rotational motion are summarised in Table 4.

Linear motion	Linear quantity	Rotational motion	Rotational quantity	Rotational units
Mass	m	Moment of inertia	I	$\mathrm{kg\,m^{-2}}$
Velocity	$v = \frac{\Delta s}{\Delta t}$	Angular velocity	$\omega = \frac{\Delta \theta}{\Delta t}$	$\mathrm{rad\,s^{-1}}$
Acceleration	$a = \frac{\Delta v}{\Delta t}$	Angular acceleration	$\alpha = \frac{\Delta \omega}{\Delta t}$	$\mathrm{rad\,s^{-2}}$
Displacement	s	Angular displacement	θ	rad
Translational kinetic energy	$E_T = \frac{1}{2}mv^2$	Rotational kinetic energy	$E_k = \frac{1}{2}I\omega^2$	J
Momentum	$p = mv$	Angular momentum	$L = I\omega$	N m s
Force	$F = ma$ $F = \frac{\Delta(mv)}{\Delta t}$	Turning moment (torque)	$T = Fr$ $T = I\alpha$ $T = \frac{\Delta(I\omega)}{\Delta t}$	N m
Work done	$W = Fs$	Work done	$W = T\theta$	J
Power	$P = Fv$	Power	$P = T\omega$	W
Impulse	$I = Ft$	Angular impulse	$\Delta L = I\omega_2 - I\omega_1$ $\Delta L = T\Delta t$	N m s

Table 4 *The correspondence between linear motion and angular motion*

ASSIGNMENT 2: ANALYSING A FLYWHEEL-DRIVEN BUS

(MS 0.1, MS 2.3)

In Switzerland, just after the Second World War, a novel type of transport system was introduced on routes that did not merit a tram or trolley-bus service. Fuel had been rationed, so a compromise was sought. The ‘gyrobus’ – a flywheel-powered bus – caught people’s attention. Read this information about the gyrobus and answer the questions at the end.

The gyrobus design was originally based on the chassis of a lorry, but was changed for a lighter design when it went into regular service in 1953. Its range was 6 km, so the first regular service was only 4.5 km, travelling at a maximum speed of 55 km per hour.

There were four recharging points along the route, where a ‘top-up’ charge took about 10 minutes (Figure A1). A motor on the bus used this electricity to spin up the flywheel on the bus. In normal motion, the spinning flywheel became the source of energy on the bus and was used to provide power for the traction motor attached to the wheels.

Figure A1 *The gyrobus being ‘charged up’*

Eventually, the gyrobuses were replaced with small diesel buses as running costs became too high, including electricity bills.

Design details

The flywheel was made of steel. It had a mass of 1.5 tons.

The flywheel had a diameter of 1.6 m.

It was spun in a hydrogen gas chamber to reduce air resistance.

The maximum spin it could achieve was 3000 rpm.

It was spun by an electric motor, provided by power from the usual mains supply, at the depots.

Questions

A1 Calculate the moment of inertia of the flywheel, using data from the 'Design details' box. Assume its shape to be a disc. [1 ton is different from 1 metric tonne (= 1000 kg); 1 ton is equivalent to 907 kg]

A2 When the flywheel was spinning fastest, how much energy would it be storing?

A3 In use, the spin speed would eventually drop down to about 2000 rpm. How much energy would have been transferred to drive the bus?

A4 It took 40 minutes to get the flywheel spinning at maximum speed, starting from rest. Suggest what they might have done in the depots to ensure a quick start in the morning.

Stretch and challenge

A5 The spinning flywheel acts as a gyroscope. What problems can you foresee for the bus as it changed direction?

A6 What could be a problem for the bearings supporting the flywheel, considering its mass?

A7 What safety concerns might there have been?

A8 Nowadays, with concerns about pollution from engines, lighter materials becoming available, as well as the introduction of magnetic bearings, could the gyrobus be a viable alternative means of transport? Discuss the advantages and disadvantages.

PRACTICE QUESTIONS

1. Flywheels have recently become the subject of intensive research as power storage devices for use in vehicles and power plants. They are able to store energy very efficiently and then release the energy over a prolonged period of time. A particular portable device has a moment of inertia of 1.2 kg m^2 and can operate safely up to 25 000 revolutions per minute.

   a. The flywheel starts at rest and receives an acceleration of 5 rad s^{-2} for 8 minutes. Calculate the final number of revolutions per minute.

   b. Show that the energy stored in the flywheel at the end of this 8 minute period is 3.5 MJ.

   c. After reaching its maximum speed, the flywheel is allowed to come to rest. The average power dissipated in overcoming the frictional forces is 15 W. Calculate the time taken before the flywheel is at rest.

   d. Determine the size of the frictional torque acting on the flywheel.

2. A small electric motor is used to start a flywheel. A clutch mechanism is used to connect the motor to the flywheel, when the clutch is said to be *engaged*.
   a. Initially the motor is running at 2400 rev min^{-1} and the flywheel is stationary. The motor and clutch system has a moment of inertia of 0.84 kg m^2. Calculate the angular momentum of the motor.
   b. How much rotational kinetic energy does the motor have?
   c. The motor is now connected to the flywheel via the clutch mechanism. The moment of inertia of the flywheel and driveshaft is 1.4 kg m^2. Explain why the speed of the motor decreases as the clutch is engaged.
   d. Calculate the common angular velocity of the motor and flywheel immediately after the clutch is engaged.
   e. Determine the angular impulse on the flywheel as the clutch engages.

3. Some motor racing cars are fitted with a kinetic energy recovery system (KERS). In this system, as the car brakes approaching a bend, instead of all the lost kinetic energy being dissipated as heat, some of the energy is used to accelerate a flywheel. When the car needs to accelerate out of the bend, the energy in the flywheel assists the engine in providing extra power.
   a. Describe and explain some of the design features of a flywheel in order for it to store maximum energy. Your answer should include consideration of the flywheel's shape, the material from which it is made and its design for high angular speeds.
   b. A KERS flywheel has a moment of inertia of 0.036 kg m^2 and rotates at its maximum angular speed of 6400 rad s^{-1}. When the flywheel is used to help accelerate the car, the flywheel's speed reduces uniformly to 3100 rad s^{-1} in a time of 6.6 s. You may assume that frictional losses in the drive mechanism are negligible.
      i. Calculate the energy transferred from the flywheel to the car.
      ii. Calculate the average power produced by the decelerating flywheel.
      iii. Calculate the decelerating torque on the flywheel, stating an appropriate unit.
      iv. Calculate the number of revolutions made by the flywheel in the time of 6.6 s.

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4. The turntable of a microwave oven has a moment of inertia of 8.2×10^{-3} kg m^2 about its vertical axis of rotation.
   a. With the drive disconnected, the turntable is set spinning. Starting at an angular speed of 6.4 rad s^{-1} it makes 8.3 revolutions before coming to rest.
      i. Calculate the angular deceleration of the turntable, assuming that the deceleration is uniform. State an appropriate unit for your answer.
      ii. Calculate the magnitude of the frictional torque acting at the turntable bearings.
   b. The turntable drive is reconnected. A circular pie is placed centrally on the turntable. The power input to the microwave oven is 900 W, and to cook the pie the oven is switched on for 270 seconds. The turntable reaches its operating speed of 0.78 rad s^{-1} almost immediately, and the friction torque is the same as in part **aii**.
      i. Calculate the work done to keep the turntable rotating for 270 s at a constant angular speed of 0.78 rad s^{-1} as the pie cooks.
      ii. Show that the ratio

$$\frac{\text{energy supplied to oven}}{\text{work done to drive turntable}}$$

is of the order of 10^5.

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2 THERMODYNAMICS

PRIOR KNOWLEDGE

You will need to build on your knowledge of the three gas laws, the concept of an ideal gas and the ideal gas equation. You will need to be confident in using the Kelvin temperature scale and to remember the concept of absolute zero. You need to understand the principles of the kinetic theory model. Refer back to Chapter 3 to ensure you are familiar with these topics.

LEARNING OBJECTIVES

In this chapter you will learn about the first law of thermodynamics and about the four types of thermodynamic processes that form the basis of engine cycles. You will learn how to represent these processes on indicator diagrams and how to interpret data, including the work done, from such diagrams.

(Specification 3.11.2.1 to 3.11.2.3)

2.1 THE FIRST LAW OF THERMODYNAMICS

A proper understanding of engines in cars (Figure 1) needs an understanding of **thermodynamics**, which is the study of the relationship between heat and other forms of energy. Thermodynamics has much wider applications than vehicle engines – it also governs the operation of some household appliances, such as fridges.

In order to lay the groundwork for the study of engine and refrigerator cycles in Engineering Physics Chapter 3, we will be dealing with gases and will not consider any change of state. Thermodynamic processes involve changes in the pressure, volume and temperature of gases. In Chapter 3 you saw that the three gas laws described the behaviour of an **ideal gas** and that these resulted in the **ideal gas equation**

$$pV = nRT$$

Figure 1 *Car engines work on thermodynamic processes – basically, the compression and expansion of hot gases*

where p is the gas pressure in $\mathrm{N\,m^{-2}}$ or Pa (pascal), V is the gas volume in $\mathrm{m^3}$, n is the number of moles of gas, R is the universal molar gas constant, $8.31\,\mathrm{J\,K^{-1}\,mol^{-1}}$ and T is the absolute temperature in K (kelvin).

If this equation is rearranged to become

$$\frac{pV}{T} = nR$$

it follows that, since nR is constant for a fixed mass of gas,

$$\frac{pV}{T} = \text{constant}$$

so

$$\frac{p_1V_1}{T_1} = \frac{p_2V_2}{T_2}$$

This gives us three simple situations, where the volume, the pressure or the temperature are kept the same, giving respectively pV = constant (Boyle's law), P/T = constant (pressure law) and V/T = constant (see section 3.4 of Chapter 3).

Here we will be approaching these situations from a different viewpoint – from the underlying laws of thermodynamics.

A gas that obeys the ideal gas equation exactly – an ideal gas – has no forces acting between the molecules, and hence the energy of the molecules is entirely kinetic and depends only on the temperature of the gas. The equation provides a good description of real gases if they are at low pressure and well above their liquefaction temperature.

For any real system, the principle of conservation of energy holds. A system refers to a fixed mass of some substance, enclosed by a boundary, outside of which are the **surroundings**. Boundaries can be moved, but processes happen within it. Heat or work can be transferred across the system boundary, causing compression and changes of state, for example.

A system has **internal energy**, denoted by U, which is the sum of the total kinetic energy of its constituent particles and (unless it is a near-ideal gas) the total potential energy of the particles. The internal energy U can be increased by ΔU, either by doing work on the system or by heating it, or by a combination of these.

The **first law of thermodynamics** relates the heat, the work and the internal energy, and is essentially the principle of the conservation of energy. It is given by the equation

$$Q = \Delta U + W$$

where Q = energy transferred *to* the system by heating, W = work done *by* the system and ΔU = increase in internal energy. All these quantities are measured in the unit of joule (J).

The convention for Q being heat *input* and W being work *output* (Figure 2) is historical, coming from a time when engineers were interested in getting heat into a system and getting work out. These were taken as the positive values. If heat is transferred out of the system, or work is done on the system, the energy values used in the equation must be negative.

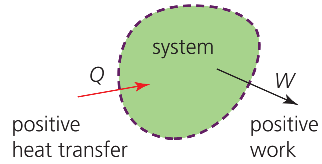

Figure 2 *The convention adopted is that positive heat is into the system and positive work is output or done by the system.*

If we rearrange the previous equation, it helps with understanding the physical situation:

$$\Delta U = Q - W$$

If energy (in the form of heat, Q) is *added* to a system, then the internal energy (U) will *increase*. However, if the gas *does work* (W) against the environment, then the energy for this will be at the expense of the internal energy, so U will *decrease*.

In summary:

- The internal energy of a gas will be *increased* if heat is *added* (Q is positive) or if it is being *compressed* (work done by the gas W is negative). The gas will get hotter, because the average molecular kinetic energy is proportional to the absolute temperature (see Chapter 3).
- The internal energy of a gas will *decrease* if heat is *lost* (Q is negative) or if the gas expands against atmospheric pressure or a piston (work done by the gas W is positive). The gas will get cooler.

Figure 3 shows the four possible scenarios.

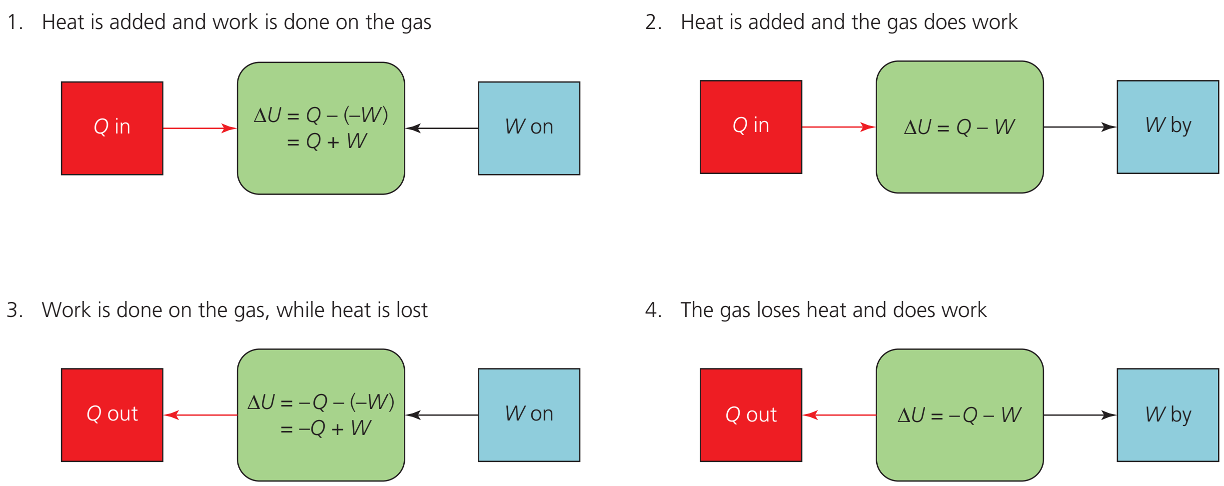

Figure 3 *The possible energy changes to a thermodynamic system. The centre box is the system itself. Notice the directions of the energy transfers.*

All we can ever calculate is how much the internal energy has *changed*. An absolute value cannot be determined, as we would need to know all the potential and kinetic energies of every particle.

Energy is a *scalar* quantity (rather like mass – we can add or subtract masses without reference to any direction, as it is not a vector quantity), so we can just add or subtract these values of energy. Consequently, the change of internal energy depends only on the *initial* and *final* states of the system, and not by how the change was actually brought about.

Worked example

An ideal gas is heated, transferring 100 J of energy to it. The gas expands, doing 20 J of work against the atmosphere. Calculate the change in internal energy of the gas. What will change as a result? Will it warm up or cool down?

The starting point is $\Delta U = Q - W$. It is always a good idea to write down the numerical values, using the accepted convention:

$$Q = +100\,\text{J}$$

$$W = +20\,\text{J}$$

So,

$$\Delta U = +100 - (+20) = 80\,\text{J}$$

The internal energy increases, so the ideal gas will warm up.

QUESTIONS

1. If the heat input to a gas is 2 MJ and the work done on the gas is 0.45 MJ, calculate the change in internal energy.
2. In a certain process, 6000 J of heat energy is added to the system, while 11000 J of work is done through expansion.
   a. Calculate the change in the system's internal energy.
   b. If the system is an ideal gas, state what has happened to its temperature.
   c. Explain why, if the system is not an ideal gas, we cannot be certain what happens to its temperature.
3. In a thermodynamic process, 5000 J of heat energy is added to the system. The internal energy of the system increases by 3000 J. Calculate the work done. Is it done on or by the system?
4. A gas loses 4 MJ of its internal energy in a process, despite 10 MJ of heat energy being added. Comment on what has happened.

KEY IDEAS

- The ideal gas equation is

  $$pV = nRT$$

  where p is the gas pressure in N m^{-2} or Pa, V is the gas volume in m^3, n is the number of moles of gas, R is the universal molar gas constant, $8.31\,\text{J K}^{-1}\,\text{mol}^{-1}$, and T is the absolute temperature in K.
- The internal energy of an ideal gas depends only on the temperature.
- The first law of thermodynamics is a statement of the conservation of energy, relating heat, work and internal energy by the equation

  $$Q = \Delta U + W$$

  where Q = energy transferred *to* the system by heating, W = work done *by* the system and ΔU = the change in the internal energy of the system.

Stretch and challenge

Reversibility

In the kinetic theory (Chapter 3), it is necessary to make a number of assumptions. By considering the concept of an ideal gas, it was mathematically possible to establish straightforward links between the kinetic energy of the particles and the temperature. Real gases will approximate to these equations, particularly monatomic gases at low temperature (well below the boiling point) and at low pressure. In other situations, more adjustments are needed. Trying to get exact mathematical relationships for real gases is almost impossible.

In thermodynamics, the derivation of the equations governing changes in gases requires further assumptions.

In order to be able to specify values of the pressure, volume and temperature of the system at any point in time, we need changes to occur *very slowly* – in theory infinitely slowly. Otherwise, in a rapid change, the pressure or temperature would not be uniform throughout the system. We need the system to pass through a series of equilibrium states. Such a change is **reversible** – a small change in conditions would make the change move in the other direction. The energy changes would also reverse. Since we need to obey the principle of conservation of energy, there can be *no dissipative effects*, such as friction or turbulence, which would transfer heat out of the system. It must be possible for the system to return to its exact initial conditions.

Changes in real engines are not reversible, because the processes usually happen far too quickly. But the theory of an ideal reversible heat engine is still very useful, as we will see.

QUESTIONS

Stretch and challenge

5. In nature as well as engineering, there are no processes that could be classed as truly reversible (they are therefore irreversible). Explain whether or not you agree with this statement.

2.2 ISOTHERMAL PROCESSES

Thermodynamic processes are sometimes called **non-flow processes**, which for gases seems a misnomer, because surely gases, as fluids, should flow by definition? The term actually means a process during which the fluid does not move in or out of the system during the process – the system is *closed*. When we consider an engine cycle, we know that fuel needs to be introduced initially (and the exhaust vented ultimately), but the process itself that generates power is a closed system that goes through a cyclical process.

There are four different non-flow thermodynamic processes; in each case, there is a certain restriction imposed on the system. First, we will consider an **isothermal process** ('iso' is from the Greek for equal, and 'thermos' is from the Greek for hot). This is a process in which the system stays at the same temperature.

For a gas that approximates to ideal, the internal energy consists only of the total kinetic energy of the molecules. At constant temperature, the average molecular kinetic energy is constant, so for a closed system (with constant number of molecules) the total internal energy must be constant. So $\Delta U = 0$.

The first law of thermodynamics therefore reduces to

$$Q = W$$

So, if a gas expands and does external work W, an amount of heat Q has to be supplied to the gas in order to maintain its temperature constant, and vice versa.

An isothermal change in a system requires the gas to be kept in a thin-walled vessel that is composed of an excellent conducting material, surrounded by a constant-temperature bath. Any expansion or contraction within the system must take place slowly, so that the pressure and volume may be specified at any point in time, and so that the temperature is always constant throughout. Under such conditions, the process could be reversed and returned to its initial state, so the process is termed 'reversible'.

For a *reversible isothermal* change, it follows from the ideal gas equation that at all stages of the process

$$pV = \text{constant}$$

because n and T are both constant. So

$$p_1V_1 = p_2V_2$$

where p_1 and V_1 are the initial pressure and volume of the gas, and p_2 and V_2 are the pressure and volume after the isothermal change. This is, of course, *Boyle's law* (see section 3.4 of Chapter 3). As pressure increases, the volume goes down in inverse proportion (Figure 4).

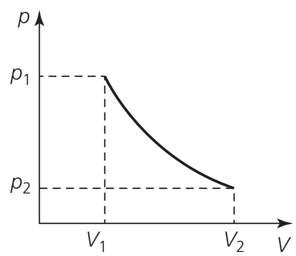

Figure 4 *An isothermal expansion: pV = constant*

The p–V curve in Figure 4 is for a constant-temperature change, that is, an isothermal change, and the curve itself is referred to as an 'isothermal'.

What happens at a different temperature? The product pV is still a constant, but a different constant: $pV \propto T$. So, for different T, we get a family of isothermals, which move progressively away from the origin as the temperature T increases (Figure 5).

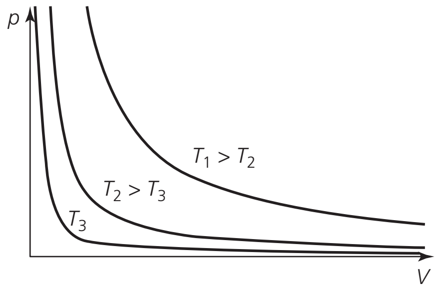

Figure 5 *Isothermals: the higher the temperature, the larger the value of nRT, so the curve is further from the origin.*

QUESTIONS

6. A gas at a pressure of $15\,\mathrm{N\,m^{-2}}$ occupies a volume of $5.0\,\mathrm{m^3}$. If it is compressed isothermally to $2.0\,\mathrm{m^3}$, calculate its new pressure.
7. A gas at a pressure of $78\,\mathrm{N\,m^{-2}}$ occupies a volume of $50\,\mathrm{cm^3}$. It is kept at constant temperature. If the pressure applied to the gas decreases to $43\,\mathrm{N\,m^{-2}}$, calculate the new volume it occupies.
8. The curve AB in Figure 6 shows the isothermal compression for 0.12 mol of an ideal gas.

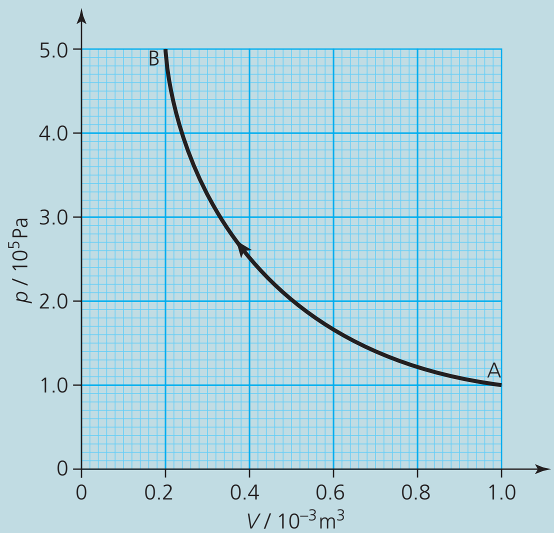

Figure 6

   a. By choosing three points on the curve, verify that the curve is isothermal.
   b. Calculate the temperature of the gas.
   c. Using the three points that you used in part a, sketch the curve on graph paper. On it, sketch also the isothermal curves for the same sample of gas at $T = 50\,\mathrm{K}$ and $T = 200\,\mathrm{K}$.

2.3 CONSTANT-PRESSURE AND CONSTANT-VOLUME PROCESSES

Constant-pressure process (work is done on/by the gas)

Consider the situation of a constant force being applied to a piston, resulting in the gas in the cylinder being compressed (Figure 7). Work is done on the gas by this force, and it is helpful to derive an expression for the numerical value of this work, in joules.

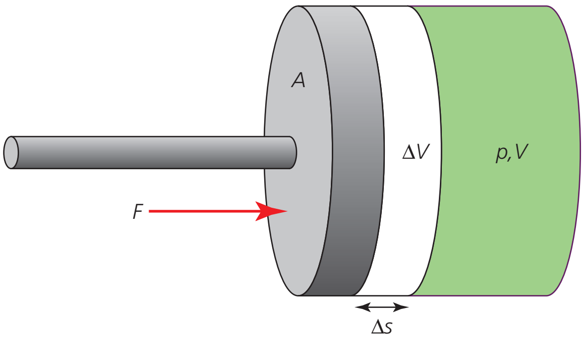

Figure 7 *Calculating the work done on a gas by a piston*

Pressure is calculated from force/area ($P = F/A$) and work done is force times distance ($W = F \times \Delta s$), so the work done during the compression is

$$W = pA\Delta s$$

where A is the cross-sectional area and Δs is the small distance moved by the piston. As $A \times \Delta s$ is the change in volume (ΔV), we can write

$$W = p\Delta V = (V_2 - V_1)$$

where V_1 and V_2 are the initial and final volumes of the gas.

For a constant-pressure process, we can therefore write the first law of thermodynamics as

$$Q = \Delta U + p\Delta V$$

For a compression, the work is done *on* the gas, so ΔV and W will be negative.

We can apply this to calculate the work done by or on a gas when the pistons move in a real engine.

Worked example

A gas is enclosed in a container which is fitted with a piston of $0.2\,m^2$ cross-sectional area. A constant pressure of $2000\,N\,m^{-2}$ is applied, which moves the piston in by 10 cm. Calculate the work done on the gas. If 10 J of energy is added by heating, calculate the total increase in internal energy of the gas.

The work done is

$$\begin{aligned} W &= \text{pressure} \times \text{change in volume} \\ &= 2000 \times 0.2 \times (-0.1) \\ &= -40\,J \end{aligned}$$

Work of 40 J is done *on* the gas, hence the negative sign. This will produce an increase in the internal energy of the gas of 40 J.

It is straightforward to see that if 10 J of energy is added by heating, in addition, then the total increase in internal energy will be 50 J.

However, we could have used the equation:

$$Q = \Delta U + p\Delta V$$

thus giving

$$\Delta U = Q - p\Delta V$$

$$= 10 - (-40) = 50\,J$$

Note the sign convention being applied here.

This was a straightforward example. In general, it is a good idea to write out the first law for the problem under consideration.

QUESTIONS

9. A gas is enclosed in a container that is fitted with a piston of $0.5\,m^2$ cross-sectional area. A constant pressure of $6000\,N\,m^{-2}$ is applied, which moves the piston in by 45 cm.
   a. Calculate the work done on the gas.
   b. If the internal energy rises by 1000 J, calculate the amount of heat energy lost during the compression.

10. Copy and complete Table 1 for a gas that is being put under pressure.

Q / J	p / $N\,m^{-2}$	ΔV / m^3	W / J	ΔU / J
100 in	50000	–0.02		
100 in	40000	0		
100 out	25000	–0.05		
500 out	30000	–0.03		

Table 1

Constant-volume process

If the volume of a system such as a gas is held constant, the system does *no* work ($W = p\Delta V = 0$), because work done by (or on) the gas would require an expansion (or compression) and hence a change of volume. The first law of thermodynamics reduces to

$$\Delta U = Q$$

If heat is absorbed by a system (that is, Q is positive), the internal energy of the system must increase, and vice versa.

QUESTIONS

11. Sketch on one set of p–V axes
    a. a constant-volume process
    b. a constant-pressure process.

    On each, draw an arrow to show the direction of the change, if the temperature increases during the process.

2.4 ADIABATIC PROCESSES

One very important type of change that can occur to a system is an **adiabatic process**. It refers to an *isolated* system, where *no energy is able to transfer in or out by heating* from or to the external environment.

So, any work being done *on* the system increases the internal energy (in other words, its temperature increases). Since W is used to show work being done *by* the system, $-W$ will be the work done *on* the system. No heat is involved in the process, so $Q = 0$. The first law of thermodynamics therefore reduces to

$$-W = \Delta U$$

If the system is allowed to do work, then this will be at the expense of the internal energy, and its temperature will decrease:

$$W = -\Delta U$$

where the $-\Delta U$ term shows the internal energy as decreasing.

For an ideal gas, the internal energy is the total molecular *translational* kinetic energy (see Chapter 3). But real gases are not composed of simple spheres dashing about (Figure 8). The molecules may not be monatomic, but diatomic or polyatomic, which means that they also have *rotational* and *vibrational* kinetic energy.

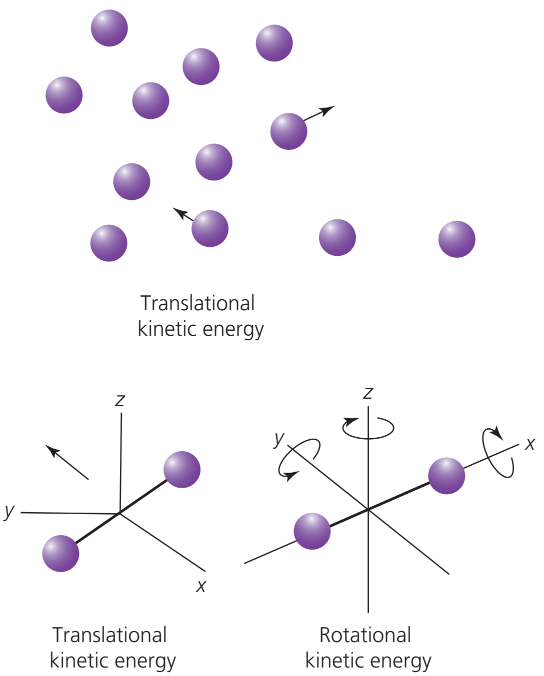

Figure 8 *The atoms of a monatomic gas can only have translational kinetic energy. But atoms that are bound in molecules are able to rotate, so they possess additional forms of kinetic energy.*

Each method by which a system can absorb energy is called a 'degree of freedom'. Monatomic gases can absorb energy in three translational directions (x, y and z), so are said to have three degrees of freedom. Diatomic molecules can also rotate in several ways, although that which is around its own axis (the x-axis in Figure 8) can be ignored. It is possible to have vibrational energy as well, but this is significant only at very high temperatures.

So, any change in the internal energy will be different depending on the **atomicity** of the gas. It can be shown that, if the gas is in other ways ideal (so that all the internal energy is kinetic energy), when it undergoes a *reversible* adiabatic expansion or contraction (to keep the equilibrium requirement), then

$$pV^{\gamma} = \text{constant}$$

So we obtain

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

where γ is a constant for the particular gas that is dependent on the degrees of freedom and hence the atomicity of the gas (Table 2). The constant γ has no unit.

γ	Gas atomicity
1.67	Monatomic
1.40	Diatomic
1.33	Polyatomic

Table 2 *The value of γ for gases of different molecular structure*

When the graph of pV^{γ} is plotted for an expansion or compression, we get the shape shown in Figure 9, showing an expansion from V_1 to V_2.

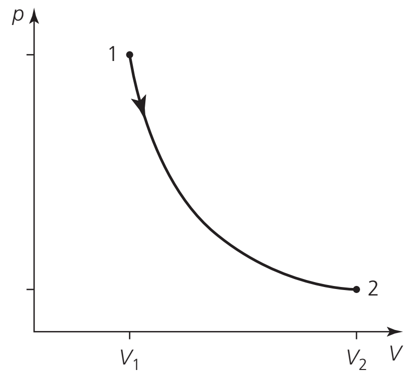

Figure 9 *An adiabatic expansion*

This p–V curve looks rather like that for an isothermal expansion (Figure 4). There is a difference, which becomes clear when the two types of curve are plotted on the same axes (Figure 10). Again, we are dealing with an expansion. As the equation has a power term within it (V^{γ}), at any point (p, V) it is steeper than the isothermal. The expansion results in cooling from T_1 to T_2.

It is clear from the physical process taking place that this will be the case: the expansion is the result of the gas doing work against the environment. The energy used is at the expense of its internal energy (no heat energy is involved), so it cools down.

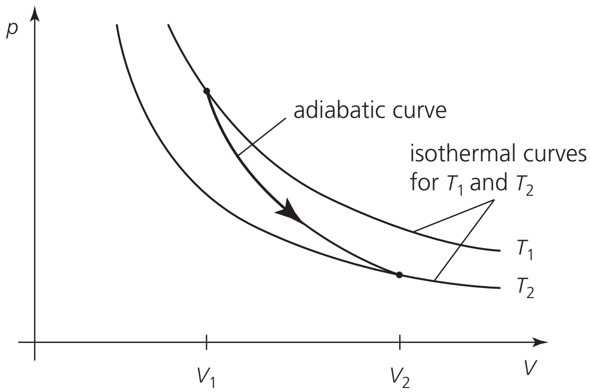

Figure 10 *An adiabatic expansion. As the curve is steeper, it will cross isothermal curves.*

A system with perfect thermal insulation would undergo an adiabatic change. Such a perfect situation is not possible in practice. But another possibility is a process that happens so rapidly that there is insufficient time for heat to transfer in or out. (Rather oddly, it is possible to consider this process as reversible: since no heat is lost out of the system, work done by the external process will equal the rise in internal energy of the working fluid. If there is a process that can restore the fluid back to its initial state by reducing its internal energy and allowing it to do external work, then this would be an overall reversible process.) This describes what happens in parts of the cycle of a real heat engine, so we can view these stages as adiabatic.

Worked example 1

A gas at a pressure of $20\,\text{N m}^{-2}$ occupies a volume of $5\,\text{m}^3$. The gas has a value of $\gamma = 1.40$. If the gas is compressed adiabatically to $2\,\text{m}^3$, calculate its new pressure.

As the compression is adiabatic, we use the equation

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

Substituting into this equation gives

$$20 \times (5)^{1.4} = p_2 \times (2)^{1.4}$$
$$20 \times 9.52 = p_2 \times 2.64$$
$$p_2 = 72.1\,\text{N m}^{-2}$$

Worked example 2

A gas at a pressure of $40\,\text{N m}^{-2}$ occupies a volume of $50\,\text{cm}^3$. The gas has $\gamma = 1.33$. If the gas is allowed to expand very quickly and its pressure drops to $10\,\text{N m}^{-2}$, calculate the new volume.

In this situation, the 'very quickly' implies an adiabatic expansion, so we use

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

Substituting the values gives

$$40 \times (50)^{1.33} = 10 \times (V_2)^{1.33}$$
$$40 \times 182 = 10 \times (V_2)^{1.33}$$
$$728 = (V_2)^{1.33}$$
$$V_2 = 142\,\text{cm}^3$$

QUESTIONS

12. A gas at a pressure of $60\,\text{N m}^{-2}$ occupies a volume of $250\,\text{cm}^3$. The gas has $\gamma = 1.67$. If the gas is pressurised very quickly and its pressure increases to $150\,\text{N m}^{-2}$, calculate the new volume. Assume no heat leaves the system.

Another form of the adiabatic equation

When the equation

$$p_1V_1^{\gamma} = p_2V_2^{\gamma}$$

is combined with the ideal gas equation

$$pV = nRT$$

and the pressure term p is eliminated, we get, for a fixed mass of gas, the result

$$TV^{(\gamma-1)} = \text{constant}$$

So we can write

$$T_1V_1^{(\gamma-1)} = T_2V_2^{(\gamma-1)}$$

This allows the final temperature to be calculated in an adiabatic expansion or contraction.

Worked example 3

A gas occupies a volume of 25 m^3 and is at a temperature of 273 K. The gas has a value of γ of 1.40. The gas is now compressed adiabatically to 10 m^3. Calculate its new temperature.

This compression is adiabatic, but we are now concerned about the new temperature, so we use the equation

$$T_1V_1^{(\gamma-1)} = T_2V_2^{(\gamma-1)}$$

Substituting into this equation gives

$$\begin{aligned} 273 \times (25)^{(1.40-1)} &= T_2 \times (10)^{(1.40-1)} \\ 273 \times (25)^{0.40} &= T_2 \times (10)^{0.40} \\ 273 \times 3.6 &= T_2 \times 2.5 \\ T_2 &= 393 \end{aligned}$$

The new temperature T_2 is 393 K.

QUESTIONS

13. A gas is initially at a temperature of 40 °C and occupies 450 cm^3. The gas has $\gamma = 1.33$. The gas expands very quickly and its temperature drops to 10 °C. Assuming no heat leaves the system, calculate its final volume.

Summary of non-flow processes

An overall summary of the application of the first law of thermodynamics to these special cases of non-flow processes is given in Table 3.

Process	Restriction	First law of thermodynamics	Ideal gas equation
Isothermal	$\Delta U = 0$	$Q = W$	pV = constant
Adiabatic	$Q = 0$	$W = -\Delta U$	pV$^\gamma$ = constant
Constant volume	$W = 0$	$\Delta U = Q$	$\frac{p}{T}$ = constant
Constant pressure	$W = p\Delta V$	$Q = \Delta U + p\Delta V$	$\frac{V}{T}$ = constant

Table 3 *A summary of the special cases of the first law of thermodynamics*

QUESTIONS

14. When you are pumping up a tyre on a bicycle (Figure 12), you will have noticed that the bicycle pump becomes warmer. Use the first law of thermodynamics to explain this effect.

Figure 11

15. A gas (whose γ is 1.4) is initially at a pressure of 40 N m^{-2} and occupies a volume of 10 m^3.

a. If it is compressed *isothermally* to 2 m^3, calculate its new pressure.

b. If, instead, it is compressed *adiabatically* to 2 m^3, calculate its new pressure.

c. Sketch a p–V graph for the changes to show the effect of the two types of compression.

Stretch and challenge

Specific heat capacities

In section 3.2 of Chapter 3 we defined the specific heat capacity (c) of a substance by $Q = mc\Delta\theta$. For a gas, which can change significantly in volume on heating or cooling, defining specific heat capacity purely in terms of unit temperature rise causes problems. So, instead, two particular cases are considered: processes that are at a constant volume and those that are at a constant pressure. Two **principal specific heat capacities** result:

- c_V is the energy required to produce unit temperature rise in unit mass of the gas at constant volume
- c_p is the energy required to produce unit temperature rise in unit mass of the gas at constant pressure.

It is found, in practice, that the ratio of these two values, c_p/c_V, is important in several physical processes in gases, including sound waves and adiabatic changes. It is equal to γ in the adiabatic equation, given previously:

$$\gamma = c_p/c_V$$

As shown in Table 2, γ depends on the atomicity of the gas – on the number of types of kinetic energy that the molecules can have (whether it is rotational, vibrational or translational).

The rapid expansion and contraction of air when sound waves pass through it is a near-adiabatic process. We should therefore not be too surprised to discover that γ is involved in the equation for the speed of sound in a gas:

$$c = \sqrt{\frac{\gamma p}{\rho}}$$

where p is pressure and ρ is the density of the gas.

KEY IDEAS

- The first law of thermodynamics states that $Q = \Delta U + W$.
- An *isothermal* process is one where the system stays at the *same temperature*. For an ideal gas, $\Delta U = 0$.

  From the first law of thermodynamics, $Q = W$.

  From the ideal gas equation, $pV = \text{constant}$.
- The work done W by a gas when its volume changes by ΔV at constant pressure p can be calculated from $W = p\Delta V$. Work done on a gas is negative.

  The first law of thermodynamics can then be written as $Q = \Delta U + p\,\Delta V$.
- An *adiabatic* process is where *no heat* is able to transfer in from or out to the external environment, $Q = 0$.

  From the first law of thermodynamics, $W = -\Delta U$.

  The equation $pV^{\gamma} = \text{constant}$ applies, where γ is a constant that depends on the atomicity of the gas.

2.5 DETERMINING THE WORK DONE FROM A p–V DIAGRAM

A p–V diagram for a thermodynamic process that has an arrow to show the direction of the change is known as an **indicator diagram**. It is an important tool in engineering, and we will look at many of these. The indicator diagram in Figure 12 shows an expansion from low V_1 to high V_2. Work is done by the expanding system, $W > 0$.

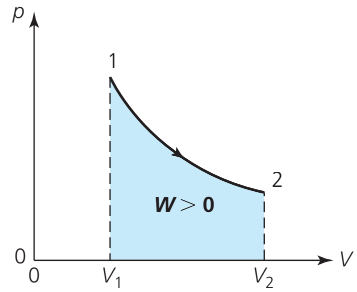

Figure 12 *A p–V diagram, or indicator diagram, showing the expansion of a gas. The work done by the gas, W > 0, is the area under the curve. If the arrow is reversed, the diagram will represent the compression of a gas and the area under the curve will be the work done on the gas. In this case, W < 0.*

We found in Engineering Physics section 2.3 that, for a gas at constant pressure, if we multiply the pressure by the change of volume, we calculate the work done. In much the same way that the area under a velocity–time graph gives the displacement (*see section 9.4 in Chapter 9 in Year 1 Student Book*), a significant feature of a p–V diagram is that the area under the curve, down to the volume axis, has a physical meaning:

The area under a p–V diagram, down to the V-axis, gives the work done.

The general rule for indicator diagrams is that, if the volume is *increasing*, then work is being done *by* the gas, and vice versa.

If the graph shape is simple, the area can be calculated using geometry. The graph in Figure 13 shows a gas increasing in volume, against a constant pressure (for example, atmospheric pressure). The work done by the gas is the area under the line, which is simply a rectangle. Note that the area under the curve must extend down to the volume axis (that is, to the $p = 0$ line).

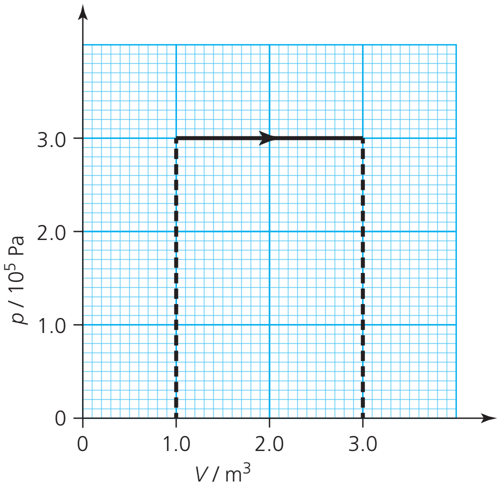

Figure 13 *A constant-pressure change*

The work done in Figure 13 is $p\Delta V = 3 \times 10^5 \times (3 - 1) = 6 \times 10^5$ J. As it is an expansion, this has a *positive* value – work is being done *by* the gas.

However, when dealing with curves, which are almost always the case in real applications, there are two options. The first is an 'estimation of area' method, which Assignment 1 takes you through. This involves counting graph squares. An averaging technique (squares that are half-filled or more are included, while squares that are less than half-filled are ignored) leads to a reasonable approximation.

However, a second, more sophisticated, technique can be applied. If the equations of the curves are known, which is generally the case, the area can be calculated using *calculus*, which will be more accurate. It can be shown that the work done, W, by the gas in changing its volume from an initial state V_1 to a final state V_2 is given by the following integral:

$$W = \int_{V_1}^{V_2} p \, dV = \text{area of shaded region below curve}$$

ASSIGNMENT 1: PLOTTING AND ANALYSING INDICATOR DIAGRAMS

(MS 0.1, MS 0.2, MS 3.1, MS 3.2, MS 3.8)

The data in Table A1 are for an isothermal change in a sample of an ideal gas, measured at certain points at a specific temperature.

	p / 10^5 Pa	V / 10^{-3} m^3
A	6.4	5.0
B	5.0	6.4
C	4.0	8.0
D	3.0	10.7
E	2.0	16.0
F	1.4	23.0
G	1.0	32.0

Table A1

Questions

A1 a. Plot these points onto graph paper accurately. Use as much of the paper as you can, but (for a later question) ensure that your pressure axis reaches 10×10^5 Pa.

b. Draw a best-fit curve through the points.

c. There are several square sizes on graph paper. For the graph paper and the axis scales that you are using, determine the numerical and physical significance of each square.

d. In order to estimate the work done, make an estimate of the number of squares under the curve. Would you choose the mm squares or the cm squares? Why?

e. What is your estimate of the total work done?

f. As it is an estimation method, state the uncertainty in your answer.

g. What will be the physical situation if the arrows on your diagram go

i. from A → G

ii. from G → A?

h. What temperature would these changes occur at, if there was 1 mol of gas? Label your curve with that temperature.

The temperature of the same gas sample is now raised by 100 K. At this new temperature, the gas undergoes an isothermal expansion.

Questions

A2 Using the same volumes as in Table A1, calculate and tabulate the corresponding pressures.

A3 Repeat the steps from question **A1** using your new data. Plot the second graph on the same axes as the first. Label your second curve with the new temperature.

A4 Will these two curves eventually cross?

What you have plotted and drawn are two of a family of curves, each unique for a particular sample of gas at a specific temperature. Moving along one of these 'isothermals' from one point (p_1, V_1) to another (p_2, V_2), in either direction, results in the same work done, either by or on the gas. But more work is done when moving between V_1 and V_2 at a higher temperature because the curve is further from the origin and so has a larger area beneath it. This is the clue we need to understand an engine cycle.

QUESTIONS

16. A gas is compressed by a sequence of constant-volume and constant-pressure processes, as shown in the indicator diagram in Figure 14. Calculate the work being done *on* the gas in total from A to E.

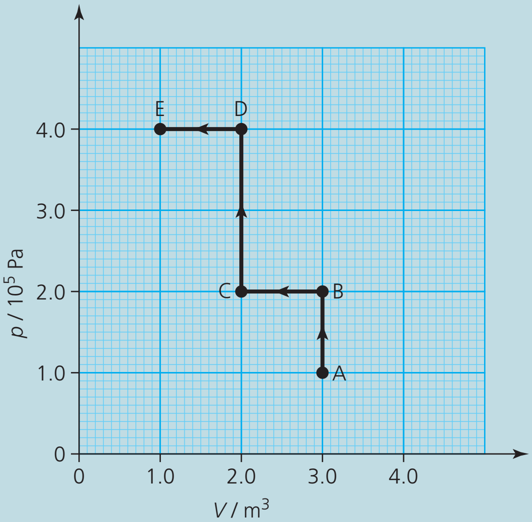

Figure 14

17. A gas does work against the environment, and its pressure and volume change as shown in Figure 15. Estimate the work done by the gas.

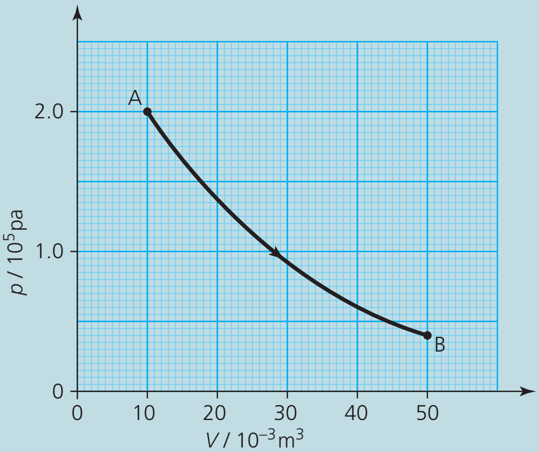

Figure 15

Figures 16 and 17 show some more p–V diagrams for different situations. Figure 16 compares an isothermal expansion and an adiabatic expansion of an ideal gas from the same initial state. The adiabatic compression curve is steeper than the isothermal one. The adiabatic expansion results in a lower final pressure and a fall in temperature as the curve crosses a lower-temperature isothermal.

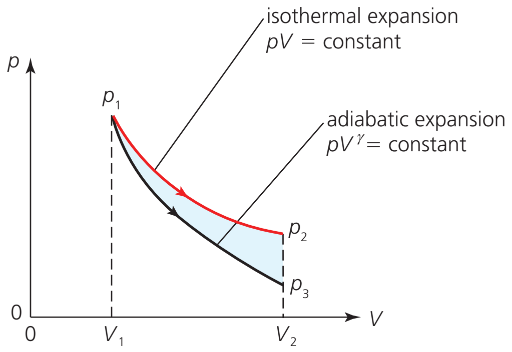

Figure 16 *A comparison of an isothermal and an adiabatic expansion. The area under the curve is greater for the isothermal expansion than for the adiabatic expansion. This means that more work is done in the isothermal expansion.*

The p–V diagram in Figure 17 shows an adiabatic expansion followed by an isothermal compression.

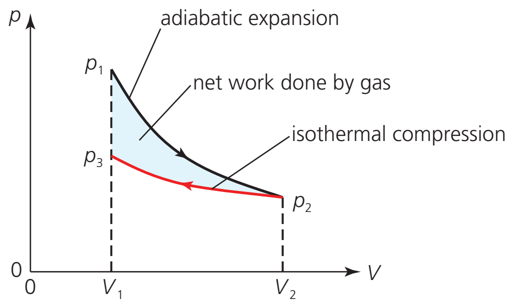

Figure 17 *A p–V diagram of a gas that has been expanded adiabatically from volume V_1 to V_2 and then compressed isothermally back to its original volume V_1. The net work done by the gas is the region between the curves.*

Cyclic processes

A **cyclic process** is one in which the system undergoes two or more consecutive changes such that the final state is the same as the initial state. This one **cycle** is repeated continuously. In the cycle shown in Figure 18, the area under the expansion curve is the work done *by* the gas. The area under the compression curve is the work done *on* the gas. As work is a scalar value, the difference between these two numbers represents the net work done *by* the gas, which corresponds to the difference in area. Therefore

$$\text{work done per cycle} = \text{area of loop}$$

This will be applied in Engineering Physics Chapter 3 when we consider heat engine cycles. Different curves connected together will produce different areas, and so differing amounts of work being done, either by the gas or on the gas. The purpose of an engine is to

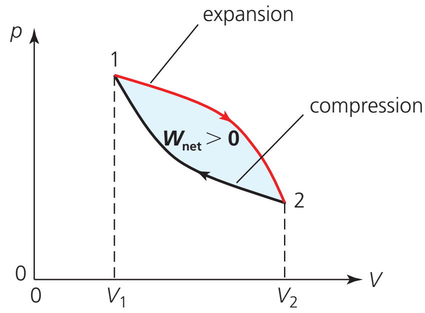

Figure 18 *A p–V diagram showing a thermodynamic cycle in which the system is taken back to its original state. The net work is positive, since the area under the expansion curve is greater than the area under the compression curve.*

force the working substance into such a cycle, so that the work done by the gas exceeds the work done on the gas as it is returned to the starting point. The net difference means that the engine is doing useful work.

This is the basis of a variety of real engines, ranging from steam engines to petrol and diesel engines, all of which underpin industrialised societies. Most were invented in order to address a specific need. One of the earliest was by Thomas Newcomen, in 1712 (Figure 19). In those days, a major problem in the mining of tin and coal was flooding. He considered ways of improving the pumping out of the water and consequently designed the first practical steam engine.

Figure 19 *A postage stamp commemorating the Newcomen steam engine. In the engine, water was heated, producing steam. As the steam condensed, it pulled down a piston attached to a large wooden beam, which rocked upon a central fulcrum. The chain attachment was linked directly to a pump at the bottom of the mine.*

QUESTIONS

18. In the cycle shown in Figure 20, curves AD and BC are both isothermals.

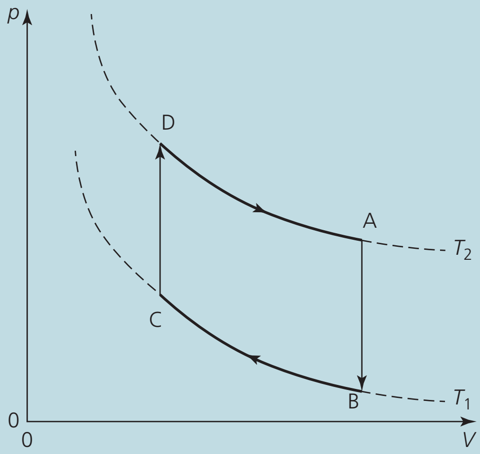

Figure 20

a. Is T_2 higher or lower than T_1?

b. For each section (DA, AB, BC, CD), write down the process that is occurring at that stage in the process.

c. For each section (DA, AB, BC, CD), write down whether work is being done *on* or *by* the gas.

d. What is the significance of the area enclosed by the cycle?

19. For the cycle in Figure 21, estimate the net work done.

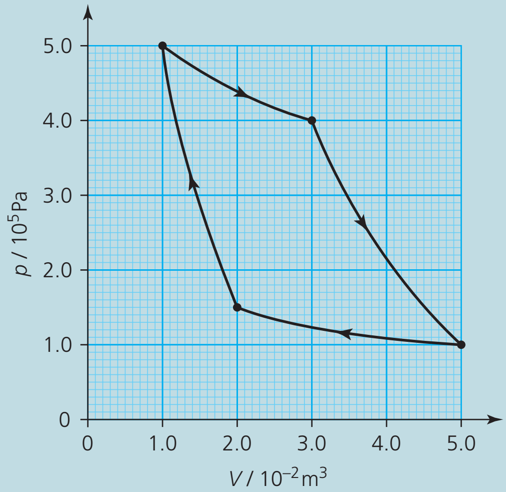

Figure 21

KEY IDEAS

- A pressure versus volume plot for a thermodynamic system is known as a p–V diagram or an indicator diagram. Arrows show the direction of the process.
- The area under the curve for an expansion is equal to the work being done by the gas.
- The area enclosed by the loop of a cyclic p–V process is the work done by (or on) the gas.

ASSIGNMENT 2: EVALUATING PERPETUAL MOTION MACHINES

The first law of thermodynamics $Q = \Delta U + W$ is a statement of the conservation of energy: energy cannot be created or destroyed. We know that perpetual motion cannot occur. Yet there has always been a fascination with the idea, and many people through the centuries have strived to create a perpetual motion machine – one that would run forever providing an inexhaustible source of energy.

This is not just the province of crackpots – it has attracted the thinking of many well-respected scientists, some of whom tried to prove and some of whom tried to disprove whether such machines could actually work, either theoretically or experimentally. Isaac Newton considered it possible; Leonardo da Vinci was against the idea, but designed machines he hoped would work. Robert Fludd tried building such machines; Johann Bessler claimed to have built several perpetual motion machines; and Bishop John Wilkins (a founder of the British Royal Society) suggested such a device. More recently, Nikolai Tesla claimed to have discovered a principle as the basis for a perpetual motion machine, but did not build it. Even in the 21st century, there have been attempts to create such devices. Richard Feynman has been involved with the case against – he used a 'Brownian ratchet' in his lectures to show why a perpetual motion machine would not work.

Sometimes, the fatal flaw behind a design is not obvious, so that at first glance it looks promising, but on closer inspection a law of physics is seen to be broken. Perpetual motion machines are sometimes classified by which physics laws they break.

1. A **perpetual motion machine of the first kind** produces work without any energy input. This violates the first law of thermodynamics.
2. A **perpetual motion machine of the second kind** turns heat energy totally into mechanical work. This does not violate the first law. However, in Engineering Physics Chapter 3 we will see that the second law of thermodynamics states that no machine can convert heat totally into useful work – so this law is violated.
3. A **perpetual motion machine of the third kind** is one with no frictional effects, so it can maintain its motion forever. Superconductors, which are real and in use, have no resistance below a 'critical temperature', so a current induced into a ring of such material will circulate (in theory, forever). This current flow produces a magnetic field, which is an energy store, but it does not continually produce energy.

Questions

A1 Your task in this assignment is to decide, as a group, whether ideas for perpetual motion are feasible or whether physics principles are violated. You will need to think critically and discuss.

Each member of the group researches one idea, makes notes and considers arguments for and against, then presents their findings to the rest of the group. As a group, discuss and decide what flaws are present or what principles are being violated. Can you classify the idea as one of the three kinds in the list?

Some suggestions as starting points for research are given here.

a. The 'over-balanced wheel' (Figure A1), first suggested in 1659.

Search terms: (shifting mass) overbalanced wheel, Leonardo da Vinci, Arabian wheel

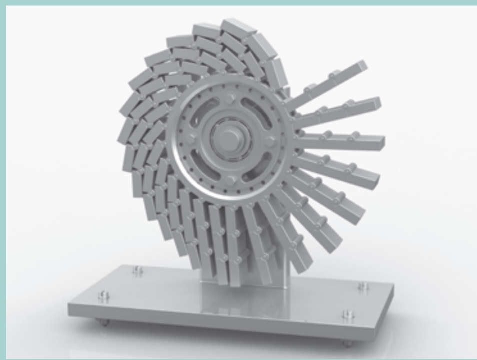

Figure A1 *The weights on the right have fallen further out from the centre so will exert a greater torque than those on the left, resulting in rotation in a clockwise direction. By attaching this to a motor, power could be produced.*

b. A ring-shaped *superconductor* carrying a current. The magnetic field due to this current could induce a current in a motor.

Search terms: superconductor, zero resistance, critical temperature, induction

c. A self-powered *electric car*, using wind turbines and solar panels.

Search terms: wind-powered car, solar-powered car, wind power for cars

d. *Magnetic levitation*. Can this be 'tapped' somehow?

Search terms: maglev, superconductors, Meissner effect, (electro)magnetic repulsion

e. The *drinking bird toy* (Figure A2). Can such nodding motion be utilised to provide energy?

Search terms: drinking bird, dipping bird, methylene chloride

Figure A2 The drinking bird seems to continue nodding up and down with no apparent energy input.

Stretch and challenge

f. *Matter/antimatter inequality*. There is more matter than antimatter in the known Universe. It seems that charge is not conserved and there is a 'handedness' in nature, called 'CP violation'. Can we take advantage of this to get 'something for nothing', or is new physics needed, beyond the standard model, to explain CP violation?

Search terms: matter, antimatter, asymmetry, CERN, CP violation

PRACTICE QUESTIONS

1. Figure Q1 shows a child's 'pop' gun in which a piston is pushed quickly along the barrel, compressing the air in the barrel. When the pressure is high enough, the cork is expelled at high speed from the end of the barrel.

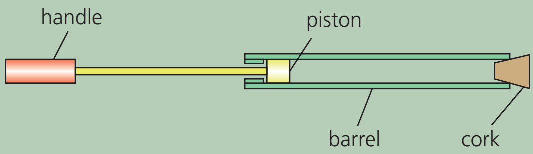

Figure Q1

Figure Q1 shows the gun before it is 'fired'. The air in the barrel is at a pressure of 1.0×10^5 Pa, a temperature of 290 K and the volume is $2.1 \times 10^{-5}\,m^3$.

a. i. The volume of air in the barrel at the instant the cork is expelled is $1.2 \times 10^{-5}\,m^3$.

Calculate the pressure of the air in the barrel at the instant the cork is expelled.

Assume that the air is compressed adiabatically. [adiabatic index, γ, for air = 1.4]

ii. Calculate the maximum temperature reached by the air in the gun. Give your answer to an appropriate number of significant figures.

b. The work needed to compress the air adiabatically from $2.1 \times 10^{-5}\,m^3$ to $1.2 \times 10^{-5}\,m^3$ is 1.4 J. Use the first law of thermodynamics to determine the change in internal energy of the air during the compression. Explain how you arrived at your answer.

c. Explain, giving your reasons, whether the volume of air in the barrel at the point when the cork leaves the gun would be less than, equal to, or greater than $1.2 \times 10^{-5}\,m^3$ if the handle of the gun had been pushed in slowly. Assume there is no leakage of air past the cork or piston. You may find it helpful to sketch a p–V diagram of the compression.

AQA Unit 5C June 2012 Q1

2. Figure Q2 shows a model rocket for demonstrating the principle of rocket propulsion.

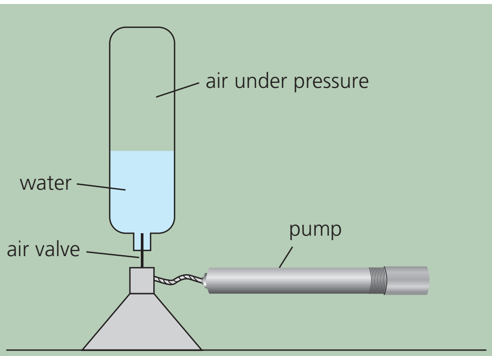

Figure Q2

Air is pumped into an upside-down plastic bottle that has been partly filled with water. When the pressure reaches 3.6×10^5 Pa (i.e. 2.6×10^5 Pa above atmospheric pressure), the air valve is forced out by the water pressure and the air in the bottle expands. The expanding air forces the water out of the neck of the bottle at high speed; this provides the thrust that lifts the bottle high into the air.

Figure Q3 shows the variation of pressure with volume for the air initially in the bottle as it expands from 3.6×10^5 Pa to atmospheric pressure, assuming the expansion is adiabatic.

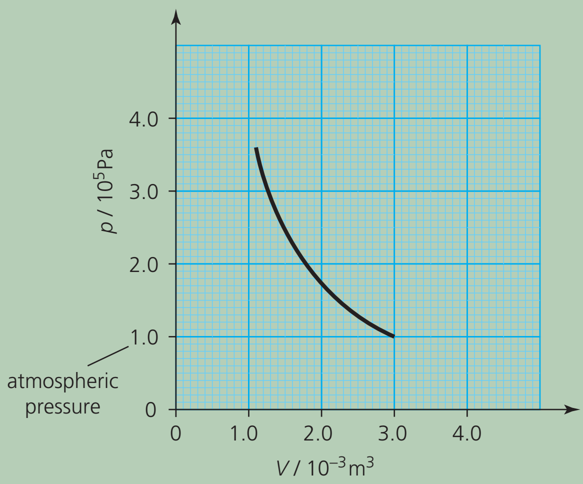

Figure Q3

a. Use the graph to estimate the work done by the air as it expands from a pressure of 3.6×10^5 Pa to atmospheric pressure.

b. With reference to Figure Q3, state and explain whether the rocket would have reached the same height if the air had expanded isothermally.

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3. In order to design engines, engineers must consider the thermodynamic processes that take place in gases. Figure Q4 is a p–V graph showing the variation of pressure and volume of a fixed mass of gas in a cylinder. A is one point in the cycle undergone by the gas.

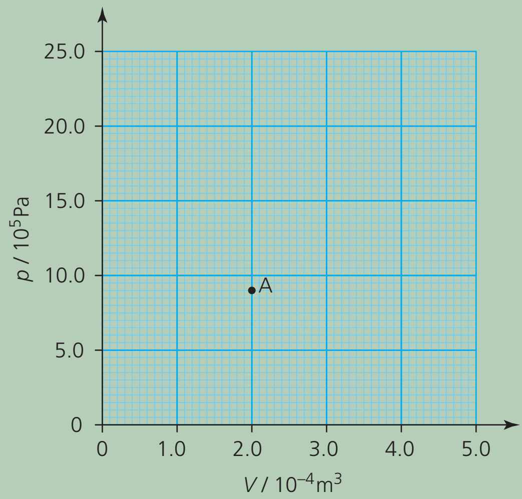


Figure Q4

a. The gas in the cylinder is compressed *isothermally* to a volume that is half of what it was when at point A. Copy the axes onto graph paper and plot point A.

Draw a line representing this isothermal compression. Plot *two* points in addition to A so that you can draw the line accurately. Label the new end of the line, B.

b. The gas undergoes an *adiabatic* expansion so that it returns to its original volume.

i. State what is meant by *adiabatic*.

ii. Without attempting further calculations, sketch the adiabatic expansion on the graph that you drew in part **a**.

iii. State and explain what happens to the temperature of a gas when it is allowed to expand adiabatically.

c. At point A, the temperature of the gas is 1100 °C. Calculate the number of moles of gas present in the cylinder.

AQA Specification B Unit 5 Jan 2013 Q1

3 HEAT ENGINES

PRIOR KNOWLEDGE

You will need your knowledge from Engineering Physics Chapter 2 about the two main thermodynamic processes involving gases – isothermal and adiabatic compression and expansion. You will need to recall the use of the *p*–*V* diagram as a means of summarising the changes that a gas undergoes as it expands or contracts. Remember that for a cyclic process the work done is equal to the area of the loop on the *p*–*V* diagram.

LEARNING OBJECTIVES

In this chapter you will extend these ideas to an understanding of real engine cycles and look at two versions in detail – the four-stroke petrol engine cycle and the diesel engine cycle. You will consider their theoretical and practical efficiencies, and their dependence on the second law of thermodynamics. You will also learn what happens if a heat engine is run backwards.

3.1 ENGINE CYCLES

Real, workable, efficient engines can appear complicated (Figure 1). However, the principles that underlie engines are actually quite straightforward. As a starting point to understanding these, we need to define some expressions.

A device or system that extracts energy from its environment in the form of heat and converts it into useful work is called a **heat engine**. At the heart of every heat engine is a **working substance**, which is the substance (usually fluid) on which the thermodynamic processes are performed in the engine, by changes of temperature, pressure and volume.

Figure 1 *The engineer has to turn engine theory into a practical design, to ensure, for example, that your car will start on a cold morning in winter.*

Typical examples of heat engines include the **internal combustion engine** (which uses a petrol–air mixture), the **diesel engine** (which uses a diesel–air mixture) and the **steam engine** (which uses water). The working substance is enclosed in a cylinder. The thermodynamic processes experienced by the fluid cause a piston in the cylinder to move up and down (back and forth), and this motion is then converted, via a crankshaft, into a rotational motion to drive a vehicle or machinery. Standard car engines usually have four cylinders (Figure 2), but some may have six, eight or even twelve.

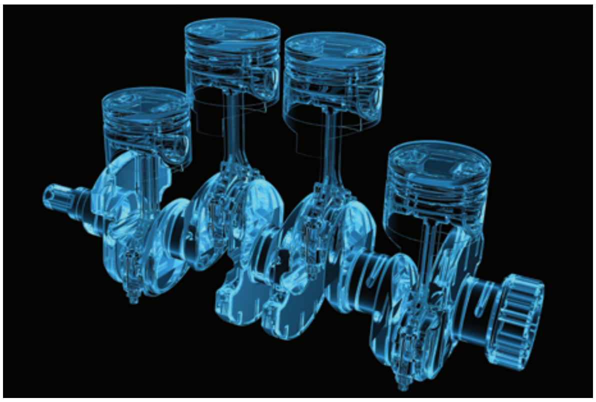


Figure 2 *X-ray image of the four pistons in a standard car engine*

For such engines to be useful, they must work continuously in a cycle, as we considered in Engineering Physics section 2.5. The working substance in each cylinder is forced through a series of thermodynamic processes, eventually returning to its starting state. The processes in each cylinder are staggered, as discussed in Engineering Physics section 1.5, so that the power output is continuous.

The four-stroke petrol engine cycle

The most common engine used in ordinary cars is the **four-stroke petrol engine**, which is one type of internal combustion engine. It was invented by a German engineer called Nikolaus Otto in the 19th century, and consequently the four-stroke cycle used is called the **Otto cycle**.

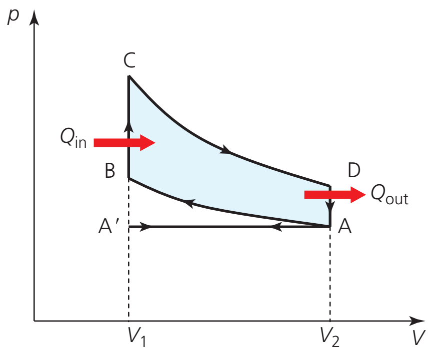


Figure 3 *The p–V diagram for an idealised four-stroke petrol engine cycle. The diagram for an actual engine has rounded corners.*

Figure 3 shows the p–V diagram, or indicator diagram, for the cycle of an idealised four-stroke petrol engine. By 'idealised' we mean that the working substance – the mixture of petrol vapour and air – is behaving in a theoretical way and the main processes occur one after another with no time delay.

The following sequence of processes take place, as illustrated in Figure 4.

A′ to A The inlet valve opens and the exhaust valve closes. The piston moves down. During this time, a mixture of typically 7% petrol vapour and 93% air at about 50 °C is drawn into each cylinder. This is the *induction stroke* (stage 1 in Figure 4).

At A The inlet valve closes.

A to B The piston moves up, compressing the gas adiabatically. The temperature rises to 300 °C. This is the *compression stroke* (stage 2 in Figure 4).

B to C A spark plug ignites the gas mixture at B, supplying heat Q_{in} and increasing the pressure at constant volume. The temperature rises to about 2000 °C.

C to D The increased pressure pushes the piston down as the gas expands adiabatically, decreasing both the pressure and temperature. This is the *power stroke* (stage 3 in Figure 4).

D to A The exhaust valve opens at D and most of the burnt gas mixture is released, removing an amount of heat Q_{out}. The pressure and temperature of the gas that remains in the cylinder decrease rapidly.

A to A′ As the piston moves up, the remaining gas mixture is expelled. This is the *exhaust stroke* (stage 4 in Figure 4).

At A′ The exhaust valve closes and the inlet valve opens. The cycle repeats.

The work done by the engine in one cycle is the area of the loop ABCD shown in Figure 3.

It is crucial to note an important difference between the indicator diagram and the real cycle of any one piston. One complete path around the indicator diagram does not correspond to one up-and-down movement of a piston. You can see from Figure 4 that, during the induction stroke, the piston is descending as it pulls on the fuel–air mixture. But it also descends during the power stroke. So we should note the following:

One thermodynamic cycle on the indicator diagram corresponds to two up-and-down motions of the piston, and hence two revolutions of the engine.

This will matter, later on, when we calculate the work done in a cycle and the power delivered by the engine.

The **thermal efficiency** of such an engine is the ratio of the maximum useful work that can be done by the engine to the energy supplied by the fuel (see Engineering Physics section 3.3). This is a measure of the theoretical performance of the engine. In practice, real engines are never as good as the theoretical maximum. (The reasons for this will be covered in Engineering Physics section 3.4.) Typical actual efficiencies are about 28%, as compared to a theoretical efficiency of about 58%.

The diesel engine cycle

The other common engine is the **diesel engine**. Rudolf Diesel was another German engineer who developed (and patented in 1892) the design of an

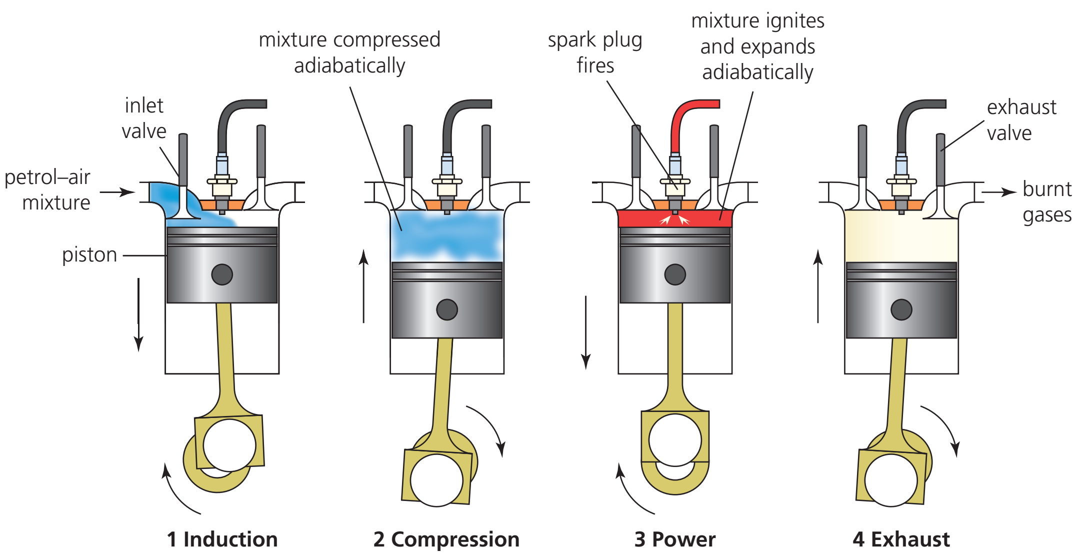


Figure 4 *The four main stages in the cycle of a four-stroke petrol engine.*

engine that compressed air, raising its temperature above the ignition point of the fuel. No spark plugs were needed. Diesel engines require heavier construction and produce a higher torque than internal combustion engines. They replaced steam piston engines for industrial uses, making Diesel a millionaire, and are commonly found today in heavy-duty machines, lorries and increasingly also in domestic vehicles.

A diesel engine cycle is similar to that of the four-stroke petrol engine. The main difference is that, in the diesel engine, there is no fuel in the cylinder during compression. Figure 5 shows the indicator diagram for the cycle of an idealised diesel engine.

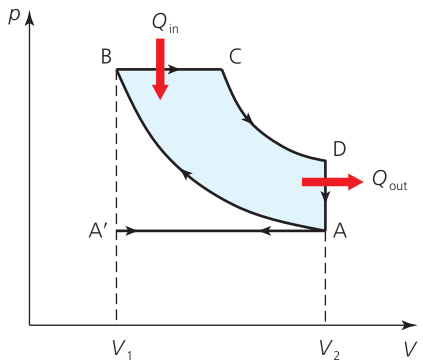


Figure 5 *The p–V diagram for an idealised diesel cycle. The diagram for an actual engine has rounded corners.*

The following sequence of processes takes place.

At A′	The inlet valve opens and the exhaust valve closes.
A′ to A	Air is drawn into each of the cylinders at atmospheric pressure as the piston moves down. This is the *induction stroke*.
At A	The inlet valve closes.
A to B	The air in the cylinder is compressed adiabatically as the piston moves up. The temperature of the air rises to 700 °C and becomes hot enough to ignite the fuel. This is called the *compression stroke*.
B to C	Diesel fuel is sprayed into the cylinder and is ignited immediately by the hot air, supplying heat Q_{in}. This forces the piston to move down at constant pressure and is the *first part of the power stroke*.
C to D	The fuel supply is cut off at C and the burnt gas expands adiabatically. This forces the piston down and the temperature falls. This is the *second part of the power stroke*.
D to A	The exhaust valve opens at D and releases the exhaust gas, removing an amount of heat Q_{out}. The pressure and temperature of the gas remaining in the cylinder decrease accordingly.
A to A′	The remainder of the gas is expelled from the cylinder as the piston moves up. This is the *exhaust stroke*.
At A′	The exhaust valve closes, the inlet valve opens and the cycle is repeated.

The work done by the engine in one cycle is the area of the loop ABCD shown in Figure 5. As with the four-stroke petrol engine, one such thermodynamic cycle involves two up-and-down motions of the piston.

Comparing petrol and diesel engines

In an engine, the **compression ratio** is the ratio of the volume enclosed in the cylinder at the beginning of the compression stroke to the volume enclosed at the end of the stroke. On a p–V diagram (Figures 3 and 5), this is the ratio of the volumes, V_2 to V_1. A diesel engine can achieve much higher compression ratios (typically 16 : 1) compared with petrol engines (about 10 : 1). The consequence of this is that diesel engines are much *more efficient* than petrol engines. The theoretical thermal efficiency of a diesel engine is about 65%, compared with 58% for a petrol engine. But, for reasons we will discuss later (in Engineering Physics section 3.4), the efficiency of real diesel engines drops to around 36% (compared with 28% for a petrol engine). Also, as mentioned above, they deliver greater torque.

But the disadvantages of diesel engines are that they operate at higher working pressures than petrol engines, which makes them more expensive to produce, as they have to be more robust. Also, they have a lower power-to-weight ratio.

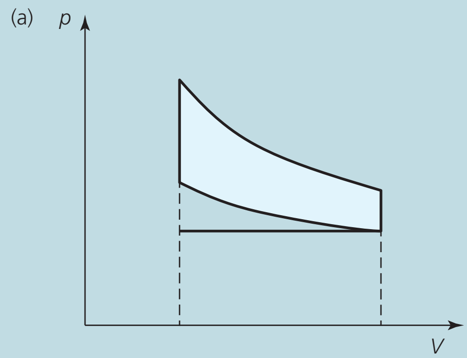
There are differences in the exhaust emissions from each type of engine. Petrol engines produce more carbon monoxide (CO), hydrocarbons (HCs), oxides of nitrogen (NO_x) and carbon dioxide (CO_2) than diesel engines. However, particulate emission (such as unburnt HCs) that occurs with diesel engines is almost non-existent in petrol engines. The situation in a petrol engine improves if a catalytic converter is installed. This oxidises pollutants, reducing CO, HC and NO_x emissions, but CO and HCs are still not reduced below the levels from a diesel engine.

KEY IDEAS

- A device or system that extracts energy from its environment in the form of heat and converts it into useful work is called a heat engine.
- The working substance works in a cycle, passing through a repeated series of thermodynamic processes.
- Indicator diagrams are used to understand the processes in heat engines.
- Common engines are the four-stroke petrol engine, running on the Otto cycle, and the diesel engine, which is also four-stroke in operation.
- In a four-stroke engine, power is produced once in every two revolutions of the engine.

QUESTIONS

1. a. Make a large copy of the diagram in Figure 6a of an indicator diagram for a four-stroke petrol engine. Add arrows to each part of the cycle and label each part with a brief description of what is happening in each cylinder.



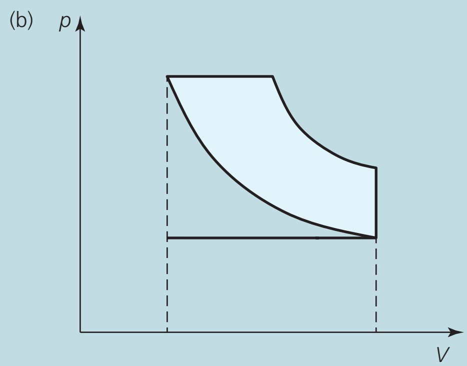


Figure 6

b. Repeat part **a** for the diesel engine cycle in Figure 6b.

2. Summarise the advantages and the disadvantages of a diesel engine compared with a petrol engine.

3.2 POWER OF AN ENGINE

When the power of an engine is being considered, what is meant is usually the *output* power, which is the driving power that is delivered to the engine's crankshaft. However, this is the 'end of the chain' of the production and transmission of power, and it masks the losses that will have occurred in this process. To find out how well the engine may be working, there are other power indicators that are used to compare the engine's performance. The following are the three key power indicators, in the order of the processes occurring in the heat engine:

1. The power that is derived from the burning of the fuel – the **input power**.
2. The theoretical power that the engine can deliver, based on the indicator diagram (recall that the area enclosed is work done) – the **indicated power**.
3. The **output power**, which may also be called the **brake power**.

We will consider these in turn.

Input power

This relates to the energy released as the fuel flows in and is subsequently burned, so is the product of the energy per kilogram and the flow rate. It is given by the relationship

$$P_{\text{input}} = \text{calorific value of fuel (J kg}^{-1}\text{)} \times \text{fuel flow rate (kg s}^{-1}\text{)}$$

It is measured in watts W (or kW or MW). The **calorific value** of a fuel is a measure of its energy density. Typical calorific values are 44.8 MJ kg^{-1} for diesel and 48.0 MJ kg^{-1} for petrol. It is simply how fast energy is being consumed and will therefore depend on factors such as the type of engine, how fast the car is moving (so air resistance will be affecting this) and others such as how it is driven and the condition of the tyres.

Worked example 1

The fuel used in an engine is propane, which has a calorific value of 49 MJ kg^{-1}. If the flow rate is 5×10^{-2} kg s^{-1}, calculate the input power.

$$\begin{aligned}\text{Input power} &= \text{calorific value} \times \text{flow rate}\\ &= 49.0 \times 5.00 \times 10^{-2}\\ &= 2.45 \times 10^{6}\ \text{W}\\ &= 2.45\ \text{MW}\end{aligned}$$

QUESTIONS

3. A car uses diesel fuel at a rate of 10 kg h^{-1}. Calculate the input power if the fuel has a calorific value of 48 MJ kg^{-1}.
4. DERV (diesel-engined road vehicle) fuel has a calorific value of 42.9 MJ kg^{-1}. Calculate the input power if the flow rate is 7.00×10^{-2} kg s^{-1}.
5. The energy content of petrol is about 44.1 MJ kg^{-1}. Calculate the input power when 7.80 kg of fuel flowed into an engine in one minute.
6. The energy content of domestic coal is 28.7 GJ tonne^{-1}. At what rate would you have to burn coal to equal the input power of the diesel from question **4**?

Indicated power

This is the *theoretical power* capability of an engine. It is based on the theoretical work done, which is the area enclosed by the *p–V* graph in each cycle. To get the power, we need to multiply by the number of cycles there are in 1 s and also by the number of cylinders. So the indicated power is given by

$$P_{\text{ind}} = \text{area of } p\text{–}V \text{ loop (J)} \times \text{number of cycles per second (s}^{-1}\text{)} \times \text{number of cylinders}$$

This assumes frictionless motion, and so is the maximum theoretical power output of an engine.

Worked example 2

The p–V diagram in Figure 7 shows the theoretical cycle for a petrol engine that is running at 1800 rpm. The engine comprises four cylinders. Calculate the indicated power of the engine.

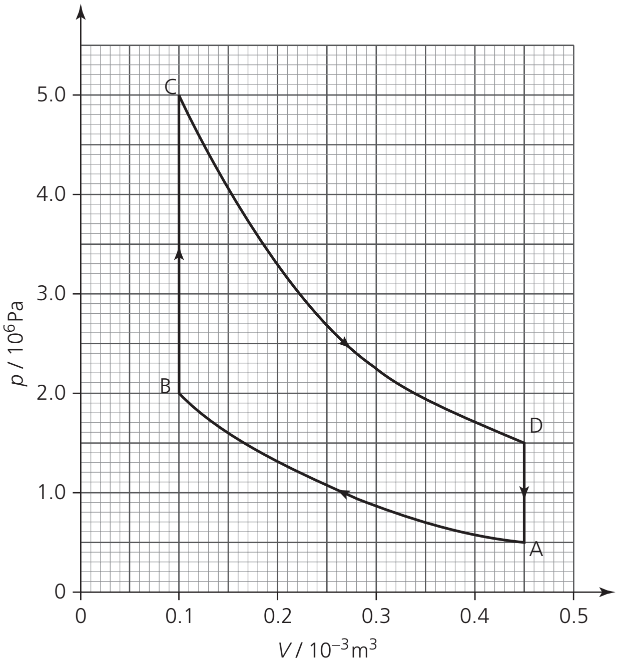


Figure 7

We know, from Engineering Physics Chapter 2, that the work done in one engine cycle is the area enclosed by the loop. Because the graph has a complicated shape, it is necessary to count the squares and calculate what the area of one square represents, which is the scaling factor. This will be in joules.

Counting the small squares gives a value of about 580. Each small square represents $(0.1 \times 10^6) \times (0.01 \times 10^{-3}) = 1$ J (the scaling factor). So the work done is the number of squares × scaling factor = 580 J for each cycle.

The engine is running at 1800 rpm (revolutions per minute), which is 30 rev s^{-1} (revolutions per second). But two complete cycles of the pistons are needed to give us one power stroke (see Engineering Physics section 3.1). So when the engine is rotating at 30 rev s^{-1}, power is being delivered 15 times each second. There are four pistons, so the indicated power is

$$P_{\text{ind}} = 580 \times 15 \times 4 = 3.5 \times 10^4 \text{ W} = 35 \text{ kW (to 2 s.f.)}$$

QUESTIONS

7. The area of an indicator diagram gives 960 J. If a four-stroke engine is rotating at 4500 rpm, calculate the indicated power per cylinder.

8. A four-stroke engine has eight cylinders. On the p–V diagram, the theoretical cycle encloses an area of 250 small squares, where the scaling factor is 7 J. If the engine is rotating at 2500 rpm, calculate the indicated power of the engine.

9. The p–V plot in Figure 8 is for a four-stroke petrol engine, which has four pistons (cylinders). This engine is rotating at 3000 rpm. Use this information and the diagram to calculate the indicated power of the engine.

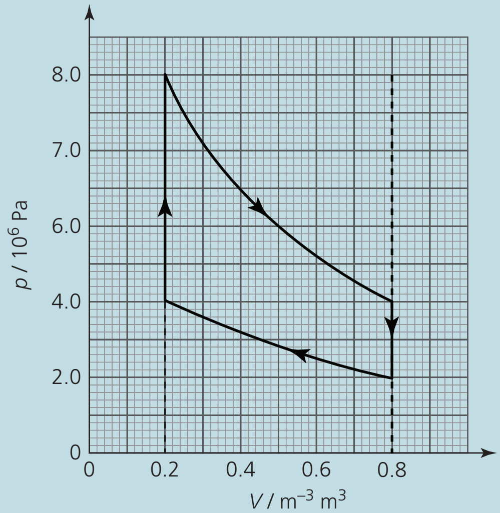


Figure 8

Output or brake power

This is a measure of the engine's power without the loss in power caused by the gearbox, alternator, differential, water pump and other auxiliary components. It is the power delivered to the engine's crankshaft (that is, the engine's flywheel).

At this point, we can re-introduce from Engineering Physics section 1.9 a relationship that is very useful when dealing with engines, as it enables you to calculate the output power of an engine:

$$P_{\text{out}} = T\omega$$

where T is the output torque in N m and ω is the angular velocity in rad s^{-1}. When dealing with engines, the angular or rotational velocity is traditionally given in rpm, so this rpm number needs to be converted to the unit rad s^{-1}.

For example, if the idling speed of an engine is 800 rpm, the angular velocity will be given by

$$\omega = \frac{800 \times 2\pi}{60}\ \text{rad s}^{-1}$$
$$= 84\ \text{rad s}^{-1}$$

The equation above for P_{out} gives the engine output power in watts (W). Among car manufacturers, the output power is often referred to as the brake power – or **brake horsepower** (**bhp**) in the old unit horsepower, which is still in common usage. The horsepower was coined by James Watt as a way of convincing breweries of the advantages of the steam engine over horses – he calculated an average power of a brewery horse.

The relationship is

$$1\ \text{horsepower} = 1\ \text{bhp} = 746\ \text{W}$$

So a car with 250 bhp has an output power of $250 \times 746\ \text{W} = 187\ \text{kW}$.

The indicated power (based on the indicator diagram) will be greater than the output power because of frictional processes in the engine and transmission. The difference is the **frictional power**:

$$P_{friction} = \text{indicated power} - \text{output (brake) power}$$
$$= P_{ind} - P_{out}$$

QUESTIONS

10. A car has an indicated power of 100 kW. Its measured power output is 75 bhp. Calculate the frictional power.

11. A typical car has a measured torque of 147 N m when the engine is rotating at 4000 rpm. The power from the engine was calculated theoretically as 110 kW. Use this information to calculate

a. the output power of the car

b. the frictional power.

12. The indicated power for a four-stroke engine is 200 kW. The car's output power was then measured. It was rotating at 3000 rpm during the test and the torque measured was 290 N m. Calculate the frictional power of the engine.

KEY IDEAS

- Input power is the energy released as the fuel burns:

$$P_{input} = \text{calorific value of fuel } (\text{J kg}^{-1}) \times \text{fuel flow rate } (\text{kg s}^{-1})$$

- Indicated power is based on the indicator diagram:

$$P_{ind} = \text{area of } p\text{–}V \text{ loop (J)} \times \text{number of cycles per second } (\text{s}^{-1}) \times \text{number of cylinders}$$

- Output (brake) power is the product of the output torque and the angular velocity.

$$P = T \times W$$

It is the power needed to maintain a constant angular momentum:

$$P_{out} = T\omega$$

- The frictional power of an engine is the difference between the output (brake) power and the indicated power:

$$P_{friction} = \text{indicated power} - \text{output (brake) power}$$
$$= P_{ind} - P_{out}$$

3.3 EFFICIENCY OF AN ENGINE

Engine designers make comparisons between cars by looking at the relative efficiency of engines. Generally, efficiency is defined as the ratio of useful output energy (or power) to input energy (or power). But since there are three key power indicators, namely input power, indicated power and output power, engine designers also consider three key indicators of efficiency, depending on what aspect they want to compare.

The **mechanical efficiency**, η, is given by

$$\eta = \frac{\text{output (brake) power}}{\text{indicated power}} = \frac{P_{out}}{P_{ind}}$$

The **thermal efficiency**, ε, is defined as

$$\varepsilon = \frac{\text{indicated power}}{\text{input power}} = \frac{P_{ind}}{P_{input}}$$

The **overall efficiency** of an engine is the ratio

$$\text{overall efficiency} = \frac{\text{output (brake) power}}{\text{input power}} = \frac{P_{out}}{P_{input}}$$

Since

$$\frac{P_{out}}{P_{input}} = \frac{P_{out}}{P_{ind}} \times \frac{P_{ind}}{P_{input}}$$

we obtain

$$\text{overall efficiency} = \frac{P_{\text{out}}}{P_{\text{input}}} = \eta \times \varepsilon$$

The overall efficiency is the product of the mechanical and thermal efficiencies.

Note that all efficiency values will be less than 1, but may be given as a percentage as an alternative.

Worked example

The output power of a car is 56 kW, the indicated power is 100 kW and the input power is 250 kW. Calculate the mechanical efficiency, the thermal efficiency and the overall efficiency.

Mechanical efficiency

$$\eta = \frac{\text{output (brake) power}}{\text{indicated power}}$$

$$= \frac{P_{\text{out}}}{P_{\text{ind}}} = \frac{56}{100} = 0.56 \text{ or } 56\%$$

Thermal efficiency

$$\varepsilon = \frac{\text{indicated power}}{\text{input power}} = \frac{P_{\text{ind}}}{P_{\text{input}}} = \frac{100}{250} = 0.40 \text{ or } 40\%$$

Overall efficiency

$$\text{overall efficiency} = \frac{\text{output (brake) power}}{\text{input power}}$$

$$= \frac{P_{\text{out}}}{P_{\text{input}}} = \frac{56}{250} = 0.22 \text{ or } 22\%$$

or expressed as the product of the mechanical and thermal efficiencies

$$\text{overall efficiency} = 0.56 \times 0.40 = 0.22 \text{ or } 22\%$$

QUESTIONS

13. Calculate the mechanical efficiency, the thermal efficiency and the overall efficiency for a real car, the data for which are as follows:

input power = 494 000 W

indicated power = 124 000 W

torque of 228 N m produced at 4000 rpm

fuel flow 0.672 kg per minute

energy density of fuel = 44.1 MJ kg^{-1}

KEY IDEAS

- Mechanical efficiency $= \eta = \dfrac{\text{output (brake) power}}{\text{indicated power}} = \dfrac{P_{\text{out}}}{P_{\text{ind}}}$
- Thermal efficiency $= \varepsilon = \dfrac{\text{indicated power}}{\text{input power}} = \dfrac{P_{\text{ind}}}{P_{\text{input}}}$
- Overall efficiency $= \dfrac{\text{output (brake) power}}{\text{input power}} = \dfrac{P_{\text{out}}}{P_{\text{input}}} = \eta \times \varepsilon$

3.4 THE FIRST LAW OF THERMODYNAMICS APPLIED TO HEAT ENGINES

In a heat engine, the energy supplied as heat from the burning fuel does work. We know that the work done cannot equal the heat supplied because of frictional losses and so on. But here we will see that, even where there are no such losses, in what is termed an 'idealised' engine, there will still be a maximum theoretical efficiency that will be less than 1. In other words, even if the output power could be equal to the indicated power (mechanical efficiency 1), there will be a maximum value of the thermal efficiency, and hence of the overall efficiency, that is less than 1. A heat engine *cannot* turn heat energy totally into work. We will be returning to this in Engineering Physics section 3.5.

The essential elements of a heat engine are shown in Figure 9. This type of diagram is very useful for analysing the input energy and output energy of engines. The energy supplied by heating is shown to come from a high-temperature 'reservoir', or **source**. The surroundings are shown as a low-temperature reservoir, or **sink**.

The arrow Q_H indicates the energy extracted as heat from the high-temperature source that is available to do work, W. The remainder of the input energy is delivered as heat Q_C to the low-temperature sink. The central loop and its cyclic nature illustrate the function of the working substance, as defined in a p–V diagram. This 'black box' approach is an overview, bypassing the detail of the engine itself.

The purpose of any engine is to transform as much as possible of the extracted energy, Q_H, into work W

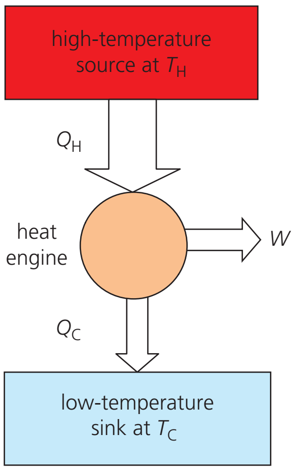


Figure 9 *The elements of a heat engine*

(usually to move something). To calculate the net work done by the engine during a cycle, we apply the first law of thermodynamics ($Q = \Delta U + W$) to the working substance. For an idealised (theoretical) heat engine, $\Delta U = 0$ for a complete cycle of the working substance, as it is in the same state at the end as it was initially. Within the system, there has been no loss of energy.

Using the first law, we now just have $W = Q_H - Q_C$ for the work done by the system.

The measure of the success of an idealised heat engine is through its thermal efficiency, ε.

$$\varepsilon = \frac{\text{indicated power}}{\text{input power}}$$

$$= \frac{\text{work done per cycle}}{\text{energy taken in as heat per cycle}}$$

So we can write

$$\varepsilon = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

It can be shown from this that the **maximum theoretical efficiency** ε_{max} is given by

$$\varepsilon_{max} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H}$$

where T_H is the temperature of the hot reservoir (source) and T_C is the temperature of the cold reservoir (sink), both in kelvin (K).

The maximum efficiency of a heat engine is always less than 1 (or less than 100%).

The theory of this maximum possible efficiency in terms of the temperatures of the hot and cold reservoirs was established by considering an ideal, theoretical cycle (consisting of two isothermal curves and two adiabatic curves) called the **Carnot cycle** after the French engineer Nicolas Carnot (see Assignment 1 at end of Engineering Physics section 3.5). Various assumptions were built in to the derivation: namely that there are no energy losses and that the process could be made to run backwards. This theoretical fully reversible process would need to happen very slowly, to maintain equilibrium.

The equation, however, is valid for *all* idealised reversible engines, irrespective of the particular cycle and the particular working substance. The differences between real and ideal engines are considered in the next subsection ('Limitations of real heat engines').

Worked example 1

An ideal heat engine is to be operated with $T_H = 300\,\text{K}$. Which one of the following values of T_C will allow the greatest maximum efficiency: 250 K, 100 K, 50 K or 10 K?

$T_C = 250\,\text{K}$:

$$\varepsilon_{max} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H} = 1 - \frac{250}{300} = 0.17 = 17\%$$

$T_C = 100\,\text{K}$:

$$\varepsilon_{max} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H} = 1 - \frac{100}{300} = 0.67 = 67\%$$

$T_C = 50\,\text{K}$:

$$\varepsilon_{max} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H} = 1 - \frac{50}{300} = 0.83 = 83\%$$

$T_C = 10\,\text{K}$:

$$\varepsilon_{max} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H} = 1 - \frac{10}{300} = 0.97 = 97\%$$

Notice that the trend is that, the lower the T_C that can be used, then the better the efficiency. However, to get 100% efficiency, we would need to have T_C at absolute zero, which is not possible. So ε_{max} will always be less than 1 (or 100%).

Worked example 2

An ideal heat engine operates between the temperatures of 850 K (T_H) and 300 K (T_C). The engine performs 1200 J of work every cycle, taking 0.25 s.

a. What is the maximum theoretical thermal efficiency of the engine?

b. What is the average power of the engine?

c. How much energy is extracted as heat from the high-temperature source every cycle?

d. How much energy is delivered as heat to the low-temperature sink every cycle?

a. The theoretical efficiency is

$$\varepsilon_{max} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H} = 1 - \frac{300}{850} = 0.647 = 65\%$$

b. Knowing the relation between power and work done gives

$$P = \frac{W}{t} = \frac{1200}{0.25} = 4800\,\text{W}$$

c. The energy extracted from the high-temperature reservoir is given by the efficiency and the work done per cycle, that is,

$$\varepsilon = \frac{W}{Q_H}$$

so that

$$Q_H = \frac{W}{\varepsilon} = \frac{1200}{0.647} = 1850\,\text{J}$$

d. Using $W = Q_H - Q_C$ gives

$$Q_C = Q_H - W = 1850 - 1200 = 650\,\text{J}$$

QUESTIONS

14. A heat engine operates between 400 °C and room temperature (20 °C). Calculate the maximum theoretical efficiency.

15. An ideal heat engine operating between 17 °C and 777 °C performs 2500 J of work every cycle, which takes 0.10 s.

- **a.** What is the maximum theoretical efficiency of the engine?
- **b.** Calculate the amount of energy per cycle that is being extracted from the high-temperature source.
- **c.** Calculate how much heat energy, per cycle, is rejected.
- **d.** Calculate the average power of the engine.

Limitations of real heat engines

How close do real engines get to an ideal engine? Consider the following three assumptions (in *italic*):

1. *The petrol–air mixture behaves as an ideal gas.*

In real systems, the petrol–air mixture does *not* behave as an ideal gas because it is a mixture of polyatomic molecules, which will sometimes be under high pressure and at high temperature, so the kinetic theory assumptions break down.

2. *The heat energy (in the compression stroke) is taken in entirely at the single temperature T_H and rejected at the single temperature T_C.*

In reality, the heat is usually taken in over a range of temperatures and rejected also over a range of temperatures. The maximum temperature is not attained, because of imperfect combustion.

3. *The processes that form the engine cycle are reversible.*

This is not true for a real system, for several reasons.

- Energy is dissipated out of the system.
- There is no equilibrium with the surroundings, as the processes are too quick.
- Inlet and exhaust valves take a finite time to open and close, and combustion is not instantaneous, so the 'sharp-edged' *p*–*V* diagrams would never occur for a real cycle.
- In the petrol engine, heating is not achieved at constant volume, because the pistons are always moving, and the expansion and compression strokes are not truly adiabatic, because heat energy is lost out of the system.

In addition to the above, the efficiency will be reduced further by frictional effects in the moving parts, and the transfer of energy out of the system by the heating of the cylinder walls.

Turbulence also plays a part. The combustion of the fuel is designed to occur in an orderly fashion at a specific rate. But, any turbulence in the vapour will change the rate and move it away from the optimum, again reducing efficiency.

Consequently, real engines have much lower efficiencies than calculations suggest. Designers are continually trying to improve engine efficiency by reducing the energy transferred out as heat (Q_C) during each cycle to make the ratio Q_C/Q_H lower.

KEY IDEAS

- It can be shown theoretically that the thermal efficiency of a heat engine can never be as high as 100%.
- The thermal efficiency links the heat flowing between the hot and cold reservoirs, with the heat engine working in between:

$$\varepsilon = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

- The maximum theoretical thermal efficiency, for an idealised engine, is given by:

$$\varepsilon_{max} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H}$$

- All real engines have an efficiency that is lower than this because of non-ideal conditions, including friction.

3.5 THE SECOND LAW OF THERMODYNAMICS

It is obvious that all hot objects naturally cool down to the ambient temperature. In fact, all objects at a temperature above absolute zero radiate electromagnetic energy. The higher the temperature, the greater the rate of energy radiated. An object's 'surroundings' are an object in this sense, too – the surroundings radiate thermal energy at a rate dependent on their temperature. So a hot object emits radiation, but also absorbs radiation from its surroundings. It is a two-way flow, but the emission from the hot object will be greater until a state of equilibrium results (Figure 10), with the rate of emission and the rate of absorption equal when the object and the surroundings are at the same final temperature.

This was described by Newton in his 'law of cooling'. He stated that the rate of loss of heat energy from a hot object is proportional to the difference in temperature between the object and its surroundings. But this 'law' is only mathematically correct in certain situations.

A fundamental truth can, however, be stated:

- Heat always flows from the hot body to the cold body, when they are brought into contact.

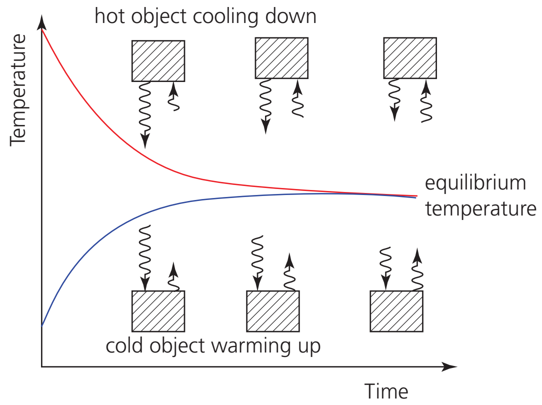


Figure 10 *Temperature against time graph for two objects of different temperatures that are in contact – for example, an object and its surroundings. As long as there is a temperature difference between the two objects, net heat will flow from the hot object to the cold object until an equilibrium state is reached.*

Another one, which we discovered in the previous section, is that:

- No heat engine can completely convert heat into work.

These two observations, when combined together, result in a statement that is one version of the **second law of thermodynamics**.

It is not possible to convert heat continuously into work without at the same time transferring some heat from a warmer to a colder body.

The first law of thermodynamics, as an energy conservation law, seems to imply that heat and work are equivalent. So why can work be entirely converted into heat, but heat cannot be converted entirely into work? And why is net heat flow always in one direction?

In the kinetic theory of gases (Chapter 3), where equations such as $pV = \frac{1}{3}Nm\overline{c^2}$ are derived, the motions of particles using the equations of motion are used. But these equations are time-reversible. What actually gives us the 'arrow of time'?

Consider the nature of heat, or thermal energy. It is the kinetic energy of the molecules of an object, which move with random speeds in random directions. We say the energy is randomised, or **disordered**. If you are converting it into something directly useful, as in an engine, then it must become more ordered somehow. A closed system will not naturally go from a state of disorder to one of order without external intervention. The normal process is the reverse – order to disorder.

A sandcastle will collapse down to a pile of sand, but the sand will not spontaneously reassemble into a sandcastle! When turning disorder to order in the case of an engine, some of the energy must be involved in the ordering process. This is what is being rejected at the cooler temperature. Consequently, the cold sink is essential, and, as this cannot be at absolute zero, the engine cannot be 100% efficient. So the following is a consequence of the second law of thermodynamics:

A heat engine needs to operate between a hot source and a cold sink.

ASSIGNMENT 1: UNDERSTANDING ENTROPY AND THE ARROW OF TIME

(MS 0.1, MS 1.1, MS 1.3, MS 2.1, MS 2.3)

In 1824, Nicholas Carnot (Figure A1) proposed a theoretical thermodynamic cycle, which was the most efficient possible. Carnot was a young French military engineer and physicist, and, because of his work, he is described as being the 'father of thermodynamics'.

Figure A1 *Nicholas Carnot created the first successful theory of the efficiency of heat engines.*

Carnot was following in the footsteps of Newcomen and James Watt, but at that point engine efficiencies were only about 3%. Steam engines had been around for quite a while, but there had been no scientific study of them. Carnot wanted to know whether the work that could be obtained was 'unbounded' and whether heat engines could be improved if steam could be replaced by some other fluid. He achieved this by idealising the heat engine cycle, which became known as the Carnot cycle. It consisted of two adiabatic changes and two isothermal changes, all of which were reversible (Figure A2). Heat was introduced at temperature T_H and rejected at the lower temperature (T_C).

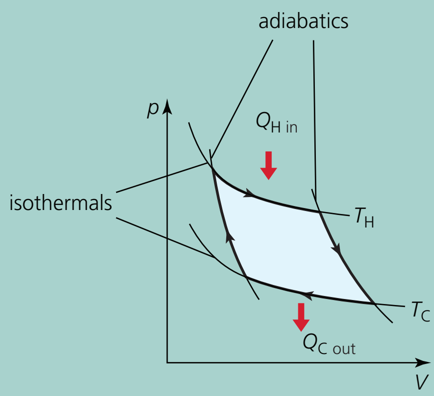


Figure A2 *The Carnot cycle*

Another key person at this time was Lord Kelvin, who suggested basing a temperature scale on such an idealised reversible heat engine. This **thermodynamic temperature scale**, or Kelvin scale, turned out to be identical to the absolute temperature scale based on the behaviour of an ideal gas (see Chapter 3). The thermodynamic temperature scale can be defined by

$$\frac{T_H}{T_C} = \frac{Q_H}{Q_C}$$

Carnot was then able to derive the expression we presented for the maximum efficiency of an idealised heat engine (which we used in Engineering Physics section 3.4) in terms of the temperatures of the source and sink:

$$\varepsilon_{max} = 1 - \frac{T_C}{T_H}$$

As we have discussed, sandcastles tend to collapse. Tidy rooms tend to get messy. Gases will diffuse and fill a room. Heat spreads out from a hot object to warm the environment. Yet, if you consider the individual motions and collisions of the atoms, then there is no reason, in principle, why the paths could not be reversed and the atoms move back to their starting points (their motion is said to be 'time-symmetric'). But we know this never occurs in everyday life: processes move from order to disorder, and we experience time moving in one direction only.

Scientists have given a name to describe and quantify the disorder, and it is called **entropy** (and given the symbol *S*). The second law of thermodynamics gives us the 'arrow of time' – it is the direction in which things seem to become more disordered, or in which entropy increases. An alternative statement of the second law is this:

It is not possible for a process to decrease the total entropy in the Universe.

There are two ways that we can deal with entropy and put it on a more mathematical footing.

Method 1 – Statistical

This method considers the likelihood of events occurring. Imagine tossing a coin. As there are two sides, head (H) and tail (T), the probability of getting a head (or a tail) is 1 in 2. This will be the same for each toss. But if we toss the coin four times, what is the likelihood of getting four heads in a row?

For four tosses, there will be 2^4 possibilities, or 16 results. But we want to find out the chance of getting four heads. Working our way through the possibilities, we get the summary in table A1.

Possible outcome	Number of times this occurs
4 heads	1
3 heads and 1 tail (in any order)	4
2 heads and 2 tails (in any order)	6
1 head and 3 tails (in any order)	4
4 tails	1
Total	*16*

Table A1

What we have been looking at is what is most likely to occur. There is a 1 in 16 chance of getting four heads, but a 6 in 16 chance of a mix of two heads and two tails. Notice that if we were, instead, to look at specific ordering in the results (for example, H,T,H,T), then this would be less likely.

Questions

A1 If you were to toss the coin 10 times, what is the likelihood of getting a string of 10 heads?

A2 The coin is tossed 100 times. What is the likelihood now of there being all 100 heads?

Relating this back to thermodynamics, we can say that the outcomes range from *highly ordered* (for example, H,H,H,H) to *highly disordered* (for example, H,H,T,T, where we have not specified the order).

The chances are that you will find the most disordered state to be the most likely, whereas complete order is statistically almost non-existent. As a further thought experiment, consider a container containing 10^{23} molecules. What is the likelihood of them all moving in the same direction at the same time – and therefore your chances of being in a room where all the molecules end up on one side, leaving behind a vacuum?!

Entropy is a measure of how likely things are to occur, and a more disordered state is more likely and so has greater entropy. It is a law of nature that the entropy of a system tends to increase.

Stretch and challenge

Method 2 – Clausius

Another very important individual in the history of thermodynamics was Rudolf Clausius, who, in 1865, suggested that the change of entropy ΔS occurring when a process happens is related to the heat that is transferred reversibly (ΔQ) as well as the temperature (T) at which it happens, by the relationship:

$$\Delta S = \frac{\Delta Q}{T}$$

The unit of S is $\mathrm{J\,K^{-1}}$.

In the Universe as a whole, or in any closed system, the overall entropy must increase, or at least stay the same. The entropy can drop in part of the system – where heat is extracted, for example – just as long as the system overall experiences an increase in entropy. So

$$\Delta S \geqslant 0$$

It is possible to work out the changes in entropy that occur because of processes such as heat engines and heat passing from hot to cold objects. For example, say 100 J of heat energy is allowed to flow from a hot reservoir at 400 K to a cold reservoir at 300 K (Figure A3).

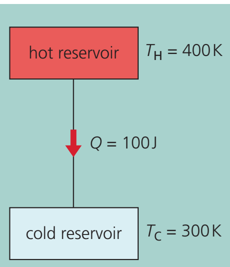


Figure A3

The change in entropy in the hot reservoir is

$$\Delta S_H = \frac{\Delta Q}{T_H} = -\frac{100}{400} = -0.25\,J\,K^{-1}$$

Notice the minus sign (entropy drops).

The change in entropy in the cooler reservoir is

$$\Delta S_C = \frac{\Delta Q}{T_C} = +\frac{100}{300} = +0.33\,J\,K^{-1}$$

This is positive (entropy increases).

The overall entropy change is

$$\Delta S_T = +0.33 - 0.25 = +0.08\,J\,K^{-1}$$

The entropy of the system has increased.

Work can only be done when heat flows from the hot to the cold reservoir, because otherwise the entropy would decrease, which is impossible.

Questions

Stretch and challenge

A3 Heat is allowed to flow from a hot reservoir to a cold reservoir. Calculate the change in entropy in the hot reservoir, the change of entropy in the cold reservoir, and the overall change in entropy, for the following values:

a. $T_H = 500\,K$, $T_C = 200\,K$, $Q = 300\,J$

b. $T_H = 350\,K$, $T_C = 300\,K$, $Q = 1050\,J$

c. $T_H = 100\,°C$, $T_C = 90\,°C$, $Q = 1000\,J$.

Comment on the changes in entropy that you have calculated.

A4 Consider a heat engine like that in Figure A4, with values:

$T_H = 500$ K, $T_C = 200$ K, $Q_H = 1000$ J

We know that the maximum thermal efficiency is given by

$$\varepsilon_{max} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H}$$

We want to know how this relates to entropy and what happens if the efficiency is altered.

a. Calculate the maximum efficiency between the two temperatures.

b. Calculate the entropy loss from the hot reservoir, the entropy gain of the cold reservoir, and what the overall change in entropy is.

c. Suppose we only get 500 J of work from the engine. Calculate the entropy change in this case.

d. Calculate the theoretical entropy change in the case where 700 J of work is done by the engine.

e. What is the pattern here and how does entropy change relate to the efficiency of a heat engine?

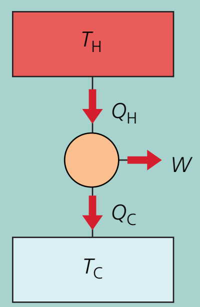


Figure A4

A5 **a.** 'Life as a process in the Universe opposes the second law of thermodynamics.' Do you agree with this statement?

b. How can you reconcile this statement with the comment that 'entropy *must* increase'?

A6 Eventually, in the Universe, everything will reach equilibrium and there will be no temperature difference anywhere. This means no further work can be done: entropy will be at a maximum and we will have the 'heat death of the Universe'. The Universe will not be able to get itself back from this position. What does this suggest about the state of entropy at the time of the Big Bang?

A7 Since the 1990s, it has been shown that the Universe is accelerating outwards. This is the reason why dark energy has been suggested. Will this hasten the heat death?

3.6 REVERSED HEAT ENGINES

Rather oddly (at least, at first glance), it is possible to take in heat energy at a low temperature and reject heat at a higher temperature: this is a **refrigerator**. This is basically a heat engine that works in reverse. In this case, work needs to be done to transfer energy from a low-temperature reservoir to a high-temperature reservoir. To achieve this practically, some external device (for example, an electric motor or compressor) has to do work on the working substance of the system, as it continuously repeats a set of thermodynamic processes, to transfer heat energy from, say, a food storage compartment (low-temperature reservoir) to a room (high-temperature reservoir). This is shown in Figure 11.

An air conditioning system (Figure 12) works in the same way. The low-temperature reservoir is the room to be cooled and the high-temperature reservoir is the warmer outdoors.

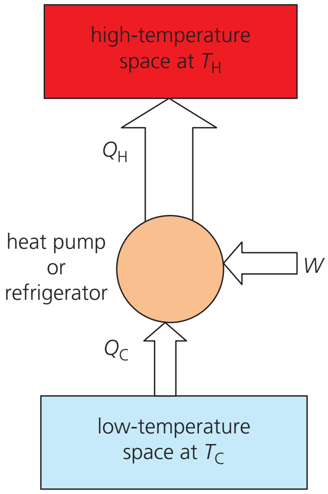


***Figure 11** The elements of a refrigerator or heat pump. Work needs to be done to move heat energy from the cold to the hot reservoir, against the usual direction of the flow of heat in nature as described by the second law of thermodynamics.*

***Figure 12** Air conditioners are an essential part of life in some hot areas of the world.*

A **heat pump** is an engine that is used to heat a building. The thermodynamic processes involved are the same as for a refrigerator or air conditioner – as shown diagrammatically in Figure 11. The room to be heated is the high-temperature reservoir and heat is transferred to it from the (cooler) outdoors.

The differences between a refrigerator and a heat pump are:

- The purpose of the refrigerator is to remove heat from the cold reservoir.
- The purpose of the heat pump is to supply heat to the high-temperature reservoir.

A recent example of a building designed to be as sustainable as possible is the Crystal in London. It is heated and cooled by ground-source heat pumps (Figure 13).

Coefficients of performance

The effectiveness of both a heat pump and a refrigerator is measured by its **coefficient of performance** (**COP**). This is the ratio of the heat extracted or supplied to the work done by the external agency. It will differ, depending on the ultimate purpose of the engine. Do not confuse the COP with efficiency – they are measuring different things. While efficiency can never exceed 1, values for the COP often greatly exceed 1. Typical household refrigerators and air conditioning units have COPs between 5 and 10. Industrial heat pumps have COP values typically between 10 and 30.

Using the first law of thermodynamics for an idealised heat engine working in reverse, as in Figure 11, and putting $\Delta U = 0$ for the working substance, as it is in the same state at the end as it was initially, we obtain

$$Q_H = W + Q_C$$

For a refrigerator, the COP is given by the ratio of the heat extracted to the work done:

$$\text{COP}_{\text{ref}} = \frac{Q_C}{Q_H - Q_C}$$

From this, it can be shown that the maximum theoretical COP for a refrigerator is given by

$$\text{COP}_{\text{ref}} = \frac{T_C}{T_H - T_C}$$

where T_C is the temperature of the cold reservoir and T_H is the temperature of the hot reservoir, both in kelvin (K).

Figure 13 *The Crystal building has solar panels to generate power and ground-source heat pumps for maintaining the interior temperature.*

For a heat pump, the COP is given by the ratio of heat delivered at the hot reservoir to the work done:

$$\mathrm{COP}_{hp} = \frac{Q_H}{Q_H - Q_C}$$

The maximum theoretical COP for a heat pump is

$$\mathrm{COP}_{hp} = \frac{T_H}{T_H - T_C}$$

Heat pumps provide an extremely low-cost and efficient form of heating, because the heat supplied, Q_H, is much greater than the work done by the external device, that is, $Q_H - Q_C$. A ground-source heat pump (which extracts heat energy from below the ground) might have a COP of 4, and so provides 4 J of heat energy for each 1 J of work put in. Contrast this with the situation of an electric fire, which will deliver at best 1 J of heat for each 1 J of electrical energy supplied. This method for heating has a lot of potential – see Assignment 2.

The value of the COP is clearly higher the closer the temperatures of the two reservoirs are to each other. As the difference in temperatures becomes smaller, so less work is needed to transfer the heat from cold to hot. This is why heat pumps work more effectively in temperate climates than in climates where the temperatures vary over a large range.

Worked example

A heat pump extracts heat from a river at 8 °C and delivers it into a room at 20 °C. Determine the coefficient of performance for such a device and explain its significance. How does this compare with a conventional electric fire?

First, we need to convert °C to kelvin: 8 °C = 281 K and 20 °C = 293 K. Then

$$\mathrm{COP}_{hp} = \frac{Q_{in}}{W} = \frac{T_H}{T_H - T_C} = \frac{293}{293 - 281} = \frac{293}{12} = 24$$

In other words, 24 J of heat would be provided with the input of only 1 J of work. Compare this with an electric heater. For every joule of electrical energy supplied, the best you will be able to get is one joule of heat.

QUESTIONS

16. a. A refrigerator maintains an internal temperature of 2 °C, while the outside ambient temperature is 25 °C. Calculate the COP.

b. The same refrigerator is operating in a hotter climate, where the ambient temperature is 35 °C. Calculate the new COP.

c. Comment on these different values.

17. A freezer is maintaining an internal temperature of −5 °C. The room it is working in is at 20 °C. Calculate the COP.

18. a. A heat pump extracts heat from a river at 3 °C and delivers it into a room at 20 °C. Calculate the COP.

b. The same heat pump now extracts heat from a river at 10 °C and delivers it into a room at 20 °C. Calculate the COP in this case.

c. The same heat pump is now used to extract heat from the ground at −5 °C and delivers it into a room at 20 °C. Calculate the COP.

d. What pattern do you see? Explain this.

KEY IDEAS

- The second law of thermodynamics states that it is not possible to convert heat continuously into work without at the same time transferring some heat from a warmer body to a colder body.
- A device or system that extracts energy from a cold reservoir and rejects it at a higher temperature is called a heat pump or refrigerator. It is a heat engine working in reverse. Work is done on the working substance, usually by a motor.
- The coefficient of performance (COP) of a reversed heat engine is the ratio of heat extracted or supplied to the work done.
- For a refrigerator,

$$\mathrm{COP}_{\mathrm{ref}} = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C}$$

- For a heat pump,

$$\mathrm{COP}_{\mathrm{hp}} = \frac{Q_H}{W} = \frac{Q_H}{Q_H - Q_C} = \frac{T_H}{T_H - T_C}$$

ASSIGNMENT 2: EVALUATING HEAT PUMPS – ARE THEY THE FUTURE FOR DOMESTIC HEATING?

Heat pumps, in the form of refrigerators or freezers, are now an essential part of everyday life. The other domestic use for a heat pump concerns the possibility of heating a house. Is this practicable?

As you saw above, if we electrically heat our houses, then for every joule of heat output, we need (at least) one joule of electrical energy input. But, by using a heat engine working in reverse, we can improve the situation quite dramatically, getting several joules of heat transferred for each joule of work put in. Notice that we are not exceeding 100% efficiency (that would be impossible), but instead we are combining energy sources, in this case not only transferring heat but doing work from a motor of some sort.

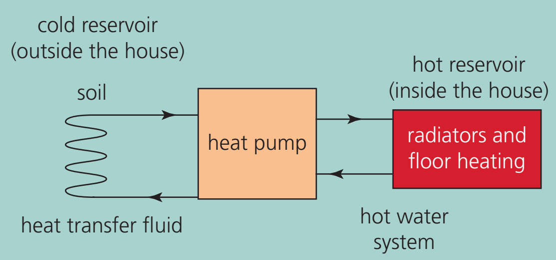


Figure A1 *A block diagram for extracting heat energy from the soil (cold reservoir) and rejecting it inside the house (hot reservoir)*

How does this work in practice?

One of the most common types of heat pump in use is the ground-source heat pump (GSHP). This is used to extract heat energy from the ground in winter and to transfer that heat into buildings. Pipes are laid in trenches at least 2 m deep in the ground – the cold reservoir. A closed loop of water and glycol (an antifreeze) is circulated, extracting heat energy. It has been cooled down to 0 °C by a section of the heat pump, so will absorb heat if the soil temperature is above this. At depths below 2 m, the ground temperature will stay fairly constant between 9 °C and 11 °C. Heat is transferred to the refrigerant in the heat pump, which is then forced through cycles, eventually transferring its heat energy to the house heating system. Meanwhile, the cooled heat transfer fluid has been pumped back out through the buried pipework for re-heating.

In the summer, the heat pump can extract heat from a hot reservoir and deliver it to the cold reservoir. So the house is being cooled and the heat deposited into the ground where it is stored for use in the winter. The usual alternative of an air conditioner just 'wastes' this heat it into the atmosphere.

How good is a heat pump?

Some advantages:

1. It uses solar energy that is stored in the ground.
2. No combustion takes place and so there are no carbon emissions on the premises.
3. Some models can be run 'backwards' in the summer, acting as an air conditioner by extracting heat from the house and storing it in the soil.
4. The ground temperature stays fairly constant throughout the year, so energy can be extracted at all times.
5. GSHPs are cheaper to run than oil boilers.
6. No fuel delivery or fuel storage is needed.
7. They have low maintenance costs and should run quietly and efficiently for about 20 years.
8. COPs, in practice, are typically about 4, so savings as an alternative to electrical heating could be substantial.
9. Financial incentives may exist because of international requirements to reduce carbon emissions.

Some disadvantages:

1. Heat pumps cost more to install than conventional heating systems.
2. The pumps need electricity for the compression and circulation (but this could be supplied by a wind turbine).
3. The COP of a heat pump is strongly influenced by the relationship between the input and output temperatures.
4. They are designed to work with a lower hot reservoir temperature than many people are normally used to (that means that the house feels cooler inside).
5. The radiators may not feel very warm to the touch. Underfloor heating may be needed as a supplement.
6. Digging deep trenches in the garden may not be practicable. The longer the loops for the heat transfer fluid, the better, as more heat is extracted.
7. Repairing leaks under the soil can be expensive, as the location may not be obvious.

Questions

You may need to do some extra research to answer some of these questions.

A1 Another type of heat pump is a water-sourced heat pump, using the heat from a body of water, such as a lake. What would be the advantages/disadvantages of this?

A2 A third version is an air-sourced heat pump. However, this is viewed as being not so efficient. Suggest a reason why not.

A3 The direction of operation of a heat pump can be reversed in summer. Why is that a strong selling point?

A4 Discuss the environmental benefits of heat pumps for commercial as well as domestic heating. Remember to consider the source of energy being used for the heat pump.

A5 GSHPs have COPs of about 4. Compare this with the theoretical value in the Worked example in Engineering Physics section 3.6, and with those calculated in questions **16** to **18**. Suggest reasons for the difference.

A6 Looking generally at the advantages and disadvantages, do you think this is an option you personally would consider? Explain your reasons. You might want to research 'renewable heat incentives'.

PRACTICE QUESTIONS

1. A petrol engine has the idealised p–V diagram (indicator diagram) in Figure Q1, in which a fixed mass of gas (air) is taken through a cycle involving four processes.
   a. A $\rightarrow$ B is an adiabatic compression from an initial temperature of 300 K. Explain what is meant by the *adiabatic compression* of a gas.
   b. B $\rightarrow$ C is the addition of 850 J of energy at constant volume. Apply the first law of thermodynamics to determine the change in internal energy of the air during this process.
   c. Describe the remaining two processes, C $\rightarrow$ D and D $\rightarrow$ A, within the cycle.
   d. Determine the number of moles of air that are taken through the cycle.
   e. Determine the work output of the cycle.
   f. Real engine cycles differ from the idealised one shown. Give two main differences between the theoretical and real cycles.

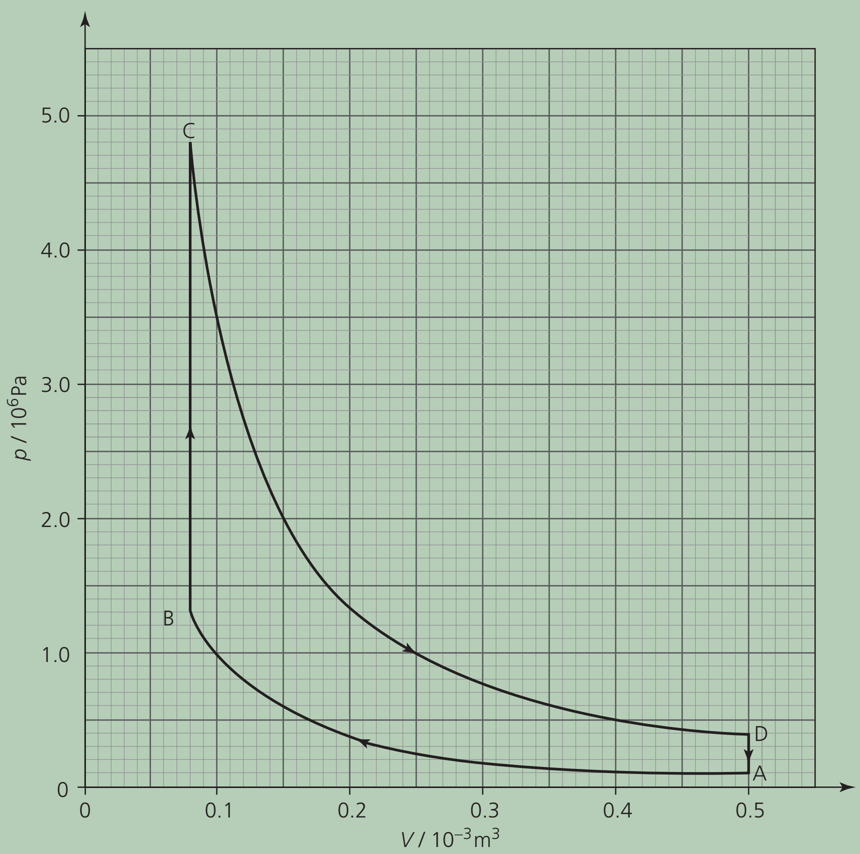


Figure Q1

2. a. Explain what is meant by the *coefficient of performance* of a heat pump.

b. The box labelled E in Figure Q2 shows a diagram of a combined heat and power scheme. The scheme provides electrical energy W from an engine-driven generator and heat Q_1 for buildings situated near to the generator. Some of the electrical energy is used to drive the heat pump shown in the box labelled P. Output Q_2 is also used to heat the buildings.

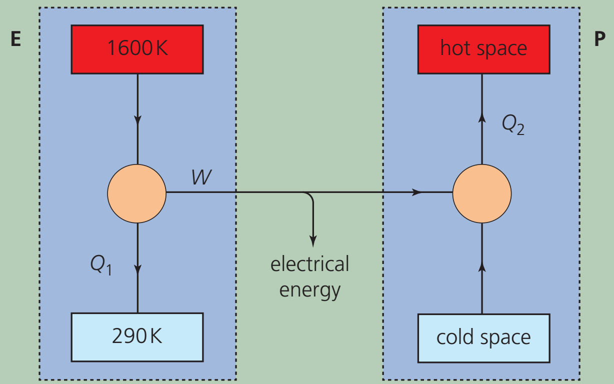


Figure Q2

You may assume that the engine runs at its maximum theoretical efficiency and that the electrical generator is 100% efficient. The output power of the engine-driven generator is 80 kW.

i. The fuel used in the engine (E) is propane of calorific value 49 MJ kg^{-1}. Calculate the rate of flow of propane into the engine. State an appropriate unit.

ii. The heat pump (labelled P) has a coefficient of performance (COP) of 2.6. The power supplied by the electrical generator to the heat pump (P) is 16 kW. Calculate the total rate at which energy is available for heating from both the engine and heat pump.

iii. The conversion of electrical energy to heat is nearly 100% efficient. Explain why the designer has proposed installing a heat pump rather than an electrical heater to provide the additional heat Q_2.

AQA Unit 5C June 2014 Q4

3. A four-stroke diesel engine with four cylinders is running at constant speed on a test bed. An indicator diagram for one cylinder is shown in Figure Q3 and other test data are given below:

Measured output power of engine (brake power) = 55.0 kW

Fuel used in 100 seconds = 0.376 litre

Calorific value of fuel = 38.6 MJ litre^{-1}

Engine speed = 4100 rev min^{-1}

a. Determine the indicated power of the engine, assuming all cylinders give the same power.

b. Calculate the overall efficiency of the engine.

AQA Unit 5C June 2012 Q4 part a

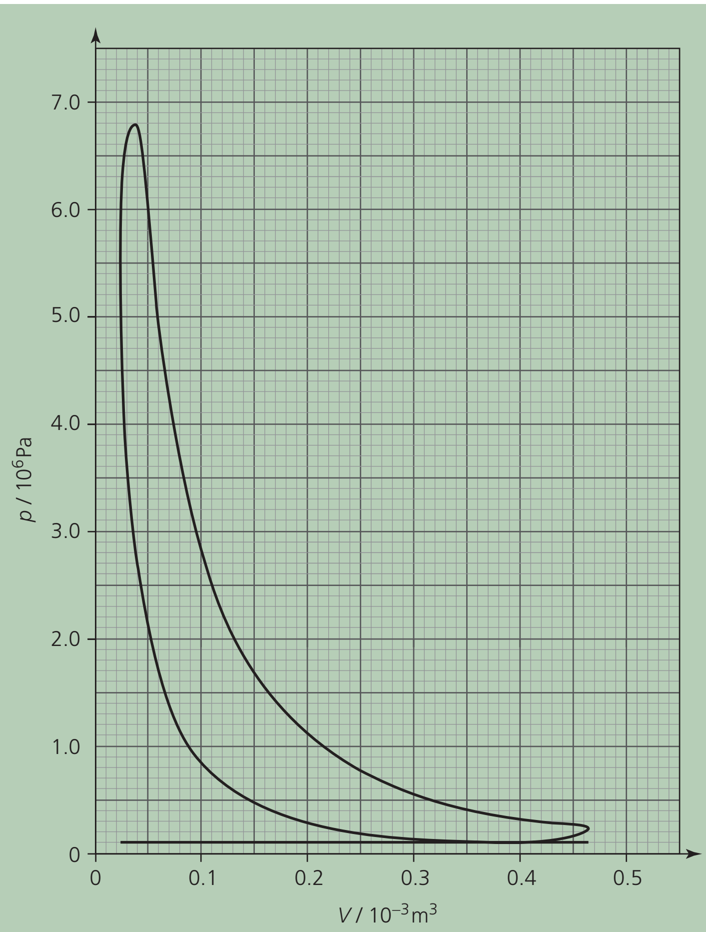


Figure Q3

4. **a.** Explain why the compression stroke of a diesel engine is considered to be an adiabatic change.

   b. Figure Q4a shows the cylinder of a diesel engine. The pressure of the air at the start of the compression stroke is 1.0×10^5 Pa and the volume above the piston is $4.5 \times 10^{-4}\,m^3$.

Figure Q4b shows the same cylinder at the instant just before the fuel is injected. The pressure above the piston is now 6.2×10^6 Pa. The compression is adiabatic with no leakage of air past the piston or valves. [adiabatic index γ for air = 1.4]

i. Calculate the volume above the piston at the instant just before the fuel is injected.

ii. The temperature of the air in the cylinder at the start of the compression stroke is 297 K. Calculate the temperature of the air at the instant just before the fuel is injected.

iii. Explain why, in a diesel engine, the fuel starts to be injected into the cylinder slightly before the piston reaches its highest point in the cylinder.

AQA Unit 5C June 2013 Q3 parts a and b

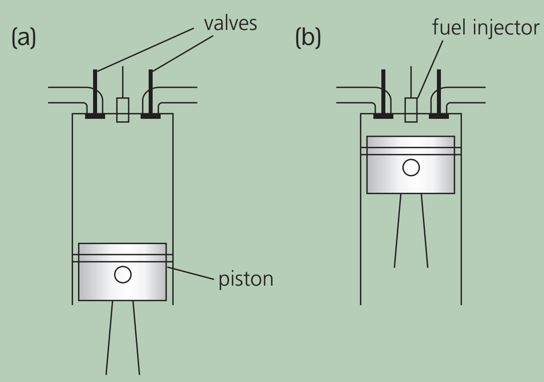


Figure Q4

ANSWERS TO IN-TEXT QUESTIONS

1 ROTATIONAL DYNAMICS

1. $30 \times 60 = 1800$ rpm

1800 rpm $= (1800 \times 2\pi)/60 = 188.5$ rad s^{-1}

Each second it turns through 188.5 rad.
In 1 minute it will have turned through $188.5 \times 60 = 11\ 310$ rad.

2. **a.** $33\frac{1}{3}$ rpm $= 33.3 \times 2\pi/60 = 3.49$ rad s^{-1}

b. Using $v = r\omega$, $v = 0.150 \times 3.49 = 0.524$ m s^{-1}

c. $v = 0.060 \times 3.49 = 0.210$ m s^{-1}

3. **a.** $\alpha =$ (change of angular velocity)/time $= 3.49/2.0 = 1.75$ rad s^{-2}

b. $a = r \times \alpha = 0.15 \times 1.75 = 0.26$ m s^{-2}

c. $\alpha = -3.49/5.0 = -0.70$ rad s^{-2}

d. $a = 0.060 \times (-0.70) = -0.042$ m s^{-2}

4. Hoop: $I = mr^2 = 0.1 \times (0.1)^2 = 0.0010$ kg m^2 (to 2 s.f.)

Disc: $I = \frac{1}{2}mr^2 = \frac{1}{2} \times 0.1 \times (0.1)^2 = 0.000\ 50$ kg m^2 (to 2 s.f.)

5. **a.** $I = \frac{1}{2}mr^2 = 0.5 \times 0.5 \times (0.075)^2 = 0.0014$ kg m^2 (to 2 s.f.)

b. $I = \frac{2}{5}mr^2 = \frac{2}{5} \times 5 \times (0.1)^2 = 0.020$ kg m^2 (to 2 s.f.)

6. $I = mr^2$ for each object. Total $I = 4 \times$ (moment of inertia of each separate object)

Total moment of inertia $I = 4 \times 0.5 \times (0.3)^2 = 0.18$ kg m^2 (to 2 s.f.)

7. In this case, we ignore objects D and B as their distance from the axis is zero.

Only A and C are included. The total $I = 2 \times 0.5 \times (0.3)^2 = 0.090$ kg m^2 (to 2 s.f.)

8. $I = \frac{1}{2}mr^2 = \frac{1}{2} \times 5.0 \times 0.50^2 = 0.625$ kg m^2 $= 0.63$ kg m^2 (to 2.s.f.)

Torque $= 30 \times 0.50 = 15$ N m

$\alpha = T/I = 15/0.625 = 24$ rad s^{-2}

9. Torque $= 30 \times 1.1 = 33$ N m

$I = T/\alpha = 33/0.30 = 110$ kg m^2

10. **a.** Torque $= 100 \times \frac{1}{2} \times 0.1 = 5.0$ N m

As $T = I \times \alpha$, $\alpha = T/I = 5.0/8.0 = 0.625 = 0.63$ rad s^{-2} (to 2 s.f.)

b. Using $\theta = \omega_1 t + \frac{1}{2}\alpha t^2$

Since $\omega_1 = 0$, $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 0.625 \times 30^2 = 281$ rad

This is about 45 rev

11. **a.** 30 rpm $= 30 \times 2\pi/60 = 3.14$ rad s^{-1}

b. $T = F \times r = 200 \times 2 = 400$ Nm

c. $\alpha = T/I = -400/500 = -0.8$ rad s^{-2}

d. $\omega_2 = \omega_1 + \alpha t$, so $t = 3.14/0.8 = 3.9$ s

$\omega_2^2 = \omega_1^2 + 2\alpha\theta$, so $\theta = (3.14)^2/(2 \times 0.8) = 6.16$ rad $= 0.98$ rev (about 1)

12. **a.** If we ignore the mass of the athlete's arm and the wire, then the moment of inertia is given by $I = mr^2 = 7.00 \times (1.3 + 0.7)^2 = 28$ kg m^2

b. We can use $\omega_2 = \omega_1 + \alpha t$ where $\omega_1 = 0$. As $v = 30$ m s^{-1}, $\omega_2 = v/r = 30/2.0 = 15$ rad s^{-1}.
So $\alpha = \omega_2/t = 15/5 = 3$ rad s^{-2}

c. $T = I\alpha = 28.0 \times 3 = 84$ N m or about 80 N m

d. By using $I = \frac{1}{2}mr^2$, we get $I = 0.5 \times 9 \times 0.35^2 = 0.55$ kg m^2

A ratio of $I_{arm}/I_{hammer} = 0.55/28 \approx 2\%$. The athlete's arm can be effectively ignored.

13. a. A: The object is initially stationary, but then its angular velocity increases to 5.0 rad s^{-1} over a time of 20 s.

B: From that value, it increases its angular velocity at a faster rate, from 5.0 to 25 rad s^{-1} in a further 20 s.

C: It maintains a constant angular velocity of 25 rad s^{-1} for 20 s.

D: It now slows down at a constant rate, eventually stopping after a further 40 s.

b. A: 0.25 rad s^{-2}

B: 1.0 rad s^{-2}

C: 0 rad s^{-2}

D: -0.63 rad s^{-2} (deceleration)

c. The total area is 1350 rad or about 215 revolutions

14. Draw tangents to the curve at points A, B and C, and find gradients using the largest triangles possible, as shown in Figure 1.

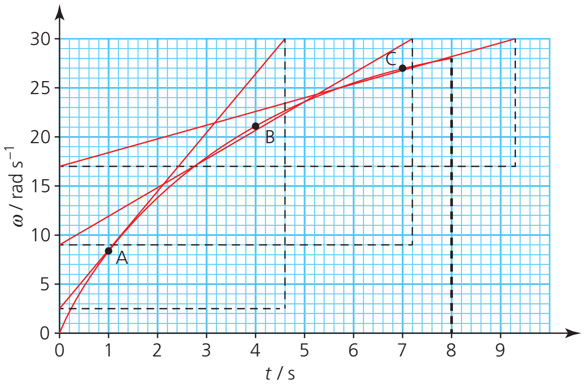


Figure 1

a. Angular acceleration at A = gradient of tangent at A $= (30 - 3)/4.6 = 5.9$ rad s^{-2}

Angular acceleration at B = gradient of tangent at B $= (30 - 9)/7.2 = 2.9$ rad s^{-2}

Angular acceleration at C = gradient of tangent at C $= (30 - 17)/9.3 = 1.4$ rad s^{-2}

b. There are about 30 cm^2 under the curve. An area of 1 cm^2 represents 5 rad. The total angular displacement is therefore 150 rad, or about 24 rev.

15. a. $\omega = (14\,000 \times 2\pi)/60 = 1.47 \times 10^3$ rad s^{-1}

b. $I = \frac{1}{2}mr^2 = \frac{1}{2} \times 272 \times 0.38^2 = 19.6$ kg m^2

$E = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 19.6 \times (1.47 \times 10^3)^2 = 2.1 \times 10^7$ J

The energy would have been transferred to kinetic energy of the disintegrating parts.

16. a. $I = \frac{2}{5}MR^2 = \frac{2}{5} \times 6.0 \times 10^{24} \times (6.4 \times 10^6)^2 = 9.8 \times 10^{37}$ kg m^2

b. Period $T = 24 \times 60 \times 60 = 86\,400$ s

$\omega = 2\pi/T = 2\pi/86\,400 = 7.3 \times 10^{-5}$ rad s^{-1}

$E_k = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 9.8 \times 10^{37} \times (7.3 \times 10^{-5})^2 = 2.6 \times 10^{29}$ J

17. 30 km per hour $= 30\,000/3600 = 8.3$ m s^{-1}. The angular velocity $= v/r = 8.3/0.30 = 28$ rad s^{-1}.

The moment of inertia $= mr^2 = 1.80 \times 0.30^2 = 0.16$ kg m^2

Linear $E_k = \frac{1}{2}mv^2 = \frac{1}{2} \times 1.8 \times 8.3^2 = 62$ J

Rotational $E_k = \frac{1}{2}I\omega^2 = \frac{1}{2} \times 0.16 \times 28^2 = 63$ J

Total energy of each wheel = 125 J

18. The tin containing the liquid fruit juice will reach the bottom first. *Reason*: The liquid will effectively slide down inside the tin, whereas the frozen juice is forced to rotate. The overall moment of inertia of the tin plus the frozen juice will be greater, so more of its gravitational energy will transferred as rotational kinetic energy – and less as translational – than in the case of the tin with the liquid juice. It will hence take longer.

19. a. To minimise frictional losses due to air resistance.

b. The car will be lighter, so better fuel economy. There are no environmental problems with disposal – batteries have environment-harming chemicals inside them.

c. Energy = power × time = 60 000 × 10 = 600 000 J (600 kJ)

d. This would be sufficient in city/town driving where there is normally a great number of stops and starts over short distances.

20. a. It would be attached to the drive motor.

b. The flywheel will have stored energy and this will take time to dissipate. The drive will therefore not stop dead in its tracks in the event of a power failure. As a consequence, by slowing down the belt gradually, the difference in tension along the belt will be reduced. This is much safer, with less chance of serious damage being done.

21. **a.** Since there is a massive reduction in diameter of the star, the moment of inertia will decrease. But, since angular momentum is conserved, there will be a corresponding huge increase in the angular velocity: it will rotate faster.

b. **i.** $\omega = 2\pi/T = 2\pi/(30 \times 24 \times 3600)$
$= 2.4 \times 10^{-6} = 2 \times 10^{-6}$ rad s^{-1} (to 1 s.f.)

ii. The starting point is $I_1\omega_1 = I_2\omega_2$. The moment of inertia of the Sun is $\frac{2}{5}mr^2$. So

$$\tfrac{2}{5}mr_1^2 \times\omega_1 = \tfrac{2}{5}mr_2^2 \times\omega_2$$

assuming the mass is the same, which simplifies down to

$$\omega_2 = \left(\frac{r_1}{r_2}\right)^2 \times \omega_1$$
$$= \left(\frac{7\times10^8}{2\times10^4}\right)^2 \times 2.4\times10^{-6}$$
$$= 2940 = 3000 \text{ rad s}^{-1} \text{ (to 1 s.f.)}$$

iii. The number of revolutions per second $= 2940/2\pi = 468 = 500$ rev s^{-1} (to 1 s.f.)

22. **a.** For each child $I = mr^2 = 40 \times 2^2$
$= 160$ kg m^2. Total moment of inertia of children $= 320$ kg m^2. Total moment of inertia of children and roundabout $= 500 + 320 = 820$ kg m^2. The assumption is that we treat each child as equivalent to a point mass.

b. $\omega = 20 \times 2\pi/60 = 2.1$ rad s^{-1}

$L = I \times \omega = 820 \times 2.1 = 1720$
$= 1700$ kg^2 m s^{-1} (to 2 s.f.)

c. Using $I_1\omega_1 = I_2\omega_2$, we find $\omega_2 = 1720/500$
$= 3.4$ rad s^{-1}

23. Students' own answers.

24. 0.8 rev s^{-1} equals $0.8 \times 2\pi = 5.0$ rad s^{-1}. The change in the angular momentum equals $40 \times 5.0 = 200$ kg^2 m s^{-1}. This is the angular impulse needed.

25. **a.** The angular impulse is equal to the change in angular momentum $\Delta(I\omega)$.

Converting 9 rev s^{-1}, we get $9.0 \times 2\pi$ $= 57$ rad s^{-1}. The moment of inertia ($I = mr^2$) of the wheel is $1.8 \times (0.30)^2 = 0.16$ kg m^2.

So, the change of angular momentum $= 0.16 \times 57 = 9.1$ kg m^2 s^{-1}. This was the angular impulse required.

b. Applied torque × time = change of angular momentum, so $T \times 3.0 = 9.1$. The torque applied was therefore 3.0 N m.

As torque equals force × perpendicular distance, it follows that the force used was $3.0/0.30 =$ 10 N.

26. Power $= T \times \omega = 25 \times 15 = 375$ W

27. The braking torque $= F \times d = 200 \times 0.10$ $= 20$ N m

Power to overcome the braking torque $= T \times \omega$ $= 20 \times 45 = 900$ W

28. **a.** Torque $= F \times d = 20 \times 0.100 = 2.0$ N m

b. Moment of inertia of whole system $= I_{wheel} + I_{clay}$.

Angular velocity $= 3.50 \times 2\pi = 22.0$ rad s^{-1}

Using $I = \frac{1}{2}mr^2$ for each part:

$I_{wheel} = \frac{1}{2} \times 40.0 \times (0.350)^2 = 2.45$ kg m^2

$I_{clay} = \frac{1}{2} \times 2.40 \times (0.10)^2 = 0.012$ kg m^2

Total moment of inertia $= 2.46$ kg m^2

As $T\Delta t = \Delta(I\omega) = I \times \Delta\omega$ in this case, then

$\Delta\omega = T \times \Delta t/I = 2.0 \times 10/2.46 = 8.1$ rad s^{-1}

In this case, $\Delta\omega$ is negative, that is, a reduction in the angular velocity, so the new value is $22.0 - 8.1 = 13.9$ rad s^{-1}.

c. This time, we use work $W = T \times \theta$. The torque is 2.0 N m, so we have to calculate the angular displacement.

Since $\theta = \frac{1}{2}(\omega_1 + \omega_2) \times t$, then
$\theta = \frac{1}{2} \times (22.0 + 13.9) \times 10 = 180$ rad

$W = 2 \times 180 = 360$ J

2 THERMODYNAMICS

1. Using $\Delta U = Q - W$, where $Q = +2$ MJ and $W = -0.45$ MJ, we find

$\Delta U = +2 - (-0.45) = 2.45$ MJ

2. **a.** Using $\Delta U = Q - W$, we have $Q = +6000$ J and $W = 11\,000$ J, so

$\Delta U = +6000 - (+11\,000) = -5000$ J

b. The temperature has decreased.

c. Only for an ideal gas is the internal energy dependent only on the temperature.

3. Use $\Delta U = Q - W$ with $\Delta U = +3000$ J and $Q = +5000$ J, so

$+3000 = +5000 - W$

giving $W = +2000$ J. The system has done work.

4. Using $\Delta U = Q - W$ with $\Delta U = -4$ MJ and $Q = +10$ MJ, we have

$-4 = +10 - W$

so $W = +14$ MJ. The gas has done 14 MJ of work against the environment.

5. This statement should be agreed with. Going by the definition of 'reversible', a requirement would be that a process would have to be performed very slowly, so that it could (in theory) change direction. Going further, complex systems are composed of molecules, and changes would result in some new configuration. Work would need to be done, and there would be some energy dissipation as a consequence. Reversibility would therefore not be possible.

6. Using $p_1V_1 = p_2V_2$, $p_2 = p_1V_1/V_2 = 15 \times 5.0/2.0 = 37.5 = 38$ N m^{-2} (to 2 s.f.)

7. Using $p_1V_1 = p_2V_2$, $V_2 = p_1V_1/p_2 = 78 \times 50/43 = 90.7 = 91$ cm^3 (to 2 s.f.)

8. **a.** pV is a constant for an isothermal.

pV for point B =
$5.0 \times 10^5 \times 0.2 \times 10^{-3} = 100$

pV for point A =
$1.0 \times 10^5 \times 1.0 \times 10^{-3} = 100$

pV for a further point:
$2.5 \times 10^5 \times 0.4 \times 10^{-3} = 100$

b. Using point B, $pV = nRT$, so $T = pV/nR = (5.0 \times 10^5 \times 0.2 \times 10^{-3})/(0.12 \times 8.31) = 100$ K.

c. See Figure 3.

9. **a.** $p\Delta V = 6000 \times (-0.45) \times 0.5 = -1350$ J

b. $Q = \Delta U + p\Delta V$

$Q = +1000 + (-1350) = -350$ J (the negative sign shows that heat is lost)

10.

Q / J	p / N m^{-2}	ΔV / m^3	W / J	ΔU / J
100 in	50 000	−0.02	−1000	1100
100 in	40 000	0	0	100
100 out	25 000	−0.05	−1250	1150
500 out	30 000	−0.03	−900	400

11. See Figure 3.

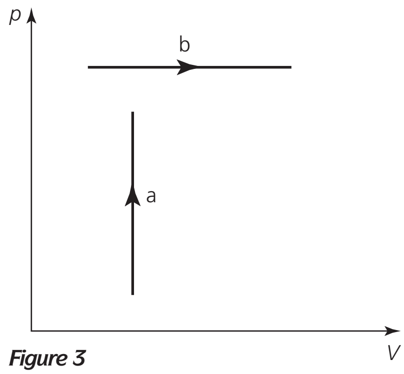


Figure 3

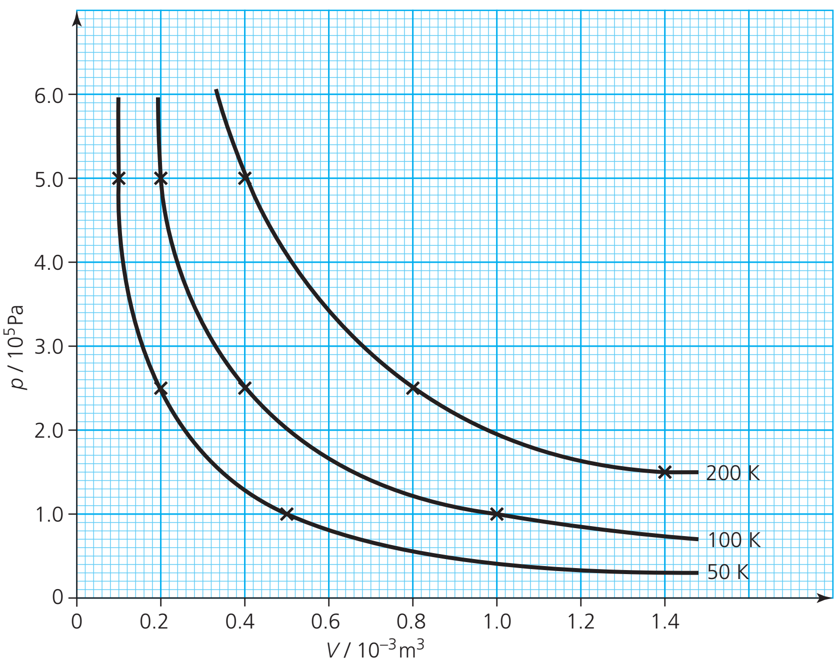


Figure 2

12. It is an adiabatic compression, so $p_1V_1^\gamma = p_2V_2^\gamma$ and substituting values gives

$$60 \times (250)^{1.67} = 150 \times (V_2)^{1.67}$$
$$60 \times 10100 = 150 \times (V_2)^{1.67}$$
$$4040 = (V_2)^{1.67}$$
$$V_2 = 144 \text{ cm}^3$$

13. Use $T_1V_1^{(\gamma-1)} = T_2V_2^{(\gamma-1)}$ for an adiabatic change. First, convert °C to K: 40 °C = 273 + 40 = 313 K; and 10 °C = 273 + 10 = 283 K. Then substitute in the equation to give

$$313 \times 450^{0.33} = 283 \times V_2^{0.33}$$
$$313 \times 7.5086 = 283 \times V_2^{0.33}$$
$$8.3046 = V_2^{0.33}$$

The final volume is 611 to 3 sf, when no rounding is used in the calculation.

14. $Q = \Delta U + W$

No energy is supplied to the air as heat, so $Q = 0$, as the pumping motion is adiabatic. Work is done *on* the air in the tube, so W is negative. The gain in internal energy is therefore equal to the amount of work done on the gas. The increased temperature of the gas is a result of the gain in internal energy of the air in the tube. This will eventually conduct through the wall of the hand pump, making it feel warm to the touch.

15. **a.** $p_1V_1 = p_2V_2$

$40 \times 10 = p_2 \times 2$

The new pressure is 200 N m^{-2}

b. $p_1V_1^\gamma = p_2V_2^\gamma$

$40 \times 10^{1.4} = p_2 \times 2^{1.4}$

The new pressure this time is 381 N m^{-2}

c. See Figure 4.

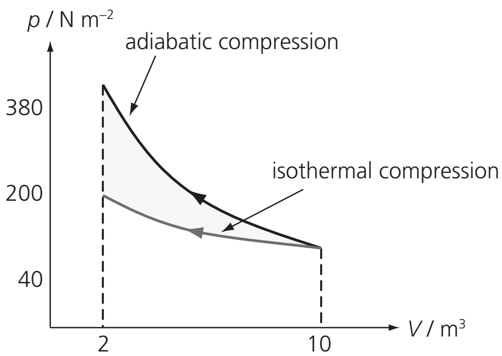


Figure 4

16. The total area under the graph:
$(4 \times 10^5 \times 1) + (2 \times 10^5 \times 1) = 6 \times 10^5$ J

17. There are 800 +/– 10 square millimetres

1 mm^2 = $(0.05 \times 10^5) \times (1 \times 10^{-3})$ = 5 J of work.

(800 +/– 10) × 5 = 4000 +/– 50 J

18. **a.** As $pV = nRT$, it follows that $T = pV/nR$. As both n and R are constants, T will depend on pV. The further from the origin, the greater the product pV will be, hence the higher the temperature. So T_2 must be higher than T_1.

b. DA: Isothermal expansion

AB: A drop of pressure at a constant volume. The temperature has dropped and heat energy has been lost from the system.

BC: Isothermal compression

CD: An increase in pressure at a constant volume. The temperature has increased as heat energy has been added to the system.

c. DA: Work is done *by* the gas

AB: No work as there is no change in volume

BC: Work is done *on* the gas

CD: No work as there is no change in volume

d. As the work done *by* the gas is *greater* than the work done *on* the gas (areas under the curves), the area enclosed will be the *net work being done*.

19. The number of (half-filled or more) square millimetres is 680 +/–10.

Each mm^2 scales to $(0.1 \times 10^5) \times (0.1 \times 10^{-2})$ = 10 J

(680 +/–10) × 10 = 6800 +/–100 J

3 HEAT ENGINES

1. See Figure 5.

2. Advantages of diesel: better fuel efficiency; greater torque; no spark plugs needed; lower greenhouse gas emissions

Disadvantages of diesel: heavier (for same power); parts need to be higher strength/more robust to withstand higher compression; more expensive to produce; harmful particulate emissions

3. 10 kg h^{-1} converts to $10/(60 \times 60)$ $= 0.0028$ kg s^{-1}.

Input power = calorific value × flow rate
$= 48 \times 10^6 \times 0.0028 = 130\,000$ W = 130 kW

4. Input power = calorific value × flow rate
$= 42.9 \times 10^6 \times 7.00 \times 10^{-2} = 3.00 \times 10^6$ W
= 3.00 MW

5. 7.80 kg min^{-1} is 0.130 kg s^{-1}.

Input power = calorific value × flow rate
$= 44.1 \times 10^6 \times 0.130 = 5.73 \times 10^6$ W = 5.73 MW

6. Converting 28.7 GJ tonne^{-1} to J kg^{-1} gives
$28.7 \times 10^9/1000 = 28.7 \times 10^6$ J kg^{-1}

From the answer to question 4, we need to produce 3.00×10^6 W

Input power = calorific value × flow rate

$3.00 \times 10^6 = 28.7 \times 10^6 \times$ flow rate

flow rate $= (3.00 \times 10^6)/(28.7 \times 10^6)$
$= 0.105$ kg s^{-1}

7. 4500 rpm is $4500/60 = 75$ rev s^{-1}. We need to divide by 2 for the number of cycles per second. The indicated power per cylinder is

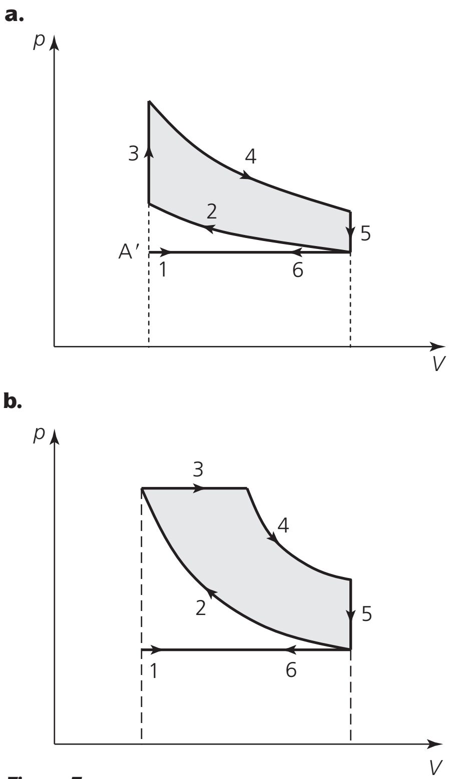
$$\begin{aligned} P_{\text{ind}} &= \text{area of } p\text{–}V \text{ loop (J)} \times \text{number of cycles per second (s}^{-1}) \\ &\quad \times \text{number of cylinders} \\ &= 960 \times (75/2) \times 1 \\ &= 36000 \text{ W} = 36 \text{ kW} \end{aligned}$$

8. 2500 rpm converts to 42 rev s^{-1}. The indicated power is

$P_{\text{ind}} = (250 \times 7) \times (42/2) \times 8 = 2.9 \times 10^5$ W

9. There are 502 small squares in the p–V loop.

The area of a small square (scaling factor) =
$(0.2 \times 10^6) \times (0.02 \times 10^{-3}) = 4$ J



a.
1 Intake of mixture of petrol vapour and air (induction stroke)
2 Piston moves up, compressing the mixture adiabatically (compression stroke)
3 Mixture ignited, increasing pressure and temperature at constant volume
4 Piston pushed down and gas expands adiabatically (power stroke)
5 Heat removed through exhaust valve, decreasing pressure and temperature of gas
6 Exhaust of burnt gas (exhaust stroke)

b.
1 Intake of air (induction stroke)
2 Piston moves up, compressing the air adiabatically (compression stroke)
3 Diesel sprayed in and ignited, piston moves down (power stroke, part 1)
4 Gas expands adiabatically, piston moves down (power stroke, part 2)
5 Heat removed through exhaust valve, decreasing pressure and temperature of gas
6 Exhaust of burnt gas (exhaust stroke)

Figure 5

The work done by the engine per cycle = 502 × 4 = 2008 J

3000 rpm converts to 50 rev s^{-1}

$$\begin{aligned} P_{ind} &= \text{area of } p\text{–}V \text{ loop (J)} \\ &\quad \times \text{number of cycles per second } (s^{-1}) \\ &\quad \times \text{number of cylinders} \\ &= 2008 \times (50 / 2) \times 4 \\ &= 200800 \text{ W} \approx 200 \text{ kW} \end{aligned}$$

10. 75 bhp is equivalent to 75 × 746 = 56 000 W. The frictional power is

$$\begin{aligned} P_{friction} &= \text{indicated power} - \text{output (brake) power} \\ &= P_{ind} - P_{out} \\ &= 100000 - 56000 \\ &= 44 \text{ kW} \end{aligned}$$

11. **a.** For the output power, we use $P = T \times \omega$, so we need the angular velocity of the engine. 4000 rpm converts to 420 rad s^{-1}.

Output power = 147 × 420 = 62 000 W = 62 kW

b. Indicated power = 110 kW

Frictional power = 110 – 62 = 48 kW

12. 3000 rpm converts to 3000 × 2π/60 = 310 rad s^{-1}.

As output power $P = T \times \omega$, P = 290 × 310 = 89 900 ≈ 90 000 W

Frictional power = 200 000 – 90 000 = 110 000W = 110 kW

13. Input power (P_{input}) is calculated by calorific value × flow rate = 44.1 × 10^6 × (0.672/60) = 494 000 W

Indicated power (P_{ind}) is given as 124 000 W

4000 rpm is 4000 × 2π/60 = 419 rad s^{-1}

Output power (P_{out}) is calculated from $T \times \omega$ = 228 × 419 = 95 500 W

Mechanical efficiency is

$$\eta = \frac{\text{output (brake) power}}{\text{indicated power}} = \frac{P_{out}}{P_{ind}} = \frac{95500}{124000}$$
$$= 0.77 \text{ or } 77\%$$

Thermal efficiency is

$$\varepsilon = \frac{\text{indicated power}}{\text{input power}} = \frac{P_{ind}}{P_{input}} = \frac{124000}{494000} = 0.25 \text{ or } 25\%$$

Overall efficiency = 0.77 × 0.25 = 0.19 or 19%

14. The temperatures must be converted to kelvin (K). We obtain

$$\varepsilon_{max} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H} = 1 - \frac{20 + 273}{400 + 273} = 1 - \frac{293}{673}$$
$$= 0.56 = 56\%$$

15. **a.** We first have to convert temperatures to kelvin. Then we get

$$\varepsilon_{max} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H} = 1 - \frac{17 + 273}{777 + 273} = 1 - \frac{290}{1050}$$
$$= 0.72 = 72\%$$

b. As $\varepsilon = \frac{W}{Q_H}$, $Q_H = \frac{W}{\varepsilon}$, so $Q_H = \frac{2500}{0.72} = 3500$ J

c. Since $Q_H = W + Q_C$, $Q_C = Q_H - W$ = 3500 – 2500 = 1000 J

d. $P = \frac{W}{t} = \frac{2500}{0.10} = 25000\text{W} = 25$ kW

16. **a.** Using $COP_{ref} = \frac{T_C}{T_H - T_C}$ for a refrigerator, where T_H = 25 + 273 = 298 K and T_C = 2 + 273 = 275 K, gives

$$COP_{ref} = \frac{275}{298 - 275} = \frac{275}{23} = 11.96 \approx 12$$

b. Using $COP_{ref} = \frac{T_C}{T_H - T_C}$, where T_H = 35 + 273 = 308 K and T_C = 2 + 273 = 275 K, gives

$$COP_{ref} = \frac{275}{308 - 275} = \frac{275}{33} = 8.3$$

c. More work is needed in the hotter climate, so the refrigerator will use more electrical energy and will cost more to run.

17. Using $COP_{ref} = \frac{T_C}{T_H - T_C}$ for a freezer, where T_H = 293 K and T_C = 268 K, gives

$$COP_{ref} = \frac{268}{293 - 268} = \frac{268}{25} = 10.7$$

18. **a.** Using $COP_{hp} = \frac{T_H}{T_H - T_C}$ for a heat pump, where T_H = 293 K and T_C = 276 K, gives

$$COP_{hp} = \frac{293}{293 - 276} = \frac{293}{17} = 17.2$$

b. Similarly for T_H = 293 K and T_C = 283 K,

$$COP_{hp} = \frac{293}{293 - 283} = 29.3$$

c. For T_H = 293 K and T_C = 268 K,

$$COP_{hp} = \frac{293}{293 - 268} = 11.7$$

d. The closer the temperatures of the hot and cold reservoirs, the greater the COP will be. Less work is needed to transfer heat energy up to the hot reservoir, so the COP is increasing.

GLOSSARY

Adiabatic process A thermodynamic process during which no heat enters or leaves a system.

Angular impulse A change in angular momentum.

Angular acceleration The rate of change of angular velocity, ω, with time $\alpha = \frac{\Delta\omega}{\Delta t}$

Angular momentum A vector quantity in rotational motion, given by $L = I\,\omega$, where L = angular momentum, I = moment of inertia and ω = angular velocity.

Angular displacement The angular displacement of a body is the angle in radians (sometimes expressed in revolutions or degrees) through which a point or line has been rotated about a specified axis. The symbol is θ.

Angular velocity The rate of change of *angular* displacement and is calculated from the relationship $\omega = \Delta\theta/\Delta t$, unit radians per second ($rads^{-1}$). Occasionally, the phrase angular speed is used, also in $rads^{-1}$.

Atomicity The number of atoms in one molecule of a substance.

Brake horsepower (bhp) The output power from an engine expressed in imperial units where 1 horsepower = 746 watts.

Brake power The output power from an engine.

Calorific value The measure of how much energy a fuel theoretically contains and releases when completely combusted. It is also known as the energy density. The unit is Jkg^{-1} or $MJkg^{-1}$.

Carnot cycle A theoretical thermodynamic cycle of two adiabatic and two isothermal changes, which is the most efficient possible.

Centripetal acceleration The acceleration, a, of an object moving at a steady speed in a circle directed towards the centre of the circle.

Centripetal force The force, f, directed towards the centre of the circle, required to keep an object moving at a steady speed in a circle.

Coefficient of performance (COP) In the context of refrigerators and heat pumps, the ratio of the heat extracted or supplied to the work done by the external agency.

Compression ratio The ratio of the volume enclosed in the cylinder of an engine at the beginning of the compression stroke to the volume enclosed at the end of the stroke.

Cycle A series of repeating pressure and volume changes describing a thermodynamic process.

Cyclic process In the context of thermodynamics, a process which continually repeats itself such as an engine running.

Diesel engine An internal combustion engine with a fairly high compression ratio, in which no fuel is introduced into the cylinder during compression.

Dielectric constant The dielectric constant or relative permittivity of a dielectric, ε_r is the factor by which the electric field between two charges is decreased by the presence of the dielectric, relative to a vacuum.

Disordered In the context of thermodynamics, the randomised kinetic energy of hot gas molecules.

Entropy A measure of the degree of disorder of a system.

Energy storage capacity The amount of total energy that can be stored mechanically, through rotation of a flywheel, for example, to be released subsequently. It will depend on the angular velocity and the moment of Inertia and calculated from the kinetic energy expression $\frac{1}{2}I\omega^2$.

First law of thermodynamics A law stating the conservation of energy. In the context of heat engines, the energy transferred to the system by heating, Q, is equal to the increase in internal energy ΔU + the work done by the system, W: $Q = \Delta U + W$.

Flybrid A mechanical system that converts a vehicle's kinetic energy into rotational energy of a flywheel during braking and reconverts the energy back to the vehicle a kinetic energy when required.

Flywheel A rotating disc or wheel which stores kinetic energy as it spins.

Four-stroke petrol engine An internal combustion engine with a high compression ratio, in which fuel is introduced into the cylinder during compression.

Frictional power The difference between the indicated power and output (brake) power of an engine.

Frictional torque A frictional force which opposes or causes rotational motion.

Gyroscope A device consisting of a wheel or disc that spins rapidly about an axis that is also free to change direction.

Heat engine A device or system that extracts energy from its environment in the form of heat and converts it into useful work.

Heat pump A device that transfers heat from a colder area to a hotter area by using mechanical energy, such as a refrigerator.

Hybrid In the context of cars, a combination of a conventional combustion engine and an electric motor used to power it.

Ideal gas A gas that obeys Boyle's law under all conditions: a gas whose molecules are infinitely small and exert no force on each other, except during collisions.

Ideal gas equation An equation describing the relationship between the pressure, volume and absolute temperature of an ideal gas: $pV = nRT$.

Indicated power The theoretical power that an engine can deliver.

Indicator diagram The pressure – volume diagram that describes the cycle of a heat engine such as a petrol or diesel engine.

Inertia The resistance of any object to a change in its state of motion.

Input power The power that is derived from the burning of the fuel in a heat engine.

Internal combustion engine A heat engine in which the fuel combines with oxygen inside the engine's cylinder.

Internal energy The sum of the total kinetic energy of its constituent particles of a system.

Isothermal process A thermodynamic process that takes place under constant temperature conditions.

Maximum theoretical efficiency For a heat engine, the difference in temperature between the hot and cold reservoirs divided by the temperature of the hot reservoir. The maximum theoretical efficiency is always less than 1

$$\varepsilon_{max} = \frac{T_H - T_C}{T_H} = 1 - \frac{T_C}{T_H}.$$

Mechanical efficiency The ratio of output (brake) power to indicated power for an engine.

Moment of inertia (I) A measure of how the mass of a rotating body is distributed about its axis of rotation, given by $I = \sum mr^2$; unit kg m^2.

Non-flow processes The four thermodynamic processes involving gases: isothermal, adiabatic, constant pressure and constant volume.

Otto cycle The indicator cycle (p–V diagram) describing the petrol engine.

Output power The driving power (also called brake power) that is delivered to the engine's crankshaft.

Overall efficiency For an engine, given by brake power/input power:

$$\frac{\text{output (brake) power}}{\text{input power}} = \frac{P_{out}}{P_{input}}$$

Perpetual motion machine of the first kind An imaginary machine which produces work without any energy input. This violates the first law of thermodynamics.

Perpetual motion machine of the second kind An imaginary machine which turns heat energy totally into mechanical work. As no machine can convert heat totally into useful work, this violates the second law of thermodynamics.

Perpetual motion machine of the third kind An imaginary machine with no frictional effects, so it can maintain its motion forever.

Power The rate at which energy is transferred or at which work is done, measured in joules per second, or watts, W.

Precession The slow movement of the axis of a spinning body around another axis due to a torque.

Principle of conversation of angular momentum The total angular momentum of a system remains constant provided there is no external torque.

Principal specific heat capacities The two principal specific heat capacities of a gas are: c_v, the energy required to produce unit temperature rise in unit mass of the gas at constant volume and c_p the energy required to produce unit temperature rise in unit mass of the gas at constant pressure.

Radians The angle subtended when the arc length is equal to the radius; 1 rad = 57.3°.

Refrigerator A heat pump that takes in heat at low temperatures and rejects heat at higher temperatures.

Regenerative braking A system to 'collect' and store energy from a car's braking motion for reuse.

Reversible In the context of thermodynamics, an infinitely slow change in the pressure, volume or temperature of a gas.

Revolution A single complete cycle or turn about an axis (through 2π radians or 360 degrees) of a rotating object.

Rotational kinetic energy The kinetic energy of an object by virtue of its rotation, given by $E_k = \frac{1}{2}I\omega^2$

RPM The rate at which an object turns is often given in terms of the number of full circles that it completes in a given time. This is the rotational frequency, f, and it is typically quoted in revolutions per minute (rpm).

Second law of thermodynamics A law stating that heat naturally flows from a hot body to a cold body until both are at the same temperature.

Sink A heat reservoir.

Source An energy or heat supply.

Steam engine A heat engine in which the fuel combines with oxygen to generate steam which is then introduced into the engine's cylinder.

Surroundings The region outside a thermodynamic system.

System A thermodynamic system refers to a fixed mass of some substance, enclosed by a boundary.

Tangential acceleration An increase in tangential velocity caused by an increase in angular velocity.

Tangential velocity The instantaneous linear velocity, v, of a rotating object: $v = r\omega$

Thermal efficiency ε, the ratio of the indicated power to the input power for an engine; also given by:

$$\varepsilon = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}.$$

Thermodynamic temperature scale The Kelvin temperature scale.

Thermodynamics The study of the movement of heat through systems.

Torque A turning effect: the moment of a couple about a point; unit N m.

Work done In the context of thermodynamics, the energy given to a system by a thermodynamic change, given the symbol, W: $Q = \Delta U + W$

Working substance The substance in a thermodynamic system that undergoes a change to its pressure, volume or temperature.

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